Mechanical properties of solids

Mechanical properties of solids: contents

at the end of this lecture you should understand....

- basic definitions: stress and strain
- elastic and plastic deformation, fracture
- macroscopic picture for elastic deformation: Young's modulus, Hooke's law, Poisson's ratio, shear stress, modulus of rigidity, bulk modulus.
- elastic deformation on the microscopic scale, forces between atoms.
- atomic explanation of shear stress / yielding to shear stress, dislocations and their movement
- plastic deformation, easy glide, work hardening, fracture
- brittle fracture, brittle-ductile transition

Basic definitions

stress: force on an object per area perpendicular to force

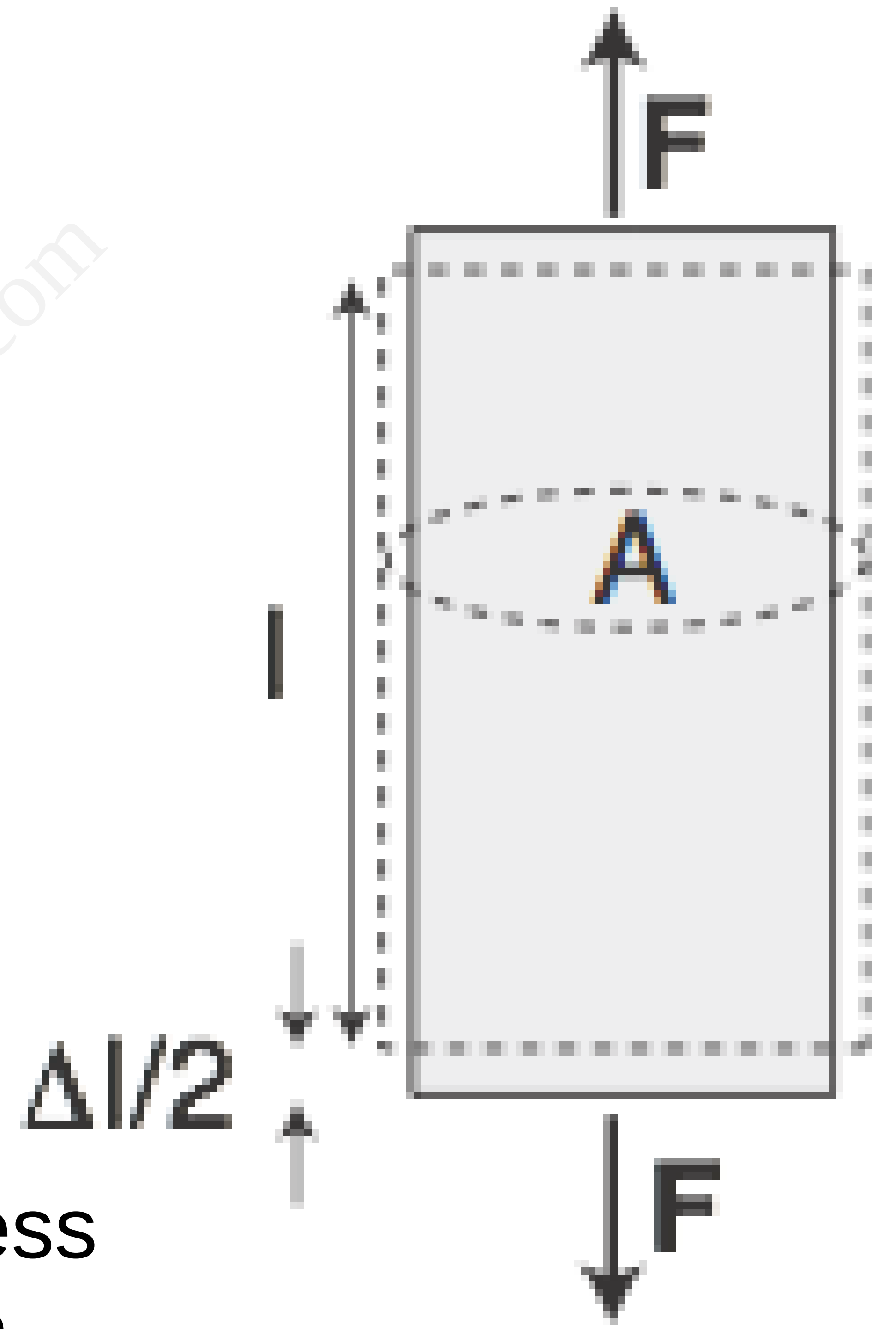
$$\sigma = \frac{F}{A}$$

unit: Pa

strain: length change relative to absolute length

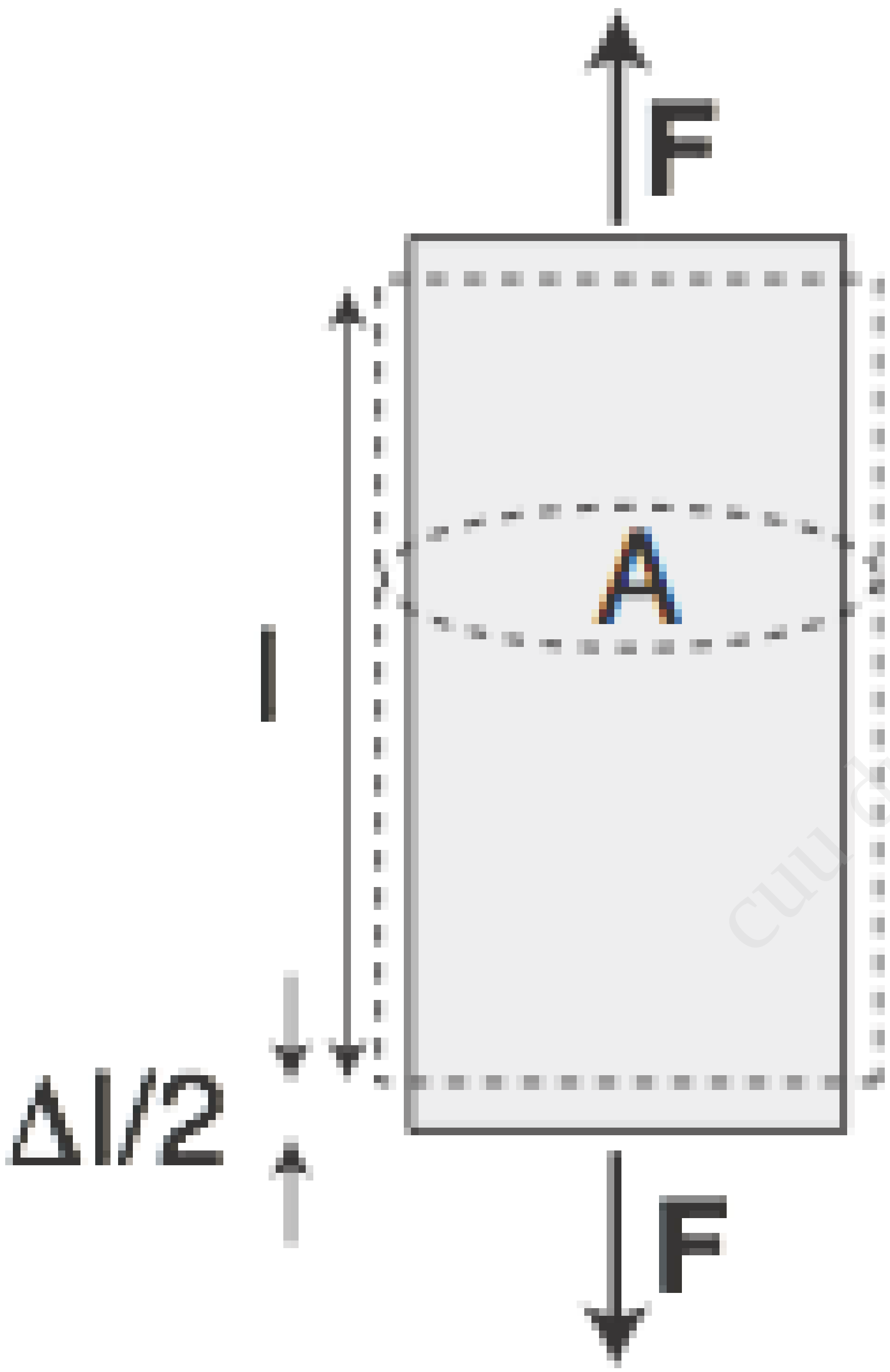
$$\epsilon = \frac{\Delta l}{l}$$

unit: dimensionless
technical: m/m

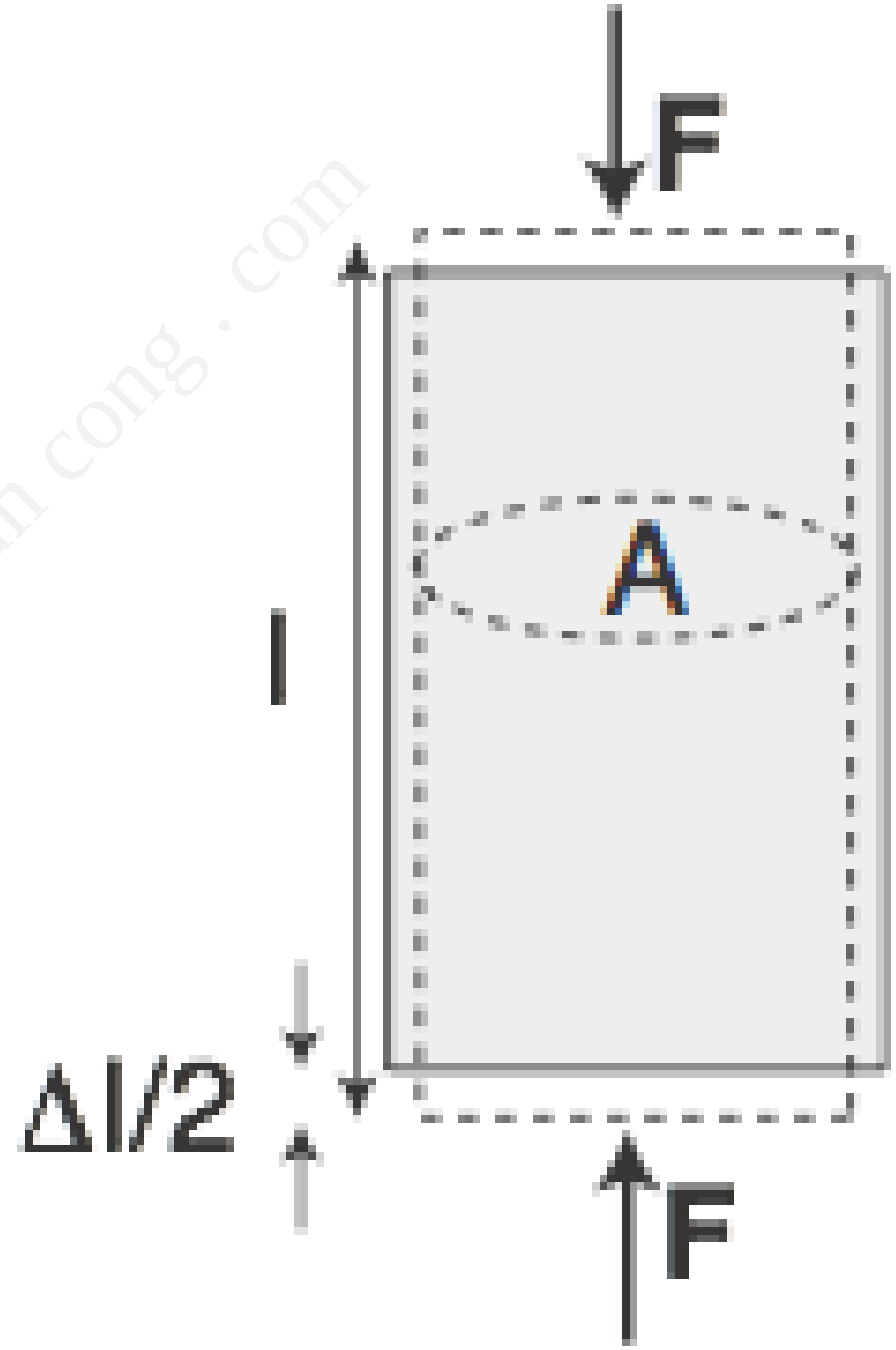


Basic definitions

tensile stress

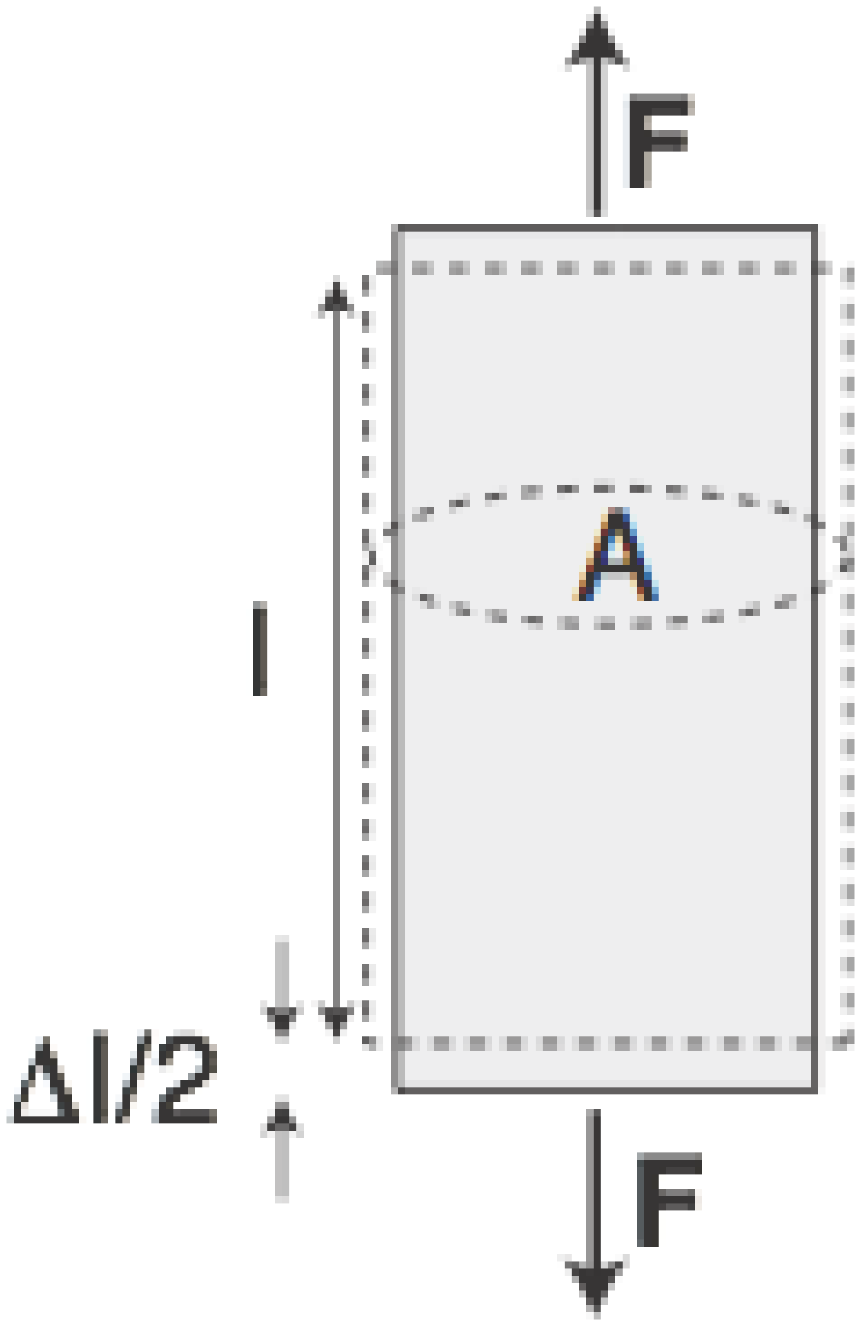


compressive stress



Elastic and plastic deformation, fracture

what happens when the tensile stress is increased?

