Artificial Intelligence

MACHINE LEARNING BASIC ALGORITHMS

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Outline

- Introduction to Machine learning
- ID3 Decision tree
- Naïve Bayesian classification

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Machine Learning

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What is machine learning?

 Machine learning involves adaptive mechanisms that enable computers to learn from experience, learn by example and learn by analogy.



Types of machine learning



Source: https://ldi.upenn.edu/sites/default/files/Introduction-to-Machine-Learning.pdf

Types of machine learning



Source: https://www.ceralytics.com/3-types-of-machine-learning/

Machine learning algorithms



Regression Algorithms



Decision Tree Algorithms



Ensemble Algorithms



Regularization Algorithms



Support Vector Machines



Algorithms





Instance-based Algorithms



Association Rule Learning Algorithms





Artificial Neural Network

Algorithms

Dimensional Reduction

Algorithms

Deep Learning Algorithms



Clustering Algorithms



Bayesian Algorithms

IF \rightarrow THEN	
IF \rightarrow THEN	
IF \rightarrow THEN	
IF \rightarrow THEN	

Learning Classifier Systems

Source: https://ldi.upenn.edu/sites/default/files/Introduction-to-Machine-Learning.pdf

Supervised learning

• Learn a function that maps an input to an output based on examples, which are pairs of input-output values.





Supervised learning: Examples



- Predict SPAM if unknown AND (money OR pills)
- Predict SPAM if 2money + 3pills 5 known > 0

Linearly separable

Supervised learning: Examples

Object detection



Indoor scene recognition



Handwritten digit recognition







Scene text recognition

Supervised learning: More examples

• Weather prediction: Predict the weather type or the temperature at any given location...



- Medicine: diagnose a disease (or response to chemo drug X, or whether a patient is re-admitted soon?)
 - Input: from symptoms, lab measurements, test results, DNA tests, ...
 - Output: one of set of possible diseases, or "none of the above"
 - E.g., audiology, thyroid cancer, diabetes, etc.



- Computational economics:
 - Predict if a user will click on an ad so as to decide which ad to show
 - Predict if a stock will rise or fall (with specific amounts)

Classification vs. Regression

- Train a model to predict a categorical dependent variable
- Case studies: predicting disease, classifying images, predicting customer churn, buy or won't buy, etc.





Binary classification vs. Multiclass classification vs. Multilabel classification

Classification vs. Regression

- Train a model to predict a continuous dependent variable
- Case studies: predicting height of children, predicting sales, forecasting stock prices, etc.



Unsupervised learning

- Infer a function to describe hidden structure from "unlabeled" data
 - A classification (or categorization) is not included in the observations.





Unsupervised learning: Examples

 Social network analysis: cluster users of social networks by interest (community detection)



Ref: Shevtsov, Alexander, et al. "Analysis of Twitter and YouTube during US elections 2020." *arXiv e-prints* (2020): arXiv-2010.

Semi-supervised learning



 The model is initially trained with a small amount of labeled data and a large amount of unlabeled data.



Reinforcement learning

• The agent learns from the environment by interacting with it and receives rewards for performing actions.



Reinforcement learning: Example



Reinforcement learning: Examples

Multi-Agent Hide and Seek

https://openai.com/blog/emergent-tool-use/

https://arxiv.org/pdf/1909.07528.pdf

Machine learning and related concepts



Source: https://blogs.nvidia.com/blog/2016/07/29/whats-difference-artificial-intelligence-machine-learningdeep-learning-ai/ 21

Machine learning and related concepts







Learning agents – Why learning?

