CHAPTER 2: INTEREST RATES AND TIME VALUE OF MONEY

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Sources:
CFA Program Curriculum – Volume 1
Brealey, R., et. al. (2009), *Fundamentals of corporate finance*, 6th Ed, McGraw-Hill Irwin
Mishkin, F.S. (2010), *The Economics of Money, Banking and Financial markets*, 9th Ed, The Addison – Wesley Series in Economics

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2. Types of interest rates
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4. The behaviour of interest rates
5. The risk and term structure of interest rates

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1. Definition of Interest Rates

- ***An interest rate*** (denoted ***r*** or ***i***): is the rate (percentage) of required compensation (the price of the right to use the fund) over the loan amount .
- Formula: $$i = \frac{interest\ payment}{principal}$$
- **Example 1.1:**
  - If $9,500 today and $10,000 a year later are equivalent in value, then $10,000 - $9,500 = $500 is the ***required compensation*** for receiving $10,000 in one year later than now.
  - ***The interest rate*** – the required compensation stated as a rate of return – is $500/$9500 = 0.0526 or 5.26%
- Note: When mentioned, interest rates usually quoted as *annually* interest rates.

1 Definition of Interest Rates

- Interest rates can be thought in three ways:
  - Required rates of return: the minimum rate of return an investor must receive in order to accept the investment
  - Discount rates: the rate used to discount the future amount to find its value today.
  - Opportunity costs: the value the investor forgo by choosing a particular course of action

1. Definition of Interest Rates

- **Example 1.1 (cont.):**
  $500/$9500 = 5.26%:
  - Is the required compensation rate (rate of return) that the investor agree for his investment
  - Is the rate at which we discount the $10,000 future amount to find its present value.
  - If the investor do not lend $9,500, but instead using it today, he would have forgone earning 5.26% on the money.

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2. Types of Interest Rates

- Fixed vs. Floating interest rates
- Nominal vs. Real interest rates
- Simple vs. Compound interest rates
- Types of interest rates by the commercial bank's operations
- Types of interest rates by the banking regulations

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FIXED VS. FLOATING INTEREST RATES

- **Fixed rate:** is the interest rate that remains unchanged at a predetermined rate for the entire term of the loan.
- **Floating rate:** is the interest rate that changes over time, the change is usually tied to the movement of an outside indicator.

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NOMINAL VS. REAL INTEREST RATES

- With the influence of inflation, Interest rates have two kinds:
  - Nominal interest rate (i_N)
  - Real interest rate (i_R)
- Formula: $(1+i_R)=(1+i_N)/(1+\Pi)$

$$i_R=(i_N-\Pi)/(1+\Pi) \qquad (2.1)$$

In which: Π: is the inflation rate

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SIMPLE VS. COMPOUND INTEREST RATES

- Interest rates could be quoted in two ways:
  - Simple interest
  - Compound interest

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SIMPLE INTEREST

- **Simple interest** is the interest earned only on the original investment; no interest is earned on interest
- Calculation of the future cash sum (FV) of a present amount of money today :

$$FV_n = PV \times (1 + n \times i) \qquad (2.2)$$

In which:

- FV_n: is the future amount at time n
- PV: is the present amount of money
- i: is the interest rate

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SIMPLE INTEREST

- **Example 2.1:** You borrow $1000 and agree to repay at 12% interest, paying annually, using simple interest in 2 year's time. Calculate the interest and principal component?
- Solution:
  - Interest owed in year 1 = 0.12 * 1000 = $120
  - Interest owed in year 2 = 0.12 * 1000 = $120
  - Total repayment = $1,000 (1 + 2 * 0.12) = $ 1,240

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SIMPLE INTEREST

- NOTE: Simple interest is usually used in calculating the interest rate with terms of payment lower than a year from annual interest rate.

E.g.: 12% p.a. (SIMPLE), is equivalent to

1% per month, or

3% per quarter, or

6% semi-annually.

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Compound Interest

- **Compound interest:** interest earned on interest.
- The mechanics for compound interest is that interest income in the previous period will be reinvested in later periods to generate more interest.

