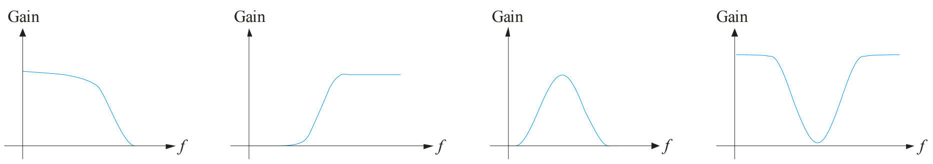
Chapter 15: Active Filters

15.1: Basic filter Responses

- A filter is a circuit that passes certain frequencies and rejects or attenuates all others.
- The **passband** is the range of frequencies allowed to pass through the filter.
- The **critical frequency**, f_c, defines the end (or ends) of the passband and is normally specified at the point where the response drops -3dB (70.7%) from the passband response.
- Basic filter responses are:

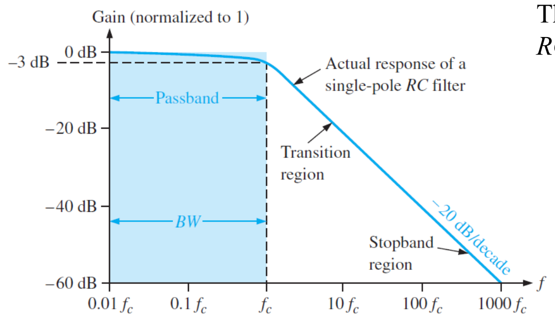


Low-pass High-pass Band-pass Band-stop

15.1: Basic filter Responses

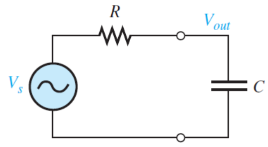
Low-Pass Filter Response

- The **low-pass filter** allows frequencies below the critical frequency to pass (from dc to f_c) and rejects other. The simplest low-pass filter is a passive *RC* circuit with the output taken across *C*.
- → The bandwidth of an ideal low-pass filter is $BW = f_c$



The critical frequency of a low-pass *RC* filter occurs when $X_C = R$ where

$$f_c = \frac{1}{2\pi RC}$$



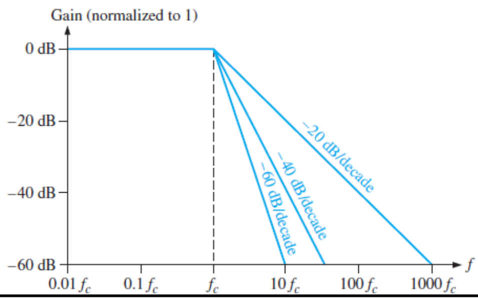
Ideal response (shaded area): ideal low-pass filter; no response for frequencies above f_c

Actual response (curved line): the gain drops rapidly after f_c with a rate decided by number of poles (number of *RC* circuits contained in the filter)

15.1: Basic filter Responses

Low-Pass Filter Response

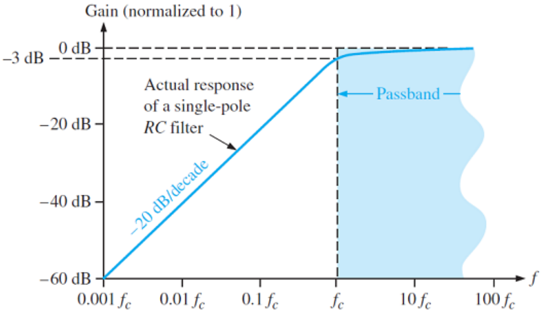
- The -20dB roll-off rate is not a particularly good filter characteristic (far from ideal filter) because too much of the unwanted frequencies (beyond the passband) are allowed through the filter
- In order to produce a more effective filter that has a steeper transition region, it is necessary to add additional poles (*RC* circuits) combined with op-amps that have frequency-selective feedback circuits → filters can be designed with roll-off rates of -40dB, -60dB or more dB/decade as shown
- Filters that include one or more op-amps in the design are called **active filters.** These filters can optimize the roll-off rate or other attribute (such as phase response) with a particular filter design.

