

Toi xin loi cac ban vi bai hoc luc sang, toi chua tinh a_0 nen ket qua ra chua chinh xac. Toi sua lai cho cac ban nhu sau:

$$f(x) = |x|, \quad f(x) = f(x+2).$$

Chung ta can tim khai trien chuoi Fourier

$$|x| = \frac{a_0}{2} + \sum_{n \geq 1} (a_n \cos \pi n x + b_n \sin \pi n x).$$

tinh toan nhu tren lop (chung ta quen tinh a_0), ta co

$$a_0 = \int_{-1}^1 |x| dx = 1, \quad b_n = 0, \quad a_n = \int_{-1}^1 |x| \cos \pi n x dx = 4 \frac{(-1)^n - 1}{\pi^2 n^2}.$$

Nhu vay ta co

$$|x| = \frac{1}{2} + \sum_{k \geq 0} \frac{-4}{\pi^2 (2k+1)^2} \cos \pi (2k+1)x.$$

Chon $x = 1$, ta co

$$1 = \frac{1}{2} + \sum_{k \geq 0} \frac{-4}{\pi^2 (2k+1)^2} \cos \pi (2k+1).$$

Va vi vay,

$$\frac{1}{2} = \sum_{k \geq 0} \frac{4}{\pi^2 (2k+1)^2}$$

hay

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}.$$

Bang cach dat nhu tren lop, tinh lai tong sau:

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}.$$