CTT310: Digital Image Processing

Image Restoration

Dr. Nguyen Ngoc Thao
Department of Computer Science, FIT, HCMUS

Outline

- A model of the image degradation/restoration process
- Noise models
- Additive noise reduction by spatial domain filtering
- Periodic noise reduction by frequency domain filtering
- Degradation functions

Section 5.1

A MODEL OF THE IMAGE DEGRADATION/RESTORATION PROCESS

What is Image restoration?

- **Image restoration** attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon
  - Restoration techniques are oriented toward modeling the degradation and applying the inverse process in order to recover the original image

- Noise reduction can be achieved by filtering in both the spatial domain and frequency domain

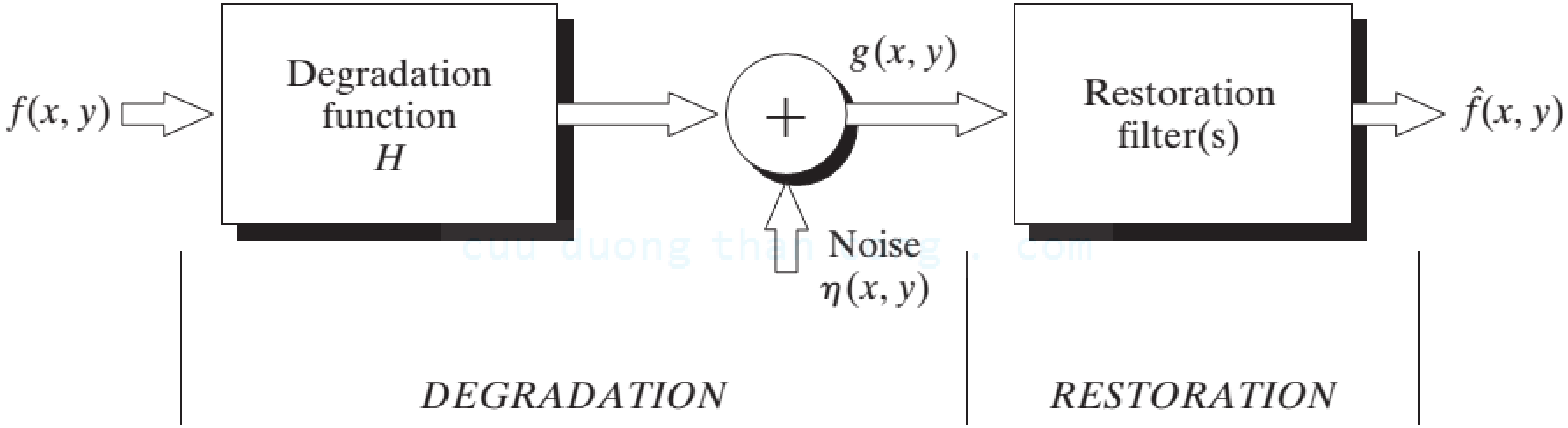
Image restoration vs. Image enhancement

- Objective process
- Formulation of a criterion of goodness that will yield an optimal estimate of the desired result
- E.g. removal of image blur by applying a deblurring function

- Subjective processes
- Heuristic procedures that take advantage of the psychophysical aspects of the human visual system
- E.g. contrast stretching – based primarily on the pleasing aspects it might present to the viewer

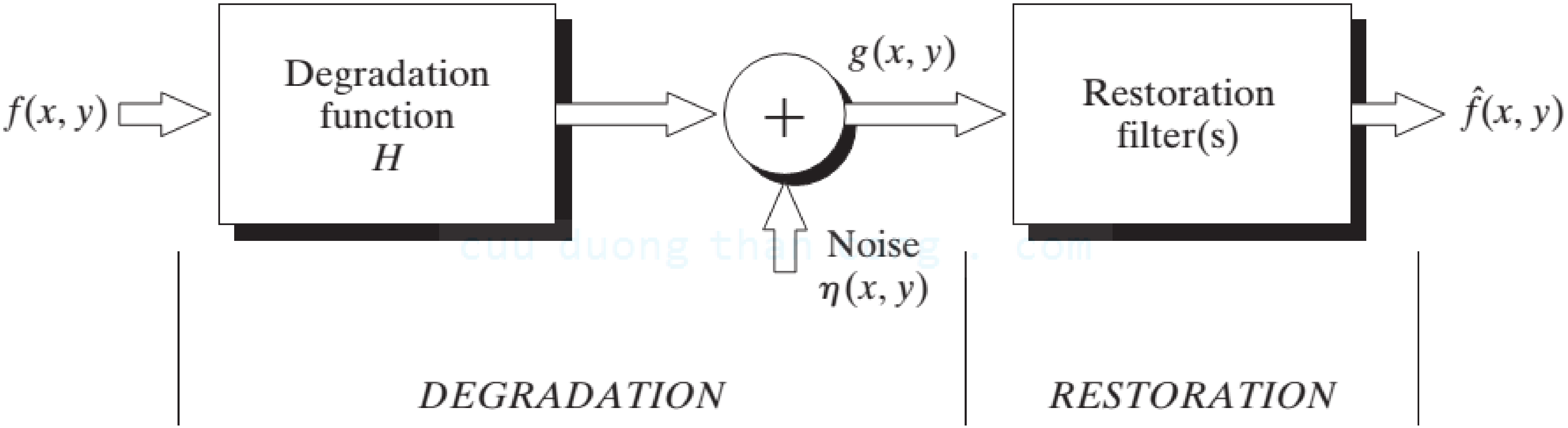
The image degradation process

- The **degradation process** is modeled as a degradation function that, together with an additive noise term, operates on an image $f(x, y)$ to produce a degraded image $g(x, y)$



The image restoration process

- Given $g(x, y)$, some knowledge about the degradation function H and the additive noise term $\eta(x, y)$
- The objective of **restoration** is to obtain an estimate $\hat{f}(x, y)$ of the original image $f(x, y)$
  - The more we know about H and η, the closer $\hat{f}(x, y)$ to $f(x, y)$



Representations of the degraded image

- If H is a linear, position-invariant process, the degraded image is given in the spatial domain by

$$g(x, y) = h(x, y) \star f(x, y) + \eta(x, y)$$

  - where $h(x, y)$ is the spatial representation of the degradation function and the symbol ★ indicates convolution

- An equivalent frequency domain representation is given by

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

- *Approaches in this lecture assume that H is the identity operator and they deal only with degradations due to noise*

Section 5.2

NOISE MODELS

Noise in digital images

- The principal sources of noise in digital images arise during image acquisition and/or transmission
- The performance of imaging sensors
  - Environmental conditions during image acquisition, the quality of the sensing elements themselves, etc.
  - E.g. for a CCD camera, light levels and sensor temperature are major factors affecting the amount of noise in the resulting image
- Corruption during transmission
  - Principally due to interference in the channel used for transmission
  - E.g. an image transmitted using a wireless network might be corrupted as a result of lightning or other atmospheric disturbance

Noise in digital images

- Noise cannot be predicted but can be approximately described in statistical way using the probability density function (PDF)

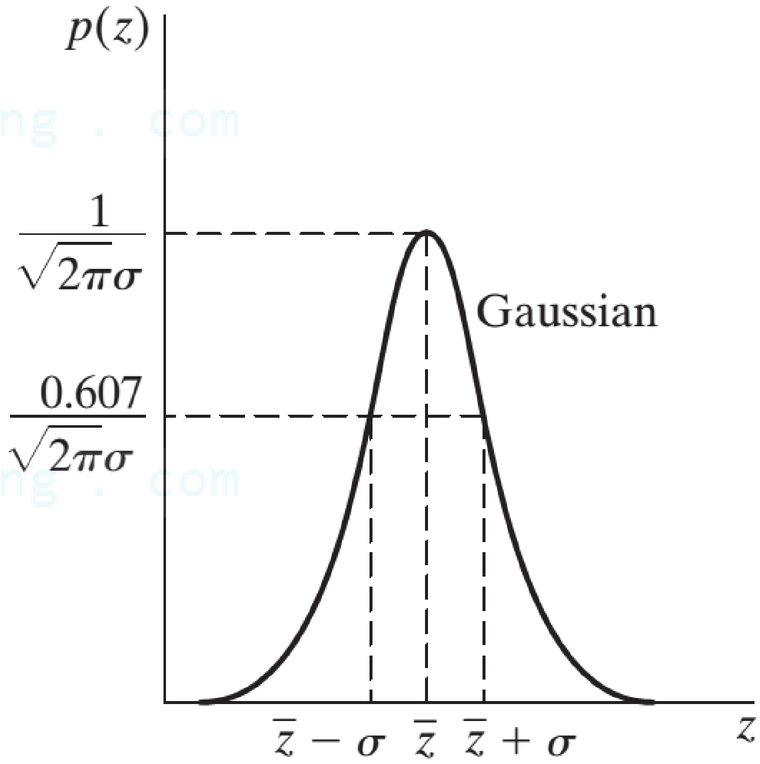
- Assumption: there is no correlation between pixel values and the values of noise components
  - These assumptions are partially invalid in some applications like quantum-limited imaging (X-ray, nuclear-medicine imaging,…)

Gaussian noise

- The PDF of **Gaussian (or normal) noise** is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})/2\sigma^2}$$

  - where z represents intensity, $\bar{z}$ is the average value z and σ is its standard deviation



Approximately 70% of its values in the range $[(\bar{z} - \sigma), (\bar{z} - \sigma)]$ and 95% of its values in the range $[(\bar{z} - 2\sigma), (\bar{z} - 2\sigma)]$

Gaussian noise

- Gaussian noise arises in an image due to factors such as electronic circuit noise and sensor noise due to poor illumination and/or high temperature
- It is used frequently in practice due to its mathematical tractability in both the spatial and frequency domains

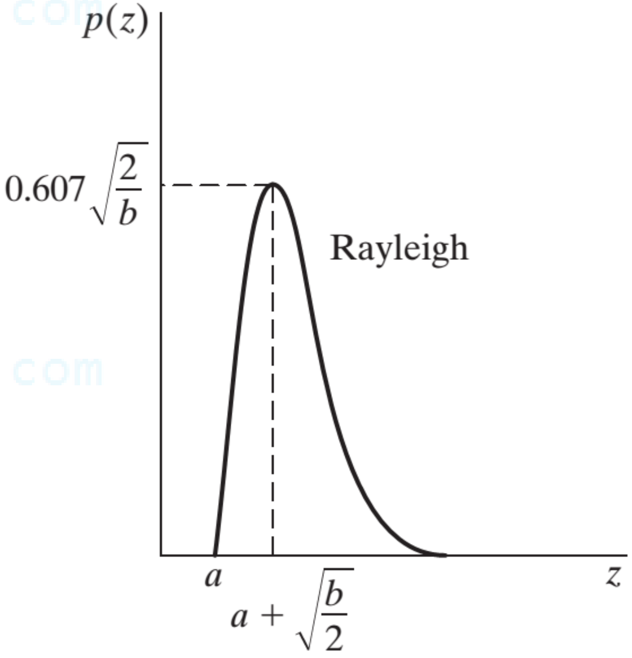
Rayleigh noise

- The PDF of **Rayleigh noise** is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & for\ z \geq a \\ 0 & for\ z < a \end{cases}$$

  - where the mean and variance of this density are given by $\bar{z} = a + \sqrt{\pi b/4}$ and $\sigma^2 = \frac{b(4-\pi)}{4}$, respectively

- The Rayleigh density is helpful in characterizing noise phenomena in range imaging

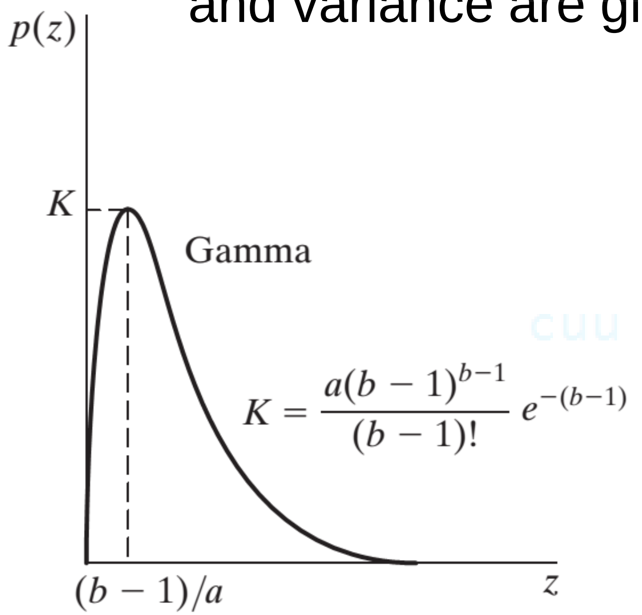


Erlang noise

- The PDF of **Erlang (gamma) noise** is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

  - where $a > 0$, b is a positive integer, "!" indicates factorial, the mean and variance are given by $\bar{z} = \frac{b}{a}$ and $\sigma^2 = \frac{b}{a^2}$, respectively



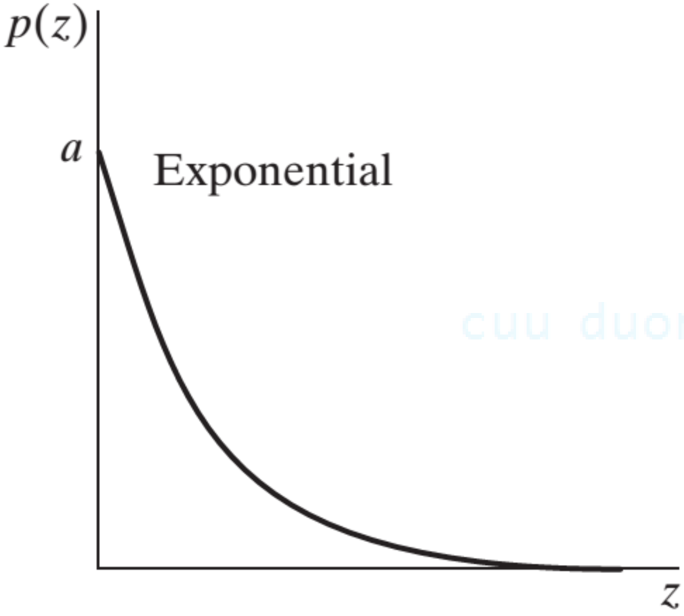
- This density is also called *gamma density* when the denominator is a gamma function $\Gamma(b)$
- Applications: laser imaging

Exponential noise

- The PDF of **exponential noise** is given by

$$p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0 & for\ z < 0 \end{cases}$$

  - where $a > 0$, the mean and variance are given by $\bar{z} = \frac{1}{a}$ and $\sigma^2 = \frac{1}{a^2}$, respectively



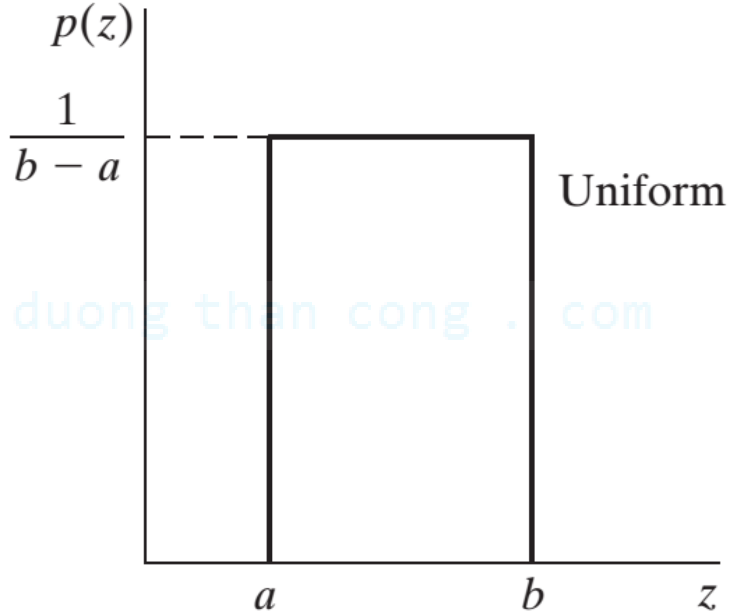
- Applications: laser imaging

Uniform noise

- The PDF of **uniform noise** is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & if\ a \le z \le b \\ 0 & otherwise \end{cases}$$

