10

Mechanical properties of solids

- How are stress and strain defined?
- Why are alloys stronger than pure metals?
- What are solid lubricants?

Cars are not made of glass, and bottles are not made of steel, although cars and bottles may be made from polymers. The reasons for this are mostly bound up with the mechanical properties of the materials. These are controlled by two interacting features of the solid, the strength of the chemical bonds in the material, and the defects present.

The effects of variation in chemical bond strength are displayed by the mechanical properties of a crystal structure. For example, the clays and the micas are sheet silicates. The structures are built from sheets of atoms linked by strong chemical bonds. These sheets are linked into stacks by weak chemical bonds. This causes a marked anisotropy in behaviour, in which the strength of the solid in the plane of the sheets is quite different from the strength perpendicular to the sheets. The slightly greasy feel of many of these types of mineral is due to the fact that when some of the solid is rubbed between finger and thumb the weakly bound sheets slide over each other, creating stacking defects. Such materials are exploited as solid lubricants. The strongest minerals are those with no weak bonds, but contain infinite three-dimensional networks of $[SiO_4]$ tetrahedra, linked to each other by all four vertices. Despite this strength, they often shatter and are brittle. Metals, in contrast, although weaker in some ways, can be hammered into sheets (are malleable) or drawn into wires (are ductile) without breaking.

Chemical bonding, however, is rarely directly responsible for the observed mechanical properties of a material. In fact, most solids are far weaker than the strength predicted on the basis of the chemical bonds present. In these cases, the defects present, especially dislocations, stacking defects and grain boundaries, control matters.

Materials reveal their mechanical properties when subjected to forces. The application of a force results in a deformation. The amount of deformation will depend on the magnitude of the force and its direction measured with respect to the crystallographic axes. Both force and deformation are vector quantities. In the discussion below, it will be assumed that all materials are isotropic in this respect and that there is no crystallographic relationship between force and deformation, which are both presumed to be scalars (numbers, See section S4.13). In fact, in much of the discussion, especially of the elastic properties of solids, the atomic structure is ignored, and the solids are treated as if they were continuous. This viewpoint cannot explain plastic deformation, and knowledge of the crystal structure of the solid is needed to understand the

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role of dislocations. In the final section, concerned with properties on a nanoscale, knowledge of the microstructure of the solid at atomic dimensions is essential.

10.1 Deformation

10.1.1 Strength

Everyone has a subjective idea of strength, and some materials, such as steel, are universally regarded as strong, whereas others, such as plastics, are considered weak. However, the strength of a material will depend exactly on how it is evaluated. A reliable measure of the strength of a solid is the amount of force that can be applied to it before it breaks (Figure 10.1).

- A material that is stretched is in tension, and suffers tensile forces (Figure 10.1a). The tensile strength of metals and polymer fibres is usually high.

- A material that is squeezed is in compression, and suffers compressive forces (Figure 10.1b). Compressive strength needs to be high in building materials that have to bear heavy loads.

- A material that is subjected to opposed forces is said to be sheared, and suffers shear forces (Fig. 10.1c). Many polymers behave like very viscous liquids and have very low shear strength.

- A material that is twisted is subjected to a torsional load (Figure 10.1d). Torsional strength is important for shafts that transmit rotation.

- A solid that is flexed or bent; (Figure 10.1e) is subject to both tension and compression, and the flexural strength gives a measure of the amount of bending that an object can sustain without fracture.

- Finally, the impact strength of a solid measures the resistance to a sudden blow (Figure 10.1f). Glass has low impact strength, whereas wooden bats have high impact strength.

A component may experience all of these forces and resultant deformations at the same time, but here each will be treated separately.

The interdependence between the forces applied and the deformations that are produced are summarised by a number of moduli, of which Young's modulus, also called the elastic modulus, is the best known (see Section 10.1.4 and Section S4.1).

10.1.2 Stress and strain

The force (often called the load) applied to an object is defined in terms of the stress on the object. Stress is measured as the force applied to a unit area of the

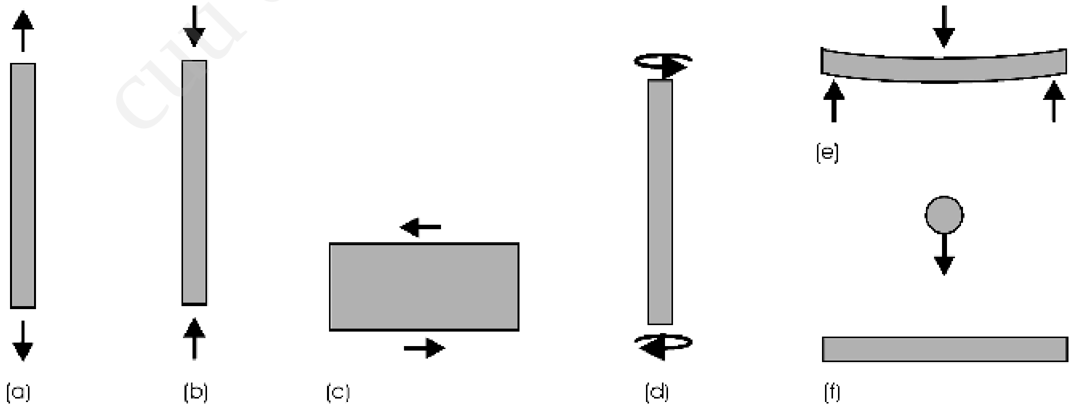

Figure 10.1 Mechanical loading: (a) tension, (b) compression, (c) shear, (d) torsion, (e) bend and (f) impact