- Unknown environments
 - A robot designed to navigate mazes must learn the layout of each new maze it encounters.
- Environment changes over time
 - An agent designed to predict tomorrow's stock market prices must learn to adapt when conditions change from boom to bust.
- No idea how to program a solution
 - The task to recognizing the faces of family members

Learning element

- Design of a learning element is affected by
 - Which *components* is to be improved
 - What *prior knowledge* the agent already has
 - What *representation* is used for the components
 - What *feedback* is available to learn these components
- Type of feedback
 - Supervised learning: correct answers for each example
 - Unsupervised learning: correct answers not given
 - Reinforcement learning: occasional rewards

Supervised learning

- Simplest form: learn a function from examples
- Given a training set of *N* example input-output pairs

 $(x_1, y_1), (x_2, y_2), \dots, (x_N, y_N)$

- where each y_i was generated by an unknown function y = f(x)
- Find a hypothesis h such that $h \approx f$
- To measure the accuracy of a hypothesis, give it a test set of examples that are different with those in the training set.



Supervised learning

- Construct h so that it agrees with f.
- The hypothesis *h* is **consistent** if it agrees with *f* on all observations.
- Ockham's razor: Select the simplest consistent hypothesis.



Supervised learning problems

• h(x) = the predicted output value for the input x



Regression vs. Classification

• Estimating the price of a house



Size of the House

- Will you pass or fail the exam?
 - 2 classes: Fail/Pass



- Is this an apple, an orange or a tomato?
 - 3 classes: Apple / Orange / Tomato



The wait@restaurant problem

Predicting whether a certain person will wait to have a seat in a restaurant.



The wait@restaurant problem

- The decision is based on the following attributes
 - **1. Alternate:** is there an alternative restaurant nearby?
 - 2. Bar: is there a comfortable bar area to wait in?
 - 3. Fri/Sat: is today Friday or Saturday?
 - 4. Hungry: are we hungry?
 - 5. Patrons: number of people in the restaurant (None, Some, Full)
 - 6. Price: price range (\$, \$\$, \$\$\$)
 - 7. Raining: is it raining outside?
 - 8. Reservation: have we made a reservation?
 - 9. Type: kind of restaurant (French, Italian, Thai, Burger)
 - **10. WaitEstimate:** estimated waiting time (0-10, 10-30, 30-60, >60)

The wait@restaurant decision tree



Learning decision trees

- Divide and conquer: Split data into smaller and smaller subsets
- Splits are usually on a single variable



• After splitting up, each outcome is a new decision tree learning problem with fewer examples and one less attribute.

Learning decision trees



Splitting the examples by testing on attributes

ID3 Decision tree algorithm

- 1. The remaining examples are **all** positive (or **all** negative), \rightarrow DONE, it is possible to answer Yes or No.
 - E.g., in Figure (b), None and Some branches
- 2. There are **some** positive and **some** negative examples \rightarrow choose the **best** attribute to split them
 - E.g., in Figure (b), Hungry is used to split the remaining examples

ID3 Decision tree algorithm

3. No examples left at a branch \rightarrow return a default value.

- No example has been observed for a combination of attribute values
- The default value is calculated from the plurality classification of all the examples that were used in constructing the node's parent.
- These are passed along in the variable parent examples
- 4. No attributes left but both positive and negative examples \rightarrow return the plurality classification of remaining ones.
 - Examples of the same description, but different classifications
 - Usually an error or noise in the data, nondeterministic domain, or no observation of an attribute that would distinguish the examples.
ID3 Decision tree: Pseudo-code

function DECISION-TREE-LEARNING(exam	ples, attributes, parent examples)
returns a tree	
if <i>examples</i> is empty	No examples left
then return PLURALITY-VALUE(pa	rent examples)
else if all <i>examples</i> have the same classified	cation
then return the classification	are all pos/all neg
else if attributes is empty	
then return PLURALITY-VALUE(examp	oles)
else	No attributes left but examples are still pos & neg

ID3 Decision tree: Pseudo-code

function DECISION-TREE-LEARNING(*examples*, *attributes*, *parent examples*) **returns** a tree

...

else

 $A \leftarrow argmax_{a \in attributes}$ IMPORTANCE(*a*, *examples*)

tree \leftarrow a new decision tree with root test *A*

for each value v_k of *A* do

 $exs \leftarrow \{e : e \in examples \text{ and } e.A = v_k\}$

 $subtree \leftarrow DECISION-TREE-LEARNING(exs, attributes - A, examples)$

add a branch to *tree* with label $(A = v_k)$ and subtree *subtree*

return tree

Decision tree: Inductive learning

• Simplest: Construct a decision tree with one leaf for every example

→ memory based learning

 \rightarrow worse generalization.



