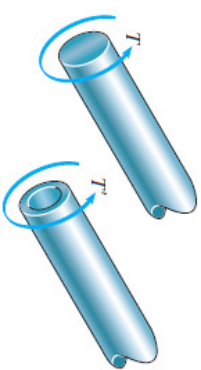


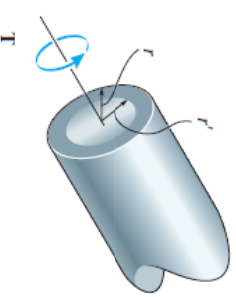


## Chương 5: Bài tập\_XOẮN

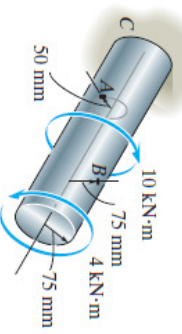
- 5-1. A shaft is made of a steel alloy having an allowable shear stress of  $\tau_{\text{allow}} = 12 \text{ ksi}$ . If the diameter of the shaft is 1.5 in., determine the maximum torque **T** that can be transmitted. What would be the maximum torque **T'** if a 1-in.-diameter hole is bored through the shaft? Sketch the shear-stress distribution along a radial line in each case.



- 5-2. The solid shaft of radius  $r$  is subjected to a torque **T**. Determine the radius  $r'$  of the inner core of the shaft that resists one-half of the applied torque ( $T/2$ ). Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



- 5-3. The solid shaft is fixed to the support at **C** and subjected to the torsional loadings shown. Determine the shear stress at points **A** and **B** and sketch the shear stress on volume elements located at these points.



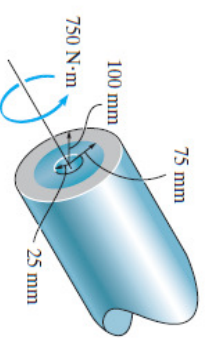
L7A\_Cơ học vật liệu (215004)

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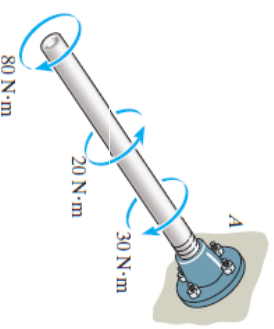


## Chương 5: Bài tập\_XOẮN

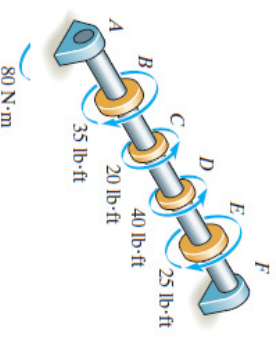
- \*5-4. The tube is subjected to a torque of  $750 \text{ N} \cdot \text{m}$ . Determine the amount of this torque that is resisted by the gray shaded section. Solve the problem two ways: (a) by using the torsion formula, (b) by finding the resultant of the shear-stress distribution.



- 5-5. The copper pipe has an outer diameter of 40 mm and an inner diameter of 37 mm. If it is tightly secured to the wall at **A** and three torques are applied to it as shown, determine the absolute maximum shear stress developed in the pipe.



- 5-6. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions **BC** and **DE** of the shaft. The bearings at **A** and **F** allow free rotation of the shaft.



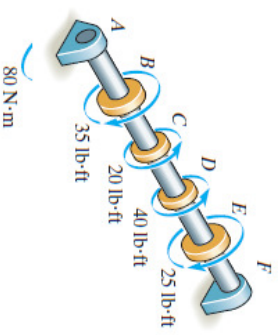
L7A\_Cơ học vật liệu (215004)

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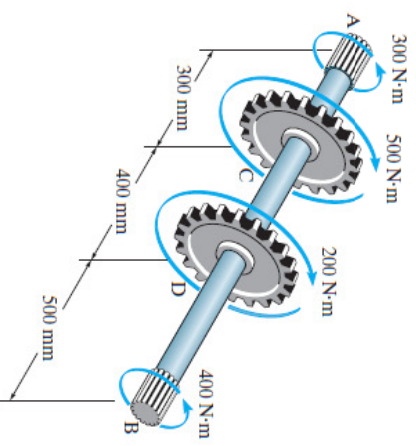


## Chương 5: Bài tập\_XOẮN

5-7. The solid shaft has a diameter of 0.75 in. If it is subjected to the torques shown, determine the maximum shear stress developed in regions  $CD$  and  $EF$  of the shaft. The bearings at  $A$  and  $F$  allow free rotation of the shaft.



\*5-8. The solid 30-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress on the shaft.



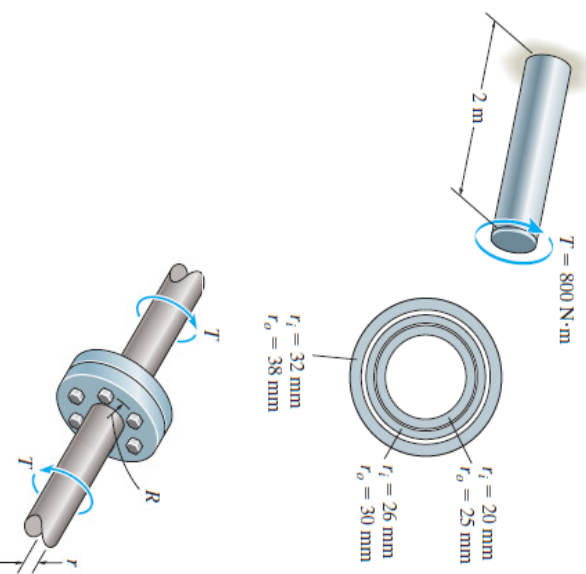
L7A\_Cơ học vật liệu (215004)

3



## Chương 5: Bài tập\_XOẮN

•5-9. The shaft consists of three concentric tubes, each made from the same material and having the inner and outer radii shown. If a torque of  $T = 800 \text{ N} \cdot \text{m}$  is applied to the rigid disk fixed to its end, determine the maximum shear stress in the shaft.



5-10. The coupling is used to connect the two shafts together. Assuming that the shear stress in the bolts is *uniform*, determine the number of bolts necessary to make the maximum shear stress in the shaft equal to the shear stress in the bolts. Each bolt has a diameter  $d$ .

$n$  is the number of bolts and  $F$  is the shear force in each bolt.

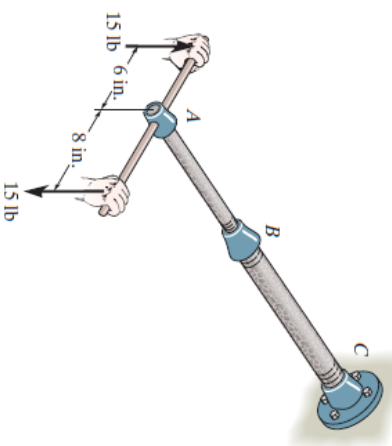
L7A\_Cơ học vật liệu (215004)

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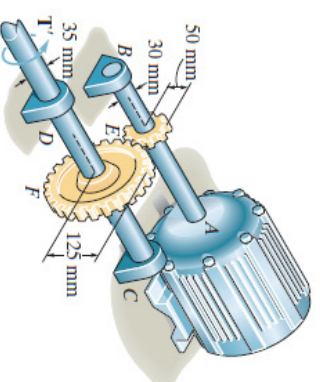


## Chương 5: Bài tập\_XOẮN

**5-11.** The assembly consists of two sections of galvanized steel pipe connected together using a reducing coupling at *B*. The smaller pipe has an outer diameter of 0.75 in. and an inner diameter of 0.68 in., whereas the larger pipe has an outer diameter of 1 in. and an inner diameter of 0.86 in. If the pipe is tightly secured to the wall at *C*, determine the maximum shear stress developed in each section of the pipe when the couple shown is applied to the handles of the wrench.



**\*5-12.** The motor delivers a torque of  $50 \text{ N} \cdot \text{m}$  to the shaft *AB*. This torque is transmitted to shaft *CD* using the gears at *E* and *F*. Determine the equilibrium torque  $T'$  on shaft *CD* and the maximum shear stress in each shaft. The bearings *B*, *C*, and *D* allow free rotation of the shafts.



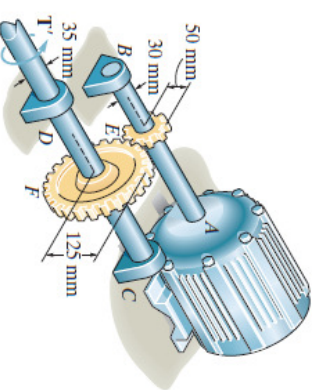
L7A\_Cơ học vật liệu (215004)

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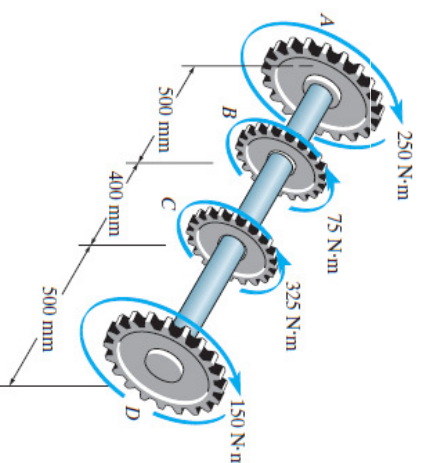


## Chương 5: Bài tập\_XOẮN

**•5-13.** If the applied torque on shaft *CD* is  $T' = 75 \text{ N} \cdot \text{m}$ , determine the absolute maximum shear stress in each shaft. The bearings *B*, *C*, and *D* allow free rotation of the shafts, and the motor holds the shafts fixed from rotating.



**5-14.** The solid 50-mm-diameter shaft is used to transmit the torques applied to the gears. Determine the absolute maximum shear stress in the shaft.



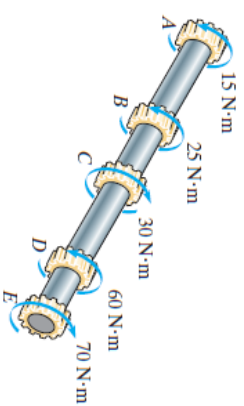
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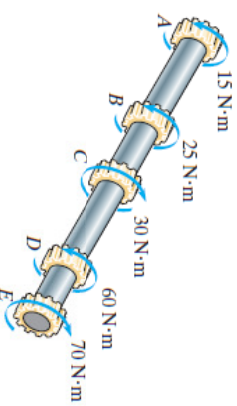


## Chương 5: Bài tập\_XOẮN

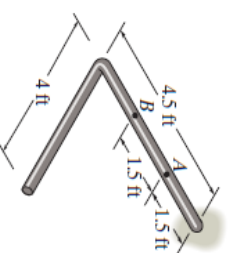
5-15. The solid shaft is made of material that has an allowable shear stress of  $\tau_{\text{allow}} = 10 \text{ MPa}$ . Determine the required diameter of the shaft to the nearest mm.



\*5-16. The solid shaft has a diameter of 40 mm. Determine the absolute maximum shear stress in the shaft and sketch the shear-stress distribution along a radial line of the shaft where the shear stress is maximum.