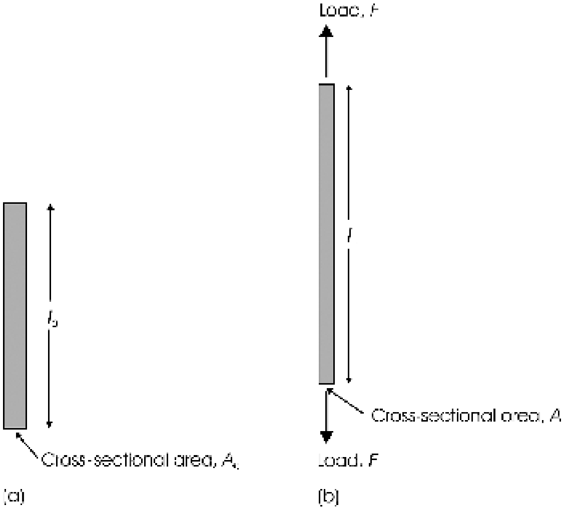

Figure 10.2 A rod in tension: (a) initial state and (b) final state

specimen. The application of a stress results in a dimensional change, which is called the strain.

For a rod-shaped specimen (Figure 10.2), used for the evaluation of metal or polymer samples:

$$\sigma_T = \frac{F}{A}$$

where σ_T is the true stress, F is the force (or load) applied to the rod, and A is the cross-sectional area subjected to the force. The force (or load) is measured in newtons, N, and the area is measured in m^2. Stress is measured in pascals, Pa (N m^{-2}), commonly cited as MPa $= 10^6$ N m^{-2}, or GPa $=$ 10^9 N m^{-2}.

For practical purposes, it is often adequate to ignore the continuous change in cross-sectional area that occurs when a force is applied. The stress so defined is called the engineering (or nominal) stress, σ:

$$\sigma = \frac{F}{A_0}$$

where F is the average force (or load) applied, and A_0 is the initial cross-sectional area of the sample.

The elongation of the rod when subjected to a force is equivalent to the strain. The increment in tensile strain experienced, $\Delta\,\varepsilon_T$, when a rod is extended, is defined as the ratio of the increase in length, $\Delta\,l$, to the total length:

$$\Delta\,\varepsilon_T = \frac{\Delta\,l}{l}$$

The total true strain is then given by:

$$\begin{aligned}\varepsilon_T &= \int_{l_0}^{l} \frac{\mathrm{d}\,l}{l_0}\\ &= \ln\left[\frac{l}{l_0}\right]\end{aligned}$$

where l is the final length of the specimen, and l_0 is the original length. As strain is a ratio, it has no units. If the incremental changes are ignored, the engineering (or nominal) strain is:

$$\varepsilon = \frac{l - l_0}{l_0}$$

where l is the final length of the specimen, and l_0 is the original length of the specimen. In the stress–strain diagrams for metals and polymers that follow, engineering stress and engineering strain are plotted.

Many materials are used under compression rather than tension. At low loads, the compressed material behaves in a similar way to materials tested under tension. In compression tests, the value of the force is taken as negative and hence we have negative values of stress and strain compared with those obtained in tension.

10.1.3 Stress–strain curves

A great deal can be learned about the mechanical properties of materials by stressing them until they fracture or break. The most common mechanical test involving metals or polymers is the tensile test, in which a sample of the solid is stretched. The test uses a standard test piece with a shape dependent on the material to be tested. Metals usually have a central cylindrical section, of known gauge length,

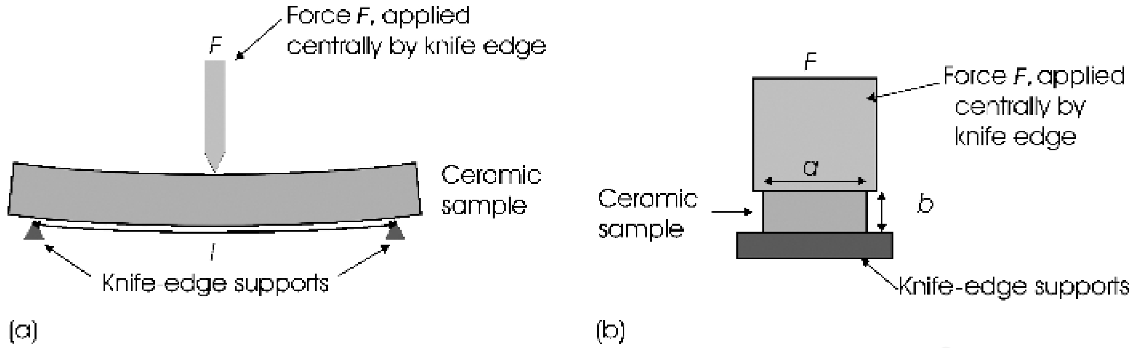

Figure 10.3 The three-point bend test for ceramic samples: (a) sideways and (b) end-on view of the test

usually 50 mm, on which the measurements are made. Polymer samples tend to be sections cut from plates and are larger in dimension. During the test, the instrument applies a force to the sample at a constant rate and simultaneously records the change in dimensions of the test piece.

