SOLID MECHANICS

Chapter 2: Deformation: Displacements and strain

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2.1 General deformations

2.2 Geometric construction of small deformation theory

2.3 Strain transformation

2.4 Principal strains

2.5 Spherical & Deviatoric strains

2.6 Strain compatibility

2.7 Curvilinear strain-displacement relations cylindrical coordinates

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Deformations: non-homogeneous

An elastic solid is said to be deformed or strained when the relative displacements between points in the body are changed.

This is in contrast to rigid-body motion where the distance between points remains the same.

Fig 1. Rigid-body motion

Fig 2. Deformed or strained

Small Deformation Theory

- Consider two neighboring material points P_0 and P connected with the *relative position vector* ***r*** as shown in Fig 3.
- Through a general deformation, these points are mapped to locations P'_0 and P' in the deformed configuration.
- In linear elasticity, only small deformation theory is necessary.

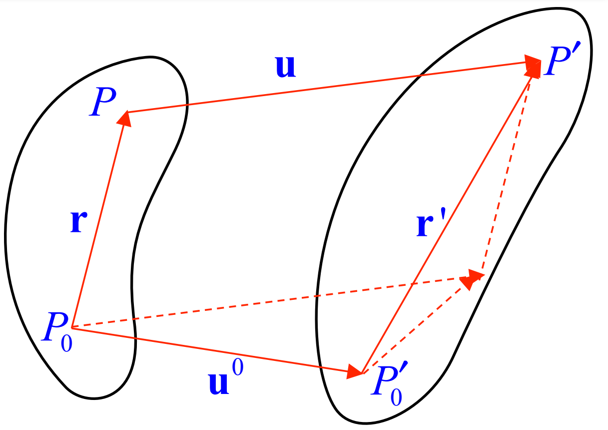

Fig 3. General deformation between two neighboring points

Taylor series expansion around point P_0 to express the components of **u** as

$$f(u_i+\Delta u_i)=f(u_i)+f_{,u_i}(u_i)\Delta u_i$$

$$u=u^o+\frac{\partial u}{\partial x}r_x+\frac{\partial u}{\partial y}r_y+\frac{\partial u}{\partial z}r_z$$
$$v=v^o+\frac{\partial v}{\partial x}r_x+\frac{\partial v}{\partial y}r_y+\frac{\partial v}{\partial z}r_z$$
$$w=w^o+\frac{\partial w}{\partial x}r_x+\frac{\partial w}{\partial y}r_y+\frac{\partial w}{\partial z}r_z$$

The change in the relative position vector **r**

$$\Delta\mathbf{r}=\mathbf{r}'-\mathbf{r}$$

prove

$$\Delta r_i=u_{i,j}r_j$$

Small Deformation Theory

$$f\left(u_i+\Delta u_i\right)=f\left(u_i\right)+f_{,u_i}\left(u_i\right)\Delta u_i$$

$$u=u^o+\frac{\partial u}{\partial x}r_x+\frac{\partial u}{\partial y}r_y+\frac{\partial u}{\partial z}r_z$$
$$v=v^o+\frac{\partial v}{\partial x}r_x+\frac{\partial v}{\partial y}r_y+\frac{\partial v}{\partial z}r_z$$
$$w=w^o+\frac{\partial w}{\partial x}r_x+\frac{\partial w}{\partial y}r_y+\frac{\partial w}{\partial z}r_z$$

$$\Rightarrow \quad u_i=u_i^0+u_{i,j}r_j$$

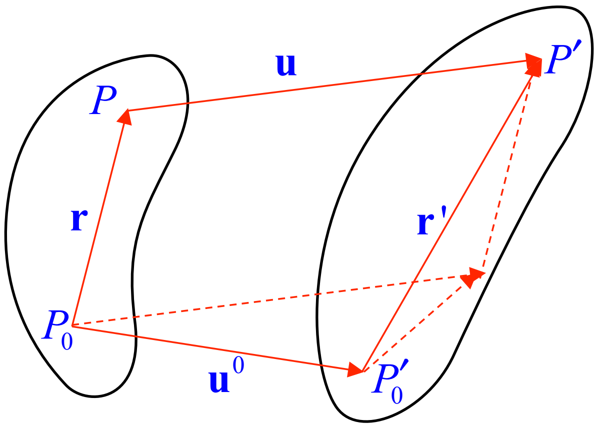

Fig 3. General deformation between two neighboring points

$$\Delta\mathbf{r}=\mathbf{r}'-\mathbf{r}$$
$$=\mathbf{u}-\mathbf{u}^0$$

$$\Delta r_x=r'_x-r_x=\frac{\partial u}{\partial x}r_x+\frac{\partial u}{\partial y}r_y+\frac{\partial u}{\partial z}r_z$$
$$\Delta r_y=r'_y-r_y=\frac{\partial v}{\partial x}r_x+\frac{\partial v}{\partial y}r_y+\frac{\partial v}{\partial z}r_z$$
$$\Delta r_z=r'_z-r_z=\frac{\partial w}{\partial x}r_x+\frac{\partial w}{\partial y}r_y+\frac{\partial w}{\partial z}r_z$$

$$\Rightarrow \quad \Delta r_i=u_{i,j}r_j$$

Small Deformation Theory

- Tensor $u_{i,j}$ is called the *displacement gradient tensor*.

$$u_{i,j} = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix} = \frac{1}{2}(u_{i,j} + u_{j,i}) + \frac{1}{2}(u_{i,j} - u_{j,i}) = e_{ij} + \omega_{ij}$$

$$\begin{cases} e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \text{ , strain tensor} \\ \omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}) \text{ , rotation tensor} \end{cases}$$

- Choose $r_i = \mathrm{d}x_i$, we can write the general result in the form

$$u_i = u_i^0 + u_{i,j} r_j = u_i^0 + e_{ij}\mathrm{d}x_j + \omega_{ij}\mathrm{d}x_j$$

- Using a dual vector $\omega_i = -1/2\ \varepsilon_{ijk}\omega_{jk}$, we have

$$\omega_1 = \omega_{32} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right) \text{ ; } \omega_2 = \omega_{13} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) \text{ ; } \omega_3 = \omega_{21} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) \Rightarrow \boldsymbol{\omega} = \frac{1}{2}(\nabla \times \mathbf{u})$$

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Consider the common deformational behavior of a rectangular element. Rigid-body motion does not contribute to the strain field, and hence does not affect the stresses. We therefore focus our study on the extensional and shearing deformation.