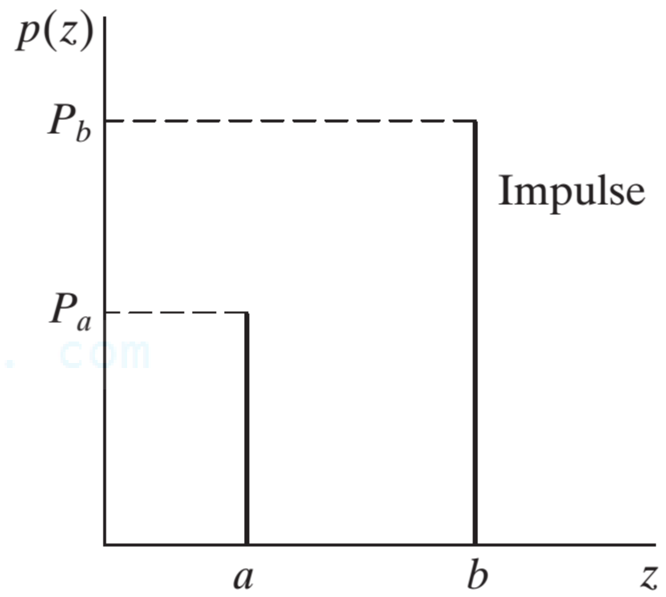
  - where the mean and variance are given by $\bar{z} = \frac{a+b}{2}$ and $\sigma^2 = \frac{(b-a)^2}{12}$, respectively



Impulse (salt-and-pepper) noise

- The PDF of **(bipolar) impulse noise** is given by

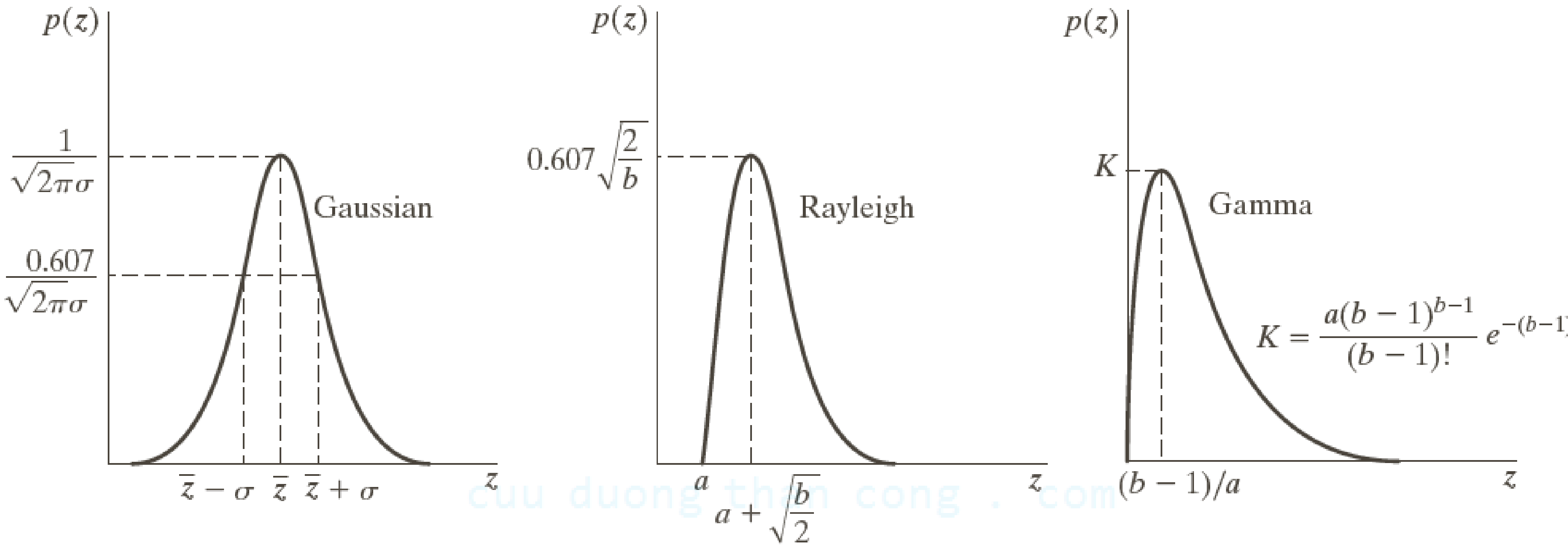
$$p(z) = \begin{cases} P_a & for\ z = a \\ P_b & for\ z = b \\ 0 & otherwise \end{cases}$$



- If $b > a$, intensity b will appear as a light dot and a as a dark dot in the image
- If either P_a or P_b is zero, the impulse noise is called unipolar
- If neither probability is zero, and especially if they are approximately equal, impulse noise values will resemble salt-and-pepper granules randomly distributed over

Impulse (salt-and-pepper) noise

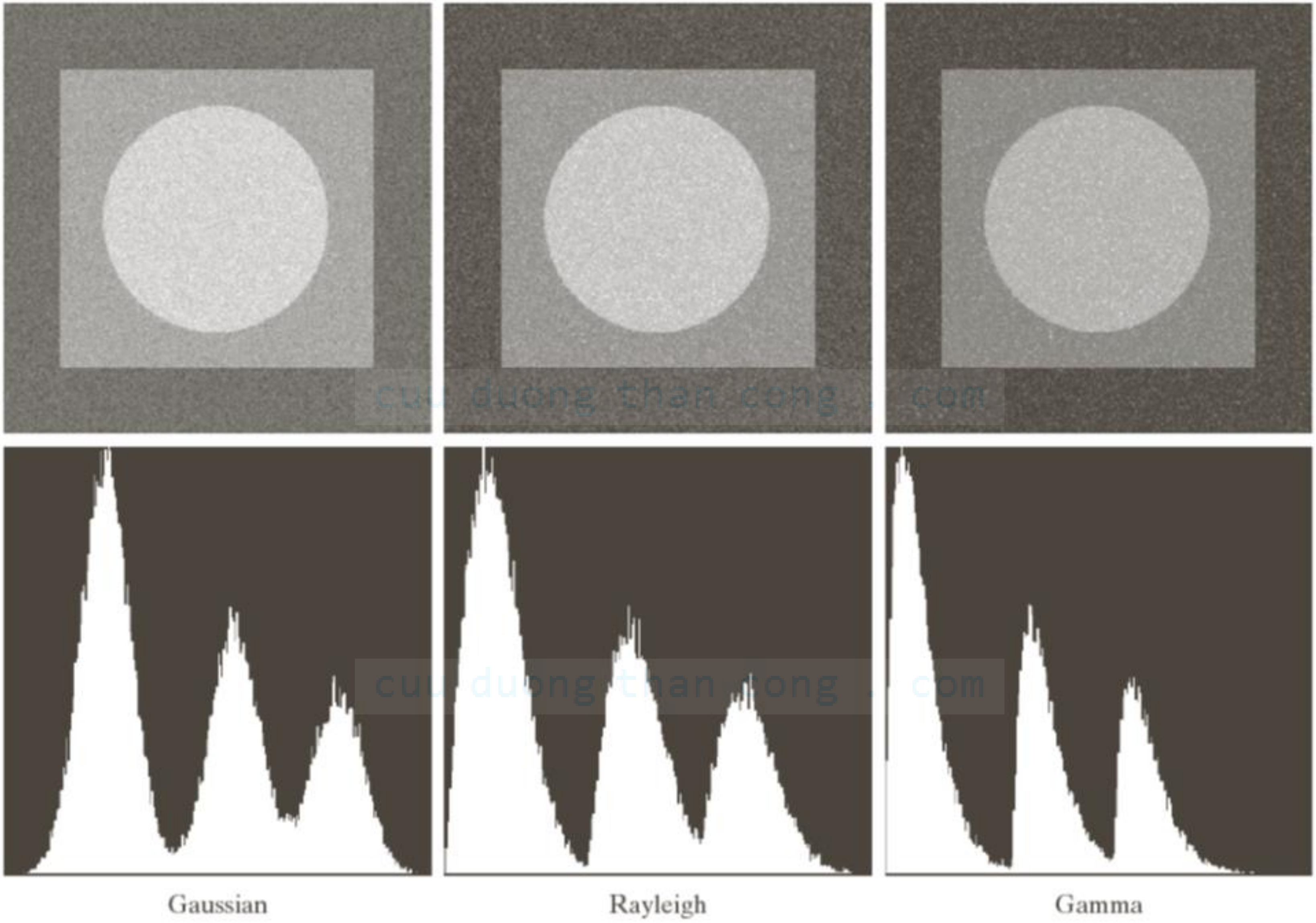
- Other terms: salt-and-pepper noise, data-drop-out noise, spike noise
- Impulse noise is found in situations where quick transients, such as faulty switching, take place during imaging


p(z)
1/√(2πσ)
0.607/√(2πσ)
Gaussian
z̄ − σ
z̄
z̄ + σ
z
p(z)
0.607√(2/b)
Rayleigh
a
a + √(b/2)
z
p(z)
K
Gamma
K = a(b − 1)^(b−1)/(b − 1)! e^−(b−1)
(b − 1)/a
z
p(z)
a
Exponential
z
p(z)
1/(b − a)
Uniform
a
b
z
p(z)
P_b
P_a
Impulse
a
b
z

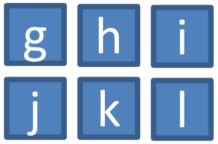
Test pattern used to illustrate the characteristics of the noise PDFs

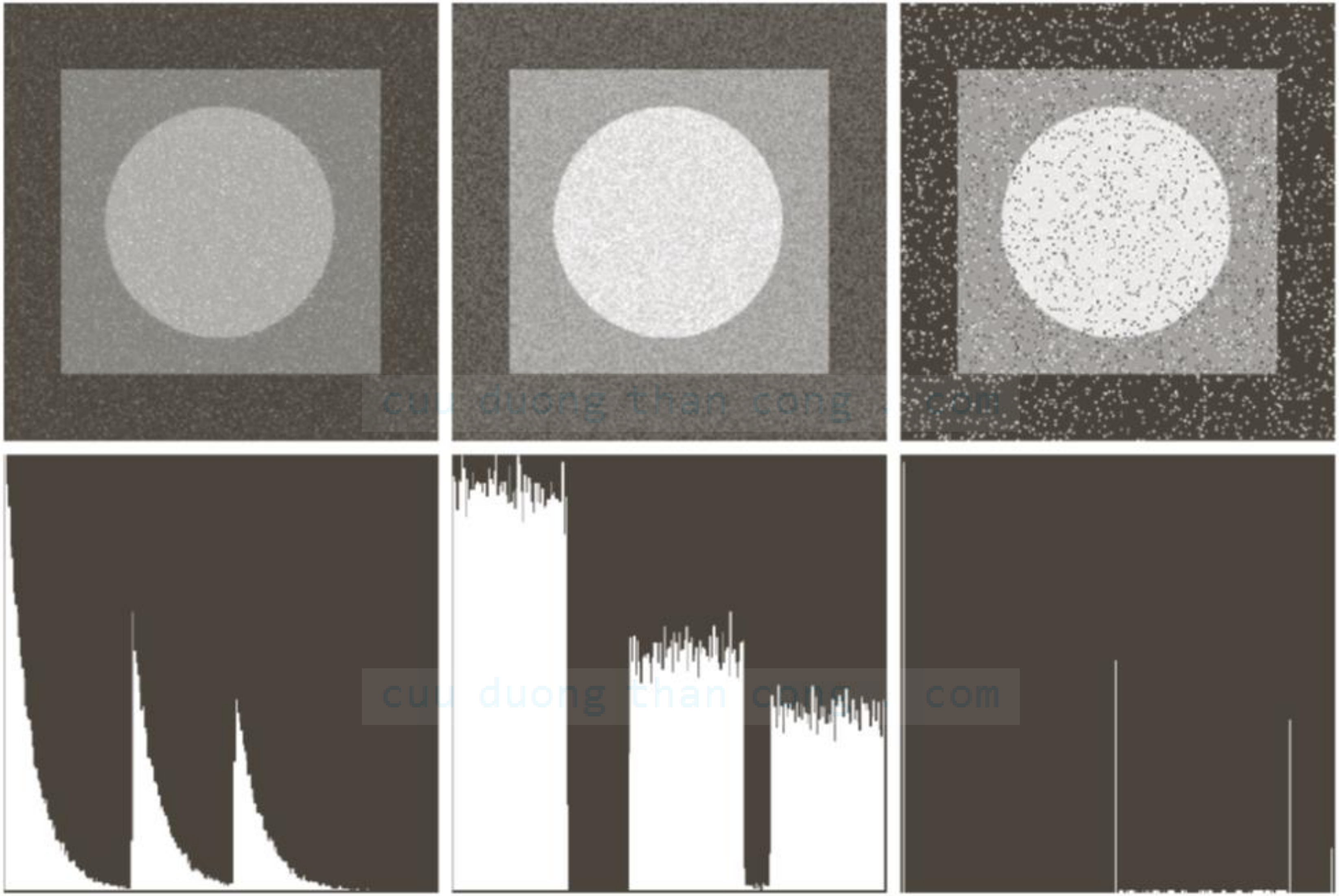
a	b	c
d	e	f

Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the test pattern in the previous slide



Images and histograms resulting from adding exponential, uniform, and salt-and-pepper noise to the test pattern in the previous slide



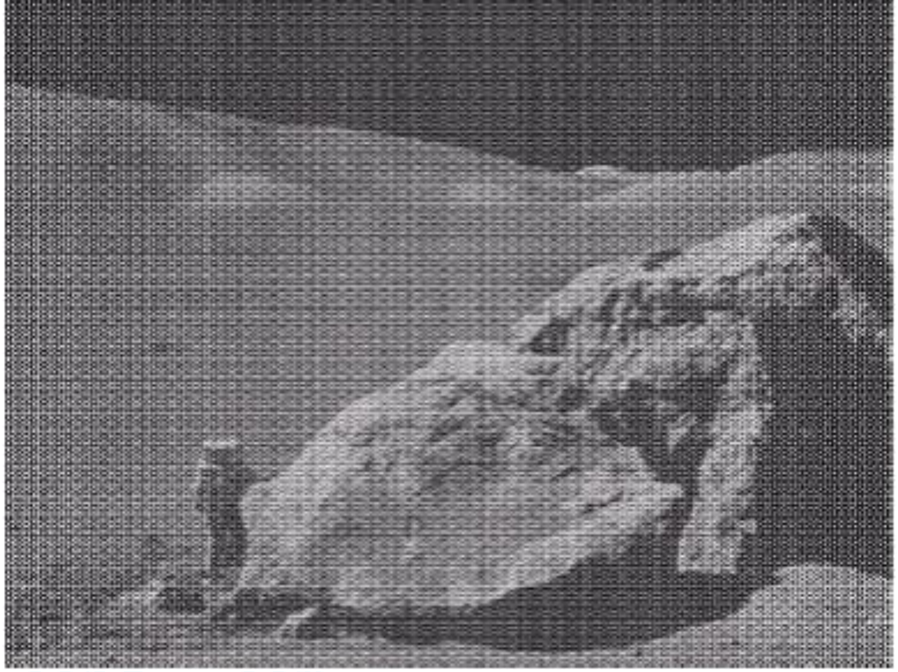


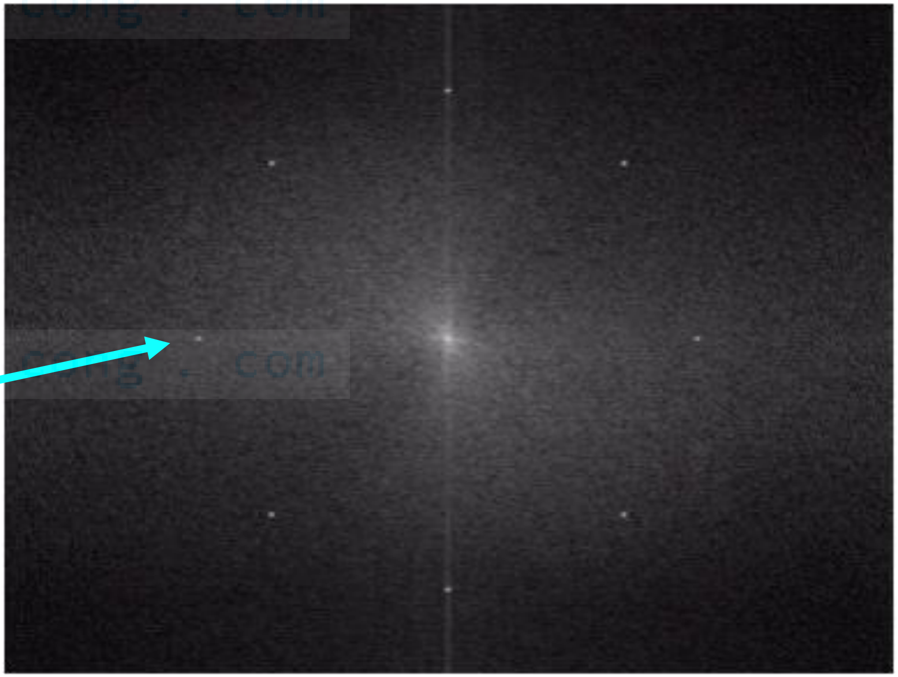
Periodic noise

- Periodic noise in an image arises typically from electrical or electromechanical interference during image acquisition.
  - It can be reduced significantly via frequency domain filtering
- *This is the only type of spatially dependent noise that will be considered in this lecture*

a
b

(a) Image corrupted by sinusoidal noise.
(b) Spectrum (each pair of conjugate impulses corresponds to one sine wave).
(Original image courtesy of NASA.)