•5-17. The rod has a diameter of 1 in. and a weight of 10 lb/ft. Determine the maximum torsional stress in the rod at a section located at A due to the rod's weight.



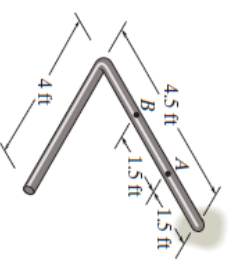
LTA\_Cơ học vật liệu (215004)

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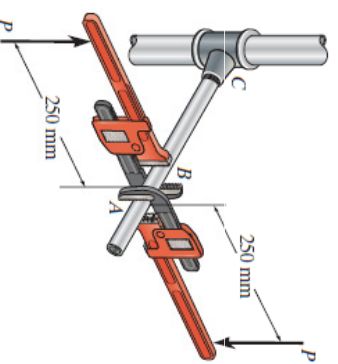


## Chương 5: Bài tập\_XOẮN

5-18. The rod has a diameter of 1 in. and a weight of 15 lb/ft. Determine the maximum torsional stress in the rod at a section located at B due to the rod's weight.



5-19. Two wrenches are used to tighten the pipe. If  $P = 300 \text{ N}$  is applied to each wrench, determine the maximum torsional shear stress developed within regions AB and BC. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm. Sketch the shear stress distribution for both cases.



LTA\_Cơ học vật liệu (215004)

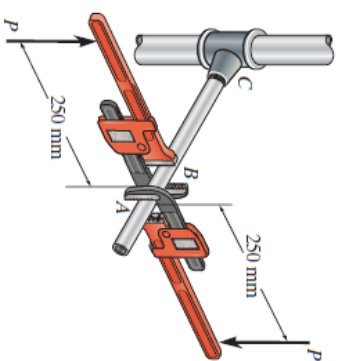
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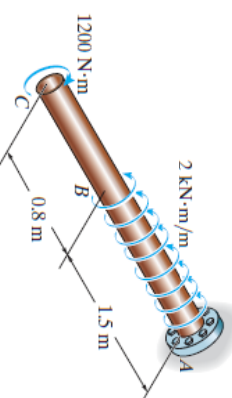


## Chương 5: Bài tập\_XOẮN

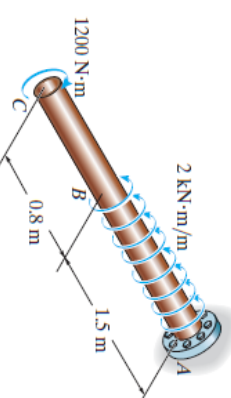
\*5-20. Two wrenches are used to tighten the pipe. If the pipe is made from a material having an allowable shear stress of  $\tau_{\text{allow}} = 85 \text{ MPa}$ , determine the allowable maximum force **P** that can be applied to each wrench. The pipe has an outer diameter of 25 mm and inner diameter of 20 mm.



•5-21. The 60-mm-diameter solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the absolute maximum and minimum shear stresses on the outer surface of the shaft and specify their locations, measured from the fixed end A.



5-22. The solid shaft is subjected to the distributed and concentrated torsional loadings shown. Determine the required diameter *d* of the shaft to the nearest mm if the allowable shear stress for the material is  $\tau_{\text{allow}} = 50 \text{ MPa}$ .



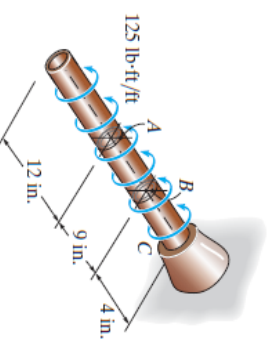
L7A\_Cơ học vật liệu (215004)

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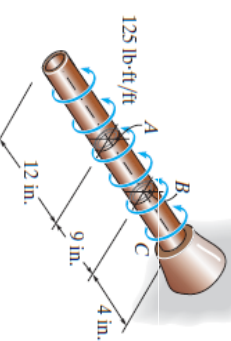


## Chương 5: Bài tập\_XOẮN

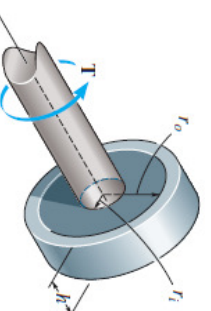
\*5-24. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and a uniformly distributed torque is applied to it as shown, determine the shear stress developed at points A and B. These points lie on the pipe's outer surface. Sketch the shear stress on volume elements located at A and B.



•5-25. The copper pipe has an outer diameter of 2.50 in. and an inner diameter of 2.30 in. If it is tightly secured to the wall at C and it is subjected to the uniformly distributed torque along its entire length, determine the absolute maximum shear stress in the pipe. Discuss the validity of this result.



5-26. A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque **T** is applied to the shaft, determine the maximum shear stress in the rubber.



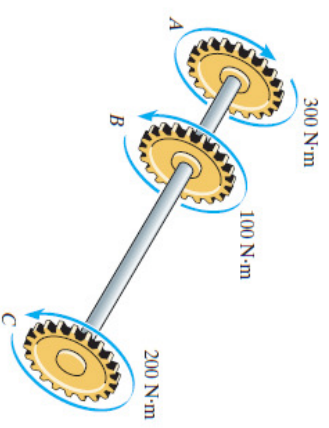
L7A\_Cơ học vật liệu (215004)

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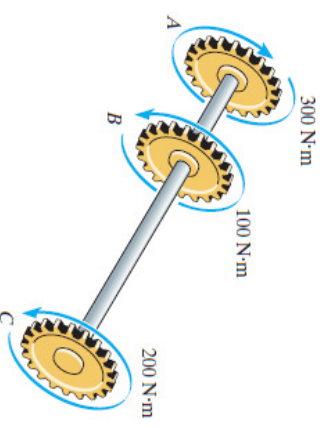


## Chương 5: Bài tập\_XOẮN

5-27. The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the maximum shear stress developed in the segments  $AB$  and  $BC$ . The shaft has a diameter of 40 mm.



\*5-28. The A-36 steel shaft is supported on smooth bearings that allow it to rotate freely. If the gears are subjected to the torques shown, determine the required diameter of the shaft to the nearest mm if  $\tau_{\text{allow}} = 60$  MPa.



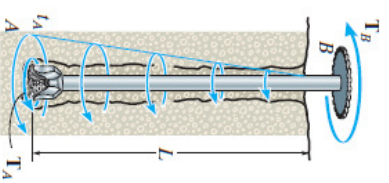
LTA\_Cơ học vật liệu (215004)

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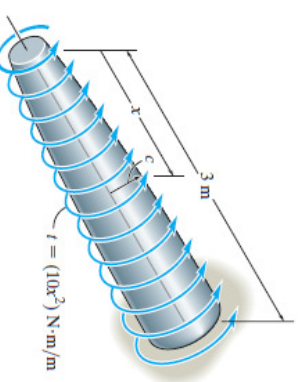


## Chương 5: Bài tập\_XOẮN

\*5-29. When drilling a well at constant angular velocity, the bottom end of the drill pipe encounters a torsional resistance  $T_A$ . Also, soil along the sides of the pipe creates a distributed frictional torque along its length, varying uniformly from zero at the surface  $B$  to  $T_A$  at  $A$ . Determine the minimum torque  $T_B$  that must be supplied by the drive unit to overcome the resisting torques, and compute the maximum shear stress in the pipe. The pipe has an outer radius  $r_o$  and an inner radius  $r_i$ .



5-30. The shaft is subjected to a distributed torque along its length of  $t = (10x^2)$  N·m/m, where  $x$  is in meters. If the maximum stress in the shaft is to remain constant at 80 MPa, determine the required variation of the radius  $c$  of the shaft for  $0 \leq x \leq 3$  m.



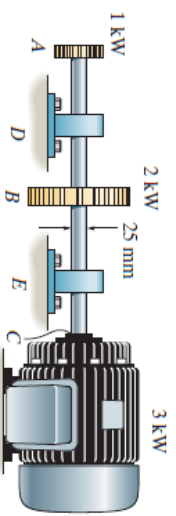
LTA\_Cơ học vật liệu (215004)

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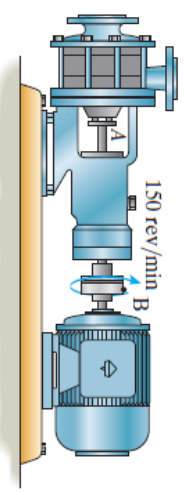


## Chương 5: Bài tập\_XOẮN

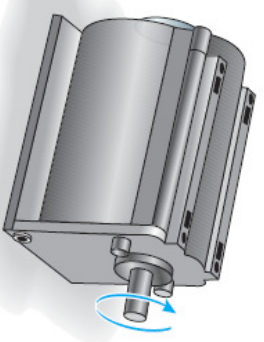
5-31. The solid steel shaft  $AC$  has a diameter of 25 mm and is supported by smooth bearings at  $D$  and  $E$ . It is coupled to a motor at  $C$ , which delivers 3 kW of power to the shaft while it is turning at 50 rev/s. If gears  $A$  and  $B$  remove 1 kW and 2 kW, respectively, determine the maximum shear stress developed in the shaft within regions  $AB$  and  $BC$ . The shaft is free to turn in its support bearings  $D$  and  $E$ .



\*5-32. The pump operates using the motor that has a power of 85 W. If the impeller at  $B$  is turning at 150 rev/min, determine the maximum shear stress developed in the 20-mm-diameter transmission shaft at  $A$ .



•5-33. The gear motor can develop 2 hp when it turns at 450 rev/min. If the shaft has a diameter of 1 in., determine the maximum shear stress developed in the shaft.



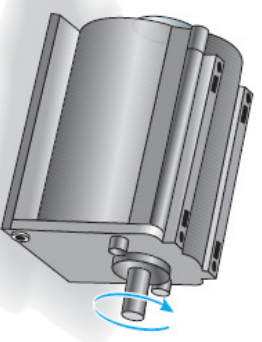
L7A\_Cơ học vật liệu (215004)

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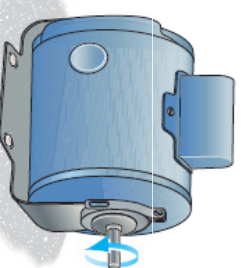


## Chương 5: Bài tập\_XOẮN

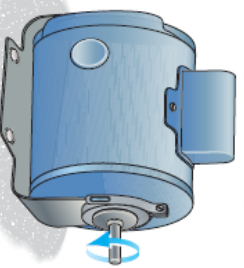
5-34. The gear motor can develop 3 hp when it turns at 150 rev/min. If the allowable shear stress for the shaft is  $\tau_{\text{allow}} = 12$  ksi, determine the smallest diameter of the shaft to the nearest  $\frac{1}{8}$  in. that can be used.