- Advanced: Split on each variable so that the purity of each split increases (i.e. either only yes or only no)
 - E.g., using Entropy to measure the purity of data

A purity measure with entropy

• Entropy is a measure of the uncertainty of a random variable V with values v_k .

$$H(V) = \sum_{k} P(v_k) \log_2 \frac{1}{P(v_k)} = -\sum_{k} P(v_k) \log_2 P(v_k)$$

• v_k is a class in V (e.g., yes/no in binary classification)

P(*v_k*) is the proportion of the number of elements in class *v_k* to the number of elements in *V*

A purity measure with entropy

- Entropy is maximal when all possibilities are equally likely.
- Entropy is zero in a pure "yes" (or pure "no") node.





Provost, Foster; Fawcett, Tom. Data Science for Business: What You Need to Know about Data Mining and Data-Analytic Thinking

• Decision tree aims to **decrease** the entropy in each node.

The wait@restaurant training data

T = True, F = False

Example					At	tributes	3				Target
Encompro	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Type	Est	WillWait
X_1	T	F	F	Т	Some	\$\$\$	F	Т	French	0–10	Т
X_2	Т	F	F	Т	Full	\$	F	F	Thai	30–60	F
X_3	F	Т	F	F	Some	\$	F	F	Burger	0–10	Т
X_4	Т	F	Т	Т	Full	\$	F	F	Thai	10–30	Т
X_5	Т	F	Т	F	Full	\$\$\$	F	Т	French	>60	F
X_6	F	Т	F	Т	Some	\$\$	Т	Т	Italian	0–10	Т
X_7	F	Т	F	F	None	\$	Т	F	Burger	0–10	F
X_8	F	F	F	Т	Some	\$\$	Т	Т	Thai	0–10	Т
X_9	F	Т	Т	F	Full	\$	Т	F	Burger	>60	F
X_{10}	Т	Т	Т	Т	Full	\$\$\$	F	Т	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	Т	Т	Т	Т	Full	\$	F	F	Burger	30–60	Т

 $H(S) = -\binom{6}{12} \log_2 \binom{6}{12} - \binom{6}{12} \log_2 \binom{6}{12} = 1$

6 True, 6 False 42



Calculate Average Entropy of attribute Alternate

 $AE_{Alternate} = P(Alt = T) \times H(Alt = T) + P(Alt = F) \times H(Alt = F)$

$$AE_{Alternate} = \frac{6}{12} \left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right) - \left(\frac{3}{6}\log_2\frac{3}{6}\right) \right] + \frac{6}{12} \left[-\left(\frac{3}{6}\log_2\frac{3}{6}\right) - \left(\frac{3}{6}\log_2\frac{3}{6}\right) \right] = 1$$



• Information Gain is the difference in entropy from before to after the set *S* is split on the selected attribute. $IG(Alternate, S) = H(S) - AE_{Alternate} = 1 - 1 = 0$



$$AE_{Bar} = \frac{6}{12} \left[-\left(\frac{3}{6}\log_2 \frac{3}{6}\right) - \left(\frac{3}{6}\log_2 \frac{3}{6}\right) \right] + \frac{6}{12} \left[-\left(\frac{3}{6}\log_2 \frac{3}{6}\right) - \left(\frac{3}{6}\log_2 \frac{3}{6}\right) \right] = 1$$
$$IG(Bar, S) = H(S) - AE_{Bar} = 1 - 1 = 0$$



$$AE_{Sat/Fri?} = \frac{5}{12} \left[-\left(\frac{2}{5}\log_2\frac{2}{5}\right) - \left(\frac{3}{5}\log_2\frac{3}{5}\right) \right] + \frac{7}{12} \left[-\left(\frac{4}{7}\log_2\frac{4}{7}\right) - \left(\frac{3}{7}\log_2\frac{3}{7}\right) \right] = 0.979$$

