Lời giải đề ĐSTT 2015 - 2016

$f(x) = ax^3 + bx^2 + cx + d$

Có: $\begin{cases} f(1) = 1 \\ f(-1) = -1 \\ f(2) = -7 \\ f(-3) = 53 \end{cases} \Leftrightarrow \begin{cases} a + b + c + d = 1 \\ -a + b - c + d = -1 \\ 8a + 4b + 2c + d = -7 \\ -27a + 9b - 3c + d = 53 \end{cases}$

Ma trận hóa hệ phương trình:

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 8 & 4 & 2 & 1 & -7 \\ -27 & 9 & -3 & 1 & 53 \end{array}\right) \xrightarrow[\substack{d_3 := d_3 - 8d_1 \\ d_4 := d_4 + 27d_1}]{d_2 := d_2 + d_1} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 2 & 0 \\ 0 & -4 & -6 & -7 & -15 \\ 0 & 36 & 24 & 28 & 80 \end{array}\right) \xrightarrow[\substack{d_3 := -d_3 \\ d_4 := \frac{1}{4}d_4}]{d_2 := \frac{1}{2}d_2} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 4 & 6 & 7 & 15 \\ 0 & 9 & 6 & 7 & 20 \end{array}\right)$$

$$\xrightarrow[d_4 := d_4 - 9d_2]{d_3 := d_3 - 4d_2} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 6 & 3 & 15 \\ 0 & 0 & 6 & -2 & 20 \end{array}\right) \xrightarrow{d_4 := d_4 - d_3} \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 6 & 3 & 15 \\ 0 & 0 & 0 & -5 & 5 \end{array}\right)$$

Hệ tương ứng $\begin{cases} a + b + c + d = 1 \\ b + d = 0 \\ 6c + 3d = 15 \\ -5d = 5 \end{cases} \Leftrightarrow \begin{cases} a = -2 \\ b = 1 \\ c = 3 \\ d = -1 \end{cases}$

Vậy $f(x) = -2x^3 + x^2 + 3x - 1$

$B = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}$, $C = \begin{pmatrix} -1 & 4 \\ 2 & 0 \\ 3 & -2 \end{pmatrix}$, $A^{-1} = \begin{pmatrix} 5 & -6 & 9 \\ -2 & 2 & -3 \\ 3 & -3 & 5 \end{pmatrix}$

a) Xét

$$\left(\begin{array}{ccc|ccc} 5 & -6 & 9 & 1 & 0 & 0 \\ -2 & 2 & -3 & 0 & 1 & 0 \\ 3 & -3 & 5 & 0 & 0 & 1 \end{array}\right) \xrightarrow{d_1 := d_1 + 2d_2} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 2 & 0 \\ -2 & 2 & -3 & 0 & 1 & 0 \\ 3 & -3 & 5 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[d_3 := d_3 - 3d_1]{d_2 := d_2 + 2d_1} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 2 & 0 \\ 0 & -2 & 3 & 2 & 5 & 0 \\ 0 & 3 & -4 & -3 & -6 & 1 \end{array}\right) \xrightarrow{d_2 := d_2 + d_3} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 2 & 0 \\ 0 & 1 & -1 & -1 & -1 & 1 \\ 0 & 3 & -4 & -3 & -6 & 1 \end{array}\right)$$

$$\xrightarrow{d_3 := d_3 - 3d_2} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 2 & 0 \\ 0 & 1 & -1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 0 & -3 & -2 \end{array}\right) \xrightarrow{d_3 := -d_3} \left(\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 2 & 0 \\ 0 & 1 & -1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 3 & 2 \end{array}\right)$$

$$\xrightarrow[d_1 := d_1 - 3d_3]{d_2 := d_2 + d_3} \left(\begin{array}{ccc|ccc} 1 & -2 & 0 & 1 & -7 & -6 \\ 0 & 1 & 0 & -1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 & 2 \end{array}\right) \xrightarrow{d_1 := d_1 + 2d_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -3 & 0 \\ 0 & 1 & 0 & -1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 3 & 2 \end{array}\right)$$

