Cấu trúc dữ liệu và giải thuật

BALANCED SEARCH TREES

Contents

2

- Balanced Search Trees
- 2-3 Trees
- 2-3-4 Trees

Balanced Search Trees

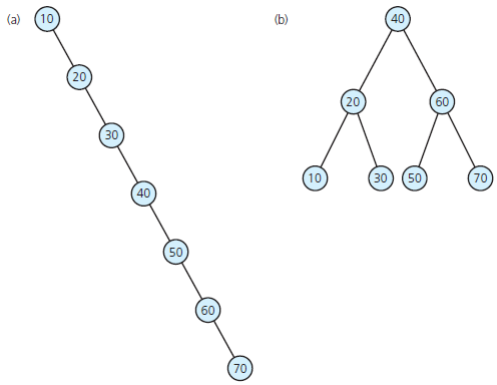
3

- Height of a binary search tree sensitive to order of insertions and removals
  - Minimum = $log_2 (n + 1)$
  - Maximum = n
- Various search trees can retain balance despite insertions and removals

Balanced Search Trees

4

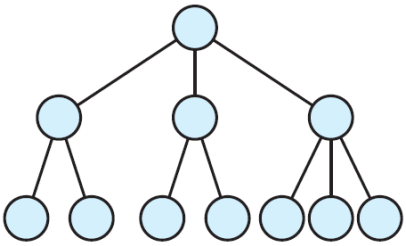
- FIGURE 19-1 (a) A binary search tree of maximum height; (b) a binary search tree of minimum height



2-3 Trees

5

- A 2-3 tree not a binary tree
- A 2-3 tree never taller than a minimum-height binary tree



2-3 Trees

6

- Placing data items in nodes of a 2-3 tree
  - A 2-node (has two children) must contain single data item greater than left child's item(s) and less than right child's item(s)
  - A 3-node (has three children) must contain two data items, S and L , such that
    - S is greater than left child's item(s) and less than middle child's item(s);
    - L is greater than middle child's item(s) and less than right child's item(s).
  - Leaf may contain either one or two data items.

2-3 Trees

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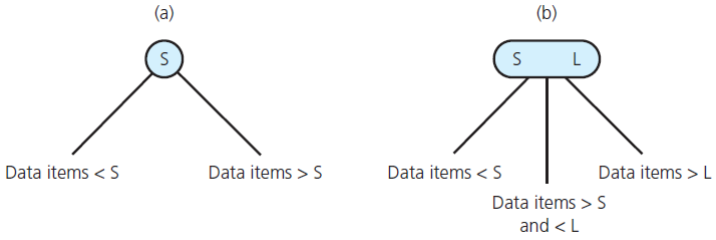
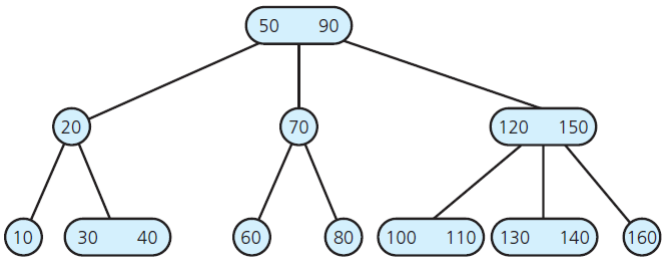
- FIGURE 19-3 Nodes in a 2-3 tree: (a) a 2-node;



2-3 Trees

8



A 2-3 tree

Traversing a 2-3 Tree

9

- Traverse 2-3 tree in sorted order by performing analogue of inorder traversal on binary tree:

```
// Traverses a nonempty 2-3 tree in sorted order.
inorder(23Tree: TwoThreeTree): void

   if (23Tree's root node r is a leaf)
      Visit the data item(s)
   else if (r has two data items)
   {
      inorder(left subtree of 23Tree's root)
      Visit the first data item
      inorder(middle subtree of 23Tree's root)
      Visit the second data item
      inorder(right subtree of 23Tree's root)
   }
   else // r has one data item
   {
      inorder(left subtree of 23Tree's root)
      Visit the data item
      inorder(right subtree of 23Tree's root)
   }
```