 $IG(Sat/Fri?, S) = H(S) - AE_{Sat/Fri?} = 1 - 0.979 = 0.021$



$$\begin{aligned} AE_{Hungry} &= \frac{7}{12} \left[-\left(\frac{5}{7}\log_2 \frac{5}{7}\right) - \left(\frac{2}{7}\log_2 \frac{2}{7}\right) \right] + \frac{5}{12} \left[-\left(\frac{1}{5}\log_2 \frac{1}{5}\right) - \left(\frac{4}{5}\log_2 \frac{4}{5}\right) \right] \\ &= 0.804 \end{aligned}$$

 $IG(Hungry, S) = H(S) - AE_{Hungry} = 1 - 0.804 = 0.196$



Example					At	tribute	8				Target
Litempre	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Type	Est	WillWait
X_1	Τ	F	F	Т	Some	\$\$\$	F	Т	French	0–10	Т
X_2	Т	F	F	Т	Full	\$	F	F	Thai	30–60	F
X_3	F	Т	F	F	Some	\$	F	F	Burger	0–10	Т
X_4	Т	F	Т	Т	Full	\$	F	F	Thai	10–30	Т
X_5	Т	F	Т	F	Full	\$\$\$	F	Т	French	>60	F
X_6	F	Т	F	Т	Some	\$\$	Т	Т	Italian	0–10	Т
X_7	F	Т	F	F	None	\$	Т	F	Burger	0–10	F
X_8	F	F	F	Т	Some	\$\$	Т	Т	Thai	0–10	Т
X_9	F	Т	Т	F	Full	\$	Т	F	Burger	>60	F
X_{10}	Т	Т	Т	Т	Full	\$\$\$	F	Т	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	Т	Т	Т	Т	Full	\$	F	F	Burger	30–60	Т

$$AE_{Raining} = \frac{4}{12} \left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right) - \left(\frac{2}{4}\log_2\frac{2}{4}\right) \right] + \frac{8}{12} \left[-\left(\frac{4}{8}\log_2\frac{4}{8}\right) - \left(\frac{4}{8}\log_2\frac{4}{8}\right) \right] = 1$$
$$IG(Raining, S) = H(S) - AE_{Hungry} = 1 - 1 = 0$$



$$AE_{Reservation} = \frac{5}{12} \left[-\left(\frac{3}{5}\log_2\frac{3}{5}\right) - \left(\frac{2}{5}\log_2\frac{2}{5}\right) \right] + \frac{7}{12} \left[-\left(\frac{3}{7}\log_2\frac{3}{7}\right) - \left(\frac{4}{7}\log_2\frac{4}{7}\right) \right] = 0.979$$

 $IG(Reservation, S) = H(S) - AE_{Reservation} = 1 - 0.979 = 0.021$



$$\begin{aligned} &AE_{Patron} \\ &= \frac{2}{12} \left[-\left(\frac{0}{2}\log_2\frac{0}{2}\right) - \left(\frac{2}{2}\log_2\frac{2}{2}\right) \right] + \frac{4}{12} \left[-\left(\frac{4}{4}\log_2\frac{4}{4}\right) - \left(\frac{0}{4}\log_2\frac{0}{4}\right) \right] \\ &+ \frac{6}{12} \left[-\left(\frac{2}{6}\log_2\frac{2}{6}\right) - \left(\frac{4}{6}\log_2\frac{4}{6}\right) \right] = 0.541 \end{aligned}$$