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Compound Interest

- **Example 2.1 (cont.):** If the interest used is compound interest, calculate the interest and principal component?
- Solution:

Original investment	\$100.00
Interest for the 1st year (\$100 x 0.12)	12.00
Interest for the 2nd year based on original investment (\$100 x 0.12)	12.00
Interest for the 2nd year based on interest earned in the 1st year (0.12 x \$12 interest on interest)	1.44
Total ([\$100 x (1+ 0.05)]x (1+ 0.05))	**\$125.44**

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COMPOUND INTEREST

- We see that with compound interest:

$$\begin{pmatrix}\text{BALANCE AT}\\ \text{END OF YEAR}\end{pmatrix} = \begin{pmatrix}\text{BALANCE AT}\\ \text{START OF YEAR}\end{pmatrix}\times(1+i)$$

- Therefore the future amount of an amount of money today after n year is :

$$FV_n = PV \times (1+i)^n \quad (2.3)$$

In which:

- FV_n: is the future amount after n year
- PV: is the present amount of money
- i: is the interest rate

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COMPOUND INTEREST

- **The Frequency of Compounding**
  - With investments paying interest more than once a year, but as stated before, people usually quote a annual interest rate. How could we compound interest in this case?
  - E.g.: Your bank state that a saving account pays a quoted interest rate 8% compounded monthly.
    - The quoted interest here is 8% per annum, therefore the monthly interest rate is: 8%/12 = 0.67%
    - If the interest is paid monthly therefore the future value factor here is: $(1 + 0.67\%)^{12} = 1.083$ not $(1 + 8\%) = 1.08$
  - With more than one compounding period per year, the future value factor is no longer (1+i)

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COMPOUND INTEREST

- **The Frequency of Compounding**
  - Therefore, with more than one compounding period per year, the future value can be expressed as:

$$FV_N = PV(1 + \frac{i}{m})^{m \times N} \quad (2.4)$$

  - In which:
    - i: is the annual interest rate
    - N: is the number of years
    - m: is the number of compounding periods per year or the frequency of compounding

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COMPOUND INTEREST

- **The Frequency of Compounding**
  - When a rate is quoted as annual rate but the interest is compound more than 1 time per year, the quoted annual rate is called a nominal or annual percentage rate (APR) (denoted as *i*).
  - If the frequency of compounding is m, then the interest rate per period is $\frac{i}{m}$, and this is an effective rate per period.
  - By applying the above formula, we could calculate the annual effective rate (AER) as follow:

$$AER = (1 + \frac{i}{m})^m - 1 \quad (2.5)$$

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Compound Interest

- **The Frequency of Compounding**
  - **Example 2.2:** If you have a credit card pays an APR of 18% per year compounded monthly. The effective (real) monthly rate is 18%/12 = 1.5% so the effective annual rate is $(1+0.015)^{12} - 1 = 19.56\%$
  - The two equal APR with different frequency of compounding have different effective annual rates:

Annual Percentage rate (APR)	Frequency of Compounding (m)	Annual Effective Rate (AER)
18	1	18.00
18	2	18.81
18	4	19.25
18	12	19.56
18	52	19.68
18	365	19.72

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Compound Interest

- **Continuous Compounding**
  - Note that as the frequency of compounding increases, so does the annual effective rate
  - What occurs as the frequency of compounding rises to infinity?
  - We know that: $FV_N = PV(1 + \frac{i}{m})^{m \times N}$ (2.4) ,
    - when m→∞ then: $\lim_{m \to \infty}(1 + \frac{i}{m})^m = e^i$
    - Then the formula for future value in the continuous compounding is: $FV_N = PVe^{i \times N}$
    - And the annual effective rate in this case is:

$$AER = \lim_{m \to \infty}(1 + \frac{i}{m})^m - 1 = e^i - 1 \quad (2.6)$$

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Compound Interest

- **Continuous Compounding**
  - **Example 2.2 (cont.):** The effective annual rate that's equivalent to an annual percentage rate of 18% is then $e^{0.18} - 1 = 19.7217\%$

Annual Percentage rate (APR)	Frequency of Compounding (m)	Annual Effective Rate (AER)
18	1	18.00
18	2	18.81
18	4	19.25
18	12	19.56
18	52	19.68
18	365	19.72
18	infinity	19.72

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Types of Interest Rates by the Commercial Bank's Operations

- **Bid rate**: is the interest rate that the bank mobilise funds from investors.
- **Ask rate**: is the interest rate that the bank lends to the borrowers.
- **Interbank rate**: is the interest rate that charged on loans made between banks on the interbank market.