Searching a 2-3 Tree

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- Retrieval operation for 2-3 tree similar to retrieval operation for binary search tree

```
// Locates the value target in a nonempty 2-3 tree. Returns either the located
// entry or throws an exception if such a node is not found.
findItem(23Tree: TwoThreeTree, target: ItemType): ItemType

   if (target is in 23Tree's root node r)
   {  // The item has been found
      treeItem = the data portion of r
      return treeItem      // Success
   }
```

Searching a 2-3 Tree

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```
else if (r is a leaf)
   throw NotFoundException  // Failure

// Else search the appropriate subtree
else if (r has two data items)
{
   if (target < smaller item in r)
      return findItem(r's left subtree, target)
   else if (target < larger item in r)
      return findItem(r's middle subtree, target)
   else
      return findItem(r's right subtree, target)
}
else // r has one data item
{
   if (target < r's data item)
      return findItem(r's left subtree, target)
   else
      return findItem(r's right subtree, target)
}
```


Searching a 2-3 Tree

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- Possible to search 2-3 tree and shortest binary search tree with approximately same efficiency, because:
  - Binary search tree with n nodes cannot be shorter than $\log_2 (n + 1)$
  - 2-3 tree with n nodes cannot be taller than $\log_2 (n + 1)$
  - Node in a 2-3 tree has at most two items

Searching a 2-3 Tree

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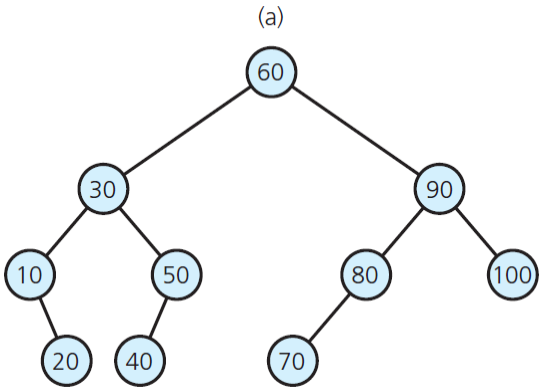