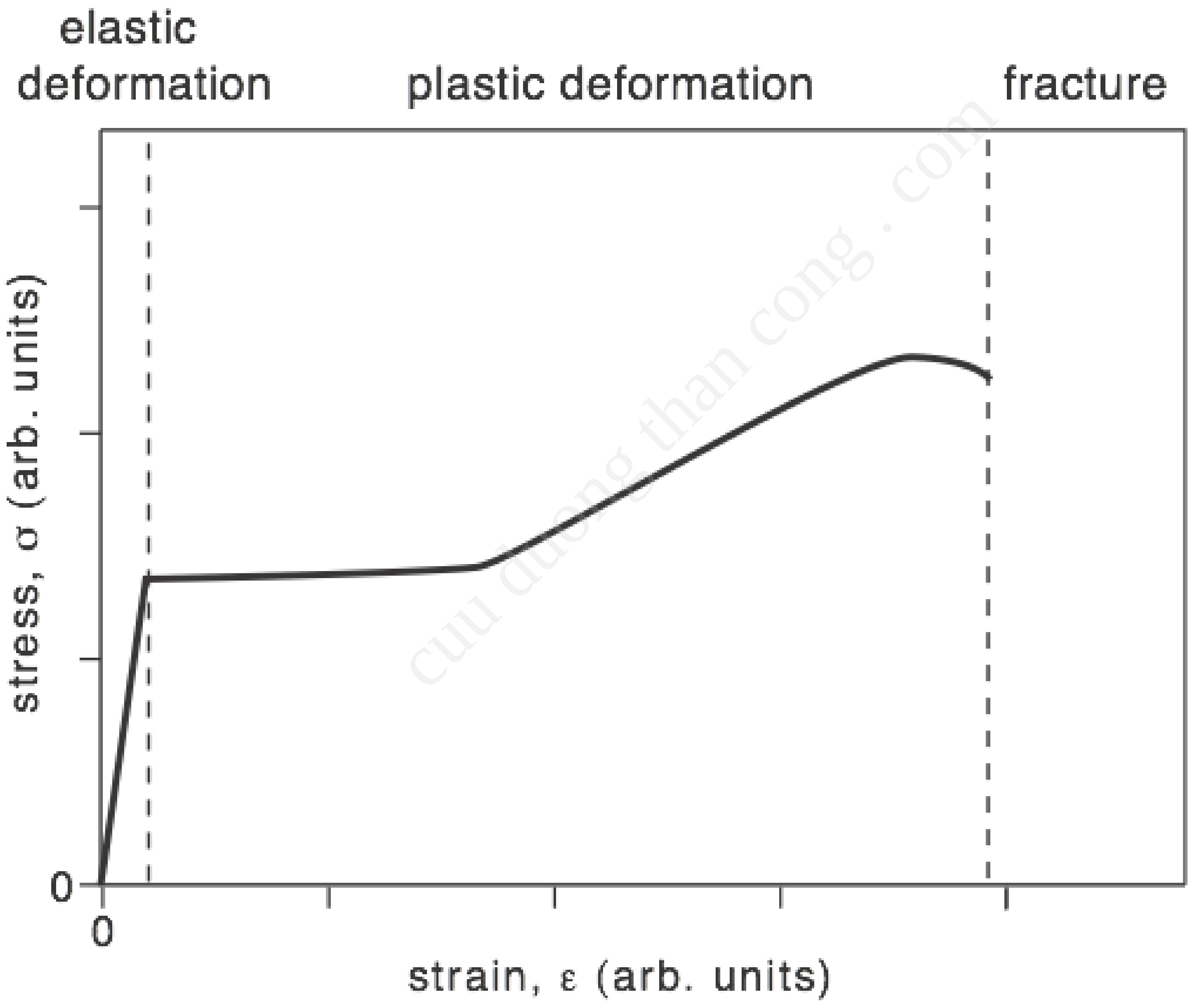


1. elastic deformation (reversible)
2. plastic deformation (irreversible)
3. fracture

Materials which show plastic deformation are called ductile.

Materials which show no plastic deformation are called brittle.

stress/strain curve for a ductile metal

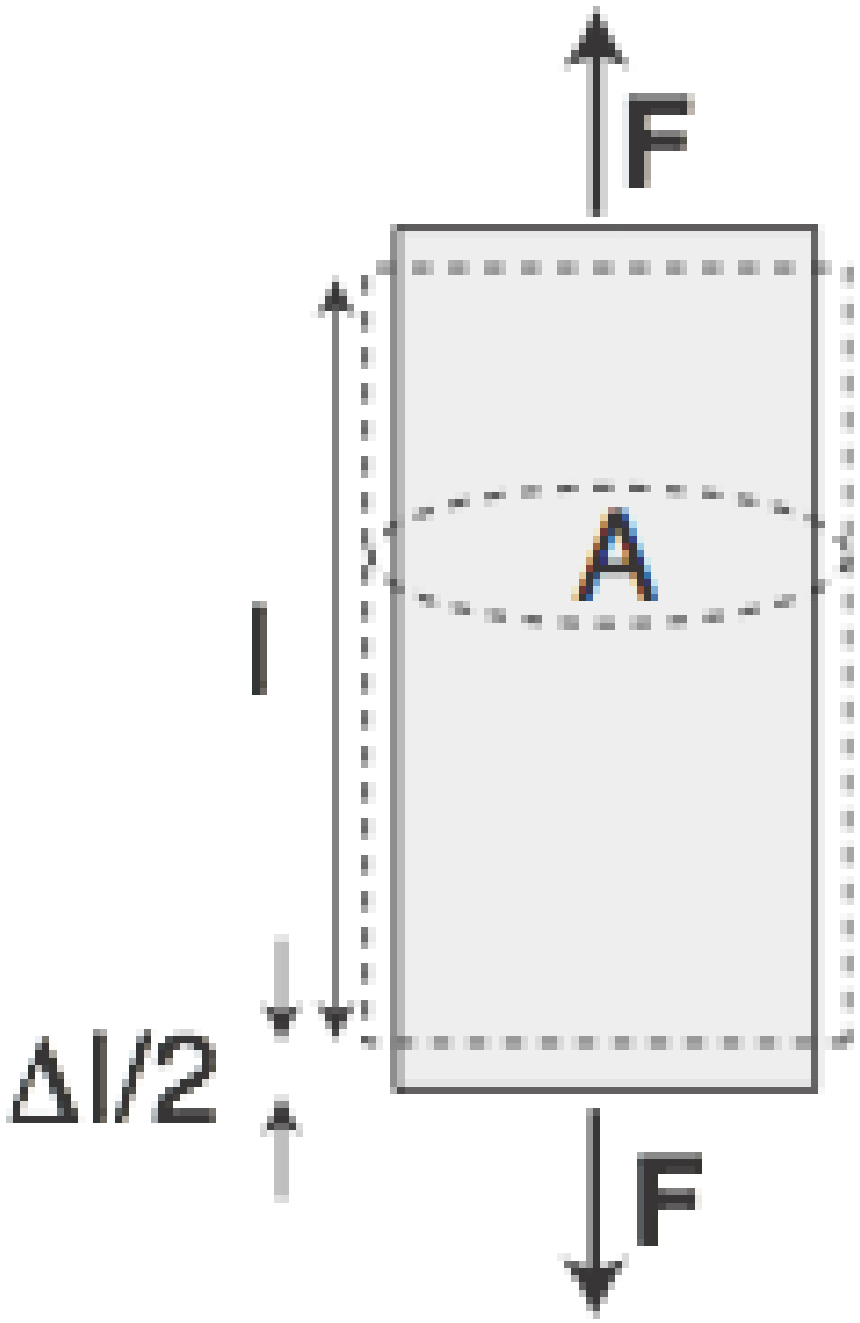


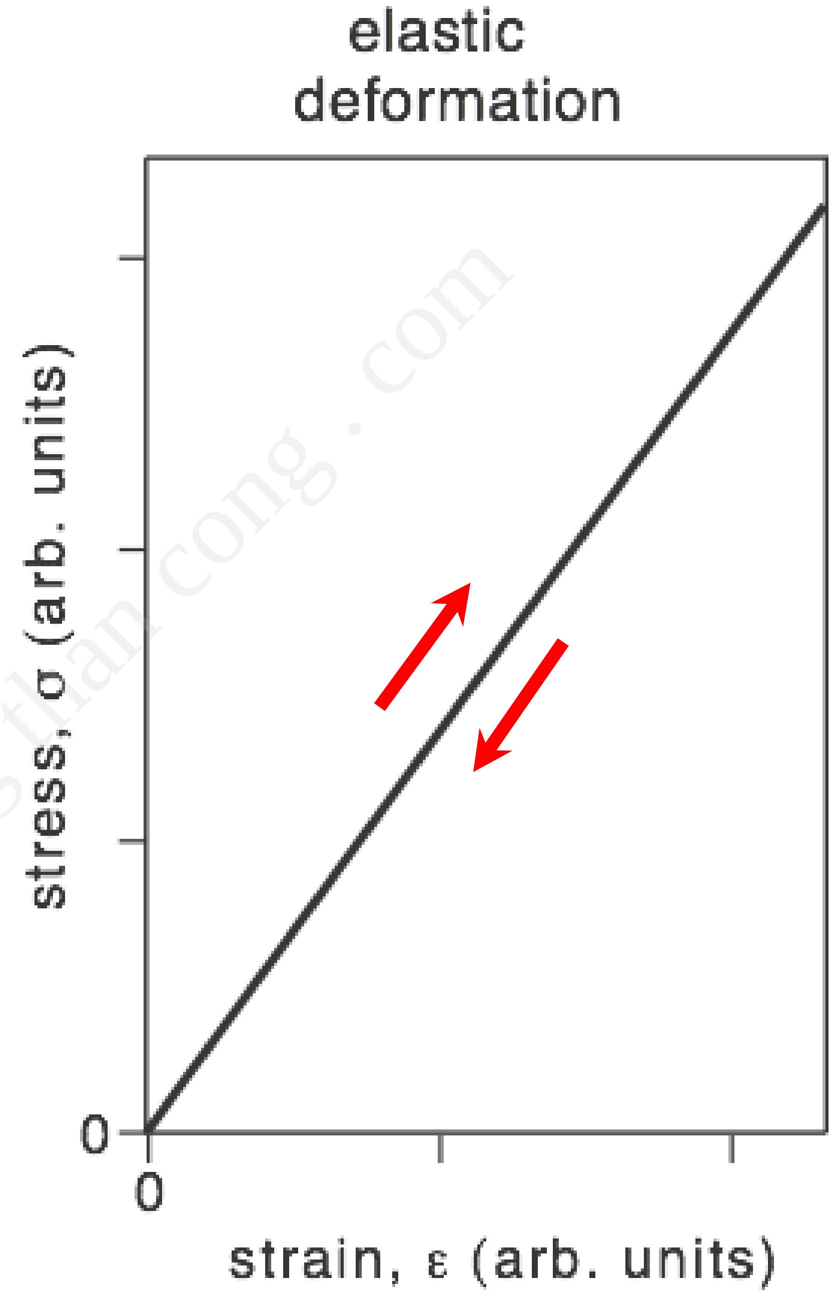
$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{\Delta l}{l}$$

