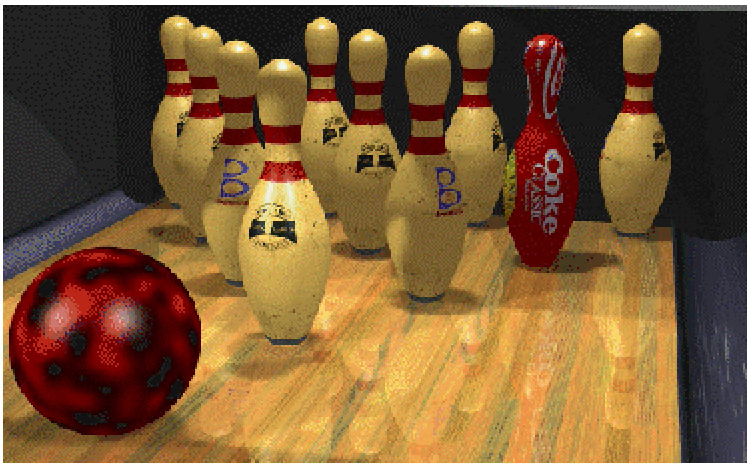
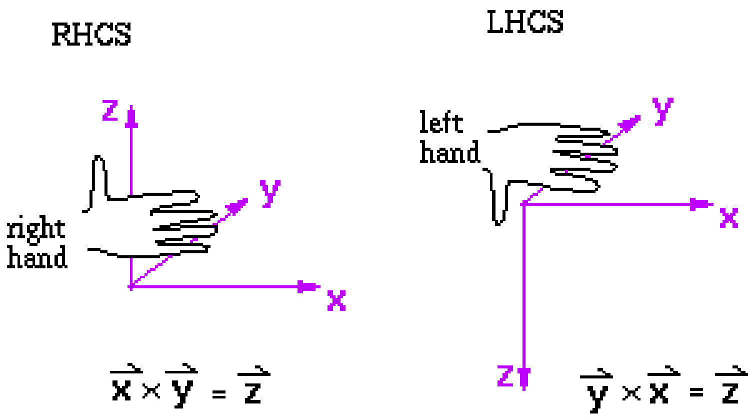
CAC PHEP BIEN ÑOI 3 CHIEU



Dan nhap

- Cung mot loai ñoi töông co the xuat hien trong nhieu canh va xuat hien nhieu lan trong mot canh vôi cac phöông vò, mau sac khac nhau.
- Neu ta co cac mo hình ñoi töông tot, ta co the phat sinh ra cac ñoi töông khac nhau tö mot mo hình duy nhat nhô cac phep bien ñoi.
- Cac phep bien ñoi quan trong nhat la cac phep bien ñoi Affine va cac phep chieu.

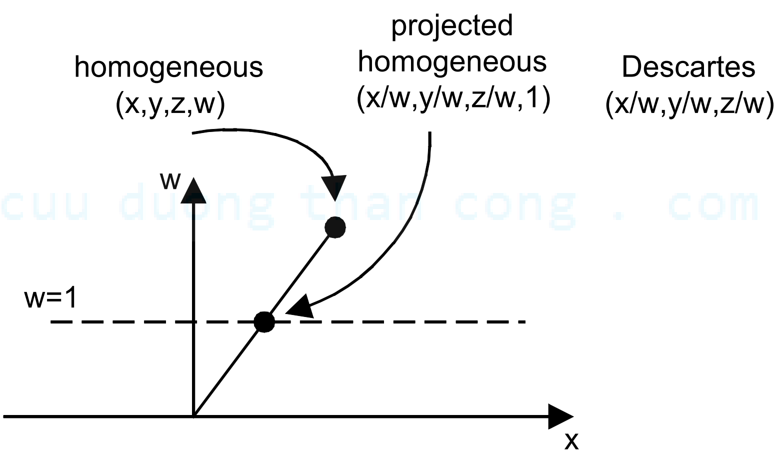
Heä toïa ñoä baøn tay phaûi/baøn tay traùi



- Heä toïa ñoä theo quy öôùc baøn tay phaûi: ñeå baøn tay phaûi sao cho ngoùn caùi höôùng theo truïc z, khi naém tay laïi, caùc ngoùn tay chuyeån ñoäng theo höôùng töø truïc x ñeán truïc y.
- Heä toïa ñoä theo quy öôùc baøn tay traùi: ñeå baøn tay phaûi sao cho ngoùn caùi höôùng theotruïc z, khi naém tay laïi, caùc ngoùn tay chuyeån ñoäng theo höôùng töø truïc x ñeán truïc y.