A balanced binary search tree



Searching a 2-3 Tree

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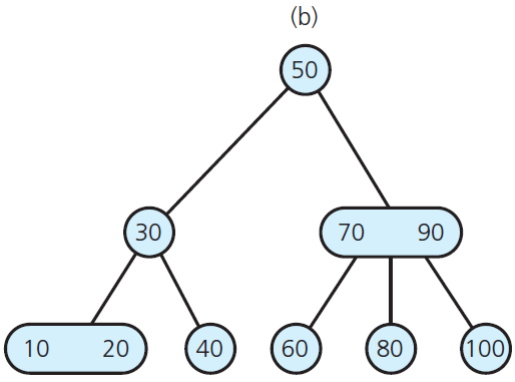
A 2-3 tree with the same entries



Inserting Data into a 2-3 Tree

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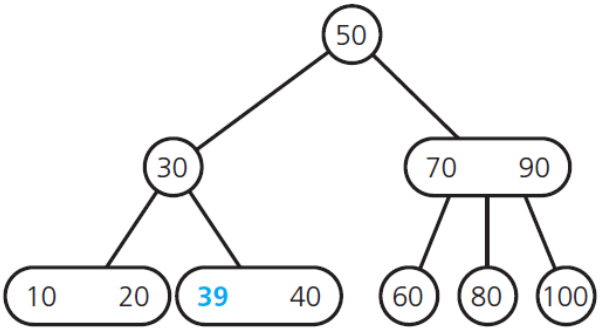
After inserting 39 into the tree

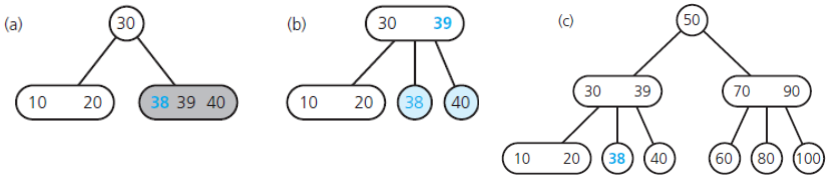


Inserting Data into a 2-3 Tree

17

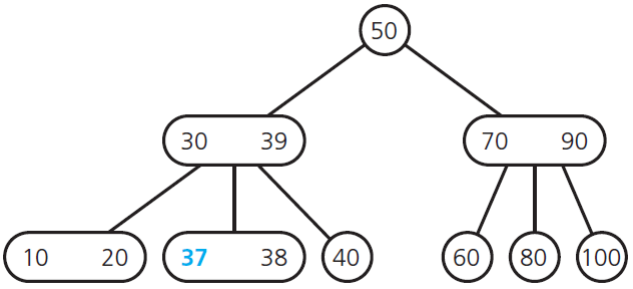
The steps for inserting 38 into the tree:

(a) The located node has no room;
(b) the node splits; (c) the resulting tree



Inserting Data into a 2-3 Tree

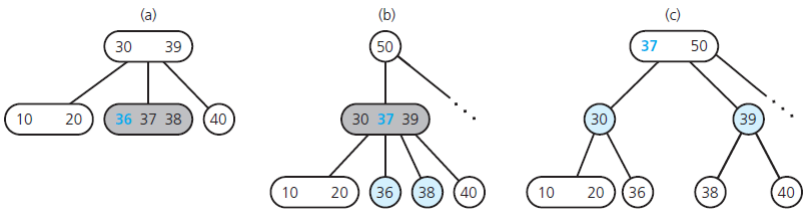
18



After inserting **37** into the tree

Inserting Data into a 2-3 Tree

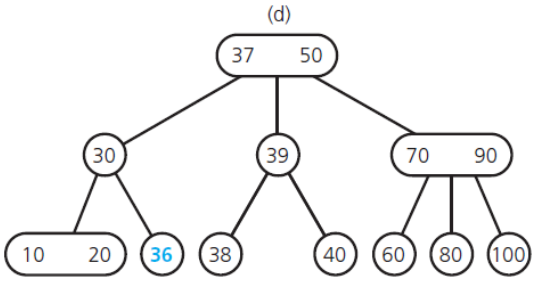
19



(a), (b), (c) The steps for inserting 36 into the tree

Inserting Data into a 2-3 Tree

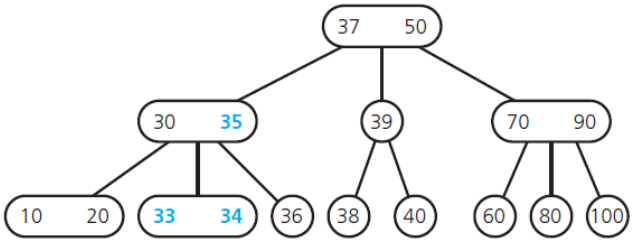
20



(d) the resulting tree

Inserting Data into a 2-3 Tree

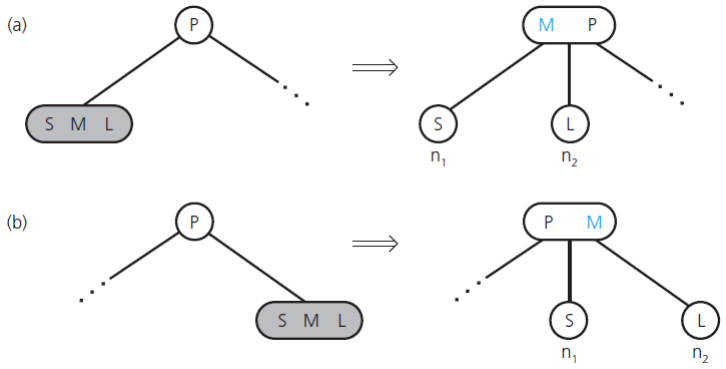
21



The tree after the insertion of 35, 34, and 33

Inserting Data into a 2-3 Tree

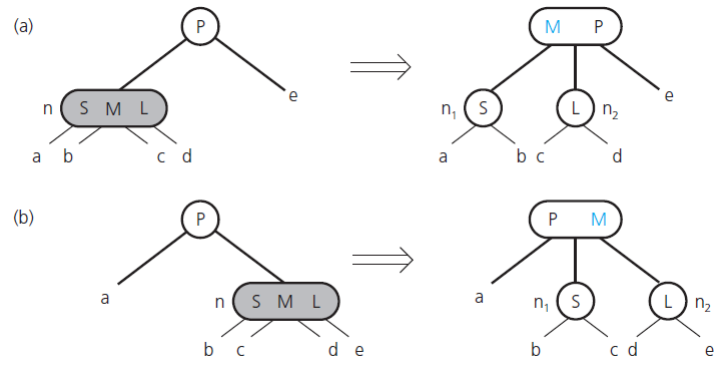
22



Splitting a leaf in a 2-3 tree when the leaf is a (a) left child; (b) right child

Inserting Data into a 2-3 Tree

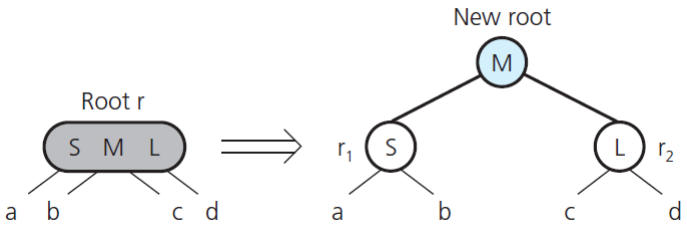
23



Splitting an internal node in a 2-3 tree when the node is a (a) left child; (b) right child

Inserting Data into a 2-3 Tree

24



Splitting the root of a 2-3 tree

Inserting Data into a 2-3 Tree

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- Summary of insertion strategy