Macroscopic picture: elastic deformation

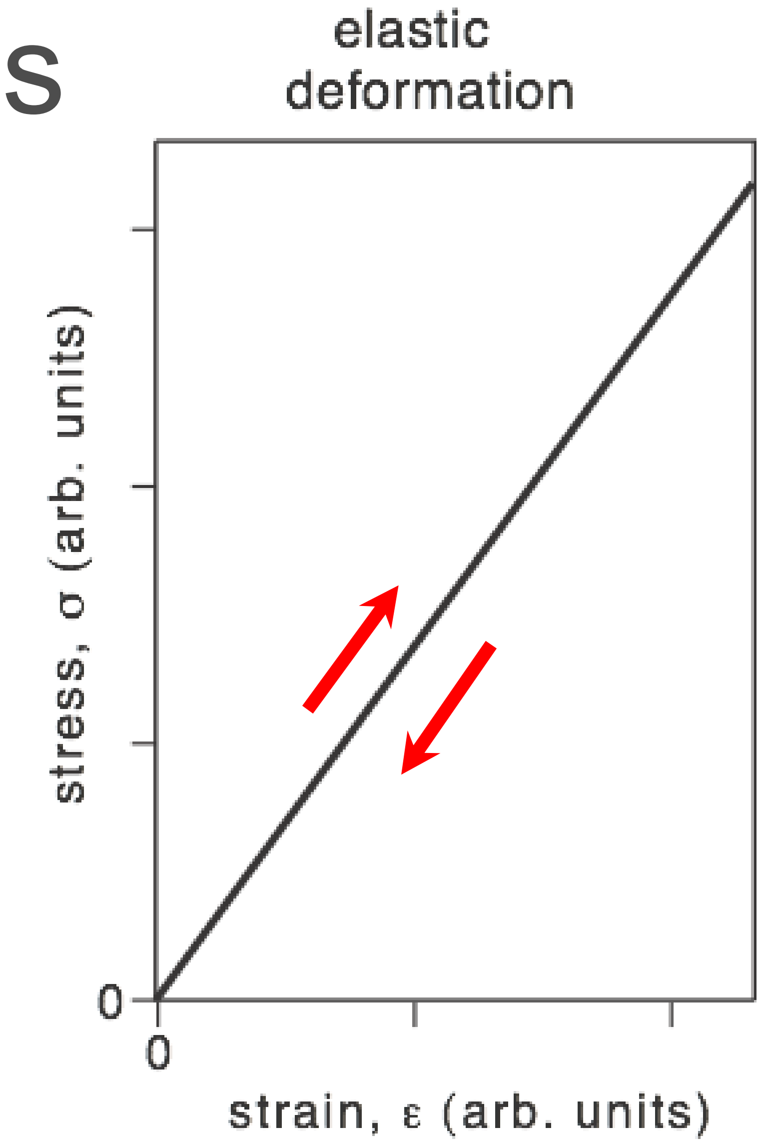
the linear region





behaviour is linear and reversible for a strain of up to 0.01 or so

Young’s modulus



stress: force on an object per area

$$\sigma = \frac{F}{A}$$

strain: length change relative to absolute length

$$\epsilon = \frac{\Delta l}{l}$$

Young’s modulus

$$Y = \frac{\sigma}{\epsilon} = \frac{F}{A}\frac{l}{\Delta l}$$

unit: Pa

Young's modulus and Hooke's law

Young's modulus

$$Y = \frac{\sigma}{\epsilon} = \frac{F}{A}\frac{l}{\Delta l}$$

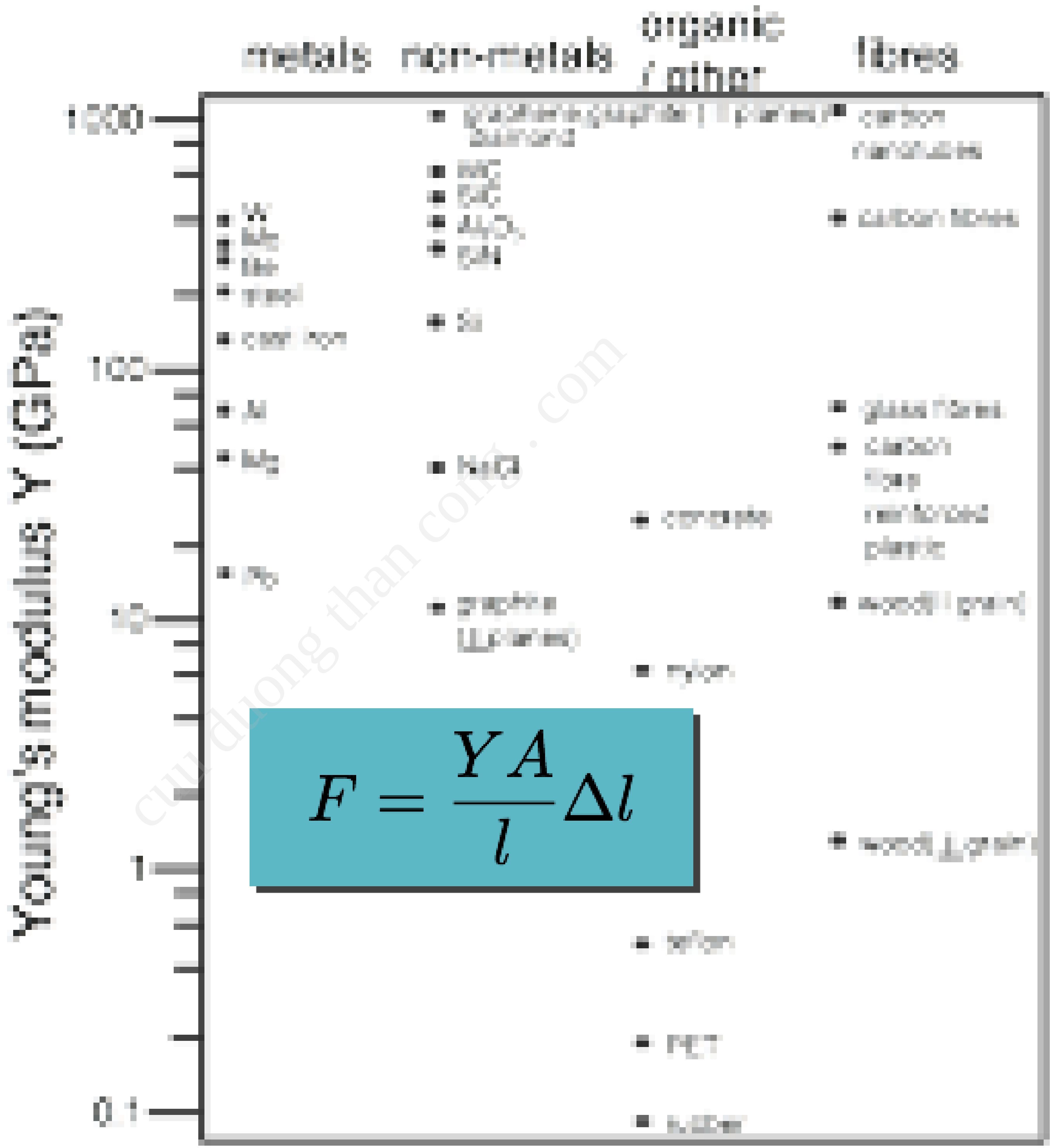
Hooke's law

$$F = \frac{YA}{l}\Delta l$$

stress $\sigma = \frac{F}{A}$

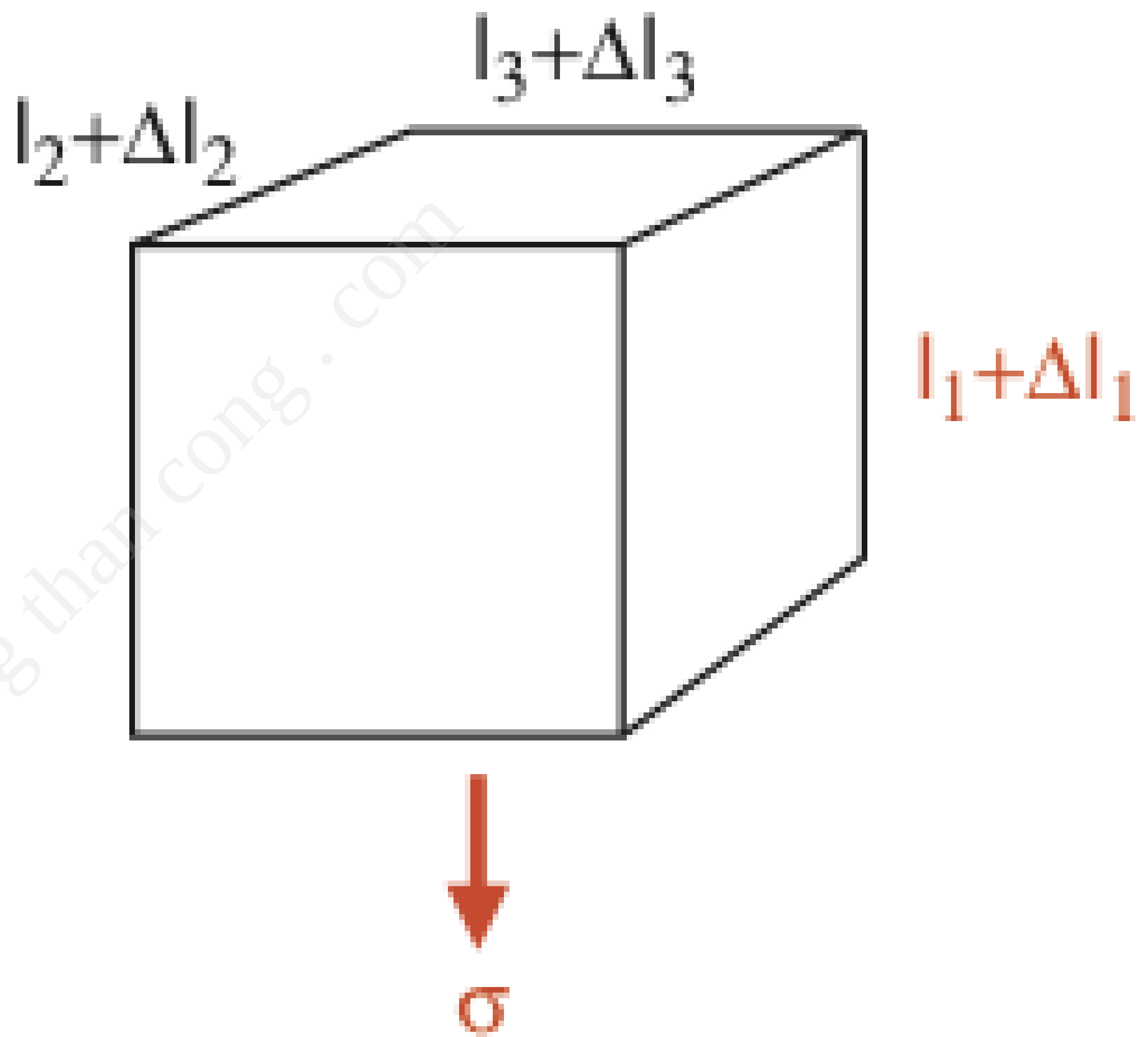
strain $\epsilon = \frac{\Delta l}{l}$

Young’s modulus



$$F = \frac{YA}{l}\Delta l$$

Poisson's ratio



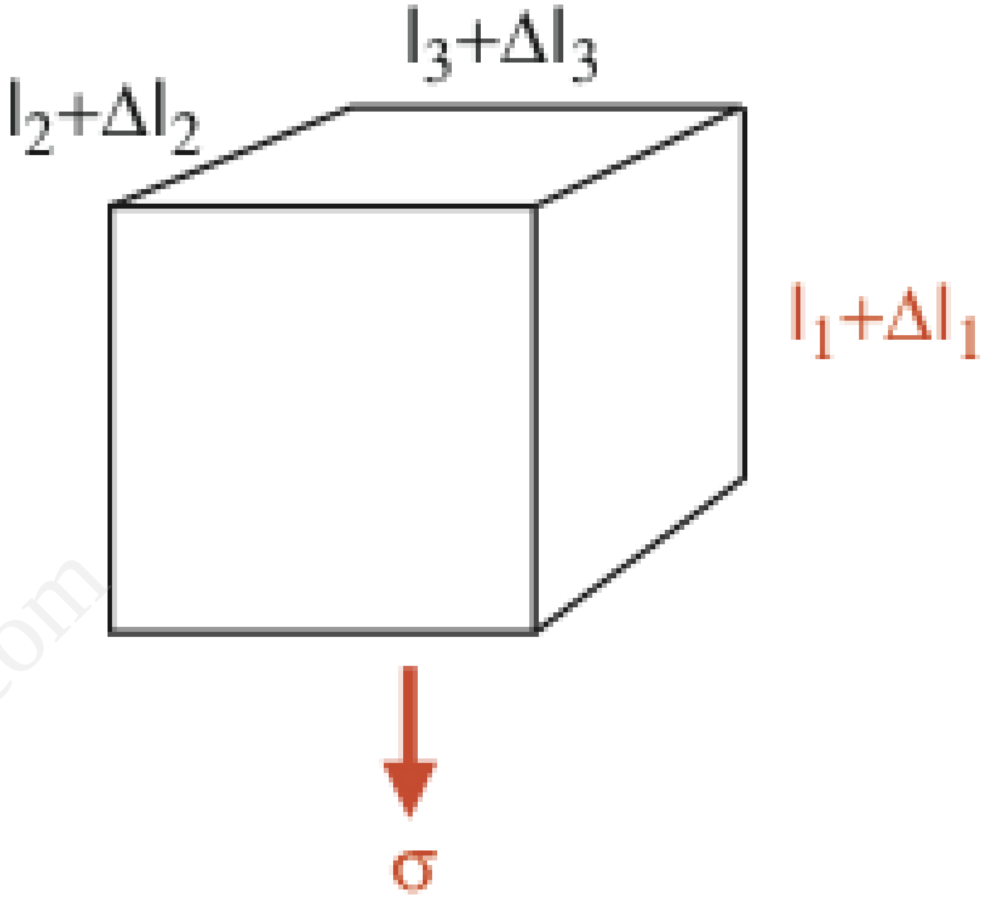
Poisson's ratio

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1}$$

$\nu \leq 0.5$

This means that the volume of the solid always increases under tensile stress

Poisson's ratio



$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1}$$

the volume is (assume the extensions are small)

$$(l_1 + \Delta l_1)(l_2 + \Delta l_2)(l_3 + \Delta l_3) \approx l_1 l_2 l_3 + \Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3$$

change in volume

$$\Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3 = \Delta l_1 l_2 l_3 + l_1(-\nu \frac{\Delta l_1}{l_1} l_2) l_3 + l_1 l_2 (-\nu \frac{\Delta l_1}{l_1} l_3)$$

$$= (1 - 2\nu) \Delta l_1 l_2 l_3$$

and since $\Delta l_1 > 0$ it follows that $\nu \leq 0.5$

Poisson's ratio

- There is also a lower limit to the Poisson ratio. We get

$-1 < v \leq 0.5$

Some examples: volume change for cube is $= (1-2\nu)\Delta l_1 l_2 l_3$

v	what happens?
v >0.5	~~tensile stress: volume decrease, compressive stress: volume increase~~
v = 0.5	no volume change, incompressible solid
0< v<0.5	"normal" case for most materials, volume increase upon tens. stress, volume decrease upon compr. stress
-1< v<0	volume increase upon tens. stress, volume decrease upon compr. stress; wires get thicker as you pull them!

Poisson ratio: examples

material	v
diamond	0.21
Al	0.33
Cu	0.35
Pb	0.4
Steel	0.29
rubber	close to 0.5
cork	close to 0

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1}$$

This is why it is possible to get a cork back into a wine bottle!

Foams with a negative poisson ratio

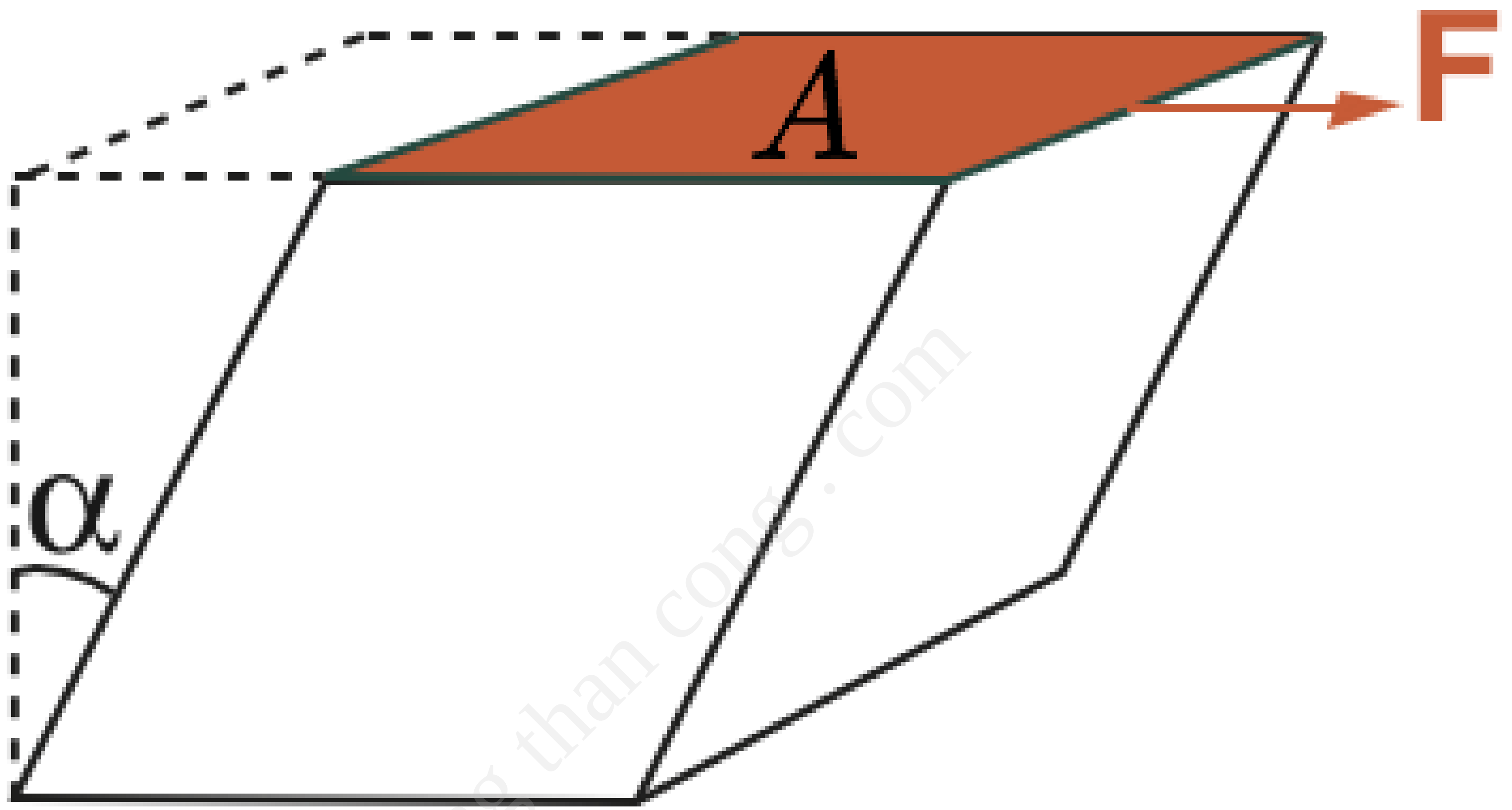
from: Exploring the nano-world with LEGO bricks
http://mrsec.wisc.edu/Edetc/LEGO/index.html

Elastic deformation: macroscopic

other deformations:

- shear stress: twisting of the sample
- hydrostatic pressure: compression
- torsion stress: torsion (not discussed here)

Shear stress / modulus of rigidity



shear stress: tangential force
on an object per area

$$\tau = \frac{F}{A}$$

unit: Pa

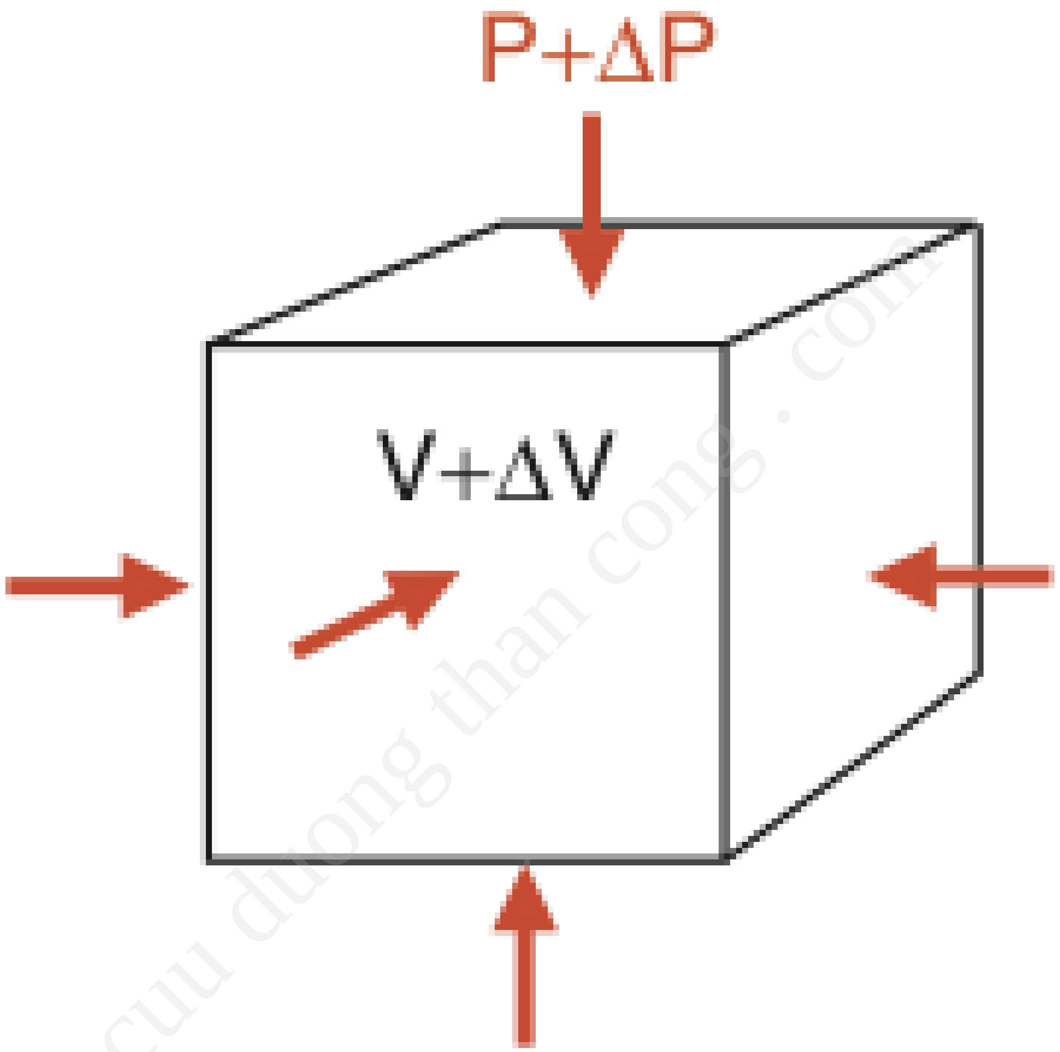
modulus of rigidity

$$G = \frac{\tau}{\alpha}$$

unit: Pa

Hydrostatic pressure / bulk modulus

exposure to hydrostatic pressure



bulk modulus

$$K = -\Delta P \frac{V}{\Delta V}$$

unit: Pa

Relation between elastic constants

- in a more formal treatment, the quantities are related:

$$G = \frac{Y}{2(1+\nu)}$$

(online note philiphofmann.net)

$$K = \frac{Y}{3(1-2\nu)}$$

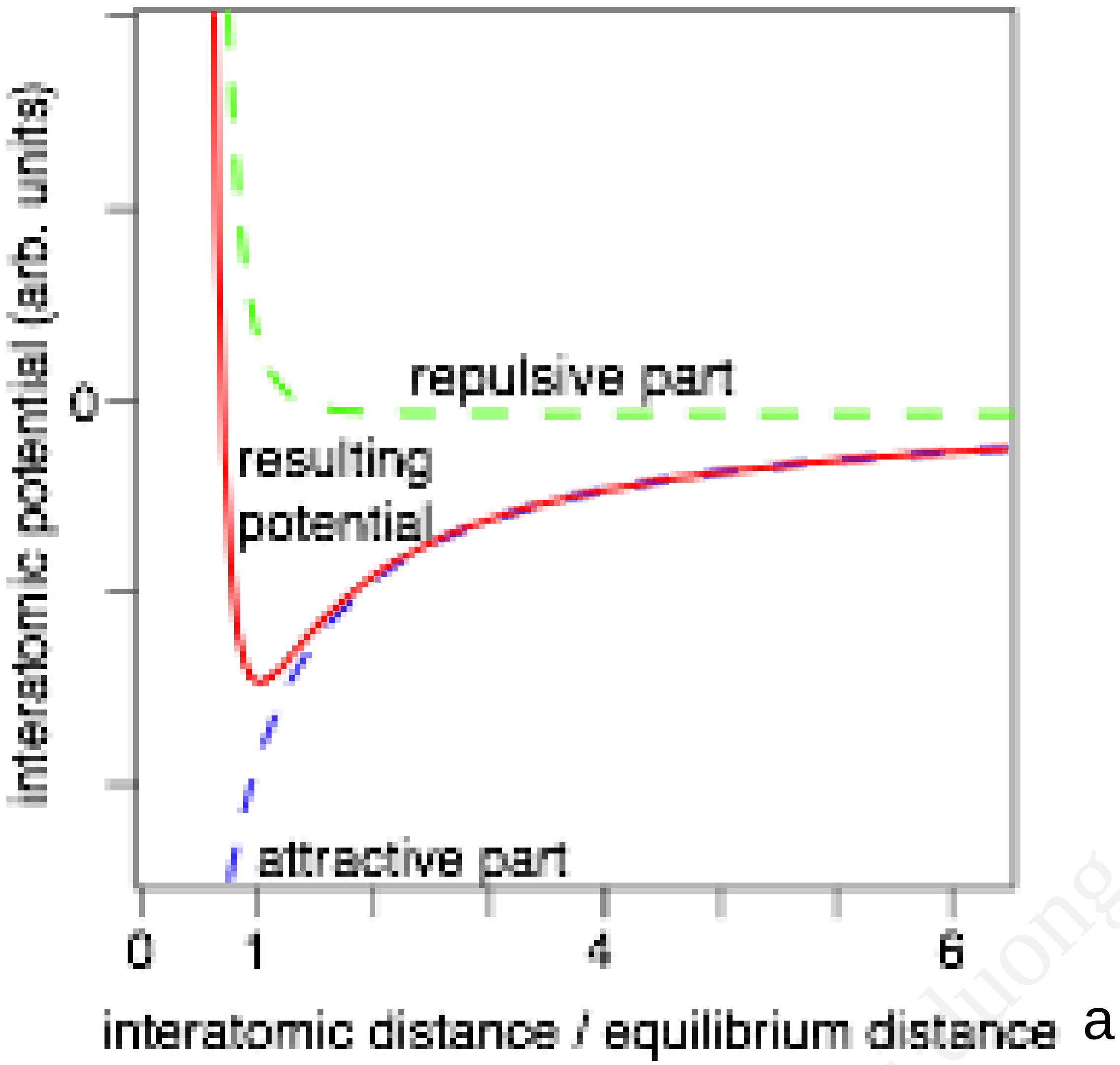
problem 3.1 in book

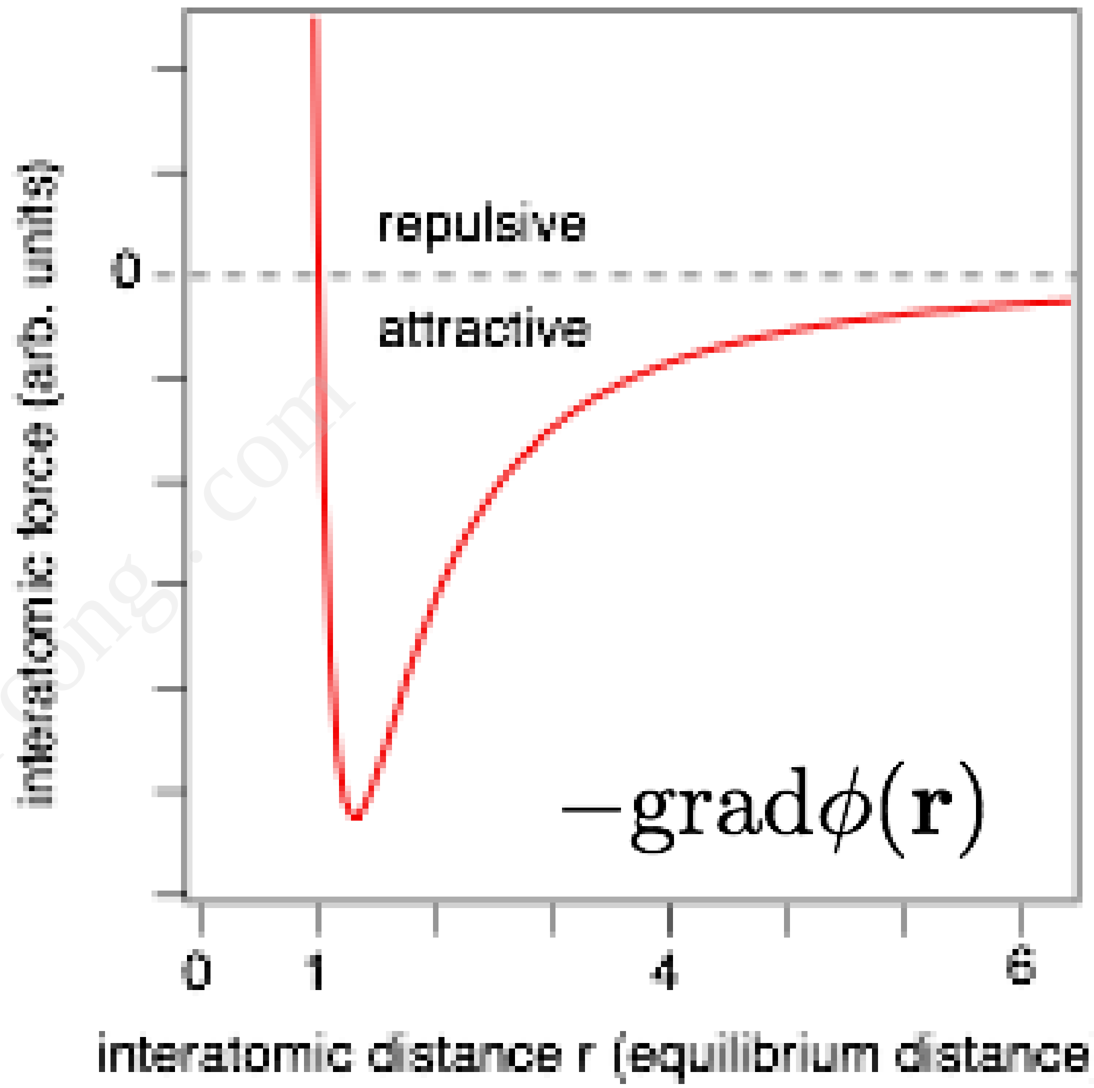
modulus of rigidity and bulk modulus as a function of Young's modulus and Poisson ratio.

Elastic deformation: microscopic

- Can we explain this behaviour on a microscopic scale?
- Can we relate the macroscopic elastic constants to the microscopic forces?

Why is the force linear?



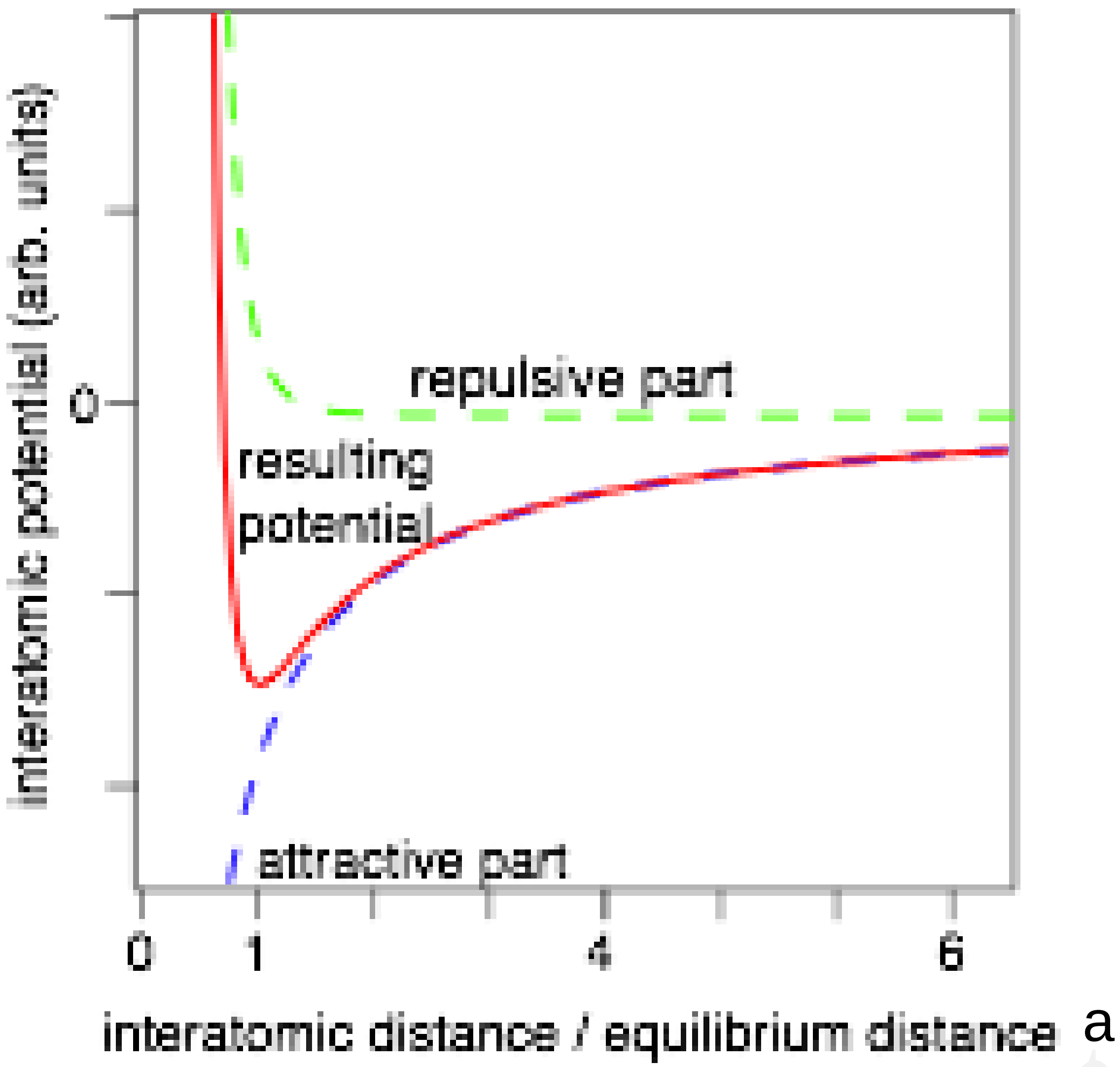


$$\phi(x) = \phi(a) + \frac{d\phi(a)}{dx}(x-a) + \frac{1}{2}\frac{d^2\phi(a)}{dx^2}(x-a)^2 + \frac{1}{6}\frac{d^3\phi(a)}{dx^3}(x-a)^3 + \ldots$$

energy offset

=0

harmonic potential linear force.



Pb:
cohesive energy: 2.3 eV / atom
interatomic distance: 3.43 Å

W:
cohesive energy: 8.9 eV / atom
interatomic distance: 2.73 Å

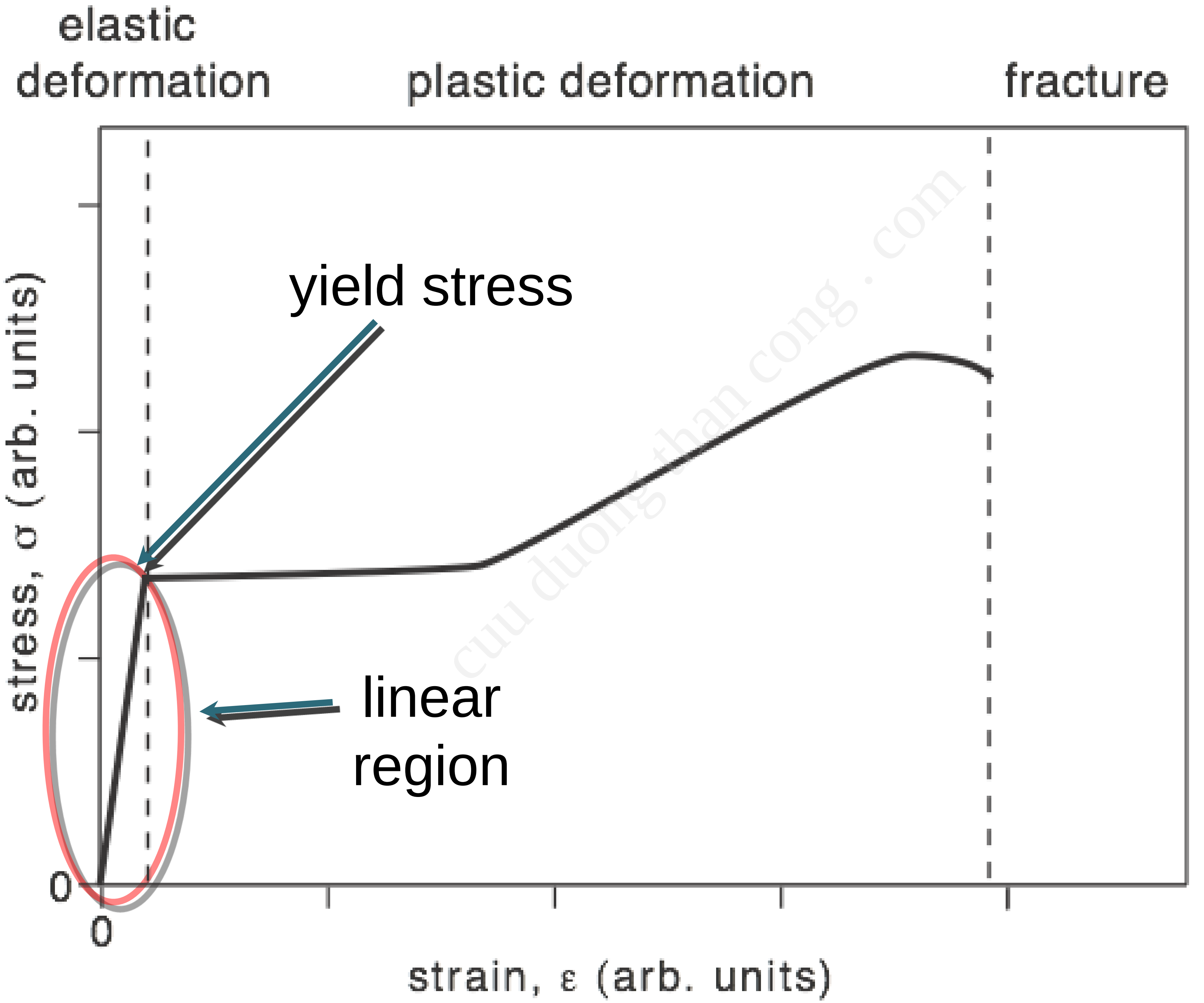
$$\phi(x) = \phi(a) + \frac{d\phi(a)}{dx}(x-a) + \frac{1}{2}\frac{d^2\phi(a)}{dx^2}(x-a)^2 + \frac{1}{6}\frac{d^3\phi(a)}{dx^3}(x-a)^3 + \dots$$

linear restoring force:

$$F = -K(x-a) \qquad K = \frac{d^2\phi(a)}{dx^2}$$

curvature of the potential, not depth. So why is it related to the cohesive energy?