Vậy $A = (A^{-1})^{-1} = \begin{pmatrix} -1 & -3 & 0 \\ -1 & 2 & 3 \\ 0 & 3 & 2 \end{pmatrix}$

Xét

$$\left(\begin{array}{cc|cc} 2 & -3 & 1 & 0 \\ 3 & -4 & 0 & 1 \end{array}\right) \xrightarrow{d_1 := d_1 - d_2} \left(\begin{array}{cc|cc} -1 & 1 & 1 & -1 \\ 3 & -4 & 0 & 1 \end{array}\right) \xrightarrow{d_2 := d_2 + 3d_1} \left(\begin{array}{cc|cc} -1 & 1 & 1 & -1 \\ 0 & -1 & 3 & -2 \end{array}\right)$$

$$\xrightarrow[d_2 := -d_2]{d_1 := -d_1} \left(\begin{array}{cc|cc} 1 & -1 & -1 & 1 \\ 0 & 1 & -3 & 2 \end{array}\right) \xrightarrow{d_1 := d_1 + d_2} \left(\begin{array}{cc|cc} 1 & 0 & -4 & 3 \\ 0 & 1 & -3 & 2 \end{array}\right) \Rightarrow B^{-1} = \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix}$$

b/ $AXB = C \Leftrightarrow X = A^{-1} C B^{-1}$

$$= \begin{pmatrix} 5 & -6 & 9 \\ -2 & 2 & -3 \\ 3 & -3 & 5 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 2 & 0 \\ 3 & -2 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 10 & 2 \\ -3 & -2 \\ 6 & 2 \end{pmatrix} \begin{pmatrix} -4 & 3 \\ -3 & 2 \end{pmatrix}.$$

Vậy $X = \begin{pmatrix} -46 & 34 \\ 18 & -13 \\ -30 & 22 \end{pmatrix}$

$$\begin{cases} x + y + z = 1 \\ my + 3z + x = 2 \\ (m-1)z + x + 2y = 0 \end{cases}$$

Ma trận hóa hệ phương trình $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & m & 3 & 2 \\ 1 & 2 & m-1 & 0 \end{array}\right)$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & m & 3 \\ 1 & 2 & m-1 \end{vmatrix} \xrightarrow[d_3 = d_3 - d_1]{d_2 = d_2 - d_1} \begin{vmatrix} 1 & 1 & 1 \\ 0 & m-1 & 2 \\ 0 & 1 & m-2 \end{vmatrix} \overset{m\,1}{=} \begin{vmatrix} m-1 & 2 \\ 1 & m-2 \end{vmatrix}$$

$$= (m-1)(m-2) - 2 = m^2 - 3m = m(m-3)$$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & m & 3 \\ 0 & 2 & m-1 \end{vmatrix} \xrightarrow{d_2 = d_2 - 2d_1} \begin{vmatrix} 1 & 1 & 1 \\ 0 & m-2 & 1 \\ 0 & 2 & m-1 \end{vmatrix} \overset{m\,1}{=} \begin{vmatrix} m-2 & 1 \\ 2 & m-1 \end{vmatrix}$$

$$= (m-2)(m-1) - 2 = m(m-3)$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 0 & m-1 \end{vmatrix} \xrightarrow[d_3 = d_3 - d_1]{d_2 = d_2 - d_1} \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & m-2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -1 & m-2 \end{vmatrix}$$

$$= m - 2 + 2 = m$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & m & 2 \\ 1 & 2 & 0 \end{vmatrix} \xrightarrow[d_3 = d_3 - d_1]{d_2 = d_2 - d_1} \begin{vmatrix} 1 & 1 & 1 \\ 0 & m-1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = \begin{vmatrix} m-1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= -(m-1) - 1 = -m$$