Examples of Continuum Motion & Deformation

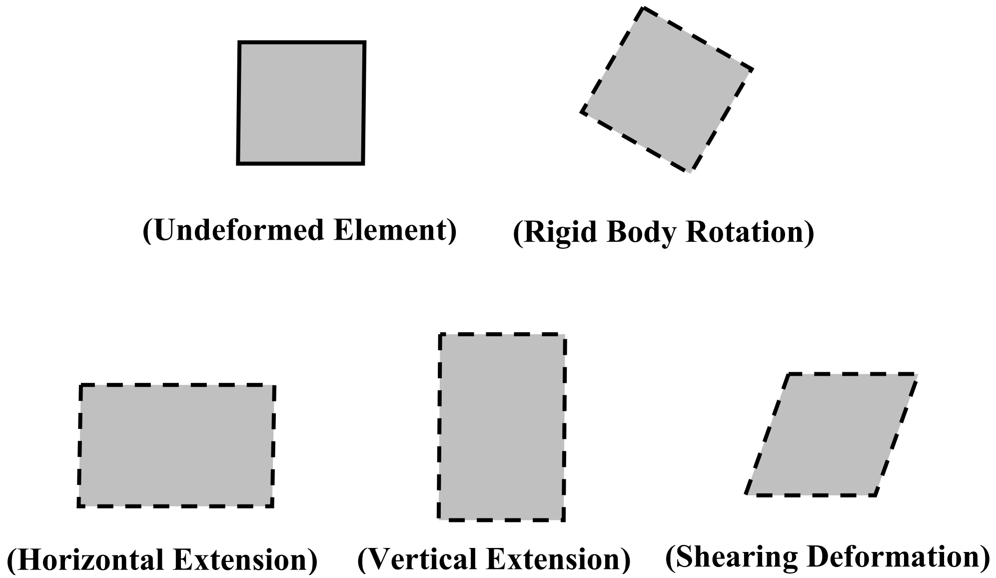

Fig 4. Typical deformations of a rectangular element

Consider a 2D deformation of a rectangular element with original dimensions $\mathrm{d}x$ by $\mathrm{d}y$. Point $A(x,y)$ with displacement components $u(x,y)$ and $v(x,y)$. Point B has displacement $u(x+\mathrm{d}x,y)$ and $v(x+\mathrm{d}x,y)$.

In small deformation theory, (Taylor series expansion)

$$u(x+\mathrm{d}x, y) \approx u(x, y) + (\partial u / \partial x)\mathrm{d}x$$

The *normal* strain in x-direction

$$\varepsilon_x = \frac{A'B' - AB}{AB}$$

From the geometry

$$A'B' = \sqrt{\left(\mathrm{d}x + \frac{\partial u}{\partial x}\mathrm{d}x\right)^2 + \left(\frac{\partial v}{\partial x}\mathrm{d}x\right)^2}$$

$$= \mathrm{d}x\sqrt{\left(1 + \frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2}$$