The stress and strain relationships for a ceramic specimen are more often determined by bending a bar, plate or cylinder of material (Figure 10.3). In this test, the lower part of the ceramic is under tension, and the upper surface is under compression. As ceramic materials are generally much stronger in compression, failure is initiated on the surface under tension. The maximum stress in the upper surface of a deformed sample, $\sigma^{\max}$, is given by:

$$\sigma_{\mathrm{T}}^{\max} = \frac{3Fl}{2ab^2}$$

where F is the applied force and l is the arc length of the deformed sample between the supports, with a bar with width a normal to the force and thickness b parallel to the force (Figure 10.3). The strain corresponding to the maximum stress is related to the maximum deflection, δ, at the centre of the bar by

$$\varepsilon_{\mathrm{T}}^{\max} = \frac{6\delta b}{l}$$

The cross-sectional area of the ceramic specimen is not greatly altered during the test, so that the true stress is measured. In testing a ceramic, the force or load is slowly increased until fracture.

A plot of load against extension, stress against strain or, more commonly, engineering stress against engineering strain, gives a good picture of the mechanical behaviour of the solid in question (Figure 10.4). The behaviour of brittle materials such as ceramics, brittle metals such as cast iron, or polymers that are chilled to well below the glass transition temperature is drawn in Figure 10.4(a). In these materials, the stress is usually directly proportional to the strain over all or most of the range up to fracture. Metals initially show a similar linear relationship, but the plot ultimately curves and extensive deformation occurs before fracture (Figure 10.4b). This type of curve is typical of a ductile solid. The curves for most polymers are very temperature-sensitive. Thermoplastic polymers above the glass transition temperature give rise to a plot that curves in the opposite way from that of a ductile metal (Figure 10.4c). Elastomers (Figure 10.4d) deform at far lower stress levels than other materials.

The linear part of the stress–strain curve is the elastic region. Here, removal of the load will allow the solid to return to its original dimensions, quite reversibly. In the case of elastomers, this reversibility is maintained over the whole of the stress–strain curve.

For all other solids, once the elastic region is passed, the deformation of the solid is not reversed when the stress is removed, and some degree of permanent deformation remains. This is called plastic deformation. For metals, the point at which elastic behaviour changes to plastic behaviour is

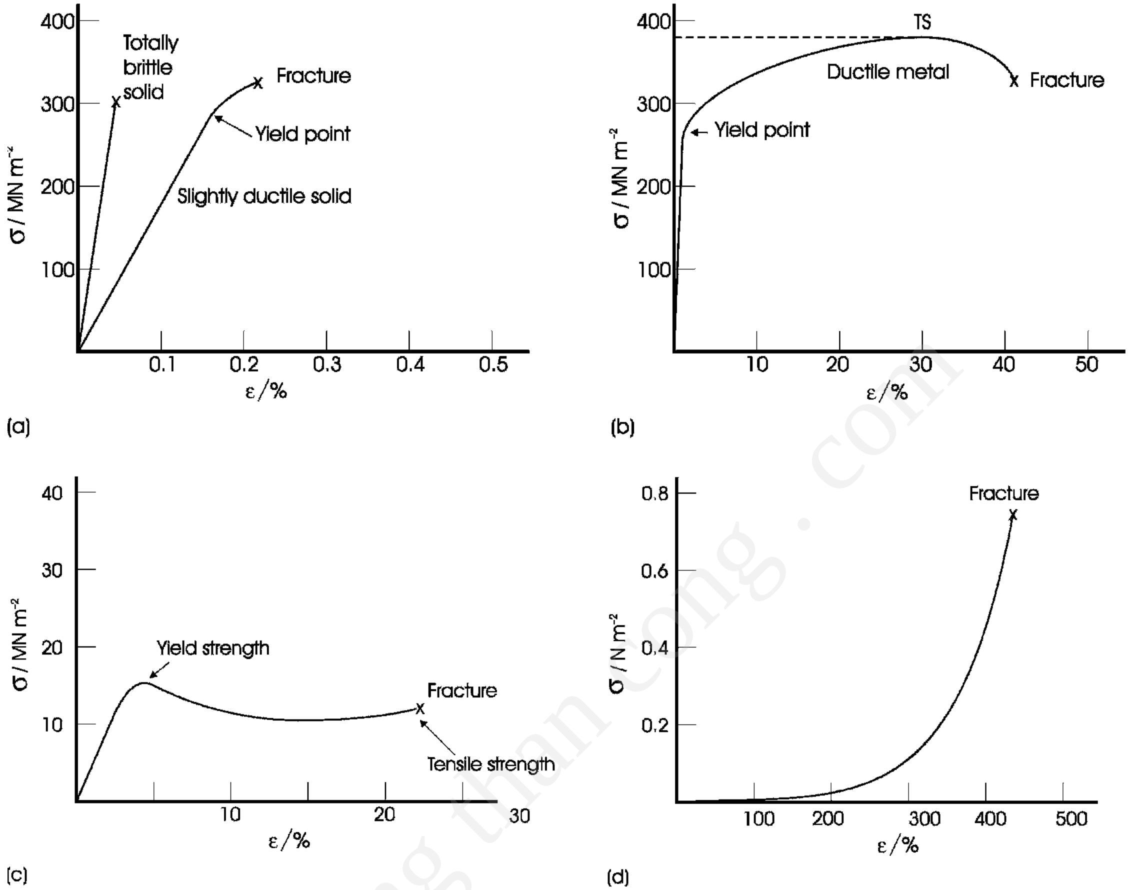

Figure 10.4 Schematic engineering stress–engineering strain (σ–ε) curves for (a) brittle and slightly ductile solids, (b) ductile metals, (c) a typical polymer, and (d) rubber, an elastomer. Note the different stress scale in part (d); point x represents fracture of the specimen; point TS is the ultimate tensile strength