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Types of interest rates used in banking regulations

- **Ceiling/Floor rates:** are highest/lowest rates that banks is authorised to make.
- **Base rate**: is the reference rate made by central bank (government) to guide commercial banks in making their own interest rates
- **Discount rate**: is the interest rate charged to commercial banks when they borrow from central bank.

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3. Interest Rates Evaluation - Time Value of Money

3.1 Time value of money
3.2 Future value
3.3 Present value
3.4 Implication on investment

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3.1 Time Value of Money

- In making financial decisions, like investment or borrowing decisions, people or corporations usually have to compare value of cash payments at different dates.
- E.g.:
  - You pay for university tuitions, that is an investment that you hope for a pay off later on in the form of a high salary. *Will your future salary be sufficient to justify the current studying expenditure?*
  - Companies pay for their investments in a new project, hoping to get higher return by borrowing money from the bank. *How much the companies have to repay the bank, will the future return offset the current interest for the bank?*

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3.1. Time Value of Money

"MONEY HAS A TIME VALUE"

as people think:

"a dollar received today is more valuable than a dollar received in the future".

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3.1. Time Value of Money

Time value of money concerns equivalence relationships between cash flows occurring on different dates.

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3.1. Time Value of Money

- **Example 3.1:**
  - You pay $10,000 today and in return receive $9,500 today. Is it alright?
  - What if you receive $9,500 today and will pay $10,000 one year later? It would be fair because a payment of $10,000 one year from now would probably be worth less than a payment of $10,000 today. Therefore, you could **discount** the $10,000 received in one year.

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3.1. TIME VALUE OF MONEY

- **Example 3.**
  - You pay eturn receive nt?
  - What if ay and will pay $10 would be fair beca ,000 one year from now would pr bly be worth less than a payment of $1 000 today. Therefore, you could **discount** the $10,000 received in one year.

To cut its value based on how much time passes before the money is paid

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3.1. TIME VALUE OF MONEY

- Why value of money today is different to that of future? (In general, value of money today is higher than value of money in the future). Because in the mean time, we could:
  - Invest the money in a risk free interest bearing account
  - Inflation will erode the purchasing power of the money
  - Bear the uncertainty of receiving back your money in the future

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3.2. Future Value

- **Future value (FV)** is the amount to which an investment will grow after earning interest.
- We will consider:
  - Future value of a single cash flow
  - Future value of multiple cash flows

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Future Value of a Single Cash Flow

- **Example 3.2**: Suppose you invest \$100 in an interest-bearing bank account paying 5% annually.
  - At the end of the first year, you will have:
    - \$100 of your initial investment; and
    - 0.05 x \$100 = \$5 interest earned.
  - To formalize this one-period example, we have:
    - PV: the initial investment (present value)
    - FV_n: future value of the investment n periods from today
    - i: rate of interest per period

      For n = 1 we have

      $FV_1 = PV(1+i)$

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FUTURE VALUE OF A SINGLE CASH FLOW

- **Example 3.2 (cont.):** Now you decide to invest the initial $100 for another year with interest earned and credited to your account annually (annual compounding). You can see how your investment grows:

Year	Balance at Start of Year	Interest Earned during Year	Balance at End of Year
1	$100	0.05 x $100 = $5	$105
2	$105	0.05 x $105 = $5.25	$110.25
3	$110.25	0.05 x $110.25 = $5.5125	$115.7625

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FUTURE VALUE OF A SINGLE CASH FLOW

- The general formula for the future value of a lump sum after n periods are:

$$FV_n = PV(1+i)^n \qquad (3.1)$$

In which:

- PV: the initial investment (present value)
- FV_n: future value of the investment n periods from today
- i: rate of interest per period

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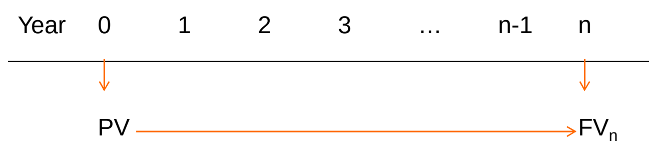
FUTURE VALUE OF A SINGLE CASH FLOW

- The relationship between the initial investment and its future value:



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FUTURE VALUE OF MULTIPLE CASH FLOWS

- Future value of multiple cash flows is the future value of series of cash flows, the cash flows could be even or uneven.
- We will consider:
  - Future value of multiple unequal cash flows
  - Future value of multiple equal cash flows

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Future Value of Multiple Cash Flows

- **Future value of multiple equal cash flows - Annuity**
  - **Annuity**: is a finite set of even sequential cash flows.
    - An **Ordinary annuity** has a first cash flow that occurs one period from now (indexed as t = 1)
    - An **Annuity due** has a first cash flow that occurs immediately (indexed at t = 0)
    - A **Perpetuity** is a perpetual annuity, or a set of even never-ending sequential cash flows, with the first cash flow occurring one period from now.
    - **Perpetual growth** is a set of never-ending sequential cash flows, with the cash flows growing year by year

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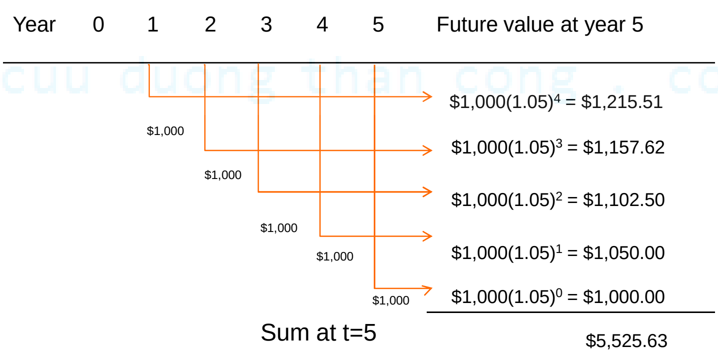
Annuity

- **Ordinary annuity**
  - **Example 3.3:** Consider an ordinary annuity paying 5% annually. Suppose we have five separate deposits of $1,000 occurring at equally intervals of one year, with the first payment occurring at t = 1. Our goal is to find future value of this ordinary annuity after the last deposit at t = 5.



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Annuity

- **Ordinary annuity**
  - Formula:

$$FV_n = A[(1+i)^{n-1} + (1+i)^{n-2} + \ldots + (1+i)^1 + (1+i)^0]$$

$$FV_n = A[\frac{(1+i)^n - 1}{i}] \qquad (3.2)$$

In which:
- FV_n: is the future value
- A: is the annuity amount
- i: is the interest rate
- n: is the number of year

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Annuity

- **Annuity due**
  - Formula:

$$FV_n = A[(1+i)^n + (1+i)^{n-1} + \ldots + (1+i)^2 + (1+i)^1]$$

$$FV_n = A[\frac{(1+i)^n}{i} - 1] \qquad (3.3)$$

In which:
- FV_n: is the future value
- A: is the annuity amount
- i: is the interest rate
- n: is the number of year

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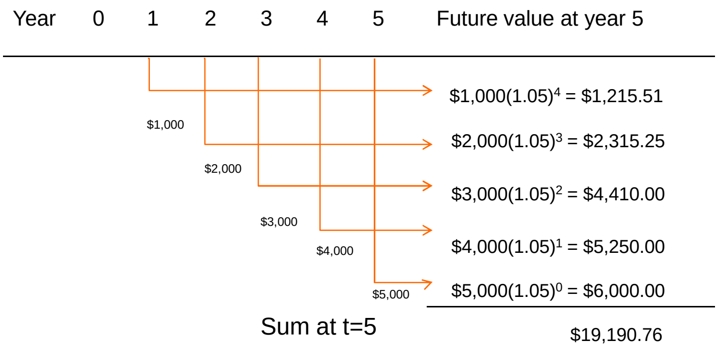
FUTURE VALUE OF MULTIPLE CASH FLOWS