```
// Inserts a new item into a 2-3 tree whose items are distinct and differ from the
// new item.
insertItem(23Tree: TwoThreeTree, newItem: ItemType)

   Locate the leaf, leafNode, in which newItem belongs
   Add newItem to leafNode

   if (leafNode has three items)
      split(leafNode)

// Splits node n, which contains two items. Note: If n is
// not a leaf, it has four children.
split(n: TwoThreeNode)

   if (n is the root)
      Create a new node p
   else
      Let p be the parent of n

   Replace node n with two nodes, n1 and n2, so that p is their parent
   Give n1 the item in n with the smallest value
   Give n2 the item in n with the largest value
```

Inserting Data into a 2-3 Tree

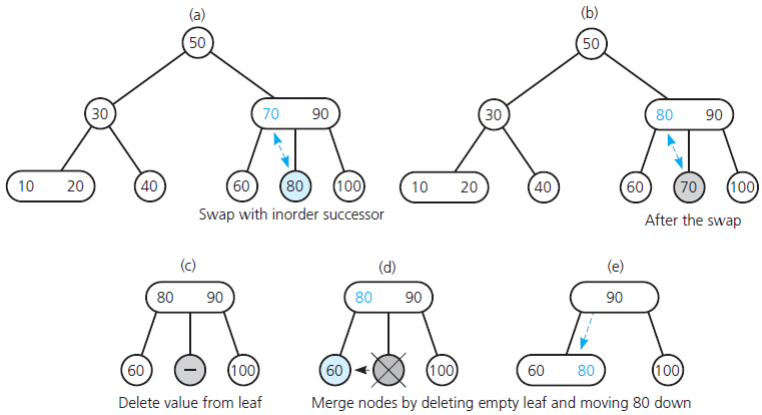
- Summary of insertion strategy

```
split(n: TwoThreeNode)

   if (n is the root)
      Create a new node p
   else
      Let p be the parent of n

   Replace node n with two nodes, n1 and n2, so that p is their parent
   Give n1 the item in n with the smallest value
   Give n2 the item in n with the largest value