Heä toaï ñoä thuaàn nhaát (Homogeneous Coordinates)

- Moïi ñieåm (x, y, z) trong khoâng gian Descartes ñöôïc bieåu dieãn bôûi moät boä boán toïa ñoä trong khoâng gian 4 chieàu thu goïn (hx, hy, hz, h). Ngöôøi ta thöôøng choïn h=1.



- $(x, y, z)_{\text{Descartes}}$ $(x, y, z, 1)_{\text{Homogeneous}}$
- $(x, y, z, w)_{\text{Homogeneous}}$ $(x/w, y/w, z/w)_{\text{Descartes}}$ $(w \neq 0)$.

Cac phep bien ñoi tuyen tính

- Phep bien ñoi tuyen tính la to hôp cua cac PBÑ:
    - Tæ le
    - Quay
    - Bien dang va
    - Ñoi xöng

$$(x' \quad y' \quad z') = (x \quad y \quad z)\begin{pmatrix} a & d & g \\ b & e & h \\ c & f & i \end{pmatrix} \quad \text{vôi} \quad \begin{vmatrix} a & d & g \\ b & e & h \\ c & f & i \end{vmatrix} \neq 0$$

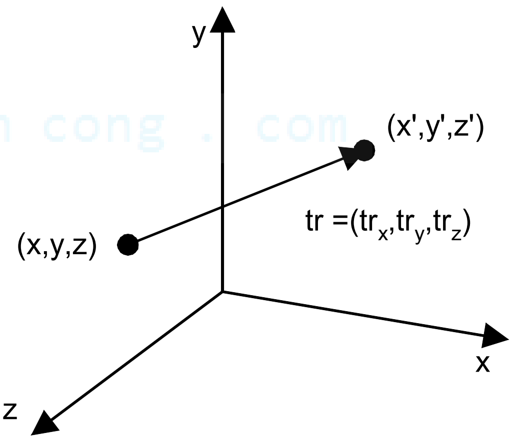
- Cac tính chat cua cac phep bien ñoi tuyen tính
    - Thoa man tính chat ve to hôp tuyen tính.

$$T(s_1P_1 + s_2P_2) = s_1T(P_1) + s_2T(P_2)$$

    - Goc toa ño la ñiem bat ñong.
    - Anh cua ñöông thang la ñöông thang.
    - Anh cua cac ñöông thang song song la cac ñöông thang song song.
    - Bao toan tæ le khoang cach
    - To hôp cac phep bien ñoi co tính phan phoi

Phep tònh tien

- Dòch chuyen mot ñiem tö vò trí ñen vò trí khac trong khong gian theo vector offset **tr.**



Phep bien ñoi Affine

- Phep bien ñoi Affine la to hôp cua cac phep bien ñoi:
  - ♦ Tuyen tính
  - ♦ Tònh tien

tæ le, quay, bien dang

tònh tien

$$(x' \quad y' \quad z' \quad 1) = (x \quad y \quad z \quad 1)\begin{pmatrix} a & b & c & 0 \\ d & e & f & 0 \\ g & h & i & 0 \\ tr_x & tr_y & tr_z & 1 \end{pmatrix}$$

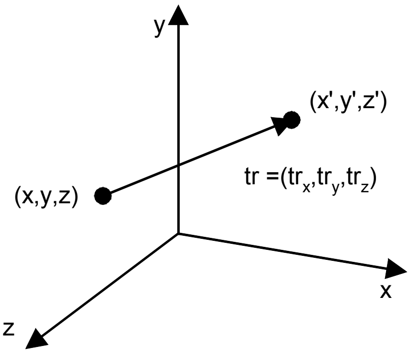
- Cac tính chat
  - ♦ Goc toa ño khong la ñiem bat ñong.
  - ♦ Anh cua ñöông thang la ñöông thang.
  - ♦ Anh cua cac ñöông thang song song la cac ñöông thang song song.
  - ♦ Bao toan tæ le khoang cach
  - ♦ To hôp cac phep bien ñoi co tính phan phoi

