Chapter 4:

Time Varying Fields

4.1: Time Varying Fields and Potentials :

cuu duong than cong . com

a) Introduction to Time Varying Fields

- **Electric charges induce electric fields and electric current induce magnetic fields.**
- **the cases for *magnetostatic* and *electrostatic* where the magnetic field and the electric field are *constant with time*.**
- **In the *magnetostatic and electrostatic cases*, the $\vec{E}$ and $\vec{D}$ fields are *independent* of $\vec{B}$ and $\vec{H}$ fields.**

- **if the charge and current sources were to *vary with time t*,**
  - **not only will the fields also vary with time .**
  - **the electric and magnetic fields become interconnected .**

- **And the coupling between them produces electromagnetic waves capable of traveling through free space and in material media .**

Model :

Maxwell's equations

$$\text{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (1)$$

$$\text{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2)$$

$$\text{div}\vec{D} = \rho_V \quad (3)$$

$$\text{div}\vec{B} = 0 \quad (4)$$

$$div\,\vec{J} = -\partial \rho_V / \partial t \quad (5)$$

Constitutive relations

$$\vec{B} = \mu\vec{H} = \mu_0(\vec{H} + \vec{M})$$

$$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\vec{E} + \vec{P}$$

$$\vec{J} = \sigma\vec{E}$$

Boundary conditons

$$H_{1t} - H_{2t} = J_S$$

$$E_{1t} - E_{2t} = 0$$

$$D_{1n} - D_{2n} = \rho_S$$

$$B_{1n} - B_{2n} = 0$$

$$J_{1n} - J_{2n} = -\frac{\partial \rho_S}{\partial t}$$

b) Time Varying Potentials :

1. Magnetic vector potential :

$$\begin{cases} \operatorname{div}\vec{B} = 0 & (4) \\ \operatorname{div}(\operatorname{rot}\vec{A}) = 0 & \text{(vector algebra)} \end{cases} \Rightarrow \vec{B} = \operatorname{rot}\vec{A}$$

2. Electric scalar potential:

$$(2): \operatorname{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\operatorname{rot}\frac{\partial \vec{A}}{\partial t}$$

$$\begin{cases} \operatorname{rot}\left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0 \\ \operatorname{rot}(grad\,\varphi) = 0 \ \text{(vector algebra)} \end{cases} \Rightarrow \vec{E} = -\operatorname{grad}\varphi - \frac{\partial \vec{A}}{\partial t}$$

3. The Lorenz condition : **multivalued → singlevalued**

$$\operatorname{div}\vec{A} + \mu\varepsilon\frac{\partial \varphi}{\partial t} = 0$$

c) D'Alembert equation for vector potential:

- (1) : $\mathrm{rot}\,\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ ⇒ $\mathrm{rot}\,\vec{B} = \mu\,\vec{J} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$

⇒ $\mathrm{rot}(\mathrm{rot}\,\vec{A}) = \mu\,\vec{J} + \mu\varepsilon \frac{\partial}{\partial t}\left(-\mathrm{grad}\,\varphi - \frac{\partial \vec{A}}{\partial t}\right)$

⇒ $\mathrm{grad}(\mathrm{div}\,\vec{A}) - \Delta\,\vec{A} = \mu\,\vec{J} - \mathrm{grad}\left(\mu\varepsilon \frac{\partial \varphi}{\partial t}\right) - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2}$

- **Using Lorenz condition :** $\mathrm{div}\,\vec{A} + \mu\varepsilon \frac{\partial \varphi}{\partial t} = 0$

  - **D'Alembert equation for vector potential:**

$$\Delta\,\vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\,\vec{J}$$

d) D'Alembert equation for scalar potential:

❖ (3) :

$$\rho_V = \operatorname{div}\vec{D} = \varepsilon.\operatorname{div}(-\operatorname{grad}\varphi - \frac{\partial \vec{A}}{\partial t}) = -\varepsilon.\Delta\varphi - \varepsilon\frac{\partial}{\partial t}(\operatorname{div}\vec{A})$$

❖ **Using Lorenz condition:** $\operatorname{div}\vec{A} + \mu\varepsilon\frac{\partial\varphi}{\partial t} = 0$

$$\Rightarrow \rho_V = -\varepsilon\Delta\varphi + \mu\varepsilon^2\frac{\partial^2\varphi}{\partial t^2}$$

➤ **D'Alembert equation for scalar potential:**

$$\Delta\varphi - \mu\varepsilon\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho_V}{\varepsilon}$$

❖ Summary :

i. Potentials φ(t) and $\vec{A}$(t) satisfy wave equations .

$$\Delta \vec{A} - \frac{1}{v^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J}$$

$$\Delta \varphi - \frac{1}{v^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho_V}{\varepsilon}$$

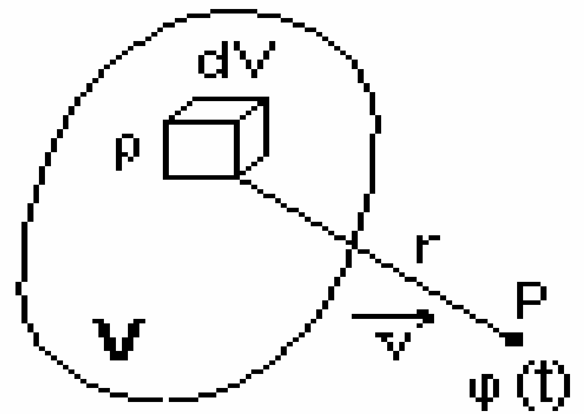
➡ Time Varying Fields propagate in medium with $v = \frac{1}{\sqrt{\mu\varepsilon}}$

➡ produce electromagnetic waves ➡ Apply for communications

ii. The solution of the wave equations :

$$\vec{A}(t) = \frac{\mu}{4\pi}\int_V \frac{\vec{J}(t - r/v)\,dV}{r}$$

$$\varphi(t) = \frac{1}{4\pi\varepsilon}\int_V \frac{\rho_V(t - r/v)\,dV}{r}$$



➡ **φ(t) and $\vec{A}$(t) : Retarded potentials**