5-35. The 25-mm-diameter shaft on the motor is made of a material having an allowable shear stress of  $\tau_{\text{allow}} = 75$  MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



\*5-36. The drive shaft of the motor is made of a material having an allowable shear stress of  $\tau_{\text{allow}} = 75$  MPa. If the outer diameter of the tubular shaft is 20 mm and the wall thickness is 2.5 mm, determine the maximum allowable power that can be supplied to the motor when the shaft is operating at an angular velocity of 1500 rev/min.



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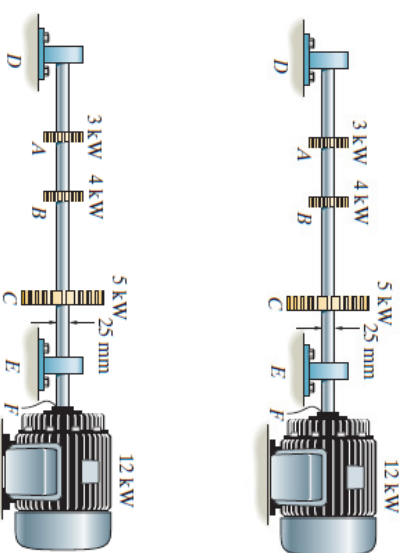
## Chương 5: Bài tập\_XOẮN

•5-37. A ship has a propeller drive shaft that is turning at 1500 rev/min while developing 1800 hp. If it is 8 ft long and has a diameter of 4 in., determine the maximum shear stress in the shaft caused by torsion.

5-38. The motor *A* develops a power of 300 W and turns its connected pulley at 90 rev/min. Determine the required diameters of the steel shafts on the pulleys at *A* and *B* if the allowable shear stress is  $\tau_{\text{allow}} = 85 \text{ MPa}$ .



5-39. The solid steel shaft *DE* has a diameter of 25 mm and is supported by smooth bearings at *D* and *E*. It is coupled to a motor at *F*, which delivers 12 kW of power to the shaft while it is turning at 50 rev/s. If gears *A*, *B*, and *C* remove 3 kW, 4 kW, and 5 kW respectively, determine the maximum shear stress developed in the shaft within regions *CF* and *BC*. The shaft is free to turn in its support bearings *D* and *E*.



\*5-40. Determine the absolute maximum shear stress developed in the shaft in Prob. 5-39.

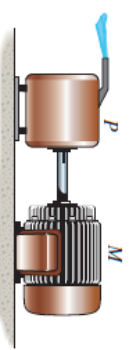
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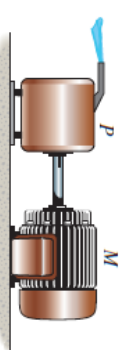


## Chương 5: Bài tập\_XOẮN

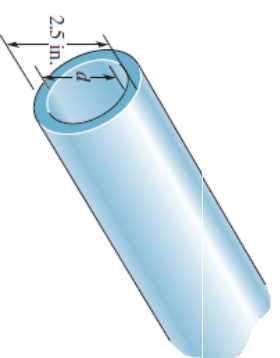
•5-41. The A-36 steel tubular shaft is 2 m long and has an outer diameter of 50 mm. When it is rotating at 40 rad/s, it transmits 25 kW of power from the motor *M* to the pump *P*. Determine the smallest thickness of the tube if the allowable shear stress is  $\tau_{\text{allow}} = 80 \text{ MPa}$ .



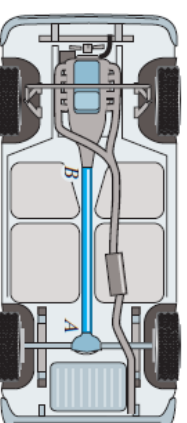
5-42. The A-36 solid tubular steel shaft is 2 m long and has an outer diameter of 60 mm. It is required to transmit 60 kW of power from the motor *M* to the pump *P*. Determine the smallest angular velocity the shaft can have if the allowable shear stress is  $\tau_{\text{allow}} = 80 \text{ MPa}$ .



5-43. A steel tube having an outer diameter of 2.5 in. is used to transmit 35 hp when turning at 2700 rev/min. Determine the inner diameter *d* of the tube to the nearest  $\frac{1}{8}$  in. if the allowable shear stress is  $\tau_{\text{allow}} = 10 \text{ ksi}$ .



\*5-44. The drive shaft *AB* of an automobile is made of a steel having an allowable shear stress of  $\tau_{\text{allow}} = 8 \text{ ksi}$ . If the outer diameter of the shaft is 2.5 in. and the engine delivers 200 hp to the shaft when it is turning at 1140 rev/min, determine the minimum required thickness of the shaft's wall.



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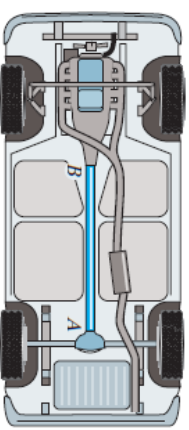
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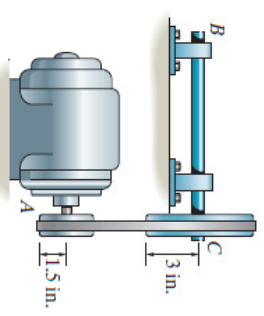


## Chương 5: Bài tập\_XOẮN

- 5-45. The drive shaft  $AB$  of an automobile is to be designed as a thin-walled tube. The engine delivers 150 hp when the shaft is turning at 1500 rev/min. Determine the minimum thickness of the shaft's wall if the shaft's outer diameter is 2.5 in. The material has an allowable shear stress of  $\tau_{\text{allow}} = 7$  ksi.



- 5-46. The motor delivers 15 hp to the pulley at  $A$  while turning at a constant rate of 1800 rpm. Determine to the nearest  $\frac{1}{8}$  in. the smallest diameter of shaft  $BC$  if the allowable shear stress for steel is  $\tau_{\text{allow}} = 12$  ksi. The belt does not slip on the pulley.



- 5-47. The propellers of a ship are connected to a A-36 steel shaft that is 60 m long and has an outer diameter of 340 mm and inner diameter of 260 mm. If the power output is 4.5 MW when the shaft rotates at 20 rad/s, determine the maximum torsional stress in the shaft and its angle of twist.

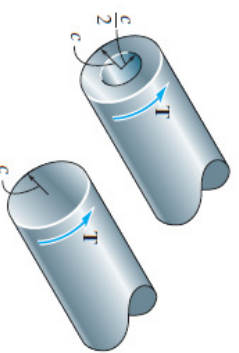
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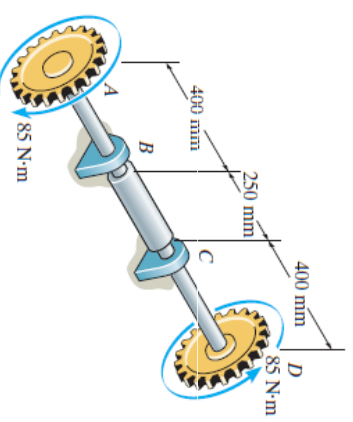


## Chương 5: Bài tập\_XOẮN

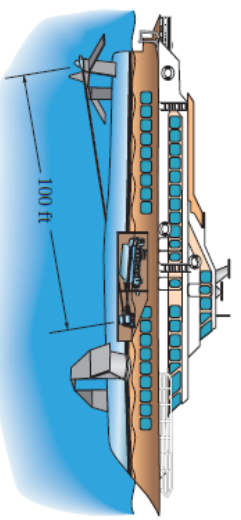
- \*5-48. A shaft is subjected to a torque  $T$ . Compare the effectiveness of using the tube shown in the figure with that of a solid section of radius  $c$ . To do this, compute the percent increase in torsional stress and angle of twist per unit length for the tube versus the solid section.



- 5-49. The A-36 steel axle is made from tubes  $AB$  and  $CD$  and a solid section  $BC$ . It is supported on smooth bearings that allow it to rotate freely. If the gears, fixed to its ends, are subjected to 85-N · m torques, determine the angle of twist of gear  $A$  relative to gear  $D$ . The tubes have an outer diameter of 30 mm and an inner diameter of 20 mm. The solid section has a diameter of 40 mm.



- 5-50. The hydrofoil boat has an A-36 steel propeller shaft that is 100 ft long. It is connected to an in-line diesel engine that delivers a maximum power of 2500 hp and causes the shaft to rotate at 1700 rpm. If the outer diameter of the shaft is 8 in. and the wall thickness is  $\frac{3}{8}$  in., determine the maximum shear stress developed in the shaft. Also, what is the “wind up,” or angle of twist in the shaft at full power?



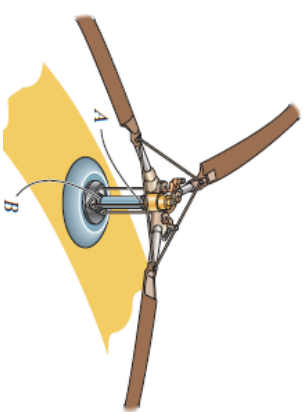
L7A\_Cơ học vật liệu (215004)

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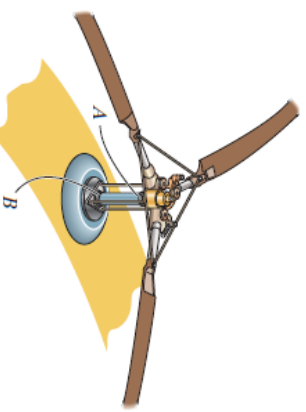


## Chương 5: Bài tập\_XOẮN

5-51. The engine of the helicopter is delivering 600 hp to the rotor shaft  $AB$  when the blade is rotating at 1200 rev/min. Determine to the nearest  $\frac{1}{8}$  in. the diameter of the shaft  $AB$  if the allowable shear stress is  $\tau_{\text{allow}} = 8$  ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



\*5-52. The engine of the helicopter is delivering 600 hp to the rotor shaft  $AB$  when the blade is rotating at 1200 rev/min. Determine to the nearest  $\frac{1}{8}$  in. the diameter of the shaft  $AB$  if the allowable shear stress is  $\tau_{\text{allow}} = 10.5$  ksi and the vibrations limit the angle of twist of the shaft to 0.05 rad. The shaft is 2 ft long and made from L2 steel.



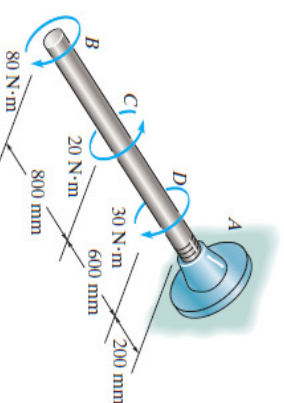
L7A\_Cơ học vật liệu (215004)

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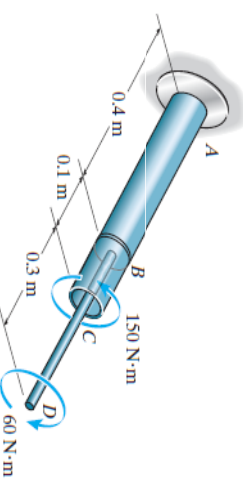


## Chương 5: Bài tập\_XOẮN

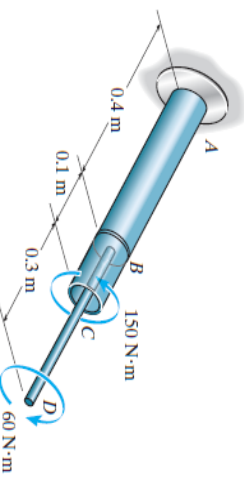
•5-53. The 20-mm-diameter A-36 steel shaft is subjected to the torques shown. Determine the angle of twist of the end  $B$ .