$$\approx \left(1 + \frac{\partial u}{\partial x}\right)\mathrm{d}x$$

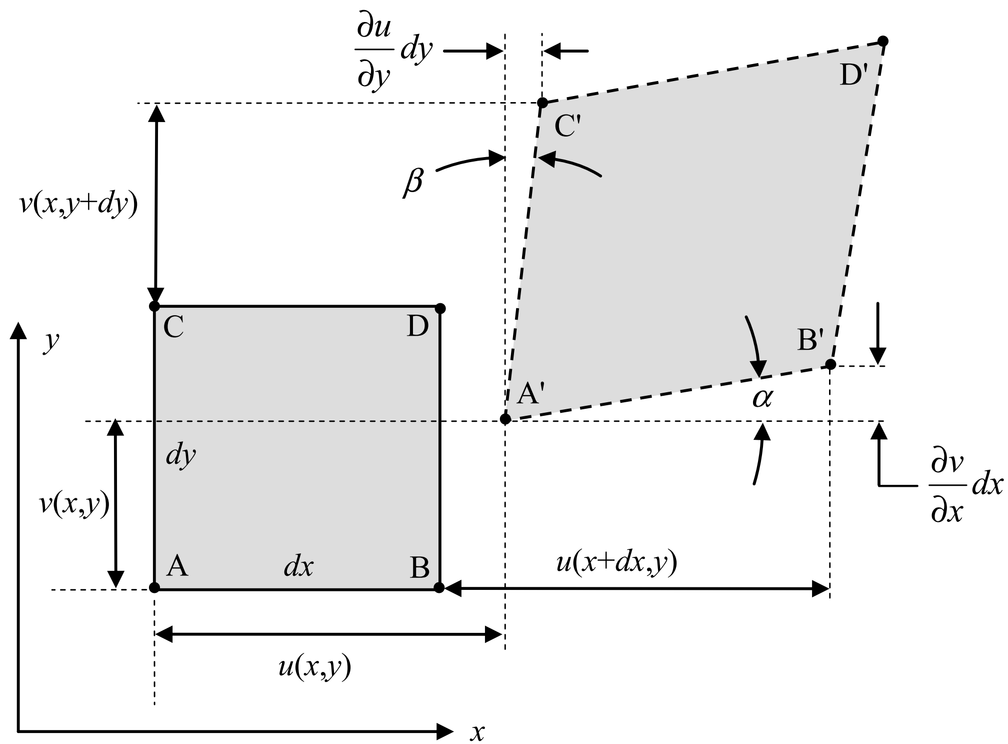

Fig 5. Two-dimensional geometric strain deformation

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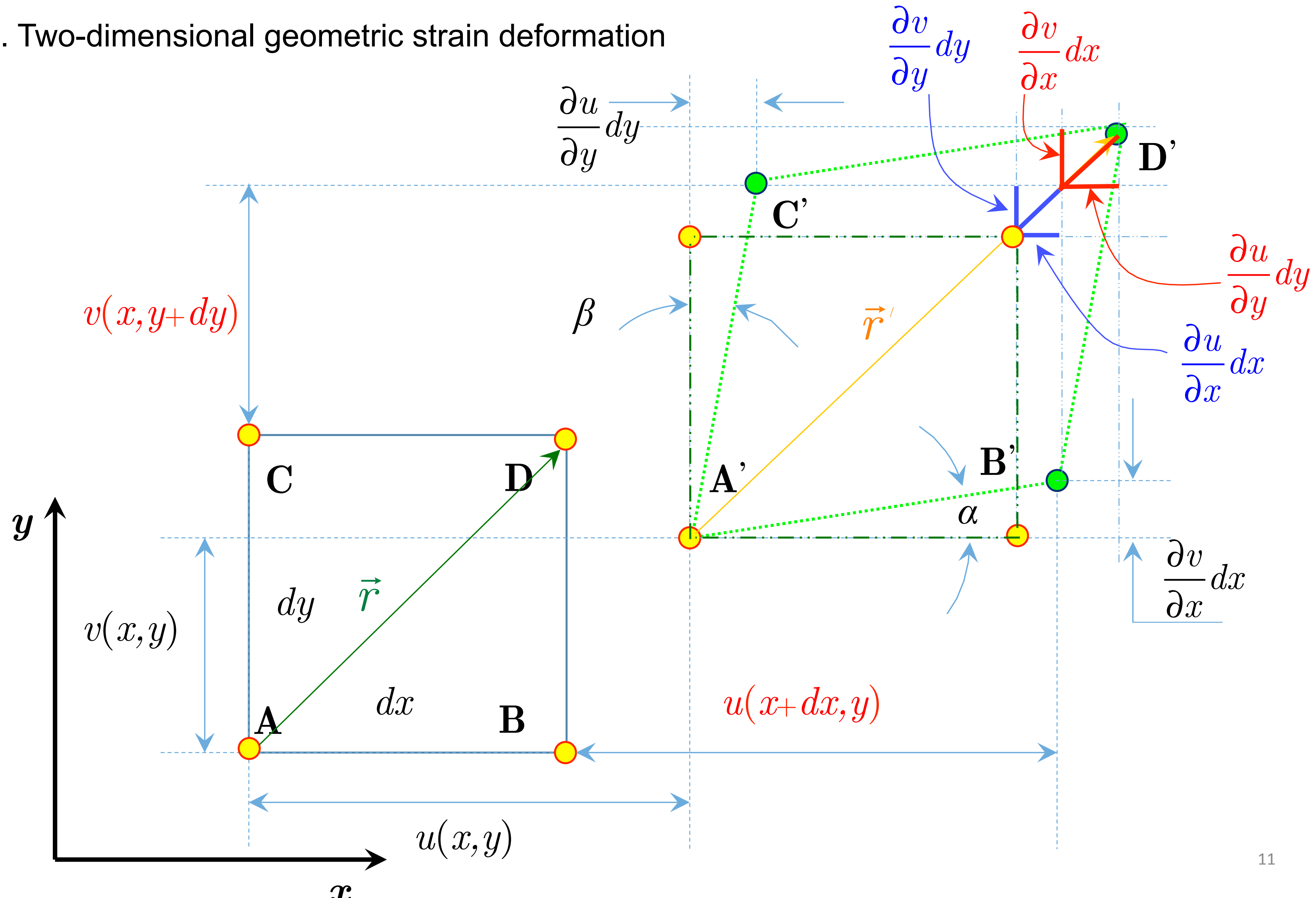

Using $AB = \mathrm{d}x$, the normal strain in x - direction reduces to $\varepsilon_x = \dfrac{\partial u}{\partial x}$

Similarly, the normal strain in y - direction $\varepsilon_y = \dfrac{\partial v}{\partial y}$

A second type of strain is shearing deformation, which involves angles changes. Shear strain is defined as the change in angle between two originally orthogonal directions in the continuum material. Measured in radians, shear strain is positive if the right angle between the positive directions of two axes decreases.

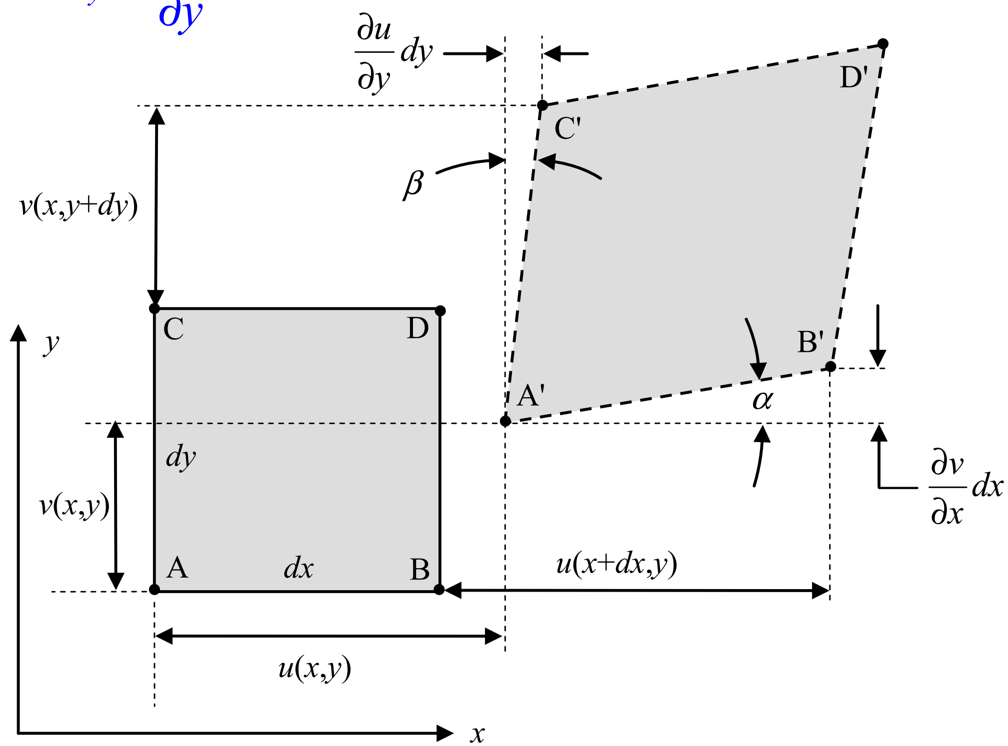

Fig 5. Two-dimensional geometric strain deformation

Shear strains in x- and y-directions can be defined as $\gamma_{xy} = \frac{\pi}{2} - \angle C'A'B' = \alpha + \beta$