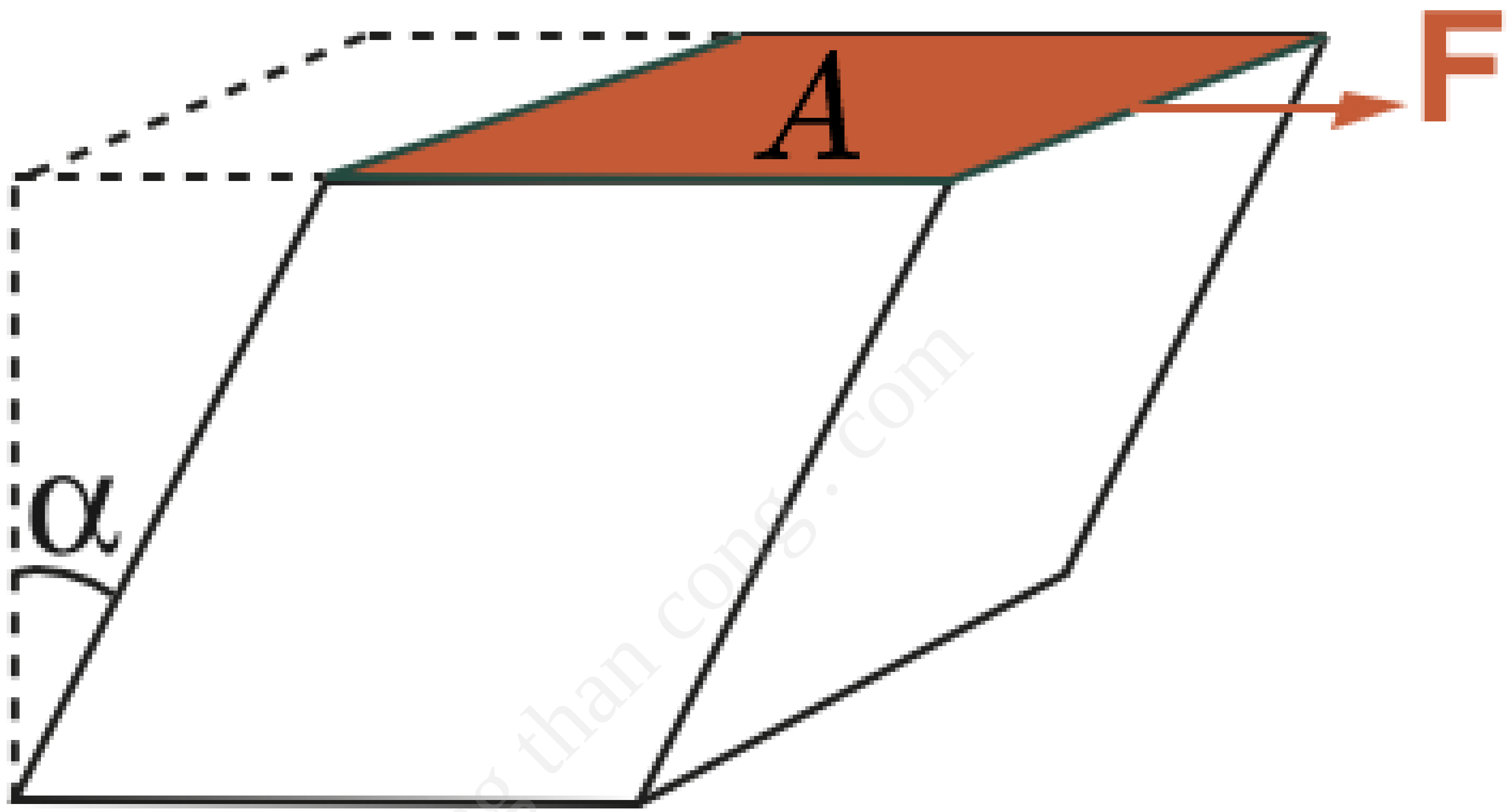
stress/strain curve for a ductile metal



$$\sigma = \frac{F}{A}$$

$$\epsilon = \frac{\Delta l}{l}$$

Shear stress / modulus of rigidity



shear stress: tangential force on an object per area

$$\tau = \frac{F}{A}$$

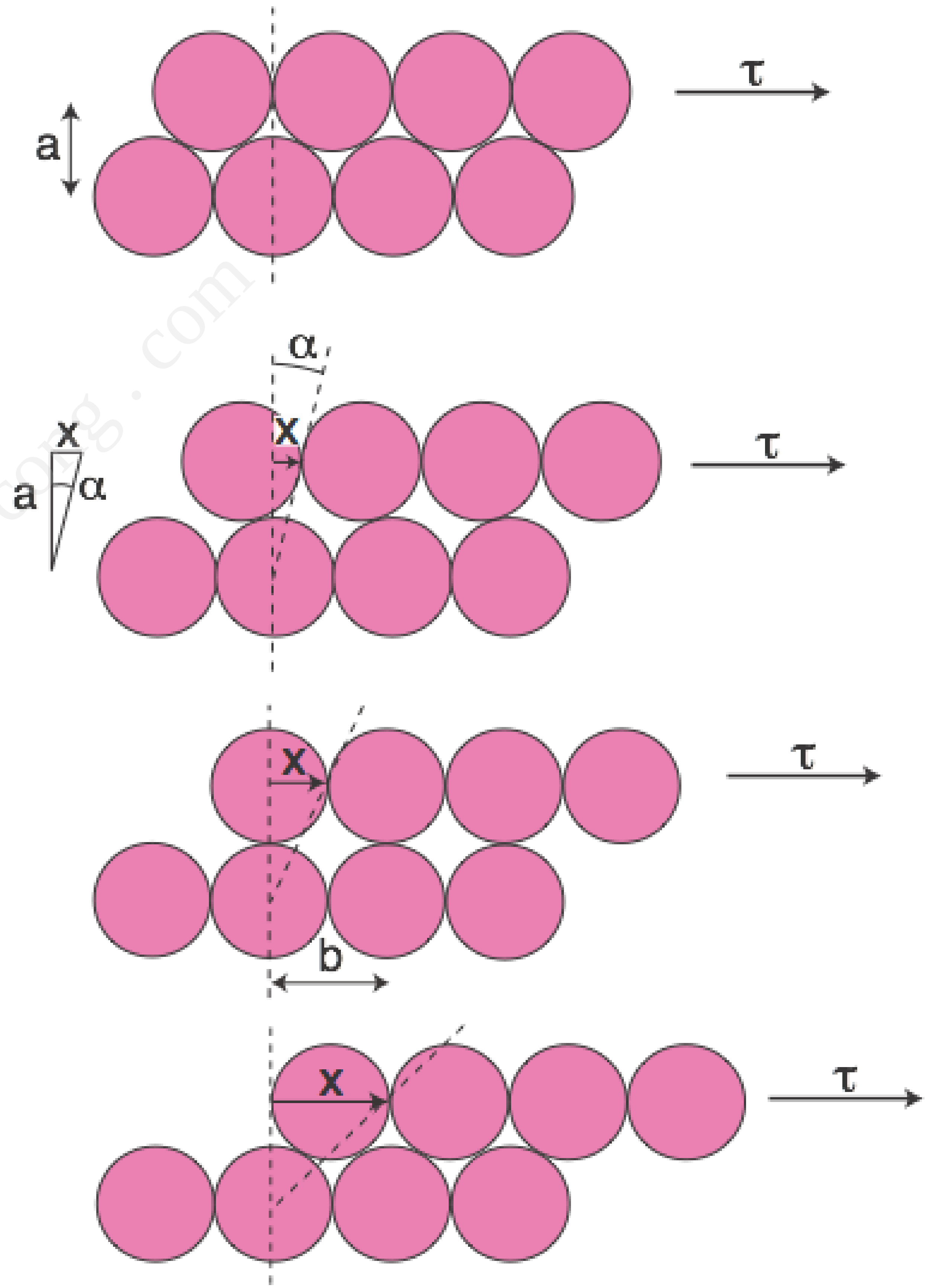
unit: Pa

modulus of rigidity

$$G = \frac{\tau}{\alpha}$$

Microscopic picture of shear stress

basic idea: pull planes across each other (here in 1D)

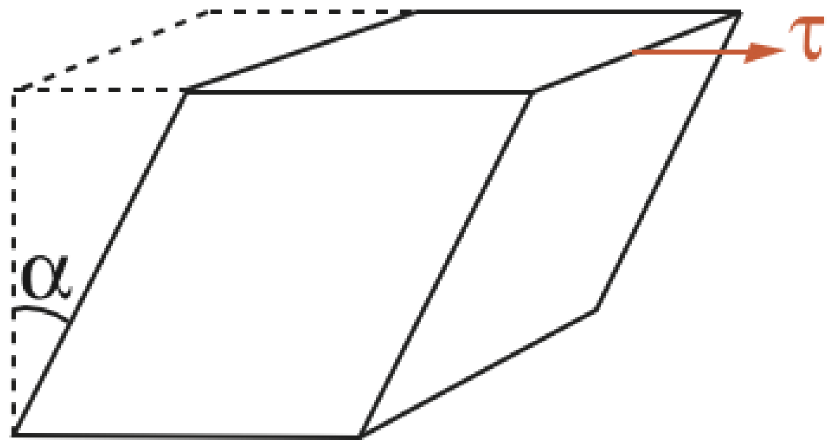


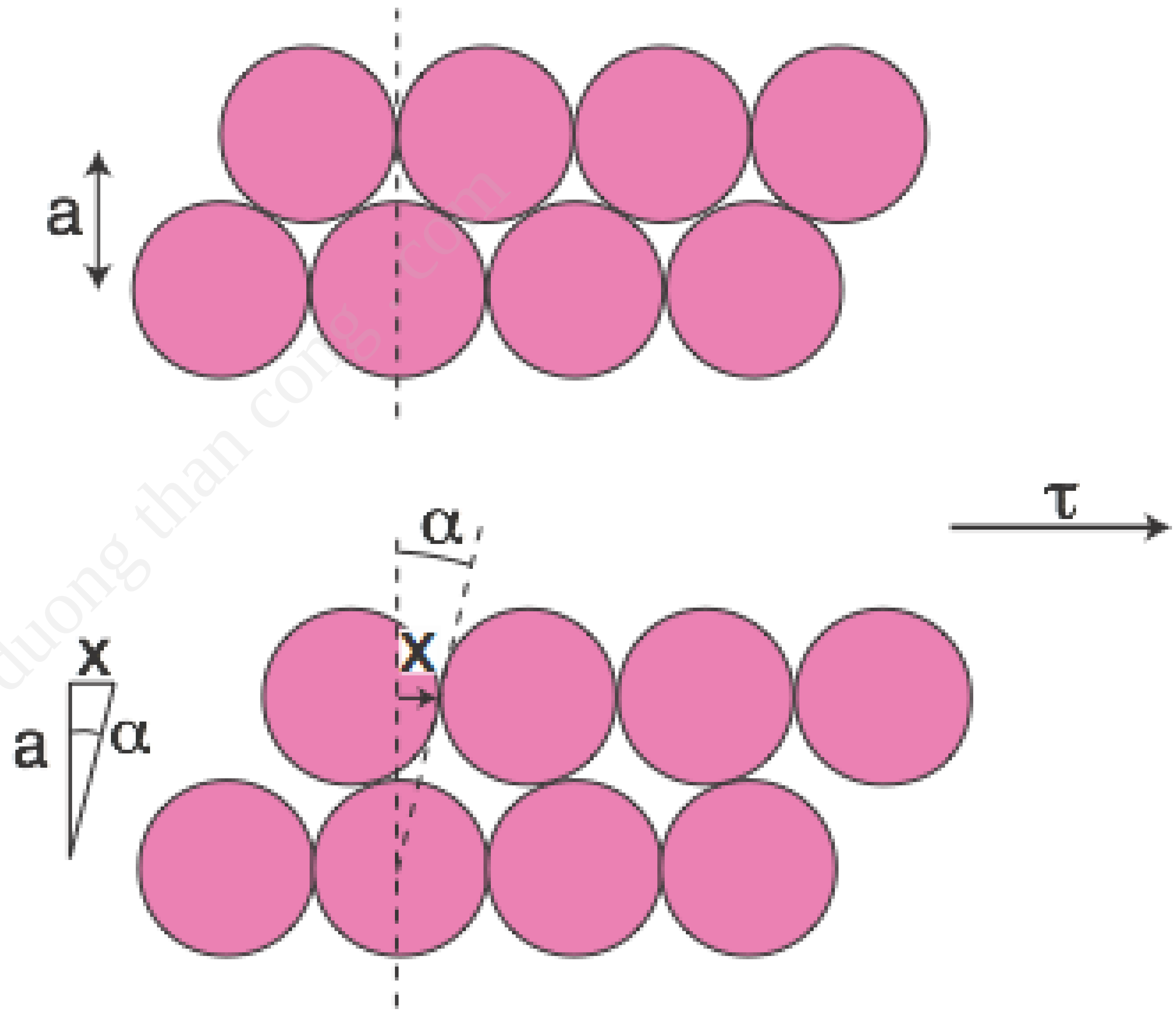
$$G = \frac{\tau}{\alpha}$$

macroscopic measurement

microscopic model

Microscopic picture of shear stress





$$\alpha = \tan^{-1}\left(\frac{x}{a}\right) \approx \frac{x}{a}$$

$$\tau = G\alpha \approx \frac{Gx}{a}$$

Relate macroscopic and microscopic changes

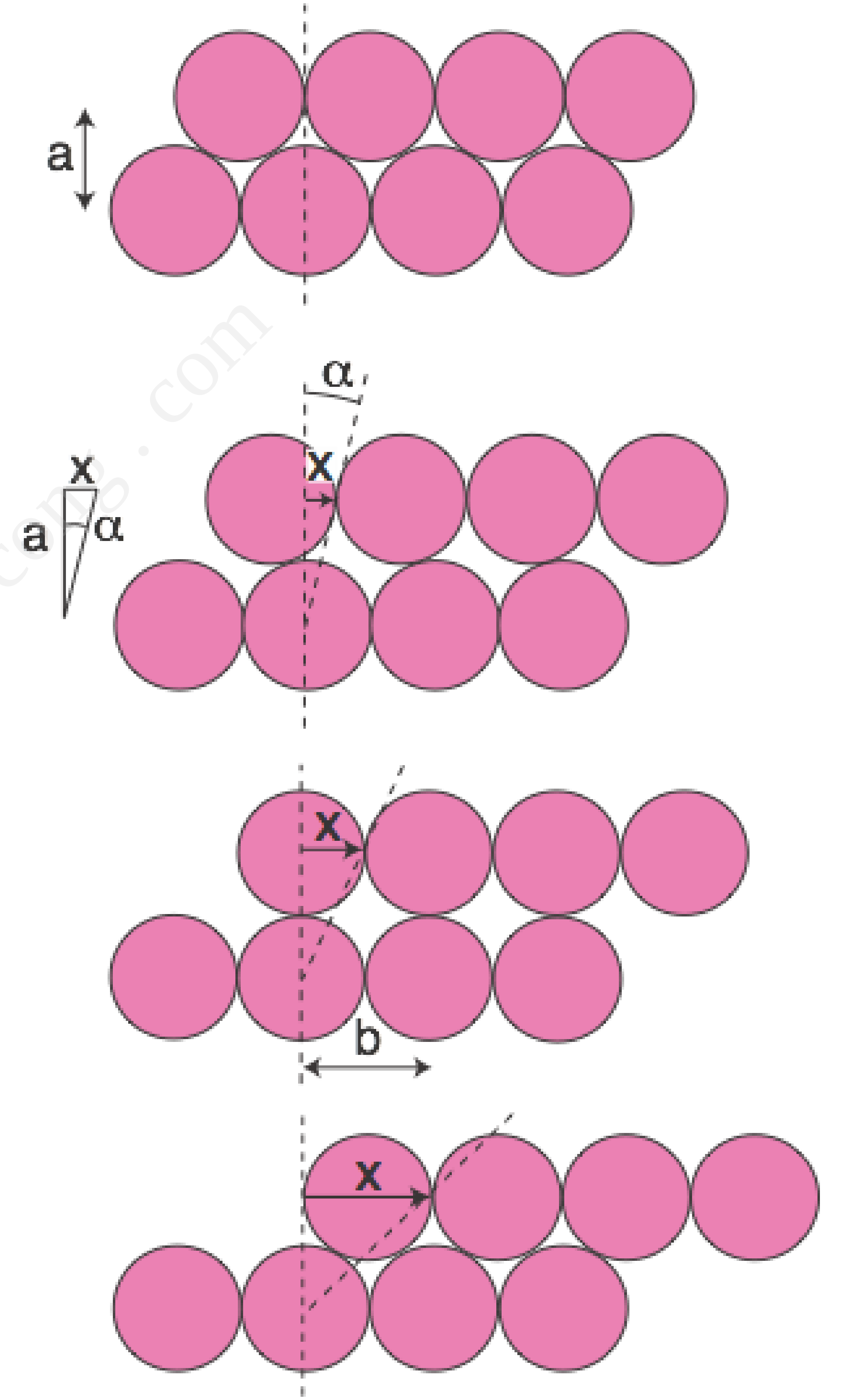
Microscopic picture of shear stress

The shear stress must have a periodic dependence

$$\tau = C \sin\left(\frac{2\pi x}{b}\right)$$

the yield stress is $\tau_Y = C$

$$\tau = \tau_Y \sin\left(\frac{2\pi x}{b}\right)$$

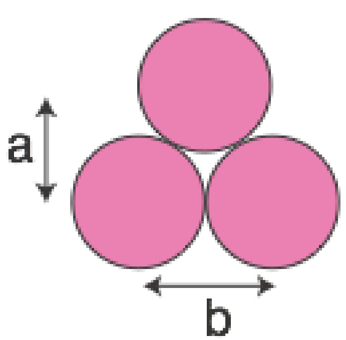


Microscopic picture of shear stress estimate of the yield stress

we have $\tau = \tau_Y \sin\left(\frac{2\pi x}{b}\right)$ and $\tau = G\alpha \approx \frac{Gx}{a}$