5-54. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at  $B$ . Determine the angle of twist at  $D$ . The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



5-55. The assembly is made of A-36 steel and consists of a solid rod 20 mm in diameter fixed to the inside of a tube using a rigid disk at  $B$ . Determine the angle of twist at  $C$ . The tube has an outer diameter of 40 mm and wall thickness of 5 mm.



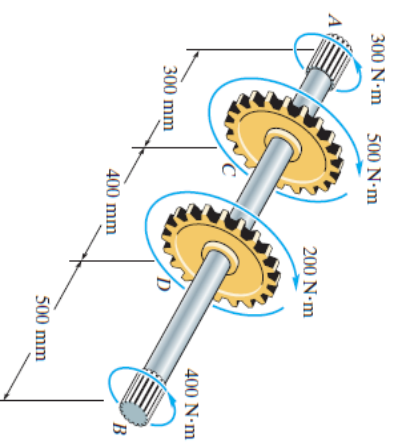
L7A\_Cơ học vật liệu (215004)

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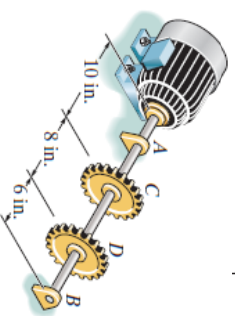


## Chương 5: Bài tập\_XOẮN

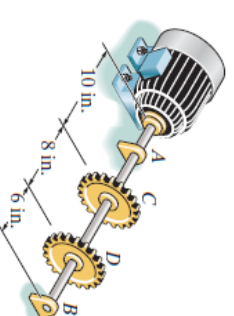
\*5-56. The splined ends and gears attached to the A-36 steel shaft are subjected to the torques shown. Determine the angle of twist of end *B* with respect to end *A*. The shaft has a diameter of 40 mm.



•5-57. The motor delivers 40 hp to the 304 stainless steel shaft while it rotates at 20 Hz. The shaft is supported on smooth bearings at *A* and *B*, which allow free rotation of the shaft. The gears *C* and *D* fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the diameter of the shaft to the nearest  $\frac{1}{8}$  in. if the allowable shear stress is  $\tau_{\text{allow}} = 8$  ksi and the allowable angle of twist of *C* with respect to *D* is  $0.20^\circ$ .



5-58. The motor delivers 40 hp to the 304 stainless steel solid shaft while it rotates at 20 Hz. The shaft has a diameter of 1.5 in. and is supported on smooth bearings at *A* and *B*, which allow free rotation of the shaft. The gears *C* and *D* fixed to the shaft remove 25 hp and 15 hp, respectively. Determine the absolute maximum stress in the shaft and the angle of twist of gear *C* with respect to gear *D*.



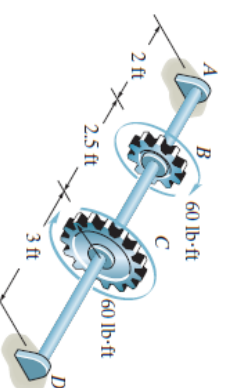
L7A\_Cơ học vật liệu (215004)

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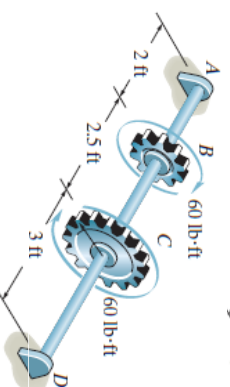


## Chương 5: Bài tập\_XOẮN

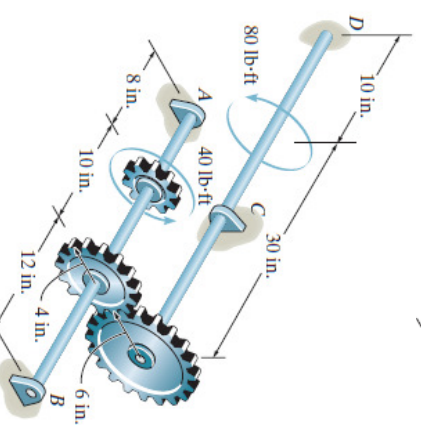
5-59. The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at *A* and *D*, which allow free rotation. Determine the angle of twist of *B* with respect to *D*.



\*5-60. The shaft is made of A-36 steel. It has a diameter of 1 in. and is supported by bearings at *A* and *D*, which allow free rotation. Determine the angle of twist of gear *C* with respect to *B*.



•5-61. The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at *A*, *B*, and *C*, which allow free rotation. If the support at *D* is fixed, determine the angle of twist of end *B* when the torques are applied to the assembly as shown.



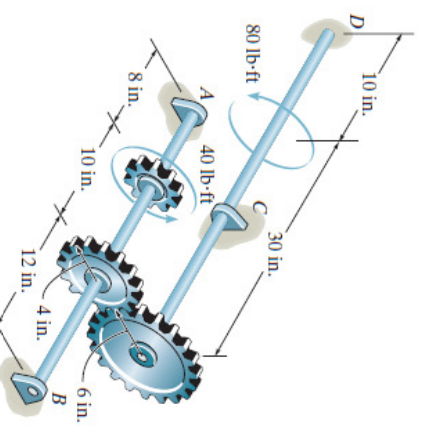
L7A\_Cơ học vật liệu (215004)

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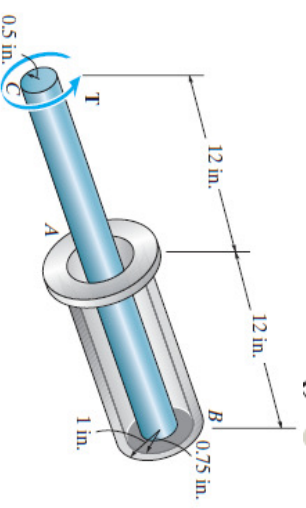


## Chương 5: Bài tập\_XOẮN

**5-62.** The two shafts are made of A-36 steel. Each has a diameter of 1 in., and they are supported by bearings at *A*, *B*, and *C*, which allow free rotation. If the support at *D* is fixed, determine the angle of twist of end *A* when the torques are applied to the assembly as shown.



**5-63.** The device serves as a compact torsion spring. It is made of A-36 steel and consists of a solid inner shaft *CB* which is surrounded by and attached to a tube *AB* using a rigid ring at *B*. The ring at *A* can also be assumed rigid and is fixed from rotating. If a torque of  $T = 2 \text{ kip} \cdot \text{in.}$  is applied to the shaft, determine the angle of twist at the end *C* and the maximum shear stress in the tube and shaft.



LTA\_Cơ học vật liệu (215004)

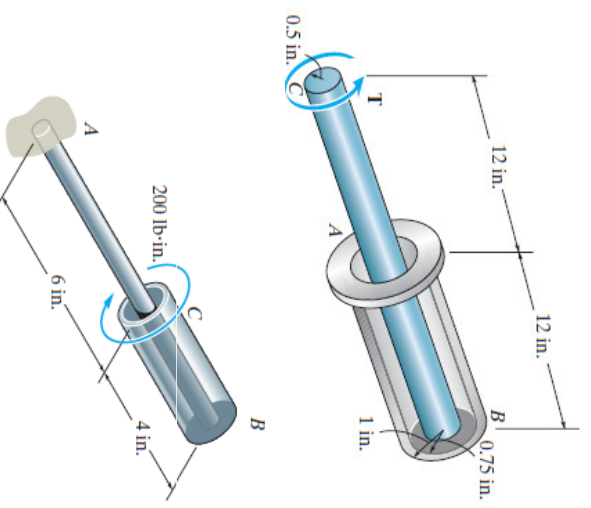
23



## Chương 5: Bài tập\_XOẮN

**\*5-64.** The device serves as a compact torsion spring. It is made of A-36 steel and consists of a solid inner shaft *CB* which is surrounded by and attached to a tube *AB* using a rigid ring at *B*. The ring at *A* can also be assumed rigid and is fixed from rotating. If the allowable shear stress for the material is  $\tau_{\text{allow}} = 12 \text{ ksi}$  and the angle of twist at *C* is limited to  $\phi_{\text{allow}} = 3^\circ$ , determine the maximum torque *T* that can be applied at the end *C*.

**\*5-65.** The A-36 steel assembly consists of a tube having an outer radius of 1 in. and a wall thickness of 0.125 in. Using a rigid plate at *B*, it is connected to the solid 1-in.-diameter shaft *AB*. Determine the rotation of the tube's end *C* if a torque of  $200 \text{ lb} \cdot \text{in.}$  is applied to the tube at this end. The end *A* of the shaft is fixed supported.



LTA\_Cơ học vật liệu (215004)

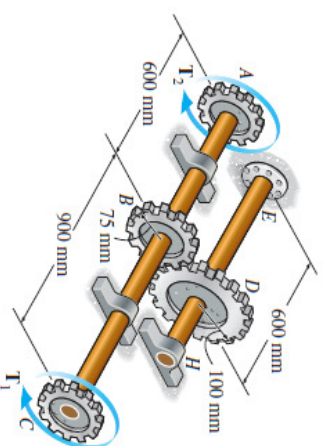
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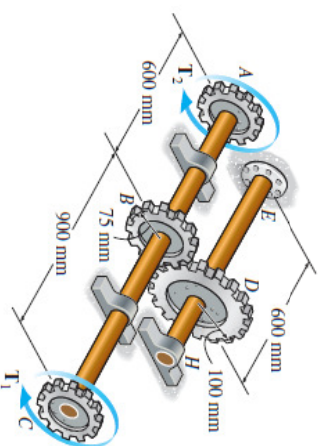


## Chương 5: Bài tập\_XOẮN

5-66. The 60-mm diameter shaft  $ABC$  is supported by two journal bearings, while the 80-mm diameter shaft  $EH$  is fixed at  $E$  and supported by a journal bearing at  $H$ . If  $T_1 = 2 \text{ kN} \cdot \text{m}$  and  $T_2 = 4 \text{ kN} \cdot \text{m}$ , determine the angle of twist of gears  $A$  and  $C$ . The shafts are made of A-36 steel.

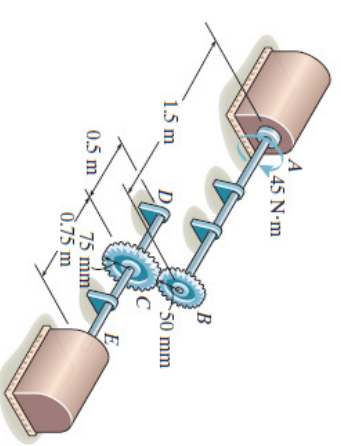


5-67. The 60-mm diameter shaft  $ABC$  is supported by two journal bearings, while the 80-mm diameter shaft  $EH$  is fixed at  $E$  and supported by a journal bearing at  $H$ . If the angle of twist at gears  $A$  and  $C$  is required to be  $0.04 \text{ rad}$ , determine the magnitudes of the torques  $T_1$  and  $T_2$ . The shafts are made of A-36 steel.