called the yield point. This occurs at a value of stress called the yield stress. In the case of slightly ductile materials (Figure 10.4a), only a small amount of plastic deformation occurs before the material breaks into two. For a ductile metal, a large amount of deformation is possible before fracture. The maximum load that can be sustained, corresponding to point TS in Figure 10.4b, is called the tensile strength (or ultimate tensile strength) of the metal. For a polymer, once the elastic region is passed, almost no increase in stress will bring about a large amount of plastic deformation (Figure 10.4c). Anyone who has carried an overloaded plastic bag for any distance will have practical experience of this. The bag will support the load for a period, and then suddenly start to stretch until it breaks. Elastomers show extensive plastic deformation under any load, but this is always reversible, and so this behaviour differs from the plastic deformation found in the other materials.

The ultimate tensile strength of a material with respect to its weight is an important engineering parameter. This is termed the specific strength:

$$\text{specific strength} = \frac{\text{tensile strength}}{\text{specific gravity}}$$

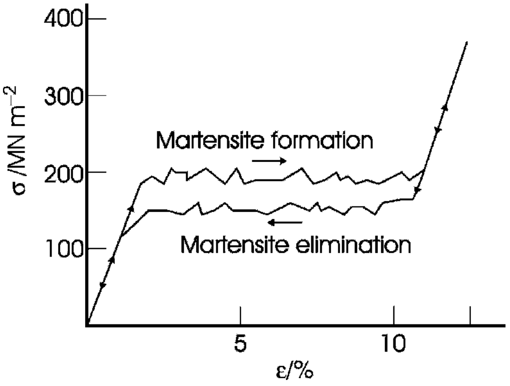

Figure 10.5 Schematic engineering stress–engineering strain (σ–ε) curve for a shape-memory alloy showing superelasticity

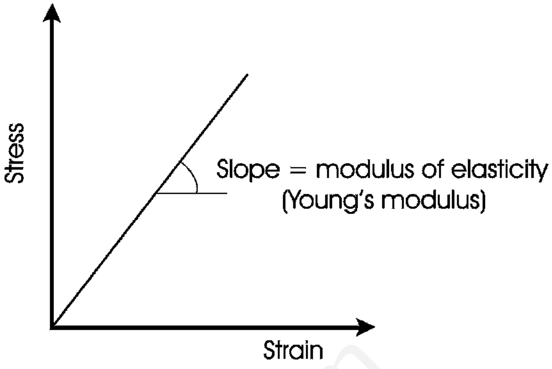

Figure 10.6 The modulus of elasticity, Young's modulus, is defined as the slope of the stress–strain curve in the linear (elastic) region

The major properties obtained from the engineering stress–engineering strain tensile curve are modulus of elasticity, yield strength, ultimate tensile strength and amount of elongation at fracture.

The stress–strain behaviour is different again in many shape-memory alloys (Section 8.3.3). In these solids, the stress–strain plot shows a region of extreme deformation (Figure 10.5). The curve returns to the original pathway on release of the stress, although rarely following exactly the same pathway, a hysteresis effect. This is called superelasticity. In the super-elastic region, martensite is forming (even at temperatures above A_f, the austenite finish temperature), under the influence of the applied stress. For example, the martensitic transformation in the shape-memory alloy Nitinol is caused by a shear of the atoms in $\{1\,0\,1\}$ planes (see Section 8.3.2). Stress has a similar shearing effect on the structure and can cause the martensitic transformation to occur above A_f. This is called a stress-induced martensitic transformation. The application of further stress causes more martensite to form and, in so doing, the stress is released. When the transformation is complete and the variant boundaries are fixed in place, the material reverts to normal behaviour. However, above A_f the martensitic form is unstable in the absence of stress. Thus, as the stress is released the martensite plates revert to the parent structure, but normally the start of the reverse transformation is initiated at a lower stress than the forward transformation, resulting in hysteresis. In practice, the degree of hysteresis is found to depend on the composition of the alloy and prior heat treatment.

10.1.4 Elastic deformation: the elastic (Young's) modulus

Elastic deformation is reversible deformation. The slope of the elastic region is a measure of the elastic modulus (Young's modulus) of the material. When the force applied to a material is relatively small and the material is subject only to elastic deformation, the stress is related to the strain by Hooke's law:

$$\sigma = E\,\varepsilon \qquad (10.1)$$

The constant of proportionality, E, is Young's modulus (Figure 10.6). As weight is an important consideration in many applications, the specific modulus is often quoted as a material parameter:

$$\text{specific modulus} = \frac{\text{modulus of elasticity}}{\text{specific gravity}}$$

The elastic deformation experienced is a result of pulling atoms apart or pushing atoms together and so is directly related to interatomic bonding. If the chemical bonds between the atoms can be accurately described in terms of energies, then the amount of deformation that will result from a given applied force can be calculated (Section S4.1.7).

