Solid Mechanics

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Suppose the point P has position $\boldsymbol{r} = \boldsymbol{r}(r, \theta)$. We now ask by how large a distance $d\boldsymbol{r}$ the head of the vector $\boldsymbol{r}$ changes if infinitesimal changes $dr, d\theta$ are made in the two polar directions.

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As seen from figure, the total change is .

$d\boldsymbol{r} = dr\boldsymbol{e}_r + rd\theta\boldsymbol{e}_\theta$.

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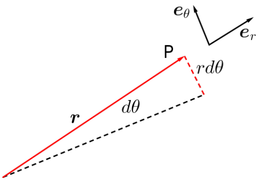

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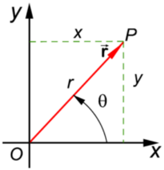
Suppose the point P has position $\boldsymbol{r} = \boldsymbol{r}(r, \theta)$. If we change r by a small amount, dr , then $\boldsymbol{r}$ moves to position $(\boldsymbol{r} + d\boldsymbol{r})$, where $d\boldsymbol{r} = (\partial \boldsymbol{r}/\partial r)dr \equiv h_r dr \boldsymbol{e}_r$.
where we have defined the unit vector $\boldsymbol{e}_r$ and the scale factor h_r by



$$h_r = |\partial \boldsymbol{r}/\partial r|, \boldsymbol{e}_r = (\partial \boldsymbol{r}/\partial r)/|\partial \boldsymbol{r}/\partial r|$$

The scale factor h_r gives the magnitude of dr when we make the change $r \to r + dr$.
For Cartesian coordinates, the scale factors are unity and the unit vectors $\boldsymbol{e}_i$ reduce to the Cartesian basis vectors that we have used throughout the course:
$\boldsymbol{r} = x\boldsymbol{e}_1 + y\boldsymbol{e}_2 + z\boldsymbol{e}_3$ so that $h_1\boldsymbol{e}_1 = (\partial \boldsymbol{r}/\partial x) = \boldsymbol{e}_1$. Similarly,we also have h_2, h_3.

(Plane) polar coordinate

$$x = r\cos(\theta), y = r\sin(\theta),$$

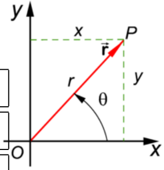
$$r = \sqrt{x^2 + y^2}, \theta = artang(y/x),$$

$$\boldsymbol{r} = r\cos(\theta)\boldsymbol{i} + r\sin(\theta)\boldsymbol{j},$$

$$d\boldsymbol{r}_r = (\partial \boldsymbol{r}/\partial r)dr = (\cos(\theta)\boldsymbol{i} + \sin(\theta)\boldsymbol{j})dr,$$

$$d\boldsymbol{r}_\theta = (\partial \boldsymbol{r}/\partial \theta)d\theta = (-r\sin(\theta)\boldsymbol{i} + r\cos(\theta)\boldsymbol{j})d\theta,$$

$$d\boldsymbol{r} = d\boldsymbol{r}_r + d\boldsymbol{r}_\theta = (\partial \boldsymbol{r}/\partial r)dr + (\partial \boldsymbol{r}/\partial \theta)d\theta,$$



Scale factors: $h_r = |\partial \boldsymbol{r}/\partial r| = 1, h_\theta = |\partial \boldsymbol{r}/\partial \theta| = r,$

Unit vectors:

$$\boldsymbol{e}_r = (\partial \boldsymbol{r}/\partial r)/|\partial \boldsymbol{r}/\partial r| = (\partial \boldsymbol{r}/\partial r)/h_r = \cos(\theta)\boldsymbol{i} + \sin(\theta)\boldsymbol{j},$$

$$\boldsymbol{e}_\theta = (\partial \boldsymbol{r}/\partial \theta)/|\partial \boldsymbol{r}/\partial \theta| = (\partial \boldsymbol{r}/\partial \theta)/h_\theta = -r\sin(\theta)\boldsymbol{i} + r\cos(\theta)\boldsymbol{j}$$

$$d\boldsymbol{r} = h_r dr \boldsymbol{e}_r + h_\theta d\theta \boldsymbol{e}_\theta$$

$$(d\boldsymbol{r})^2 = (h_r dr)^2 + (h_\theta d\theta)^2 = dr^2 + (rd\theta)^2$$

Scalar fields in orthogonal curvilinear coordinates.

Scalar fields can of course be expressed in orthogonal curvilinear coordinate: they are simply written as function $f(\xi_1, \xi_2, \xi_3)$ or for brevity $f(\xi_i)$.

Vector differentiation with respect to position: GRADIENT OPERATOR.

Give scalar field $f(\xi_1, \xi_2, \xi_3)$ in the orthogonal curvilinear coordinate (ξ_1, ξ_2, ξ_3), we have

$$df = \nabla f . d\mathbf{r}$$

For the change caused by an infinitesimal position change $d\mathbf{r}$

♣ Using Taylor theorem in 3-dimension,

$$\Longrightarrow df = \frac{\partial f}{\partial \xi_1} d\xi_1 + \frac{\partial f}{\partial \xi_2} d\xi_2 + \frac{\partial f}{\partial \xi_3} d\xi_3$$

♣ From the previous definition of the unit vectors, we have

$$d\mathbf{r} = h_1 d\xi_1 \mathbf{e}_1 + h_2 d\xi_2 \mathbf{e}_2 + h_3 d\xi_3 \mathbf{e}_3$$

We define $\nabla f = (\nabla f)_1 \mathbf{e}_1 + (\nabla f)_2 \mathbf{e}_2 + (\nabla f)_3 \mathbf{e}_3$

$$\Longrightarrow df = \nabla f . d\mathbf{r} = h_1 (\nabla f)_1 d\xi_1 + h_2 (\nabla f)_2 d\xi_2 + h_3 (\nabla f)_3 d\xi_3$$

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Main source [1]

Other r

References

Martin H Sadd.
Elasticity: theory, applications, and numerics.
Academic Press, 2009.

Thank you for listening!