 $IG(Patron, S) = H(S) - AE_{Patron} = 1 - 0.541 = 0.459$



Example					At	tributes	3				Target
Lineart	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Type	Est	WillWait
X_1	Т	F	F	T	Some	\$\$\$	F	T	French	0–10	T
X_2	Т	F	F	Т	Full	\$	F	F	Thai	30–60	F
X_3	F	Т	F	F	Some	\$	F	F	Burger	0–10	Т
X_4	Т	F	Т	Т	Full	\$	F	F	Thai	10–30	Т
X_5	Т	F	Т	F	Full	\$\$\$	F	Т	French	>60	F
X_6	F	Т	F	Т	Some	\$\$	Т	Т	Italian	0–10	Т
X_7	F	Т	F	F	None	\$	Т	F	Burger	0–10	F
X_8	F	F	F	Т	Some	\$\$	Т	Т	Thai	0–10	Т
X_9	F	Т	Т	F	Full	\$	Т	F	Burger	>60	F
X_{10}	Т	Т	Т	Т	Full	\$\$\$	F	Т	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	Т	Т	T	Т	Full	\$	F	F	Burger	30–60	Т

$$\begin{aligned} &AE_{Price} \\ &= \frac{6}{12} \left[-\left(\frac{3}{6}\log_2 \frac{3}{6}\right) - \left(\frac{3}{6}\log_2 \frac{3}{6}\right) \right] + \frac{2}{12} \left[-\left(\frac{2}{2}\log_2 \frac{2}{2}\right) - \left(\frac{0}{2}\log_2 \frac{0}{2}\right) \right] \\ &+ \frac{4}{12} \left[-\left(\frac{1}{4}\log_2 \frac{1}{4}\right) - \left(\frac{3}{4}\log_2 \frac{3}{4}\right) \right] = 0.770 \\ &IG(Price,S) = H(S) - AE_{Price} = 1 - 0.770 = 0.23 \end{aligned}$$



Example					At	tributes	3				Target
Litempre	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Type	Est	WillWait
X_1	Т	F	F	Т	Some	\$\$\$	F	T	French	0–10	T
X_2	Т	F	F	Т	Full	\$	F	F	Thai	30–60	F
X_3	F	Т	F	F	Some	\$	F	F	Burger	0–10	Т
X_4	Т	F	Т	Т	Full	\$	F	F	Thai	10–30	Т
X_5	Т	F	Т	F	Full	\$\$\$	F	Т	French	>60	F
X_6	F	Т	F	Т	Some	\$\$	Т	Т	Italian	0–10	Т
X_7	F	Т	F	F	None	\$	Т	F	Burger	0–10	F
X_8	F	F	F	Т	Some	\$\$	Т	T	Thai	0–10	T
X_9	F	Т	Т	F	Full	\$	Т	F	Burger	>60	F
X_{10}	Т	Т	T	Т	Full	\$\$\$	F	T	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	Т	Т	Т	Т	Full	\$	F	F	Burger	30–60	Т

$$\begin{split} &AE_{Type} \\ &= \frac{2}{12} \left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right) - \left(\frac{1}{2}\log_2\frac{1}{2}\right) \right] + \frac{2}{12} \left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right) - \left(\frac{1}{2}\log_2\frac{1}{2}\right) \right] \\ &+ \frac{4}{12} \left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right) - \left(\frac{2}{4}\log_2\frac{2}{4}\right) \right] + \frac{4}{12} \left[-\left(\frac{2}{4}\log_2\frac{2}{4}\right) - \left(\frac{2}{4}\log_2\frac{2}{4}\right) \right] = 1 \\ &IG(Type, S) = H(S) - AE_{Type} = 1 - 1 = 0 \end{split}$$