Periodic noise looks like **dots** in the frequency domain

Estimation of noise parameters

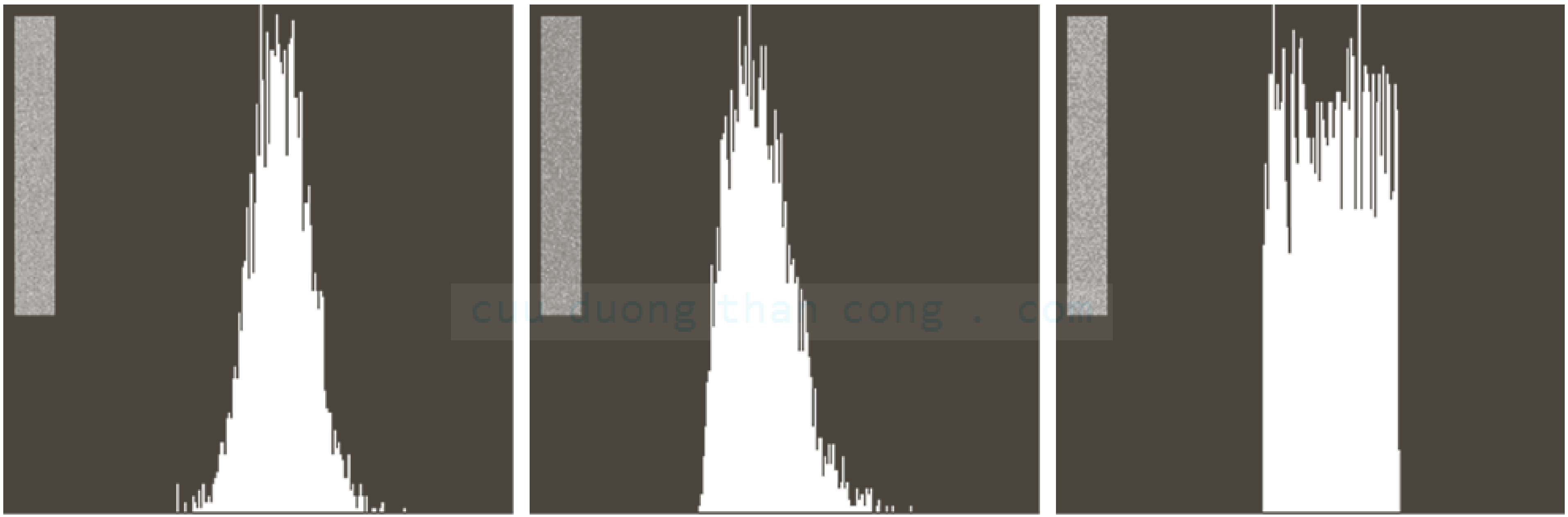
- The parameters of periodic noise typically are estimated by inspection of the Fourier spectrum of the image
  - The frequency spikes can be often detected even by visual analysis
- Inference of the periodicity of noise components directly from the image
  - Possible only in simplistic cases
- Automated analysis
  - Possible in situations in which the noise spikes are either exceptionally pronounced, or when knowledge is available about the general location of the frequency components of the interference

Estimation of noise parameters

- The parameters of noise PDFs may be known partially from sensor specifications
- It is often necessary to estimate them for a particular imaging arrangement
  - Capture a set of images of “flat” environments
  - For example, in the case of an optical sensor, this is as simple as imaging a solid gray board that is illuminated uniformly

a b c

Histograms computed using small strips of size 150×20 pixels (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images



The shapes of these histograms correspond quite closely to the shapes of the original histograms.
Their heights are different due to scaling, but the shapes are unmistakably similar

Estimation of noise parameters

- Consider a strip (subimage) denoted by S
- Let $p_s(z_i), i = 0, 1, 2, \ldots, L-1$, be the probability estimates (normalized histogram values) of the pixel intensities in S
- The mean and variance of the pixels in S as follows

$$\bar{z} = \sum_{i=0}^{L-1} z_i \, p_s(z_i)$$

and

$$\sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_s(z_i)$$

Estimation of noise parameters

- The shape of histogram identifies the closest PDF match
- Estimation of parameters for different noise PDFs
  - Gaussian noise: the mean and variance are all we need
  - Impulse noise: The actual probability of occurrence of white and black pixels. The heights of the peaks corresponding to black and white pixels are the estimates of P_a and P_b
    - This requires that both black and white pixels be visible, so a midgray, relatively constant area is needed in the image
  - Other noises: use the mean and variance to solve for the parameters a and b

Section 5.3

ADDITIVE NOISE REDUCTION BY SPATIAL DOMAIN FILTERING

Presence of noise only

- When the only degradation present in an image is noise, the formulation of the degraded image becomes

$$\boldsymbol{g(x,y) = f(x,y) + \eta(x,y)}$$

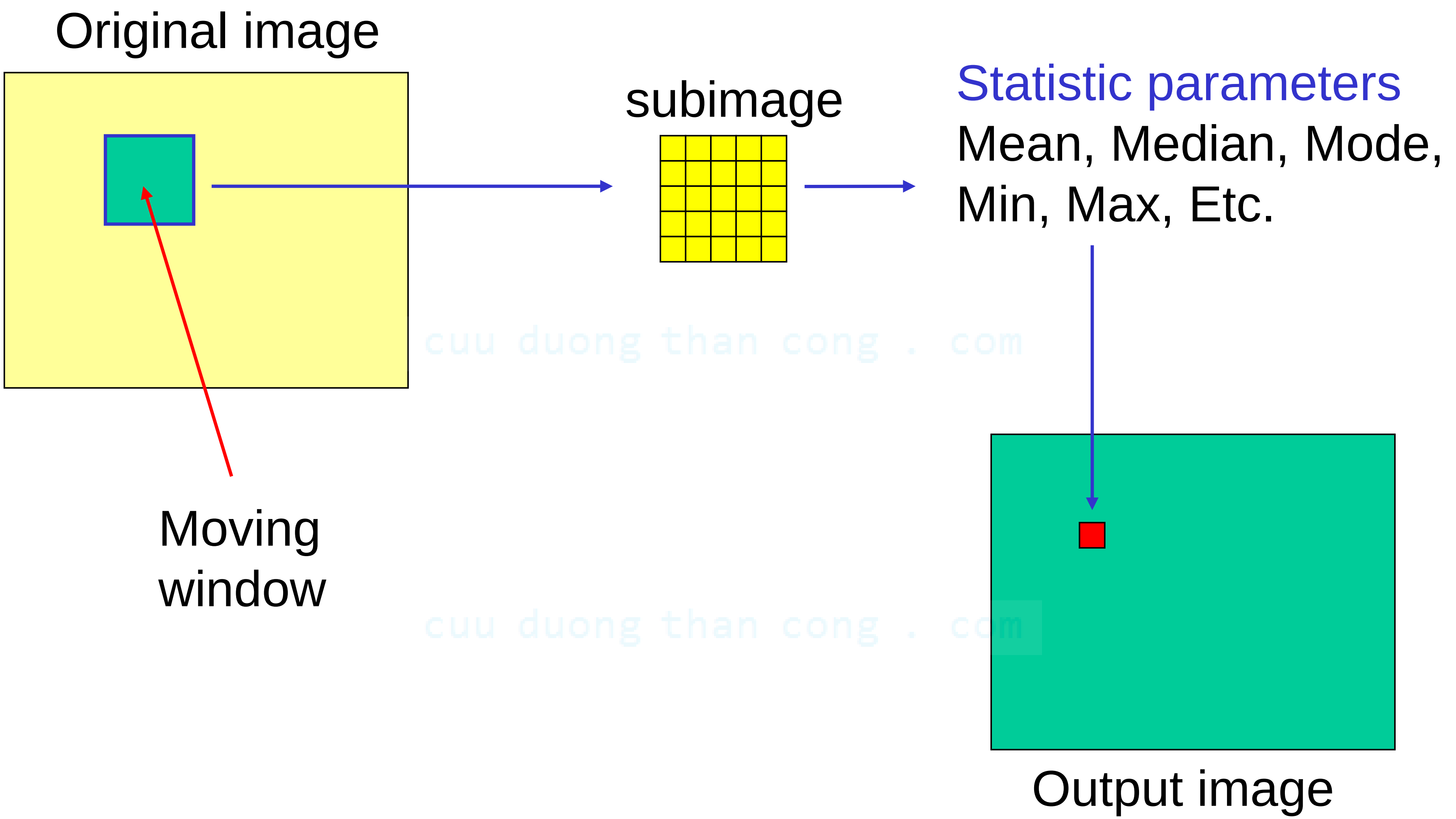
  and

$$\boldsymbol{G(u,v) = F(u,v) + N(u,v)}$$

- Subtracting the noise terms from $g(x,y)$ or $G(u,v)$ is unrealistic because these terms are unknown
  - Exception: it usually is possible to estimate the periodic noise $N(u,v)$ from the spectrum of $G(u,v)$

- **Spatial filtering** is good for situations when there is only **additive random noise**

Spatial filtering



Mean filters: Arithmetic filter

- Let S_{xy} represent the set of coordinates in a rectangular subimage window (neighborhood) of size $m \times n$ centered at point (x, y)
- The **arithmetic mean filter** computes the average value of the corrupted image $g(x, y)$ in the area defined by S_{xy}

$$\hat{f}(x,y) = \frac{1}{mn} \sum_{(s,t)\in S_{xy}} g(s,t)$$

  - This is the simplest of the mean filters

- A mean filter smooths local variations in an image, and noise is reduced as a result of blurring

Mean filters: Geometric mean filter

- An image restored using a **geometric mean filter** is given by the expression

$$\hat{f}(x, y) = \left[\prod_{(s,t)\in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

- This filter achieves smoothing comparable to the arithmetic mean filter, but it tends to lose less image detail

Mean filters: Harmonic mean filter

- The harmonic mean filter is given by

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} g(s,t)}$$

- This filter works well for salt noise, but fails for pepper noise. It does well also with other types of noise like Gaussian noise.

Mean filters: Contraharmonic mean filter

- The contraharmonic mean filter is given by

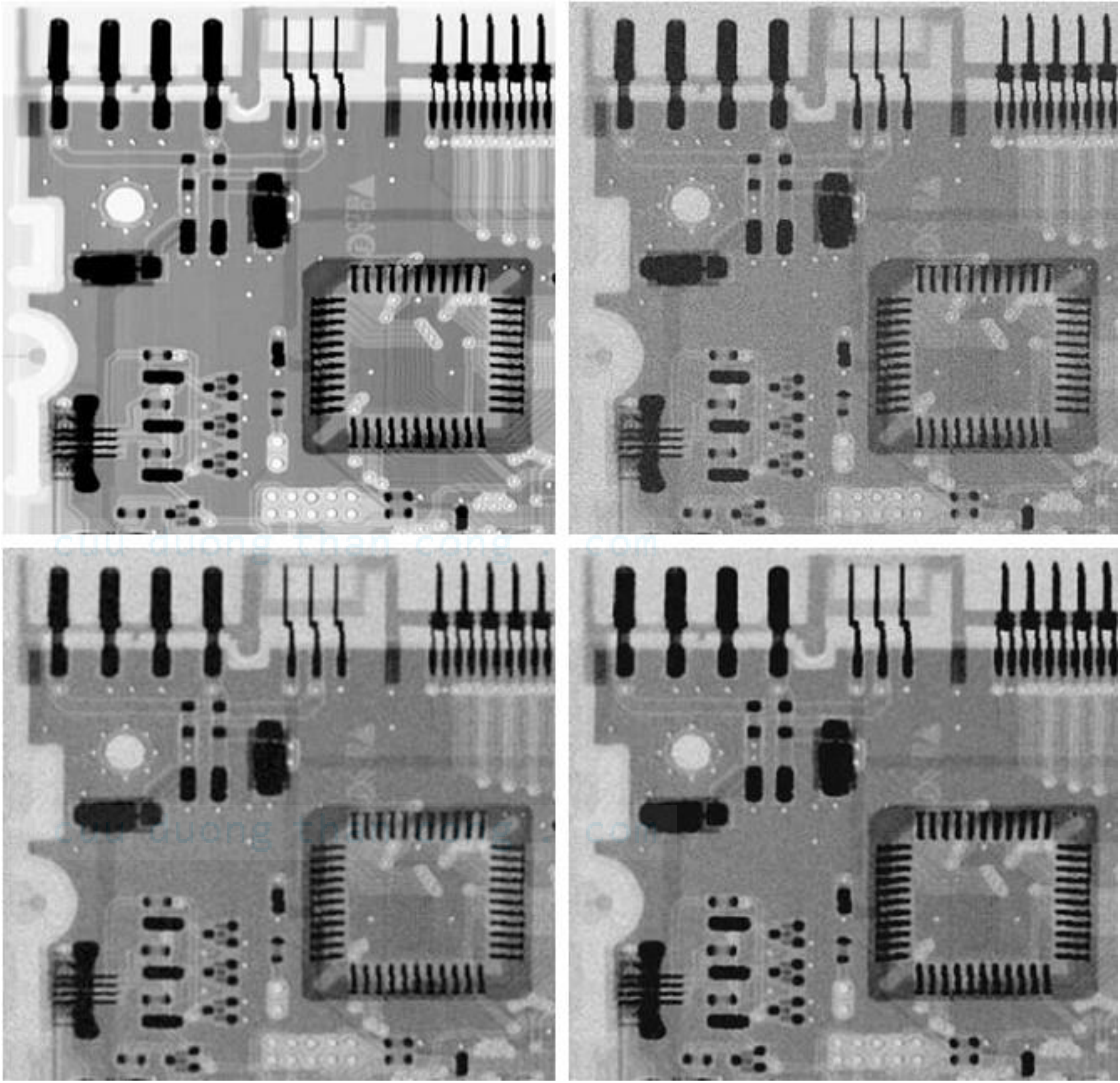
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

  - where Q is called the order of the filter

- It is well suited for reducing or virtually eliminating the effects of salt-and-pepper noise
  - $Q > 0$: eliminate pepper noise, $Q < 0$: eliminate salt noise. It cannot do both simultaneously
  - $Q = 0$: arithmetic filter, $Q = -1$: harmonic filter

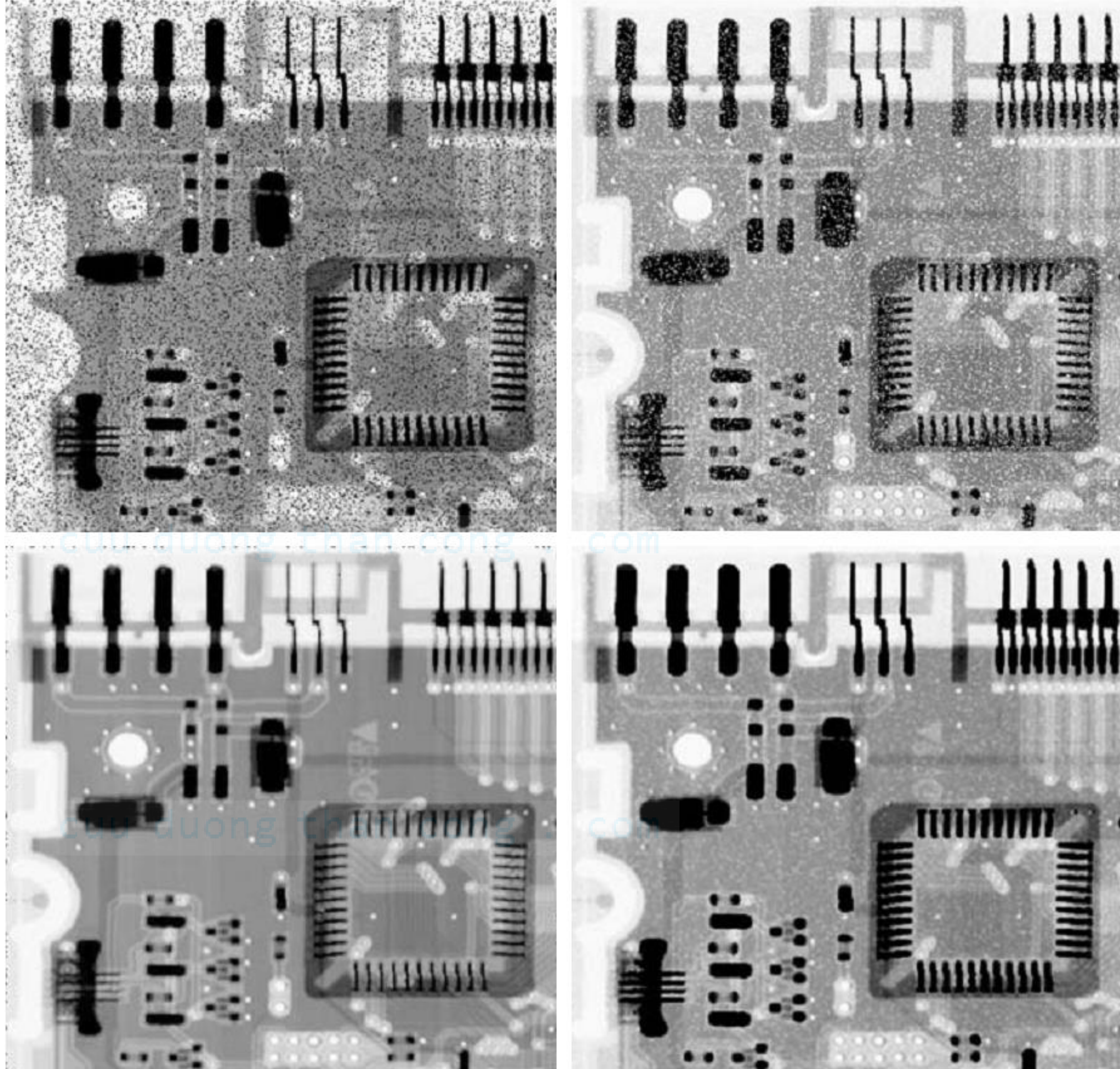
a	b
c	d

(a) X-ray image. (b) Image corrupted by additive Gaussian noise (zero mean and variance 400). (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



