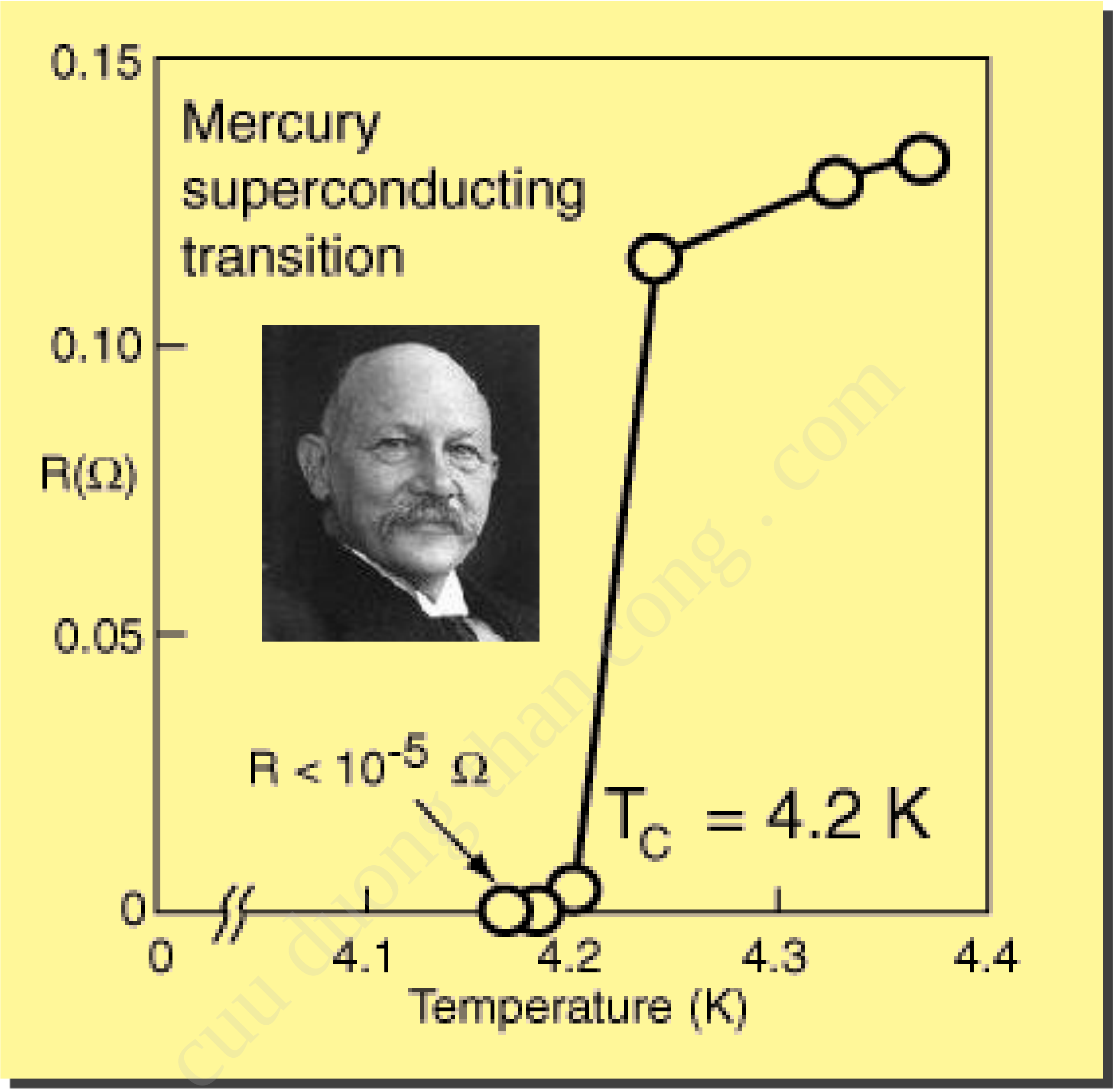
Superconductivity

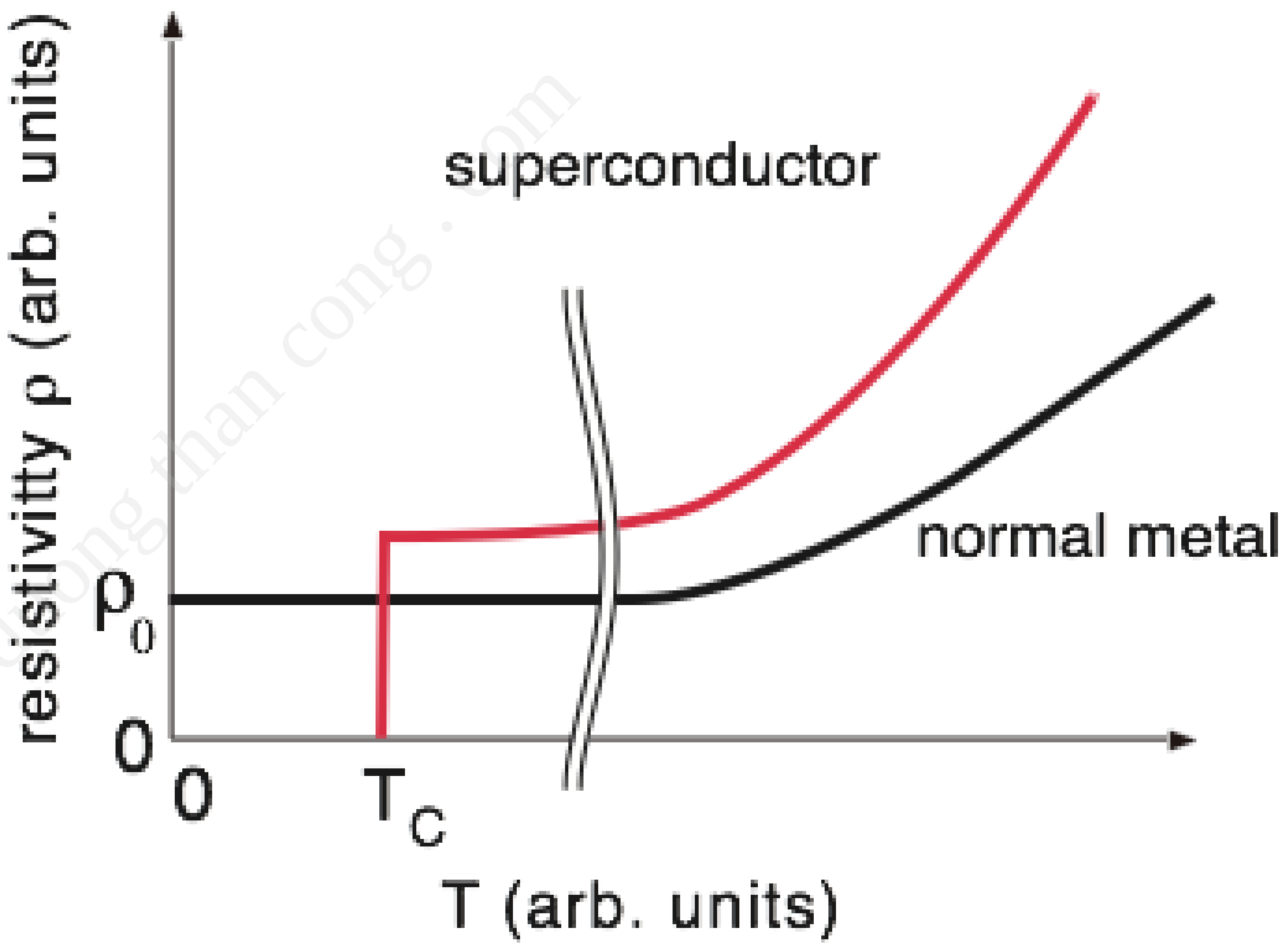
The discovery of superconductivity



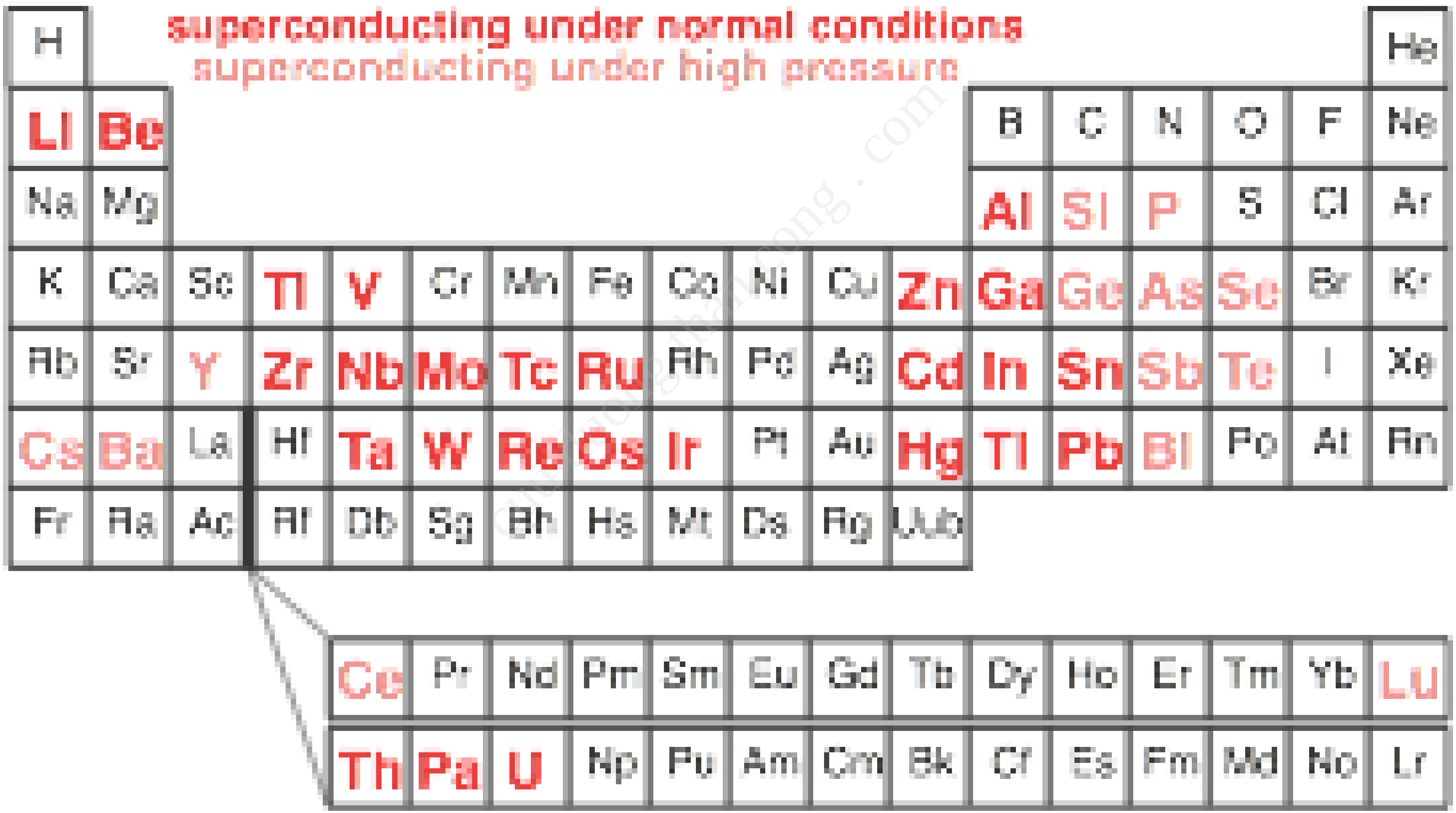
- Resistance of Mercury at low temperatures (Heike Kamerlingh Onnes in 1911, Nobel price in 1913).
- At 4.2 K the resistance drops to an unmeasurably small value. This seems to be a phase transition with a transition range of around 10^{-3} K.

What about the low-temperature conductivity of metals?

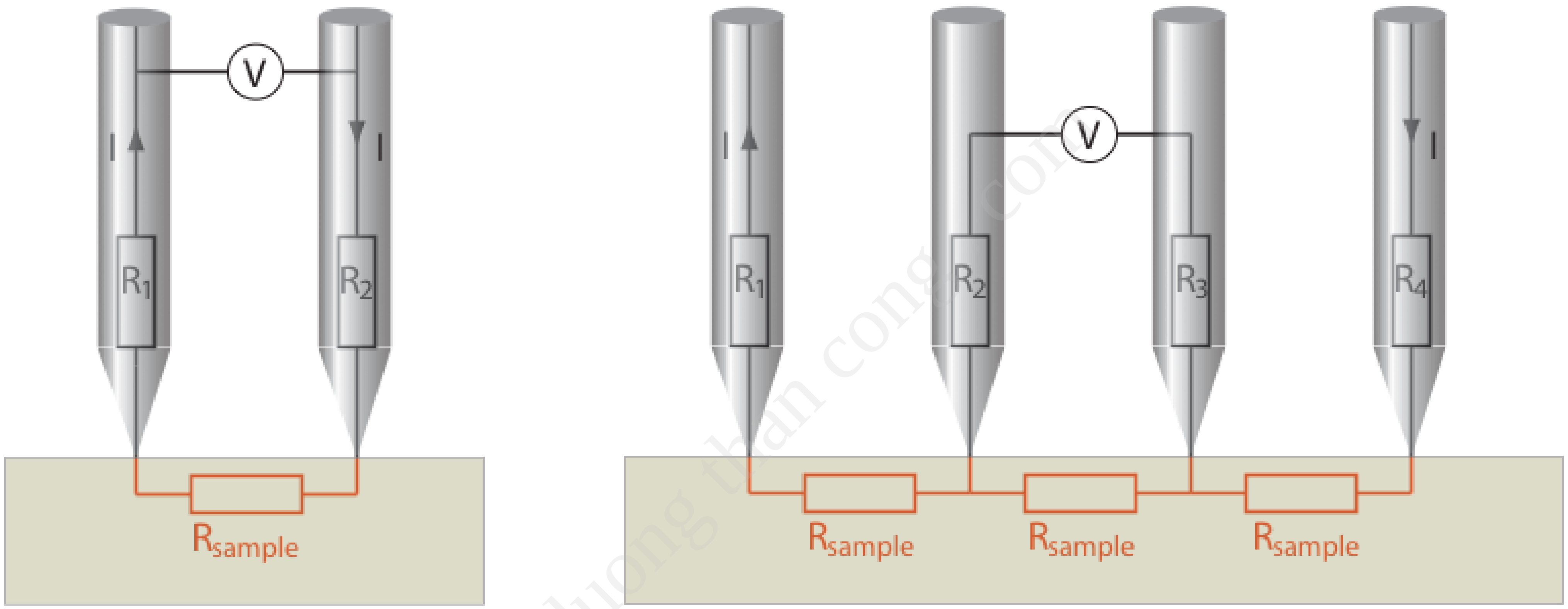
- Metals should have a finite resistance at low temperatures.



Periodic table of superconducting elements



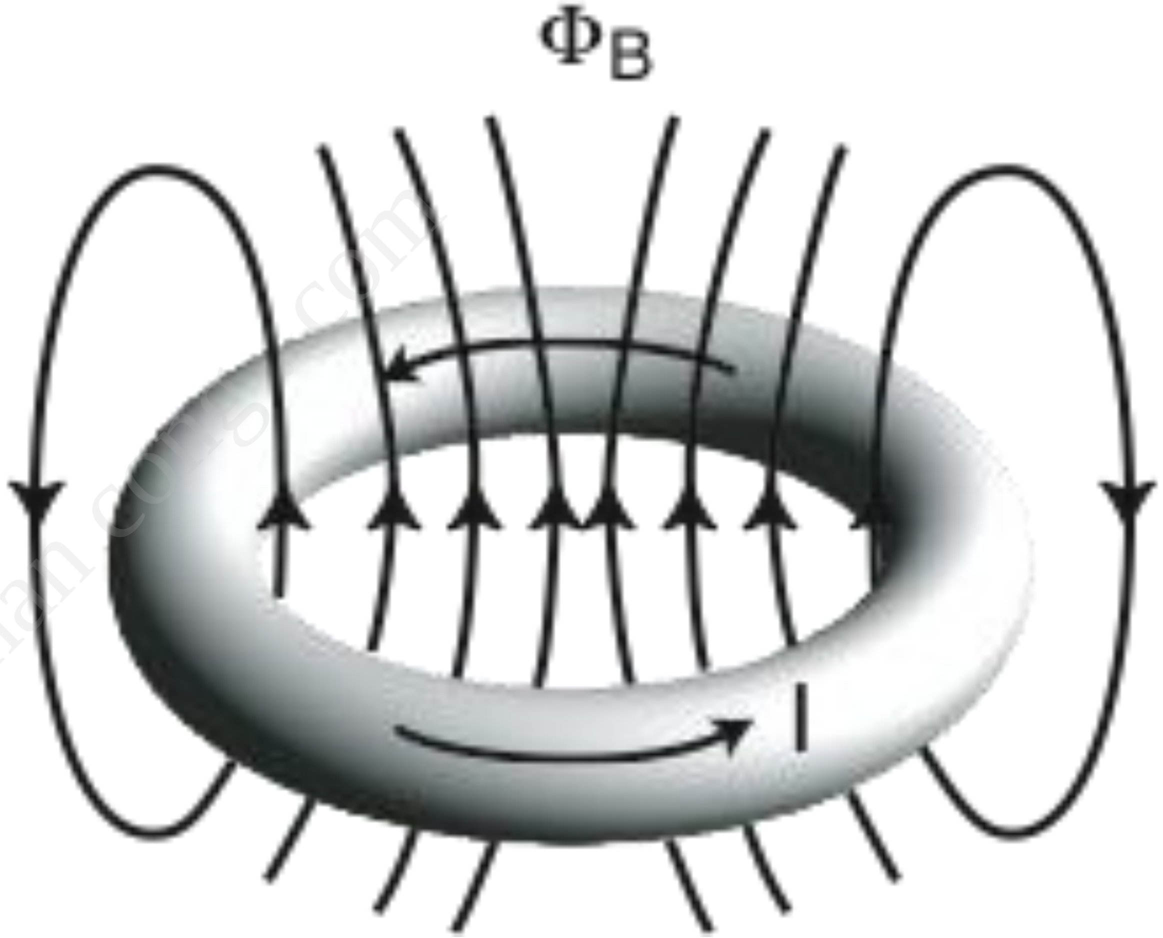
Is the resistivity of a superconductor really zero or just very small?