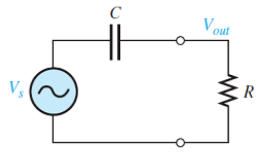


15.1: Basic filter Responses

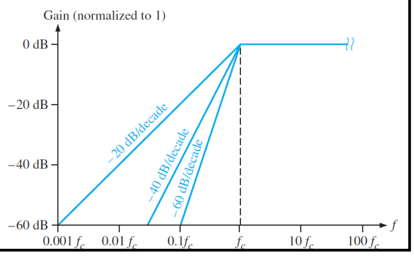
High-Pass Filter Response

- The high-pass filter passes all frequencies above a critical frequency and rejects all others. The simplest high-pass filter is a passive *RC* circuit with the output taken across *R*.





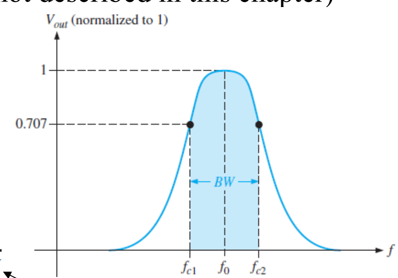
- Responses that are steeper than -20dB in the transition region are also possible with active high-pass filters; the particular response depends on the type of filter and the number of poles.



15.1: Basic filter Responses

Band-Pass Filter Response

- A **band-pass filter** passes all frequencies between two critical frequencies. The **bandwidth** is defined as the difference between the two critical frequencies. The band-pass filter can be obtained by joining the high-pass filter with low-pass filter or by RLC circuit (not described in this chapter)
- The bandwidth is $BW = f_{c2} - f_{c1}$
- The center frequency f_0 about which the bandpass is centered can be calculated from $f_0 = \sqrt{f_{c1} f_{c2}}$
- The **quality factor (*Q*)** of a band-pass filter is the ratio of the center frequency to the bandwidth. $Q = \frac{f_0}{BW}$



SEE EXAMPLE 15-1

The lower the Q the better the band selection

- If Q > 10 → narrow band-pass filter If Q < 10 → wide bandpass filter
- The quality factor (*Q*) can also be expressed in terms of the damping factor (*DF*) of the filter as $Q = \frac{1}{DF}$

15.1: Basic filter Responses

Band-Stop Filter Response

- A **band-stop filter** rejects frequencies between two critical frequencies; the bandwidth is measured between the critical frequencies. The band-pass filter can be obtained by joining the low-pass filter with high-pass filter or by RLC circuit (not described in this chapter)

- The bandwidth is

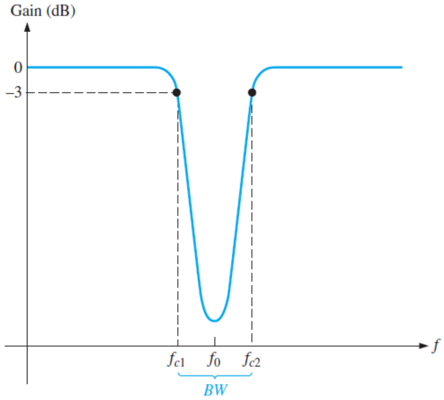
$$BW = f_{c2} - f_{c1}$$

- The center frequency f_0 is

$$f_0 = \sqrt{f_{c1} f_{c2}}$$

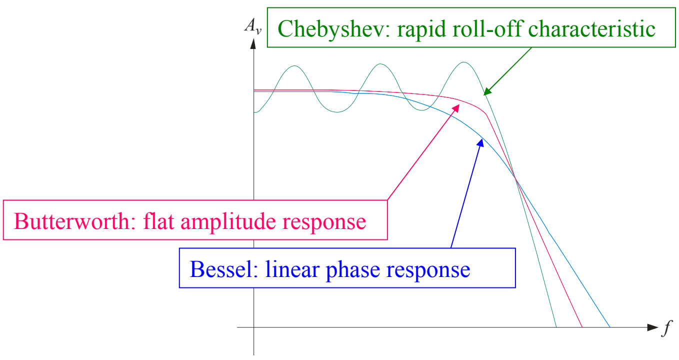
- quality factor (Q)