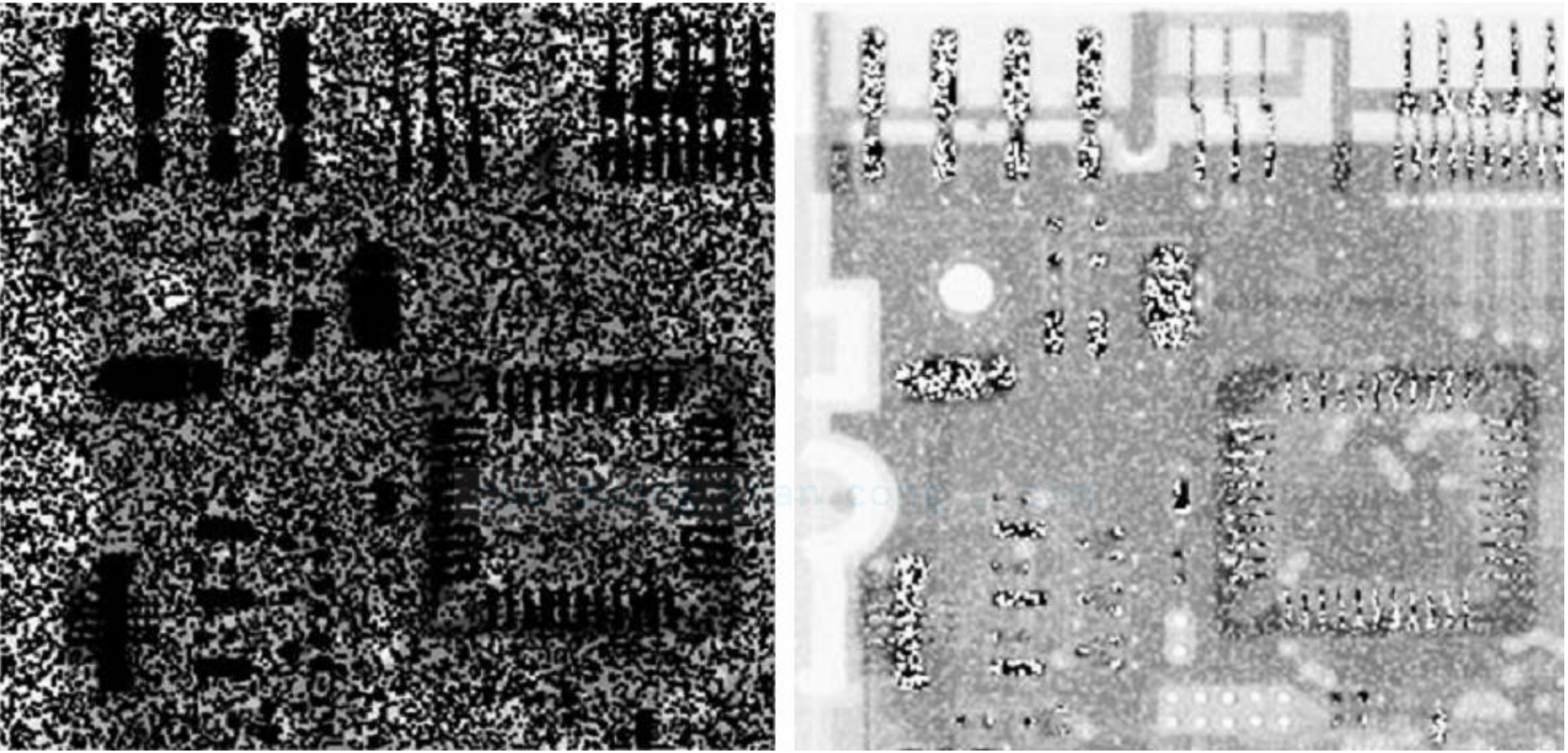
a	b
c	d

(a) Image corrupted by pepper noise with a probability of 0.1.
(b) Image corrupted by salt noise with the same probability.
(c) Result of filtering (a) with a contraharmonic filter of order 1.5.
(d) Result of filtering (b) with Q = -1.5.



Mean filters: Summary

- Arithmetic and geometric mean filters (particularly the latter) are well suited for random noise (e.g. Gaussian, uniform)
- The contraharmonic filter is well suited for impulse noise. However, whether the noise is dark or light must be known in order to select the proper sign for Q



a b

Results of selecting the wrong sign in contraharmonic filtering.
(a) Result of filtering the image corrupted by pepper noise with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering the image corrupted by salt noise with $Q = 1.5$

Order-statistic filters: Median filter

- The **median filter** replaces the value of a pixel by the median of intensity levels in the neighborhood of that pixel

$$\hat{f}(x,y) = median_{(s,t)\in S_{xy}}\{g(s,t)\}$$

  - The value of the pixel at (x, y) is included in the computation

- Excellent noise-reduction capabilities for certain types of random noise, with considerably **less blurring** than linear smoothing filters of similar size
- **Particularly effective for bipolar and unipolar impulse noise**

Order-statistic filters: Min/max filters

- The max filter finds the brightest points in an image

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- The min filter finds the darkest points in an image

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\}$$

- Pepper noise is reduced by the max filter, while salt noise is by the min filter

Order-statistic filters: Midpoint filter

- The **midpoint filter** computes the midpoint between the maximum and minimum values in the area encompassed by the filter

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

- This filter combines order statistics and averaging
- It works best for randomly distributed noise, like Gaussian or uniform noise.

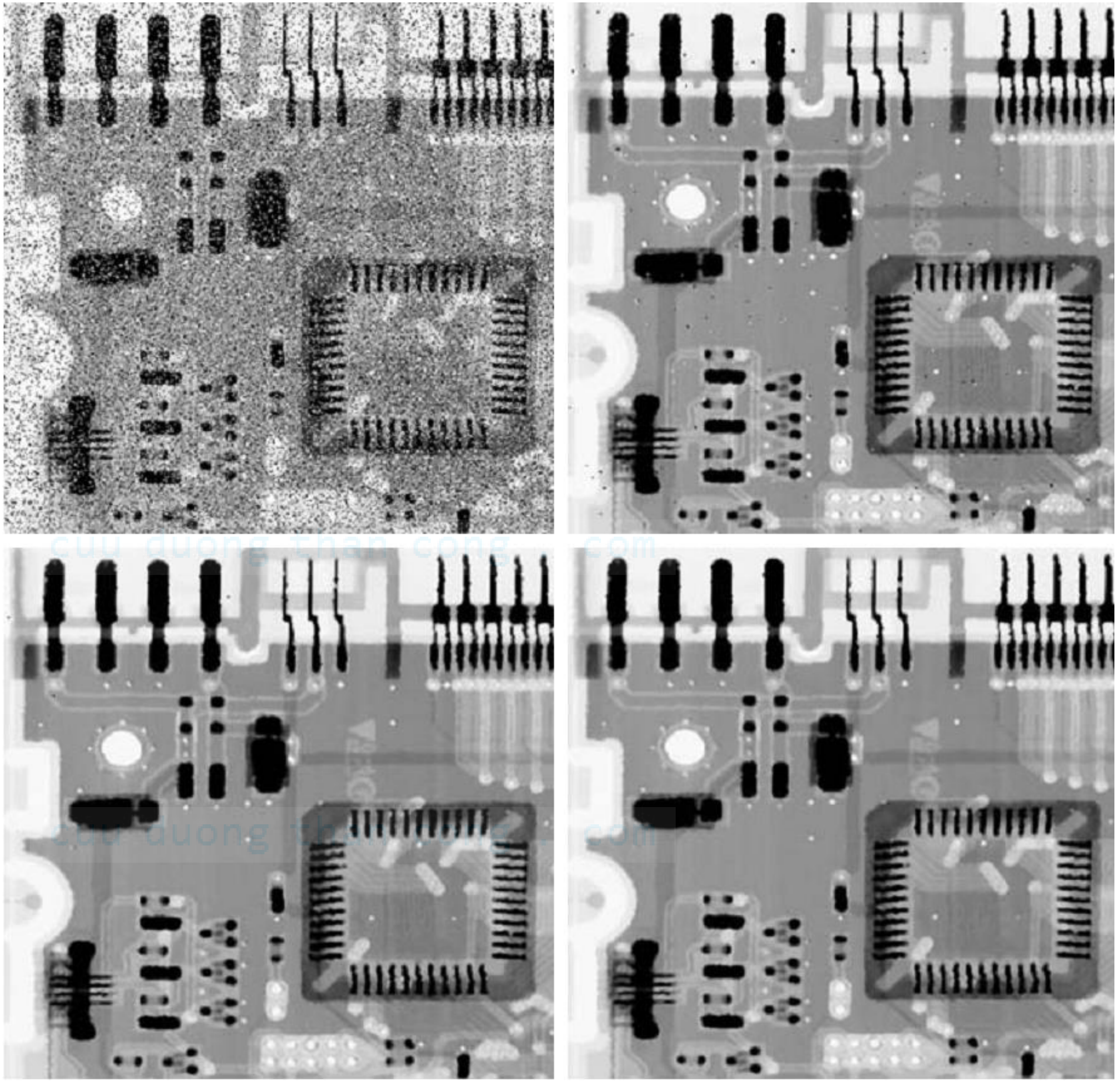
Order-statistic filters: Alpha-trimmed mean filter

- Suppose that the $d/2$ lowest and the $d/2$ highest intensity values of $g(s,t)$ in the neighborhood S_{xy} are deleted
- Let $g_r(s,t)$ represent the remaining $mn - d$ pixels
- The **alpha-trimmed mean filter** is given by

$$\hat{f}(x,y) = \frac{1}{mn-d} \sum_{(s,t)\in S_{xy}} g_r(s,t)$$

  - where the value of d can range from 0 (arithmetic mean filter) to $mn - 1$ (median filter)
  - For other values of d: useful in situations involving multiple types of noise, e.g. a combination of salt-and-pepper and Gaussian noise

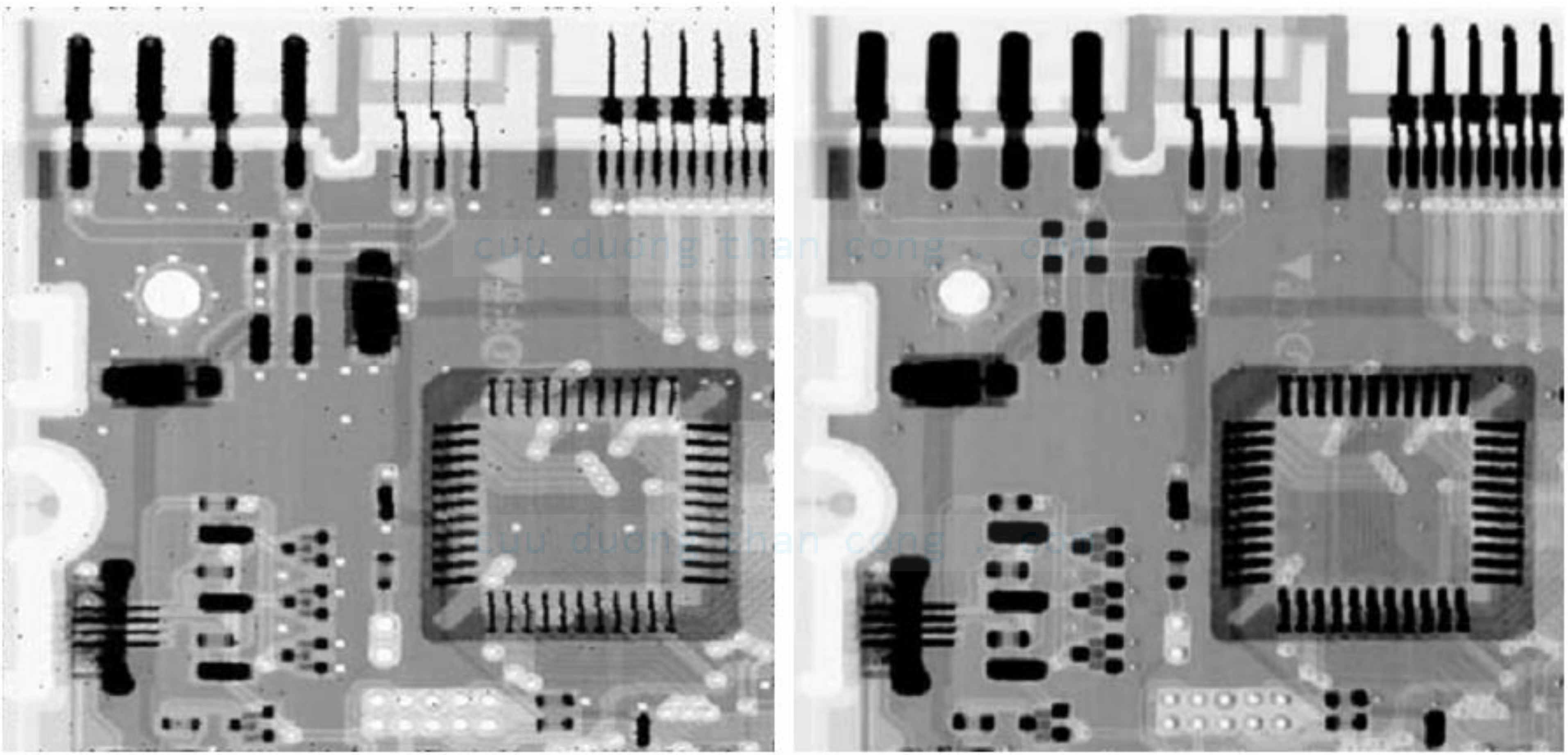
a	b
c	d



(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

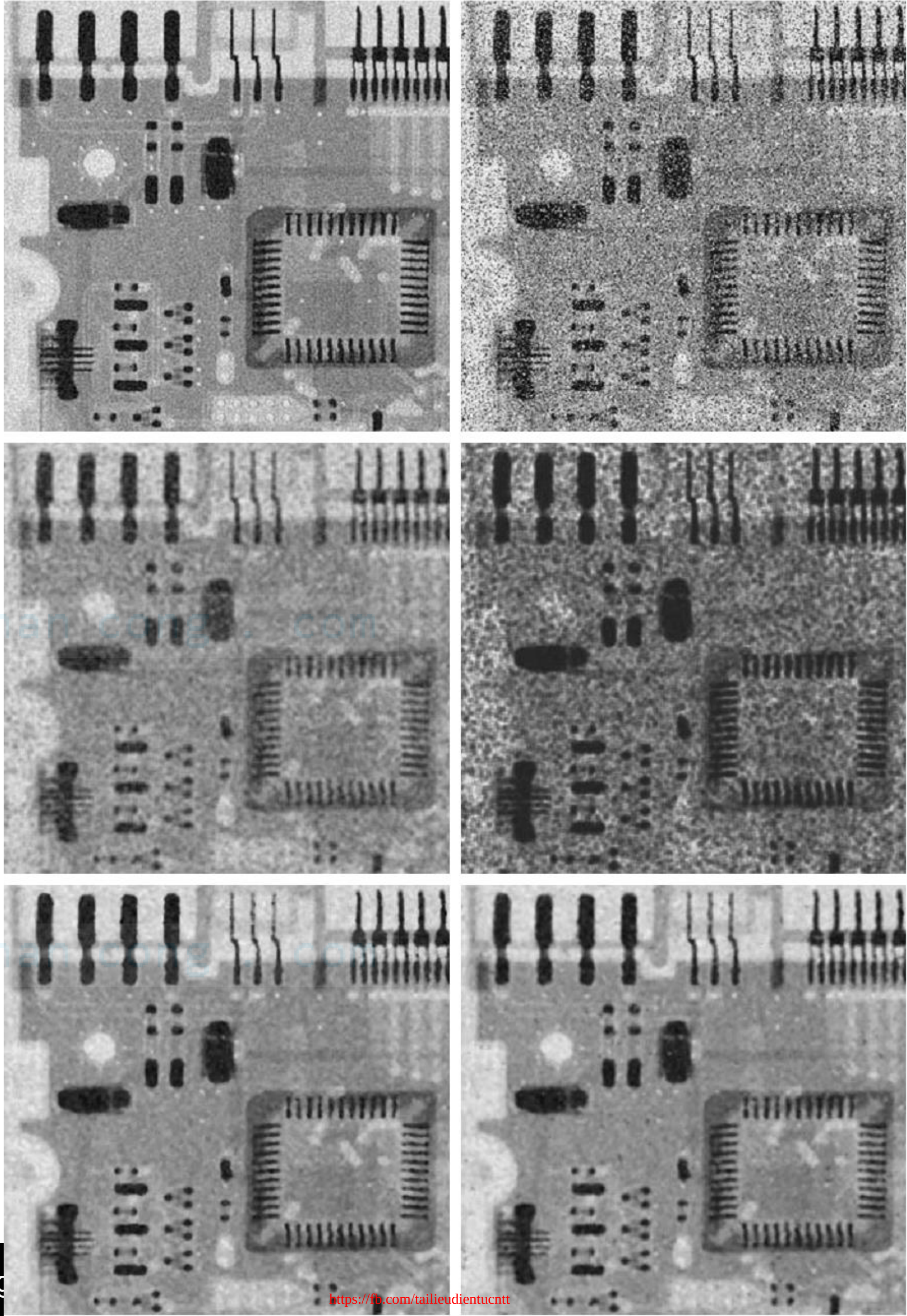
a b

(a) Result of filtering the image corrupted by pepper noise with a max filter of size 3 × 3.
(b) Result of filtering the image corrupted by salt noise with a min filter of the same size.



a	b
c	d
e	f

(a) Image corrupted by additive uniform noise (zero mean and variance 800).
(b) Image additionally corrupted by additive salt-and-pepper noise $P_a = P_b = 0.1$
Image (b) filtered with a 5×5 (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.