- **Future value of multiple unequal cash flows**
  - **Example 3.4:** Consider unequal cash flow streams, paying 5% annually. Suppose we have five separate deposits of \$1,000, \$2,000; \$3,000; \$4,000; \$5,000 occurring at equally intervals from year 1 to year 5 respectively. Our goal is to find future value of these cash flows at t = 5.



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FUTURE VALUE OF MULTIPLE CASH FLOWS

- **Future value of multiple unequal cash flows**
  - **Formula:**

$$FV_n = \sum_{t=1}^{n} C_t(1+i)^{n-t} \quad (3.4)$$

In which:

- FV_n is future value of cash flows at year n
- C_t is amount of money invested at year t
- i is the interest rate
- t is the number of year

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3.3. Present Value

- **Present value (PV)** is the value today of a future cash flow.
- There is a close relationship between the **present value** (also known as *initial investment*), which earns a rate of return (the interest rate per period), and its **future value**, which will be received n years or periods from today.
- We will consider:
  - Present value of a single cash flow
  - Present value of multiple cash flows

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Present Value of a Single Cash Flow

- Example 3.5: How much do you have to invest today to generate a future payoff of $105 in 6 years with a 5% interest rate?

Year	0	1	2	3	4	5	6
	PV?						$105

- Solution: From the formula (3.1) we have:

$$FV = PV(1+i)^n$$

therefore $\$105 = PV(1+0.05)^6$

$$PV = \frac{\$105}{(1+0.05)^6} = 78.35$$

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Present Value of a Single Cash Flow

- **Formula:**

$$PV = \frac{FV_n}{(1+i)^n} \quad (4.1)$$

In which:

- **PV: the initial investment (present value)**
- **FV_n: future value of the investment n periods from today**
- **i: rate of interest per period**

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Present Value of Multiple Cash Flows

- Present value of multiple cash flows is the present value of series of cash flows, the cash flows could be even or uneven.
- We will consider the present value of:
  - Multiple equal cash flows
  - Multiple unequal cash flows

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Present Value of Multiple Cash Flows

- **Present value of multiple equal cash flows – Annuity**
  - **Present value of ordinary annuity**

Formula:

$$PV = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} + \dots + \frac{A}{(1+i)^{n-1}} + \frac{A}{(1+i)^n}$$

$$PV = A\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right] \qquad (4.2)$$

In which:
- PV: is the present value
- A: is the annuity amount
- i: is the interest rate
- n: is the number of year

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Present Value of Multiple Cash Flows

- **Present value of multiple equal cash flows – Annuity**
  - **Present value of annuity due**

$$PV = A + \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \dots + \frac{A}{(1+i)^{n-2}} + \frac{A}{(1+i)^{n-1}}$$

$$PV = A\left[\frac{1-\frac{1}{(1+i)^{n-1}}}{i} + 1\right] \qquad \textbf{(4.3)}$$

In which:

PV: is the present value

A: is the annuity amount

i: is the interest rate

n: is the number of years

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Present Value of Multiple Cash Flows

- **Present value of multiple equal cash flows – Annuity**
  - **Present value of perpetuity**

From the formula (4.2) we have the present value of an ordinary annuity is

$$PV = A\left[\frac{1-\frac{1}{(1+i)^n}}{i}\right]$$

When $n \to \infty$ then $PV = \frac{A}{i}$ (4.4)

In which:

PV: is the present value

A: is the annuity amount

i: is the interest rate

Present Value of Multiple Cash Flows

- **Present value of multiple unequal cash flows**
  - **Example 3.6:** Consider unequal cash flow streams, paying 5% annually. Suppose we have five separate deposits of \$1,000, \$2,000; \$3,000; \$4,000; \$5,000 occurring at equally intervals from year 1 to year 5 respectively. Our goal is to find present value of these cash flows at t = 0.

