EE310

Solved Problems on PN Junction

Sedra/Smith 5^{th}/6^{th} ed.

Turki Almadhi,

EE Dept., KSU,

Riyadh, Saudi Arabia

10/05/40

Guidelines for dealing with circuits that contain diodes to be solved using the *ideal diode model*:

1. Start with an assumption about the state of each diode in the circuit. In the ideal diode model: a diode could be either turned on ($I > 0$, where I is in the forward direction) or cut off ($V < 0$, where V is the voltage of the *anode* with respect to the *cathode*).
2. Replace each diode you have assumed to be turned on with a short circuit and each diode you have assumed to be cut off with an open circuit. Solve for the forward currents of all the diodes that you have assumed turned on and the forward voltage drops across all diodes that you have assumed cut off.
3. **Verification:**

   Make sure that each and every diode you have assumed to be turned on has $I > 0$, where I is in the forward direction. Also, make sure that each and every diode you have assumed to be cut off has $V < 0$, where V is the voltage of the *anode* with respect to the *cathode*.

4. If the results of the verification step are consistent with your assumption in step 1 that means you have obtained the correct solution, otherwise you have to try another assumption and restart the process.

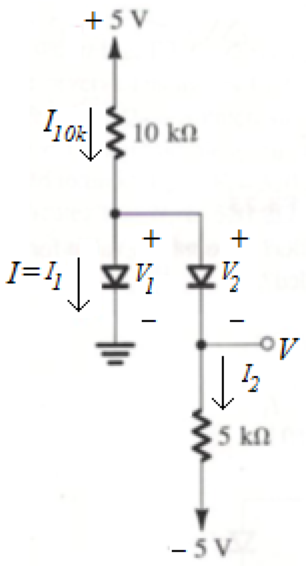
Guidelines for dealing with circuits that contain diodes to be solved using the *constant-voltage-drop model*:

1. Start with an assumption about the state of each diode in the circuit. In the *constant-voltage-drop* model: a diode could be either turned on ($I > 0$, where I is in the forward direction) or cut off ($V < 0.7$, where V is the voltage of the *anode* with respect to the *cathode*).
2. Replace each diode you have assumed to be turned on with a short circuit and each diode you have assumed to be cut off with an open circuit. Solve for the forward currents of all the diodes that you have assumed turned on and the forward voltage drops across all diodes that you have assumed cut off.
3. **Verification:**

   Make sure that each and every diode you have assumed to be turned on has $I > 0$, where I is in the forward direction. Also, make sure that each and every diode you have assumed to be cut off has $V < 0.7$, where V is the voltage of the *anode* with respect to the *cathode*.

4. If the results of the verification step are consistent with your assumption in step 1 that means you have obtained the correct solution, otherwise you have to try another assumption and restart the process.

3.9 Assuming that the diodes in the circuits of Fig. P3.9 are ideal, find the values of the labeled voltages and currents.



Assume that both D1 and D2 are cut off:

In that case, $I_{10k\Omega} = 0 \Rightarrow V_1 = 5$ V **!** Also, $V_2 = 5 - (-5) = 10$ V **!**

but both V_1 and V_2 must be $< 0 \Rightarrow$ our assumption is incorrect.

Assume that both D1 and D2 are turned on:

In that case, $I_{10k\Omega} = \frac{(5-0)}{(10\text{ k})} = 0.5$ mA. $I_2 = \frac{(0-(-5))}{(5\text{ k})} = 1$ mA $\Rightarrow I_1 = I_{10k\Omega} - I_2 = -0.5$ mA **!**

$\because I_1 < 0$ which contradicts the condition of a turned-on diode $\Rightarrow$ our assumption is incorrect.

Assume that D1 is on and D2 is off:

In that case, $I_{10k\Omega} = I_1 = \frac{(5-0)}{(10\text{ k})} = 0.5$ mA. $V_2 = 0 - (-5) = 5$ V **!**

$\because V_2 > \Rightarrow$ our assumption is incorrect.