Adaptive filters

- The adaptive filters changes their behaviors based on statistical characteristics of the image inside the filter region defined by the $m \times n$ rectangular window S_{xy}
- **Performance superior** to that of the filters discussed thus far, yet **filter complexity increased**

Adaptive, local noise reduction filter

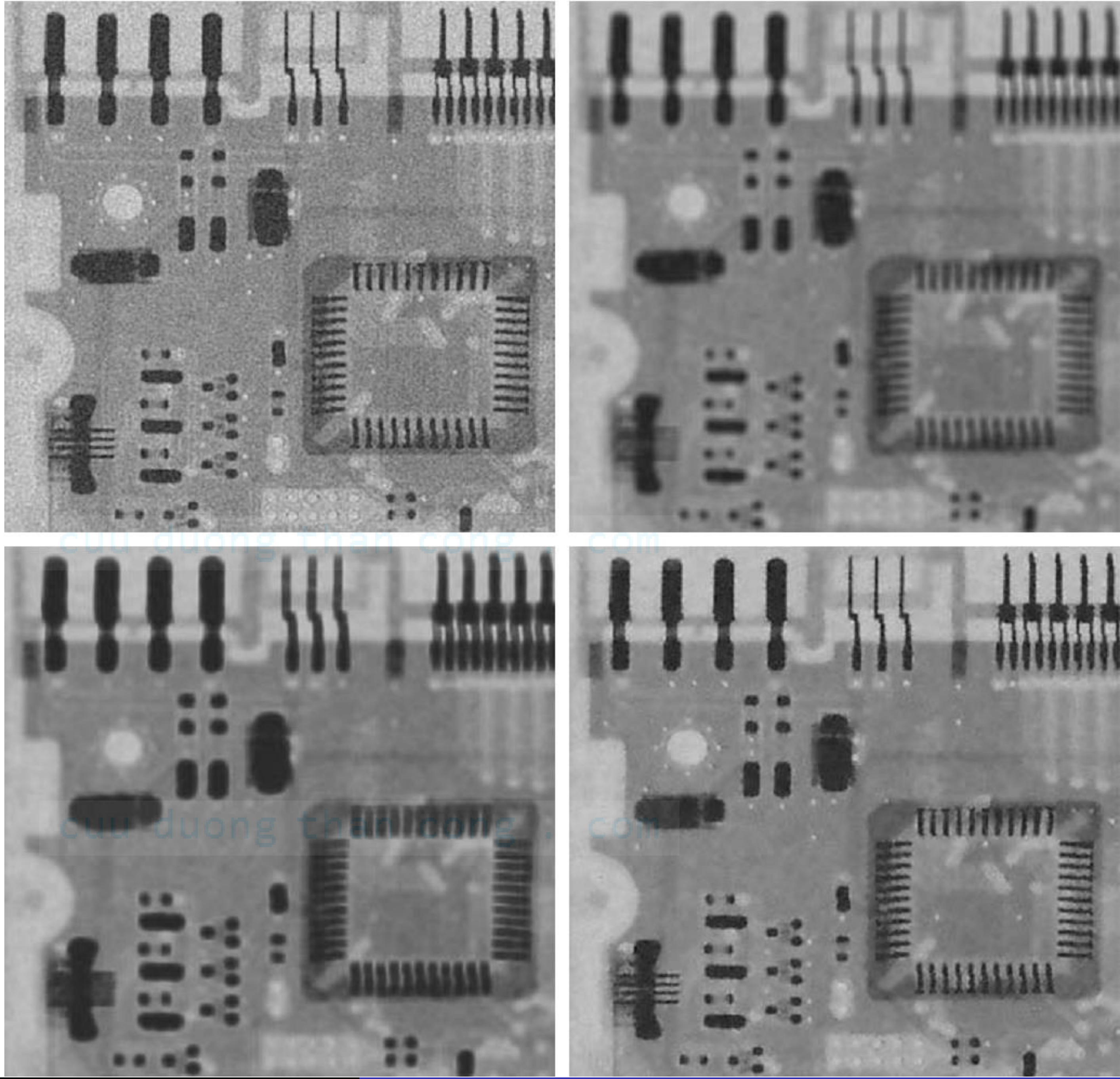
- The response of the filter at any point (x, y) on which the region S_{xy} is centered is to be based on four quantities:
  a) $g(x, y)$: the value of the noisy image at (x, y)
  b) σ_η^2: the variance of the noise corrupting $f(x, y)$ to form $g(x, y)$
  c) $m_L = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$: the local mean of the pixels in S_{xy}
  d) $\sigma_L^2 = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} (g(s, t) - m_L)^2$: the local variance of the pixels in S_{xy}

Adaptive, local noise reduction filter

- The formula of the **adaptive, local noise reduction filter** is given as

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L]$$

  - $\sigma_\eta^2 = 0$: no noise, the filter should return $g(x,y)$
  - σ_L^2 is high relative to σ_η^2: edges (should be preserved), the filter should return the value close to $g(x,y)$
  - $\sigma_L^2 = \sigma_\eta^2$: areas inside objects, the filter should return the arithmetic mean value m_L



a b
c d

(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering ($\sigma_\eta^2 = 1000$). All filters were of size 7 × 7

Adaptive median filter

- The **adaptive median filter** is superior to the traditional median filter in the sense that
  - Impulse noise with P_a and/or $P_b > 0.2$ can be handled
  - Details are largely preserved while non-impulse noises are smoothed

- This filter also works in a rectangular window area S_{xy}, however, **the size of S_{xy} is increased** during the filtering

Adaptive median filter

- Let z_{min} the minimum gray level value in S_{xy}

  z_{max} the maximum gray level value in S_{xy}

  z_{median} the median gray level value in S_{xy}

  z_{xy} the gray level value at pixel (x, y)

  S_{max} the maximum allowed size of S_{xy}

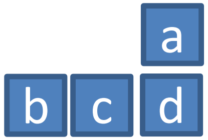
Level *A*: A1= $z_{median} - z_{min}$
A2= $z_{median} - z_{max}$
If A1 > 0 and A2 < 0, goto level B
Else → Window is not big enough
increase window size
If window size <= S_{max} repeat level A
Else return z_{xy}

Determine whether z_{median} is an impulse or not

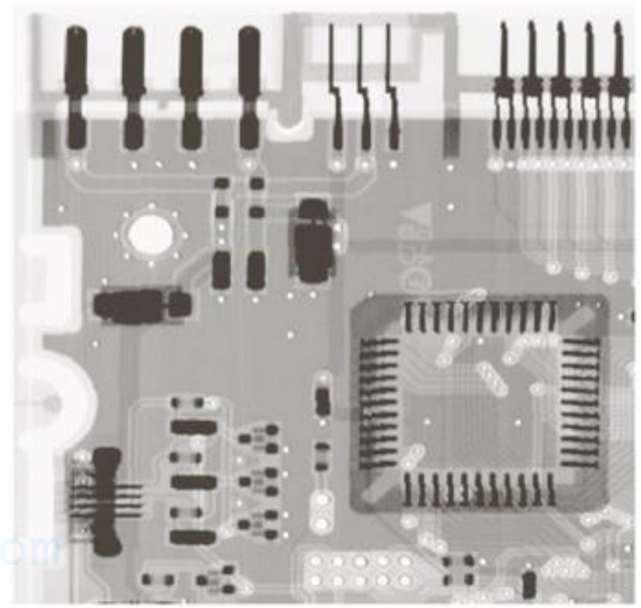
Level *B*: → z_{median} is not an impulse

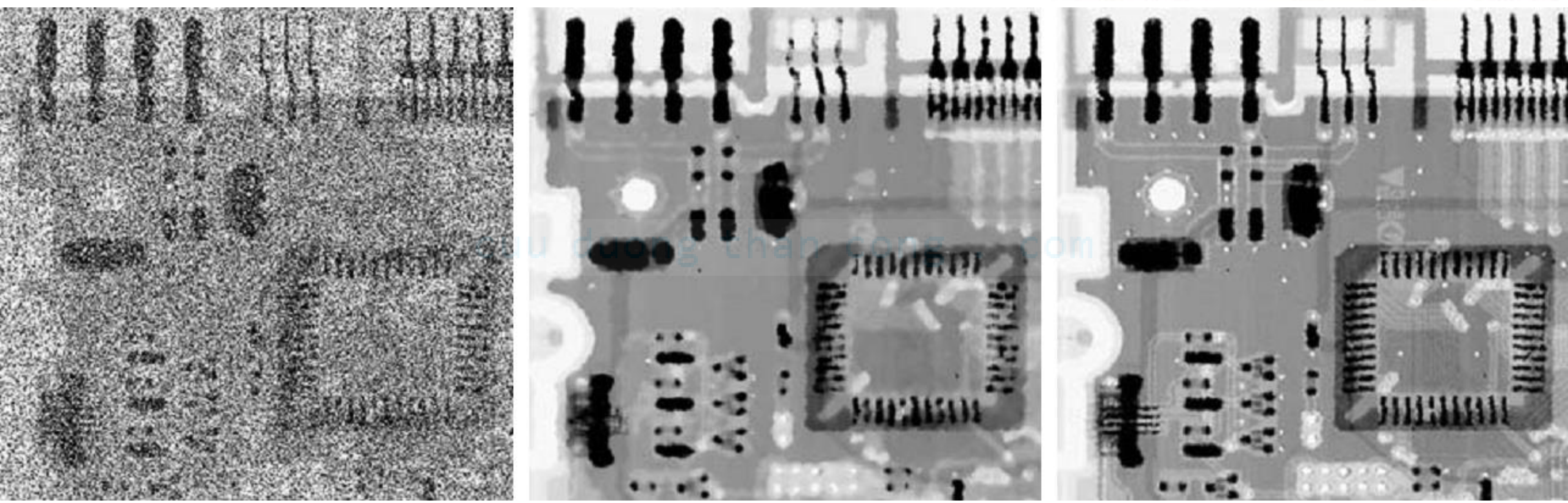
B1= $z_{xy} - z_{min}$
B2= $z_{xy} - z_{max}$
If B1 > 0 and B2 < 0, → z_{xy} is not an impulse
return z_{xy} → to preserve original details
Else
return z_{median} → to remove impulse

Determine whether z_{xy} is an impulse or not



(a) Original image. (b) Corrupted image of (a) by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$.
(c) Result of filtering with a 7×7 median filter.
(d) Result of adaptive median filtering with $S_{max} = 7$





Section 5.4

PERIODIC NOISE REDUCTION BY FREQUENCY DOMAIN FILTERING

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Characteristics of periodic noise

- Periodic noise appears as concentrated bursts of energy in the Fourier transform, at locations corresponding to the frequencies of the periodic interference
- The approach is to use a selective filter to isolate the noise
  - Bandreject, bandpass, and notch filter (introduced in Lecture 4 – P2)
  - Optimum notch filtering

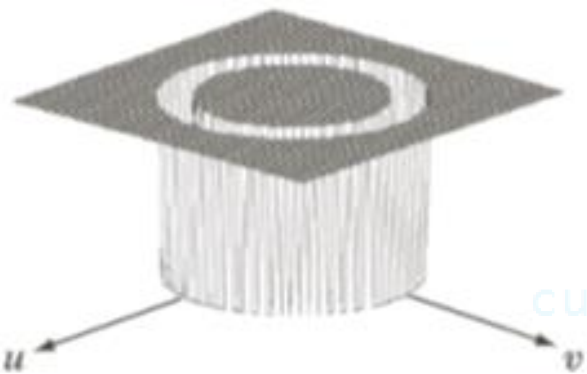
Bandreject Filters

- Let D is the width of the band, D is the distance $D(u,v)$ from the center of the filter, and D_0 is the cut-off frequency

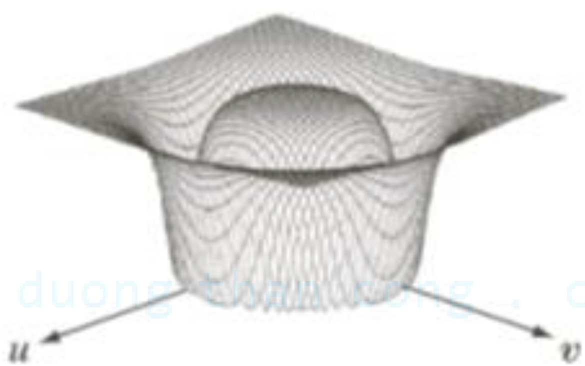
$$H(u,v) = \begin{cases} 0 & if\, D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & otherwise \end{cases}$$