Example					At	tributes	3				Target
Litempre	Alt	Bar	Fri	Hun	Pat	Price	Rain	Res	Type	Est	WillWait
X_1	Т	F	F	Т	Some	\$\$\$	F	Т	French	0–10	Т
X_2	Т	F	F	Т	Full	\$	F	F	Thai	30–60	F
X_3	F	Т	F	F	Some	\$	F	F	Burger	0–10	Т
X_4	Т	F	Т	Т	Full	\$	F	F	Thai	10–30	Т
X_5	Т	F	Т	F	Full	\$\$\$	F	Т	French	>60	F
X_6	F	Т	F	Т	Some	\$\$	Т	Т	Italian	0–10	Т
X_7	F	Т	F	F	None	\$	Т	F	Burger	0–10	F
X_8	F	F	F	Т	Some	\$\$	Т	Т	Thai	0–10	Т
X_9	F	Т	T	F	Full	\$	Т	F	Burger	>60	F
X_{10}	Т	Т	T	Т	Full	\$\$\$	F	Т	Italian	10–30	F
X_{11}	F	F	F	F	None	\$	F	F	Thai	0–10	F
X_{12}	Т	T	T	T	Full	\$	F	F	Burger	30–60	Т

 $AE_{Est.waiting time}$

$$= \frac{6}{12} \left[-\left(\frac{4}{6}\log_2\frac{4}{6}\right) - \left(\frac{2}{6}\log_2\frac{2}{6}\right) \right] + \frac{2}{12} \left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right) - \left(\frac{1}{2}\log_2\frac{1}{2}\right) \right] \\ + \frac{2}{12} \left[-\left(\frac{1}{2}\log_2\frac{1}{2}\right) - \left(\frac{1}{2}\log_2\frac{1}{2}\right) \right] + \frac{2}{12} \left[-\left(\frac{0}{2}\log_2\frac{0}{2}\right) - \left(\frac{2}{2}\log_2\frac{2}{2}\right) \right] = 0.792$$

 $IG(Est.waiting time, S) = H(S) - AE_{Est.waiting time} = 1 - 0.792$ = 0.208

 Largest Information Gain (0.459) / Smallest Entropy (0.541) achieved by splitting on Patrons.



• Continue making new splits, always purifying nodes

ID3 Decision tree algorithm



Performance measurement

- How do we know that $h \approx f$?
 - Use theorems of computational or statistical learning theory 1.
 - 2. Try *h* on a new test set of examples
 - Use the same distribution over example space as training set



Quiz 01: ID3 decision tree

- The data represent files on a computer system. Possible values of the class variable are "infected", which implies the file has a virus infection, or "clean" if it doesn't.
- Derive decision tree for virus identification.

No.	Writable	Updated	Size	Class
1	Yes	No	Small	Infected
2	Yes	Yes	Large	Infected
3	No	Yes	Med	Infected
4	No	No	Med	Clean
5	Yes	No	Large	Clean
6	No	No	Large	Clean



Naïve Bayesian classification



Bayesian classification

- A statistical classifier performs probabilistic prediction, i.e., predicts class membership probabilities
- Foundation: Based on Bayes' Theorem



 $P(c \mid \mathbf{X}) = P(x_1 \mid c) \times P(x_2 \mid c) \times \cdots \times P(x_n \mid c) \times P(c)$

Bayesian classification

• Performance

• A simple Bayesian classifier (e.g., naïve Bayesian), has comparable performance with decision tree and selected neural networks.

Incremental

- Each training example can incrementally increase/decrease the probability that a hypothesis is correct
- That is, prior knowledge can be combined with observed data.

• Standard

 Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured

The buying computer dataset

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
3140	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
3140	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
3140	medium	no	excellent	yes
3140	high	yes	fair	yes
>40	medium	no	excellent	NO 61

Bayes' Theorem

- Total Probability Theorem: $P(B) = \sum_{i=1}^{M} P(B|A_i)P(A_i)$
- Let **X** be a data sample ("evidence") with unknown class label and *H* be a hypothesis that **X** belongs to class *C*
- Bayes' Theorem: $P(H \mid X) = \frac{P(X \mid H)P(H)}{P(X)}$
- Classification is to determine $P(H \mid \mathbf{X})$, the probability that the hypothesis *H* holds given the observed data sample **X**.