- A two-point probe measurement will not work because the the contact resistance and the resistance of the wires dominates everything.
- A four point probe measurement can be tried but it does not work either: one is limited by the smallest voltage one can measure.

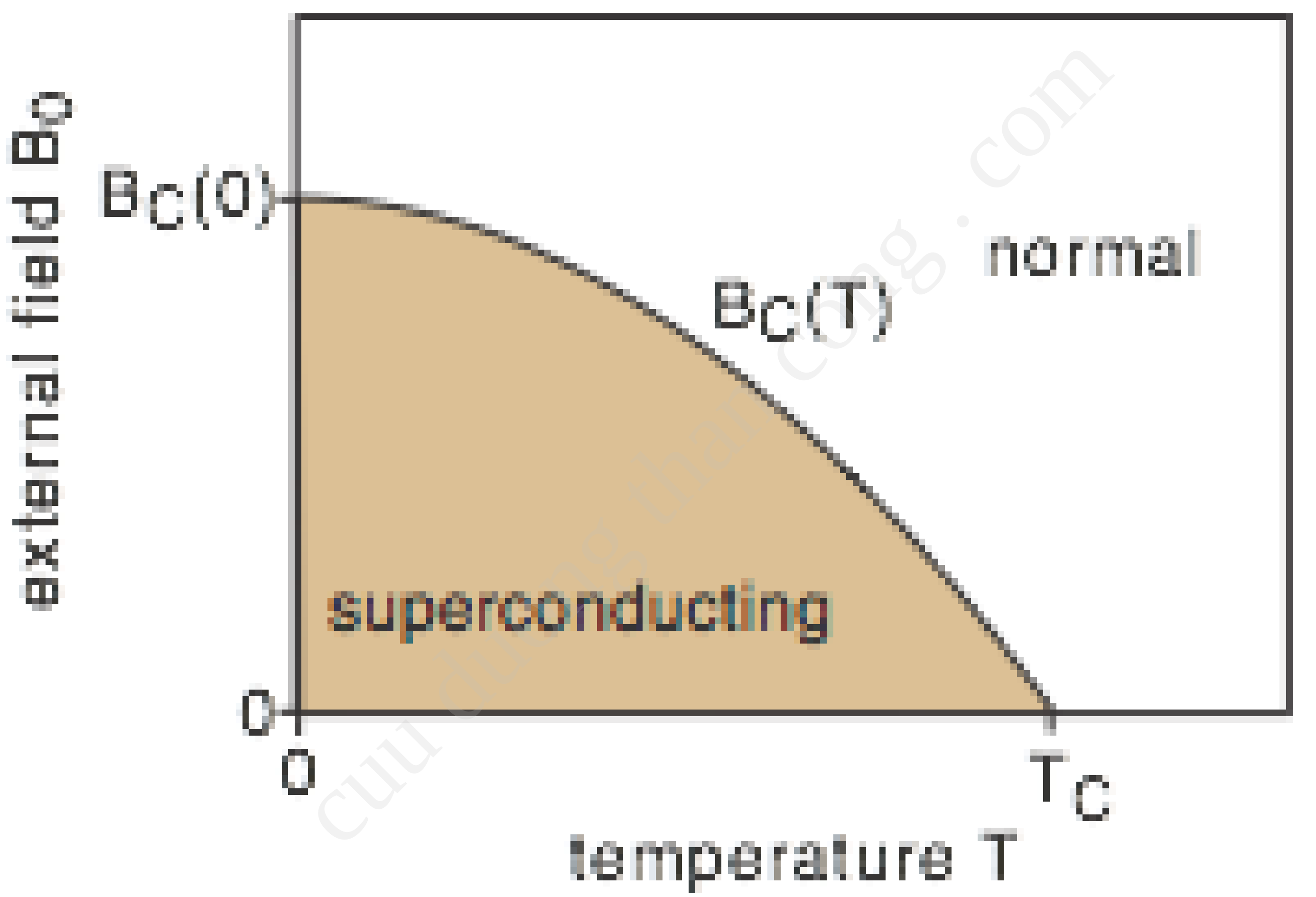
Is the resistivity of a superconductor really zero or just very small?

$$I(t) = I_0 e^{-t/\tau}$$



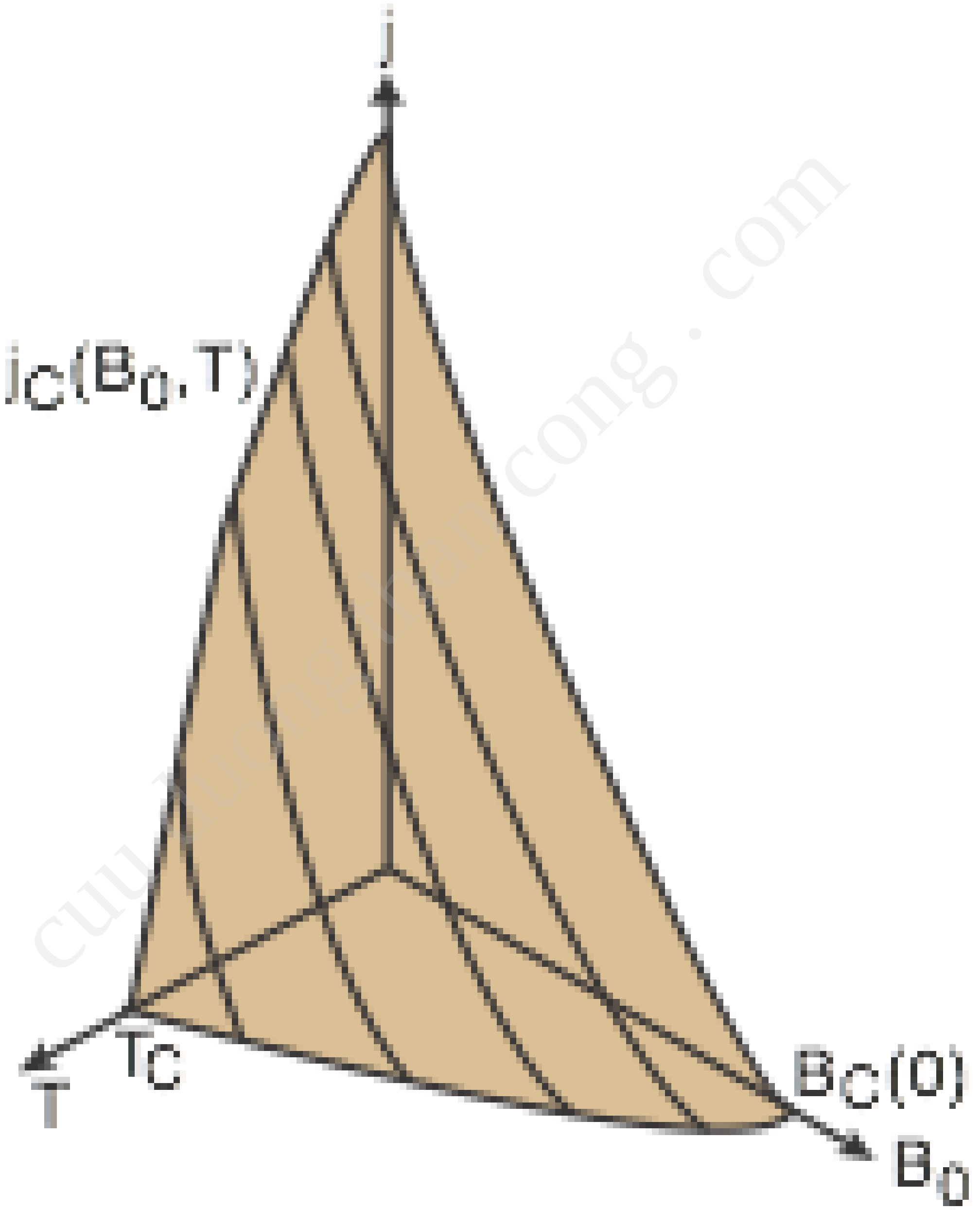
- A super-current can be induced in a ring.
- The decay of the current is given by the relaxation time (10^{-14} s for a normal metal). For a superconductor it should be infinite. Experiments suggest that it is not less than 100.000 years.

Destruction by magnetic fields

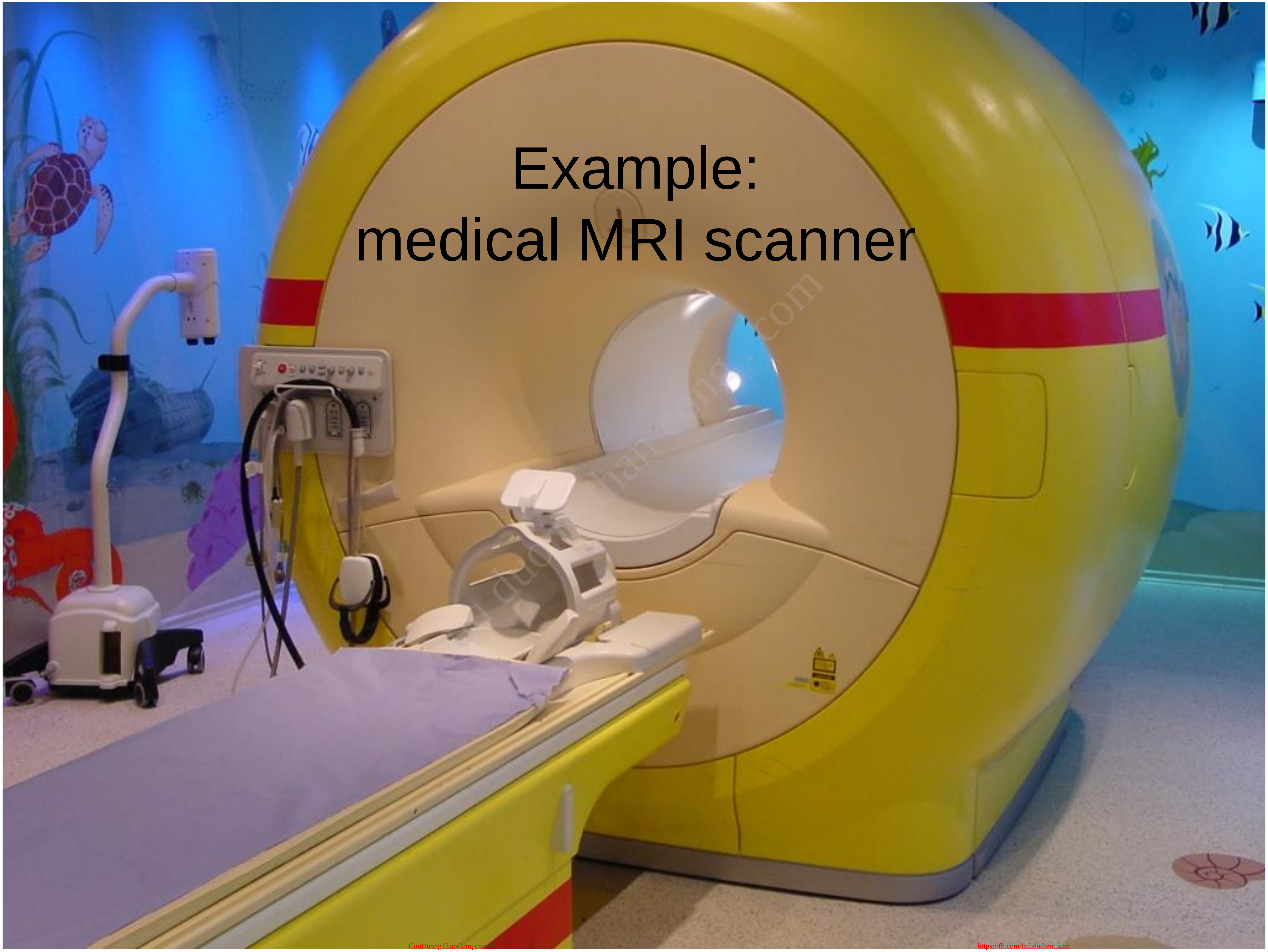


$$B_{0C}(T) = B_{0C}(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

Destruction by magnetic fields / current density



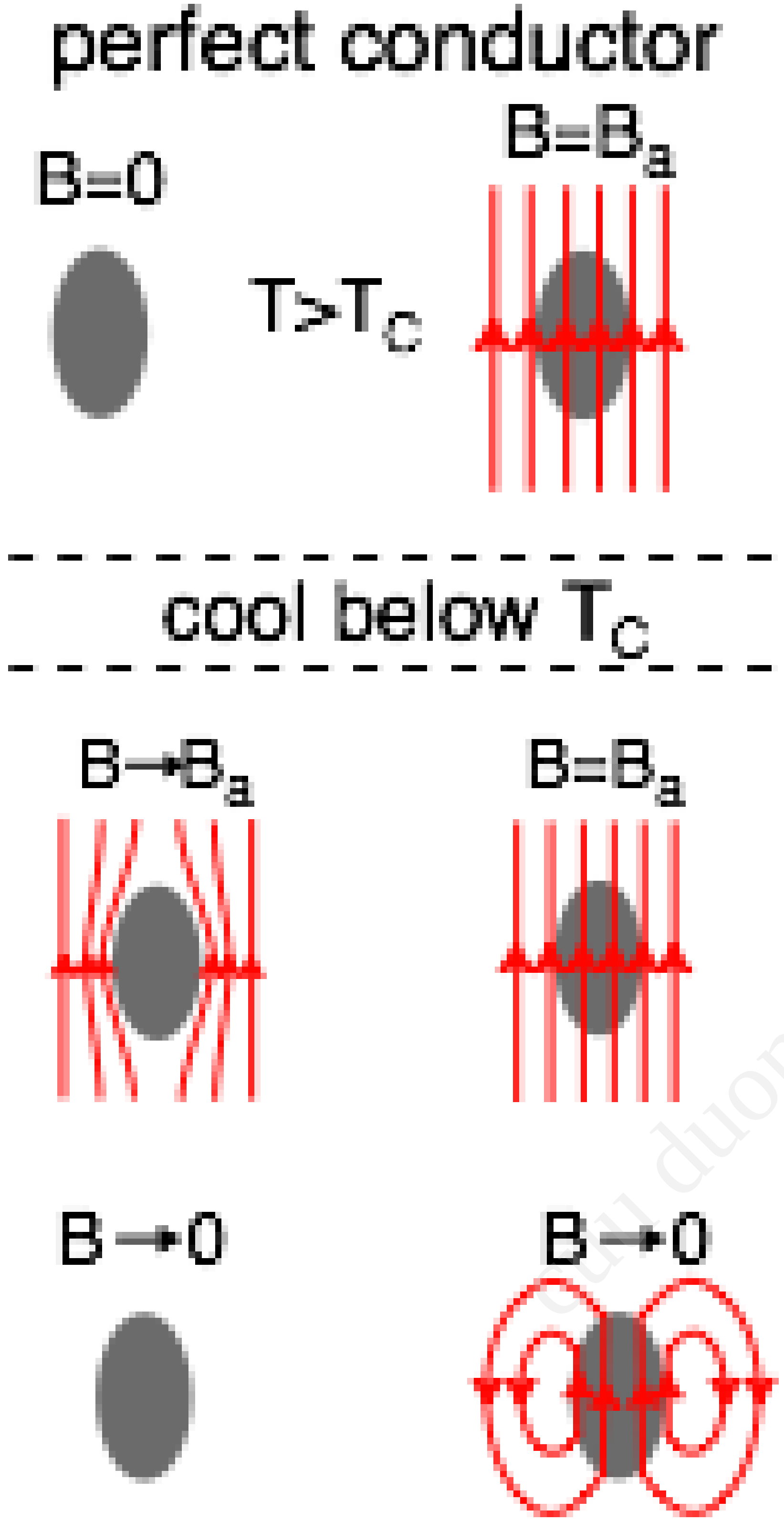
Example:
medical MRI scanner

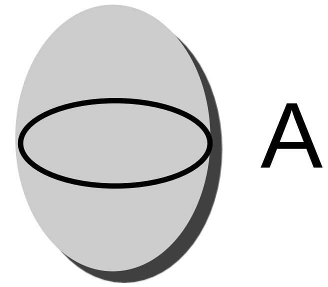


The Meissner effect

- Superconductors (type I) are perfect diamagnets (with χ_m=-1) which always expel the external field.
- This so-called Meissner effect goes beyond mere perfect conductivity.