and for a small x, we have $\tau = \tau_Y \frac{2\pi x}{b}$

combining with the other equation $\tau_Y \frac{2\pi x}{b} = \frac{Gx}{a}$



$a = b\sqrt{3}/2$

$$\tau_Y = \frac{Gb}{2\pi a} = \frac{G}{\sqrt{3}\pi}$$

Microscopic picture of shear stress

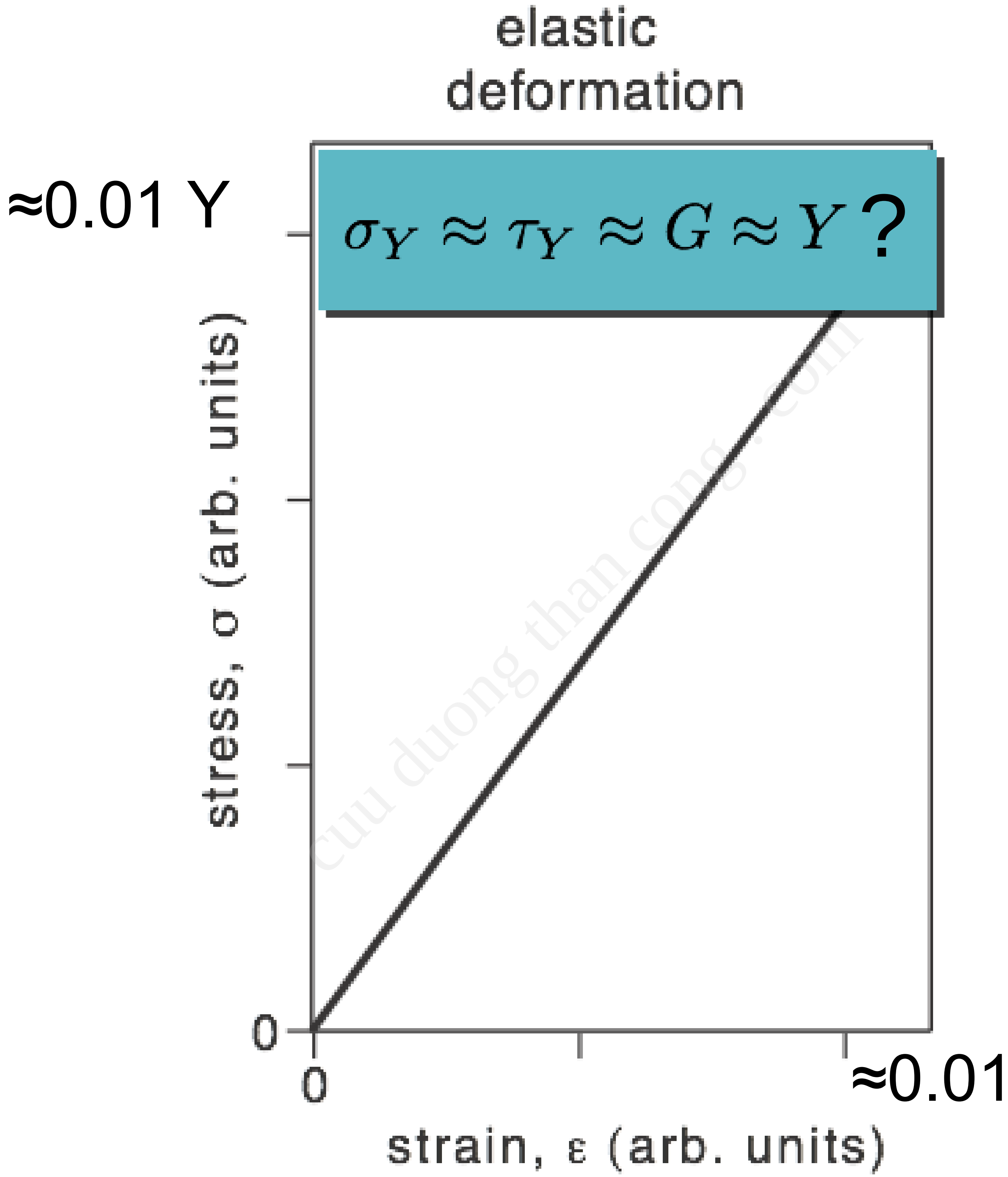
remember

$$\tau_Y = C = \frac{Gb}{2\pi a} = \frac{G}{\sqrt{3}\pi}$$

$$G = \frac{Y}{2(1+\nu)}$$

Given our crude simplifications, this is essentially

$$\tau_Y \approx G \approx Y$$


elastic
deformation
$\sigma_Y \approx \tau_Y \approx G \approx Y$?
≈0.01 Y
stress, σ (arb. units)
0
0
≈0.01
strain, ε (arb. units)

Defects

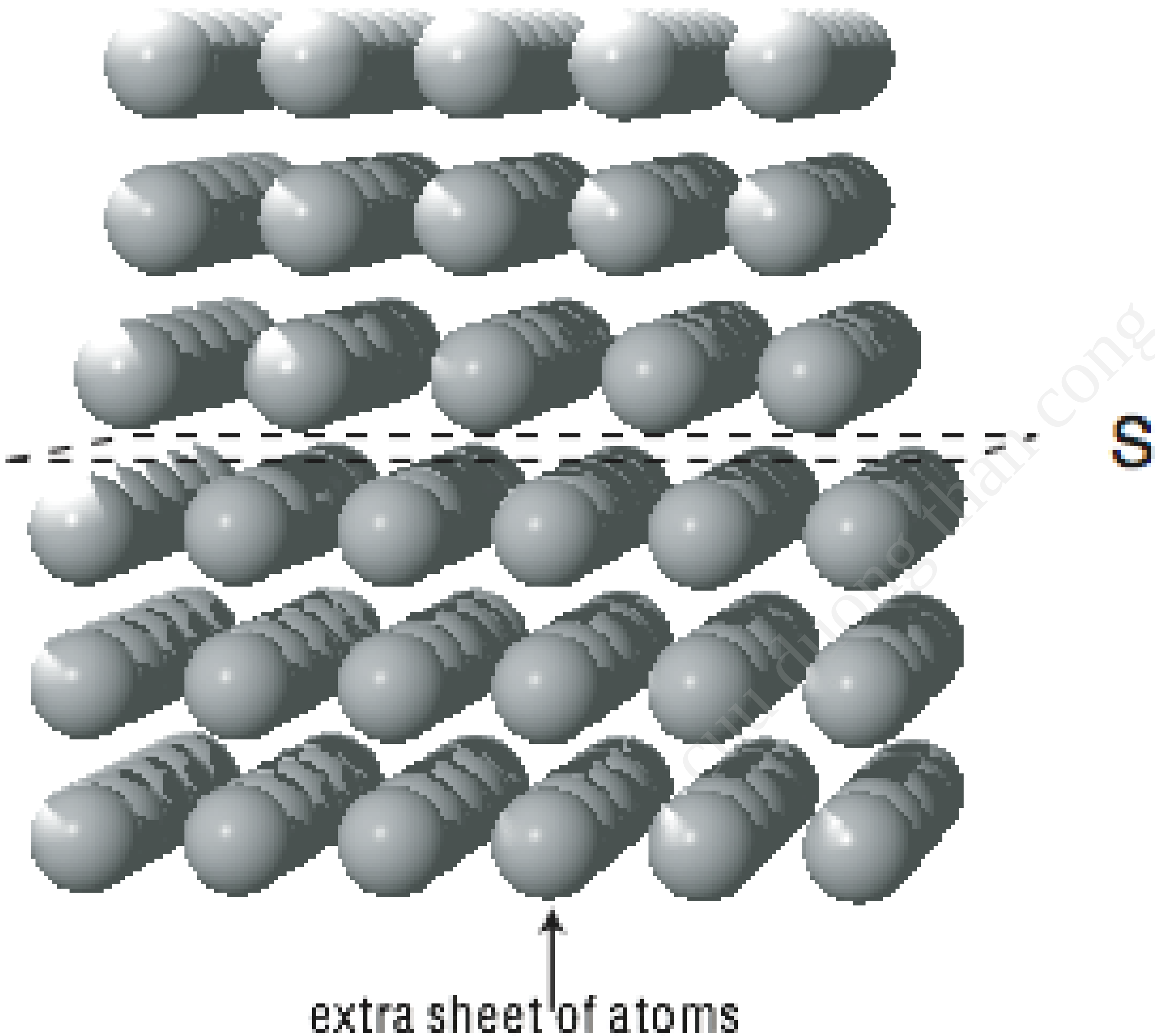
- Point defects: foreign atoms, missing atoms, substitutional, interstitial...
- Extended defects: dislocations...

Dislocations

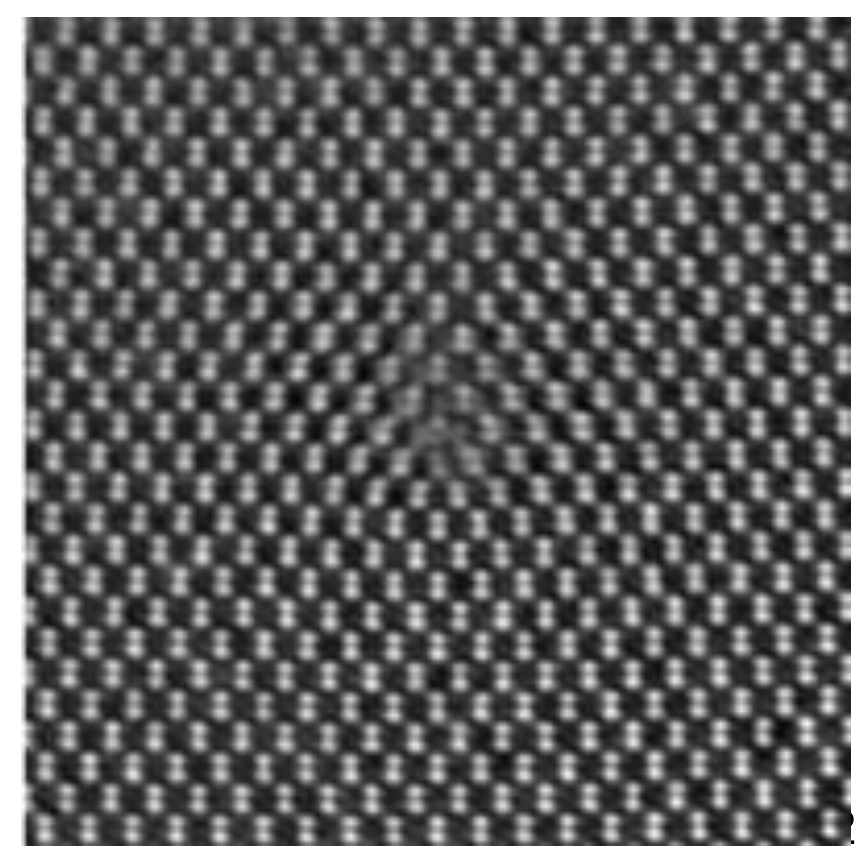
an edge dislocation

one extra sheet of atoms

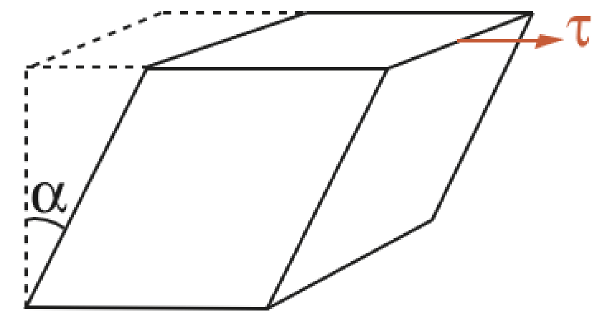
the dislocation reaches over long distances

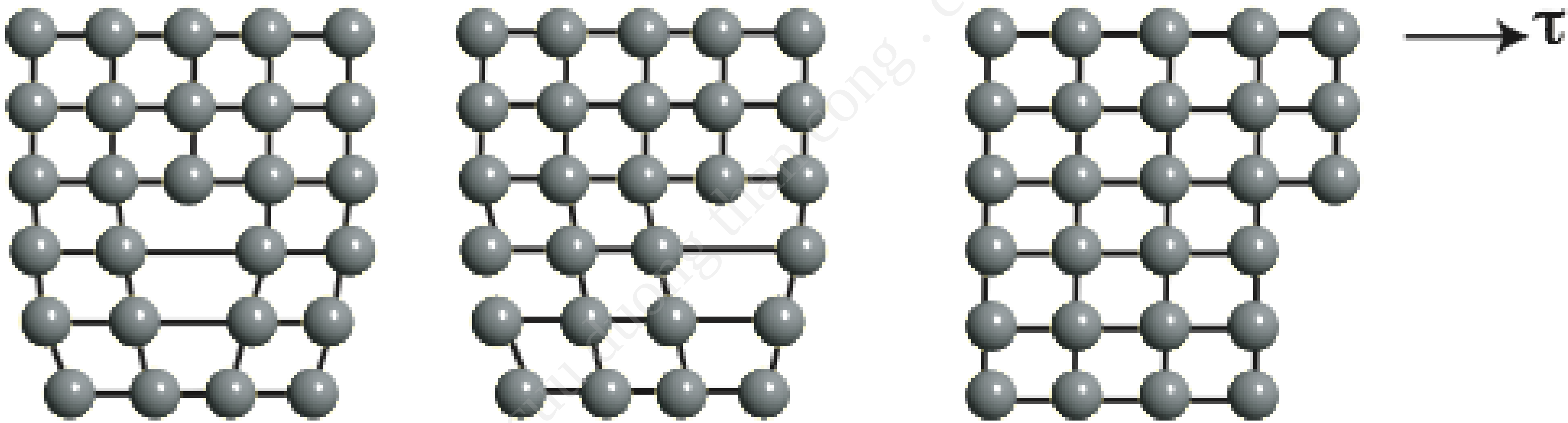


slip plane



Moving of a dislocation

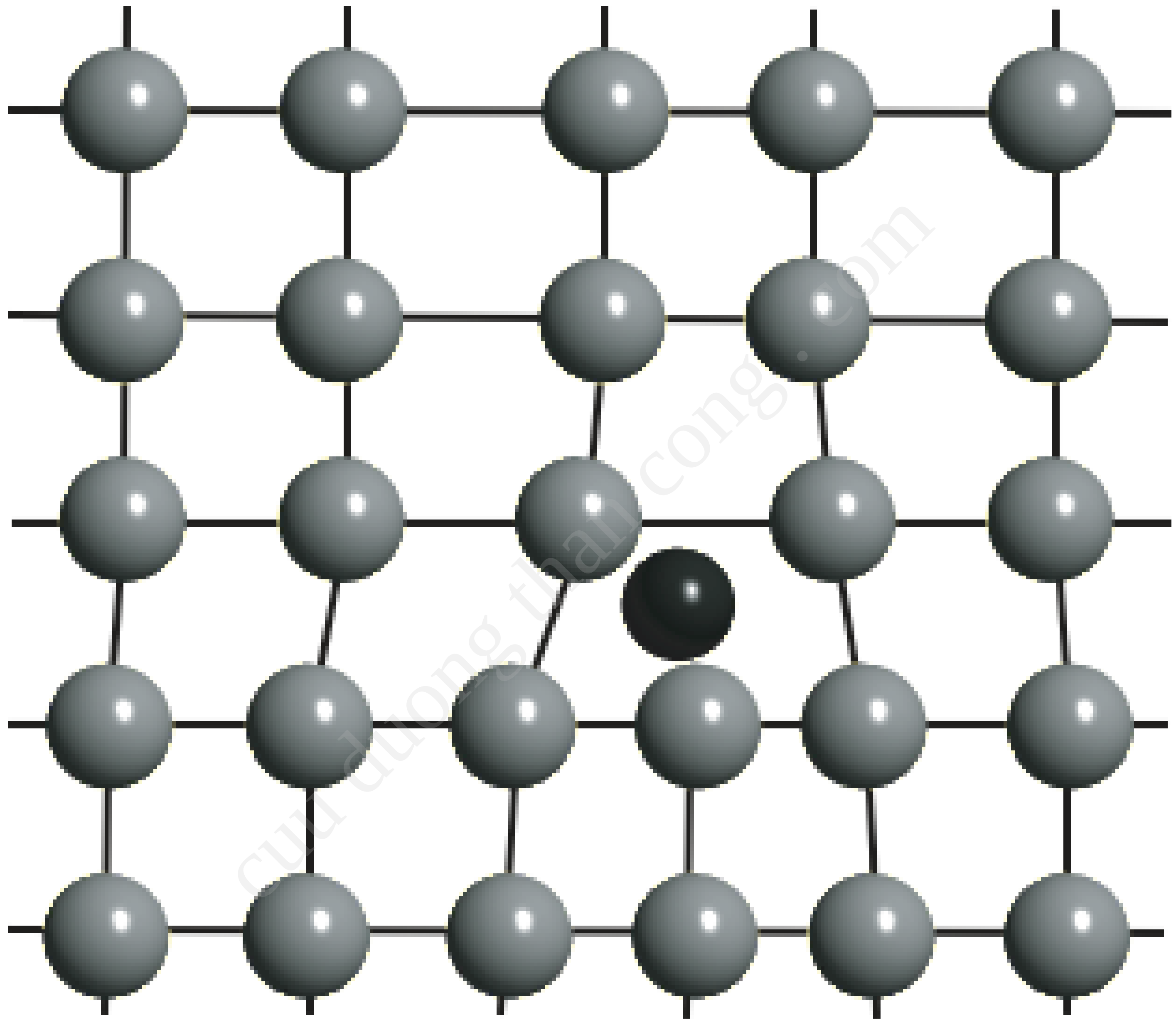




- One does not have to break all the bonds at the same time, but only one at a time to slide the plane.