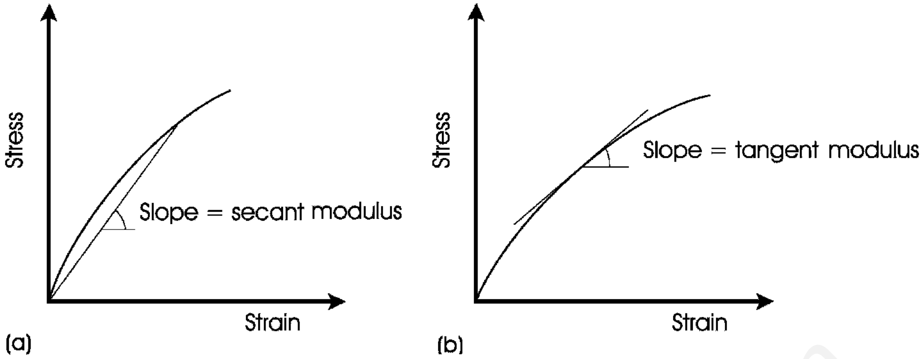

Figure 10.7 The (a) secant modulus and (b) tangent modulus of a material

In many materials, especially those in which one (or more) component(s) of the bonding is (are) relatively weak, Hooke's Law is not obeyed exactly. Instead, the stress–strain curve shows a distinct curve (Figure 10.7). This is termed nonlinear behaviour. In such cases, a single value for Young's modulus cannot be obtained. As an approximation, it is possible to measure the tangent to the curve at any point, to obtain the tangent modulus or, alternatively, the secant modulus can be used (Figure 10.7). This is the slope of the line drawn from the origin to intersect the stress–strain curve at a specified value of the strain.

Representative values of the modulus of elasticity are given in Table 10.1.

10.1.5 Poisson's ratio

Although measures of engineering stress and engineering strain assume constant cross-sectional area of the rod being stressed, a material deformed elastically longitudinally (in compression or tension) has an accompanying lateral dimensional change. This is described by Poisson's ratio, ν. If a tensile stress σ_z produces an axial strain $+\varepsilon_z$ and lateral contractions $-\varepsilon_x$ and $-\varepsilon_y$, (in isotropic materials $-\varepsilon_x = -\varepsilon_y$),

$$\nu = \frac{-\varepsilon(\text{lateral})}{\varepsilon(\text{longitudinal})} = \frac{-\varepsilon_x}{\varepsilon_z} \qquad (10.2)$$

The negative sign is to ensure that the numerical value of Poisson's ratio is positive. For isotropic materials, the theoretical value of ν is $\frac{1}{2}$. Most metals show values in the range 0.25–0.35. Some materials, counterintuitively, have a negative Poisson's ratio, and get thicker when under tension. They are discussed later in this chapter. Representative values of Poisson's ratio are given in Table 10.1.

Table 10.1 Representative values of the modulus of elasticity, E, and of the Poisson ratio, ν

Material	E/GPa	ν
Aluminium	70.3	0.345
Copper	129.8	0.343
Iron (cast)	≈152	≈0.27
Magnesium	44.7	0.291
Nickel	219.2	0.306
Titanium	115.7	0.321
Tungsten	411.0	0.280
Brass	≈100	≈0.35
Bronze	≈105	≈0.34
Steel, mild	≈212	≈0.29
Alumina	379.2	0.22
Magnesium oxide	210.3	0.23
Silicon carbide	468.9	0.17
Silica glass	72.4	0.17
Epoxy resin	≈3.2	≈0.35
Nylon 6,6	≈2.0	≈0.39
Polycarbonate	≈2.4	≈0.36
Polystyrene	≈3.5	≈0.33

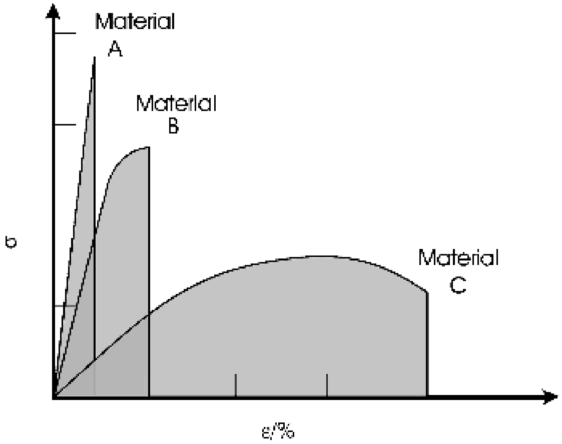

Figure 10.8 The toughness of a solid can be represented by the area under a engineering stress–engineering strain (σ–ε) curve to fracture. Material A has the highest elastic modulus, but material C has the greatest toughness

10.1.6 Toughness and stiffness

Toughness is often a much more important material property than strength. Toughness can loosely be defined as the amount of energy absorbed by a material before it fractures. A tough material has a high resistance to the propagation of cracks, and so tough materials are both strong and ductile. The toughness can be estimated by the area under the stress–strain curve (Figure 10.8).