Present value at year 0 0 1 2 3 4 5

$1,000(1/1.05)^1 = $952.38 ← $1,000

$2,000(1/1.05)^2 = $1,814.06 ← $2,000

$3,000(1/1.05)^3 = $2,591.51 ← $3,000

$4,000(1/1.05)^4 = $3,290.81 ← $4,000

$5,000(1/1.05)^5 = $3.917.63 ← $5,000

Sum at t=5 $12,566.39

PRESENT VALUE OF MULTIPLE CASH FLOWS

- **Present value of multiple unequal cash flows**
  - Formula (4.5)

$$PV = \sum_{t=1}^{n} \frac{C_t}{(1+i)^t}$$

In which:
PV is the present value
C_t is the amount of money invested in year t
i is the interest rate
n is the number of years

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PRESENT VALUE OF MULTIPLE CASH FLOWS

- **Present value of multiple unequal cash flows**
  - **Present value of perpetual growth**

$$PV = \frac{A}{(i-g)} \qquad (4.6)$$

In which:
PV is the present value
A is the amount of money invested in year 1
i is the interest rate
g is the growth rate

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3.4 Implications on Investment

- Making investment decisions by using:
  - Net present value (NPV)
  - Internal rates of return (IRR)
- Valuing bond price
- Valuing stock price

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Net Present Value

- **Net present value (NPV)** of an investment is the present value of its cash inflows minus the present value of its cash outflows.
- The **NPV rule:**
  - If the investment's NPV is positive, an investor should undertake it;
  - If the investment's NPV is negative, an investor should not undertake it;
  - If an investor has two candidates for investment but can only invest in one, the investor should choose the candidate with the higher positive NPV.

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NET PRESENT VALUE

- Steps in computing NPV and applying the NPV rule:

1. Identify all cash inflows and cash outflows of the investment
2. Determine the discount rate or opportunity cost, r, for the investment project
3. Find the present value of each cash flow, using that discount rate (outflows have a negative sign, inflows have a positive sign)
4. Sum all present values. This is NPV
5. Applying the NPV rule

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NET PRESENT VALUE

- Formula:

$$NPV = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t} \quad (5.1)$$

- In which:
  - CF_t is the expected net cash flow at time t
  - n is the investment's projected life
  - r is the discount rate or opportunity cost of capital

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Net Present Value

- **Example 3.7**: You are considering to construct an office block, with the cost of the land and the construction total up $350,000. You expect to sell the block for $400,000 a year later. You could invest in construction project if the present value of $400,000 payoff is greater than the investment of $350,000. Mean while, you could invest your money on Treasury note which offers the interest of 7%. This could be considered the opportunity cost for the project. Therefore, the present value of future expected cash flow $400,000 is:

$$\$400{,}000 \times \frac{1}{1.07} = \$400{,}000 \times 0.9346 = \$373{,}832$$

- The block is worth $373,832 today and you have to invest $350,000 today to it, therefore the **Net present value** of it is:

$$NPV = \$373{,}832 - \$350{,}000 = \$23{,}832$$

- The NPV>0 then you should accept the project

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Internal Rate of Return

- **Internal rate of return (IRR)** is the discount rate that make the net present value equal to zero.
- The **IRR rule**: “Accept projects or investments for which the IRR is greater than the opportunity cost of capital”
  - If the opportunity cost of capital is equal to the IRR, then NPV = 0
  - If the project’s opportunity cost is less than the IRR, then NPV > 0

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INTERNAL RATE OF RETURN

- Formula:

$$NPV = CF_0 + \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_n}{(1+IRR)^n} = 0$$

- In which:
  - CF_t is the expected net cash flow at time t
  - n is the investment's projected life
  - IRR is the internal rate of return

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INTERNAL RATE OF RETURN

- **Example 3.8**: You invest $350,000 to get back a cash flow of $400,000 in 1 year, in a 1 period project like this, it's easy to calculate the internal rate of return:

$$NPV = 0 = \frac{\$400{,}000}{1+IRR} - \$350{,}000$$

- therefore: $IRR = \frac{\$400{,}000 - \$350{,}000}{\$350{,}000} = 0.1429 \approx 14.3\%$
- The opportunity cost for this project is the treasury note's interest of 7%. Thus the return on the project is higher than the opportunity cost of capital, therefore you should invest in this project

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VALUING BONDS

- Bondholders could receive cash flows from two sources: **coupon** payments each period and **face value** paid at maturity.