For small deformations, $\alpha \approx \tan\alpha$ and $\beta \approx \tan\beta$, and then

$$\gamma_{xy} = \frac{\frac{\partial v}{\partial x}dx}{dx + \frac{\partial u}{\partial x}dx} + \frac{\frac{\partial u}{\partial y}dy}{dy + \frac{\partial v}{\partial y}dy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

By interchange of x & y, u & v

$$\gamma_{xy} = \gamma_{yx}$$

Similarly, we have

$$\varepsilon_x = \frac{\partial u}{\partial x}, \quad \varepsilon_y = \frac{\partial v}{\partial y}, \quad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y},$$

$$\gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

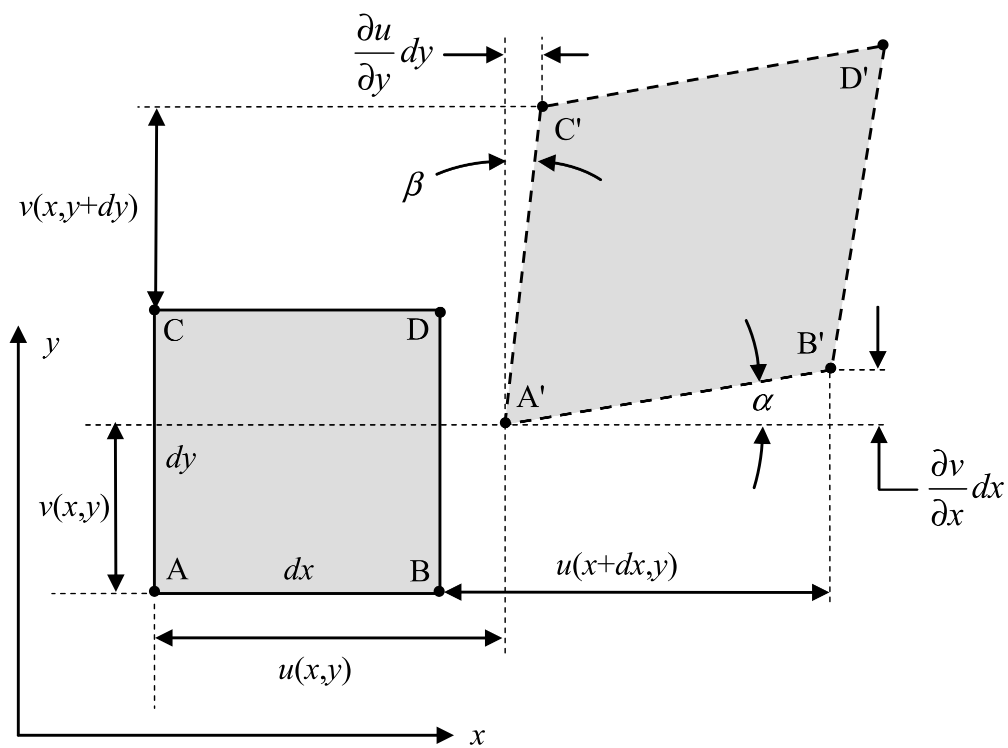

Fig 5. Two-dimensional geometric strain deformation

Using the strain tensor e_{ij}, the strain-displacement relations can be expressed as

$$e_x = \frac{\partial u}{\partial x},\ e_y = \frac{\partial v}{\partial y},\ e_z = \frac{\partial w}{\partial z},\ e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), e_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), e_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

Using tensor and matrix notation

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

$$\mathbf{e} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right]$$

The strain is a symmetric second-order tensor ($e_{ij} = e_{ji}$)

$$\mathbf{e} = \left[e_{ij}\right] = \begin{bmatrix} e_x & e_{xy} & e_{xz} \\ e_{yx} & e_y & e_{yz} \\ e_{xz} & e_{yz} & e_z \end{bmatrix}$$

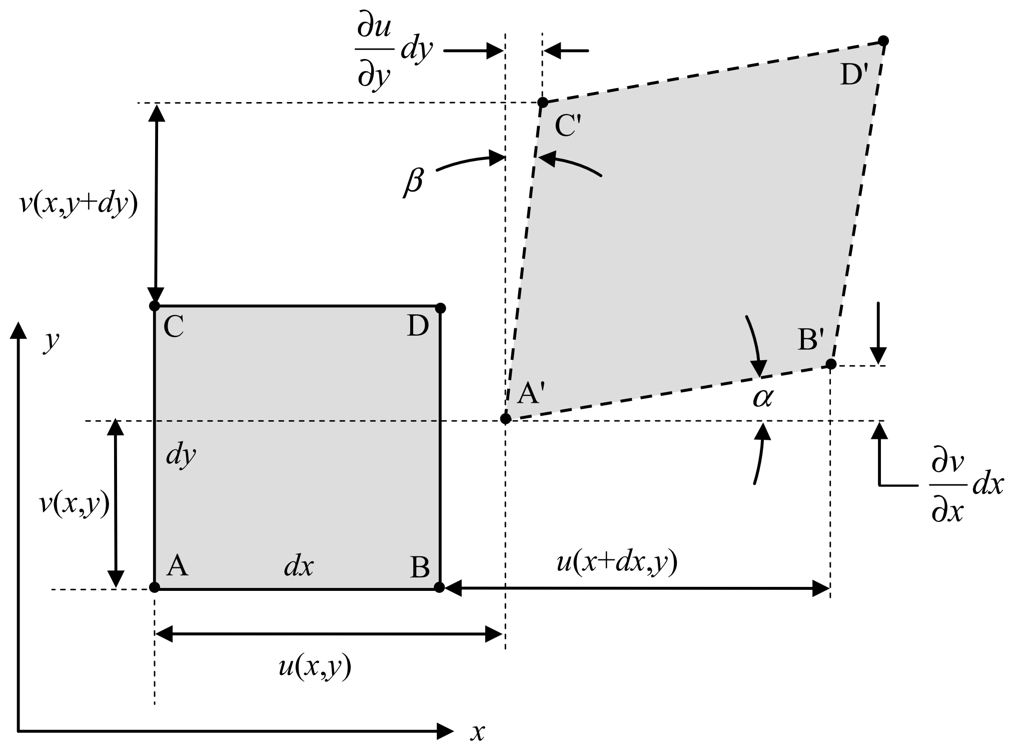

Fig 5. Two-dimensional geometric strain deformation

Example 2-1: Strain and Rotation Examples

Determine the displacement gradient, strain and rotation tensors for the following displacement field: $u = Ax^2y$, $v = Byz$, $w = Cxz^3$, where *A, B,* and *C* are arbitrary constants. Also calculate the dual rotation vector $\boldsymbol{\omega} = (1/2)(\nabla\times\boldsymbol{u})$.