Cac phep bien ñoi Affine cô sô

- Phep bien ñoi Affine co the xem la to hôp cua cac phep bien ñoi cô sô:
  - ♦ Tònh tien
  - ♦ Tæ le (tam tæ le ñat tai goc toa ño)
  - ♦ Quay quanh truc x
  - ♦ Quay quanh truc y
  - ♦ Quay quanh truc z
  - ♦ Ñoi xöng qua truc x, y, z*
  - ♦ Bien dang* (tam bien dang ñat tai goc toa ño)

- Phep tònh tien

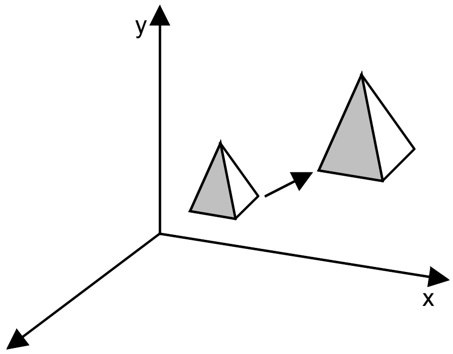
$$Tr(Tr_x, Tr_y, Tr_z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ Tr_x & Tr_y & Tr_z & 1 \end{bmatrix}$$



- Phep bien ñoi tæ le

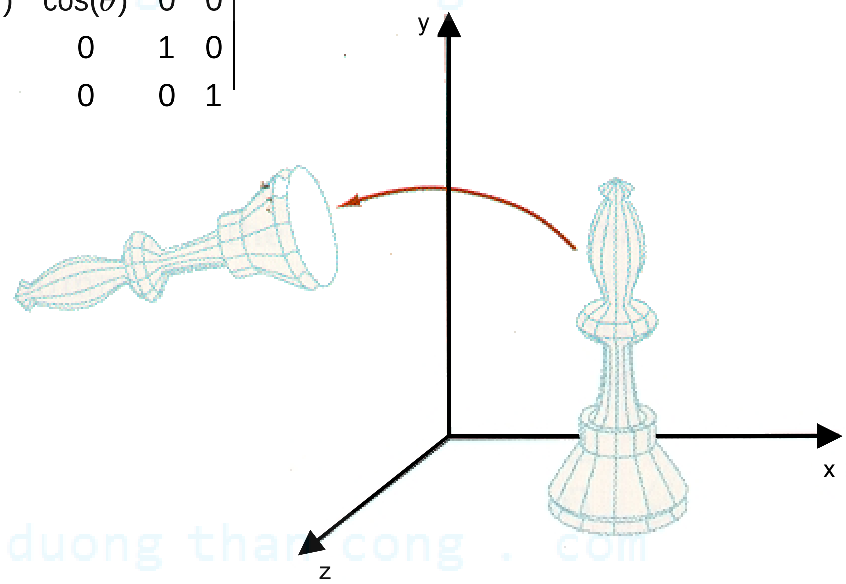
$$S(s_x, s_y, s_z) = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Khi $s_x = s_y = s_z$: ***phep ñong dang***



- Phep quay quanh truc z

$$R(z, \theta) = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Phep quay quanh truc x

$$R(x, \theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Phep quay quanh truc y