* $\Delta = 0 \Leftrightarrow m = 0 \vee m = 3$:

. $m = 3$ có $\Delta = 0$ nhưng $\Delta_2 \neq 0 \Rightarrow$ hệ vô nghiệm

. $m = 0$ có $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$, xét

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & 3 & 2 \\ 1 & 2 & -1 & 0 \end{array}\right) \xrightarrow[d_3 := d_3 - d_1]{d_2 := d_2 - d_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & 1 & -2 & -1 \end{array}\right) \xrightarrow[d_1 := d_1 + d_2]{d_3 := d_3 + d_2} \left(\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Hệ tương ứng $\begin{cases} x + 3z = 2 \\ -y + 2z = 1 \end{cases} \Leftrightarrow \begin{cases} x = 2 - 3a \\ y = 2a - 1 \\ z = a \in \mathbb{R} \end{cases}$

* ~~Vậy~~ $\Delta \neq 0 \Leftrightarrow m \neq 0 \wedge m \neq 3$:

Hệ có nghiệm $x = \frac{\Delta_1}{\Delta} = \frac{m(m-3)}{m(m-3)} = 1$

$$y = \frac{\Delta_2}{\Delta} = \frac{m}{m(m-3)} = \frac{1}{m-3}$$

$$z = -y = \frac{-1}{m-3}$$

Vậy $m = 0$, hệ có vô số nghiệm $(2-3a, 2a-1, a)$, $a \in \mathbb{R}$

$m = 3$, hệ vô nghiệm

$m \neq 0, m \neq 3$, hệ có nghiệm

$$\left(1, \frac{1}{m-3}, \frac{-1}{m-3}\right)$$

Câu 4:

a/ * $H = \{X = (u, v, w) \in \mathbb{R}^3 / 6|4u - v + 2w| \leq -5(u + 3v - 7w)^2\}$

Do $\begin{cases} 6|4u - v + 2w| \geq 0 \ \forall X \in \mathbb{R}^3 \\ -5(u + 3v - 7w)^2 \leq 0 \ \forall X \in \mathbb{R}^3 \end{cases}$ $\Rightarrow H = \{X / \cancel{4u - v + 2w = ...}\}$

$\Rightarrow H = \{(u, v, w) \in \mathbb{R}^3 / 4u - v + 2w = u + 3v - 7w = 0\}$

$\forall a, b \in H, \alpha \in \mathbb{R}$, xét:

$\alpha a + b = (\alpha a_1 + b_1, \alpha a_2 + b_2, \alpha a_3 + b_3)$

thỏa $\begin{cases} 4(\alpha a_1 + b_1) - (\alpha a_2 + b_2) + 2(\alpha a_3 + b_3) \\ = \alpha(4a_1 - a_2 + 2a_3) + (4b_1 - b_2 + 2b_3) = \alpha 0 + 0 = 0 \end{cases}$

và $(\alpha a_1 + b_1) + 3(\alpha a_2 + b_2) - 7(\alpha a_3 + b_3)$

$= \alpha(a_1 + 3a_2 - 7a_3) + (b_1 + 3b_2 - 7b_3) = \alpha 0 + 0 = 0$

Vậy $H \leq \mathbb{R}^3$

* $K = \{X = (u, v, w) \in \mathbb{R}^3 / (2u + 5v - w)(-3u + 8v + 9w) = 0\}$

Xét $(1, 1, 7) \in K$ và $(3, 0, 1) \in K$ nhưng

$(1, 1, 7) + (3, 0, 1) = (4, 1, 8) \notin K$

Vậy $K \not\leq \mathbb{R}^3$.

b/ $V = \{X = (a + 2b + c + 2d, a - b - c + d, 2a - 5b - 4c + d, 4a + 2b + 6d) / a, b, c, d \in \mathbb{R}\} = \{a(1, 1, 2, 4) + b(2, -1, -5, 2) + c(1, -1, -4, 0) + d(2, 1, 1, 6) / a, b, c, d \in \mathbb{R}^4\} = \langle\{(1, 1, 2, 4), (2, -1, -5, 2), (1, -1, -4, 0), (2, 1, 1, 6)\}\rangle$