The ease with which a material can be extended, the stiffness of the solid, is represented by the slope of the stress–strain curve in the initial, elastic region. A stiff material shows only a small strain for a given stress. The stiffness is thus equivalent to the modulus of elasticity. It is seen from Figure 10.8 that the material with the highest modulus of elasticity is not necessarily the toughest. In this case, material C is the toughest of the three, although it has the lowest stiffness. The toughness of a specimen will depend on specimen geometry and the way in which the stress is applied.

10.1.7 Brittle fracture

Many materials are elastic and brittle, especially at lower temperatures. Single crystals frequently fracture by cleavage along crystal planes in which the bonding is relatively weak (Figure 10.9a). A polycrystalline material can fracture in two ways. Fracture across the constituent crystallites is akin to crystal cleavage and is called transgranular or transcrystalline fracture. [Single crystals can fracture only in a transgranular fashion.] In some materials, the weakest part is the region between crystallites, and so the fracture surface runs along the boundaries between the constituent crystallites. This is called intergranular fracture (Figure 10.9b). Amorphous materials such as glass or brittle polymers fracture to produce a smooth surface resembling the inside of a seashell. This is called conchiodal fracture (Figure 10.9c). Materials containing voids or several phases, such as porcelain, which contains glass, crystals, and voids, frequently fracture in the neighbourhood of these defects (Figure 10.9d).

Polymer fibres are rather different in that they can be thought of as aligned molecules. Many polymer fibres, such as Kevlar®, carbon fibre and ultrahigh-molecular-weight polythene have tensile strengths comparable to or better than steel. Rigid polymers, such as methyl methacrylate, polycarbonate, or many thermoplastics and elastomers at low temperatures, behave in a brittle fashion.

The elastic deformation in all these materials can be successfully explained by considering the strength of the chemical bonds between the constituent atoms of the solid or the fibre molecule, and it is logical to conclude that fracture takes place when the tensile stress is greater than the chemical bond strength (Section S4.1.7). Rapid failure occurs when a crack propagates through the solid by the breaking of successive chemical bonds. There is a fundamental difference between metals and ceramics, on the one hand, and polymers, on the other, in this respect. In metals and ceramics, the bonding within the solid is more or less uniform and consists of fairly strong bonds, metallic, ionic or covalent. In a brittle polymer, the material is made up of crystals joined by amorphous regions in which the bonding is weak, often hydrogen bonds. Although the fracture of the material proceeds in the same way as in a metal or ceramic, the weakness of the bonds that have to be broken makes these materials much less