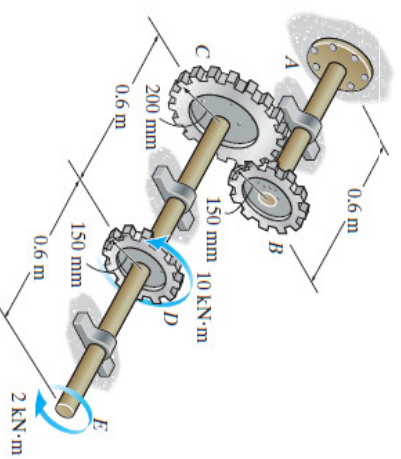


## Chương 5: Bài tập\_XOẮN

\*5-68. The 30-mm-diameter shafts are made of L2 tool steel and are supported on journal bearings that allow the shaft to rotate freely. If the motor at  $A$  develops a torque of  $T = 45 \text{ N} \cdot \text{m}$  on the shaft  $AB$ , while the turbine at  $E$  is fixed from turning, determine the amount of rotation of gears  $B$  and  $C$ .



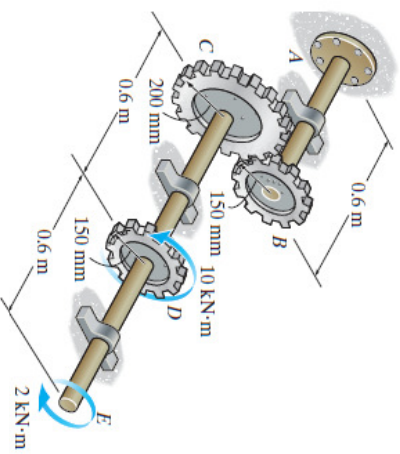
•5-69. The shafts are made of A-36 steel and each has a diameter of 80 mm. Determine the angle of twist at end  $E$ .



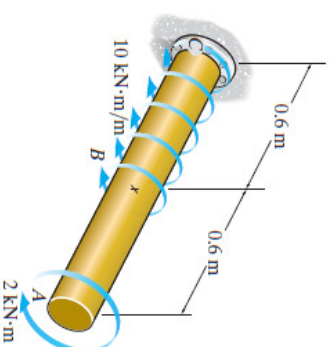


## Chương 5: Bài tập\_XOẮN

5-70. The shafts are made of A-36 steel and each has a diameter of 80 mm. Determine the angle of twist of gear  $D$ .



\*5-72. The 80-mm diameter shaft is made of 6061-T6 aluminum alloy and subjected to the torsional loading shown. Determine the angle of twist at end  $A$ .



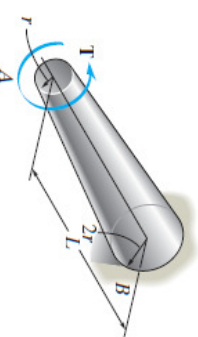
L7A\_Cơ học vật liệu (215004)

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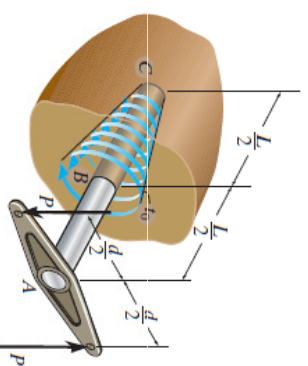


## Chương 5: Bài tập\_XOẮN

•5-73. The tapered shaft has a length  $L$  and a radius  $r$  at end  $A$  and  $2r$  at end  $B$ . If it is fixed at end  $B$  and is subjected to a torque  $T$ , determine the angle of twist of end  $A$ . The shear modulus is  $G$ .



5-74. The rod  $ABC$  of radius  $c$  is embedded into a medium where the distributed torque reaction varies linearly from zero at  $C$  to  $t_0$  at  $B$ . If couple forces  $P$  are applied to the lever arm, determine the value of  $t_0$  for equilibrium. Also, find the angle of twist of end  $A$ . The rod is made from material having a shear modulus of  $G$ .



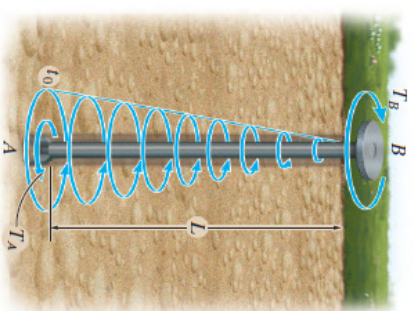
L7A\_Cơ học vật liệu (215004)

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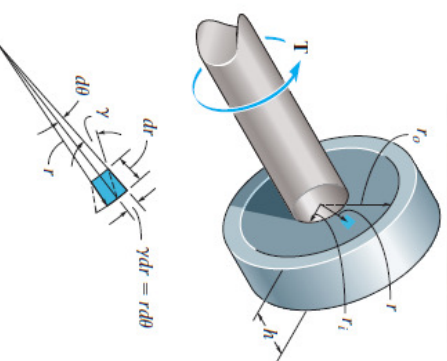


## Chương 5: Bài tập\_XOẮN

5-75. When drilling a well, the deep end of the drill pipe is assumed to encounter a torsional resistance  $T_A$ . Furthermore, soil friction along the sides of the pipe creates a linear distribution of torque per unit length, varying from zero at the surface  $B$  to  $t_0$  at  $A$ . Determine the necessary torque  $T_B$  that must be supplied by the drive unit to turn the pipe. Also, what is the relative angle of twist of one end of the pipe with respect to the other end at the instant the pipe is about to turn? The pipe has an outer radius  $r_o$  and an inner radius  $r_i$ . The shear modulus is  $G$ .



\*5-76. A cylindrical spring consists of a rubber annulus bonded to a rigid ring and shaft. If the ring is held fixed and a torque  $T$  is applied to the rigid shaft, determine the angle of twist of the shaft. The shear modulus of the rubber is  $G$ . *Hint:* As shown in the figure, the deformation of the element at radius  $r$  can be determined from  $r d\theta = dr \gamma$ . Use this expression along with  $\tau = T/(2\pi r^2 h)$  from Prob. 5-26, to obtain the result.



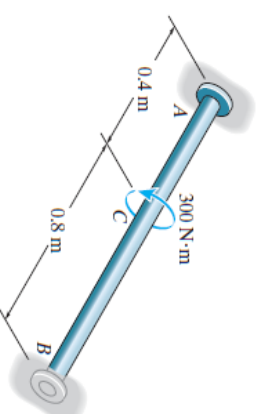
L7A\_Cơ học vật liệu (215004)

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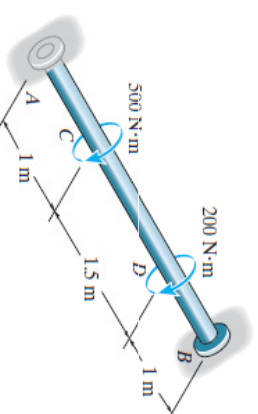


## Chương 5: Bài tập\_XOẮN

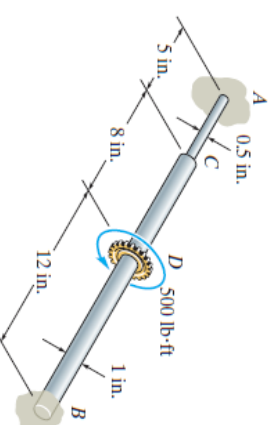
•5-77. The A-36 steel shaft has a diameter of 50 mm and is fixed at its ends  $A$  and  $B$ . If it is subjected to the torque, determine the maximum shear stress in regions  $AC$  and  $CB$  of the shaft.



5-78. The A-36 steel shaft has a diameter of 60 mm and is fixed at its ends  $A$  and  $B$ . If it is subjected to the torques shown, determine the absolute maximum shear stress in the shaft.



5-79. The steel shaft is made from two segments:  $AC$  has a diameter of 0.5 in. and  $CB$  has a diameter of 1 in. If it is fixed at its ends  $A$  and  $B$  and subjected to a torque of determine the maximum shear stress in the shaft.  $G_{st} = 10.8(10^3)$  ksi.



L7A\_Cơ học vật liệu (215004)

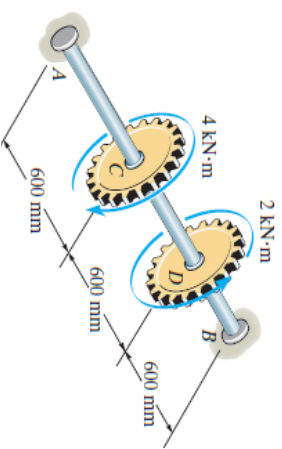
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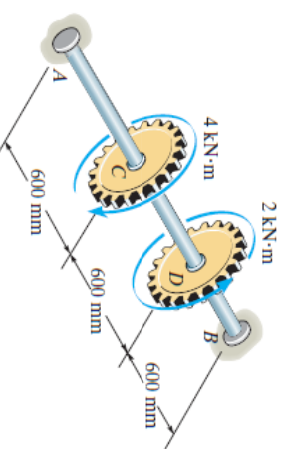


## Chương 5: Bài tập\_XOẮN

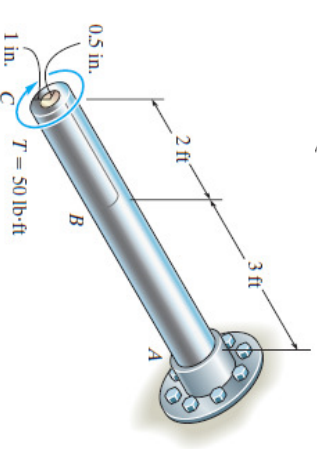
\*5–80. The shaft is made of A-36 steel, has a diameter of 80 mm, and is fixed at *B* while *A* is loose and can rotate 0.005 rad before becoming fixed. When the torques are applied to *C* and *D*, determine the maximum shear stress in regions *AC* and *CD* of the shaft.



•5–81. The shaft is made of A-36 steel and has a diameter of 80 mm. It is fixed at *B* and the support at *A* has a torsional stiffness of  $k = 0.5 \text{ MN} \cdot \text{m/rad}$ . If it is subjected to the gear torques shown, determine the absolute maximum shear stress in the shaft.



5–82. The shaft is made from a solid steel section *AB* and a tubular portion made of steel and having a brass core. If it is fixed to a rigid support at *A*, and a torque of  $T = 50 \text{ lb} \cdot \text{ft}$  is applied to it at *C*, determine the angle of twist that occurs at *C* and compute the maximum shear stress and maximum shear strain in the brass and steel. Take  $G_{st} = 11.5(10^3) \text{ ksi}$ ,  $G_{br} = 5.6(10^3) \text{ ksi}$ .