Assume that D1 is off and D2 is on:

In that case, $I_{10k\Omega} = I_2 = \frac{(5-(-5))}{(15\text{ k})} = (2/3)$ mA. $V_1 = [5 - (10\text{ k}) \times (2/3\text{ m})] - 0 = -1.667$ V ✓

$\because V_1 < 0$ and $I_2 > 0$ that confirms our assumption. ☺

$\Rightarrow I = I_1 = 0$; and $V = V_1 = (5\text{ k}) \times (2/3\text{ m}) + (-5) = -1.667$ V

Let us resolve the above problem using a constant voltage drop model with V_D = 0.7 V;

Assume that both D1 and D2 are cut off:

In that case, $I_{10k\Omega} = 0 \Rightarrow V_1 = 5$ V **!** Also, $V_2 = 5 - (-5) = 10$ V **!**

$\Rightarrow$ our assumption is incorrect.

Assume that both D1 and D2 are turned on:

In that case, $I_{10k\Omega} = \frac{(5 - 0.7)}{10\text{ k}} = 0.43$ mA. $I_2 = \frac{(0.7 - 0.7 - (-5))}{5\text{ k}} = 1.0$ mA $\Rightarrow I_1 = I_{10k\Omega} - I_2 = -0.57$ mA **!**

$\because I_1 < 0 \Rightarrow$ our assumption is incorrect.

Assume that D1 is on and D2 is off:

In that case, $I_{10k\Omega} = I_1 = \frac{5 - 0.7}{10\text{ k}} = 0.43$ mA. $V_2 = 0.7 - (-5) = 5.7$ V **!**

$\because V_2 > 0.7 \Rightarrow$ our assumption is incorrect.

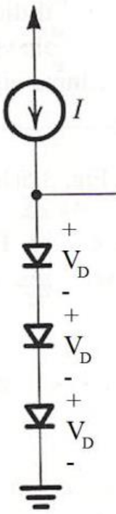
Assume that D1 is off and D2 is on:

In that case, $I_{10k\Omega} = I_2 = \frac{(5 - 0.7 - (-5))}{15\text{ k}} = 0.62$ mA. $V_1 = [5 - (10\text{ k}) \times (0.62\text{ m})] - 0 = -1.2$ V ✓

$\because V_1 < 0.7$ and $I_2 > 0$ that confirms our assumption. ☺

$\Rightarrow I = I_1 = 0$; and $V = (5\text{ k}) \times (0.62\text{ m}) + (-5) = -1.9$ V

3.23 The circuit in Fig. P3.23 utilizes three identical diodes having $n = 1$ and $I_S = 10^{-14}$ A. Find the value of the current I required to obtain an output voltage $V_O = 2$ V. If a current of 1 mA is drawn away from the output terminal by a load, what is the change in output voltage?



∵ the three diodes are identical and conduct the same current (I),

The voltage drop across each diode will be the same = V_D.

Now we have $3V_D = 2 \Rightarrow V_D \approx 0.667$ V or 667 mV.

$\Rightarrow I = 1\times10^{-14} \times e^{\frac{667}{1\times25.8}} \approx 1.69$ mA

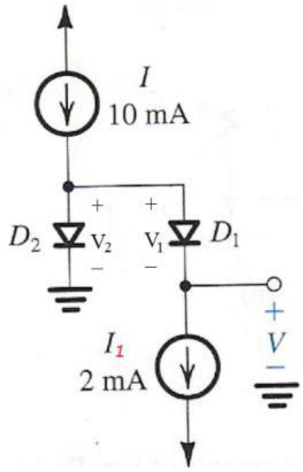
If a current of 1 mA is drawn away from the output terminal that leaves the diodes with 0.69 mA.

The change in a single diode voltage will be $\Delta V_D = 2.3 \times n \times V_T \times \log\frac{0.69}{1.69} = -23.34\ mV$

$\Rightarrow \Delta V = 3\ \Delta V_D = -70$ mV

i.e., $V_{loaded} = V + \Delta V = 1930$ mV = 1.93 V.

3.25 In the circuit shown in Fig. P3.25, both diodes have $n = 1$, but D_1 has 10 times the junction area of D_2. What value of V results? To obtain a value for V of 50 mV, what current I_1 is needed?