Bayes' Theorem

- **P(H)** (prior probability): the initial probability
 - E.g., X will buy computer, regardless of age, income, ...
- $P(\mathbf{X})$: the probability that sample data is observed
 - E.g., X is 31..40 and has a medium income, regardless of the buying
- P(X | H) (likelihood): the probability of observing the sample
 X, given that the hypothesis holds
 - E.g., given that **X** will buy computer, the probability that **X** is 31..40 and has a medium income
- $P(H \mid X) = \frac{P(X \mid H)P(H)}{P(X)}$ (posterior probability)
 - E.g., given that **X** is 31..40 and has a medium income, the probability that **X** will buy computer

Bayes' Theorem

- Informally, $P(H \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid H)P(H)}{P(\mathbf{X})}$ can be viewed as posteriori = likelihood * prior / evidence
- X belongs to C_i iff the probability $P(C_i | \mathbf{X})$ is the highest among all the $P(C_k | \mathbf{X})$ for all the k classes

Practical difficulty

- Require initial knowledge of many probabilities
- Significant computational cost involved

Classification with Bayes' Theorem

- Let *D* be a training set of tuples and associated class labels
- Each tuple is represented by a *n*-attribute $\mathbf{X} = (x_1, x_2, ..., x_n)$
- Suppose there are m classes C_1, C_2, \dots, C_m
- Classification is to derive the maximum posteriori $P(C_i | \mathbf{X})$ from **Bayes' theorem**

$$P(C_i \mid \mathbf{X}) = \frac{P(\mathbf{X} \mid C_i)P(C_i)}{P(\mathbf{X})}$$

P(X) is constant for all classes, only P(X | C_i)P(C_i) needs to be maximized.

Naïve Bayesian classification

- Class-conditional independence: There are no dependence relationships among the attributes
- The **naïve Bayesian classification** formula is written as $P(\mathbf{X} \mid C_i) = \prod_{k=1}^{n} P(x_k \mid C_i) = P(x_1 \mid C_i) \times P(x_2 \mid C_i) \times \dots \times P(x_n \mid C_i)$
 - A_k is categorical: $P(x_k | C_i)$ is the number of tuples in C_i having value x_k for A_k divided by $|C_{i,D}|$ (# of tuples of C_i in D)
 - A_k is continuous: $P(x_k | C_i) = g(x_k, \mu_{C_i}, \sigma_{C_i})$ with the Gaussian

distribution
$$g(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

• Count class distributions only \rightarrow computation cost reduced

Naïve Bayesian classification: An example

P(buys_computer = "yes")	9/14
P(buys_computer = "no")	5/14

	buys_computer = "yes"	buys_computer = "no"
age = "<=30"	2/9	3/5
age = "31…40"	4/9	0/5
age = ">40"	3/9	2/5
income = "low"	3/9	1/5
income = "medium"	4/9	2/5
income = "high"	2/9	2/5
student = "yes"	6/9	1/5
student = "no"	3/9	4/5
credit_rating = "fair"	6/9	2/5
credit_rating = "excellent"	3/9	3/5

Naïve Bayesian classification: An example

age	income	student	credit_rating	buys_computer
<=30	medium	yes	fair	?
	7 \			

- $P(\mathbf{X}|C_i)$
 - $P(X \mid buys_computer = "yes") = 2/9 * 4/9 * 6/9 * 6/9 = 0.044$
 - $P(X \mid buys_computer = "no") = 3/5 * 2/5 * 1/5 * 2/5 = 0.019$
- $P(\mathbf{X}|C_i) * P(C_i)$
 - $P(X \mid buys_computer = "yes") * P(buys_computer = "yes") = 0.028$
 - $P(\mathbf{X}| buys_computer = "no") * P(buys_computer = "no") = 0.007$
- $P(C_i \mid \mathbf{X})$
 - $P(buys_computer = "yes" | \mathbf{X}) = 0.8$
 - $P(buys_computer = "no" | \mathbf{X}) = 0.2$

Therefore, X belongs to class ("buys_computer = yes")

Avoiding the zero-probability issue

 The naïve Bayesian prediction requires each conditional probability be non-zero.

$$P(\mathbf{X} \mid C_i) = \prod_{k=1}^n P(x_k \mid C_i)$$

- Otherwise, the predicted probability will be zero
- For example,

age	income	student	credit_rating	buys_computer
3140	medium	yes	fair	?