   if (n is not a leaf)
   {
      n1 becomes the parent of n's two leftmost children
      n2 becomes the parent of n's two rightmost children
   }
   Move the item in n that has the middle value up to p
   if (p now has three items)
      split(p)
```


Removing Data from a 2-3 Tree

(a) A 2-3 tree;

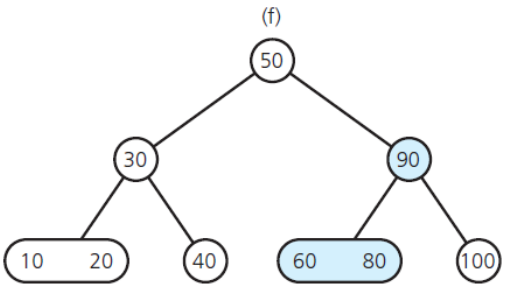
(b), (c), (d), (e) the steps for removing 70;



Removing Data from a 2-3 Tree

28

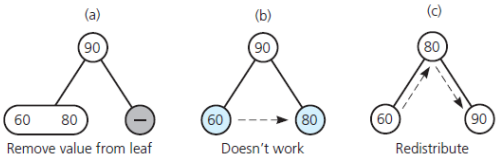
(f) the resulting tree;

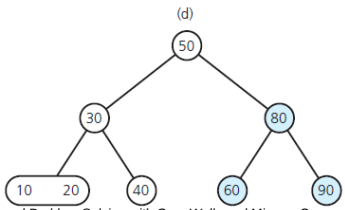


Removing Data from a 2-3 Tree

29

- (a), (b), (c) The steps for removing 100 from the tree in Figure 19-15f; (d) the resulting tree

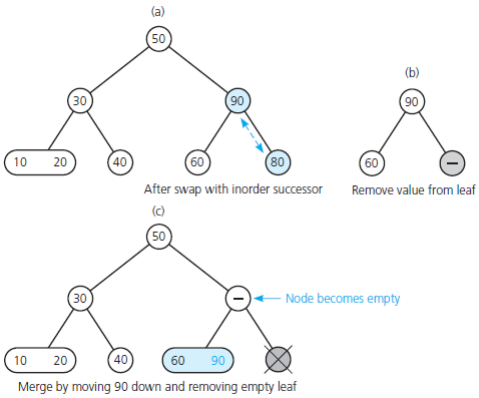




Removing Data from a 2-3 Tree

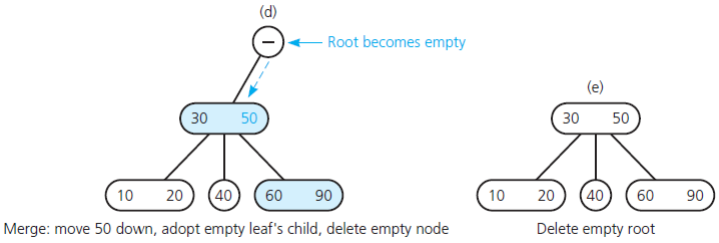
30

- FIGURE 19-17 The steps for removing 80 from the tree in Figure 19-16d



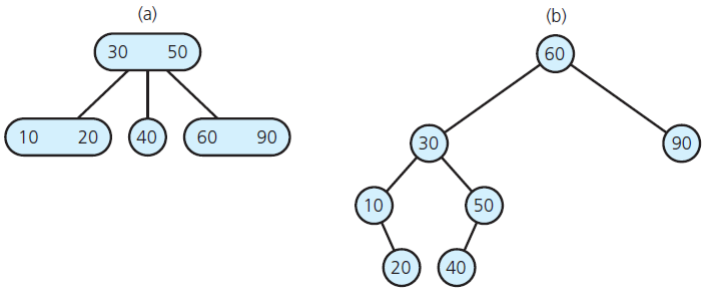
Removing Data from a 2-3 Tree

31



- FIGURE 19-17 The steps for removing 80 from the tree in Figure 19-16d

Removing Data from a 2-3 Tree



- FIGURE 19-18 Results of removing 70, 100, and 80 from (a) the 2-3 tree of Figure 19-15 a and (b) the binary search tree of Figure 19-5 a

Removing Data from a 2-3 Tree

- Algorithm for removing data from a 2-3 tree

```
// Removes the given data item from a 2-3 tree. Returns true if successful
// or false if no such item exists.
removeItem(23Tree: TwoThreeTree, dataItem: ItemType): boolean

   Attempt to locate dataItem
   if (dataItem is found)
   {
      if (dataItem is not in a leaf)
         Swap dataItem with its inorder successor, which will be in a leaf leafNode

      // The removal always begins at a leaf
      Remove dataItem from leaf leafNode
      if (leafNode now has no items)
         fixTree(leafNode)
      return true
   }
   else
      return false
```

Removing Data from a 2-3 Tree

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- Algorithm for removing data from a 2-3 tree