In general,

$$\frac{I_2}{I_1} = \frac{I_{s2}}{I_{s1}} e^{\frac{(V_2 - V_1)}{nV_T}}$$

$$V_2 - V_1 = nV_T \ln\left(\frac{I_2}{I_1} \times \frac{I_{s1}}{I_{s2}}\right)$$

$$V_2 - V_1 = 2.3nV_T \log\left(\frac{I_2}{I_1} \times \frac{I_{s1}}{I_{s2}}\right)$$

For this particular problem, $A_{D1} = 10\ A_{D2} \Rightarrow I_{S1} = 10\ I_{S2} \Rightarrow \frac{I_{S1}}{I_{S2}} = 10$

Also by inspection $I_2 = I - I_1 = 8$ mA.

Writing a node equation starting from the cathode of D_2 and going clockwise,

$V_2 - V_1 - V = 0 \Rightarrow V = V_2 - V_1 = 60 \times \log(\frac{8}{2}\times10)$ mV = 96 mV.

$50 = 60 \times \log(\frac{10-I_1}{I_1}\times10) \Rightarrow \log(\frac{10-I_1}{I_1}\times10) = 0.8334 \Rightarrow 10^{0.8334} = \frac{100}{I_1} - 10 \Rightarrow \frac{100}{I_1} = 17.814$

$$\Rightarrow I_1 = 5.61\ mA$$

3.27 Several diodes having a range of sizes, but all with $n = 1$, are measured at various temperatures and junction currents as noted below. For each, estimate the diode voltage at 1 mA and 25°C.

(a) 620 mV at 10 μA and 0°C
(b) 790 mV at 1 A and 50°C
(c) 590 mV at 100 μA and 100°C
(d) 850 mV at 10 mA and –50°C
(e) 700 mV at 100 mA and 75°C

a)

$$V|_{T=25^\circ C, I=10\mu A} = V|_{T=0^\circ C, I=10\mu A} + (25-0)\times(-2\text{ mV})$$

$$V|_{T=25^\circ C, I=10\mu A} = 620\text{ mV} + (-50\text{ mV})$$

$$V|_{T=25^\circ C, I=10\mu A} = 570\text{ mV}$$

$$V|_{T=25^\circ C, I=1mA} = V|_{T=25^\circ C, I=10\mu A} + 2.3nV_T\log\left(\frac{1\times10^{-3}}{10\times10^{-6}}\right)$$

$$V|_{T=25^\circ C, I=1mA} = (570+120)\text{ mV} = 690\text{ mV} \Leftarrow$$

b)

$$V|_{T=25^\circ C, I=1A} = V|_{T=50^\circ C, I=1A} + (25-50)\times(-2\text{ mV})$$

$$V|_{T=25^\circ C, I=1A} = 790\text{ mV} + 50\text{ mV}$$

$$V|_{T=25^\circ C, I=1A} = 830\text{ mV}$$ because the current is very large!

$$V|_{T=25^\circ C, I=1mA} = V|_{T=25^\circ C, I=1A} + 2.3nV_T\log\left(\frac{1\times10^{-3}}{1}\right)$$

$$V|_{T=25^\circ C, I=1mA} = (830-180)\text{ mV} = 650\text{ mV} \Leftarrow$$

3.34 A "1-mA diode" (i.e., one that has $v_D = 0.7$ V at $i_D = 1$ mA) is connected in series with a 200-Ω resistor to a 1.0-V supply.

(a) Provide a rough estimate of the diode current you would expect.
(b) If the diode is characterized by $n = 2$, estimate the diode current more closely using iterative analysis.

a)

$$I_2 = \frac{V_S - V_1}{R} = \frac{1-0.7}{200} = 1.5 \text{ mA}$$

b)

$$V_2 = V_1 + 2.3nV_T \log(\frac{I_2}{I_1}) = 0.7 + 0.119 \times \log(\frac{1.5}{1}) = 0.72095$$

$$I_3 = \frac{V_S - V_2}{R} = \frac{1-0.72095}{200} = 1.39525 \text{ mA}$$

$$V_3 = V_2 + 2.3nV_T \log(\frac{I_3}{I_2}) = 0.72095 + 0.119 \times \log(\frac{1.39525}{1.5}) = 0.71721$$

$$I_4 = \frac{V_S - V_3}{R} = \frac{1-0.71721}{200} = 1.41395 \text{ mA}$$

$$V_4 = V_3 + 2.3nV_T \log(\frac{I_4}{I_3}) = 0.71721 + 0.119 \times \log(\frac{1.41395}{1.39525}) = 0.71789$$