- $P(\mathbf{X} \mid buys_computer = "no") = 0 * 2/5 * 1/5 * 2/5 = 0$
- Therefore, the conclusion is always yes regardless the value of P(X | buys_computer = "yes")

Avoiding the zero-probability issue

• Laplacian correction (or Laplacian estimator)

$$P(C_i) = \frac{|C_i| + 1}{|D| + m} \qquad P(x_k | C_i) = \frac{|x_k \cup C_i| + 1}{|C_i| + r}$$

- where *m* is the number of classes, $|x_k \cup C_i|$ denotes the number of tuples contains both $A_k = x_k$ and C_i , and *r* is the number of values of attribute A_k
- The "corrected" probability estimates are close to their "uncorrected" counterparts

Naïve Bayesian classification: An example

P(buys_computer = "yes")	10/16
P(buys_computer = "no")	6/16

	buys_computer = "yes"	buys_computer = "no"
age = "<=30"	3/12	4/8
age = "31…40"	5/12	1/8
age = ">40"	4/12	3/8
income = "low"	4/12	2/8
income = "medium"	5/12	3/8
income = "high"	3/12	3/8
student = "yes"	7/11	2/7
student = "no"	4/11	5/7
credit_rating = "fair"	7/11	3/7
credit_rating = "excellent"	4/11	4/7

Naïve Bayesian classification: An example

age	income	student	credit_rating	buys_computer
3140	medium	yes	fair	?

- $P(\mathbf{X}|C_i)$
 - $P(\mathbf{X} \mid buys_computer = "yes") = 5/12 * 5/12 * 7/11 * 7/11 = 0.070$
 - $P(X \mid buys_computer = "no") = 1/8 * 3/8 * 2/7 * 3/7 = 0.006$
- $P(\mathbf{X}|C_i) * P(C_i)$
 - $P(X \mid buys_computer = "yes") * P(buys_computer = "yes") = 0.044$
 - $P(\mathbf{X}| buys_computer = "no") * P(buys_computer = "no") = 0.002$
- $P(C_i \mid \mathbf{X})$
 - $P(buys_computer = "yes" | \mathbf{X}) = 0.953$
 - $P(buys_computer = "no" | \mathbf{X}) = 0.047$

Therefore, X belongs to class ("buys_computer = yes")
Handling missing values

- If the values of some attributes are missing, these attributes are omitted from the product of probabilities
- As a result, the estimation is less accurate
- For example,

age	income	student	credit_rating	buys_computer
?	medium	yes	fair	?

Naïve Bayesian classification: Evaluation

- Advantages
 - Easy to implement
 - Good results obtained in most of the cases
- Disadvantages
 - Class conditional independence \rightarrow loss of accuracy
 - Practically, dependencies exist among variables, which cannot be modeled by Naïve Bayes
 - E.g., in medical records, patients' profile (age, family history, etc.), symptoms (fever, cough etc.), disease (lung cancer, diabetes, etc.)
- How to deal with these dependencies?
 - Bayesian Belief Networks

Quiz 02: Naïve Bayesian classification

- The data represent files on a computer system. Possible values of the class variable are "infected", which implies the file has a virus infection, or "clean" if it doesn't.
- Derive naïve Bayesian probabilities for virus identification in either cases, with or without Laplacian correction.

No.	Writable	Updated	Size	Class
1	Yes	No	Small	Infected
2	Yes	Yes	Large	Infected
3	No	Yes	Med	Infected
4	No	No	Med	Clean
5	Yes	No	Large	Clean
6	No	No	Large	Clean



THE END