VALUING BONDS

- Bondholders could receive cash flows from two sources: **coupon** payments each period and **face value** paid at maturity.

The interest payments paid to the bondholder

VALUING BONDS

- Bondholders could receive cash flows from two sources: **coupon** payments each period and **face value** paid at maturity.

Payment at the maturity of the bond. Also called *principal* or *par value*

VALUING BONDS

- Bond price can be calculated as the present value of all **coupon** and **face value** payments that the bondholders receive.

$$PV = PV(coupons) + PV(face\ value)$$

$$\Rightarrow PV = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n} + \frac{FV}{(1+i)^n}$$

Valuing Bonds

- Recall annuity formula in chapter 2 we have the formula for value of bonds:

$$PV = C[\frac{1 - \frac{1}{(1+i)^n}}{i}] + \frac{FV}{(1+i)^n}$$

- In which:
  - PV: is the value of the bond
  - C: value of coupon payment
  - i: interest rate
  - n: number of coupon payment period
  - FV: face value of the bond

Valuing Bonds

- **Interest rate and coupon rate:**
  - **Coupon rate:** is the annual interest payment as a percentage of face value
  - When the market interest rate exceeds the coupon rate, bonds sell for less than face value, and vice versa.
  - When the interest rate rises, the present value of the payments to be received by the bondholder fall and bond prices fall, and vice versa

Valuing Bonds

- **Yield to maturity:** is the interest rate for which the present value of the bond's payments equals the price.
- **Bond rates of return**

$$Bond\ rates\ of\ return = \frac{coupon\ income + price\ change}{investment}$$

- **Current yield:**

$$Current\ yield = \frac{Annual\ coupon\ payment}{Bond\ price}$$

- **Yield curve:** plot of relationship between bond yields to maturity and time to maturity.
  - Normal yield curve
  - Steep yield curve
  - Flat or humped yield curve
  - Inverted yield curve

Valuing Stocks

- **Valuing stocks using dividend discount model:**
  - **Dividend discount model:** discounted cash flow model which states that today's stock price equals the present value of all expected future dividends

$$P_0 - \frac{DIV_1}{(1+r)} + \frac{DIV_1}{(1+r)^2} + \cdots + \frac{DIV_1}{(1+r)^t} + \cdots$$

  - In which:
    - P_0 is the stock value
    - DIV_i is the dividend in year i
    - r is the rate of return

Valuing Stocks

- **Valuing stocks using dividend discount model:**
  - **Dividend discount model with no growth**

$$P_0 = \frac{DIV}{r}$$

  - In which:
    - P_0 is the stock value
    - DIV is the dividend each year
    - r is the rate of return

Valuing Stocks

- **Valuing stocks using dividend discount model:**
  - **Constant-growth dividend discount model**

$$P_0 = \frac{DIV}{r \quad g}$$

  - In which:
    - P_0 is the stock value
    - DIV_1 is the dividend in year 1
    - r is the rate of return
    - g is the growth rate

VALUING STOCKS

- **Growth stocks** vs. **Income stocks**
- If a company earns a constant return on its equity and plows back a constant proportion of earnings, then:
  - **Sustainable growth rate**: the steady rate at which firm can grow is:

$$g = \textit{sustainable growth rate} = \textit{return on equity} \times \textit{plow back ratio}$$

VALUING STOCKS

- **Growth stocks** vs. **Income stocks**
- If a company earns a constant return on its equity and plows back a constant proportion of earnings, then:
  - S... ady rate at which firm c...