$$Q = \frac{f_0}{BW}$$



15.2: Filter Response Characteristics

- **Active filters:** include one or more op-amps in the design. These filters can provide much better responses than the passive filters illustrated befor. Active filter designs optimize various parameters such as amplitude response, roll-off rate, or phase response.
- Each type of filter response (low-pass, high-pass, band-pass, or band-stop) can be tailored by circuit component values to have either a Butterworth, Chebyshev, or Bessel characteristic .

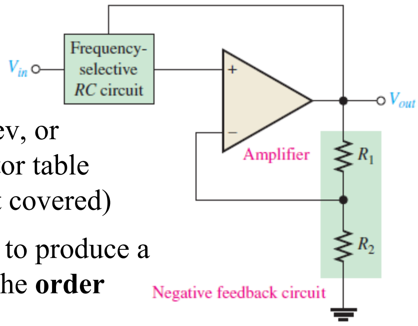


15.2: Filter Response Characteristics

The Damping Factor

- The damping factor primarily determines if the filter will have a Butterworth, Chebyshev, or Bessel response.
- The damping factor in the shown general diagram of active filter is determined by the feedback resistors R_1 and R_2 and is defined by:

$$DF = 2 - \frac{R_1}{R_2}$$



- Every filter type (Butterworth, Chebyshev, or Bessel response) has it's own damping factor table derived using a advanced mathematics (not covered)
- The value of the damping factor required to produce a desired response characteristic depends on the **order** (number of poles) of the filter.
- A ***pole*** is simply a circuit with one resistor and one capacitor. The more poles a filter has, the faster its roll-off rate is.

15.2: Filter Response Characteristics

The Damping Factor

- Because of its maximally flat response, the Butterworth characteristic is the most widely used → we will limit our coverage to the Butterworth response
- Parameters for Butterworth filters up to four poles are given in the following table. (See text for larger order filters).

Table for Butterworth filter values

Order	Roll-off dB/decade	1st stage Poles	1st stage DF	1st stage R_1/R_2	2nd stage Poles	2nd stage DF	2nd stage R_1/R_2
1	−20	1	*Optional*				
2	−40	2	1.414	0.586			
3	−60	2	1.00	1.00	1	1.00	1.00
4	−80	2	1.848	0.152	2	0.765	1.235

For example, To achieve a second-order Butterworth response → damping factor must be 1.414. →

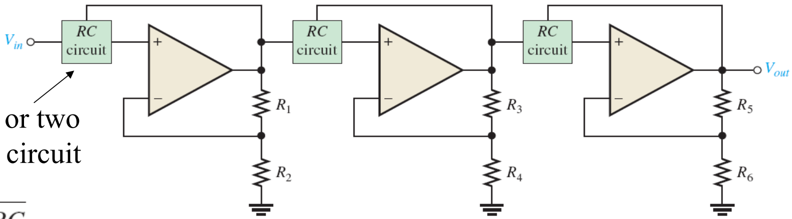
$$\frac{R_1}{R_2} = 2 - DF = 2 - 1.414 = 0.586$$

→ The gain $A_{cl(NI)} = \frac{R_1}{R_2} + 1 = 0.586 + 1 = 1.586$ which is 1 more than the resistor ratio

15.2: Filter Response Characteristics

Critical Frequency and Roll-Off Rate

- the ***order*** number is the number of poles (*RC* circuits) that must be included in the circuit of the active filter
- The number of poles determines the roll-off rate of the filter. A Butterworth response produces -20 dB/decade/pole → a first-order (one-pole) filter has a roll-off of -20 dB/decade; a second-order (two-pole) filter has a roll-off rate of -40 dB/decade; a third-order (three-pole) filter has a roll-off -60 dB/decade and so on.
- Generally, to obtain a filter with three poles or more, ***one-pole*** or ***two-pole*** filters are cascaded in stages as shown.