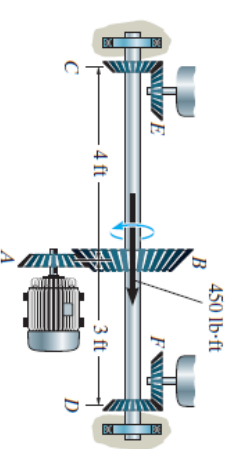
L7A\_Cơ học vật liệu (215004)

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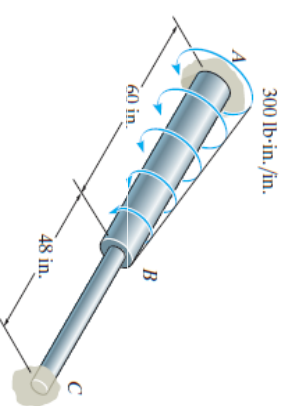


## Chương 5: Bài tập\_XOẮN

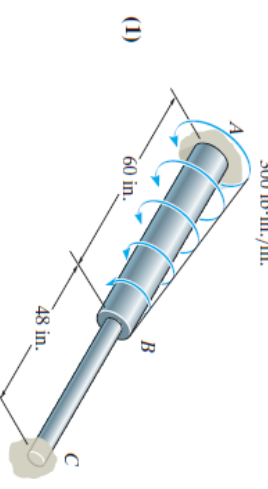
5–83. The motor *A* develops a torque at gear *B* of  $450 \text{ lb} \cdot \text{ft}$ , which is applied along the axis of the 2-in.-diameter steel shaft *CD*. This torque is to be transmitted to the pinion gears at *E* and *F*. If these gears are temporarily fixed, determine the maximum shear stress in segments *CB* and *BD* of the shaft. Also, what is the angle of twist of each of these segments? The bearings at *C* and *D* only exert force reactions on the shaft and do not resist torque;  $G_{st} = 12(10^3) \text{ ksi}$ .



\*5–84. A portion of the A-36 steel shaft is subjected to a linearly distributed torsional loading. If the shaft has the dimensions shown, determine the reactions at the fixed supports *A* and *C*. Segment *AB* has a diameter of 1.5 in. and segment *BC* has a diameter of 0.75 in.



•5–85. Determine the rotation of joint *B* and the absolute maximum shear stress in the shaft in Prob. 5–84.



L7A\_Cơ học vật liệu (215004)

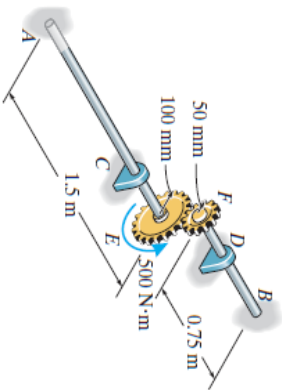
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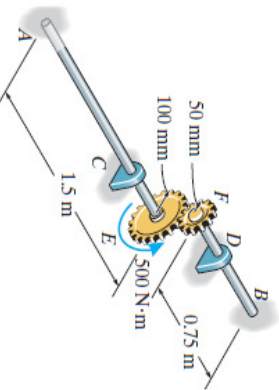


## Chương 5: Bài tập\_XOẮN

5-86. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at  $A$  and  $B$ . They are also supported by journal bearings at  $C$  and  $D$ , which allow free rotation of the shafts along their axes. If a torque of  $500 \text{ N} \cdot \text{m}$  is applied to the gear at  $E$  as shown, determine the reactions at  $A$  and  $B$ .

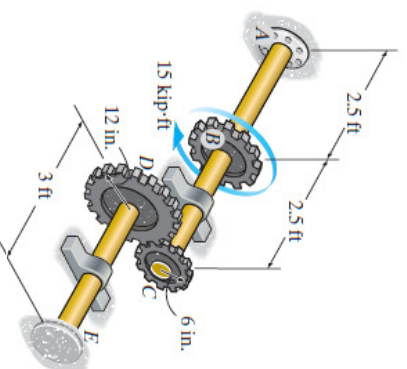


5-87. Determine the rotation of the gear at  $E$  in Prob. 5-86.

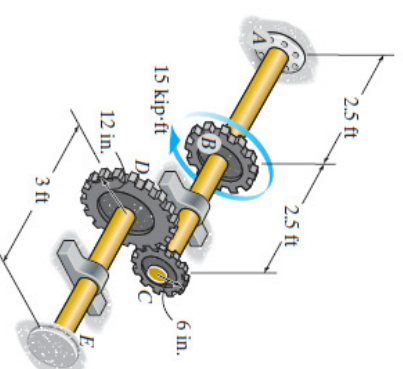


## Chương 5: Bài tập\_XOẮN

\*5-88. The shafts are made of A-36 steel and have the same diameter of 4 in. If a torque of 15 kip · ft is applied to gear  $B$ , determine the absolute maximum shear stress developed in the shaft.



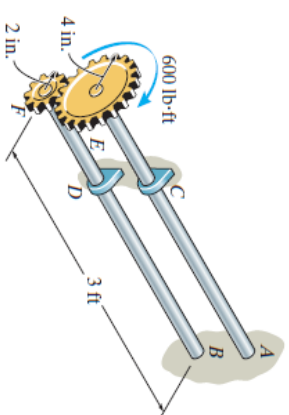
•5-89. The shafts are made of A-36 steel and have the same diameter of 4 in. If a torque of 15 kip · ft is applied to gear  $B$ , determine the angle of twist of gear  $B$ .



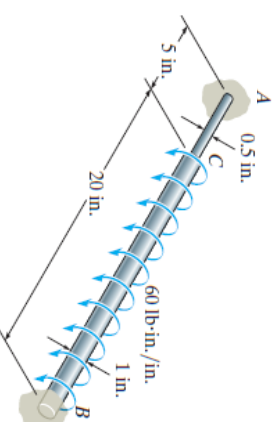


## Chương 5: Bài tập\_XOẮN

**5-90.** The two 3-ft-long shafts are made of 2014-T6 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at *A* and *B*. They are also supported by bearings at *C* and *D*, which allow free rotation of the shafts along their axes. If a torque of 600 lb·ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.

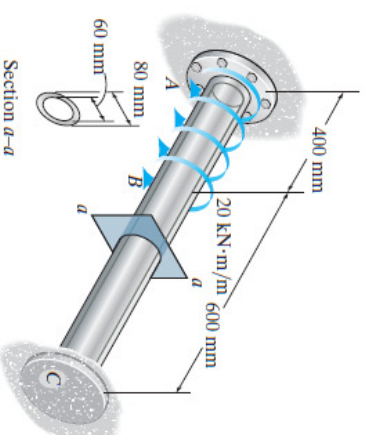


**5-91.** The A-36 steel shaft is made from two segments: *AC* has a diameter of 0.5 in. and *CB* has a diameter of 1 in. If the shaft is fixed at its ends *A* and *B* and subjected to a uniform distributed torque of 60 lb·in./in. along segment *CB*, determine the absolute maximum shear stress in the shaft.

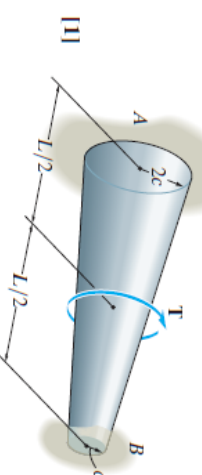


## Chương 5: Bài tập\_XOẮN

**\*5-92.** If the shaft is subjected to a uniform distributed torque of  $t = 20 \text{ kN} \cdot \text{m/m}$ , determine the maximum shear stress developed in the shaft. The shaft is made of 2014-T6 aluminum alloy and is fixed at *A* and *C*.



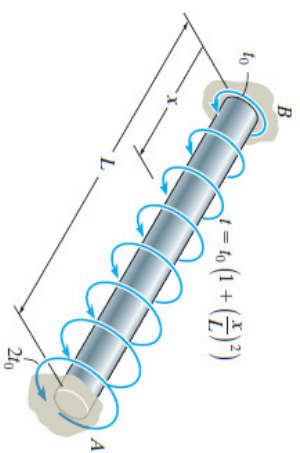
**\*5-93.** The tapered shaft is confined by the fixed supports at *A* and *B*. If a torque **T** is applied at its mid-point, determine the reactions at the supports.



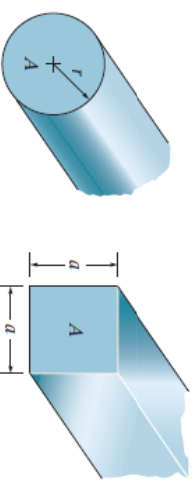


## Chương 5: Bài tập\_XOẮN

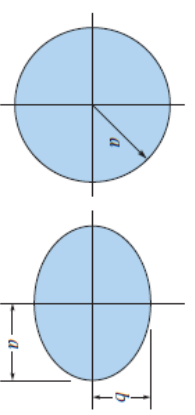
5-94. The shaft of radius  $c$  is subjected to a distributed torque  $t$ , measured as torque/length of shaft. Determine the reactions at the fixed supports  $A$  and  $B$ .



5-95. Compare the values of the maximum elastic shear stress and the angle of twist developed in 304 stainless steel shafts having circular and square cross sections. Each shaft has the same cross-sectional area of  $9 \text{ in}^2$ , length of  $36 \text{ in}$ , and is subjected to a torque of  $4000 \text{ lb} \cdot \text{in}$ .



\*5-96. If  $a = 25 \text{ mm}$  and  $b = 15 \text{ mm}$ , determine the maximum shear stress in the circular and elliptical shafts when the applied torque is  $T = 80 \text{ N} \cdot \text{m}$ . By what percentage is the shaft of circular cross section more efficient at withstanding the torque than the shaft of elliptical cross section?



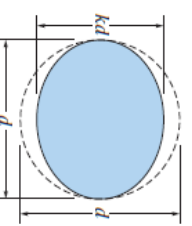
L7A\_Cơ học vật liệu (215004)

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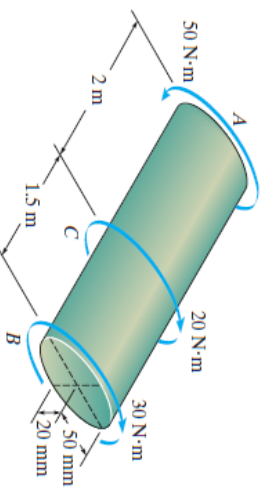
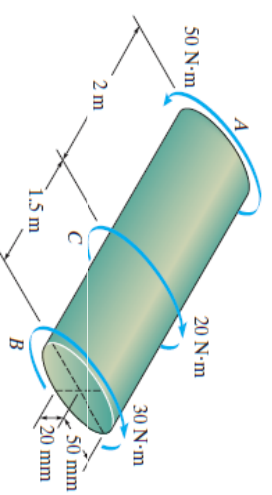


## Chương 5: Bài tập\_XOẮN

•5-97. It is intended to manufacture a circular bar to resist torque; however, the bar is made elliptical in the process of manufacturing, with one dimension smaller than the other by a factor  $k$  as shown. Determine the factor by which the maximum shear stress is increased.