The Meissner effect

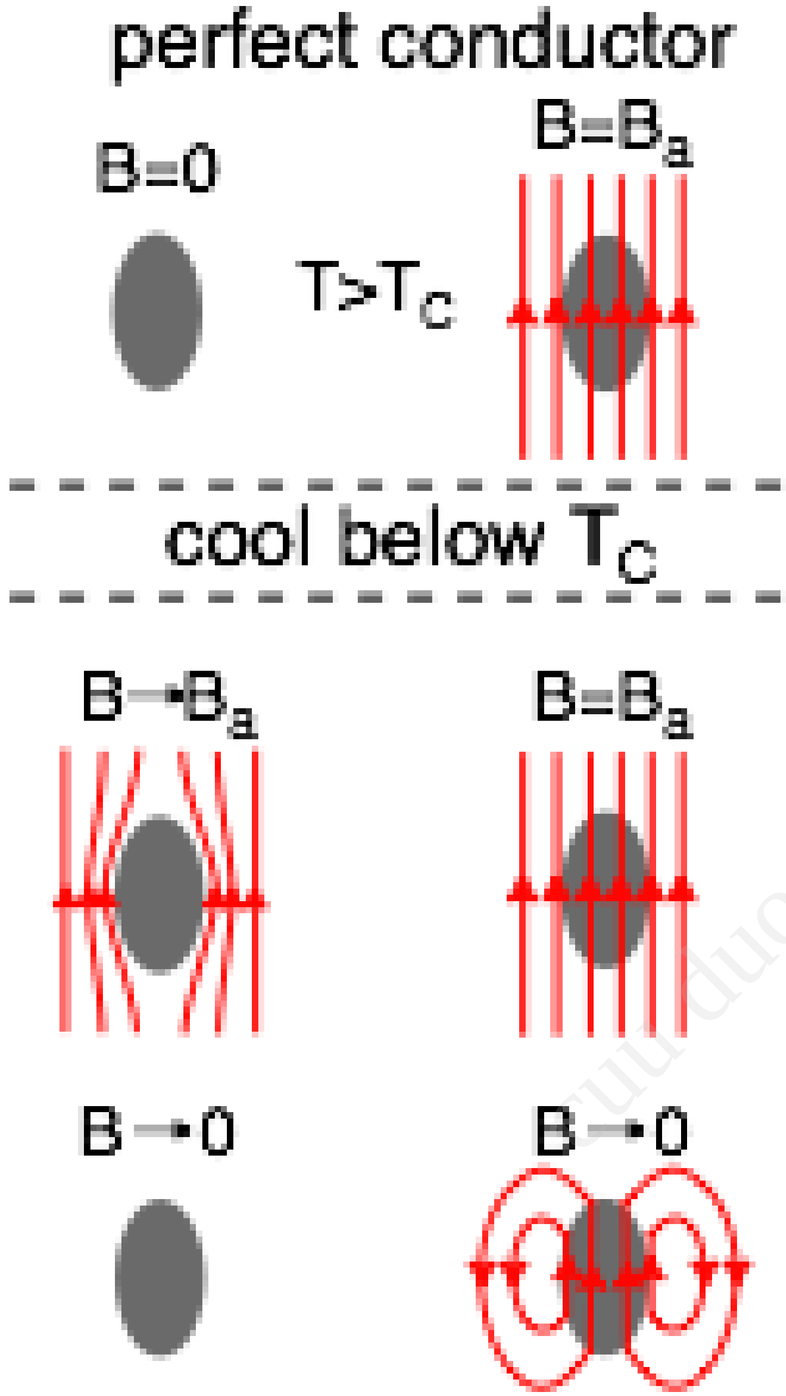




$$\oint \mathcal{E}\, d\mathbf{l} = -\frac{d\Phi_B}{dt}$$

- For a perfect conductor, the magnetic field inside the conductor cannot change. An enclosed field is possible.

The Meissner effect



movement without friction

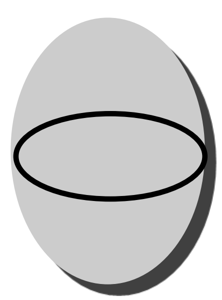
$$m\frac{\partial \mathbf{v}}{\partial t} = q\boldsymbol{\mathcal{E}}$$

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{n_s q^2}{m}\boldsymbol{\mathcal{E}}$$

with Maxwell equation

$$\mathrm{curl}\boldsymbol{\mathcal{E}} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\frac{\partial}{\partial t}\left(\frac{m}{n_s q^2}\mathrm{curl}\mathbf{j} + \mathbf{B}\right) = 0$$

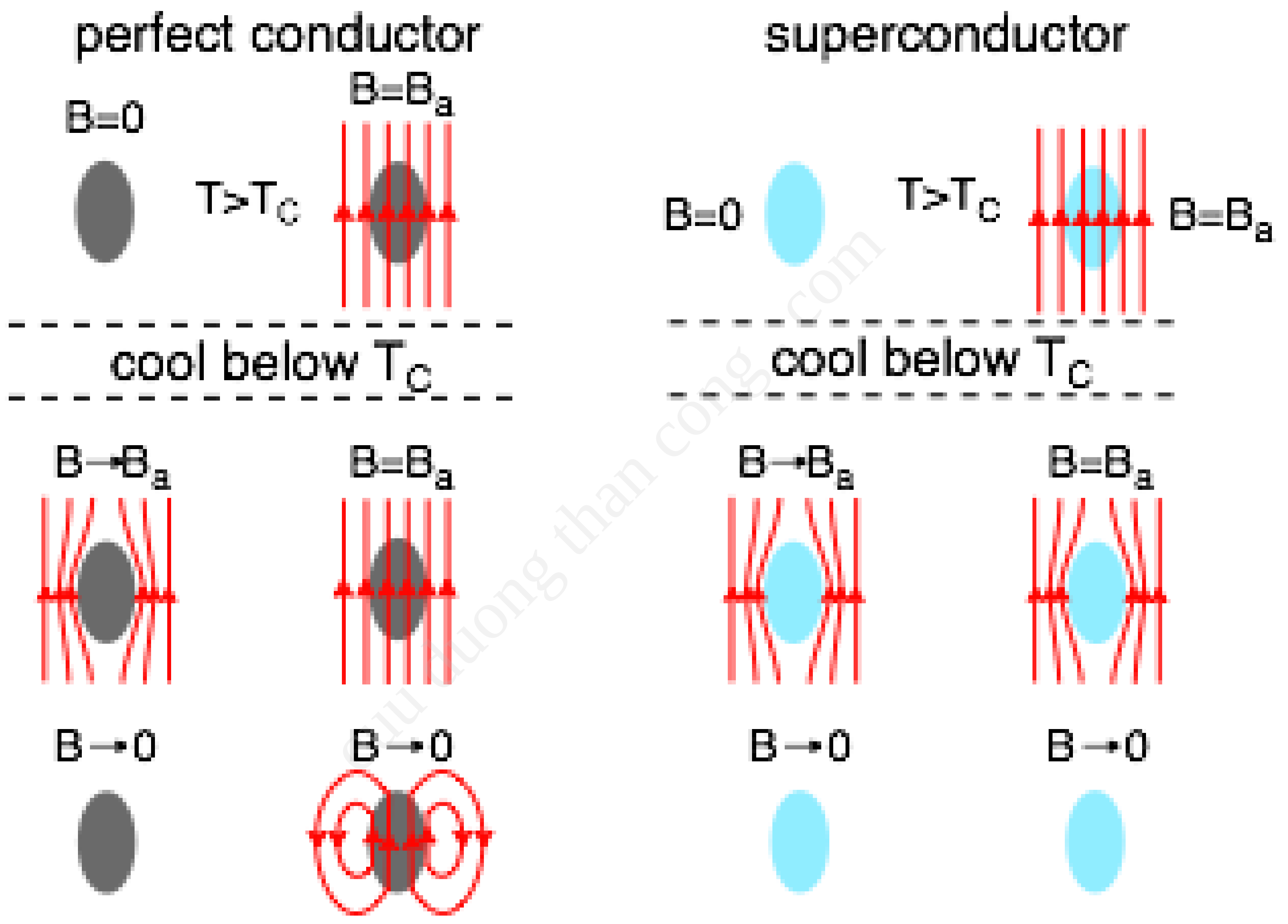


A

mag. flux by j   mag. flux by ext. field

$$\frac{\partial}{\partial t}\left(\int \frac{m}{n_s q^2}\mathrm{curl}\mathbf{j}\,d\mathbf{A} + \int \mathbf{B}\,d\mathbf{A}\right) = \frac{\partial}{\partial t}\left(\oint \frac{m}{n_s q^2}\mathbf{j}\,dl + \int \mathbf{B}\,d\mathbf{A}\right) = 0.$$

The Meissner effect

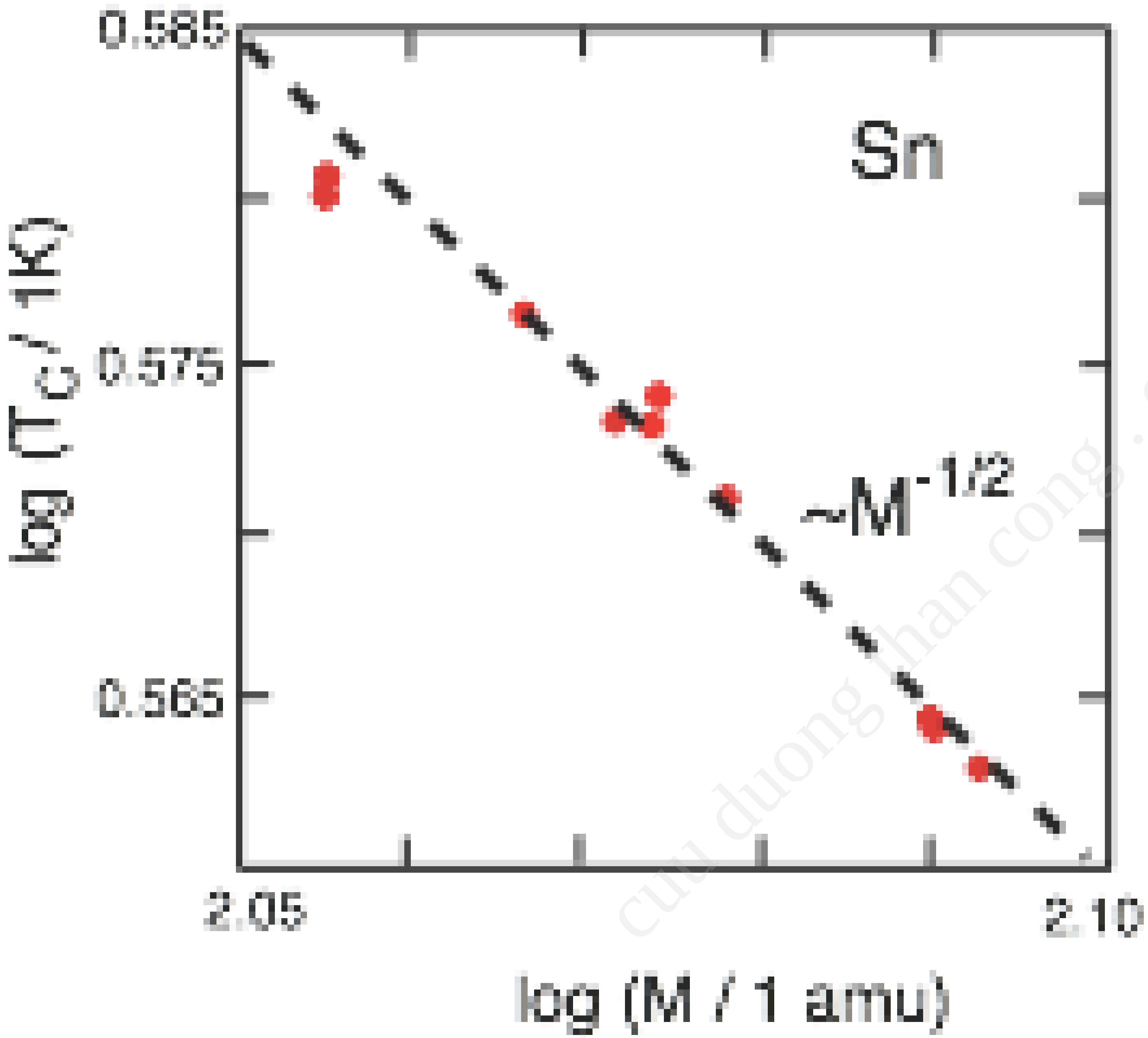


- The essence of the Meissner effect is that the superconductor ALWAYS EXPELS THE MAGNETIC FIELD (ideal diamagnet).

Why levitation?

- The levitation stems from the same reason as ordinary diamagnetic levitation: a combination of gravitational force and a magnetic force due to the inhomogeneous field.
- The only difference is the different strength because of the much higher (negative) susceptibility.

Isotope effect



remember: energy for an harmonic oscillator

$$\omega = \sqrt{\frac{\gamma}{M}}.$$

- Changing the isotope of a material should have a very minor influence on the electronic states.
- The isotope effect suggests that the lattice vibrations are involved in the superconductivity.

London equations (1935)

zero resistance $$\frac{\partial \mathbf{j}}{\partial t} = \frac{n_s q^2}{m_s}\boldsymbol{\mathcal{E}}$$ 1. London equation

$$\frac{\partial}{\partial t}\left(\frac{m_s}{n_s q^2}\mathrm{curl}\mathbf{j} + \mathbf{B}\right) = 0$$

mag. flux by j

mag. flux by ext. field

$$\frac{\partial}{\partial t}\left(\int \frac{m_s}{n_s q^2}\mathrm{curl}\mathbf{j}d\mathbf{A} + \int \mathbf{B}d\mathbf{A}\right) = \frac{\partial}{\partial t}\left(\oint \frac{m_s}{n_s q^2}\mathbf{j}dl + \int \mathbf{B}d\mathbf{A}\right) = 0$$

Meissner effect $$\frac{m_s}{n_s q^2}\mathrm{curl}\mathbf{j} + \mathbf{B} = 0$$ 2. London equation

$$\oint \frac{m_s}{n_s q^2}\mathbf{j}dl + \int \mathbf{B}d\mathbf{A} = 0$$

Field penetration

now we have

$$\frac{m_s}{n_s q^2}\mathrm{curl}\mathbf{j} + \mathbf{B} = 0$$

and the Maxwell equation (inside the superconductor)

$$\mathrm{curl}\mathbf{B} = \mu_0 \mathbf{j}$$

combining this gives

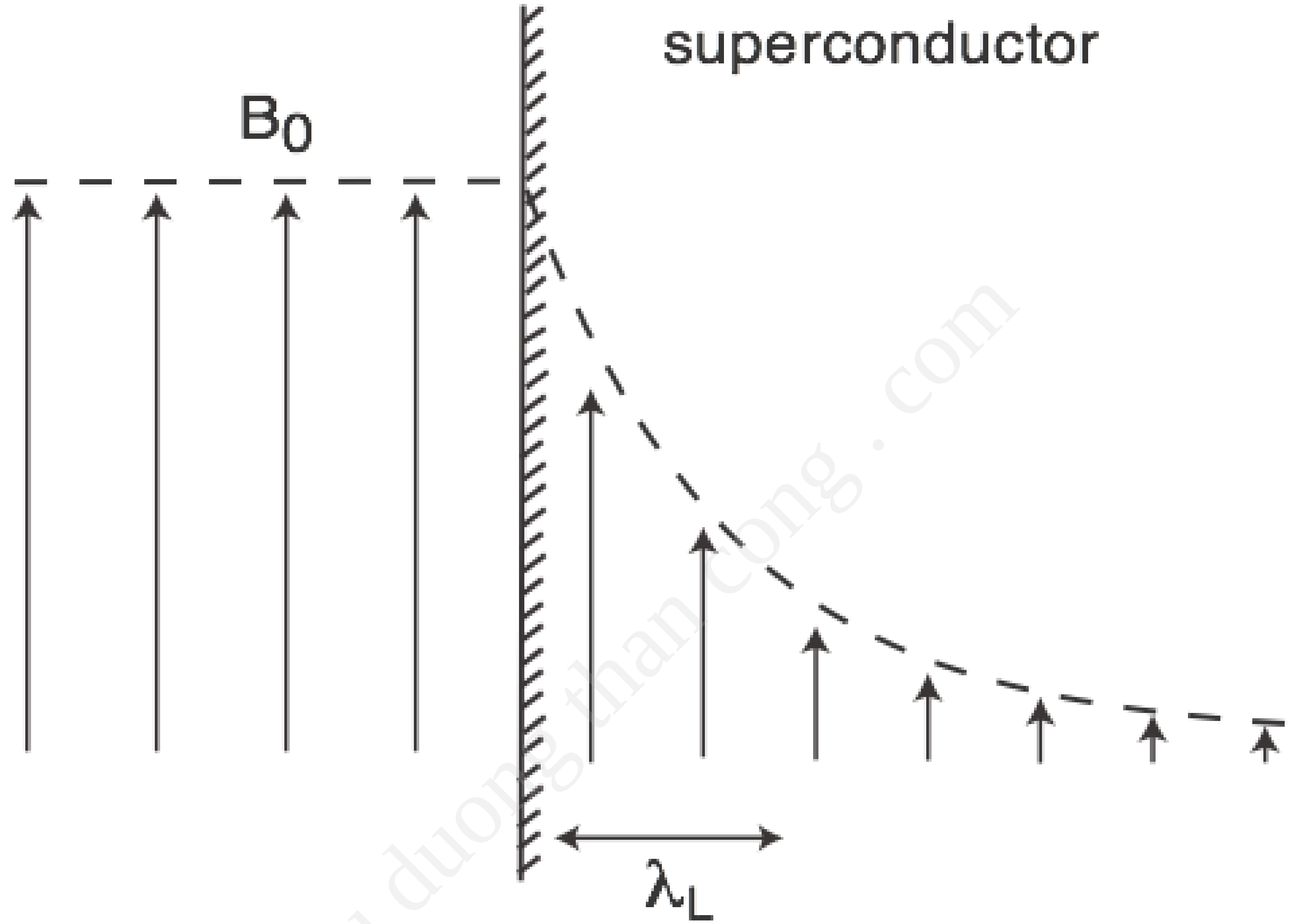
$$\mathrm{curlcurl}\mathbf{B} = \mu_0 \mathrm{curl}\mathbf{j} = -\frac{\mu_0 n_s q^2}{m}\mathbf{B}$$

$$\mathrm{curlcurl}\mathbf{B} = \mathrm{graddiv}\mathbf{B} - \Delta\mathbf{B} = -\frac{\mu_0 n_s q^2}{m}\mathbf{B}$$

$$\Delta\mathbf{B} = \frac{\mu_0 n_s q^2}{m}\mathbf{B}$$

This permits only an exponentially damped B field in the superconductor (exercise)

Field penetration



$$\triangle \mathbf{B} = \frac{\mu_0 n_s q^2}{m} \mathbf{B}$$

$$\triangle \mathbf{j} = \frac{\mu_0 n_s q^2}{m} \mathbf{j}$$

$$\lambda_L = \sqrt{m / \mu_0 n_s q^2}$$

The decay length is in the order of 1000 Å.