$$H(u,v) = \frac{1}{1 + \left[\frac{DW}{D^2 - D_0^2}\right]^{2n}}$$

$$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$$

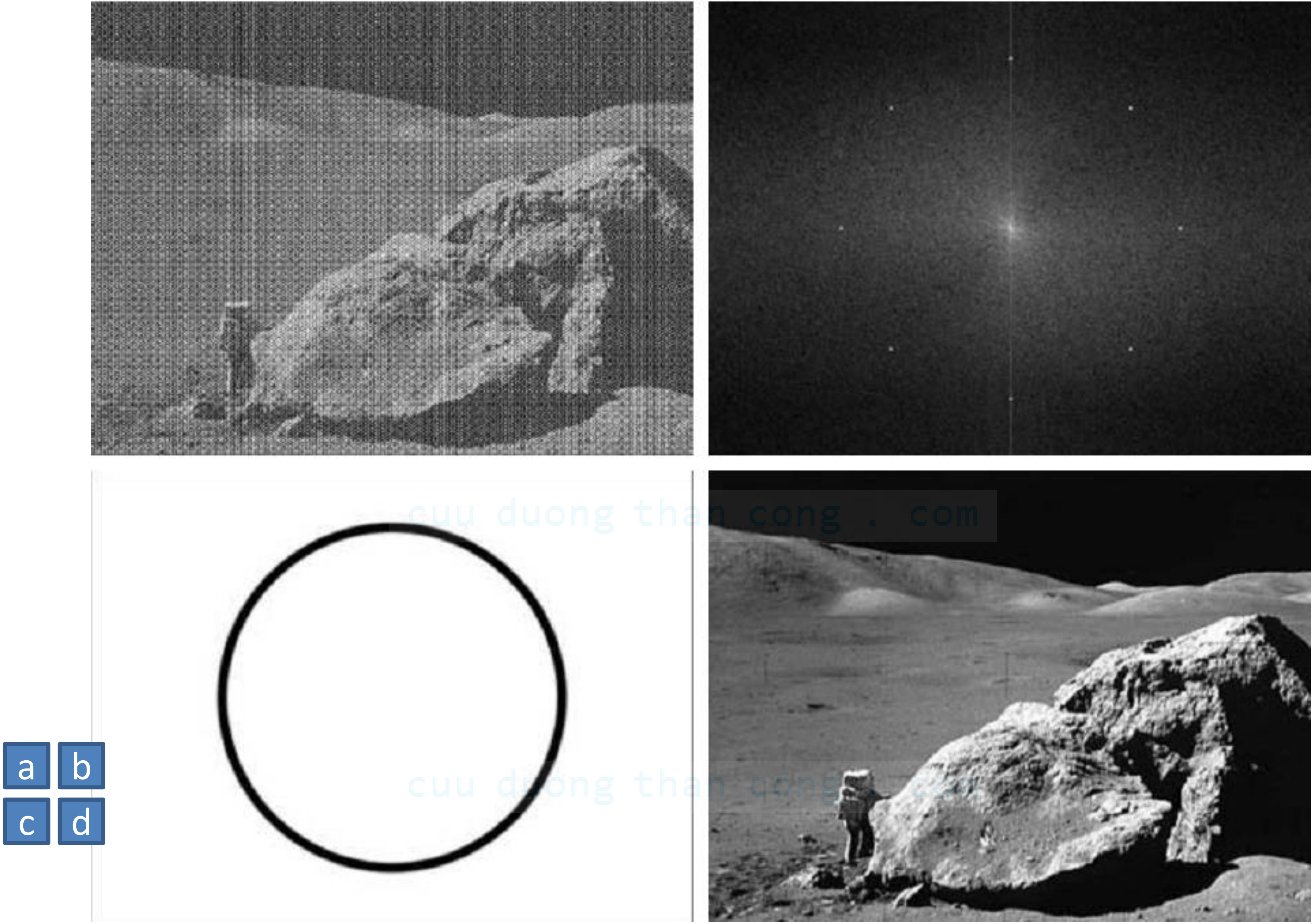


Ideal bandreject filter



Butterworth bandreject filter of order 1

Gaussian bandreject filter

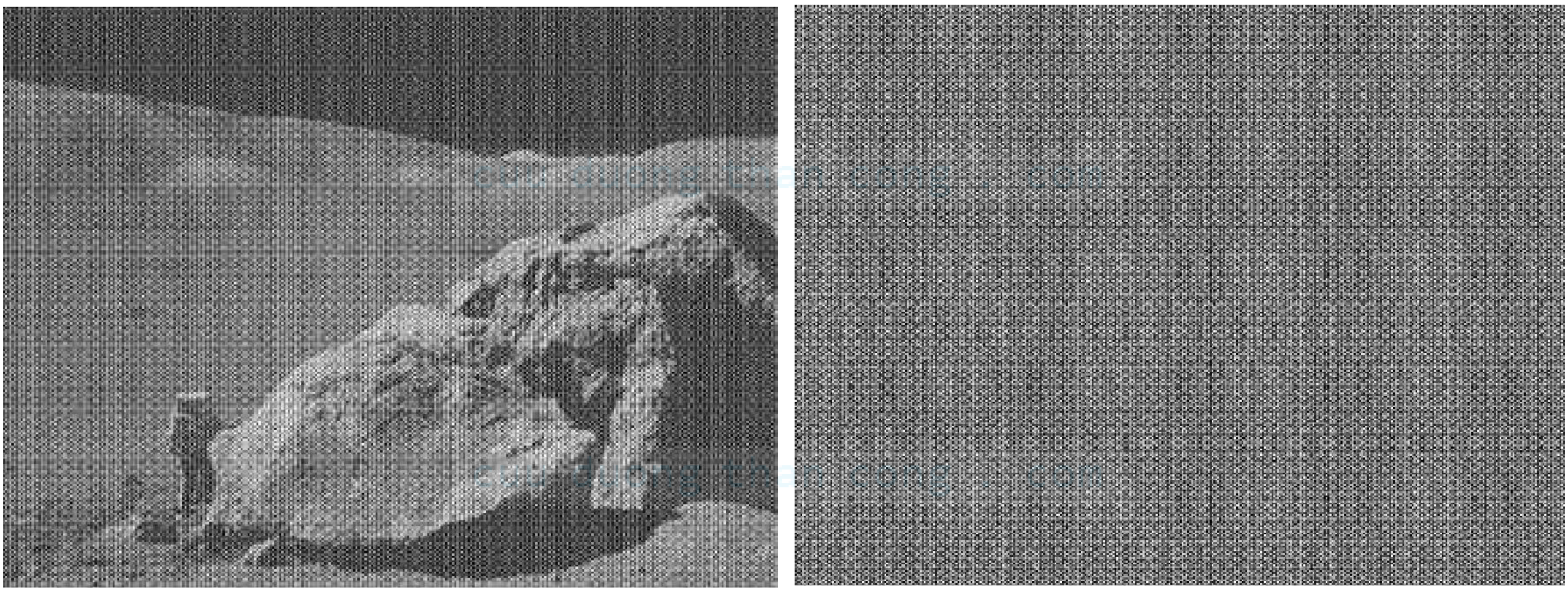


(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter of order 4. (d) Result of filtering. (Original image courtesy of NASA.)

Bandpass filters

- A bandpass filter performs the opposite operation of a bandreject filter

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$



a b (a) Image corrupted by sinusoidal noise. (b) Noise pattern of (a) obtained by bandpass filtering

Notch filters

- A notch filter rejects/passes frequencies in a predefined neighborhoods about a center frequency
- Notch reject filters are products of highpass filters whose centers have been translated to the centers of the notches

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v)H_{-k}(u,v)$$

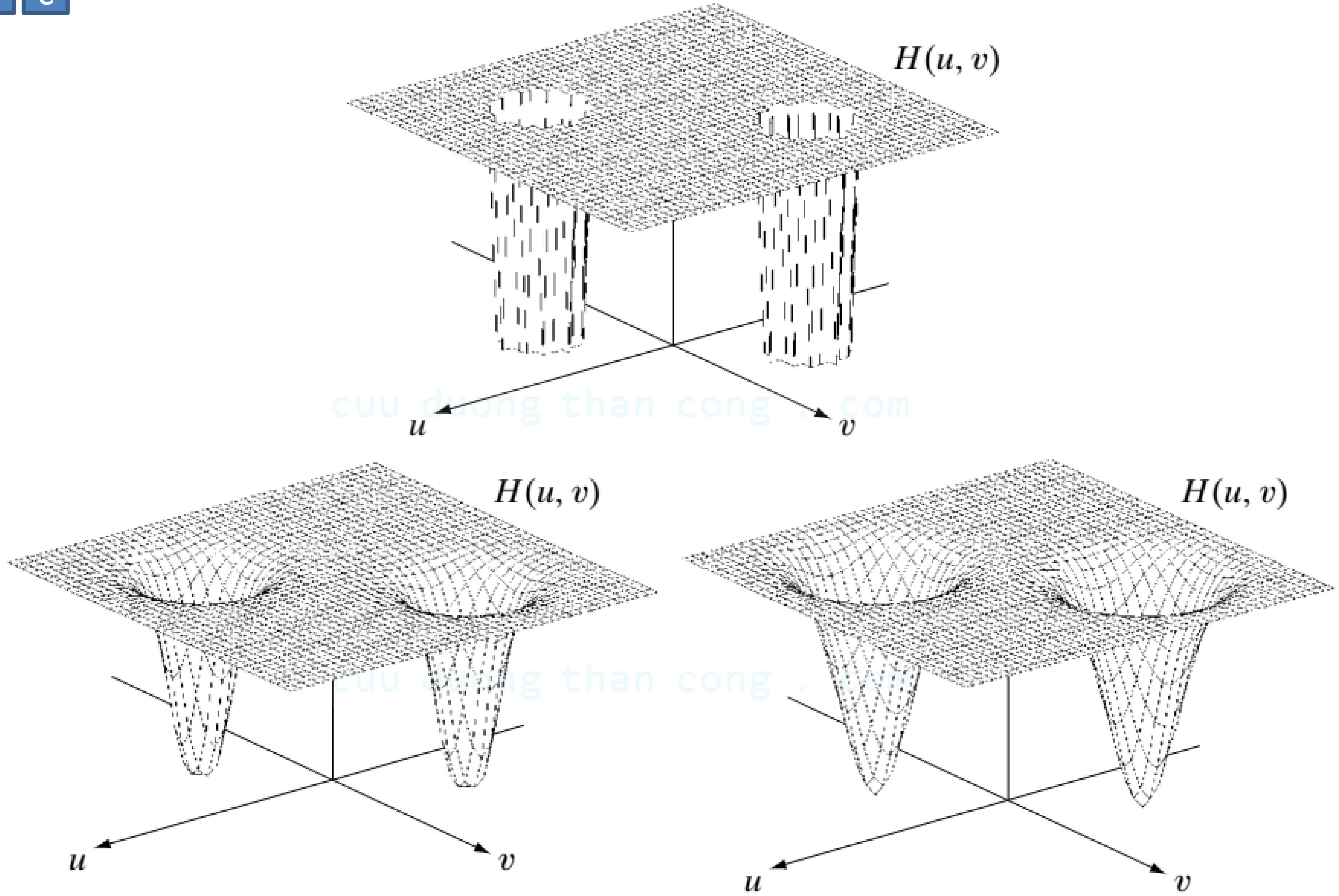
  - where $H_k(u,v)$ and $H_{-k}(u,v)$ are highpass filters whose centers are at (u_k, v_k) and $(-u_k, -v_k)$, respectively, and Q is the number of notch pairs
- Notch pass filter $H_{NP}(u,v) = 1 - H_{NR}(u,v)$

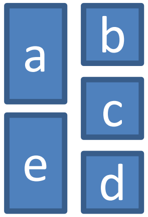
Notch filters

- Notch pass filters pass the frequencies contained in the notch areas while notch reject filters suppress those frequencies
- Notch filters must appear in symmetric pairs about the origin in order to obtain meaningful results.
  - It is due to the symmetry of the Fourier transform,
  - Exception: if the notch filter is located at the origin, in which case it appears by itself
- The number of pairs of notch filters is arbitrary. The shape of the notch areas also can be arbitrary (e.g., rectangular).

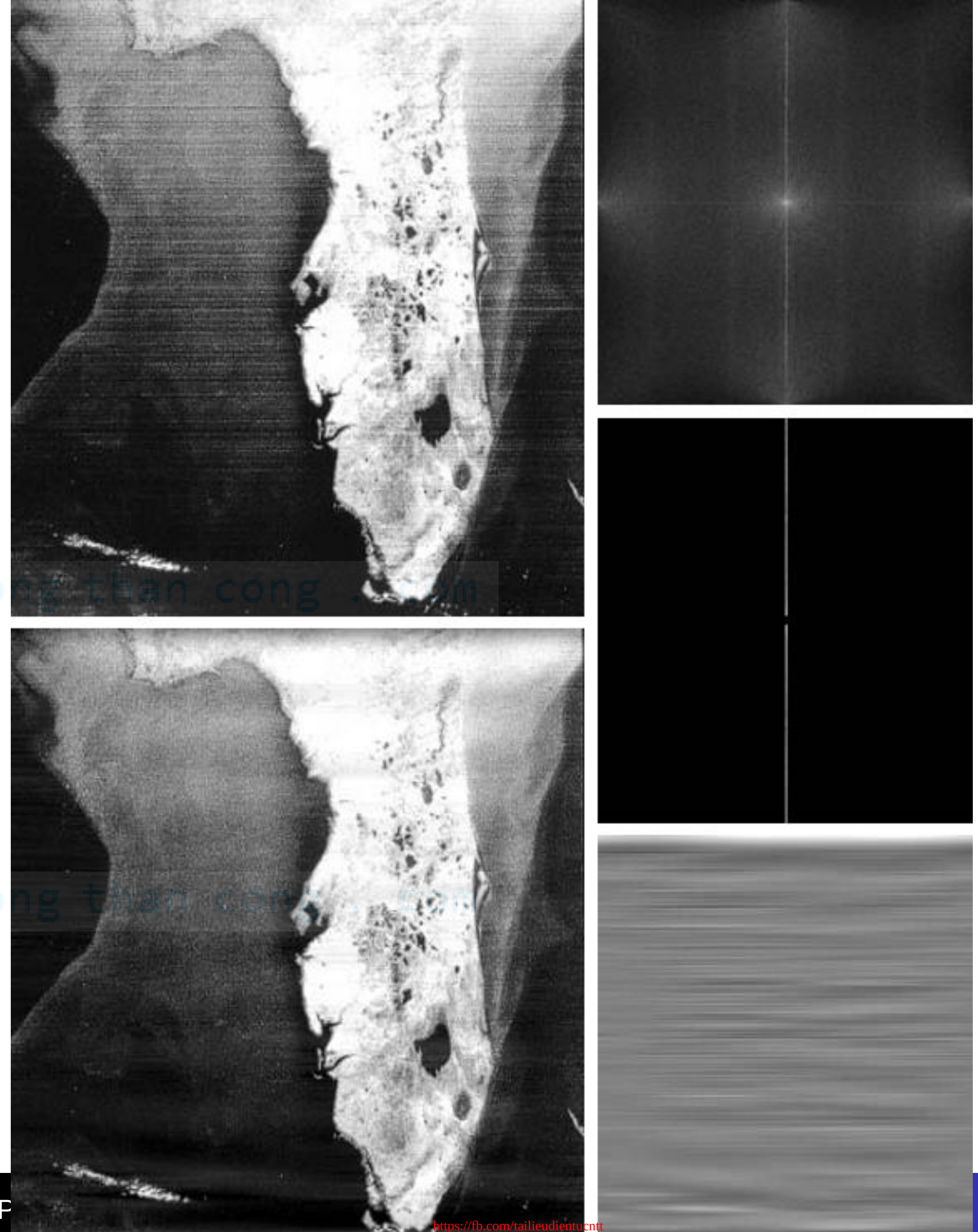
a b c

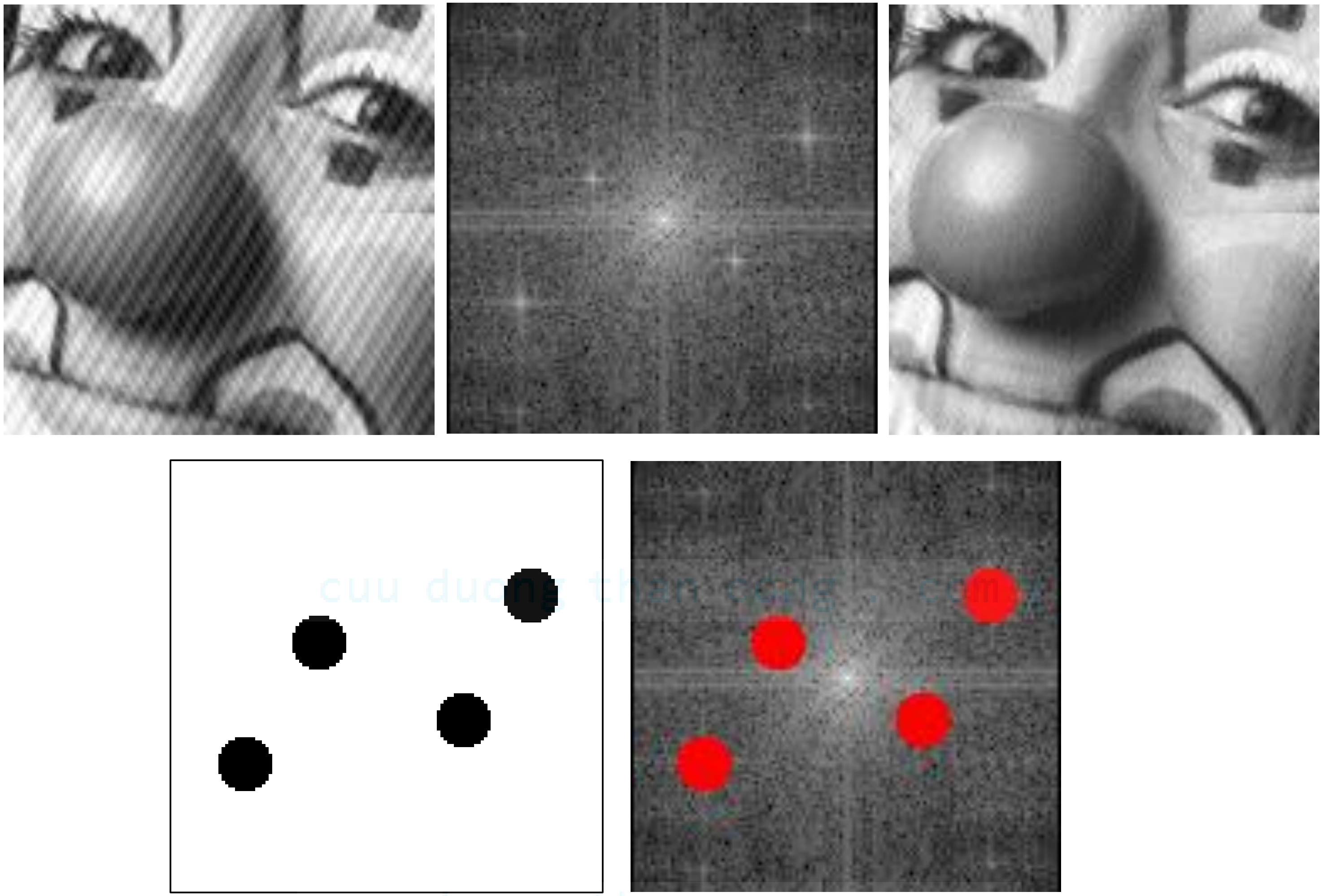
Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