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Determine the displacement gradient, strain and rotation tensors for the following displacement field: $u = Ax^2y$, $v = Byz$, $w = Cxz^3$, where *A*, *B*, and *C* are arbitrary constants. Also calculate the dual rotation vector $\boldsymbol{\omega} = (1/2)(\nabla\times\boldsymbol{u})$.

$$u_{i,j} = \begin{bmatrix} 2Axy & Ax^2 & 0 \\ 0 & Bz & By \\ Cz^3 & 0 & 3Cxz^2 \end{bmatrix}$$

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) = \begin{bmatrix} 2Axy & Ax^2/2 & Cz^3/2 \\ Ax^2/2 & Bz & By/2 \\ Cz^3/2 & By/2 & 3Cxz^2 \end{bmatrix}$$

$$\omega_{ij} = \frac{1}{2}\left(u_{i,j} - u_{j,i}\right) = \begin{bmatrix} 0 & Ax^2/2 & -Cz^3/2 \\ -Ax^2/2 & 0 & By/2 \\ Cz^3/2 & -By/2 & 0 \end{bmatrix}$$

$$\boldsymbol{\omega} = \frac{1}{2}(\nabla\times\mathbf{u}) = \frac{1}{2}\begin{vmatrix} \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ax^2y & Byz & Cxz^3 \end{vmatrix} = \frac{1}{2}\left(-By\mathbf{e}_1 - Cz^3\mathbf{e}_2 - Ax^2\mathbf{e}_3\right)$$

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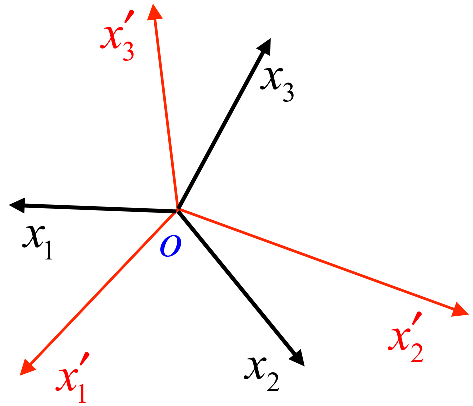
$$\mathbf{e} = \begin{bmatrix} e_x & e_{xy} & e_{xz} \\ e_{xy} & e_y & e_{yz} \\ e_{xz} & e_{yz} & e_z \end{bmatrix}$$



$$\mathbf{e}' = \begin{bmatrix} e'_x & e'_{xy} & e'_{xz} \\ e'_{xy} & e'_y & e'_{yz} \\ e'_{xz} & e'_{yz} & e'_z \end{bmatrix}$$

Fig 6. 3D rotational transformation

$e'_{ij} = Q_{ip} Q_{jq} e_{pq}$ where $Q_{ij} = \cos\left(x'_i, x_j\right)$

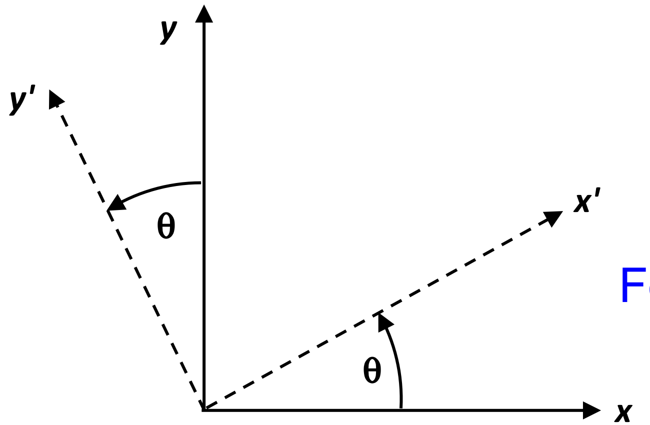


Fig 7. 2D rotational transformation

For 2D case, prove

$$\begin{cases} e'_x = \dfrac{e_x + e_y}{2} + \dfrac{e_x - e_y}{2}\cos 2\theta + e_{xy}\sin 2\theta \\ e'_y = \dfrac{e_x + e_y}{2} - \dfrac{e_x - e_y}{2}\cos 2\theta - e_{xy}\sin 2\theta \\ e'_{xy} = \dfrac{e_y - e_x}{2}\sin 2\theta + e_{xy}\cos 2\theta \end{cases}$$

$$Q_{ij} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$e'_{ij} = Q_{ip} Q_{jq} e_{pq} \quad \Rightarrow \quad \begin{cases} e'_x = e_x \cos^2\theta + e_y \sin^2\theta + 2e_{xy} \sin\theta\cos\theta \\ e'_y = e_x \sin^2\theta + e_y \cos^2\theta - 2e_{xy} \sin\theta\cos\theta \\ e'_{xy} = -e_x \sin\theta\cos\theta + e_y \sin\theta\cos\theta + e_{xy}(\cos^2\theta - \sin^2\theta) \end{cases}$$

$$\Rightarrow \quad \begin{cases} e'_x = \dfrac{e_x + e_y}{2} + \dfrac{e_x - e_y}{2}\cos 2\theta + e_{xy}\sin 2\theta \\ e'_y = \dfrac{e_x + e_y}{2} - \dfrac{e_x - e_y}{2}\cos 2\theta - e_{xy}\sin 2\theta \\ e'_{xy} = \dfrac{e_y - e_x}{2}\sin 2\theta + e_{xy}\cos 2\theta \end{cases}$$

