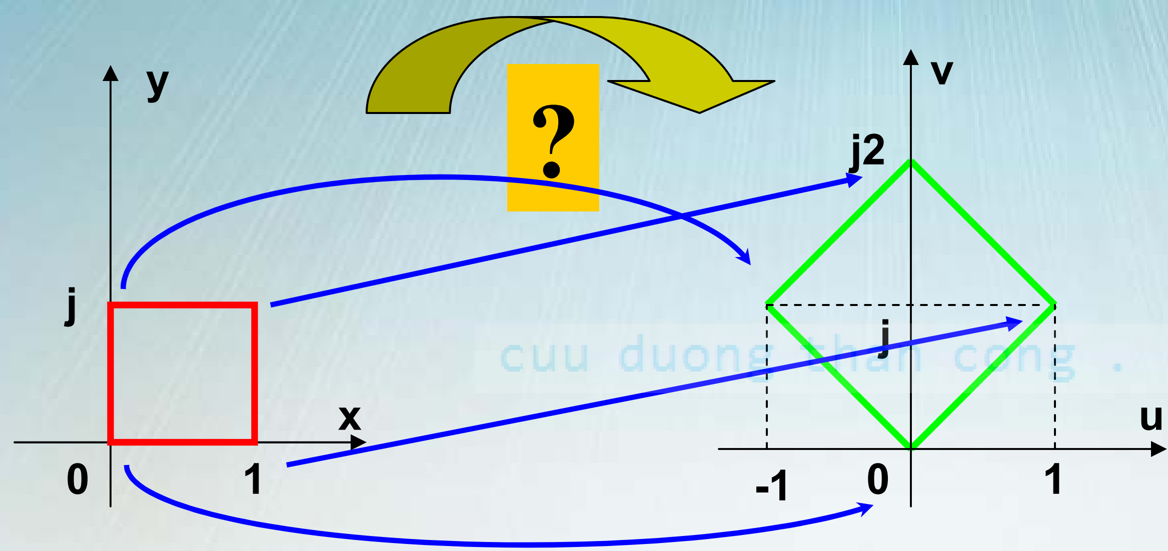
Chapter 12:

Conformal Mapping

12.1 Mapping or Transformation :

❖ Complex function w = f(z) : define a transformatin between poins in the z-plane and points in the w-plane.

❖**Example1:** Consider the transformation w = (1+j)z on the rectangular region ?



z		w
0	→	0
1+j0	→	1+j
1+j	→	j2
0+j	→	-1+j

❖ Some General Transformations:

i. **Translation:** $w = z + a$

Figures displaced in the direction of a .

ii. **Rotation:** $w = e^{j\theta}.z$

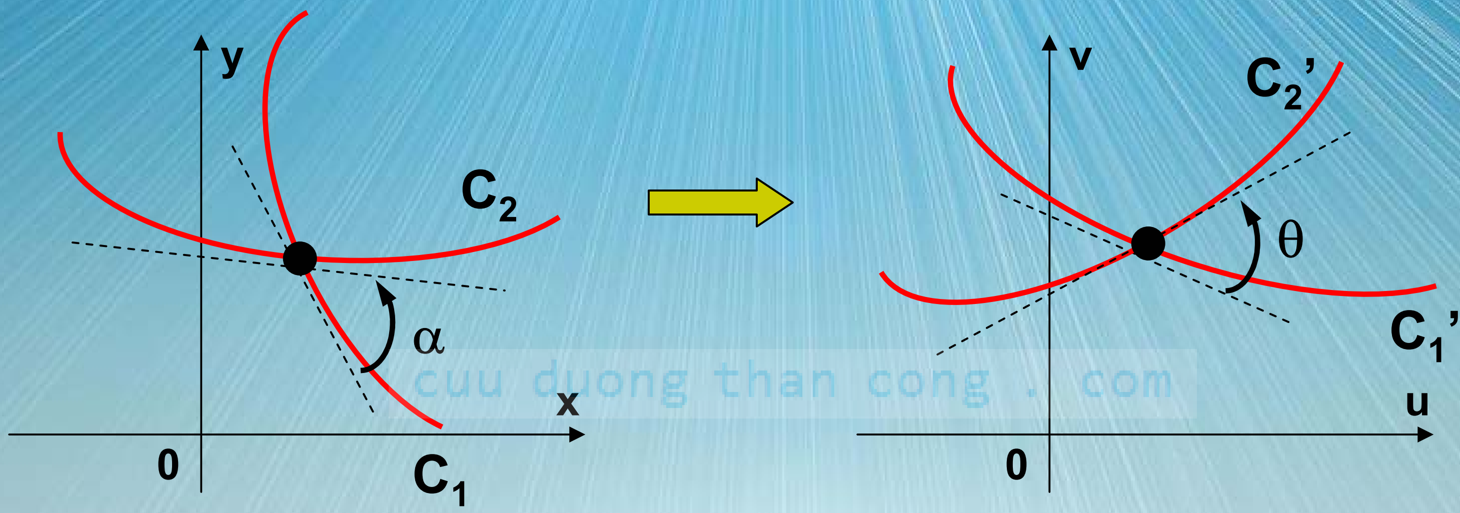
Figures are rotated in CCW if $\theta > 0$ and CW if $\theta < 0$.