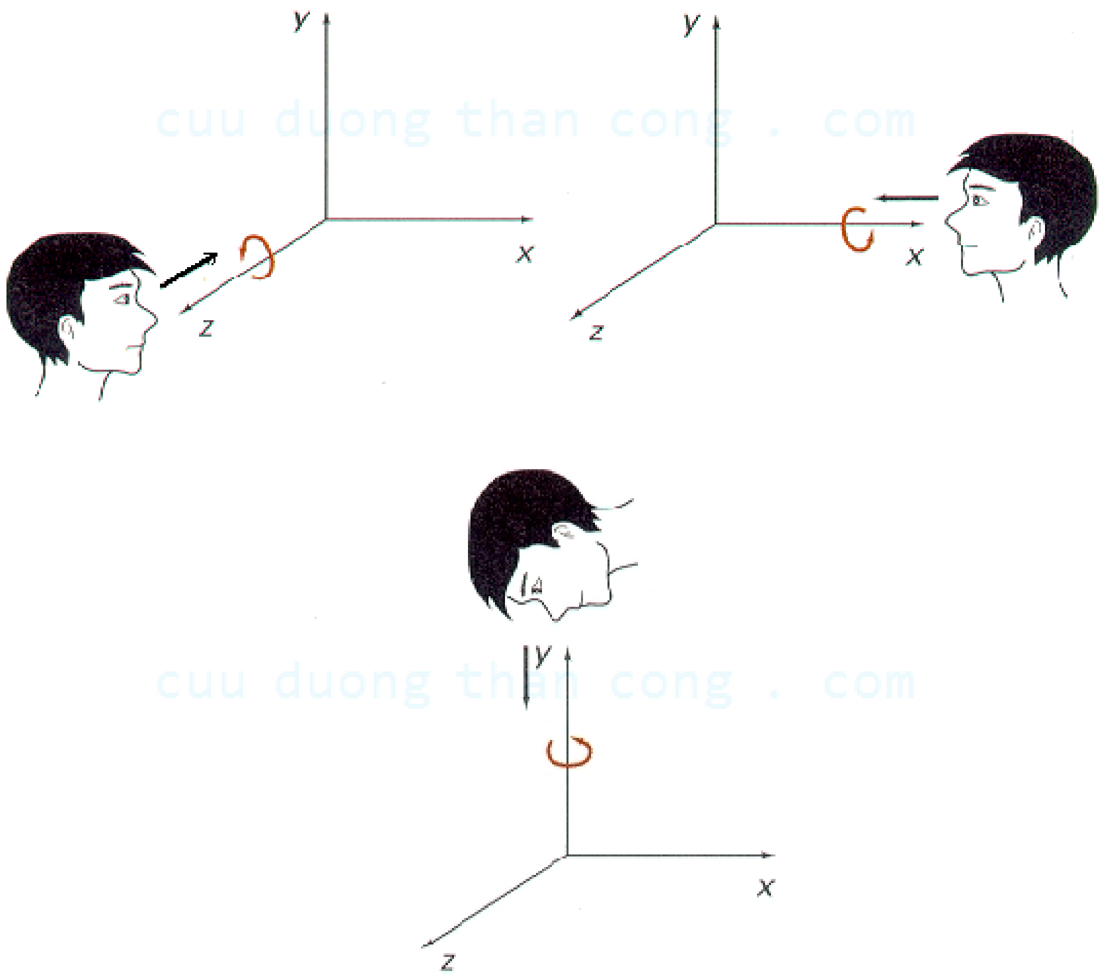
$$R(y, \theta) = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\theta) & 0 & \cos(\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Cach xac ñònh chieu döông trong cac phep quay

  Cac ñònh nghóa ve chieu quay ñöôc dung chung cho ca he toa ño theo quy öôc ban tay phai va ban tay trai. Cu the chieu döông ñöôc ñònh nghóa nhö sau:

  - Quay quanh truc x: tö truc döông y ñen truc döông z.
  - Quay quanh truc y: tö truc döông z ñen truc döông x.
  - Quay quanh truc z: tö truc döông x ñen truc döông y.

- Ví du, xet tren he toa ño ban tay trai, khi nhìn doc tö phía truc quay ve goc toa ño, chieu döông se la chieu ngöôc chieu kim ñong ho

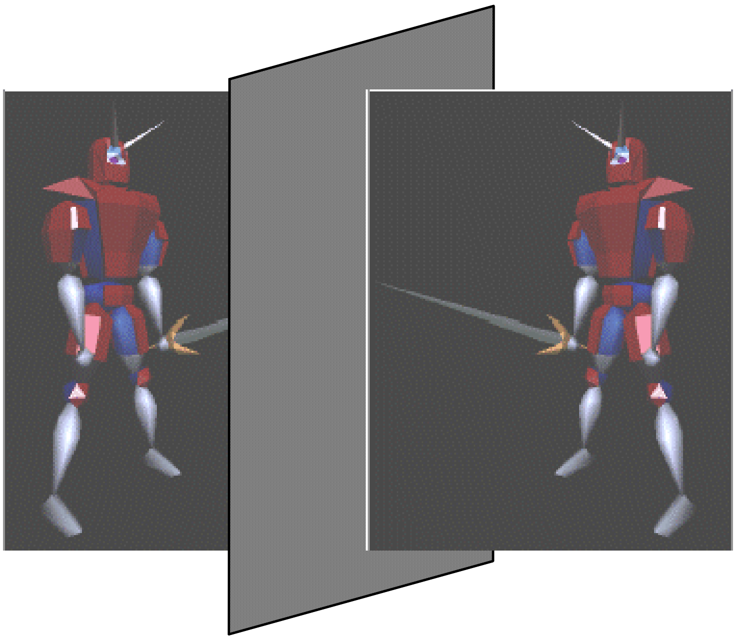


- Phep ñoi xöng qua mat phang yOz, zOx va xOy

$$Mr(x) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Mr(y) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Mr(z) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