$$I_5 = \frac{V_S - V_4}{R} = \frac{1-0.71789}{200} = 1.41055 \text{ mA}$$

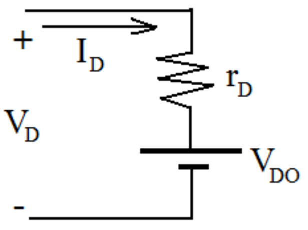
3.37 Find the parameters of a piecewise-linear model of a diode for which $v_D = 0.7$ V at $i_D = 1$ mA and $n = 2$. The model is to fit exactly at 1 mA and 10 mA. Calculate the error in millivolts in predicting v_D using the piecewise-linear model at $i_D = 0.5$, 5, and 14 mA.

$$\left| \frac{\Delta V_D}{decade(\Delta I_D)} \right| = 2.3 n V_T$$

$$\left| \frac{\Delta V_D}{decade(\Delta I_D)} \right| = 0.12 \text{ V/decade @ RT for n = 2}$$

I_D (mA)	V_D (V)
1	0.7
10	0.7+0.12=0.82

$$r_D = \frac{\Delta V_D}{\Delta I_D} = \frac{0.12}{(10-1)\times 10^{-3}} = 13.34\ \Omega$$



$$V_D = I_D \times r_D + V_{DO} \Rightarrow V_{DO} = V_D - I_D \times r_D$$

$$V_{DO} = 0.7 - 1\times 10^{-3} \times 13.34 = 0.687 \text{ V}$$

To calculate V_D exactly, we need to find I_S :

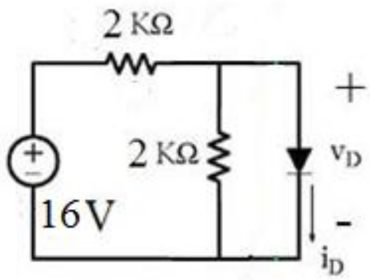
$$I_S = I_D \times e^{-(\frac{V_D}{nV_T})} = 1\times 10^{-3} \times e^{-(\frac{0.7}{2\times 0.0258})} = 1.2835\times 10^{-9} \text{ A}$$

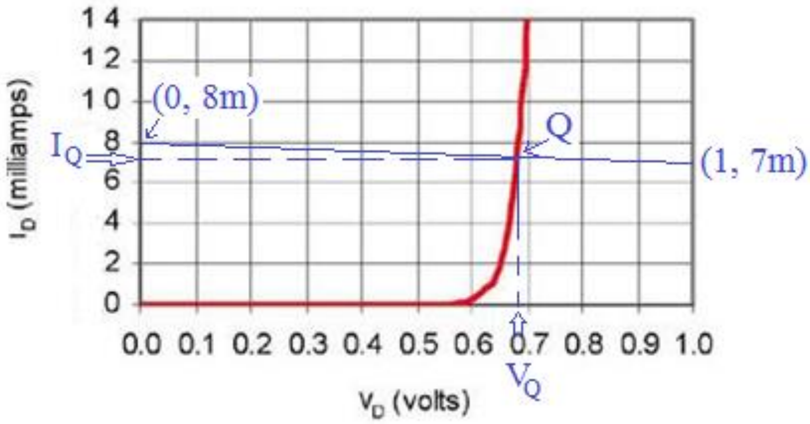
I_D (mA)	$V_D = nV_T \times \ln(\frac{I}{I_S}+1)$	V_D using Piecewise-linear model	Difference (mV)
0.5	0.6642	0.6937	29.5
5	0.7831	0.7537	-29.4
14	0.8362	0.8738	37.6

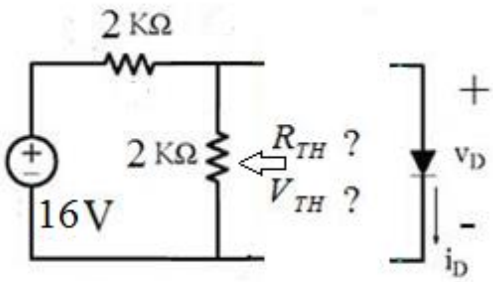
Graphical Analysis (Load Line Method):

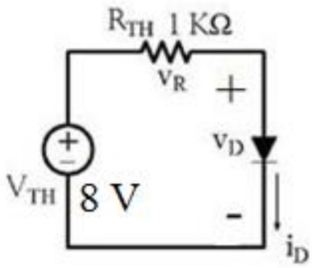
- Example 1:

The diode used in the circuit below has the i-v characteristic shown to the right. Find the the dc operating (bias) point Q.