Ginzburg-Landau theory (1950)

order parameter

$$\Psi(\mathbf{r}) = \Psi_0(\mathbf{r})e^{i\phi(\mathbf{r})}$$

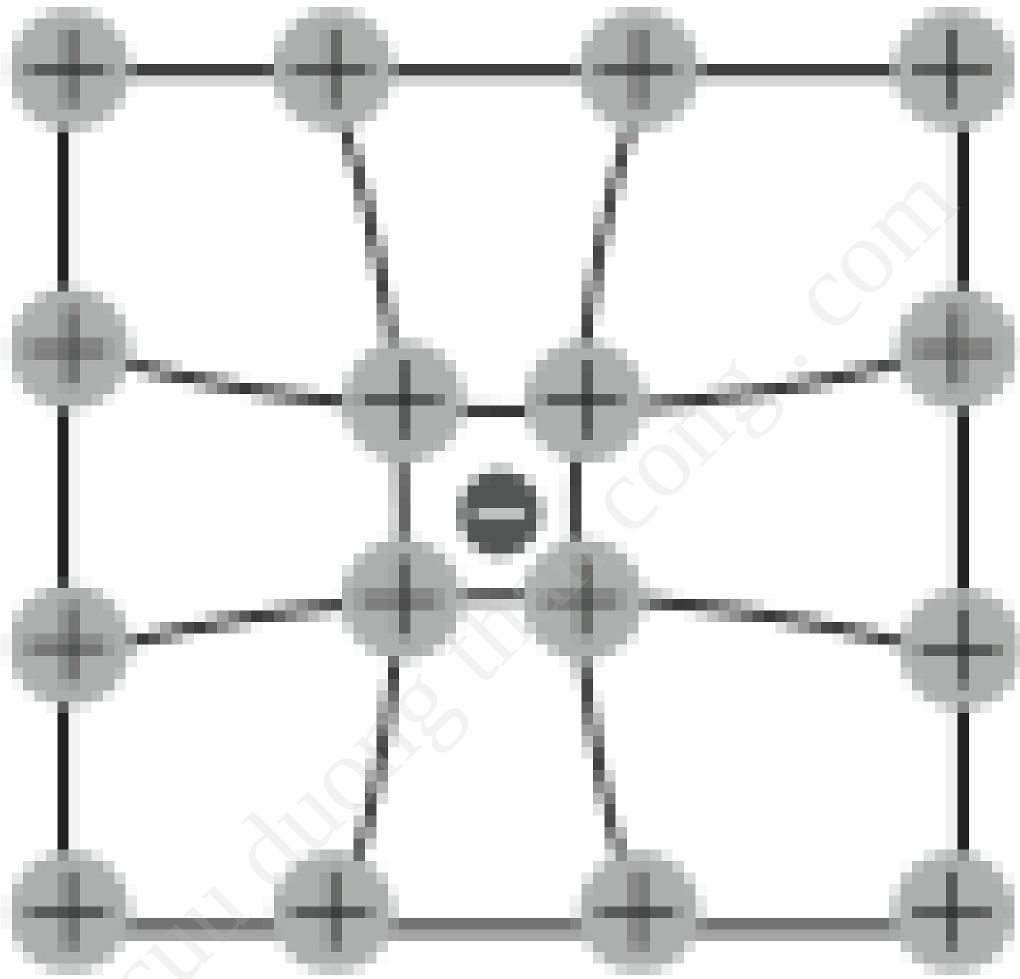
$$n_s = |\Psi^*\Psi| = \Psi_0^2$$

coherence length ξ
(length over which $\Psi(\mathbf{r})$
can change appreciably)

Key-ideas of the BCS theory of 1957 (Bardeen, Cooper, Schrieffer)

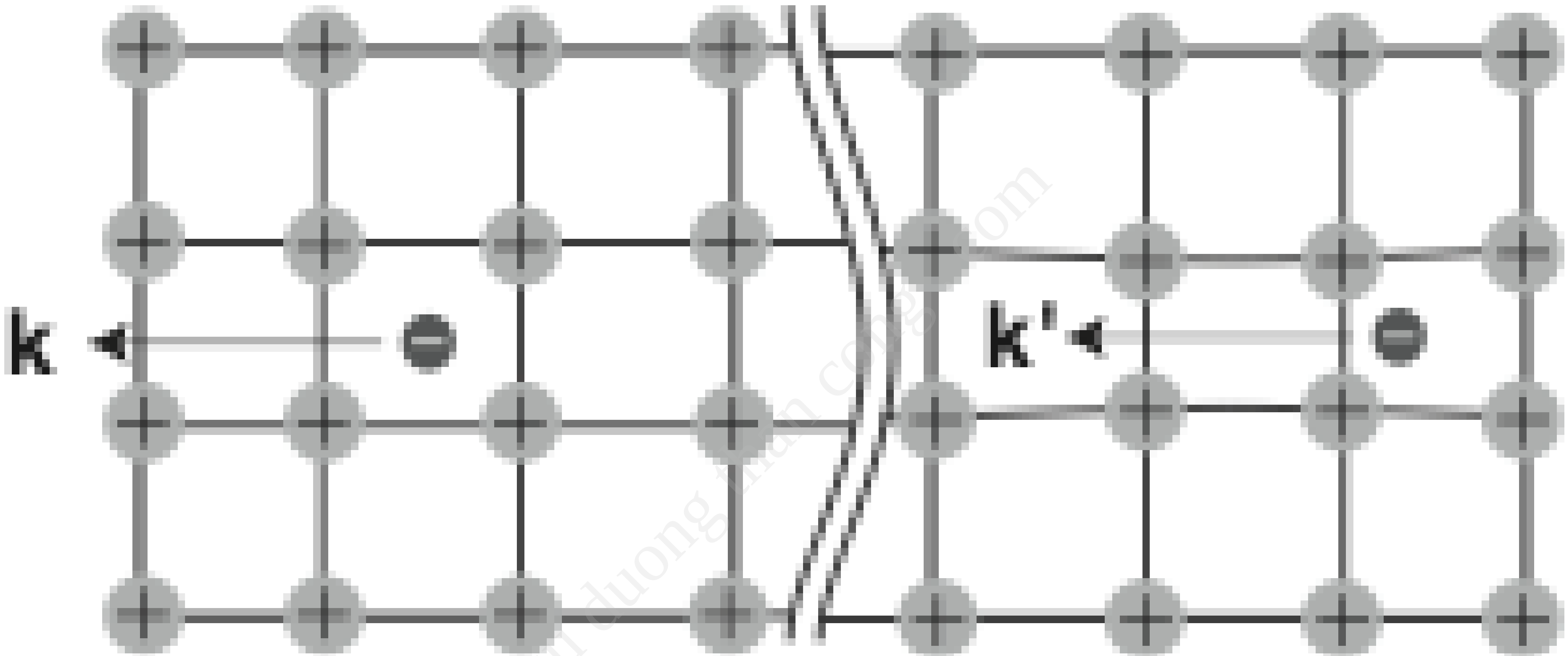
- The interaction of the electrons with lattice vibrations (phonons) must be important (isotope effect, high transition temperature for some metals which are poor conductors at room temperature).
- The electronic ground state of a metal at 0 K is unstable if one permits a net attractive interaction between the electrons, no matter how small.
- The electron-phonon interaction leads to a new ground state of Bosonic electron pairs (Cooper pairs) which shows all the desired properties.

The electron-phonon interaction/ Cooper pairs



- Polarization of the lattice by one electron leads to an attractive potential for another electron.

The electron-phonon interaction / Cooper pairs



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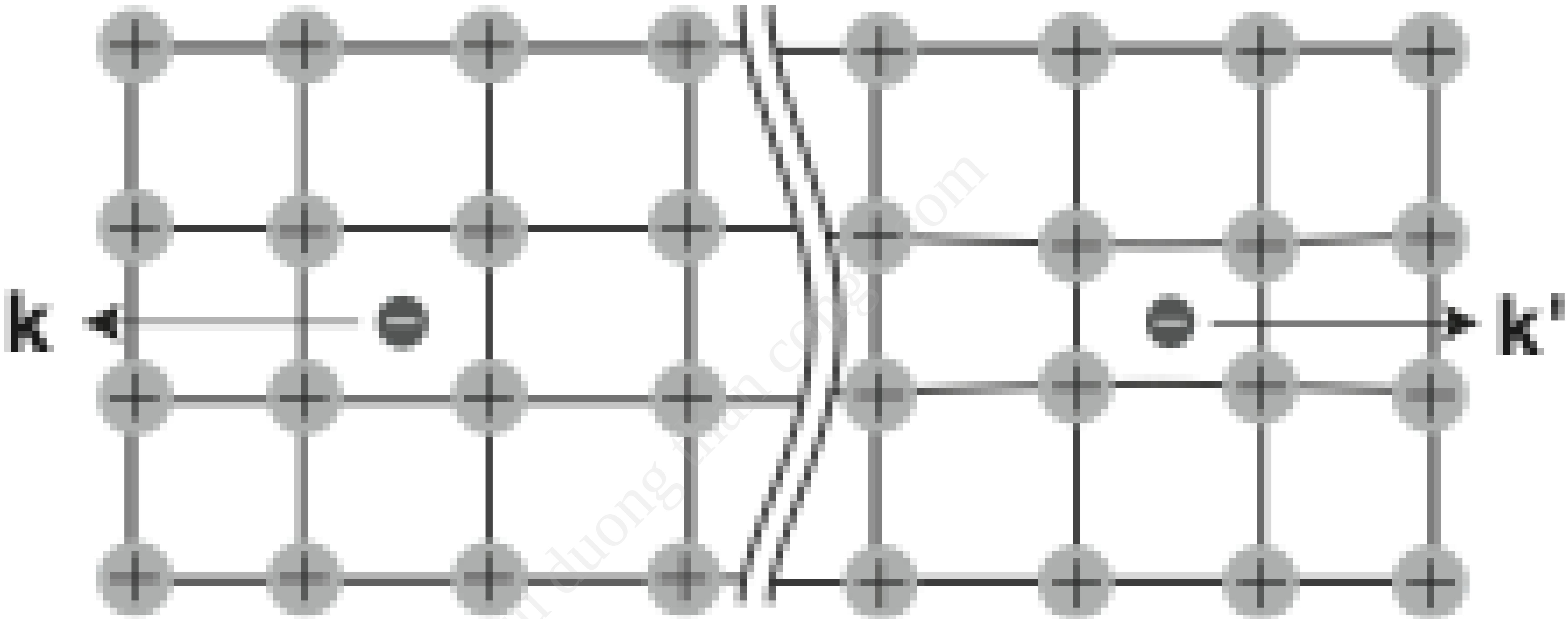
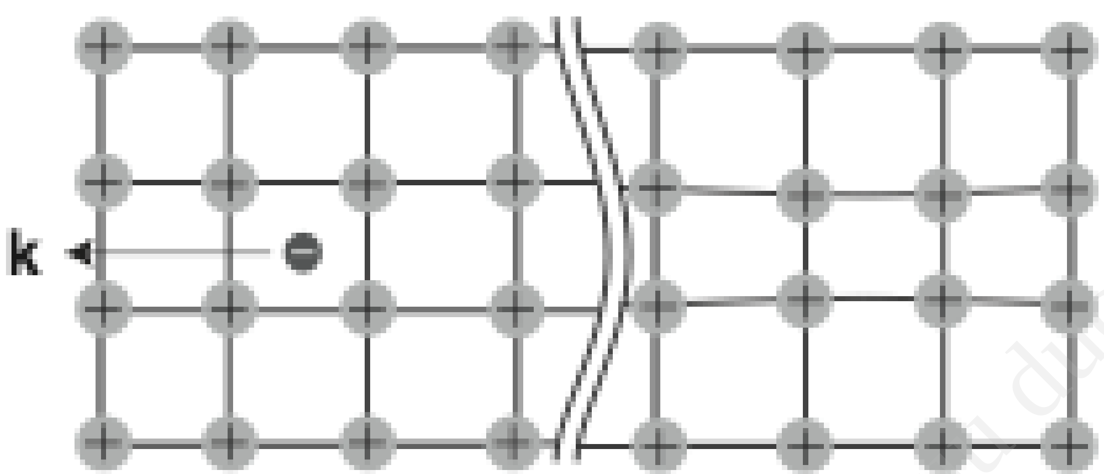


- Polarization of the lattice by one electron leads to an attractive potential for another electron.

What is the “distance” between the electrons forming Cooper pairs?



The lattice atoms move much slower than the electrons. The polarization is retarded

Establishing the polarization takes

$$\frac{2\pi}{\omega_D} \approx 10^{-13}\mathrm{s}$$

the electrons move with

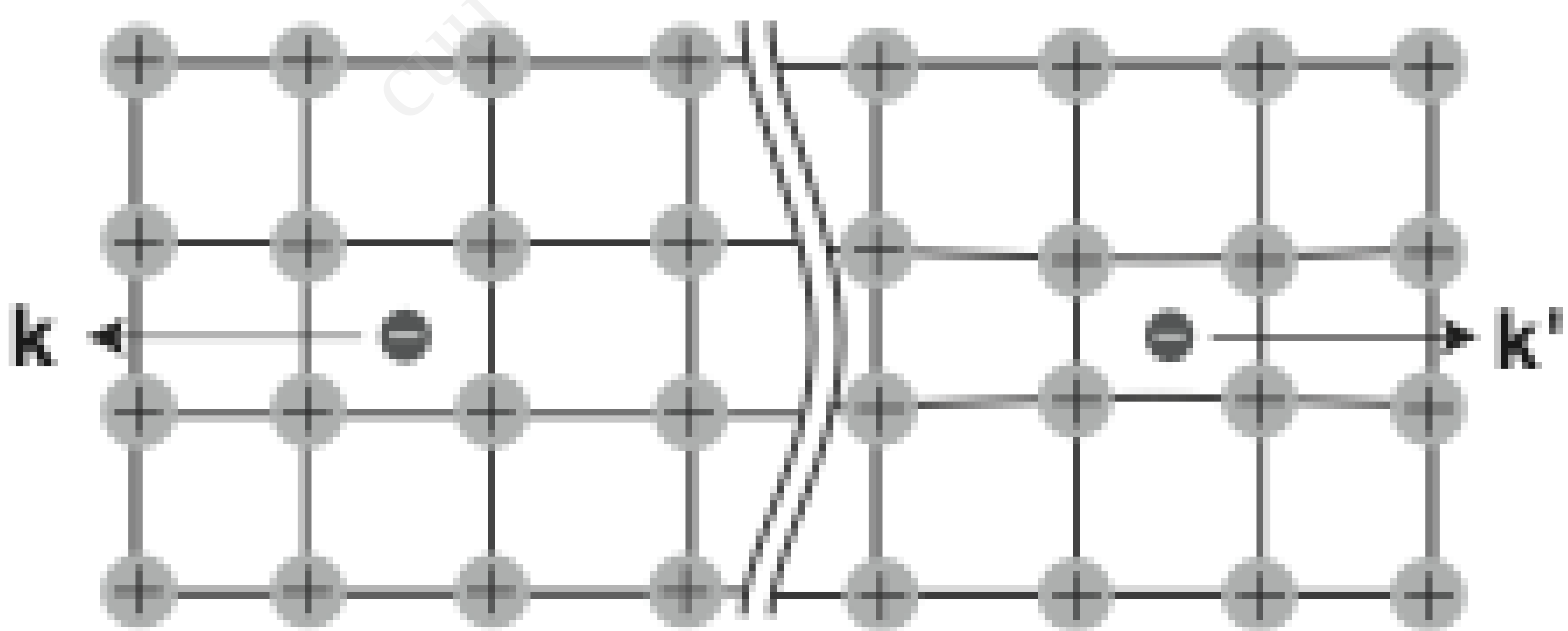
$$v_F \approx 10^6 \mathrm{ms}^{-1}$$

so distance between the electrons forming the Cooper pair is around

$$10^{-7}\mathrm{m}$$

Coherence of the superconducting state

- The electrons paired in this way are called Cooper pairs. They have opposite k vectors (a total k=0) and (in most cases) opposite spins, such that S=0.
- Cooper pairs behave like Bosons and can condense all in a single ground state with a macroscopic wave function. This leads to an energy gain with a size depending on how many Cooper pairs already are in the ground state.
- Cooper pairs are example of “quasiparticles” appearing in solids (like holes or electrons with an effective mass or phonons)