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$$e_{ij}n_j = en_i$$

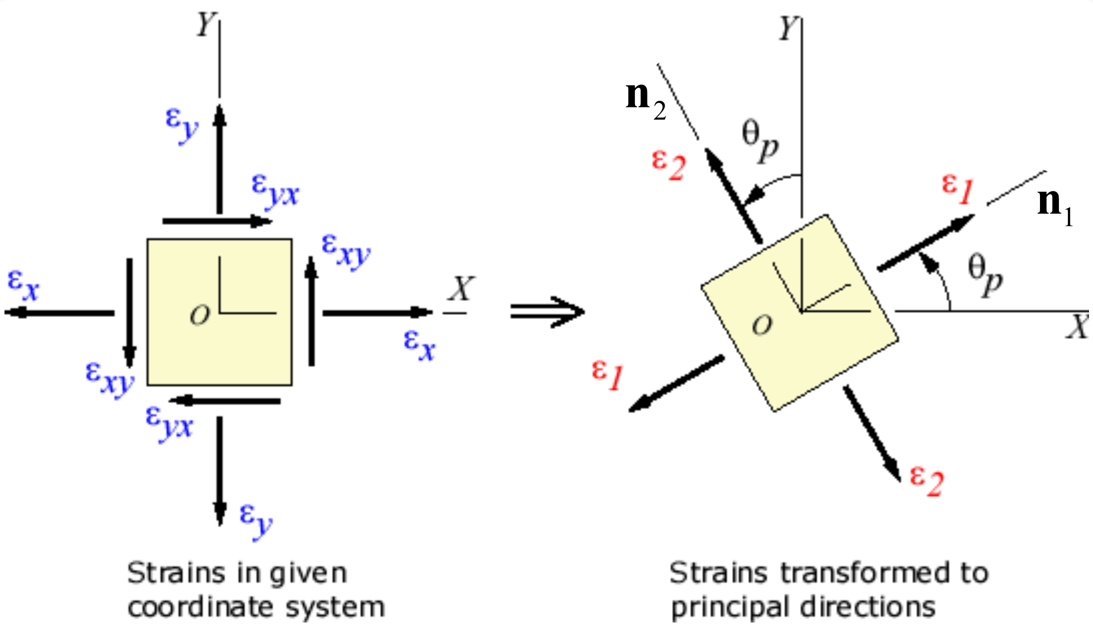
$$\det\left[e_{ij} - e\delta_{ij}\right] = 0$$

$$\Rightarrow -e^3 + \vartheta_1 e^2 - \vartheta_2 e + \vartheta_3 = 0$$

$$\vartheta_1 = e_1 + e_2 + e_3$$

$$\vartheta_2 = e_1e_2 + e_2e_3 + e_3e_1$$

$$\vartheta_3 = e_1e_2e_3$$



=> Principle strains $e_{ij} = \begin{bmatrix} e_1 & 0 & 0 \\ 0 & e_2 & 0 \\ 0 & 0 & e_3 \end{bmatrix}$

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In particular applications it is convenient to decompose the strain tensor into two parts called *spherical* and *deviatoric* strain tensors

$$e_{ij} = \underbrace{\left(e_{ij} - \frac{1}{3} e_{kk} \delta_{ij} \right)}_{\hat{e}_{ij}} + \underbrace{\frac{1}{3} e_{kk} \delta_{ij}}_{\tilde{e}_{ij}}$$

The spherical strain $\tilde{e}_{ij} = \frac{1}{3} e_{kk} \delta_{ij}$ represents only volumetric deformation

The deviatoric strain $\hat{e}_{ij} = e_{ij} - \frac{1}{3} e_{kk} \delta_{ij}$ accounts for changes in shape of material elements

Note: principal directions of the deviatoric strain are the same as those of the strain tensor

Example 2-2: Determine the principle, spherical, deviatoric strains of the following state of strain

$$e_{ij} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & -4 & 1 \\ 0 & 1 & 6 \end{bmatrix}$$

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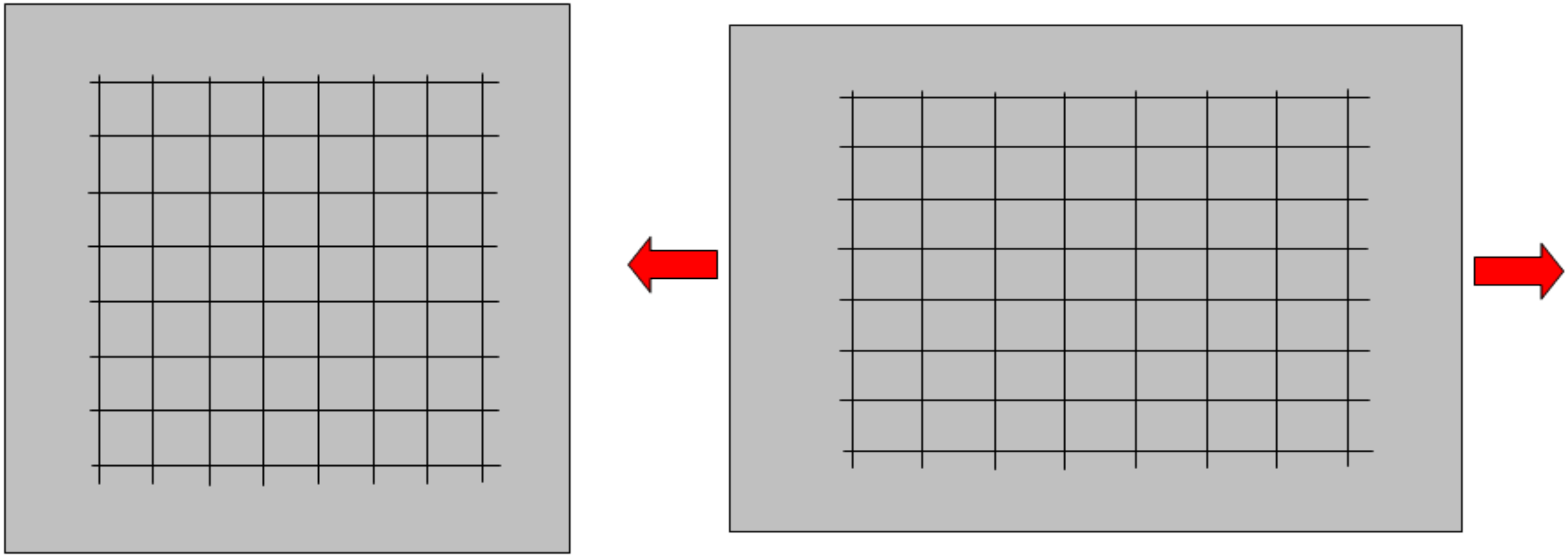
Normally we want continuous single-valued displacements; i.e. a mesh that fits perfectly together after deformation