One pole or two poles RC circuit

$$f_c = \frac{1}{2\pi RC}$$

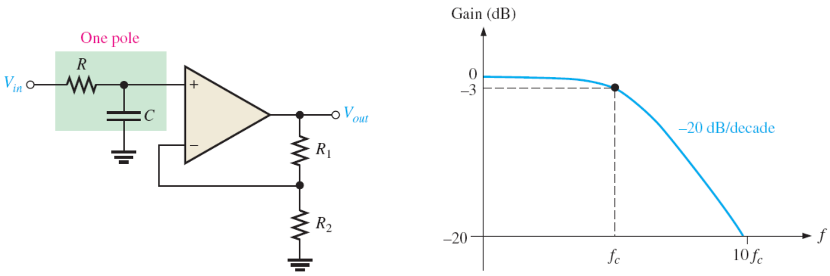
Stage 1 Stage 2 Stage 3

15.3: Active Low-Pass Filters

- Filters that use op-amps as the active element provide several advantages over passive filters (*R*, *L*, and *C* elements only). The op-amp provides gain, so the signal is not attenuated as it passes through the filter.

A Single-Pole Low-Pass Filter

- active filter with a single low-pass *RC* frequency-selective circuit that provides a roll-off of -20 dB/decade



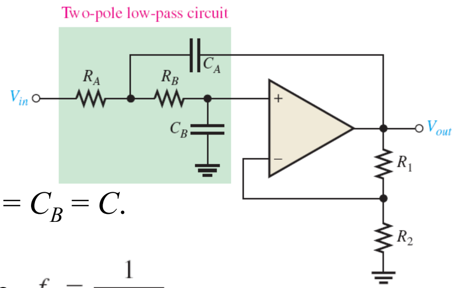
- The critical frequency $f_c = \frac{1}{2\pi RC}$
- The closed-loop voltage gain $A_{cl(NI)} = \frac{R_1}{R_2} + 1$

15.3: Active Low-Pass Filters

The Sallen-Key Low-Pass Filter (Double-Pole Low-Pass Filter)

- The Sallen-Key is one of the most common configurations for a second-order (two-pole) filter.
- It is an active filter with a two low-pass *RC* circuits that provides a roll-off of -40 dB/decade
- The critical frequency

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$



If we choose $R_A = R_B = R$ and $C_A = C_B = C$.

→ critical frequency simplifies to $f_c = \frac{1}{2\pi RC}$

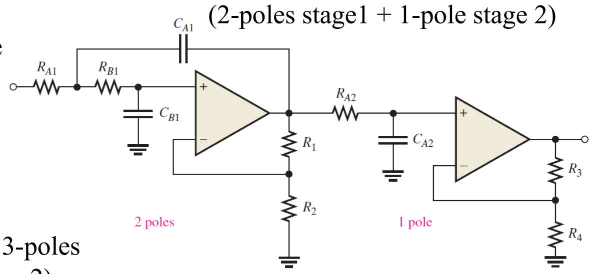
- The closed-loop voltage gain $A_{cl(NI)} = \frac{R_1}{R_2} + 1$