Ginzburg-Landau theory (1950)

order parameter

$$\Psi(\mathbf{r}) = \Psi_0(\mathbf{r})e^{i\phi(\mathbf{r})}$$

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coherence length ξ
(length over which $\Psi(\mathbf{r})$
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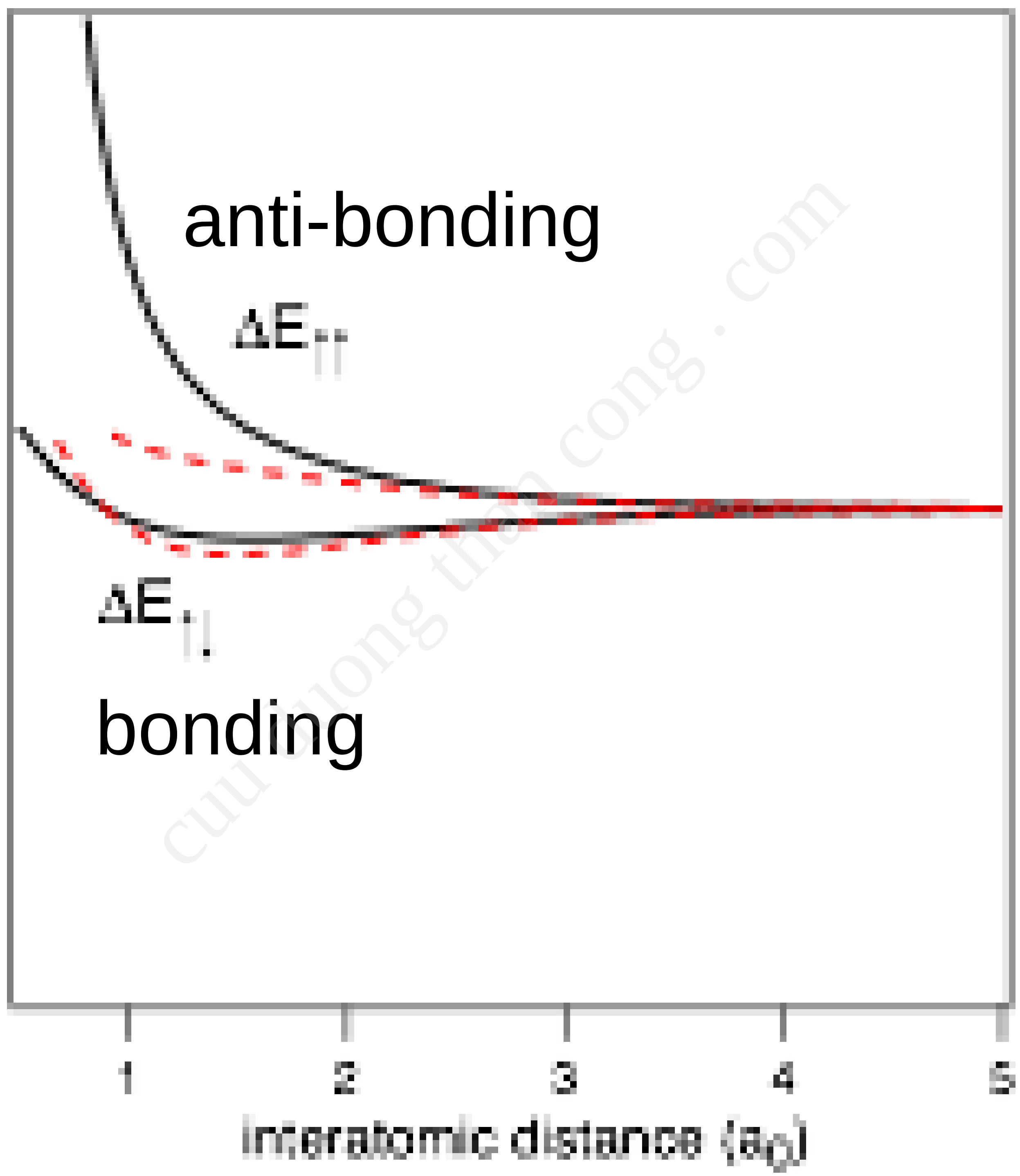
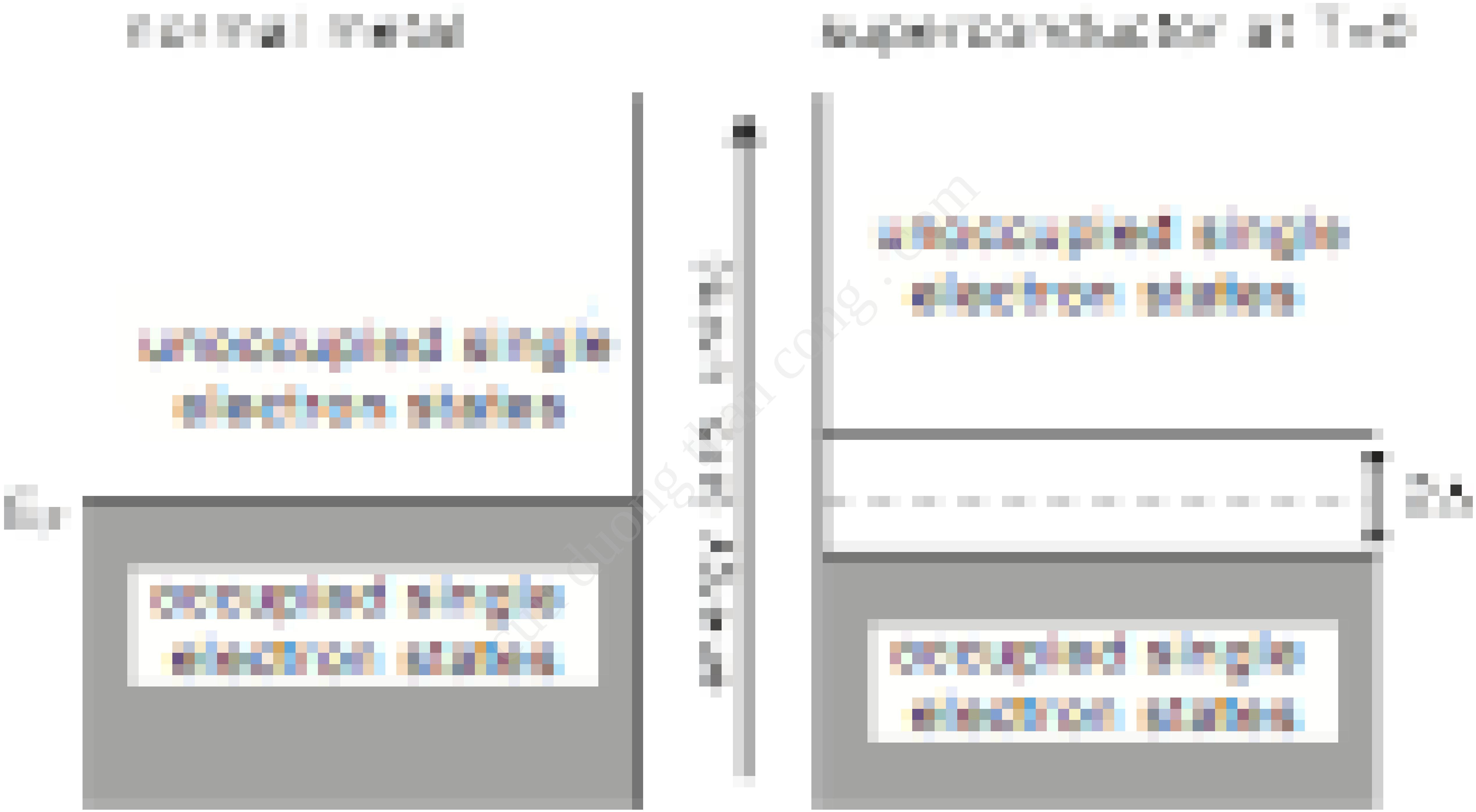
The BCS ground state

$$\Delta = 3.53 k_B T_C$$

Hydrogen molecule



The BCS ground state



$$\Delta = 3.53 k_B T_C$$

The BCS ground state

transition temperature

$$T_C = 1.13\Theta_D \exp \frac{-1}{g(E_F)V}$$

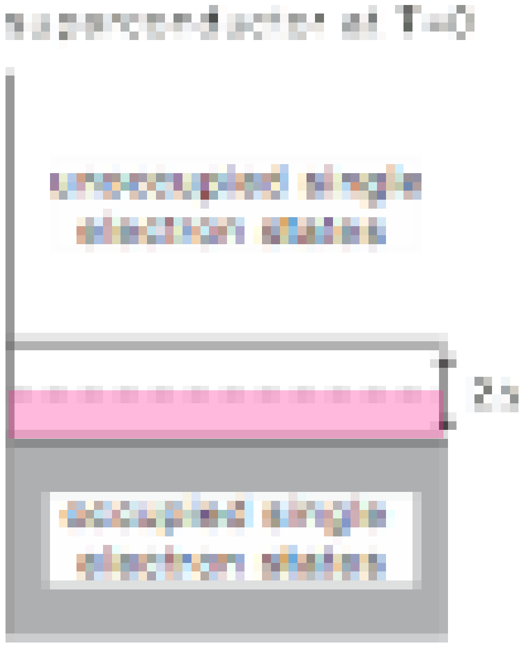
Debye temp. (Θ_D)

density of states ($g(E_F)$)

el-ph interaction strength (V)