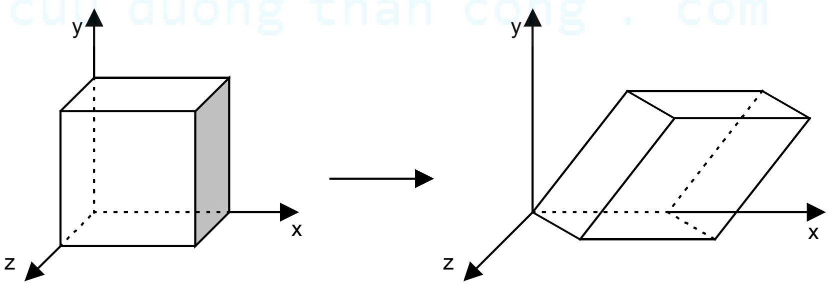


- Phep ñoi xöng qua truc x, y va z

$$M_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_y = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad M_z = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Phep bien dang

$$Sh = \begin{bmatrix} 1 & h_{yx} & h_{zx} & 0 \\ h_{xy} & 1 & h_{zy} & 0 \\ h_{xz} & h_{yz} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

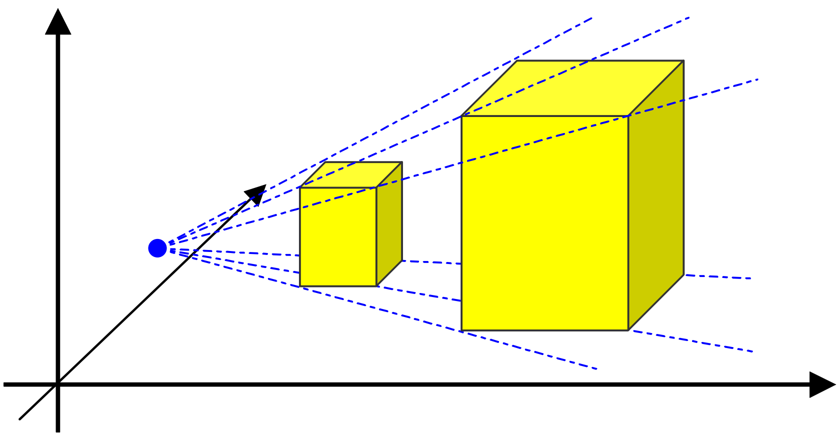


Caùc pheùp bieán ñoåi Affine toång quaùt

- Toå hôïp caùc pheùp bieán ñoåi Affine laø moät pheùp bieán ñoåi Affine.
- Moïi pheùp bieán ñoåi Affine ñeàu coù theå phaân raõ thaønh toå hôïp caùc pheùp bieán ñoåi Affine cô sôû.

Pheùp tæ leä vôùi taâm baát kyø

- Pheùp tæ leä vôùi taâm ñaët taïi ñieåm (x_f, y_f, z_f) coù theå xeùt nhö toå hôïp cuûa caùc pheùp bieán ñoåi cô sôû:
    - Tònh tieán ñieåm baát ñoäng $\left(x_f, y_f, z_f\right)$ veà goác toïa ñoä.
    - Thöïc hieän pheùp bieán ñoåi tæ leä vôùi taâm laø goác toaï ñoä.
    - Tònh tieán ngöôïc ñieåm baát ñoäng töø goác toïa ñoä trôû veà vò trí ban ñaàu.



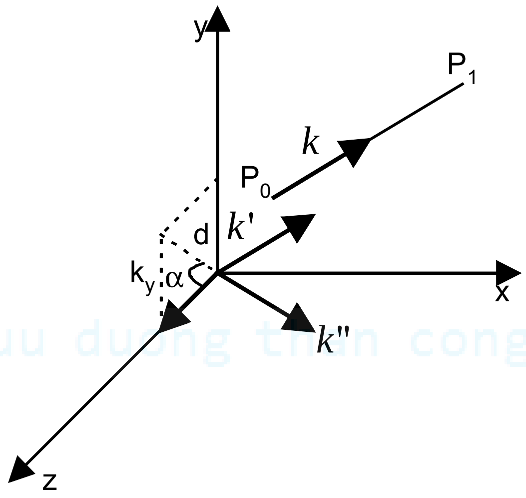
- Ma traän bieán ñoåi seõ laø: $S_f(s_x,s_y,s_z)=\begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ (1\text{-}s_x)x_f & (1\text{-}s_y)y_f & (1\text{-}s_z)z_f & 1 \end{pmatrix}$