15.3: Active Low-Pass Filters

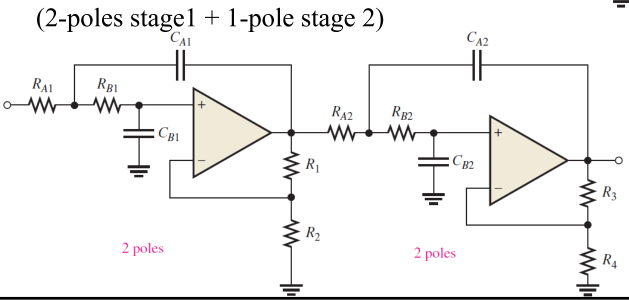
Cascaded Low-Pass Filters

- Third-order or higher low-pass response (-60 dB/decade or lower) can be done by cascading a single pole and/or two-pole low-pass filter

Third order configuration; 3-poles (2-poles stage1 + 1-pole stage 2)

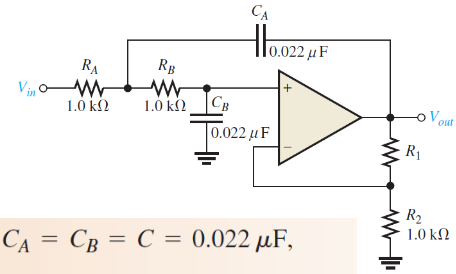


Fourth order configuration; 3-poles (2-poles stage1 + 1-pole stage 2)



15.3: Active Low-Pass Filters: Example

- Determine the critical frequency of the Sallen-Key low-pass filter in Figure, and set the value of R_1 for an approximate Butterworth response.



Since $R_A = R_B = R = 1.0\,\text{k}\Omega$ and $C_A = C_B = C = 0.022\,\mu\text{F}$,

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1.0\,\text{k}\Omega)(0.022\,\mu\text{F})} = \mathbf{7.23\,kHz}$$

For a Butterworth response, $R_1/R_2 = 0.586$.

$$R_1 = 0.586R_2 = 0.586(1.0\,\text{k}\Omega) = \mathbf{586\,\Omega}$$

15.3: Active Low-Pass Filters: Example

For the four-pole filter in Figure before in cascaded filters determine the capacitance values required to produce a critical frequency of 2680 Hz if all the resistors in the *RC* low-pass circuits are 1.8 kΩ. Also select values for the feedback resistors to get a Butterworth response

Both stages must have the same f_c. Assuming equal-value capacitors,

$$f_c = \frac{1}{2\pi RC}$$

$$C = \frac{1}{2\pi Rf_c} = \frac{1}{2\pi(1.8\,\text{k}\Omega)(2680\,\text{Hz})} = 0.033\,\mu\text{F}$$

$$C_{A1} = C_{B1} = C_{A2} = C_{B2} = \mathbf{0.033\,\mu F}$$

Also select $R_2 = R_4 = 1.8\,\text{k}\Omega$ for simplicity. Refer to Table 15–1. For a Butterworth response in the first stage, $DF = 1.848$ and $R_1/R_2 = 0.152$. Therefore,

$$R_1 = 0.152R_2 = 0.152(1800\,\Omega) = \mathbf{274\,\Omega}$$

Choose $R_1 = 270\,\Omega$.

In the second stage, $DF = 0.765$ and $R_3/R_4 = 1.235$. Therefore,

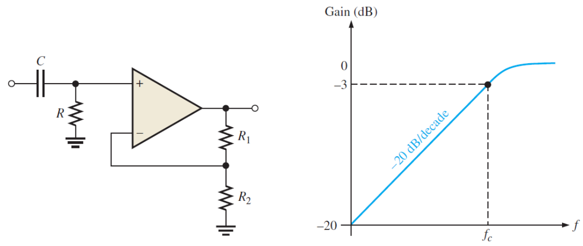
$$R_3 = 1.235R_4 = 1.235(1800\,\Omega) = \mathbf{2.22\,k\Omega}$$