isotope effect

$$\Theta_D \propto \omega_D \propto M^{-1/2}$$



number of electrons in Cooper pairs

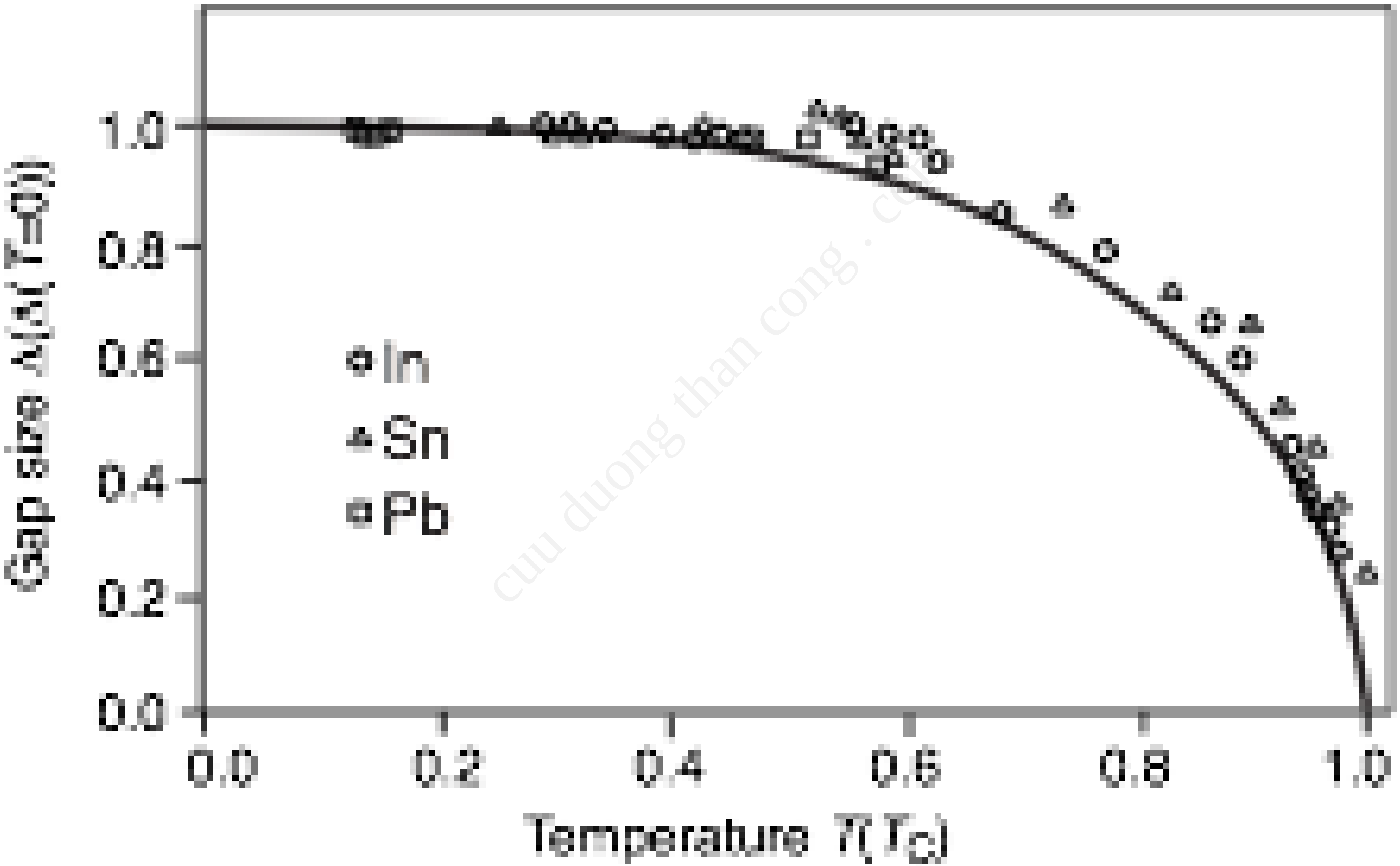
$$g(E_F)\Delta$$

$$\Delta = 3.53 k_B T_C$$

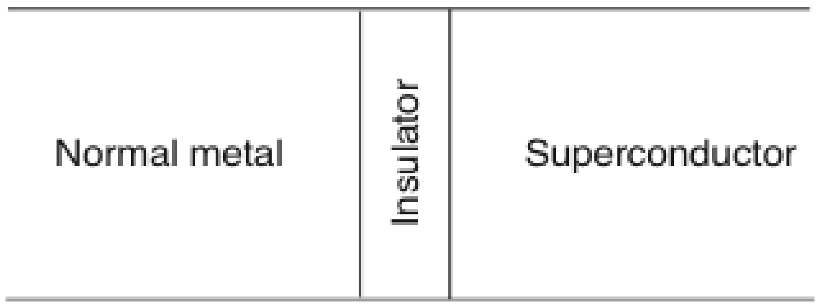
total energy gain

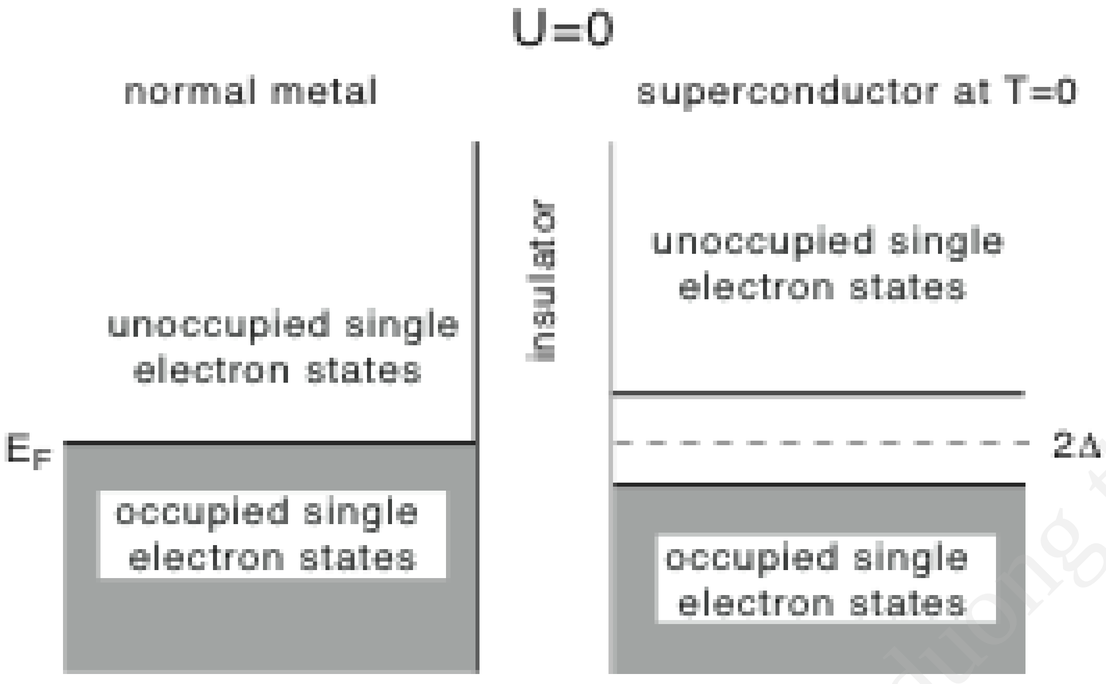
$$g(E_F)\Delta^2$$

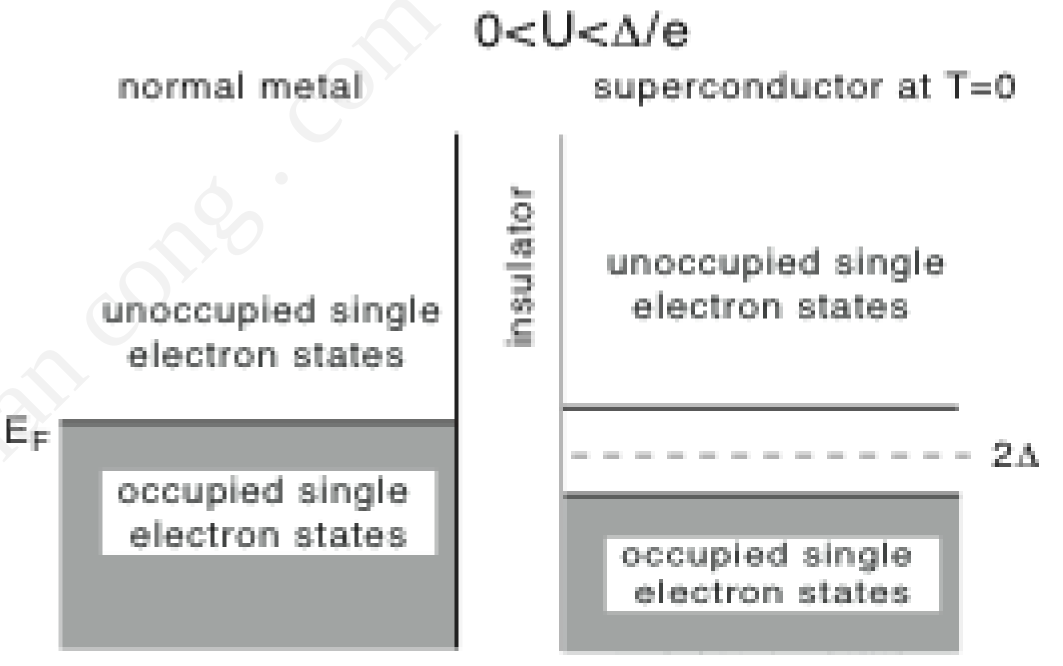
The gap at higher temperatures

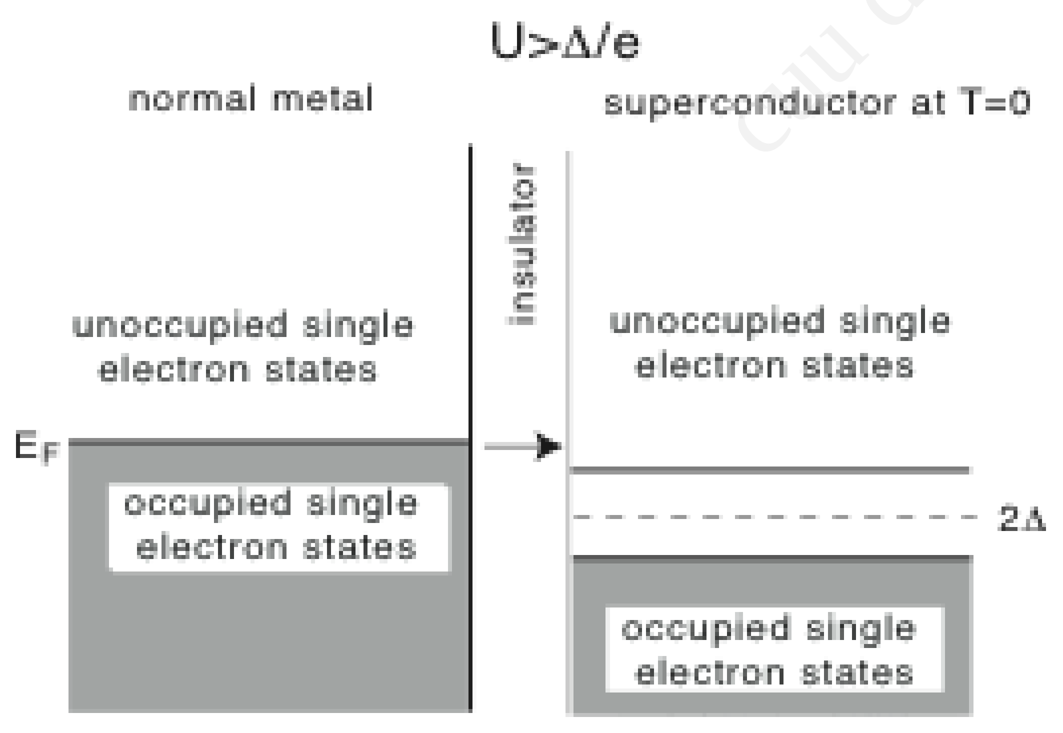


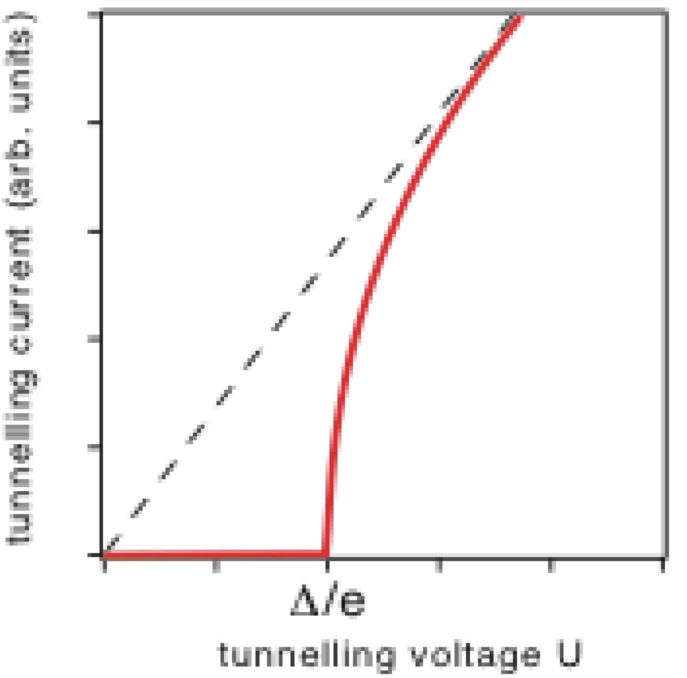
Detection of the gap: tunnelling



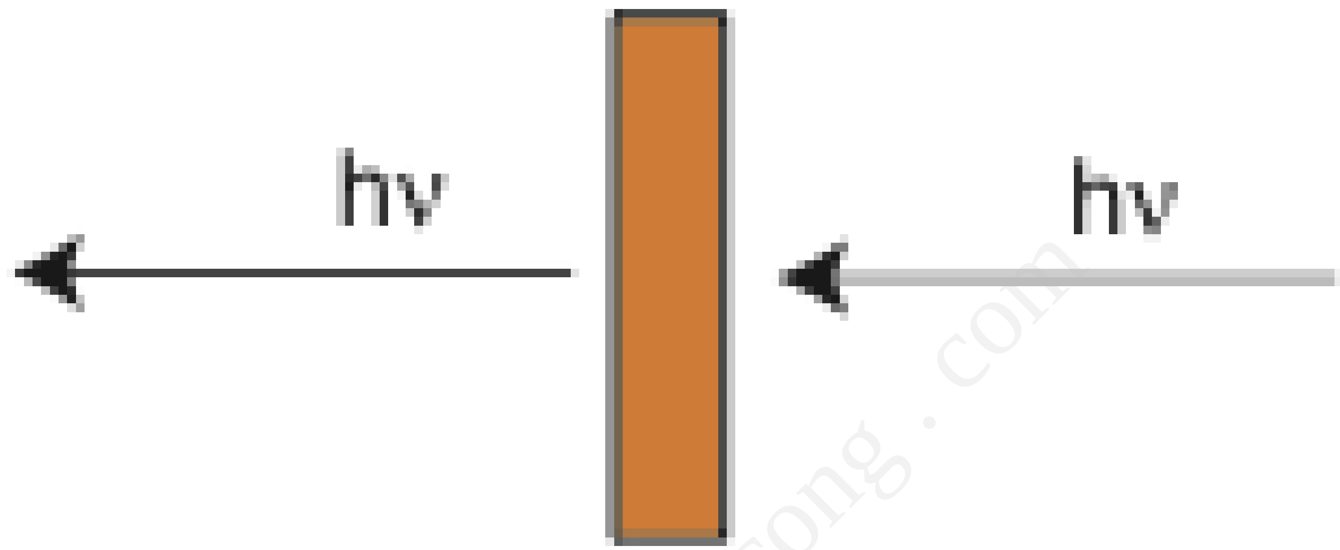






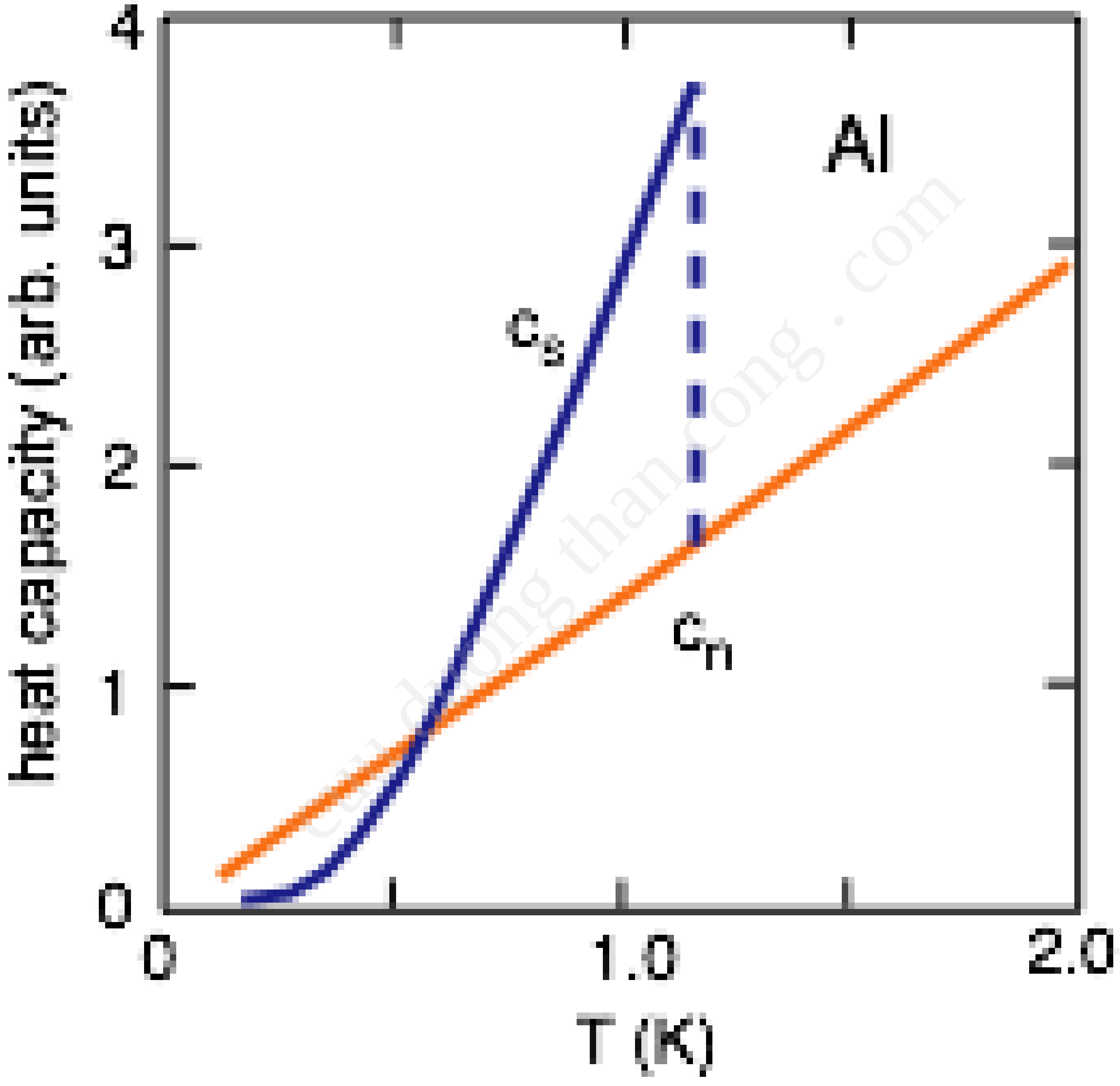


Detection of the gap: optical



- For radiation below hν < 2Δ no absorption, light is transmitted through a thin sheet of superconductor.
- For radiation below hν > 2Δ absorption takes place.
- Relevant light wavelength in the infrared.

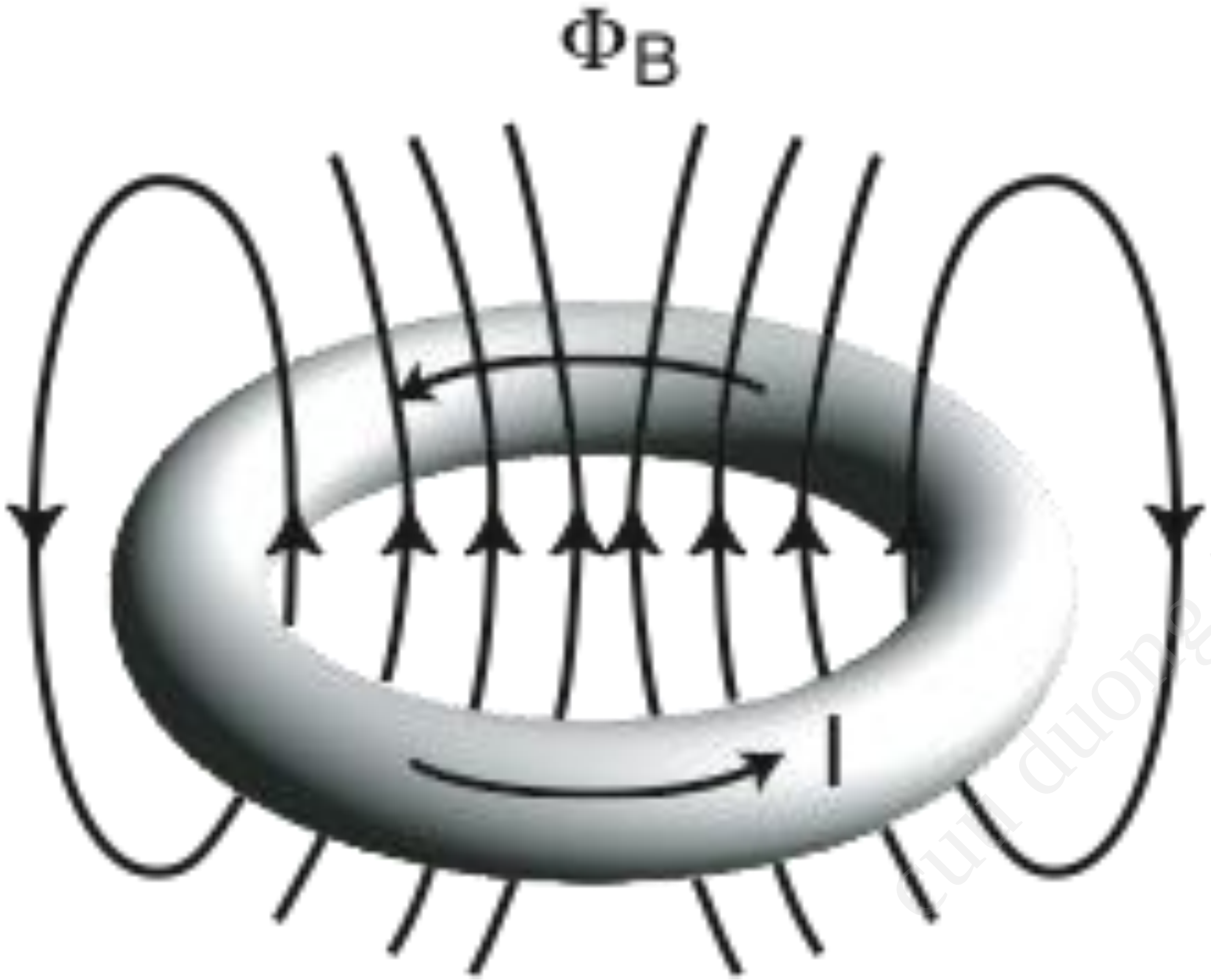
Detection of the gap: heat capacity



Zero Resistance

- All Cooper pairs are in the same state and have THE SAME total k. For k=0, no current. For $k \neq 0$ supercurrent.
- For a normal metal, single electrons can be scattered and be lost for the current. For a superconductor, one can only change the k of all Cooper pairs at the same time. This is VERY unlikely.
- Cooper pairs can be split up but this costs 2Δ and they might condensate quickly again. At high current densities, there is enough energy available to break them up. Also at high magnetic field densities.

Superconducting current in a ring



$$\lambda = \frac{h}{p}$$

Macroscopic coherence:
apply Bohr-Sommerfeld quantization

$$2\pi r = n\lambda = n\frac{h}{p}$$

n = 0,1,2...