Pheùp quay quanh moät truïc baát kyø

- Giaû söû truïc quay xaùc ñònh bôûi 2 ñieåm P_1 vaø P_2 (chieàu döông höôùng töø P_1 ñeán P_2 theå hieän bôûi vector **k**).

- Ap duing qui taic phain rai, ta coi thei bieiu diein quay quanh k moit goic θ thanh day caic phep biein ñoi cô sôi sau:

    - Tònh tiein truic k vei goic toia ñoi: Tr(-P0) (thanh truic k')
    - Quay quanh truic x ñei ñait truic k' nam trein mait phaing xOz: R(x,α) (thanh truic k'').
    - Goic quay ñöôic xaic ñònh döia trein chieiu cuia k' lein mait phaing yOz. Ta khoing cain tính α cui thei. Thay vao ñoi ta tính sin(α) va cos(α) moit caich tröic tieip.

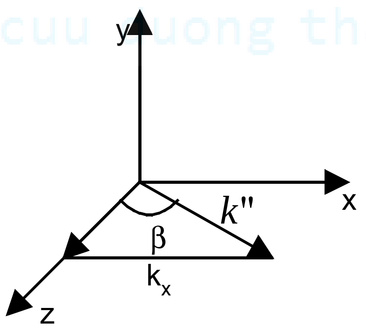


$$k = \frac{\overrightarrow{P_0P_1}}{\left\|P_0P_1\right\|},$$

$$d = \sqrt{k_y^2 + k_z^2}$$

$$\cos(\alpha) = \frac{k_z}{d}, \sin(\alpha) = \frac{k_y}{d}$$

- Quay quanh truic y ñei ñöa truic k' vei truic z: R(y,-β). Töông töi böôic tröôic, ta khoing cain tính cui thei β.
- Thöic hiein phep quay quanh truic z moit goic θ: R(z,θ)
- Thöic hiein chuoi caic phep biein ñoi ngöôic lai qua trình trein.



$$\cos(\beta) = \frac{d}{1} = d,$$

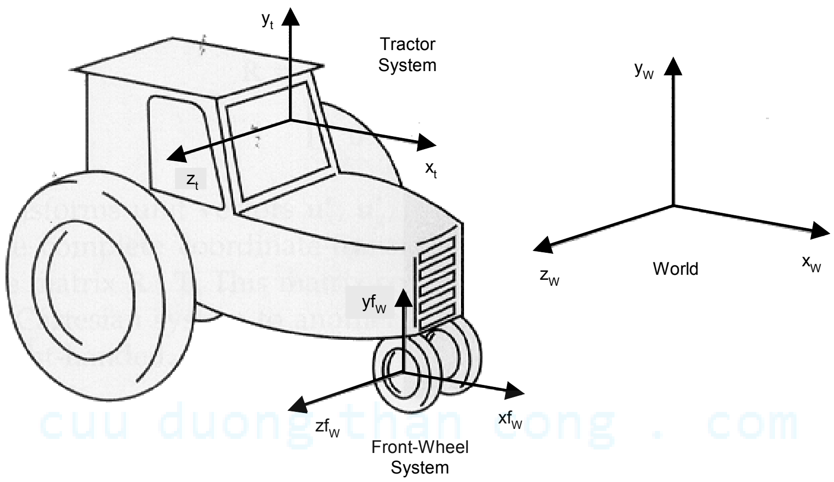
$$\sin(\beta) = \frac{k_x}{1} = k_x$$

- Nhö vay, phep quay quanh 1 truc bat ky co the ñöôc phan ra thanh chuoi cac bien ñoi cô sô sau:

$$Tr(-P_0)\ R(x,\alpha)\ R(y,-\beta)\ R(z,\theta)\ R(y,\beta)\ R(x,-\alpha)\ Tr(P_0)$$