Load Line Equation

$$V_{TH} = I_D \times R_{TH} + V_D$$

$$8 = I_D \times 1000 + V_D$$

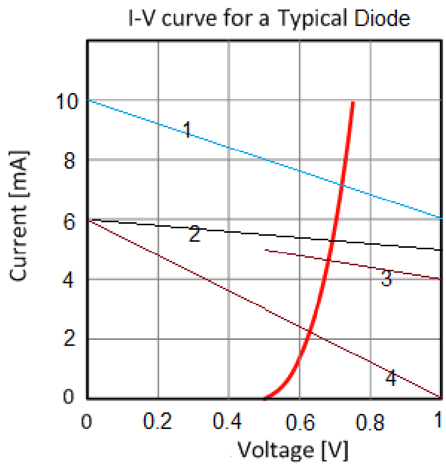
$$@V_D = 0\,\text{V} \rightarrow I_D = 8\text{ mA}$$

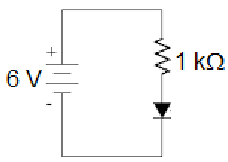
$$@V_D = 1\,\text{V} \rightarrow I_D = 7\text{ mA}$$

- Example 2

In the given i-v characteristic below, which load line represent the given circuit?

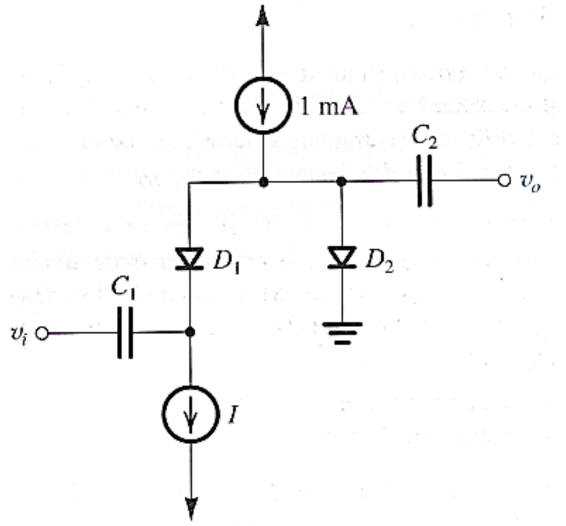
Answer: Number 2.





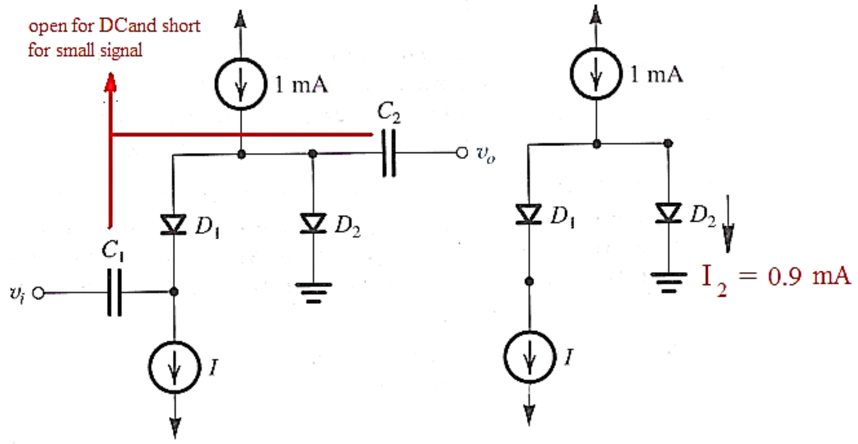
3.56 In the capacitor-coupled attenuator circuit shown in Fig. P3.56, I is a dc current that varies from 0 mA to 1 mA, D_1 and D_2 are diodes with $n = 1$, and C_1 and C_2 are large coupling capacitors. For very small input signals, find the values of the ratio v_o/v_i for I equal to:

(a) 0 μA
(b) 1 μA
(c) 10 μA
(d) 100 μA
(e) 500 μA
(f) 600 μA
(g) 900 μA
(h) 990 μA
(i) 1 mA



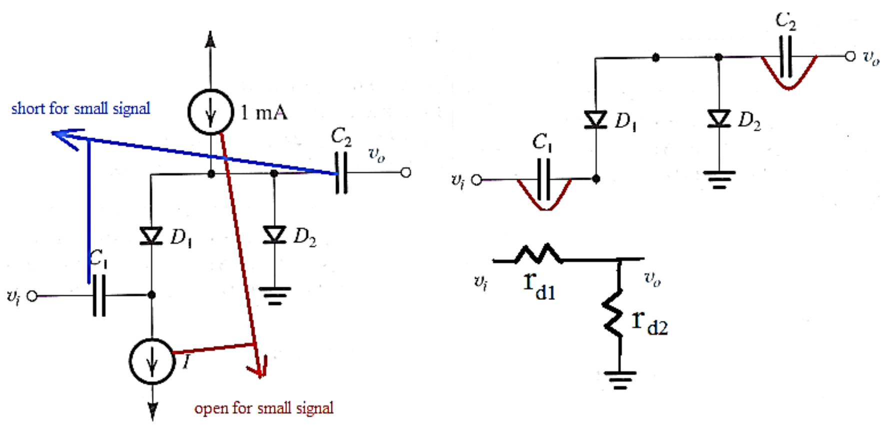
d) I=100 µA

DC Equivalent circuit:



Small Signal Equivalent circuit:

$$r_{d1} = \frac{nV_T}{I_1} = \frac{1\times.0259}{100\times10^{-6}} = 259\ \Omega \qquad r_{d2} = \frac{nV_T}{I_2} = \frac{1\times.0259}{0.9\times10^{-3}} = 28.78\ \Omega$$



$$V_o/V_i = \frac{r_{d2}}{r_{d2}+r_{d1}} = \frac{28.78}{28.78+259} = 0.1\ V/V$$

3.107)
A young designer, aiming to develop intuition, concerning conducting paths within an integrated circuit, examines the end-to-end resistance of a connecting bar 10 μm long, 3 μm wide, 1 μm thick, made of various materials. The designer considers:

a) intrinsic silicon
b) *n*-doped silicon with $N_D = 1 \times 10^{16}$/cm^3
c) *n*-doped silicon with $N_D = 1 \times 10^{18}$/cm^3
d) *p*-doped silicon with $N_D = 1 \times 10^{10}$/cm^3
e) aluminum with resistivity of 2.8 μΩ·cm

Find the resistance in each case. For intrinsic silicon, use μ_n = 1350 cm^2/V·s, μ_p = 480 cm^2/V·s, and $n_i = 1.5 \times 10^{10}$/cm^3. For doped silicon, assume $\mu_n \cong 2.5\mu_p$ = 1200 cm^2/V·s. (Recall that $R = \rho L/A$.)

In general,

$$n = \frac{N_D - N_A}{2} + \sqrt{(\frac{N_D - N_A}{2})^2 + {n_i}^2} \qquad ;\text{and} \quad p = \frac{N_A - N_D}{2} + \sqrt{(\frac{N_A - N_D}{2})^2 + {n_i}^2}$$

$$a)\ n = p = n_i \quad \Rightarrow R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{[q(n\mu_n + p\mu_p)]A} = \frac{L}{[qn_i(\mu_n + \mu_p)]A} \Rightarrow$$

$$R = \frac{10\times10^{-4}}{1.602\times10^{-19}\times1.5\times10^{10}(1350+480)\times3\times10^{-4}\times1\times10^{-4}} = 7.58 \times 10^9\ \Omega\ =\ 7.59\ \text{G}\Omega$$

$$b)\ n = N_D; p = \frac{{n_i}^2}{N_D} \Rightarrow R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{q\left(N_D\mu_n + \frac{{n_i}^2}{N_D}\mu_p\right)A} \approx \frac{L}{q(N_D\mu_n)A}$$

$$(note: N_D\mu_n \gg \frac{{n_i}^2}{N_D}\mu_p\ ) \quad \Rightarrow$$

$$R = \frac{10 \times 10^{-4}}{1.602 \times 10^{-19} \times 1 \times 10^{16} \times 1200 \times 3 \times 10^{-4} \times 1 \times 10^{-4}} = 17361\ \Omega = 17.361\ k\Omega$$