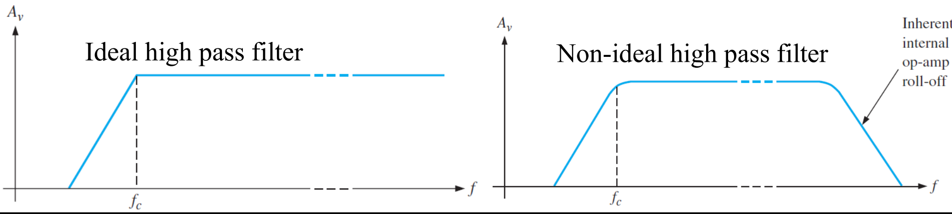
Choose $R_3 = 2.2\,\text{k}\Omega$.

15.4: Active High-Pass Filters

A Single-Pole Filter

- A high-pass active filter with a roll-off -20 dB/decade is shown in Figure. Notice that the input circuit is a single high-pass *RC* circuit. The negative feedback circuit is the same as for the low-pass filters previously discussed. The high-pass response curve is shown in Figure 15–13(b).

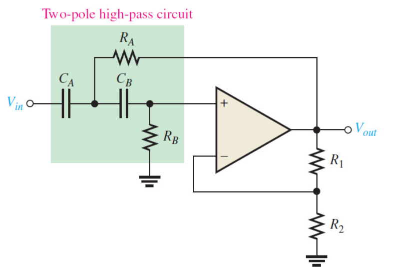




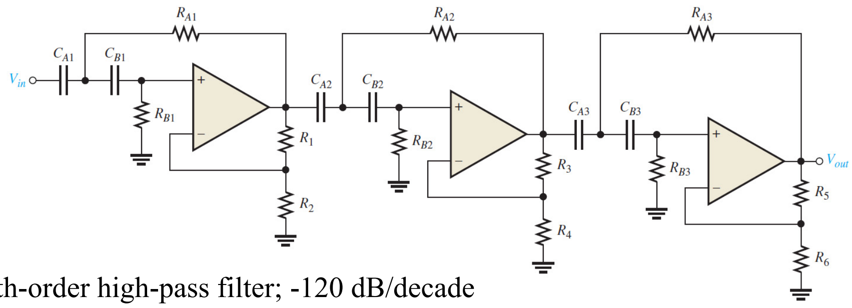
15.4: Active High-Pass Filters

The Sallen-Key High-Pass Filter (Double-Pole High-Pass Filter)

- It is an active filter with a two high-pass *RC* circuits that provides a roll-off of -40 dB/decade



Cascading High-Pass Filters



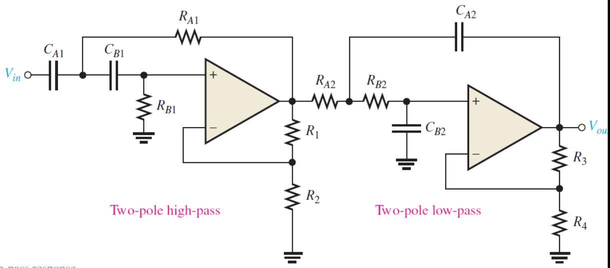
Sixth-order high-pass filter; -120 dB/decade

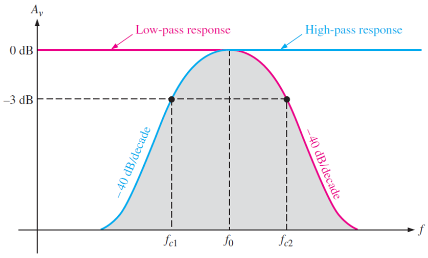
15.5: Active Band-Pass Filters

■ As mentioned, band-pass filters pass all frequencies bounded by a lower-frequency limit and an upper-frequency limit and reject all others lying outside this specified band

Cascaded High-Pass and Low-Pass Filters

■ implementing a band-pass filter can be done by cascading arrangement of a high-pass filter and a low-pass filter, as shown in Figure





$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}}$$