Undeformed State Deformed State

Mathematical Concepts Related to Deformation Compatibility

Strain-Displacement Relations

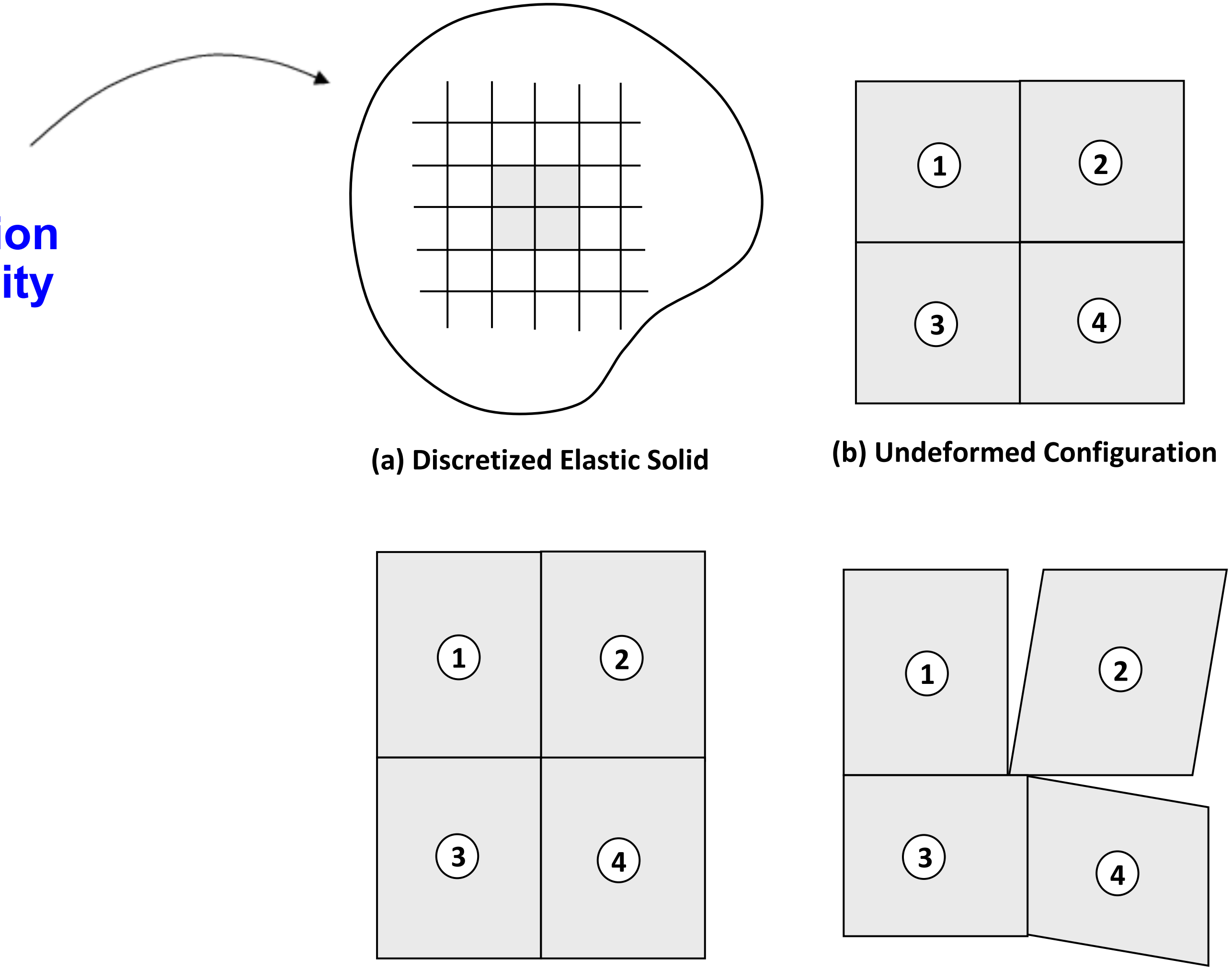
$$e_x = \frac{\partial u}{\partial x}, \; e_y = \frac{\partial v}{\partial y}, \; e_z = \frac{\partial w}{\partial z}, \; e_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \; e_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right), \; e_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

Given the Three Displacements:
We have six equations to easily determine the six strains

Given the Six Strains:
We have six equations to determine three displacement components. This is an *over-determined system* and in general will not yield continuous single-valued displacements unless the strain components satisfy some additional relations

→ The strains must satisfy additional relations called *integrability* or *compatibility equations*

Physical Interpretation of Strain Compatibility



(a) Discretized Elastic Solid

(b) Undeformed Configuration

(c) Deformed Configuration
Continuous Displacements

(d) Deformed Configuration
Discontinuous Displacements

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \Rightarrow \begin{cases} e_{ij,kl} = \frac{1}{2}\left(u_{i,jkl} + u_{j,ikl}\right) \\ e_{kl,ij} = \frac{1}{2}\left(u_{k,lij} + u_{l,kij}\right) \\ e_{jl,ik} = \frac{1}{2}\left(u_{j,lik} + u_{l,jik}\right) \\ e_{ik,jl} = \frac{1}{2}\left(u_{i,kjl} + u_{k,ijl}\right) \end{cases} \Rightarrow e_{ij,kk} + e_{kk,ij} - e_{ik,jk} - e_{jk,ik} = 0$$

Saint Venant Compatibility Equations

81 individual equations, most are either simple identities or repetitions, and only 6 are meaningful.