```
    else
        return false

// Completes the removal when node n is empty by either deleting the root,
// redistributing values, or merging nodes. Note: If n is internal, it has one child.
fixTree(n: TwoThreeNode)

   if (n is the root)
       Delete the root
   else
   {
      Let p be the parent of n
      if (some sibling of n has two items)
      {
          Distribute items appropriately among n, the sibling, and p
```


Removing Data from a 2-3 Tree

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- Algorithm for removing data from a 2-3 tree

```
          if (n is internal)
              Move the appropriate child from sibling to n
      }
      else // Merge the node
      {
          Choose an adjacent sibling s of n
          Bring the appropriate item down from p into s
          if (n is internal)
              Move n's child to s
          Remove node n
          if (p is now empty)
              fixTree(p)
      }
   }
```

Removing Data from a 2-3 Tree

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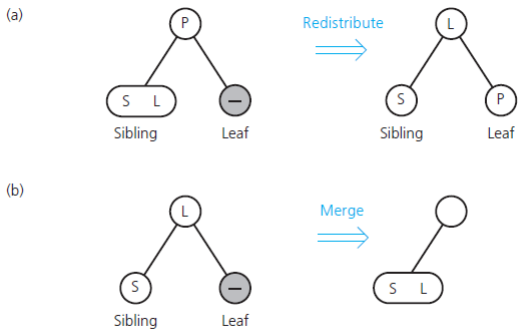
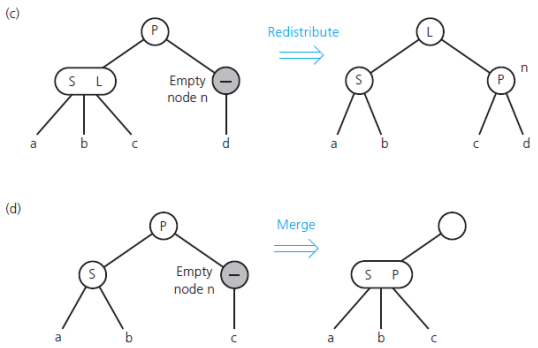
- FIGURE 19-19 (a) Redistributing values; (b) merging a leaf;



Removing Data from a 2-3 Tree

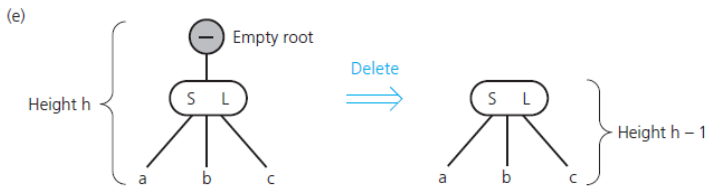
37



- FIGURE 19-19 (c) redistributing values and
- children; (d) merging internal nodes

Removing Data from a 2-3 Tree

38



- FIGURE 19-19 (e) deleting the root

2-3-4 Trees

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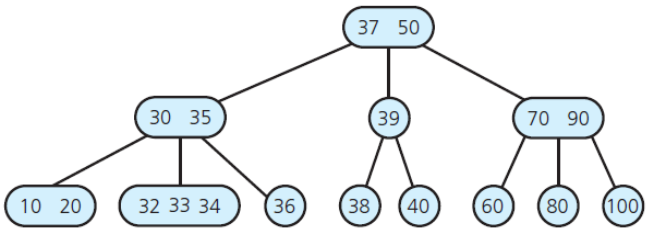
- FIGURE 19-20 A 2-3-4 tree with the same data items as the 2-3 tree in Figure 19-6 b



2-3-4 Trees

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- Rules for placing data items in the nodes of a 2-3-4 tree
  - 2-node (two children), must contain a single data item that satisfies relationships pictured in Figure 19-3 a.
  - 3-node (three children), must contain a single data item that satisfies relationships pictured in Figure 19-3 b.
  - . . .

2-3-4 Trees

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- 4-node (four children) must contain three data items S , M , and L that satisfy:
  - S is greater than left child's item(s) and less than middle-left child's item(s)
  - M is greater than middle-left child's item(s) and less than middle-right child's item(s);
  - L is greater than middle-right child's item(s) and less than right child's item(s).
- A leaf may contain either one, two, or three data items

2-3-4 Trees

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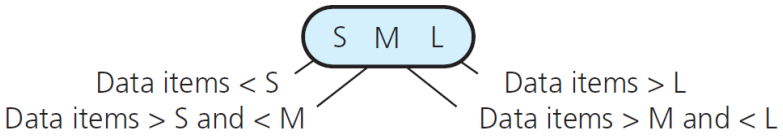
- FIGURE 19-21 A 4-node in a 2-3-4 tree



2-3-4 Trees

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- Has more efficient insertion and removal operations than a 2-3 tree
- Has greater storage requirements due to the additional data members in its 4-nodes