$d)$ *Since* N_A is comparable to n_i,

$$p = \frac{N_A - N_D}{2} + \sqrt{(\frac{N_A - N_D}{2})^2 + {n_i}^2} = 2.08 \times 10^{10}\ cm^{-3}; \qquad n = \frac{{n_i}^2}{p} = 1.08 \times 10^{10}\ cm^{-3}$$

$$R = \frac{\rho L}{A} = \frac{L}{\sigma A} = \frac{L}{q(n\mu_n + p\mu_p)A}$$

$$R = \frac{10 \times 10^{-4}}{1.602 \times 10^{-19}(2.08 \times 10^{10} \times 480 + 1.08 \times 10^{10} \times 1200) \times 3 \times 10^{-4} \times 1 \times 10^{-4}} = 9.07\ G\Omega$$

$$e)\ \ R = \frac{\rho L}{A} = \frac{2.8 \times 10^{-6} \times 10 \times 10^{-4}}{3 \times 10^{-4} \times 1 \times 10^{-4}} = \ 0.0934\ \ \Omega = 93.4\ m\Omega$$

3.108)

Holes are being steadily injected into a region of n-type silicon. In the Steady state, the excess-hole concentration profile shown in Fig P3.108 Is established in the n-type silicon region. Here "excess" means over and above the concentration p_{no}.

If $N_D = 10^{16}\ /cm^3$, $n_i = 1.5 \times 10^{10}\ /cm^3$ and $W = 5\mu m$,
find the density of the current that will flow in the x direction.

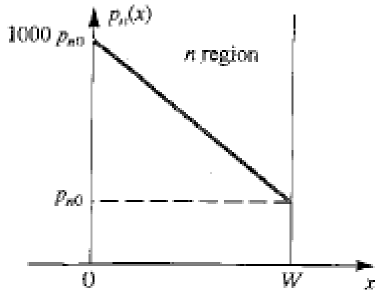


FIGURE P3.108

$$J_p = -qD_p\frac{dp}{dx}$$

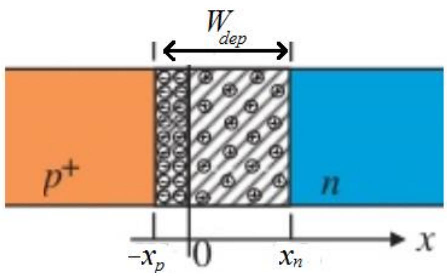
$$p_{no} = \frac{{n_i}^2}{n_{no}} = \frac{{n_i}^2}{N_D} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4\ cm^{-3}$$

$$\frac{dp}{dx} = \frac{p_{no} - 1000p_{no}}{W - 0} = \frac{-999p_{no}}{W} = \frac{-999 \times 2.25 \times 10^4}{5 \times 10^{-4}} = -4.4955 \times 10^{10}\ cm^{-4}$$

$$J_p = -qD_p\frac{dp}{dx} = -1.602 \times 10^{-19} \times 12 \times (-4.4955 \times 10^{10}) = 8.64 \times 10^{-8}\ A/cm^2$$

3.115)

If for a particular junction, the acceptor concentration is $1 \times 10^{16}/\text{cm}^3$ and the donor concentration is $1 \times 10^{15}/\text{cm}^3$, find the junction built-in voltage (barrier voltage). Assume $n_i = 1 \times 10^{10}/\text{cm}^3$. Also, find W_{dep} and its extent in each of the p and n regions when the junction is reverse biased with $V_R = 5$ V. At this value of the reverse bias, calculate the *magnitude* of the charge stored on either side of the junction. Assume the junction area is 400 μm². Also, calculate C_j.