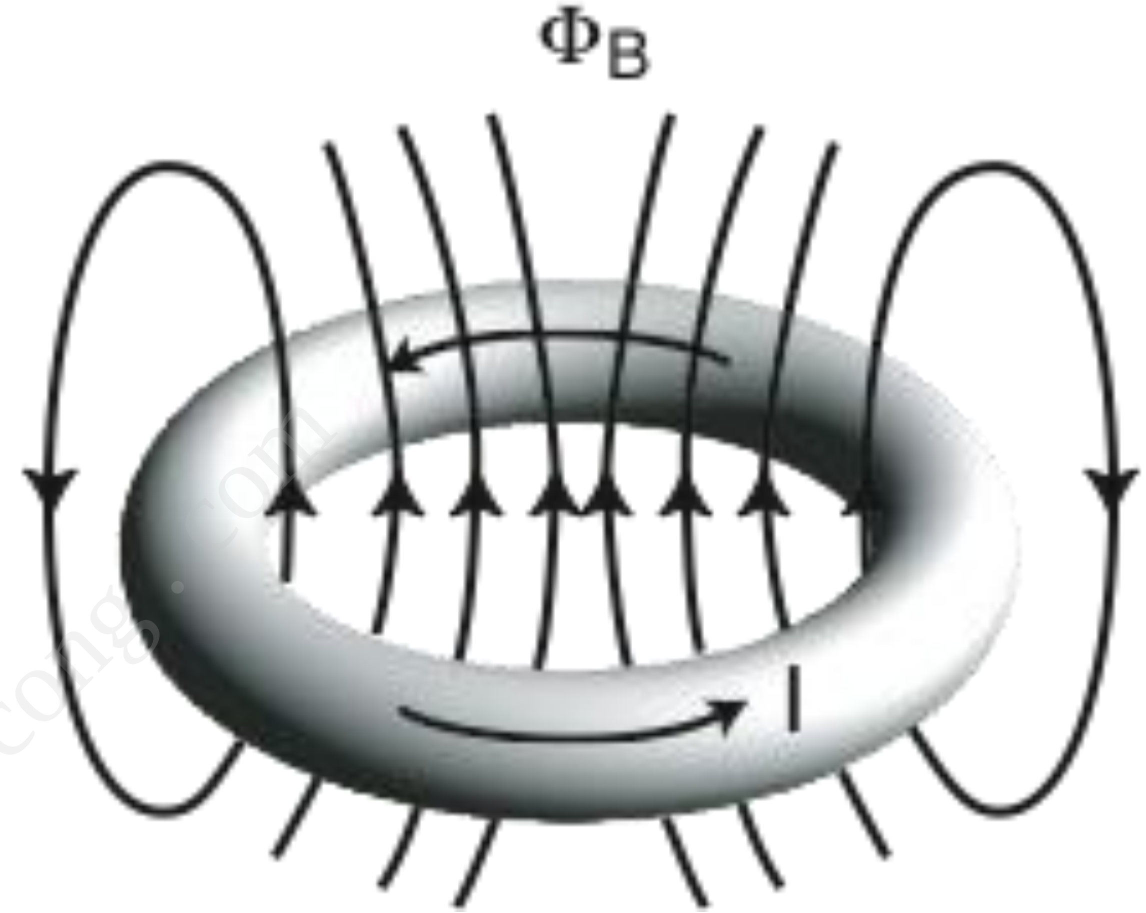
$$2\pi r p = nh$$

more general

$$\oint \mathbf{p} d\mathbf{r} = nh$$

Superconducting current in a ring

$$\oint \mathbf{p} d\mathbf{r} = nh$$



$$\oint \mathbf{p} - q\mathbf{A} d\mathbf{r} = hn$$

$$\mathbf{p} = m_s \mathbf{v}$$
$$\mathbf{j} = n_s q \mathbf{v}$$

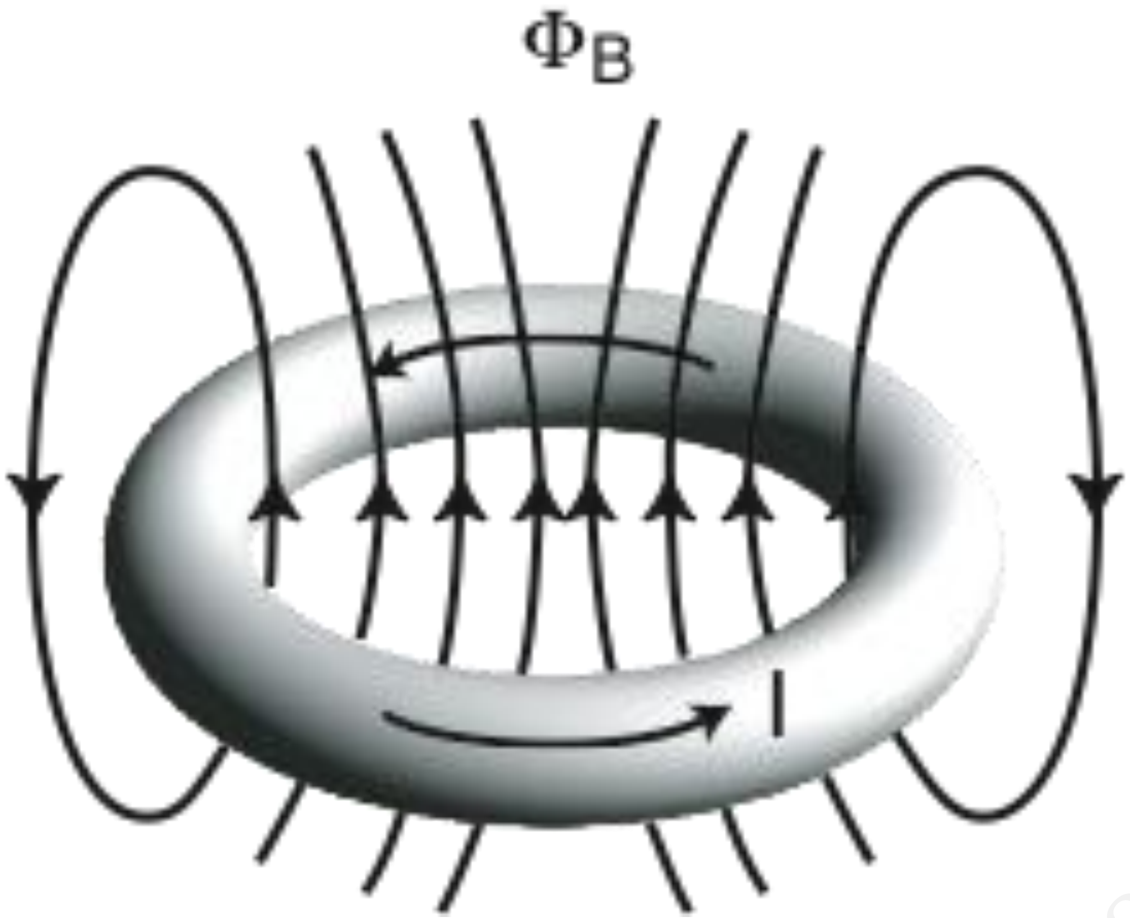
$$\frac{m_s}{n_s q} \oint \mathbf{j} d\mathbf{r} - q \oint \mathbf{A} d\mathbf{r} = nh$$

Stoke's integral theorem

$$\oint \mathbf{A} d\mathbf{r} = \int \mathrm{curl} \mathbf{A} d\mathbf{a} = \int \mathbf{B} d\mathbf{a} = \Phi_B$$

$$\frac{m_s}{n_s q^2} \oint \mathbf{j} d\mathbf{r} - \Phi_B = n \frac{h}{q}$$

Superconducting current in a ring



$$\frac{m_s}{n_s q^2}\oint \mathbf{j}d\mathbf{r} - \Phi_B = n\frac{h}{q}$$

integration path inside the ring material

$$\Phi_B = n\frac{h}{q}$$

The magnetic flux through the ring is quantized. *q* turns out to have a value of -2*e* in the experiment. So one flux quantum is

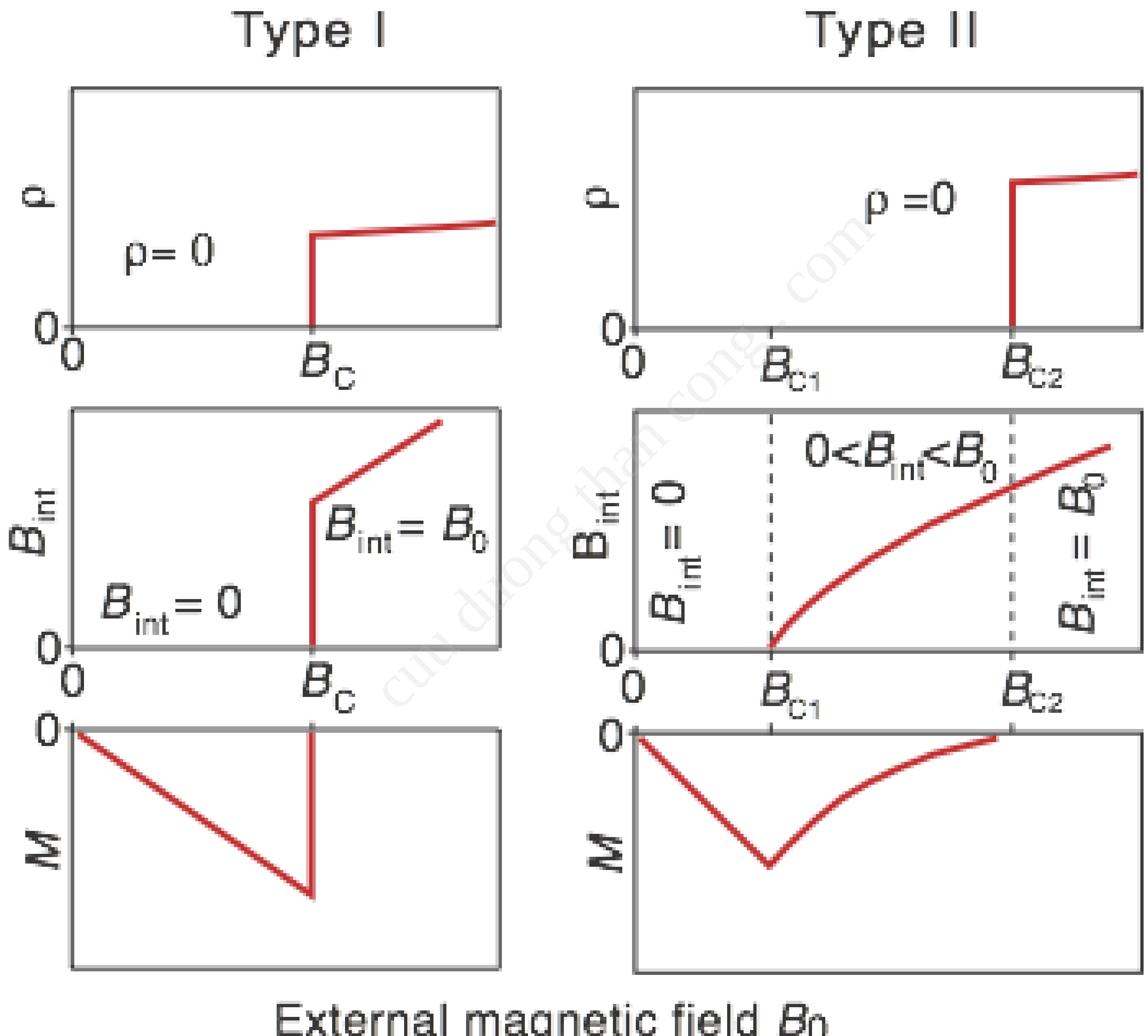
$$\frac{h}{2e} \quad = \quad 2.067\times10^{-15}\mathrm{Tm}^2$$

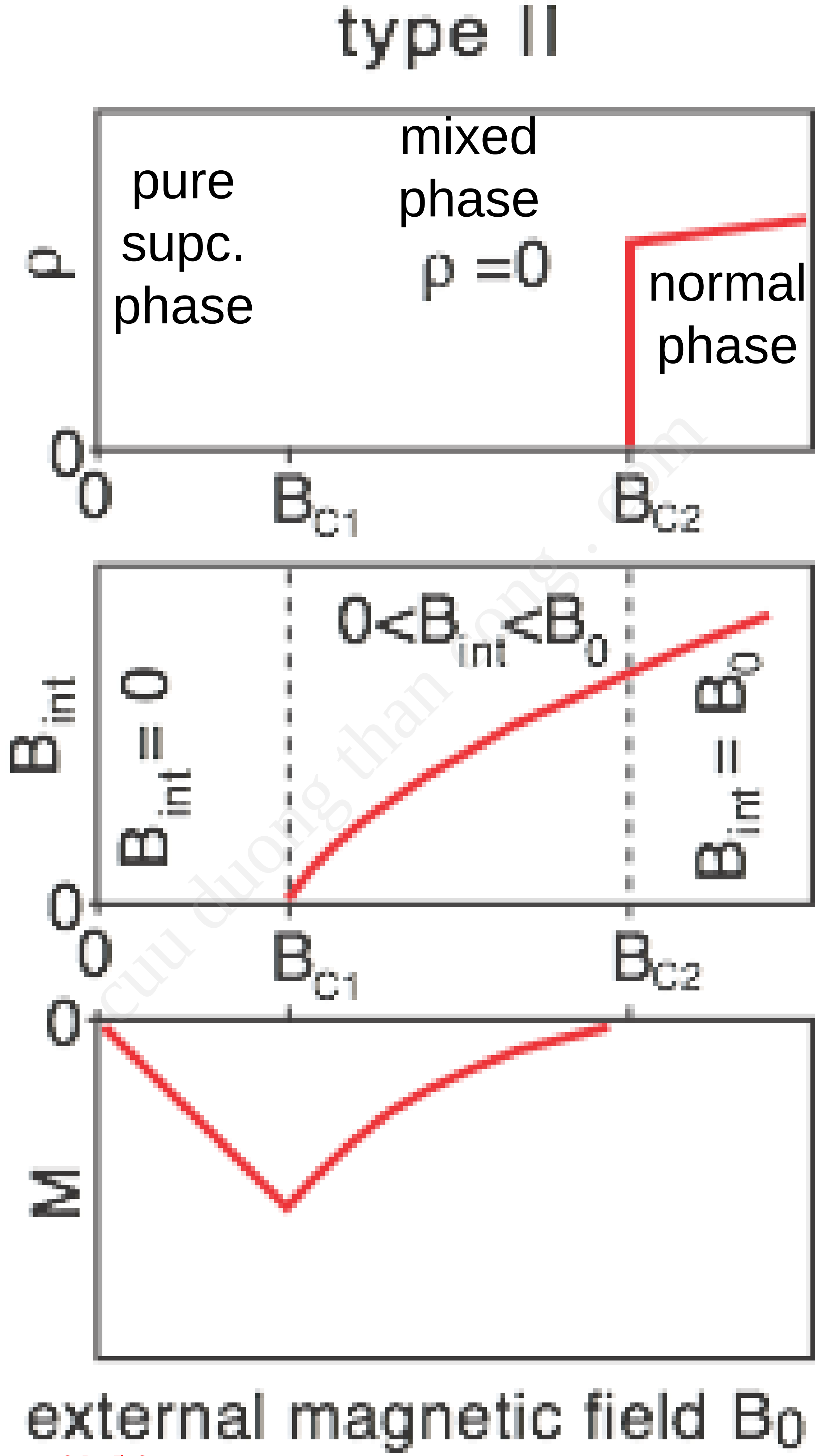
movie of superconducting maglev train
from IFW Dresden

Many open questions

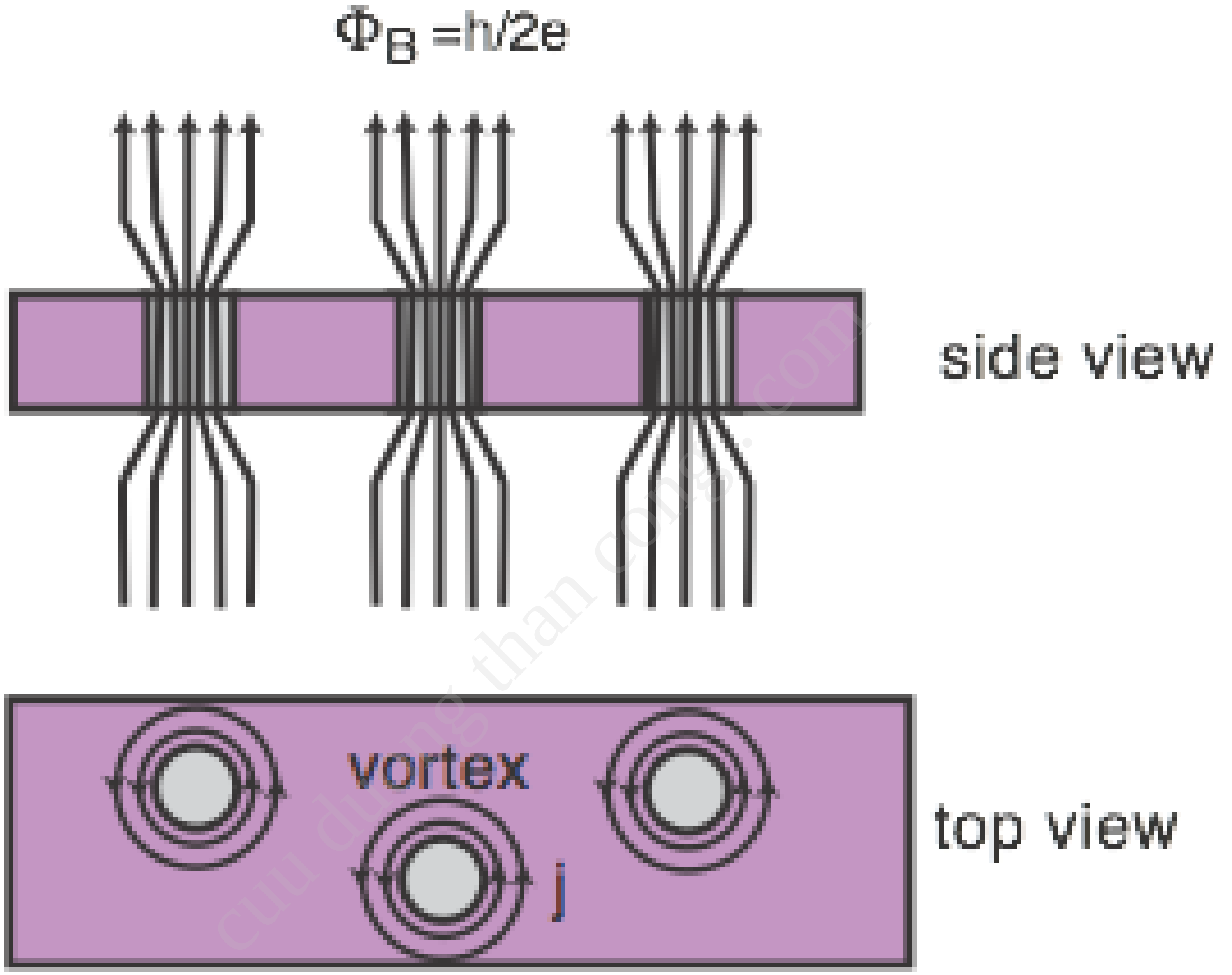
- Why does the train follow the tracks?
- Why does the train not fall down when turned upside down?
- How does the train “remember” the distance it has to be above the tracks?