But: An estimate of the yield stress for this is too small.

Pinning of dislocations by impurities



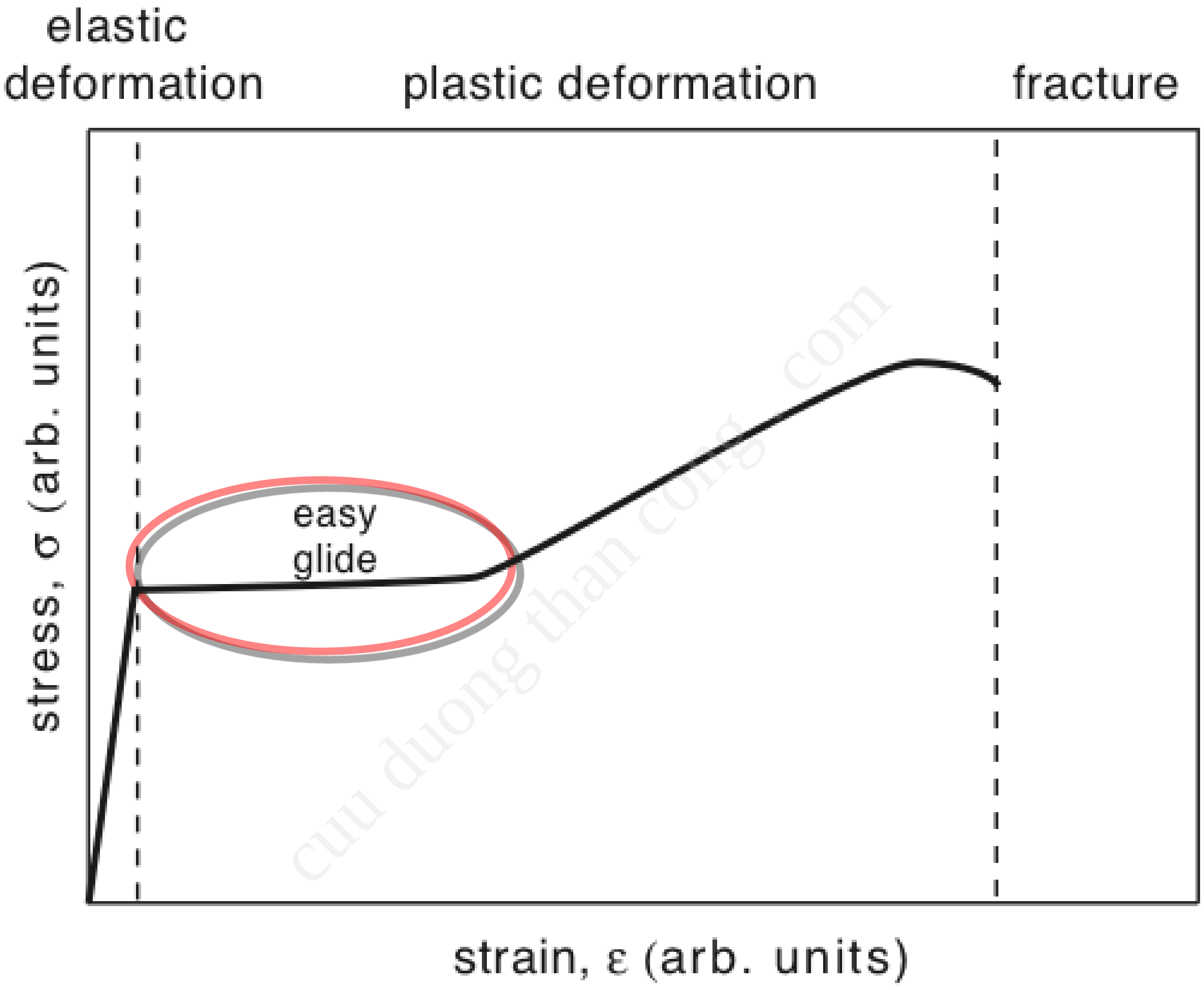
- This tends to increase the elastic limits of alloys.
- Steel with a small carbon content is tougher than pure iron.

The effect of temperature / creep

- The movements of the dislocations are facilitated by higher temperature -> the yield stress decreases.

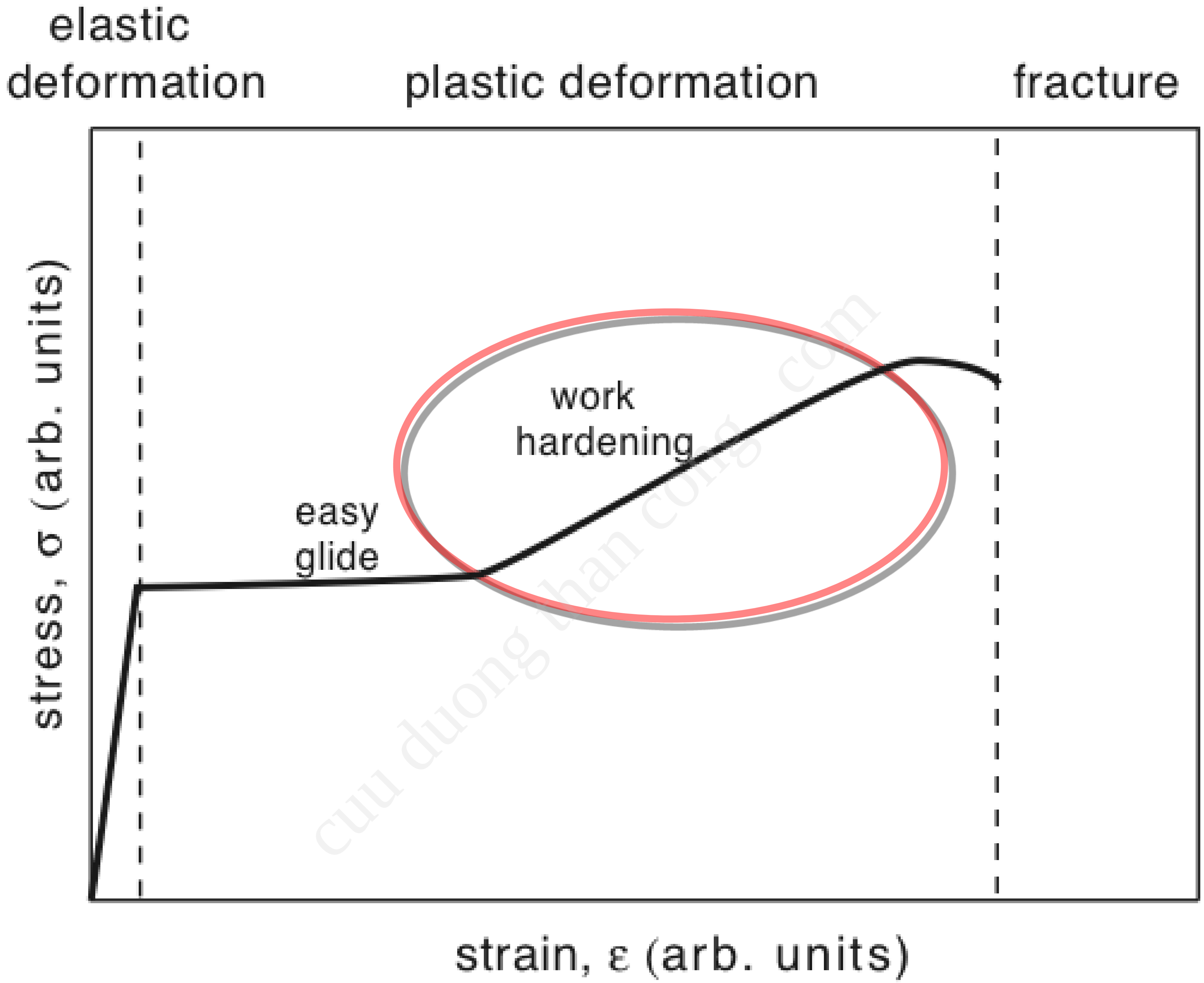
- At high temperature (50% of the melting temperature), the thermally elevated movement of dislocations gives rise to creep (permanent deformation). Can be important because accumulative (in jet engines, walls of fusion reactors....).

Plastic deformation: Easy glide



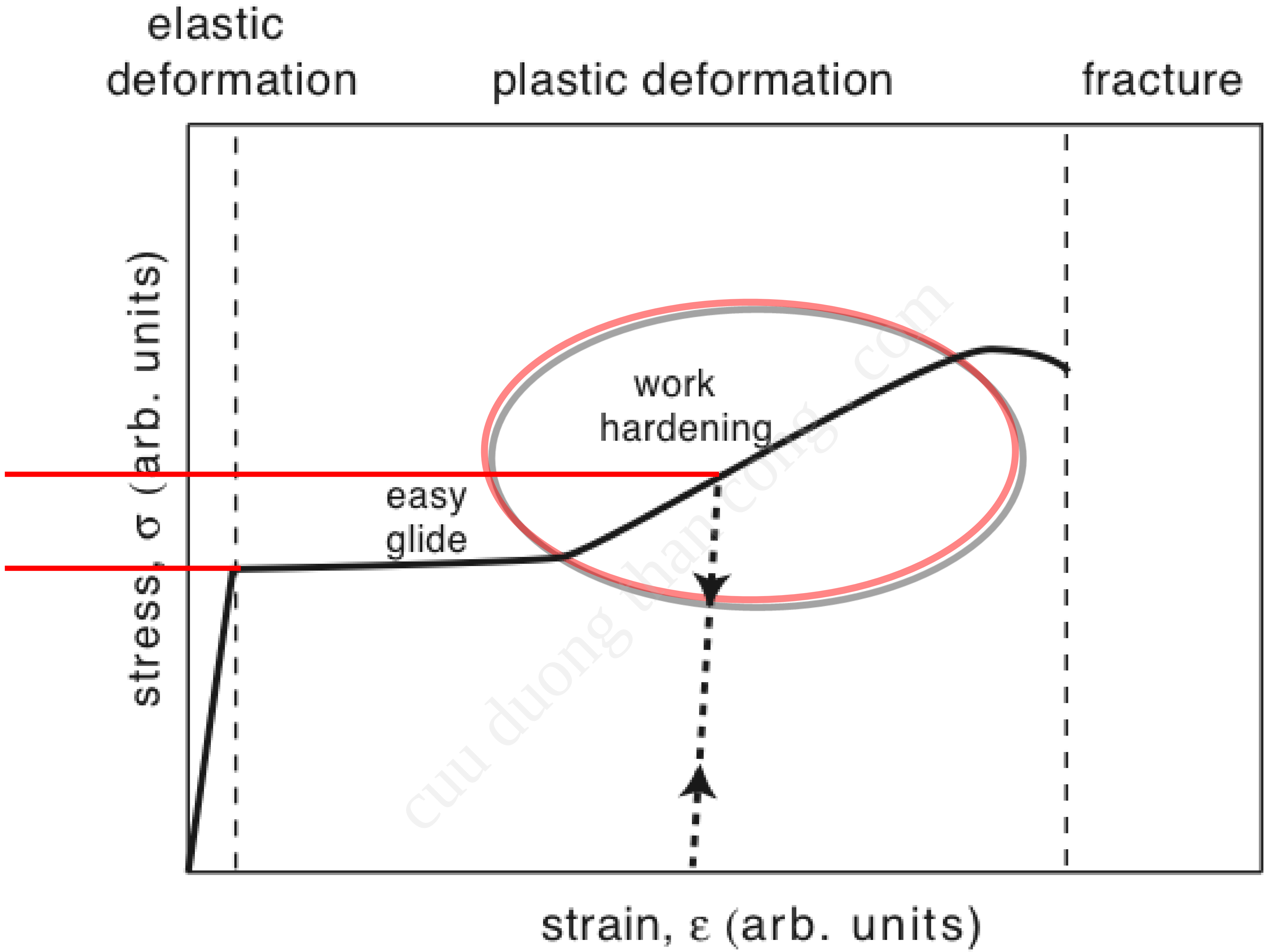
- once the yield stress is overcome, dislocation-assisted glide sets in.
- the stress increases only slightly.

Plastic deformation: work hardening



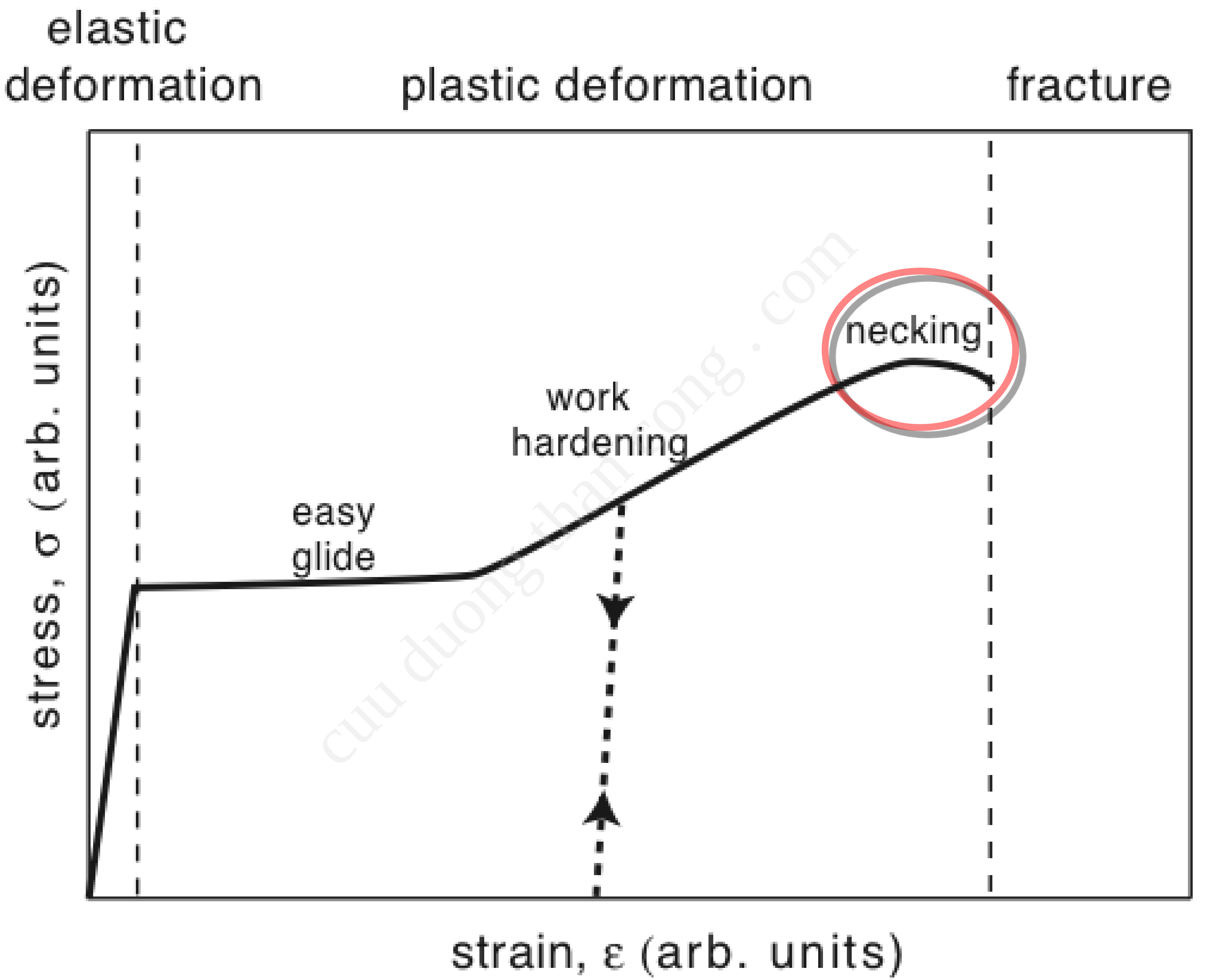
- In the work hardening zone, the stress is increasing again.
- It is as if the easy glide process doesn't work anymore.