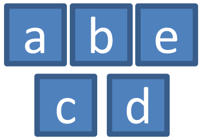


(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)





Source: http://www.imagemagick.org/Usage/fourier/



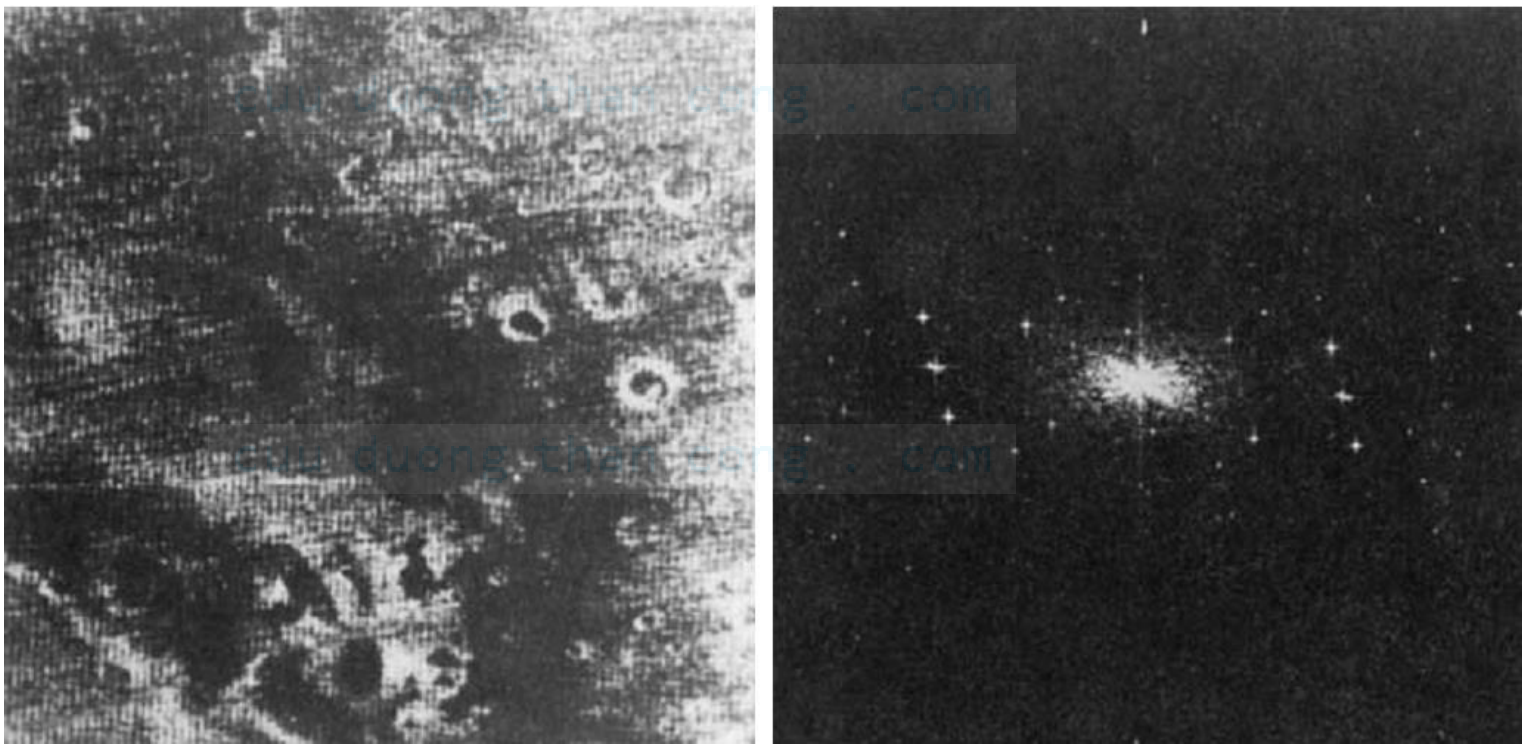
(a) Image corrupted by periodic noise. (b) Spectrum of (a). (c) A notch reject filter designed for (b). (d) Notch reject filter superimposed on (b). (e) Result of notch reject filtering

Optimum notch filtering

- The **optimum notch filtering** is useful when several interference components are present
  - The filtering process is optimum in the sense that it minimizes local variances of the restored estimate $\hat{f}(x, y)$

a b

(a) Image of the Martian terrain taken by Mariner 6. (b) Fourier spectrum showing periodic interference.



Optimum notch filtering

- The filtering process first isolates the principal contributions of the interference pattern and then subtracts a variable, weighted portion of the pattern from the corrupted image.

$$\hat{f}(x,y) = g(x,y) - \boldsymbol{w(x,y)}\eta(x,y)$$

  - where $\hat{f}(x,y)$ is the estimate of $f(x,y)$, $g(x,y)$ is the corrupted image, $\eta(x,y)$ is the interference noise pattern and $w(x,y)$ is called a weighting or modulation function

Optimum notch filtering

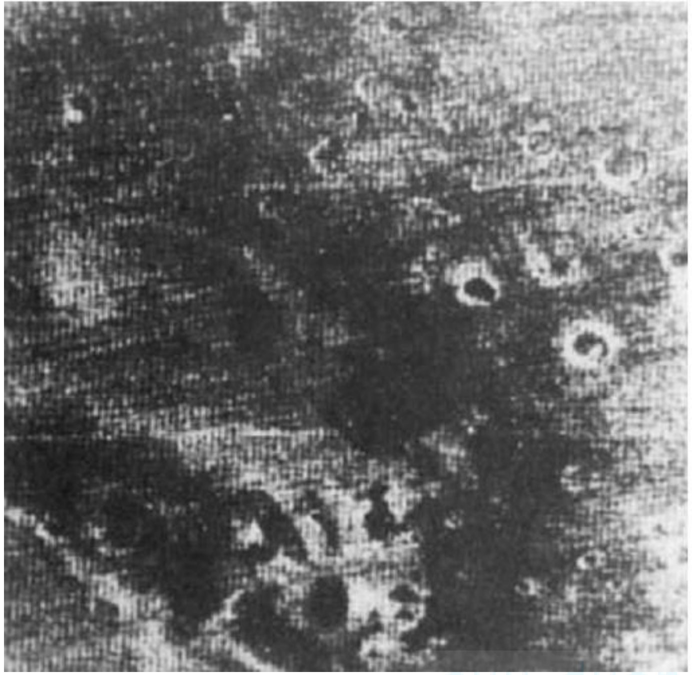
- The principal frequency components of the interference pattern can be extracted by placing a notch pass filter at the location of each spike

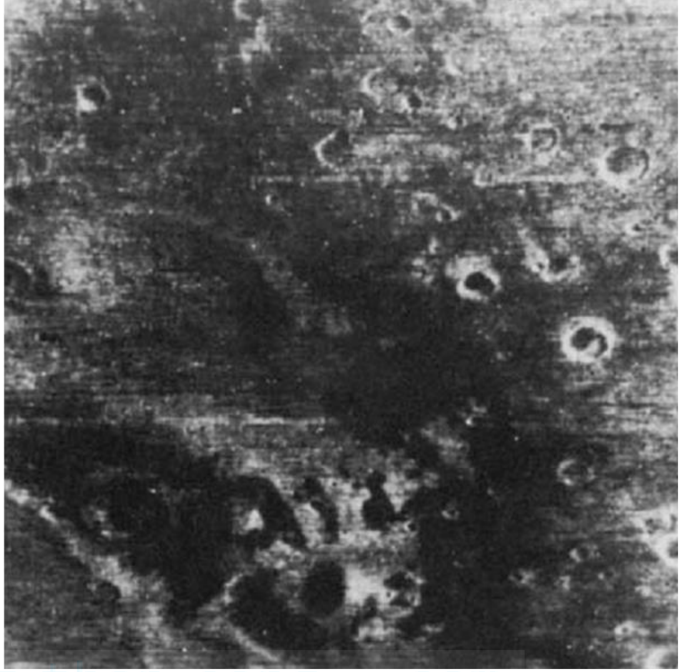
$$\eta(x,y) = \mathcal{F}^{-1}\{H_{NP}(u,v)G(u,v)\}$$

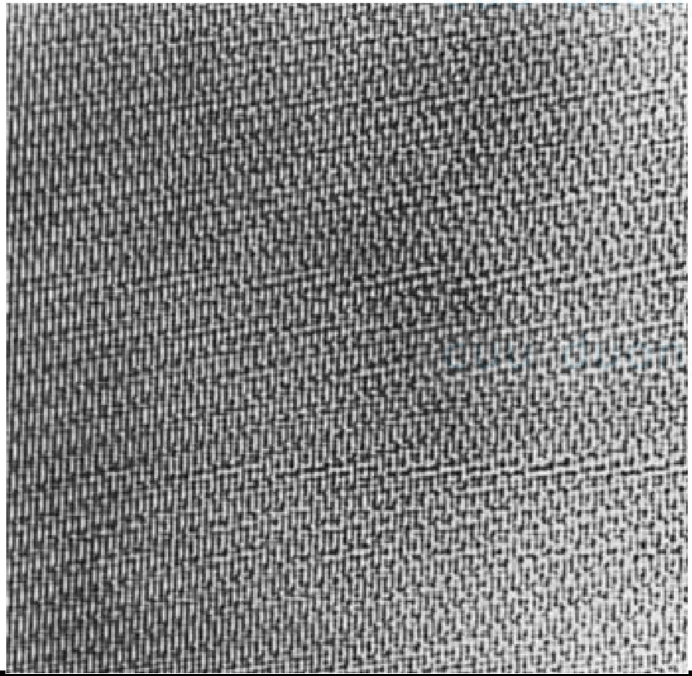
- $w(x,y)$ is selected to minimize the variance of $\hat{f}(x,y)$ over a specified neighborhood of every point (x,y)

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{n^2}(x,y) - \bar{\eta}^2\,(x,y)}$$

  - $w(x,y)$ is assumed to be constant in a neighborhood of size $(2a+1)\times(2b+1)$
  - This can be done by solving $\frac{\partial\sigma^2(x,y)}{\partial w(x,y)}$







a b
c

(a) Original image. (b) Processed image. (c) Noise interference pattern $\eta(x, y)$

Section 5.5

DEGRADATION FUNCTIONS

Linear position-invariant degradation

- A linear, spatially-invariant degradation system with additive noise can be modeled as follows

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- Many degradations can be approximated by linear, position-invariant processes
  - Linear system theory is available for the image restoration problems
  - Nonlinear and position-dependent techniques, although more general (and usually more accurate), introduce difficulties that often have no known solution or are very difficult to solve computationally.
- Keywords: image deconvolution, deconvolution filter

Estimating the degradation function

- Estimation by image observation
- Estimation by experimentation
- Estimation by mathematical modeling

- Restoration using an estimated degradation function is sometimes called **blind deconvolution**, due to the fact that the true degradation function is seldom known completely

Estimation by image observation

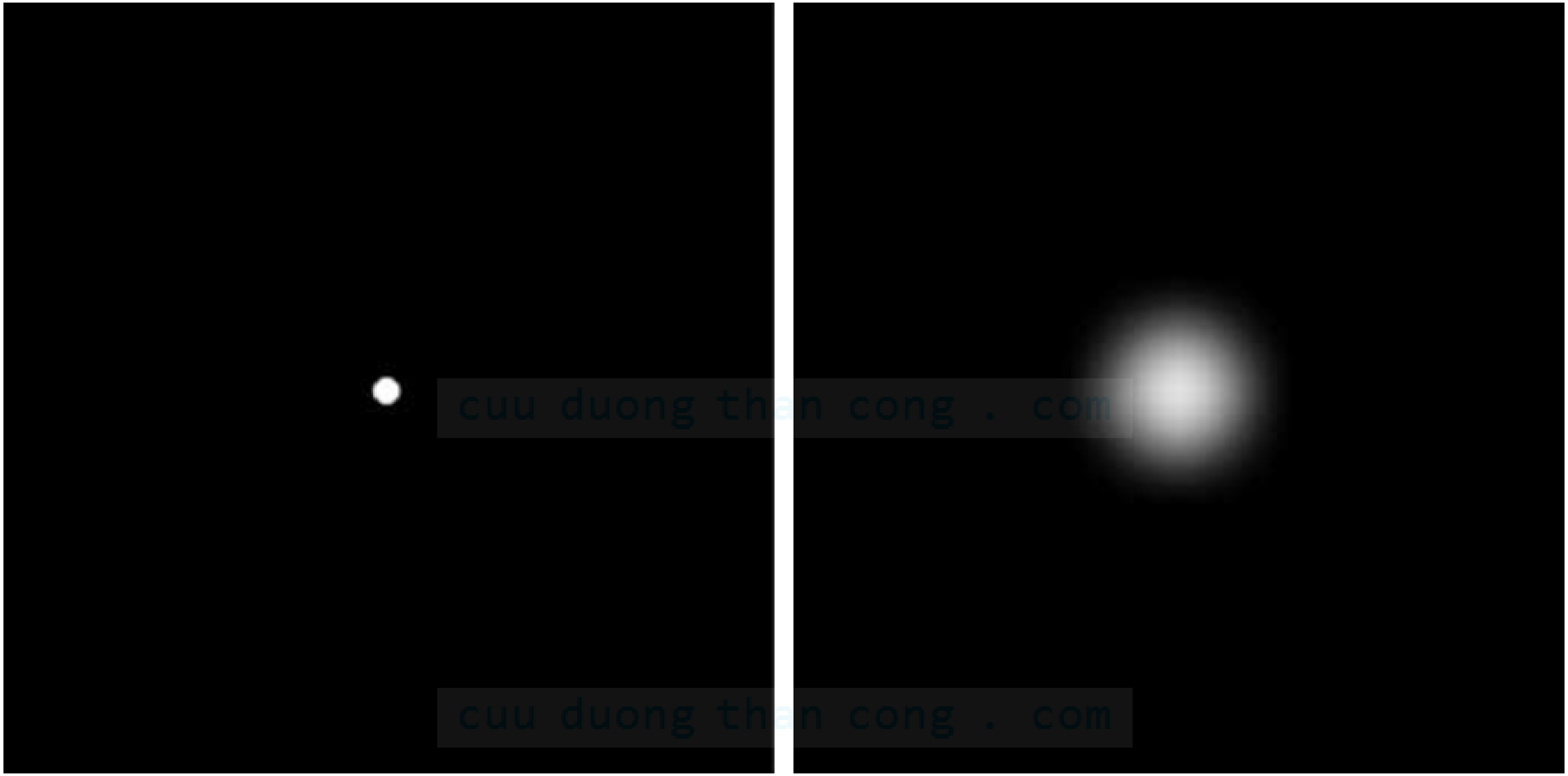
- Estimate H by gathering information from the image itself
- Let $g_s(x, y)$ be the observed subimage
  - Rectangular, containing sample structures, strong signal content.
- Let $\hat{f}_s(x, y)$ be the processed subimage
  - It is the estimation of the original image in the predefined area
- Assume that the effect of noise is negligible due to the strong-signal area. It is then $H_s(u, v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$
- The complete degradation function $H(u, v)$ can be deduced based on the assumption of position invariance.

- Laborious process used only in very specific circumstances like restoring an old photograph of historical value.

Estimation by experimentation

- If equipment similar to the equipment used to acquire the degraded image is available, it is possible in principle to obtain an accurate estimate of the degradation
  - Images are degraded as closely as possible to the image we wish to restore using different system settings
- The impulse response of the degradation is obtained by imaging an impulse (small dot of light) using the same system settings.
- The degradation function $H(u,v) = \frac{G(u,v)}{A}$
  - where $G(u,v)$ is the Fourier transform of the observed image and A is a constant describing the strength of the impulse.