Type I and type II superconductors




type II
pure
supc.
phase
mixed
phase
ρ =0
normal
phase
ρ
0
0
B C1
B C2
0<B int<B 0
B int = 0
B int = B 0
B int
M
external magnetic field B0

Vortices in type II superconductors



- The field penetrates through filaments of normal material, the rest remains superconducting.
- These filaments are enclosed by vortices of superconducting current.

How does the magnetic flux penetrate the type II superconductor

- The magnetic field enters in the form of flux quanta in normal-state regions. These regions are surrounded by vortices of supercurrent.
- This can be made visible by Bitter method, optical techniques, scanning tunnelling techniques....

25 x 25 μm (educated guess)

Vortices in type II superconductors

- The vortices repel each other.
- If they can move freely, they form a hexagonal lattice.
- But if there are defects which “pin” the vortices, they cannot change their position.

25 x 25 μm (educated guess)

Vortices in type II superconductors

- Here the vortices can move quite freely.
- Their number changes as the external field is changed.
- For a very high field, the vortices move so close together that the superconductivity in between breaks up.

25 x 25 μm

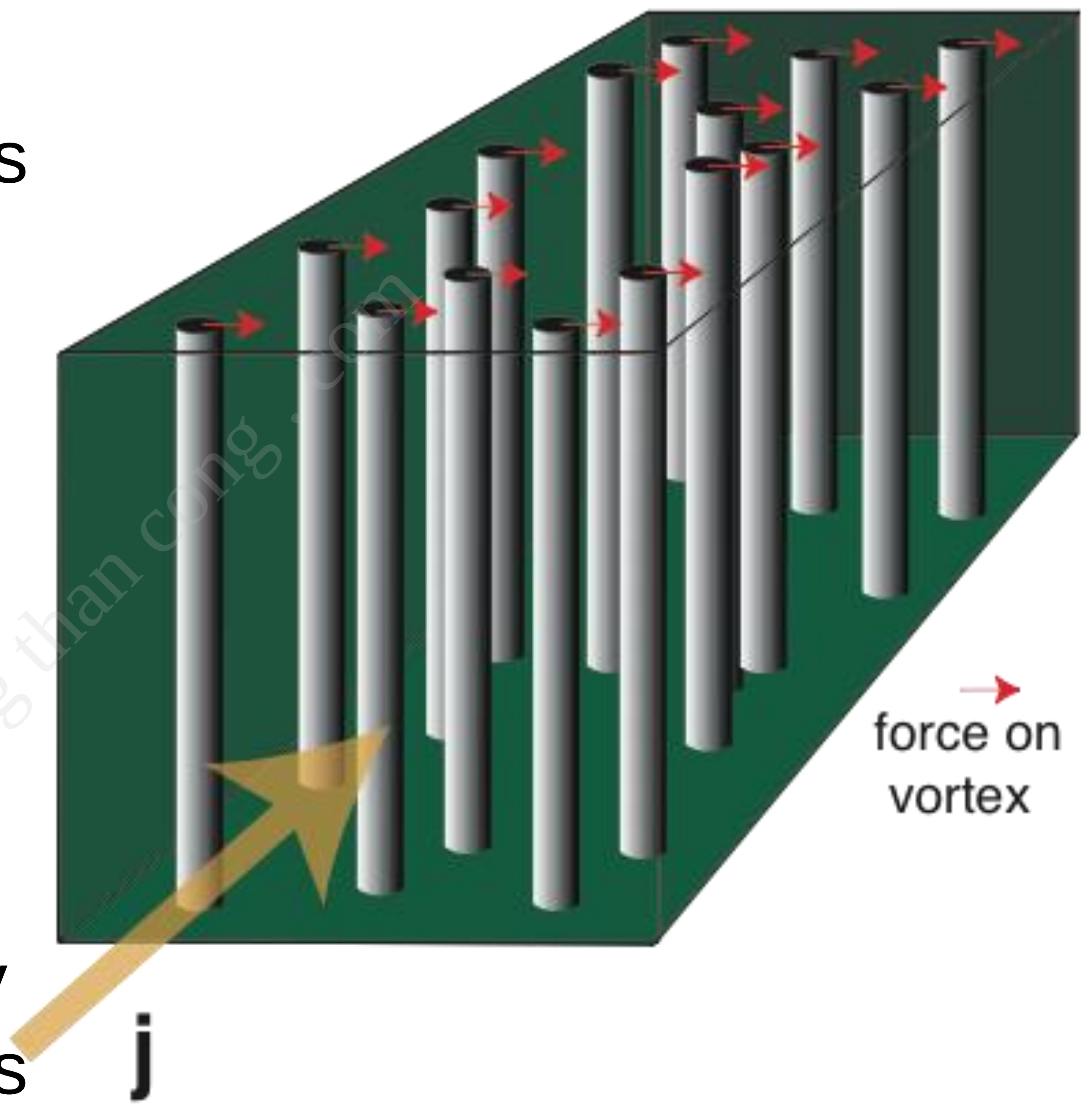
25 x 35 μm

Vortices in type II superconductors

- The superconductor used in the train movie is a type II superconductor.
- It “pins” the vortices very strongly. Therefore the magnetic field it experienced while cooling through the phase transition is “frozen in”.
- This can explain the effects we did not understand before (staying on the tracks, train upside down, train on walls...).

Current-induced vortex movement

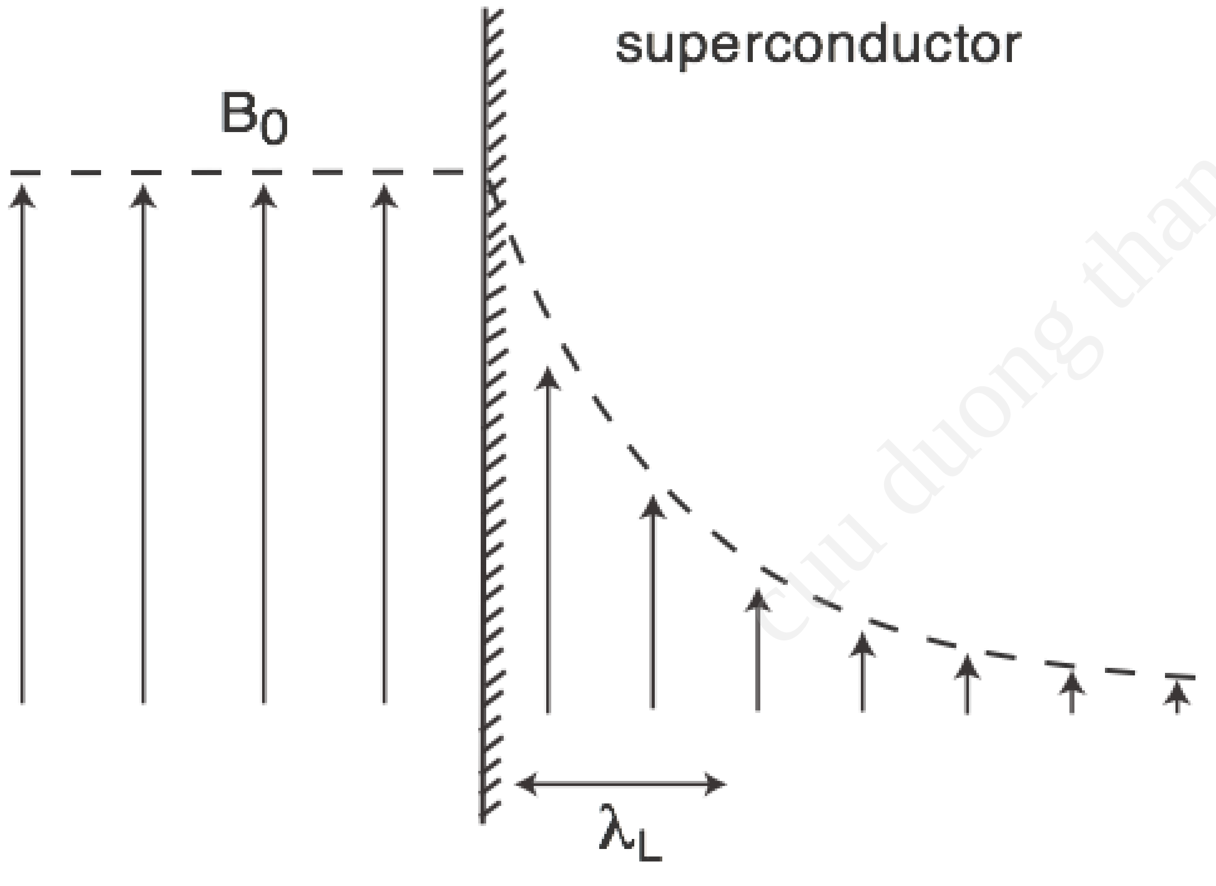
- A current through the material causes the vortices to move perpendicular to the current and to the magnetic field.
- This is very reminiscent of the Lorentz force.
- Vortex movement leads to energy dissipation and therefore resistance at very small current densities! This can be avoided by pinning the vortices.



$$\mathbf{F} = \mathbf{j} \times \Phi_B$$

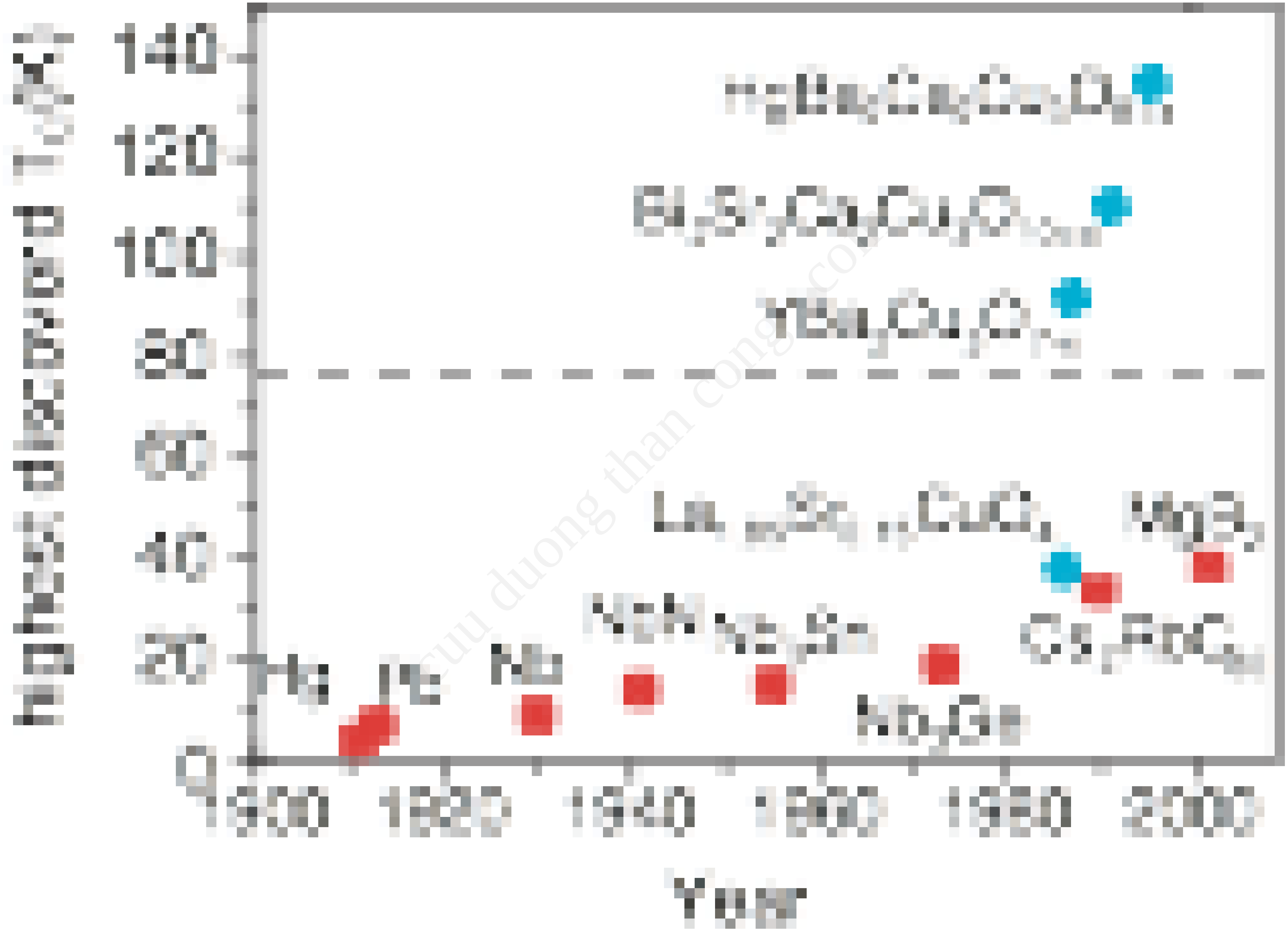
Characteristic length scales
type I or type II superconductor

The penetration depth λ



The coherence length ξ

High-temperature superconductors



- Discovery of ceramic-like high-temperature superconductors

Structure

- Superconductivity in the Cu+O planes. Very anisotropic

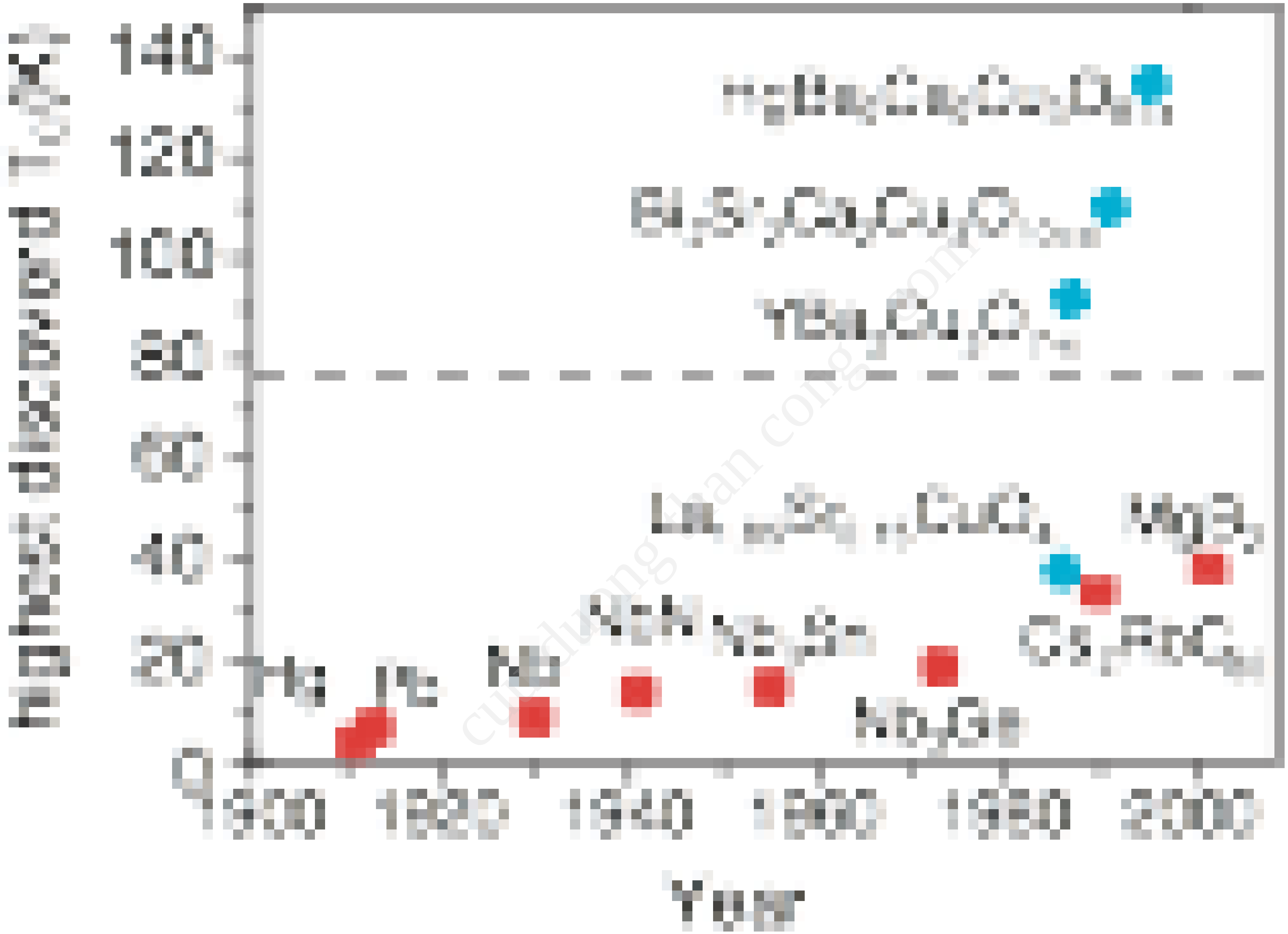
Some properties

- Extreme type II behaviour (this is why the train worked).
- Low critical current densities (because of the anisotropy)
- Ceramic-like: difficult to turn into wires

What makes the high-temperature superconductors superconduct?

- One is reasonably sure that Cooper pairs are also responsible here.
- But there is a big dispute on how they are formed: interaction of electrons with phonons, magnetic excitations (magnons), collective excitations of the electron gas (plasmons) or something else?

High-temperature superconductors



BCS

XYZ

Conclusions

- Very broad field. An exception to almost every rule we have discussed.
- Ferromagnetism and superconductivity apparently mutually exclusive.
- But we have seem some similarities in the phase transition.
- Both are ways to lower the energy of the electrons at the Fermi level. The relative energy gain may seem small but this is only because the Fermi energy is so high.
- There are also other mechanisms for this and interesting situations arise when several compete.