Plastic deformation: work hardening



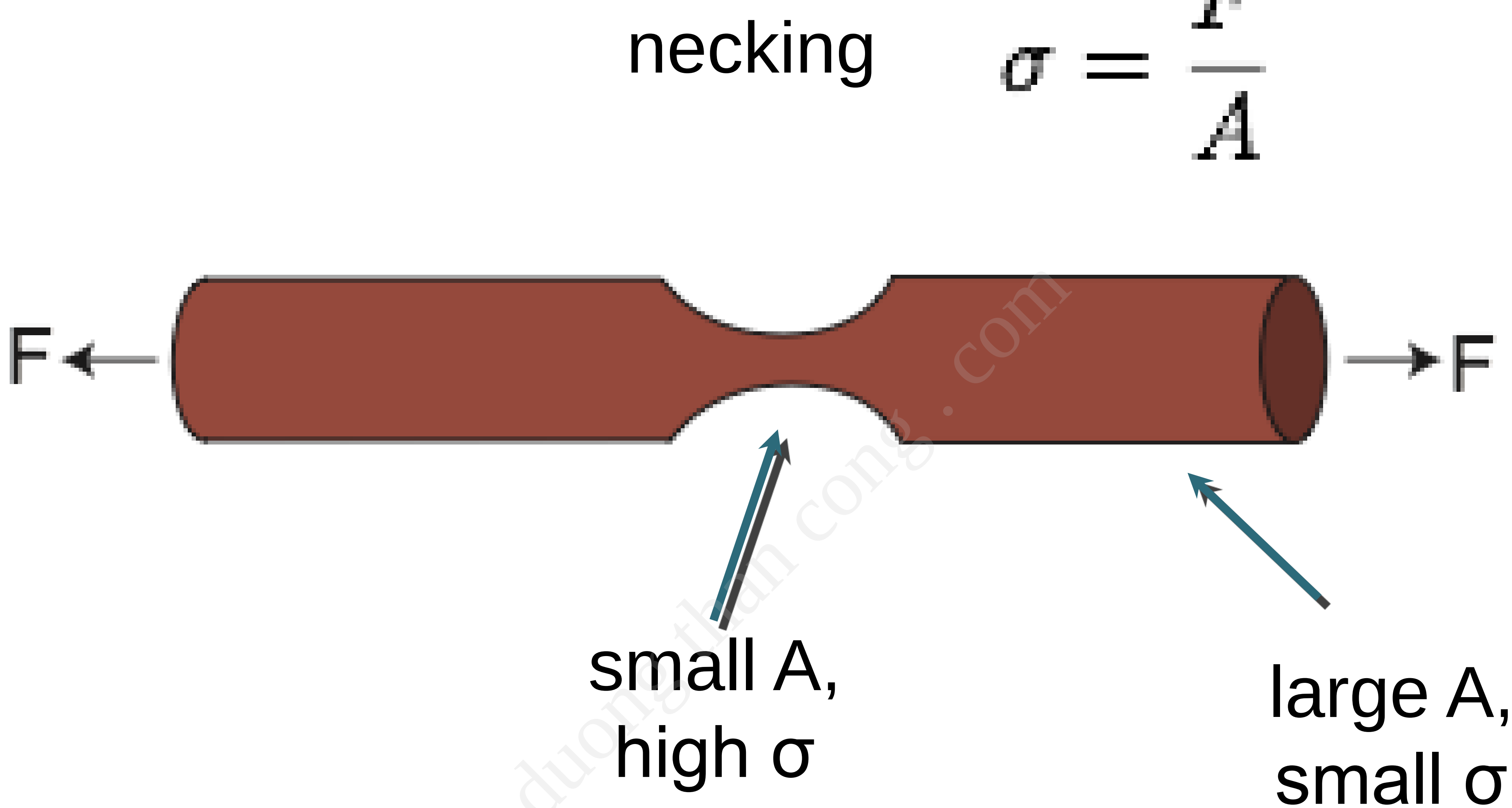
- pre-straining a material can be used to increase the yield stress (the elastic limit).

Plastic deformation: Fracture



- Close to fracture the stress is actually reduced. Why?

Plastic deformation: Fracture

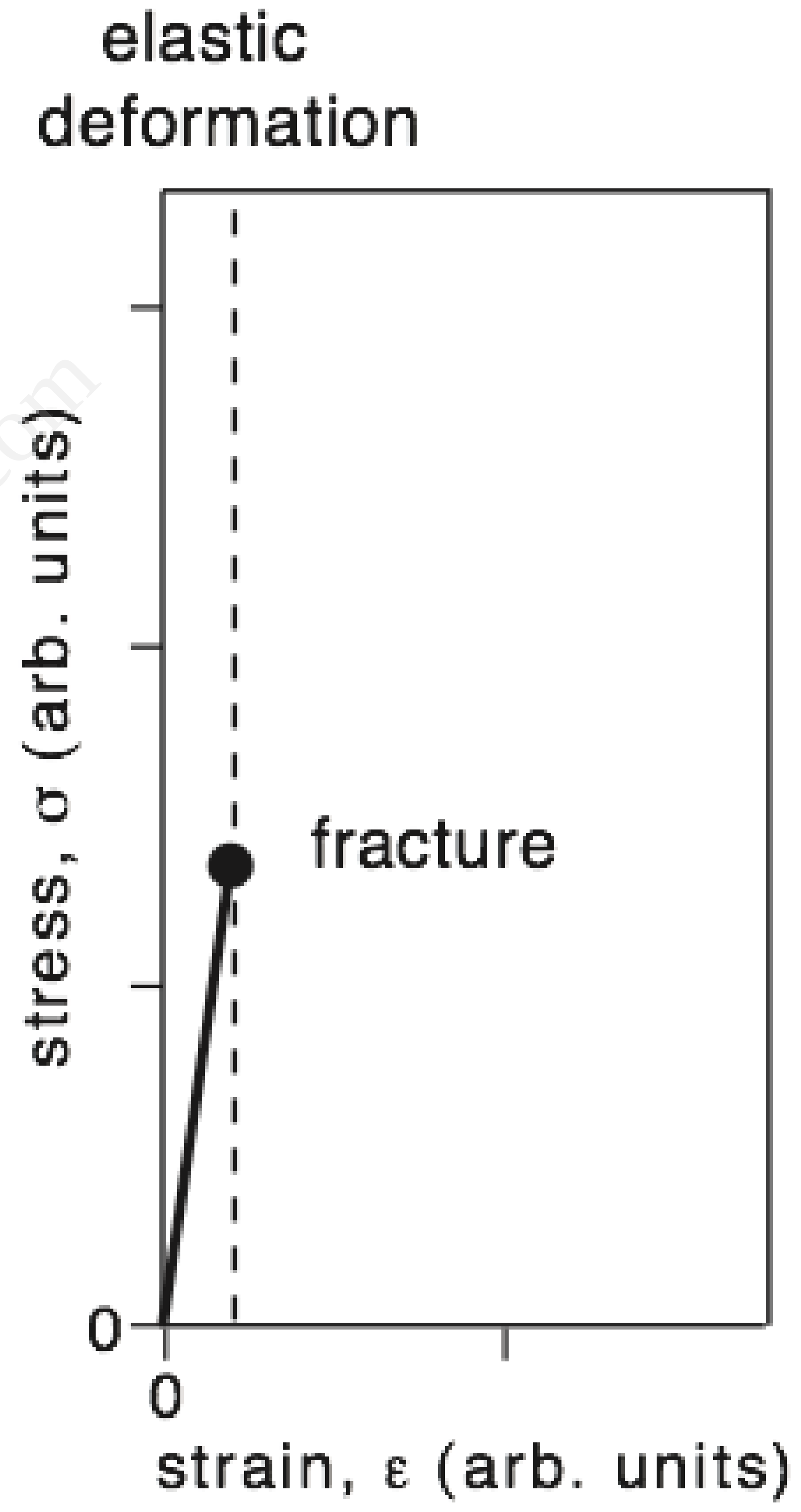


$$\sigma = \frac{F}{A}$$

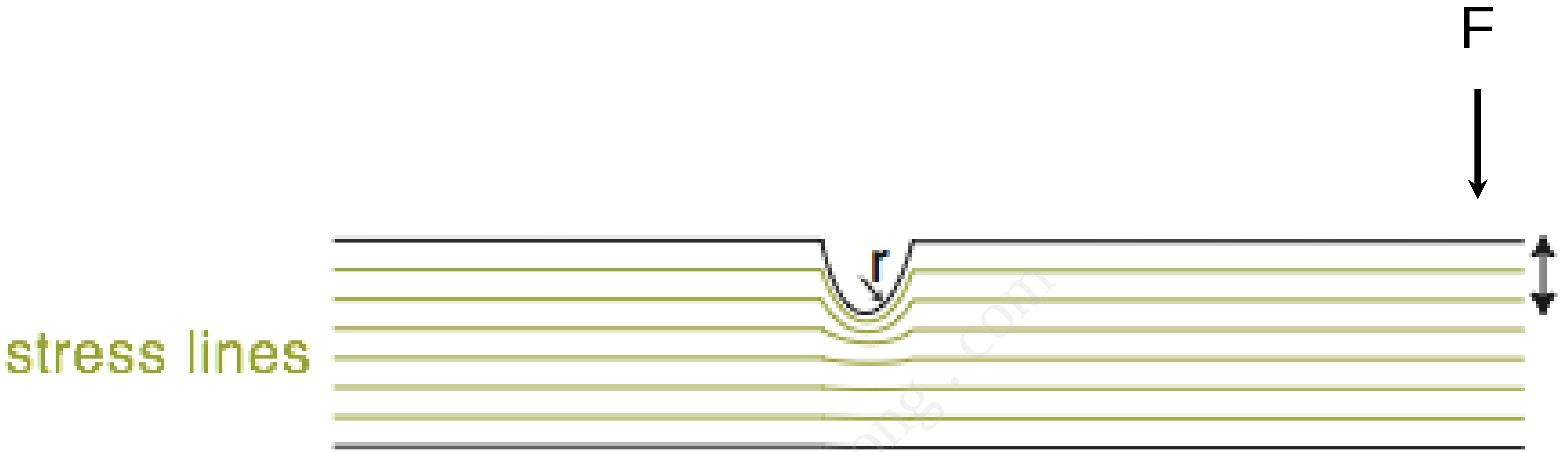
- Higher stress at the neck even if the overall stress is reduced.
- This is also why necks are self-amplifying.

Brittle fracture

- No transition to plastic deformation before fracture.
- Fracture stress should correspond to pulling the atomic layers apart but it is often much smaller. Why?

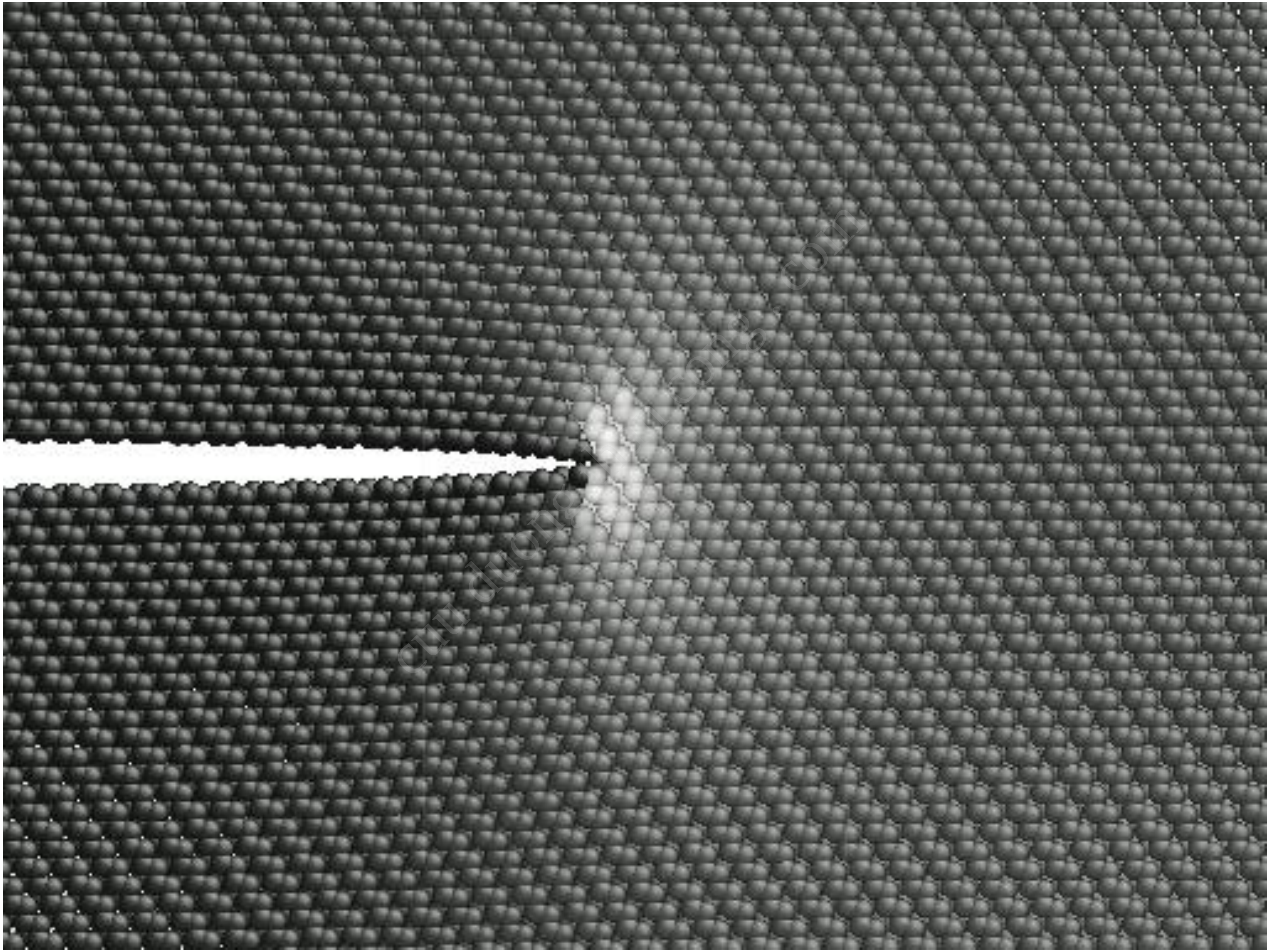


Brittle fracture: crack propagation



- Close to a crack of radius *r* and depth *l*, the stress is locally increased, approximately by a factor $2\sqrt{\frac{l}{r}}$
- This is not the same as necking but a local phenomenon!
- It is self-amplifying and if the stress is high enough, the crack propagates with a very high speed.

Stress close to a very small crack



Brittle or ductile?

- Competition between stress relieve by propagating cracks and stress relieve by moving a dislocation.
- Dislocation movement easy in metals or when molecules can be shifted against each other. Difficult for ionic or strongly covalent materials.
- Dislocation movement strongly temperature dependent but crack propagation not: materials can be ductile at high temperature and brittle at low temperature (for example, glass or steel).

Finally a word of caution...

- We have consider only the basic properties in a very simple way.
- We have looked at simple stress and shear stress. In a more formal treatment these become different aspects of the same thing.
- We only looked at an isotropic solid (ok for metals but not form many other materials, e.g. graphite or wood).

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