Estimation by experimentation



a b Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

Estimation by modeling

- Degradation modeling has been widely used because of its meaningful insights into the image restoration problem.
- The model can take into account environmental conditions that cause degradations.
  - E.g. Hufnagel and Stanley [1964] is based on the physical characteristics of atmospheric turbulence

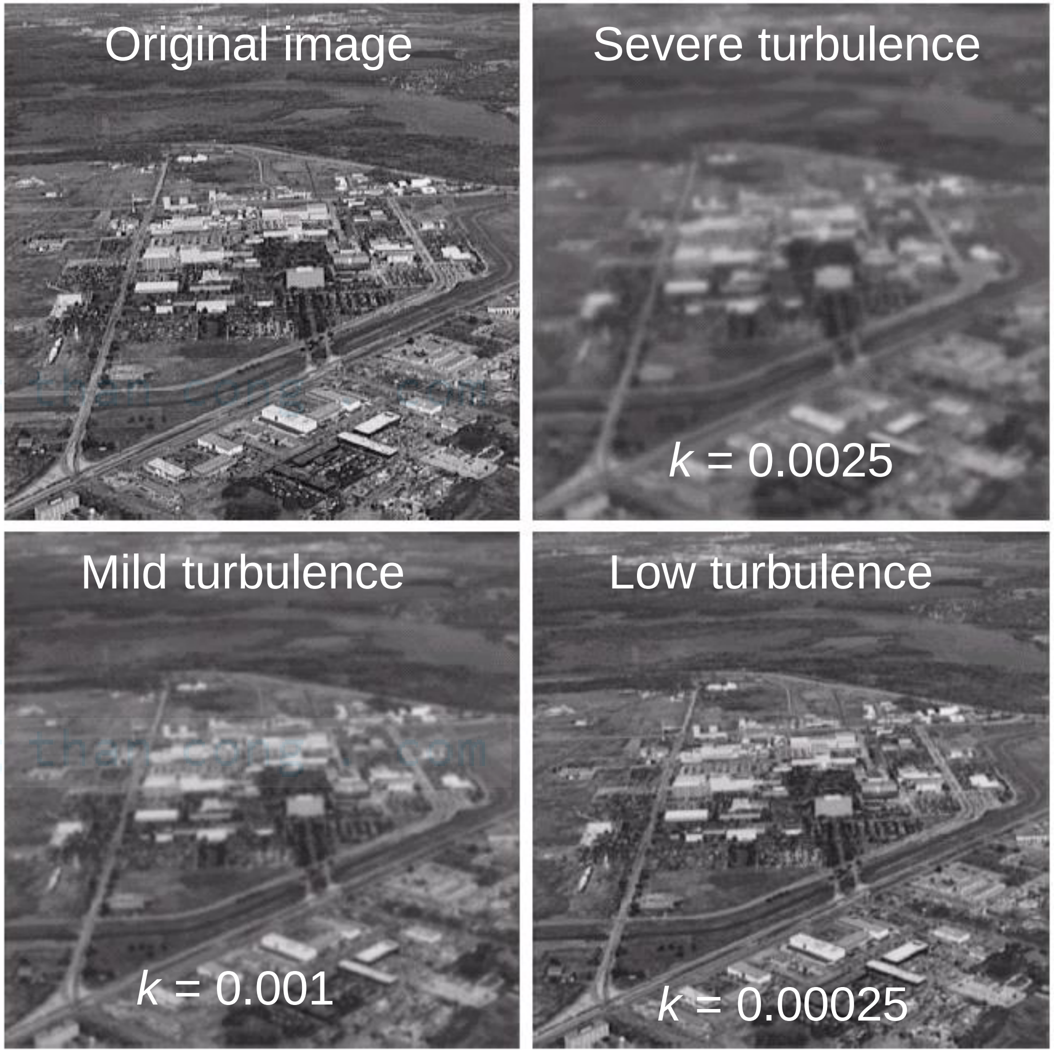
- The model can also be derived from a mathematical model starting from basic principles
  - E.g. an image has been blurred by uniform linear motion between the image and the sensor during image acquisition

Estimation by modeling

- Atmospheric Turbulence model [Hufnagel & Stanley 1964]

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

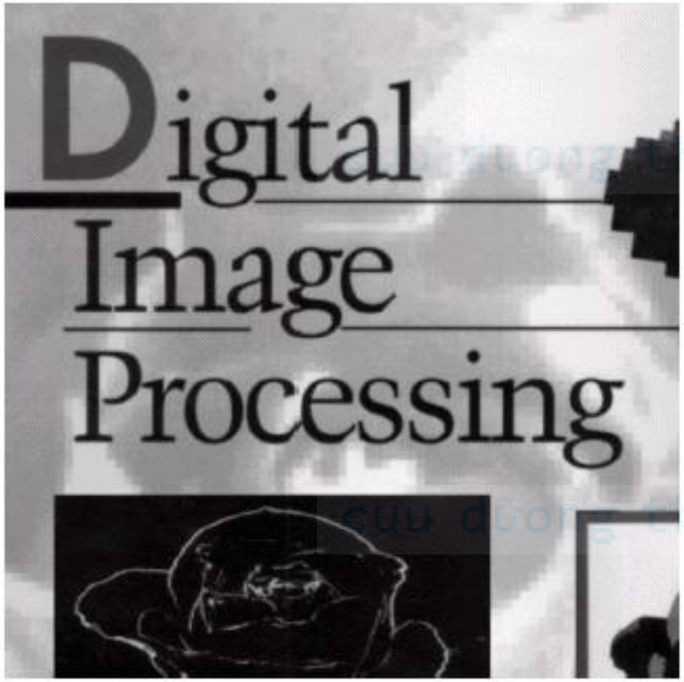
- where the constant k depends on the nature of the turbulence



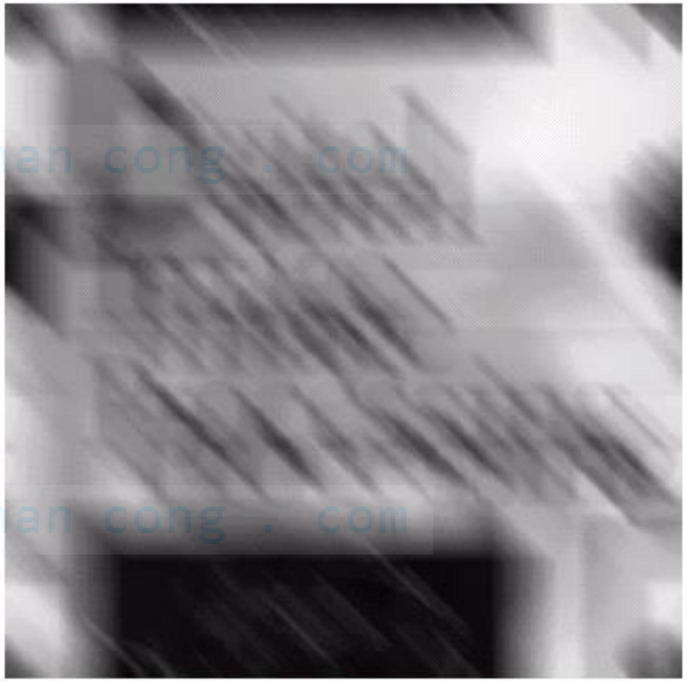
Estimation by modeling

- Motion blurring for a constant motion

$$H(u,v) = \frac{T}{\pi(ua+vb)} sin(\pi(ua+vb))e^{-j\pi(ua+vb)}$$



Original image



Motion blurred image
$a = b = 0.1, T = 1$

Inverse filtering

- From the degradation model:

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- After obtaining $H(u,v)$, $F(u,v)$ can be estimated by the **inverse filter**

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

Noise is enhanced when $H(u,v)$ is small

To avoid the side effect of enhancing noise, this formulation is applied to frequency component (u,v) with in a radius D_0 from the center of $H(u,v)$

- In practical, the inverse filter is not popularly used


Original image
Result of applying
the full filter
Result of applying
the filter with D_0=40
Blurred image
Due to Turbulence
Result of applying
the filter with D_0=70
Result of applying
the filter with D_0=85

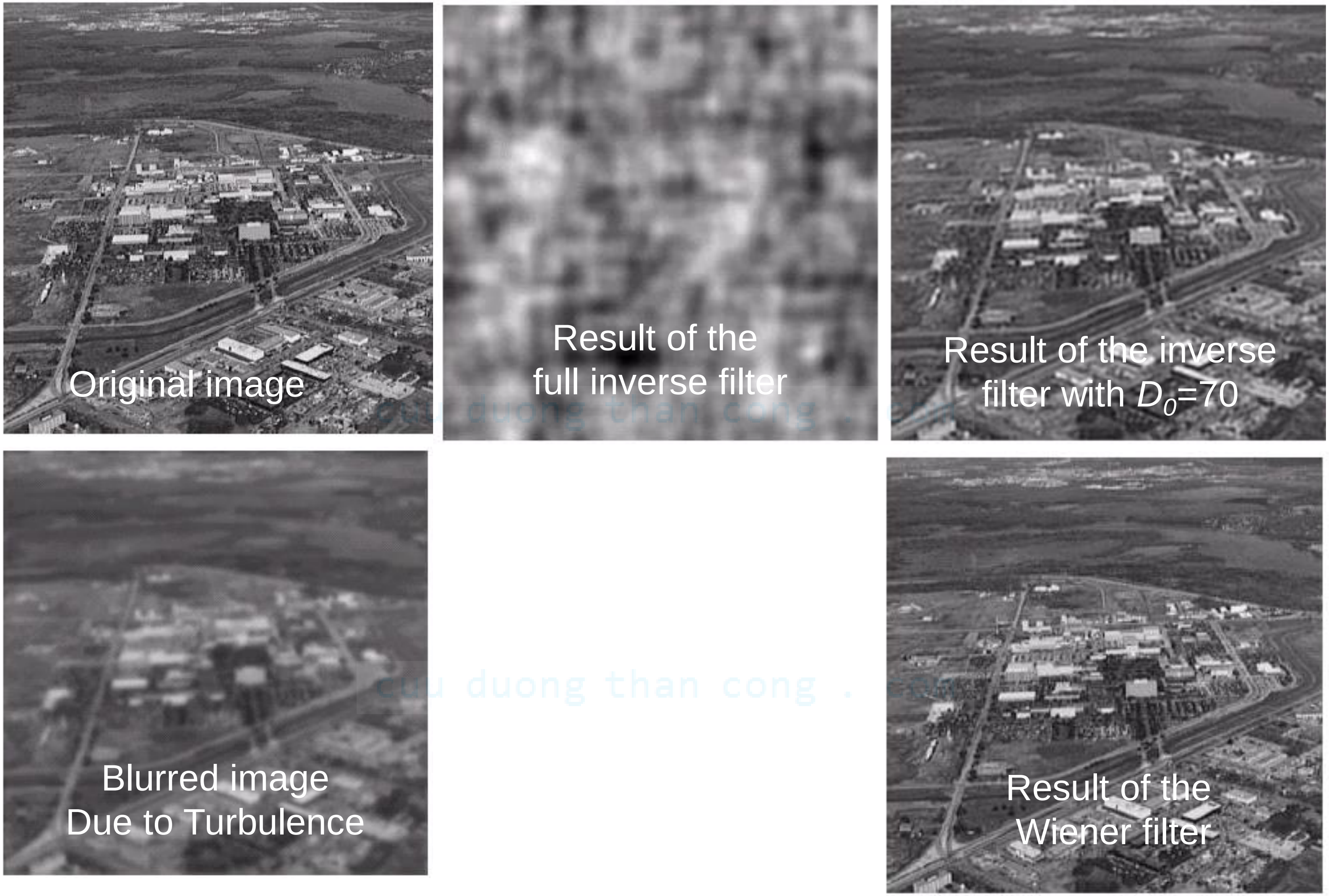
$$k = 0.0025$$

Wiener filter

- The **Wiener filter** (or Minimum Mean Square Error) is estimated from the following formula

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right]G(u,v)$$

  - where $H(u,v)$ is the degradation function,
    $|H(u,v)|^2 = H^*(u,v)H(u,v)$
    $H^*(u,v)$ is the complex conjugate of $H(u,v)$
    K is chosen manually to obtained the best visual result


Original image
Result of the
full inverse filter
Result of the inverse
filter with D_0=70
Blurred image
Due to Turbulence
Result of the
Wiener filter

Motion blur + Gaussian noise (zero mean variance 650)

Constrained least squares filter

- The **constrained least squares filter** is given as

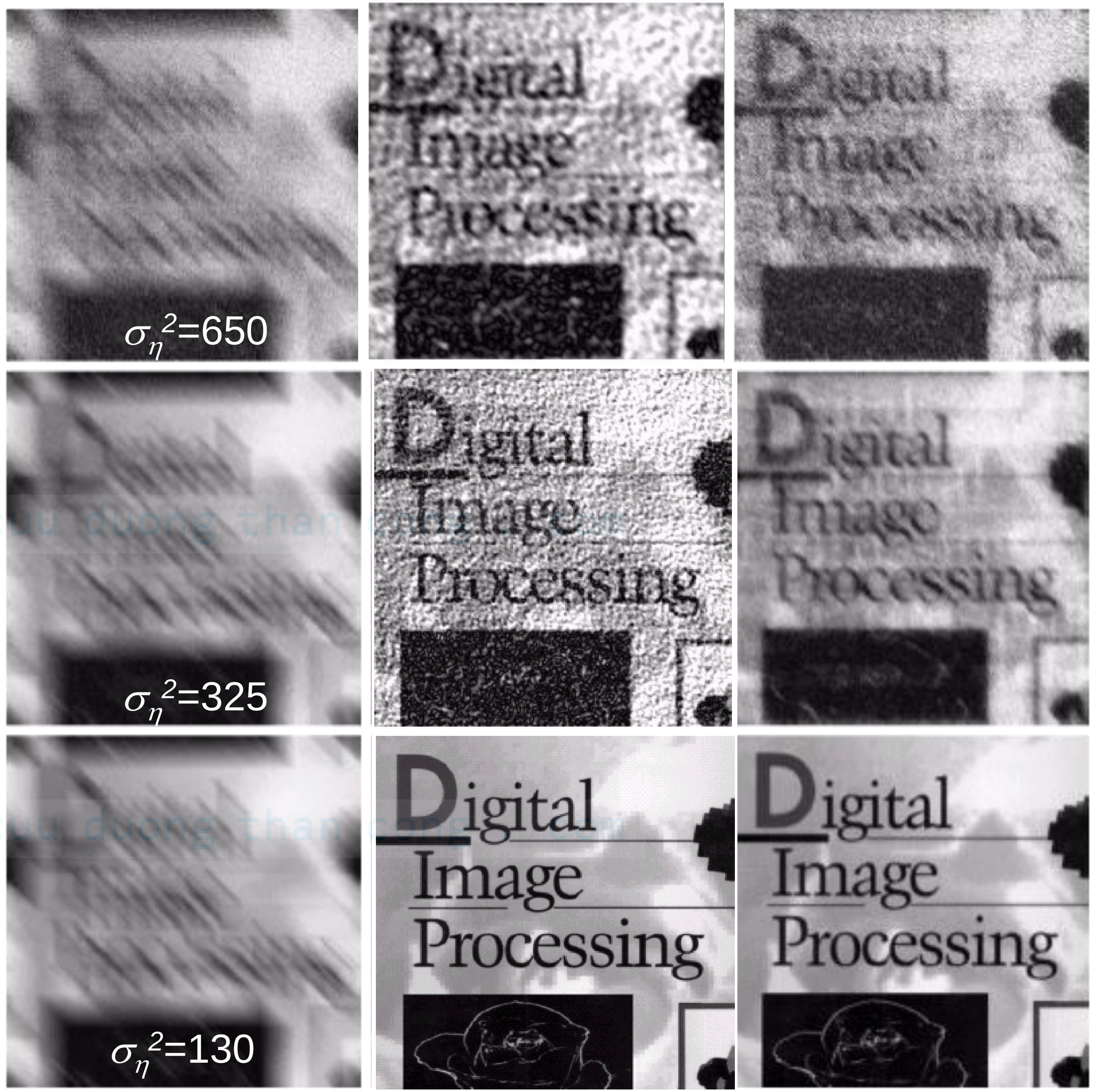
$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma|P(u,v)|^2}\right] G(u,v)$$

  - where $P(u,v)$ is the Fourier Transform of $p(x,y)$

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Constrained least squares filter **Wiener filter**

Motion blur + Gaussian noise (zero mean variance 650)



Geometric mean filter

- The **geometric mean filter** is given as

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} G(u,v)$$

  - with α and β being positive, real constants
  - $S_\eta(u,v) = |N(u,v)|^2$: power spectrum of the noise
  - $S_f(u,v) = |F(u,v)|^2$: power spectrum of the undegraded noise

Geometric mean filter

- The equation represents a family of filters combined into a single expression.
- $\alpha = 1$: inverse filter
- $\alpha = 0$: the so-called parametric Wiener filter, which reduces to the standard Wiener filter when $\beta = 1$
- $\alpha = 1/2$: the filter is a product of the two quantities raised to the same power $\rightarrow$ the definition of geometric mean
- With $\beta = 1$, the filters of $\alpha < 1/2$ behave toward the inverse filter, while those of $\alpha > 1/2$ toward the Wiener filter
- $\alpha = 1/2$ and $\beta = 1$: *the spectrum equalization filter*

References

- Rafael C. Gonzalez, Richard E. Woods, “Digital Image Processing”, 3rd edition, 2008. Chapter 5
- gear.kku.ac.th/~nawapak/178353/Chapter05.ppt

- Images are obtained from the above materials and Google