These six relations may be determined by letting $k = l$, and in scalar notation, they become

$$\frac{\partial^2 e_x}{\partial y^2} + \frac{\partial^2 e_y}{\partial x^2} = 2\frac{\partial^2 e_{xy}}{\partial x \partial y}; \quad \frac{\partial^2 e_x}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial e_{yz}}{\partial x} + \frac{\partial e_{zx}}{\partial y} + \frac{\partial e_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 e_y}{\partial z^2} + \frac{\partial^2 e_z}{\partial y^2} = 2\frac{\partial^2 e_{yz}}{\partial y \partial z}; \quad \frac{\partial^2 e_y}{\partial z \partial x} = \frac{\partial}{\partial y}\left(-\frac{\partial e_{zx}}{\partial y} + \frac{\partial e_{xy}}{\partial z} + \frac{\partial e_{yz}}{\partial x}\right)$$

$$\frac{\partial^2 e_z}{\partial x^2} + \frac{\partial^2 e_x}{\partial z^2} = 2\frac{\partial^2 e_{zx}}{\partial z \partial x}; \quad \frac{\partial^2 e_z}{\partial x \partial y} = \frac{\partial}{\partial z}\left(-\frac{\partial e_{xy}}{\partial z} + \frac{\partial e_{yz}}{\partial x} + \frac{\partial e_{zx}}{\partial y}\right)$$

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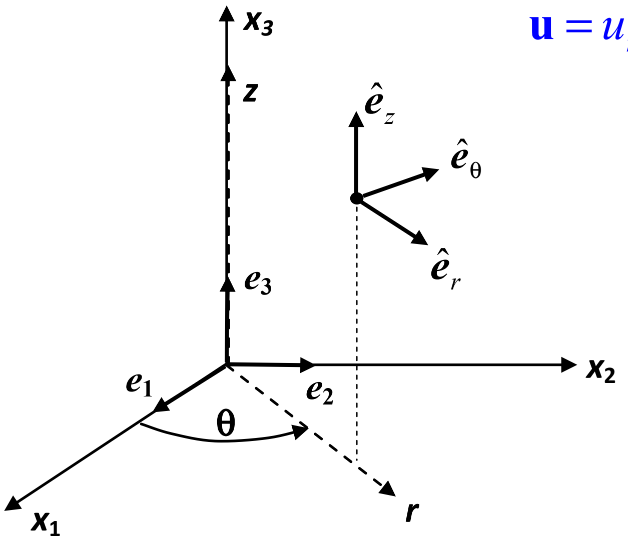
The cylindrical coordinate system



$$\mathbf{u} = u_r\mathbf{e}_r + u_\theta\mathbf{e}_\theta + u_z\mathbf{e}_z \quad \text{where} \quad \mathbf{e} = \begin{bmatrix} e_r & e_{r\theta} & e_{rz} \\ e_{r\theta} & e_\theta & e_{\theta z} \\ e_{rz} & e_{\theta z} & e_z \end{bmatrix}$$

$$e_r = \frac{\partial u_r}{\partial r}$$

$$e_\theta = \frac{1}{r}\left(u_r + \frac{\partial u_\theta}{\partial \theta}\right)$$

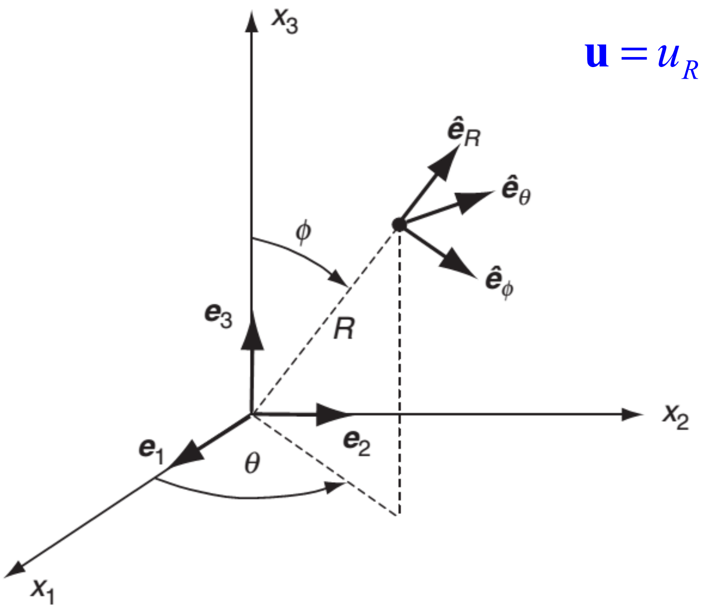
$$e_z = \frac{\partial u_z}{\partial z}$$

$$e_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right)$$

$$e_{\theta z} = \frac{1}{2}\left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r}\frac{\partial u_z}{\partial \theta}\right)$$

$$e_{zr} = \frac{1}{2}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right)$$

The spherical coordinate system



$$\mathbf{u} = u_R \mathbf{e}_R + u_\phi \mathbf{e}_\phi + u_\theta \mathbf{e}_\theta$$

where

$$\mathbf{e} = \begin{bmatrix} e_R & e_{R\phi} & e_{R\theta} \\ e_{R\phi} & e_\phi & e_{\phi\theta} \\ e_{R\theta} & e_{\phi\theta} & e_\theta \end{bmatrix}$$

$$e_R = \frac{\partial u_R}{\partial R}$$

$$e_\phi = \frac{1}{R}\left(u_R + \frac{\partial u_\phi}{\partial \phi} \right)$$

$$e_\theta = \frac{1}{R\sin\phi}\left(\frac{\partial u_\theta}{\partial \theta} + \sin\phi u_R + \cos\phi u_\phi \right)$$

$$e_{R\phi} = \frac{1}{2}\left(\frac{1}{R}\frac{\partial u_R}{\partial \theta} + \frac{\partial u_\phi}{\partial R} - \frac{u_\phi}{R} \right)$$

$$e_{\phi\theta} = \frac{1}{2R}\left(\frac{1}{\sin\phi}\frac{\partial u_\phi}{\partial \theta} + \frac{\partial u_\theta}{\partial \phi} - \cos\phi u_\theta \right)$$

$$e_{\theta R} = \frac{1}{2}\left(\frac{1}{R\sin\phi}\frac{\partial u_R}{\partial \theta} + \frac{\partial u_\theta}{\partial R} - \frac{u_\theta}{R} \right)$$

THANK YOU