$$g = \textit{sust}\ldots \textit{on equity} \times \textit{plow back ratio}$$

The fraction of earnings retained by the firm
Plow back ratio = 1- payout ratio

4. The Behaviour of Interest Rates

- Factors affect the demand for bonds
- Factors affect the supply of bonds

Factors affect the demand for bonds

	Change in variables	Change in quantity demanded at each bond price	Interest rate
Wealth	↑	↑	↓
Riskiness of bonds relative to other assets	↑	↓	↑
Expected returns on bonds relative to alternative assets	↑	↑	↓
Liquidity of bonds relative to alternative assets	↑	↑	↓

Factors affect the supply for bonds

	Change in variables	Change in quantity supplied at each bond price	Interest rate
Expected inflation	↑	↑	↑
Expected profitability of investment opportunities	↑	↑	↑
Government deficit	↑	↑	↑

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5. The Risk and Term Structure of Interest Rates

- Risk structure vs. Term structure of interest rates
- Risk structure of interest rates
- Term structure of interest rates

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RISK STRUCTURE VS. TERM STRUCTURE OF INTEREST RATES

- The relationship between those different interest rates of different bonds with the same maturity is called **risk structure of interest rates**
- **Term structure of interest rates** is the relationship between those different interest rates of bonds with different maturities.

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RISK STRUCTURE OF INTEREST RATES

- Bonds with the same maturity could have different interest rates because they may be influent by risky factors in different levels.
- These factors could be:
  - Default risk
  - Liquidity
  - Income tax consideration

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DEFAULT RISK

- **The risk of default** occurs when the issuer of the bond is unable or unwilling to make interest payments when promised or pay off the face value when the bond matures.
- The bond with the least default risk may have lowest interest rates.
- The government bond usually considered **default-free bond**, and have lowest interest rate.
- Other bond with higher default risk will have higher interest rate, and the difference between this interest rate and the rate of government bond is called **risk premium**, which is considered the cost for accepting higher default risk for the investors.

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LIQUIDITY

- The more liquid an asset is, the more it is desired by investors, and so does the bonds.
- The more liquid bonds will have lower interest.

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INCOME TAX CONSIDERATIONS

- The interest payments on some bonds are taxed while others' are not. This lead to different attraction between bonds and so does their interest rates.

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TERM STRUCTURE OF INTEREST RATES

- Bonds with identical default risk, liquidity, and tax characteristics may have different interest rates because their time remaining to maturity is different.
- The term structure of interest rates is described by the **yield curve**

A plot of the yields on bonds with different term to maturities with the same risk, liquidity and tax considerations

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TERM STRUCTURE OF INTEREST RATES

- Various theories have been used to explain the relationship between maturities and bond yield:
  - Pure expectations theory
  - Segmented markets theory
  - Liquidity premium theory
- Besides explaining why yield curves take on different shapes at different times, a good theory of the term structure of interest rates must explain 3 empirical facts:
  - Fact 1: interest rates on bonds of different maturities move together over time.
  - Fact 2: When short-term interest rate are low, yield curves are more likely to have an upward slope and vice versa
  - Fact 3: Yield curve almost always slope upward

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EXPECTATION THEORY

- The **expectation theory** of the term structure states that: the interest rate on a long-term bond will equal an average of the short-term interest rates that people expect to occur over the life of the long-term bond.
- The expectation theory explains that interest rates of different maturity bonds are different because short term interest rate are expected to have different value at future dates.
- The expectation theory also explains fact 1 and 2 but can not explain fact 3.

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SEGMENTED MARKETS THEORY

- The **segmented markets theory** sees markets for different-maturity bonds as completely separate and segmented. The interest rate for each bond with a different maturity is then determined by the supply of and demand for that bond, with no effects from expected returns on other bonds with other maturities.
- The segmented markets theory can explain fact 3 but cannot explain fact 1 and 2.

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LIQUIDITY PREMIUM AND PREFERRED HABITAT THEORIES

- **The liquidity premium and preferred habitat theories** state that the interest rate on a long-term bond will equal an average of short-term interest rates expected to occur over the life of the long-term bond plus a liquidity premium that responds to supply and demand conditions for that bond.
- They actually like the combination of the ideas of expectation theory and segmented markets theory and can explain fact 1, 2 and 3.

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