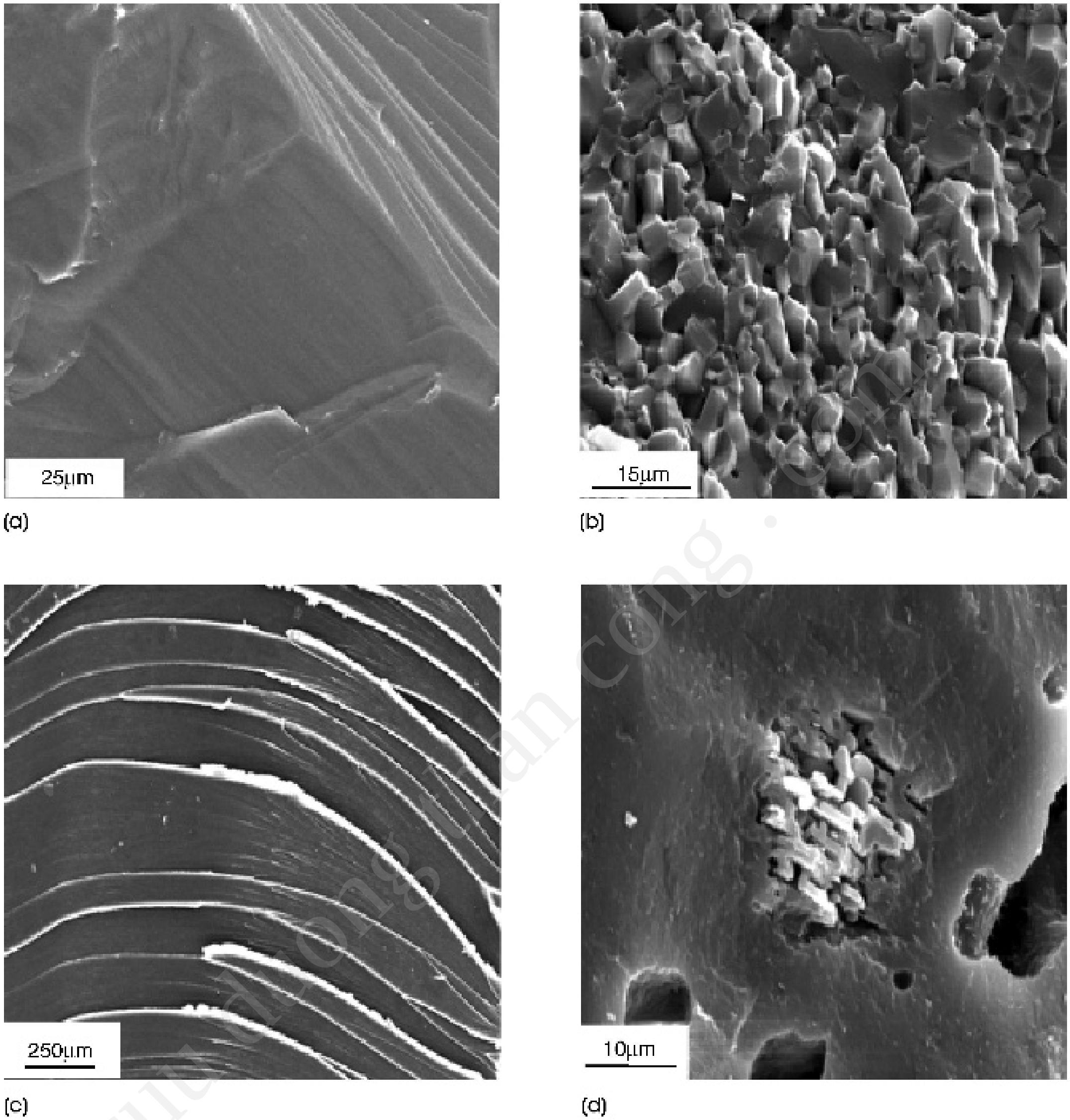

Figure 10.9 (a) Cleavage fracture of single crystalline silicon, (b) transgranular fracture of a fine-grained polycrystalline alumina ceramic crucible. (c) Conchoidal fracture of brittle poly(methyl methacrylate). (d) Fracture of porcelain showing a cluster of crystallites and pores in a glass-like matrix

strong. In the case of polymer fibres, of course, the bonds that have to break are strong covalent bonds, and thus these materials are very strong, at least in the fibre direction.

It is possible to estimate the critical stress required for fracture from the interatomic forces (Section S.4.2.1). The critical stress, σ_c, is given by:

$$\sigma_c = \left(\frac{\gamma E}{r_0}\right)^{1/2} \tag{10.3}$$

where r_0 is the equilibrium spacing of the atoms in the material, E is the modulus of elasticity, and γ is the surface energy of the solid. The theoretical strength, determined from Equation (10.3), is calculated to be about $E/6$.

Measurements consistently reveal that the real strength of solids is much less than this value, lying somewhere in the region of $E/100$ to $E/1000$. In order to resolve the anomaly, it is necessary to consider the distribution of stress in a solid in the region of a crack tip. This was determined in the early years of the 20th century for elliptical cracks, a geometry chosen to make calculations feasible in the period before computers were available. It was found that the stress in the region of a crack tip (or, in fact, at many other objects such as voids, sharp corners or sharp grain boundaries) is much greater than the applied stress. Such flaws are called stress raisers. The amount that the stress is increased at a crack tip with an elliptical cross-section, the stress concentration factor, K_t, is given by:

$$K_t = 1 + 2\left(\frac{a}{\rho}\right)^{1/2}$$

where a is the length of a surface crack (Figure 10.10a) or half the length of an internal crack (Figure 10.10b), and ρ is the radius of curvature of the crack tip. For a relatively long crack with a sharp tip (Figure 10.10c).

$$K_t \approx 2\left(\frac{a}{\rho}\right)^{1/2} \qquad (10.4)$$

Clearly, for a long crack with a sharp tip, the stress at the tip can be many times that of the nominal applied stress.

When the stress at the crack tip reaches the strength of the material as calculated from chemical bond strength, bonds will break and the crack will lengthen, leading to fracture. The critical stress for a material with a crack is found to be (Section S4.2.2):

$$\sigma_c = \left(\frac{E\gamma}{2a}\right)^{1/2} \qquad (10.5)$$

This indicates that the strength of the solid, as measured by the average applied stress, decreases as the length of the crack in the material increases.

The role of cracks in the process of brittle fracture was first investigated quantitatively by Griffith some 90 years ago. In this theory, called the Griffith theory of brittle fracture, fracture was considered to be due to the presence of microscopic cracks or flaws, called Griffith flaws, distributed throughout the solid. Griffith suggested that failure occurred when the energy introduced into the solid as a result

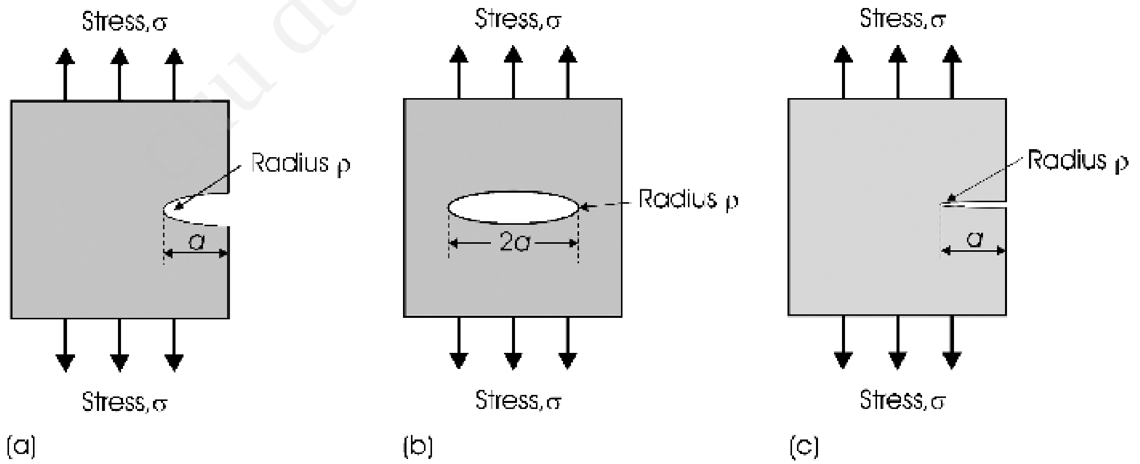

Figure 10.10 Stress at (a) an elliptical groove in a solid, (b) an elliptical pore in a solid, and (c) a sharp crack in a solid