Vậy $V = \langle S \rangle$ với $S = \{(1, 1, 2, 4), (2, -1, -5, 2), (1, -1, -4, 0), (2, 1, 1, 6)\}$

STARBOOK

Xét $\begin{pmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & -5 & 2 \\ 1 & -1 & -4 & 0 \\ 2 & 1 & 1 & 6 \end{pmatrix} \xrightarrow[\substack{d_3 := d_3 - d_1 \\ d_4 := d_4 - 2d_1}]{d_2 := d_2 - 2d_1} \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -9 & -6 \\ 0 & -2 & -6 & -4 \\ 0 & -1 & -3 & -2 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 2 \end{pmatrix}$

Do $u_2 = (2, 1, -5, 2) = 2u_1 - u_4$ $B = \{(1, 1, 2, 4), (0, 1, 3, 2)\}$ là một cơ sở của V.

Câu 5

$f(x, y, z) = (2x + 2y - 2z, 5x + 2y + z, x - 2y + 5z)$

$Ker(f) = \{ u / f(u) = 0 \}$ là không gian nghiệm $f(u) = 0$

Xét $A = \begin{pmatrix} 2 & 2 & -2 \\ 5 & 2 & 1 \\ 1 & -2 & 5 \end{pmatrix} \xrightarrow{d_1 \leftrightarrow d_3} \begin{pmatrix} 1 & -2 & 5 \\ 5 & 2 & 1 \\ 2 & 2 & -2 \end{pmatrix} \xrightarrow[d_3 := d_3 - 2d_1]{d_2 := d_2 - 5d_1} \begin{pmatrix} 1 & -2 & 5 \\ 0 & 12 & -24 \\ 0 & 6 & -12 \end{pmatrix}$

$\xrightarrow[d_3 := \frac{1}{6} d_3]{d_2 := \frac{1}{12} d_2} \begin{pmatrix} 1 & -2 & 5 \\ 0 & 1 & -2 \\ 0 & 1 & -2 \end{pmatrix} \xrightarrow[d_1 := d_1 + 2d_2]{d_3 := d_3 - d_2} \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$

$f(u) = 0 \Leftrightarrow \begin{cases} x + z = 0 \\ y - 2z = 0 \end{cases} \Leftrightarrow \begin{cases} x = -a \\ y = 2a \\ z = a \in \mathbb{R} \end{cases}$

$Ker(f) = \{ (-a, 2a, a) / a \in \mathbb{R} \} = \{ a(-1, 2, 1) / a \in \mathbb{R} \}$
$= \langle (-1, 2, 1) \rangle = \langle B \rangle$

Do $B = \{(-1, 2, 1)\}$ độc lập tuyến tính nên b/ $[g]_{C,B} = \left([g(x_1)]_B \; [g(x_2)]_B \; [g(x_3)]_B \right) = \begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 3 \\ 1 & 4 & -2 \end{pmatrix}$

$$[g]_B = [g]_{C,B} (C \to B) = \begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 3 \\ 1 & 4 & -2 \end{pmatrix} \begin{pmatrix} 5 & -6 & 9 \\ -2 & 2 & -3 \\ 3 & -3 & 5 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2 & -3 & 1 \\ -1 & 0 & 3 \\ 1 & 4 & -2 \end{pmatrix} \begin{pmatrix} -1 & -3 & 0 \\ -1 & 2 & 3 \\ 0 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -9 & -7 \\ 1 & 12 & 6 \\ -5 & -1 & 8 \end{pmatrix}$$

$\Rightarrow f(x,y,z) = (x - 9y - 7z,\; x + 12y + 6z,\; -5x - y + 8z)$