5-98. The shaft is made of red brass C83400 and has an elliptical cross section. If it is subjected to the torsional loading shown, determine the maximum shear stress within regions  $AC$  and  $BC$ , and the angle of twist  $\phi$  of end  $B$  relative to end  $A$ .



5-99. Solve Prob. 5-98 for the maximum shear stress within regions  $AC$  and  $BC$ , and the angle of twist  $\phi$  of end  $B$  relative to  $C$ .

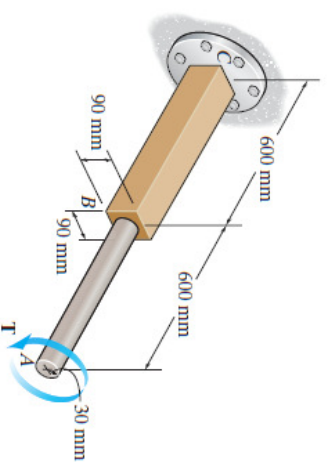
L7A\_Cơ học vật liệu (215004)

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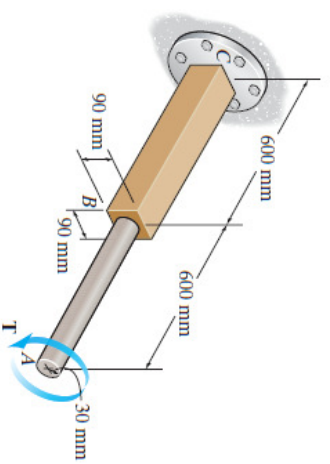


## Chương 5: Bài tập\_XOẮN

\*5-100. Segments  $AB$  and  $BC$  of the shaft have circular and square cross sections, respectively. If end  $A$  is subjected to a torque of  $T = 2 \text{ kN} \cdot \text{m}$ , determine the absolute maximum shear stress developed in the shaft and the angle of twist of end  $A$ . The shaft is made from A-36 steel and is fixed at  $C$ .



•5-101. Segments  $AB$  and  $BC$  of the shaft have circular and square cross sections, respectively. The shaft is made from A-36 steel with an allowable shear stress of  $\tau_{\text{allow}} = 75 \text{ MPa}$ , and an angle of twist at end  $A$  which is not allowed to exceed  $0.02 \text{ rad}$ . Determine the maximum allowable torque  $T$  that can be applied at end  $A$ . The shaft is fixed at  $C$ .



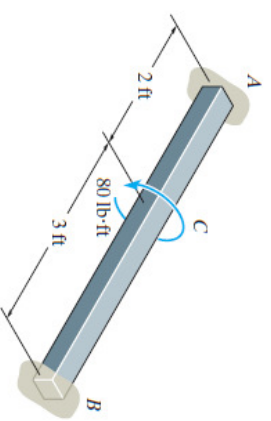
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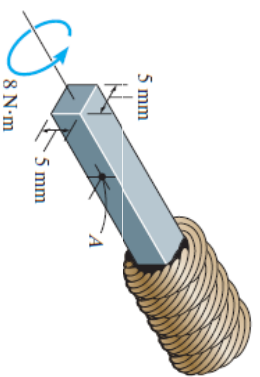


## Chương 5: Bài tập\_XOẮN

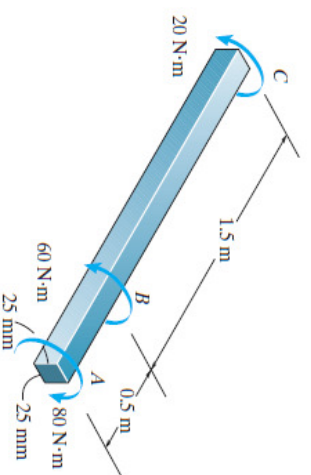
5-102. The aluminum strut is fixed between the two walls at  $A$  and  $B$ . If it has a 2 in. by 2 in. square cross section, and it is subjected to the torque of  $80 \text{ lb} \cdot \text{ft}$  at  $C$ , determine the reactions at the fixed supports. Also, what is the angle of twist at  $C$ ?  $G_{\text{al}} = 3.8(10^3) \text{ ksi}$ .



5-103. The square shaft is used at the end of a drive cable in order to register the rotation of the cable on a gauge. If it has the dimensions shown and is subjected to a torque of  $8 \text{ N} \cdot \text{m}$ , determine the shear stress in the shaft at point  $A$ . Sketch the shear stress on a volume element located at this point.



\*5-104. The 6061-T6 aluminum bar has a square cross section of 25 mm by 25 mm. If it is 2 m long, determine the maximum shear stress in the bar and the rotation of one end relative to the other end.



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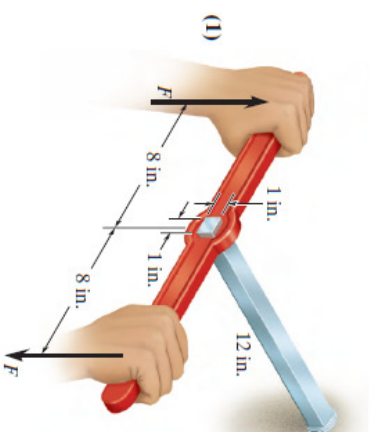
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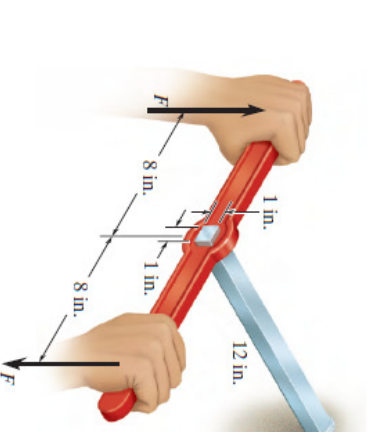


## Chương 5: Bài tập\_XOẮN

- 5-105.** The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the largest couple forces  $F$  that can be applied to the shaft without causing the steel to yield.  $\tau_y = 8$  ksi.



- 5-106.** The steel shaft is 12 in. long and is screwed into the wall using a wrench. Determine the maximum shear stress in the shaft and the amount of displacement that each couple force undergoes if the couple forces have a magnitude of  $F = 30$  lb,  $G_{st} = 10.8(10^3)$  ksi.



L7A\_Cơ học vật liệu (215004)

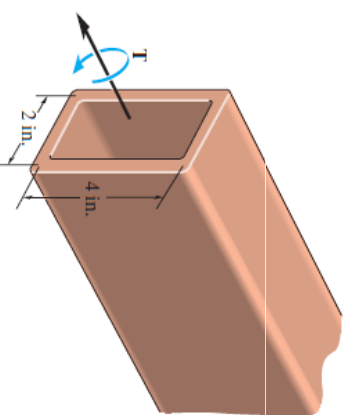
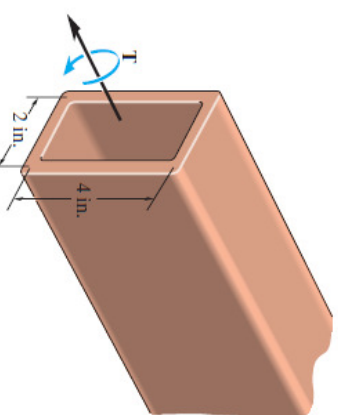
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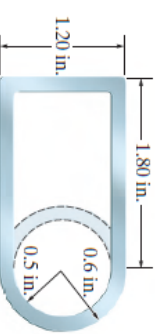
## Chương 5: Bài tập\_XOẮN

- 5-107.** Determine the constant thickness of the rectangular tube if the average shear stress is not to exceed 12 ksi when a torque of  $T = 20$  kip  $\cdot$  in. is applied to the tube. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown.

- \*5-108.** Determine the torque  $T$  that can be applied to the rectangular tube if the average shear stress is not to exceed 12 ksi. Neglect stress concentrations at the corners. The mean dimensions of the tube are shown and the tube has a thickness of 0.125 in.



- 5-109.** For a given maximum shear stress, determine the factor by which the torque carrying capacity is increased if the half-circular section is reversed from the dashed-line position to the section shown. The tube is 0.1 in. thick.



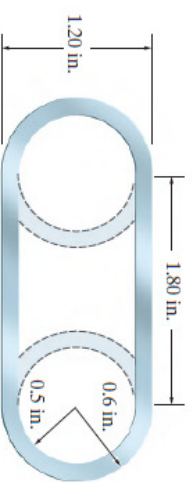
L7A\_Cơ học vật liệu (215004)

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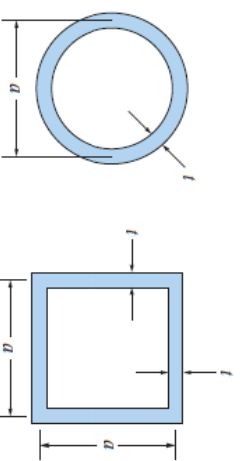


## Chương 5: Bài tập\_XOẮN

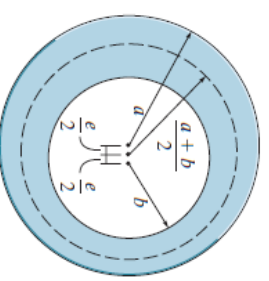
5-110. For a given average shear stress, determine the factor by which the torque-carrying capacity is increased if the half-circular sections are reversed from the dashed-line positions to the section shown. The tube is 0.1 in. thick.



5-111. A torque  $T$  is applied to two tubes having the cross sections shown. Compare the shear flow developed in each tube.



\*5-112. Due to a fabrication error the inner circle of the tube is eccentric with respect to the outer circle. By what percentage is the torsional strength reduced when the eccentricity  $e$  is one-fourth of the difference in the radii?



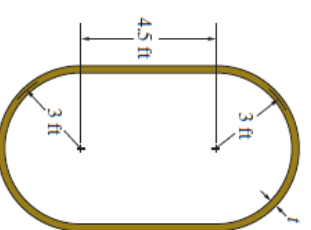
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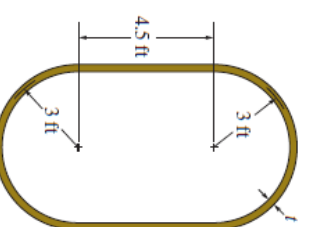


## Chương 5: Bài tập\_XOẮN

•5-113. The mean dimensions of the cross section of an airplane fuselage are shown. If the fuselage is made of 2014-T6 aluminum alloy having allowable shear stress of  $\tau_{\text{allow}} = 18$  ksi, and it is subjected to a torque of 6000 kip  $\cdot$  ft, determine the required minimum thickness  $t$  of the cross section to the nearest 1/16 in. Also, find the corresponding angle of twist per foot length of the fuselage.



5-114. The mean dimensions of the cross section of an airplane fuselage are shown. If the fuselage is made from 2014-T6 aluminum alloy having an allowable shear stress of  $\tau_{\text{allow}} = 18$  ksi and the angle of twist per foot length of fuselage is not allowed to exceed 0.001 rad/ft, determine the maximum allowable torque that can be sustained by the fuselage. The thickness of the wall is  $t = 0.25$  in.