$$V_O = V_T \times \ln \frac{N_A \times N_D}{{n_i}^2}$$

$$V_O = 0.0258 \times \ln \frac{10^{16} \times 10^{15}}{(10^{10})^2}$$

$$V_O = 0.653\ V$$

$$W_{dep} = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_O + V_R)}$$

$$W_{dep} = \sqrt{\frac{2 \times 11.7 \times 8.854 \times 10^{-14}}{1.602 \times 10^{-19}}\left(\frac{1}{10^{16}} + \frac{1}{10^{15}}\right)(0.653 + 5)} = 2.836 \times 10^{-4} \text{ cm} = 2.836\ \mu\text{m}$$

$$W_{dep} = x_p + x_n \rightarrow x_p + x_n = 2.836 \quad ① \quad ; \quad \frac{x_p}{x_n} = \frac{N_D}{N_A} \rightarrow \frac{x_p}{x_n} = \frac{10^{15}}{10^{16}} \rightarrow \frac{x_n}{x_p} = 10 \quad ②$$

Solving ① and ② simultaneously yields:

$$x_p = \frac{2.836}{11} = 0.258\ \mu\text{m} \qquad \text{and } x_n = 2.578\ \mu\text{m}$$

$$q_j = qN_D x_n A = qN_A x_p A = 1.602 \times 10^{-19} \times 10^{16} \times 0.2578 \times 10^{-4} \times 400 \times 10^{-8} = 1.652 \times 10^{-13}\ C$$

$$C_j = \frac{\varepsilon_s \times A}{W_{dep}} = \frac{11.7 \times 8.854 \times 10^{-14} \times 400 \times 10^{-8}}{2.836 \times 10^{-4}} = 1.461 \times 10^{-14}\ F = 14.61\ fF$$

3.121)

A p^+-n diode is one in which the doping concentration in the *p* region is much greater than that in the *n* region. In such a diode, the forward current is mostly due to the hole injection across the junction. Show that:

$$I \approx I_p = Aqn_i{}^2\left(\frac{D_p}{L_pN_D}\right)\left(e^{\frac{V}{V_T}} - 1\right)$$

For the specific case in which $N_D = 5 \times 10^{16}/cm^3$, $D_p = 10\ cm^2/s$, $\tau_p = 0.1\ \mu s$, and $A = 10^4\ \mu m^2$, find I_S and the voltage V obtained when $I = 0.2\ mA$. Assume operation at 300 K where $n_i = 1.5 \times 10^{10}/cm^3$. Also, calculate the excess minority-carrier charge and the value of the diffusion capacitance at $I = 0.2\ mA$.

$$I = I_S\left(e^{\frac{V}{V_T}} - 1\right) = I_p + I_n$$

Where $I_S = Aqn_i{}^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right) \Rightarrow I = Aqn_i{}^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)\left(e^{\frac{V}{V_T}} - 1\right)$

$$I_p = Aqn_i{}^2\left(\frac{D_p}{L_pN_D}\right)\left(e^{\frac{V}{V_T}} - 1\right)\ and\ \ I_n = Aqn_i{}^2\left(\frac{D_n}{L_nN_A}\right)\left(e^{\frac{V}{V_T}} - 1\right)$$

$\because$ the junction is p+-n $\Rightarrow N_A \gg N_D \Rightarrow I_p \gg I_n \Rightarrow I \approx I_p$

$$I_p = Aqn_i{}^2\left(\frac{D_p}{L_pN_D}\right)\left(e^{\frac{V}{V_T}} - 1\right) \approx I = 0.2\ mA$$

$$L_p = \sqrt{D_p\tau_p} = \sqrt{10 \times 0.1 \times 10^{-6}} = 1 \times 10^{-3}\ cm$$

$$I_S \approx Aqn_i{}^2\left(\frac{D_p}{L_pN_D}\right) = 10^4 \times 10^{-8} \times 1.602^{-19} \times (1.5 \times 10^{10})^2 \frac{10}{1 \times 10^{-3} \times 5 \times 10^{16}} = 0.72 \times 10^{-15}\ A$$

$$V \approx V_T \ln(\frac{I_p}{I_S} + 1) = 0.0258 \times 26.35 = 0.6798\ V$$

$$Q_p = \tau_pI_p = 0.1 \times 10^{-6} \times 0.2 \times 10^{-3} = 2 \times 10^{-11}\ C = 20\ pC$$

$$C_d = \frac{Q}{V_T} = \frac{Q_p + Q_n}{V_T} \approx \frac{Q_p}{V_T} = \frac{\tau_pI_p}{V_T} = 7.752 \times 10^{-10} = 0.7752\ nF$$