



4.2 Time – Harmonic Fields

a) Introduction :

- ❖ Time – harmonic Field varies sinusoidal with time.

$$\begin{aligned}\vec{E}(x,y,z,t) = & E_{mx}(x,y,z) \cos[\omega t + \psi_x(x,y,z)] \vec{a}_x \\ & + E_{my}(x,y,z) \cos[\omega t + \psi_y(x,y,z)] \vec{a}_y \\ & + E_{mz}(x,y,z) \cos[\omega t + \psi_z(x,y,z)] \vec{a}_z\end{aligned}$$

- ❖ Time – harmonic Field : practical value.

➡ If not, Fourier techniques can be used .

b) The phasor:

❖ The phasor is defined: complex function

Time domain: $\vec{E} = E_{\text{mx}}(z) \cos[\omega t + \psi_x(z)] \vec{a}_x$

Phasor domain: $\dot{\vec{E}} = E_{\text{mx}}(z) \cdot e^{j\psi(z)} \vec{a}_x = E_{\text{mx}}(z) \angle \psi(z) \cdot \vec{a}_x$

cuu duong than cong . com

❖ Relation between the field and the phasor :

$$\dot{\vec{E}}(z) \Leftrightarrow \vec{E}(z,t) = \text{Re} \{ \dot{\vec{E}}(z) * e^{j\omega t} \}$$

cuu duong than cong . com

❖ Property : $\frac{\partial \vec{E}(z,t)}{\partial t} \Leftrightarrow j\omega * \dot{\vec{E}}(z)$

c) Maxwell's equations in phasor form:

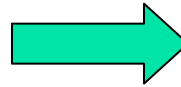
❖ In material media : $\sigma, \epsilon, \mu = \text{const}$, Maxwell's equations :

$$\text{rot } \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

$$\text{rot } \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$$

$$\text{div } \vec{E} = \rho_v / \epsilon$$

$$\text{div } \vec{H} = 0$$



$$\text{rot } \dot{\vec{H}} = (\sigma + j\omega\epsilon) \dot{\vec{E}}$$

$$\text{rot } \dot{\vec{E}} = -j\omega\mu \dot{\vec{H}}$$

$$\text{div } \dot{\vec{E}} = \dot{\rho}_v / \epsilon$$

$$\text{div } \dot{\vec{H}} = 0$$

❖ And constitutive relations :

$$\dot{\vec{J}} = \sigma \dot{\vec{E}} ; \dot{\vec{D}} = \epsilon \dot{\vec{E}} ; \dot{\vec{B}} = \mu \dot{\vec{H}}$$

❖ Example1: Maxwell's equations in phasor

In a medium characterized by $\sigma = 0$, $\mu = \mu_0$, ϵ_0 , and

$$\vec{E} = 20 \sin(10^8 t - \beta z) \vec{a}_y \text{ V/m}$$

calculate β and \vec{H} .

○ Method 1: Solve directly in time domain (see 1.7) .

○ Method 2: Using phasors : $\vec{E}(z,t) \rightarrow \dot{\vec{E}} = 20.e^{-j\beta z} \vec{a}_y \text{ (V/m)}$

$$\rightarrow \text{rot } \dot{\vec{E}} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 20.e^{-j\beta z} & 0 \end{vmatrix} = j20\beta.e^{-j\beta z} \vec{a}_x$$

$$\rightarrow \dot{\vec{H}} = -\frac{1}{j\omega\mu_0} \text{rot } \dot{\vec{E}} = -\frac{20\beta}{\omega\mu_0}.e^{-j\beta z} \vec{a}_x$$

❖ Example1: Maxwell's equations in phasor

$$\rightarrow \text{rot} \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -20\beta \cdot e^{-j\beta z} / \omega\mu_0 & 0 & 0 \end{vmatrix} = \frac{j20\beta^2}{\omega\mu_0} \cdot e^{-j\beta z} \vec{a}_y$$

Notice that $\text{rot} \vec{H} = j\omega\epsilon_0 \vec{E} = j\omega\epsilon_0 20 \cdot e^{-j\beta z} \vec{a}_y$

$$\rightarrow \beta = \omega \sqrt{\mu_0 \epsilon_0} = 10^8 / 3 \cdot 10^8 = 1 / 3$$

$$\rightarrow \vec{H} = -\frac{1}{2\pi} \cdot e^{-jz/3} \vec{a}_x$$

$$\rightarrow \vec{H}(z, t) = -\frac{1}{2\pi} \cdot \cos(10^8 t - z / 3) \vec{a}_x \text{ (A/m)}$$