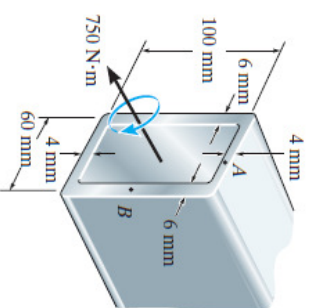
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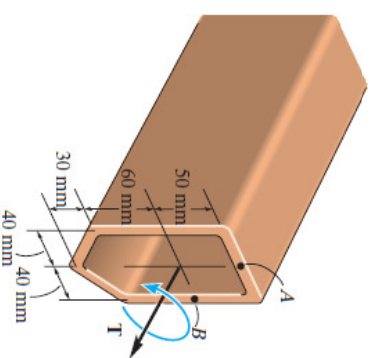


## Chương 5: Bài tập\_XOẮN

5-115. The tube is subjected to a torque of  $750 \text{ N} \cdot \text{m}$ . Determine the average shear stress in the tube at points  $A$  and  $B$ .



\*5-116. The tube is made of plastic, is 5 mm thick, and has the mean dimensions shown. Determine the average shear stress at points  $A$  and  $B$  if it is subjected to the torque of  $T = 5 \text{ N} \cdot \text{m}$ . Show the shear stress on volume elements located at these points.



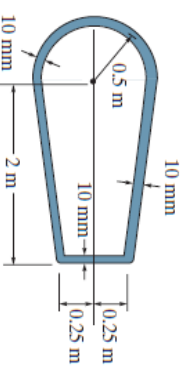
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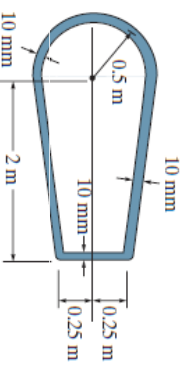


## Chương 5: Bài tập\_XOẮN

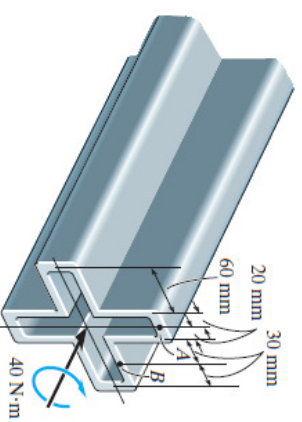
•5-117. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is made of 2014-T6 aluminum alloy having an allowable shear stress of  $\tau_{\text{allow}} = 125 \text{ MPa}$  and the wall thickness is 10 mm, determine the maximum allowable torque and the corresponding angle of twist per meter length of the wing.



5-118. The mean dimensions of the cross section of the leading edge and torsion box of an airplane wing can be approximated as shown. If the wing is subjected to a torque of  $4.5 \text{ MN} \cdot \text{m}$  and the wall thickness is 10 mm, determine the average shear stress developed in the wing and the angle of twist per meter length of the wing. The wing is made of 2014-T6 aluminum alloy.



5-119. The symmetric tube is made from a high-strength steel, having the mean dimensions shown and a thickness of 5 mm. If it is subjected to a torque of  $T = 40 \text{ N} \cdot \text{m}$ , determine the average shear stress developed at points  $A$  and  $B$ . Indicate the shear stress on volume elements located at these points.



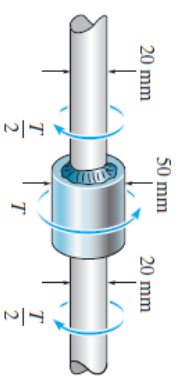
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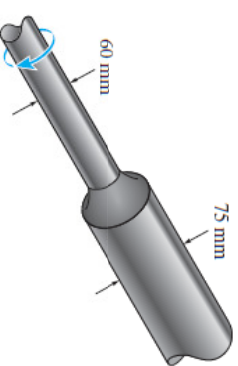


## Chương 5: Bài tập\_XOẮN

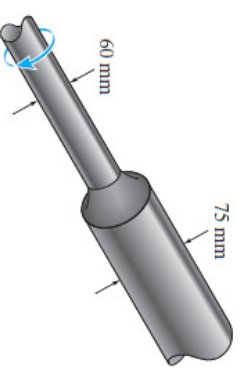
\*5-120. The steel used for the shaft has an allowable shear stress of  $\tau_{\text{allow}} = 8 \text{ MPa}$ . If the members are connected with a fillet weld of radius  $r = 4 \text{ mm}$ , determine the maximum torque  $T$  that can be applied.



•5-121. The built-up shaft is to be designed to rotate at 720 rpm while transmitting 30 kW of power. Is this possible? The allowable shear stress is  $\tau_{\text{allow}} = 12 \text{ MPa}$ .



5-122. The built-up shaft is designed to rotate at 540 rpm. If the radius of the fillet weld connecting the shafts is  $r = 7.20 \text{ mm}$ , and the allowable shear stress for the material is  $\tau_{\text{allow}} = 55 \text{ MPa}$ , determine the maximum power the shaft can transmit.



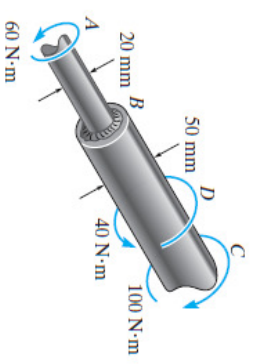
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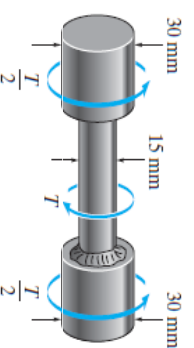


## Chương 5: Bài tập\_XOẮN

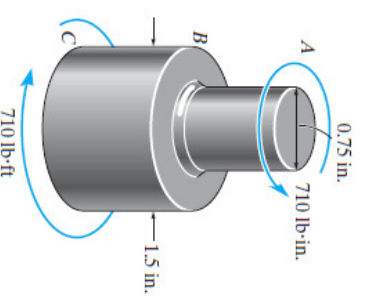
5-123. The steel shaft is made from two segments:  $AB$  and  $BC$ , which are connected using a fillet weld having a radius of 2.8 mm. Determine the maximum shear stress developed in the shaft.



\*5-124. The steel used for the shaft has an allowable shear stress of  $\tau_{\text{allow}} = 8 \text{ MPa}$ . If the members are connected together with a fillet weld of radius  $r = 2.25 \text{ mm}$ , determine the maximum torque  $T$  that can be applied.



•5-125. The assembly is subjected to a torque of  $710 \text{ lb} \cdot \text{in}$ . If the allowable shear stress for the material is  $\tau_{\text{allow}} = 12 \text{ ksi}$ , determine the radius of the smallest size fillet that can be used to transmit the torque.



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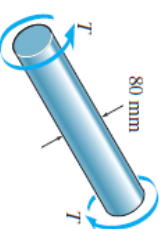
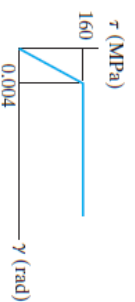
## Chương 5: Bài tập\_XOẮN

5-126. A solid shaft is subjected to the torque  $T$ , which causes the material to yield. If the material is elastic-plastic, show that the torque can be expressed in terms of the angle of twist  $\phi$  of the shaft as  $T = \frac{4}{3}T_Y(1 - \phi^3/4\phi^3)$ , where  $T_Y$  and  $\phi_Y$  are the torque and angle of twist when the material begins to yield.

5-127. A solid shaft having a diameter of 2 in. is made of elastic-plastic material having a yield stress of  $\tau_Y = 16$  ksi and shear modulus of  $G = 12(10^3)$  ksi. Determine the torque required to develop an elastic core in the shaft having a diameter of 1 in. Also, what is the plastic torque?

\*5-128. Determine the torque needed to twist a short 3-mm-diameter steel wire through several revolutions if it is made from steel assumed to be elastic-plastic and having a yield stress of  $\tau_Y = 80$  MPa. Assume that the material becomes fully plastic.

•5-129. The solid shaft is made of an elastic-perfectly plastic material as shown. Determine the torque  $T$  needed to form an elastic core in the shaft having a radius of  $\rho_Y = 20$  mm. If the shaft is 3 m long, through what angle does one end of the shaft twist with respect to the other end? When the torque is removed, determine the residual stress distribution in the shaft and the permanent angle of twist.



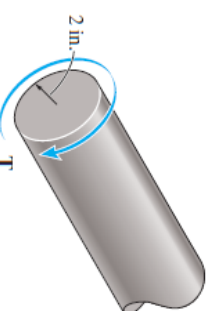
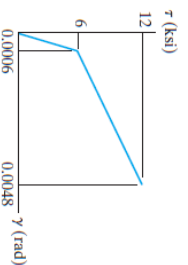
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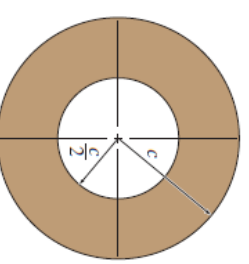
## Chương 5: Bài tập\_XOẮN

5-130. The shaft is subjected to a maximum shear strain of 0.0048 rad. Determine the torque applied to the shaft if the material has strain hardening as shown by the shear stress-strain diagram.



5-131. An 80-mm diameter solid circular shaft is made of an elastic-perfectly plastic material having a yield shear stress of  $\tau_Y = 125$  MPa. Determine (a) the maximum elastic torque  $T_Y$ ; and (b) the plastic torque  $T_p$ .

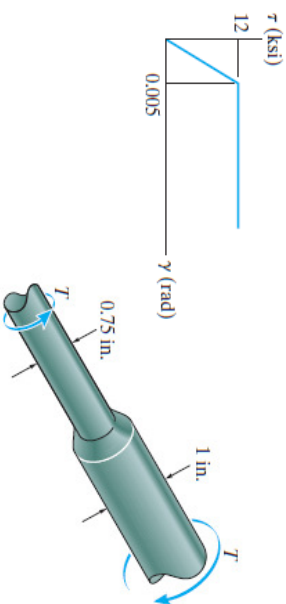
\*5-132. The hollow shaft has the cross section shown and is made of an elastic-perfectly plastic material having a yield shear stress of  $\tau_Y$ . Determine the ratio of the plastic torque  $T_p$  to the maximum elastic torque  $T_Y$ .



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**5-133.** The shaft consists of two sections that are rigidly connected. If the material is elastic plastic as shown, determine the largest torque  $T$  that can be applied to the shaft. Also, draw the shear-stress distribution over a radial line for each section. Neglect the effect of stress concentration.



**5-134.** The hollow shaft is made of an elastic-perfectly plastic material having a shear modulus of  $G$  and a yield shear stress of  $\tau_y$ . Determine the applied torque  $\mathbf{T}_p$  when the material of the inner surface is about to yield (plastic torque). Also, find the corresponding angle of twist and the maximum shear strain. The shaft has a length of  $L$ .