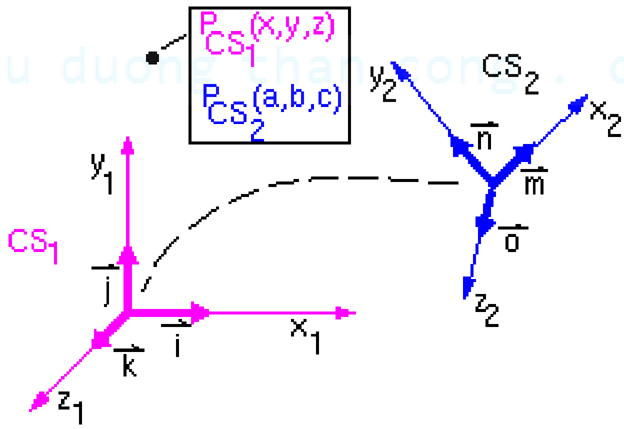
Modeling transformation

- Bien ñoi tö He toa ño ñoi töông sang He toa ño the giôi thöc.



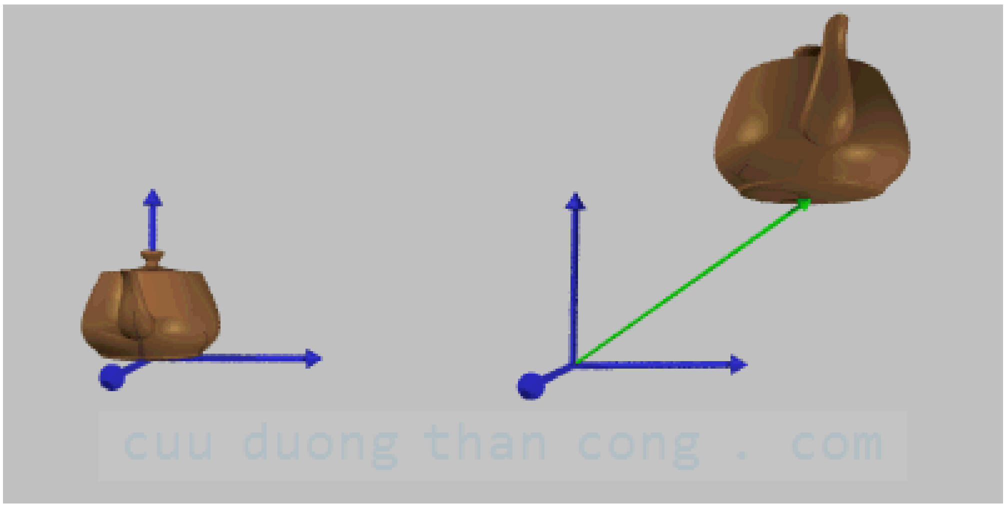
Phep bien ñoi He toa ño

- Can thöc hien mot phep quay va mot phep tình tien (goi la Rigid boby transformation).
- Neu chuyen ñoi giöa hai he toa ño ban tay trai va ban tay phai thì can them mot phep ñoi xöng nöa.

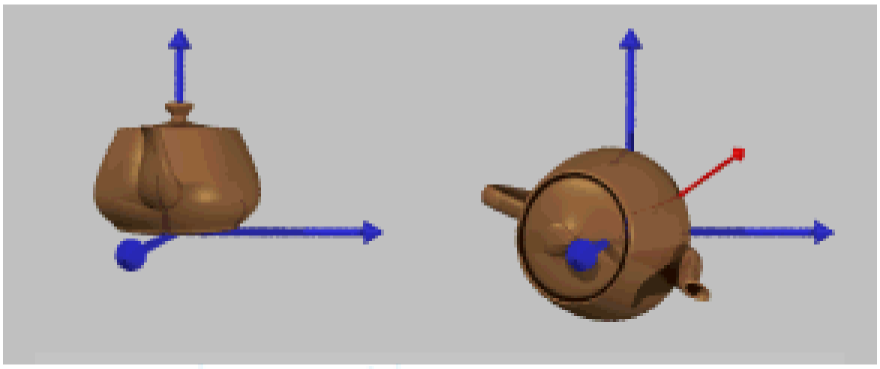


Rigid boby transformation

- Bao gom phep tònh tien va phep quay va cac to hôp cua chung.
- Do khong lam thay ñoi hình dang va kích thöôc ñoi töông, chæ lam thay ñoi vò trí, phöông höông cua chung trong khong gian.



Ví du ve phep tònh tien



Ví du ve phep quay