iii. **Stretching:** $w = \alpha z$

Figures are stretched.

iv. **Inversion:** $w = 1/z$

12.2 Conformal Mapping :



❖ Given a transformation $w = f(z)$, $C_1 \rightarrow C_1'$, $C_2 \rightarrow C_2'$. If $\theta = \alpha$ (magnitude and sense), w = conformal mapping.

❖ If f(z) analytic anf $f'(z) \neq 0$: w = conformal mapping.

12.3 Linear Fractional Mapping :

a) Definition:

- Linear Fractional Mapping is a transformation defined by:

$$\boxed{T(z) = \frac{az+b}{cz+d}} \longrightarrow \textit{(is also called bilinear transformation)}$$

- Domain of this transformation : all complex z such that $z \neq -d/c$.

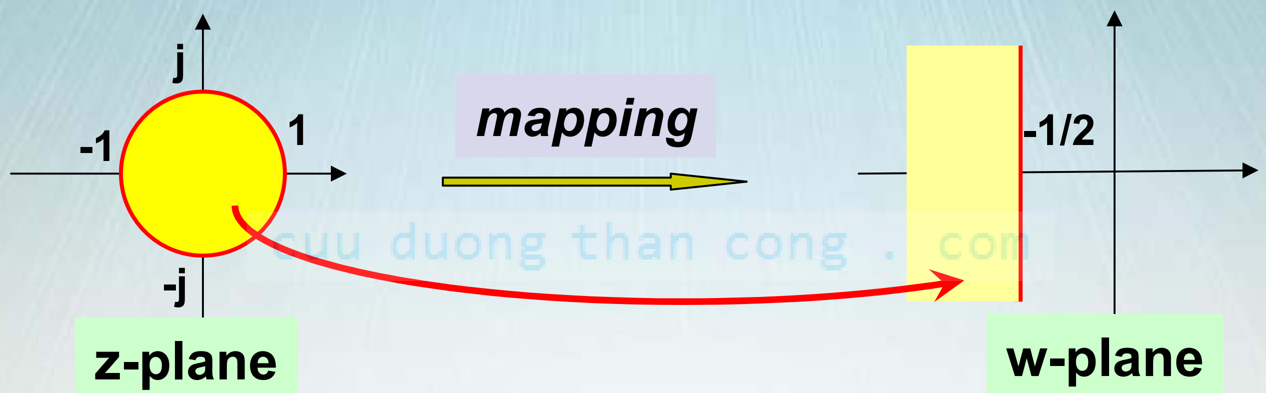
- $T'(z) = [ad - bc]/(cz+d)^2$. Require $(ad - bc) \neq 0$: the mapping is conformal.

b) Properties of Bilinear Mapping:

❖ The image of a circle C (in z-plane) is a circle C’ (in w-plane) and is a line C’ if only { $c \neq 0$ and the pole $z = -d/c$ on C} .

❖**Example1:** Find the image of the circle $|z| = 1$ under bilinear transformation $T(z) = (z + 2)/(z – 1)$? The image of the interior $|z| < 1$?

- We have: $T(j) = – 1/2 – j3/2$; $T(-1) = -1/2$; $T(-j) = – 1/2 + j3/2$.



- Find image of interior , use test point: $T(z=0) = - 2$: left $u = -1/2$.

c) Cross Ratio:

- **Method to construct a bilinear mapping which maps three given points z_1, z_2, z_3 in boundary D to three given points w_1, w_2, w_3 in boundary D'.**

- **Cross Ratio of 4 complex numbers: z, z_1, z_2, z_3 is a complex number:**

$$\frac{z-z_1}{z-z_3}\,\frac{z_2-z_3}{z_2-z_1}$$

(be careful with the order of the number !)

- **If $w = T(z)$ = bilinear mapping that maps z_1, z_2, z_3 onto w_1, w_2, w_3 then:**

$$\frac{z-z_1}{z-z_3}\,\frac{z_2-z_3}{z_2-z_1} = \frac{w-w_1}{w-w_3}\,\frac{w_2-w_3}{w_2-w_1}$$