```
template<class ItemType>
class QuadNode
{
private:
   ItemType smallItem, middleItem, largeItem; // Data portion
   QuadNode<ItemType>* leftChildPtr;          // Left-child pointer
   QuadNode<ItemType>* leftMidChildPtr;       // Middle-left-child pointer
   QuadNode<ItemType>* rightMidChildPtr;      // Middle-right-child pointer
   QuadNode<ItemType>* rightChildPtr;         // Right-child pointer

   // Constructors, accessor methods, and mutator methods are here.

   . . .

}; // end QuadNode
```

2-3-4 Trees

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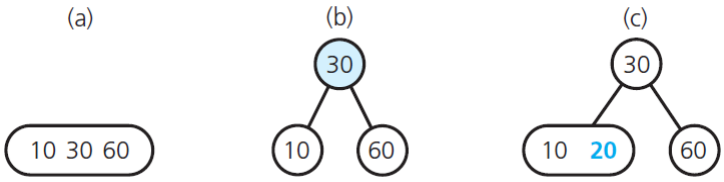
- Searching and Traversing a 2-3-4 Tree
  - Simple extensions of the corresponding algorithms for a 2-3 tree
- Inserting Data into a 2-3-4 Tree
  - Insertion algorithm splits a node by moving one of its items up to its parent node
  - Splits 4-nodes as soon as it encounters them on the way down the tree from the root to a leaf

2-3-4 Trees

45



- FIGURE 19-22 Inserting 20 into a one-node 2-3-4 tree (a) the original tree; (b) after splitting the node; (c) after inserting 20

2-3-4 Trees

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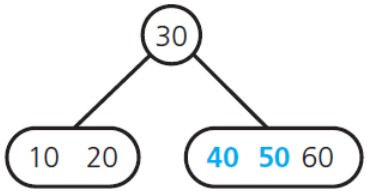

- FIGURE 19-23 After inserting 50 and 40 into the tree in Figure 19-22c

2-3-4 Trees

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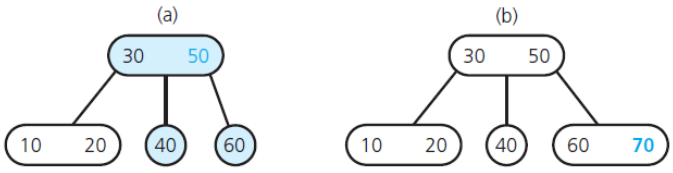

- FIGURE 19-24 The steps for inserting 70 into the tree in Figure 19-23: (a) after splitting the 4-node; (b) after inserting 70

2-3-4 Trees

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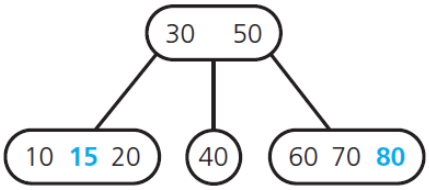

- FIGURE 19-25 After inserting 80 and 15 into the tree in Figure 19-24b

2-3-4 Trees

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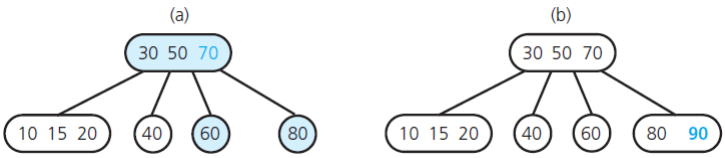
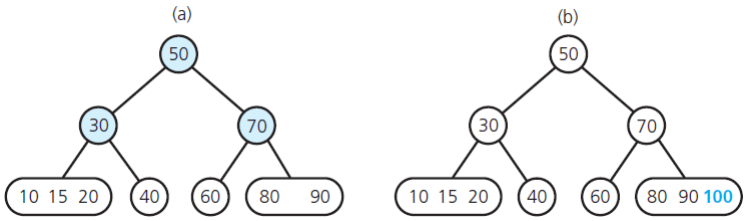


- FIGURE 19-26 The steps for inserting 90 into the tree in Figure 19-25

2-3-4 Trees

50



- FIGURE 19-27 The steps for inserting 100 into the tree in Figure 19-26b

2-3-4 Trees

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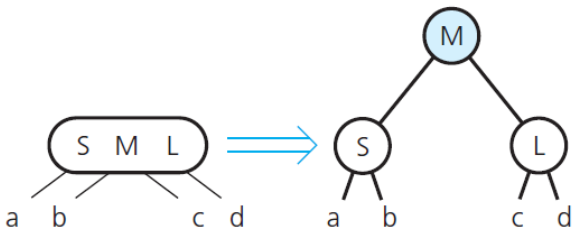
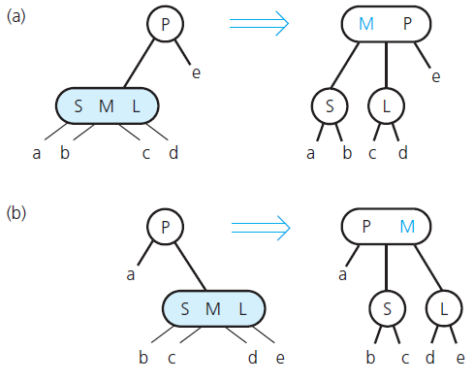


- FIGURE 19-28 Splitting a 4-node root during insertion into a 2-3-4 tree

2-3-4 Trees

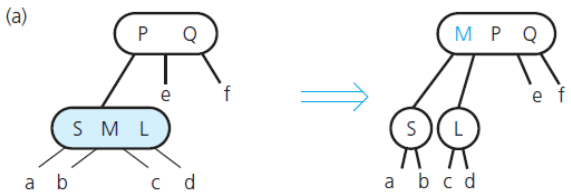
52



- FIGURE 19-29 Splitting a 4-node whose parent is a 2-node during insertion into a 2-3-4 tree, when the 4-node is a (a) left child; (b) right child

2-3-4 Trees

53

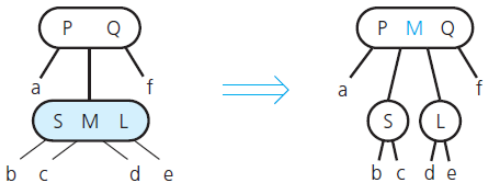


- FIGURE 19-30 Splitting a 4-node whose parent is a 3-node during insertion into a 2-3-4 tree, when the 4-node is a (a) left child

2-3-4 Trees

54

(b)

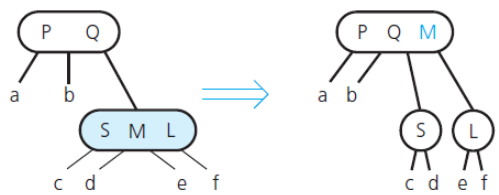


- FIGURE 19-30 Splitting a 4-node whose parent is a 3-node during insertion into a 2-3-4 tree, when the 4-node is a (b) middle child

2-3-4 Trees

55

(c)



- FIGURE 19-30 Splitting a 4-node whose parent is a 3-node during insertion into a 2-3-4 tree, when the 4-node is a (c) right child

2-3-4 Trees

- Removing Data from a 2-3-4 Tree
  - Removal algorithm has same beginning as removal algorithm for a 2-3 tree
  - Locate the node *n* that contains the item *I* you want to remove
  - Find *I* 's inorder successor and swap it with *I* so that the removal will always be at a leaf
  - If leaf is either a 3-node or a 4-node, remove *I* .