of the tension was equal to the energy needed to create two new surfaces on either side of a growing crack. [Remember that at this time the concept of chemical bonds was still being worked out.] Using the energy approach, the critical stress, σ_c, to fracture the solid was determined to be

$$\sigma_c = \left(\frac{2\gamma E}{\pi a}\right)^{1/2} \tag{10.6}$$

where γ is the surface energy of the solid (per m^2), E is the modulus of elasticity, and a is the length of a surface crack or half the length of an internal crack (Figure 10.10). Equation (10.6) is known as the Griffith equation. This equation gives the maximum stress that a solid containing an internal crack of length $2a$, or a surface crack of length a, that the solid can sustain without the crack growing. The longer the crack, the lower the stress needed to cause fracture.

Equations (10.5) and (10.6) can be made identical by setting

$$\frac{\rho}{4r_0} = \frac{2}{\pi}$$

This gives an approximate value for the crack radius, ρ:

$$\rho \approx 2.5\,r_0$$

This gives a rule-of-thumb measure for the separation of the two surfaces. The surfaces that are formed as the crack opens can be considered separate when the crack has opened to about $5\,r_0$, or about 1 nm.

10.1.8 Plastic deformation of metals and ceramics

Metals are normally ductile, and plastic deformation can be considerable. This is a valuable property of metals and is used, for example, to fabricate dish shapes of one sort or another by impressing a die into a sheet of metal. The metal deforms plastically and retains the dished shape when the load on the die is removed. Ceramics are regarded as brittle, and normally the same fabrication process applied to a ceramic would shatter it. However, at high temperatures, many ceramic materials can deform plastically, and at low temperatures many metals become brittle and lose ductility. Thus, it is convenient to treat these two apparently dissimilar crystalline materials together. Polymers, which contain crystalline and amorphous regions, are considered in Section 10.1.11.

In the case of an ordinary metal, once the loading in a tensile test is continued past the yield stress (point A, Figure10.11a), the material will be permanently

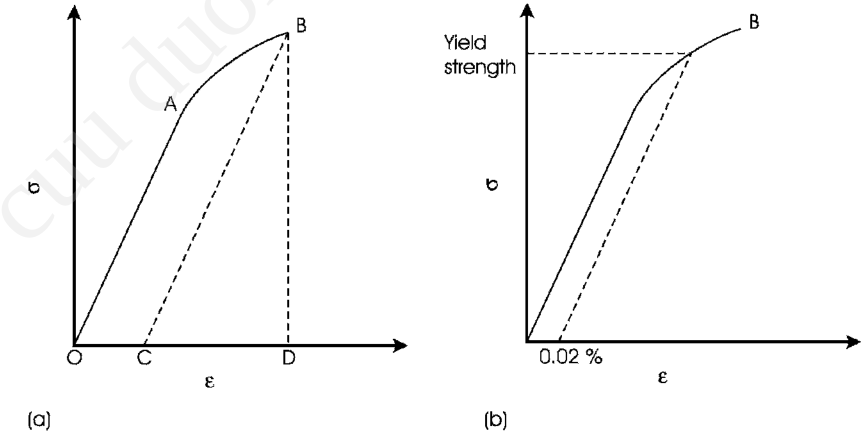

Figure 10.11 (a) Engineering stress–engineering strain (σ–ε) curve for plastic deformation, which results in a permanent distortion of the material, represented by OC. (b) The yield strength, determined by the intersection of an offset line, here at 0.02 %, parallel to the σ–ε curve Note: A, yield stress; B, load release point; D, strain under load; C strain at release of load

deformed. As the load is released, the plot of stress against strain will follow the line BC, which is parallel to AO. When the stress falls to zero, CD is the amount of elastic strain recovered, and OC represents the plastic or permanent deformation that has occurred. A rod so stressed will not return to its original thickness, but remains thinner than at the start. This property, used in drawing rods down into wires, is called plastic deformation.

The yield strength (a load) or yield stress (a stress value) is the point at which plastic deformation occurs (point A, Figure 10.11a). For most materials, the transition from elastic to plastic behaviour is rarely abrupt, and a single 'point' does not mark the boundary between elastic and plastic deformation. In order to obtain a guide as to when a stress value passes that required for plastic deformation to occur, it is usual to select a value of the stress that leads to 0.2 % plastic strain (0.002 strain). This is also called the 0.2 % offset yield strength. [This value is arbitrary, and any value could be chosen, such as 0.1% offset yield strength.] The yield strength is determined by drawing a line from the 0.02 % strain point parallel to the elastic section of the curve, to intersect the stress–strain curve (Figure 10.11b).

10.1.9 Dislocation movement and plastic deformation

From an atomic point of view, the origin of plastic deformation is most readily understood by studying single crystals. The deformation is revealed as a series of steps or lines, where parts of the crystal have moved relative to each other. The process can be one of slip, in which atom planes have slid sideways, or one of twinning, known as mechanical twinning. In slip, a small slice of crystal is moved sideways (Figure 10.12a). The slices are usually the order of a few hundred atoms wide and, during slip,

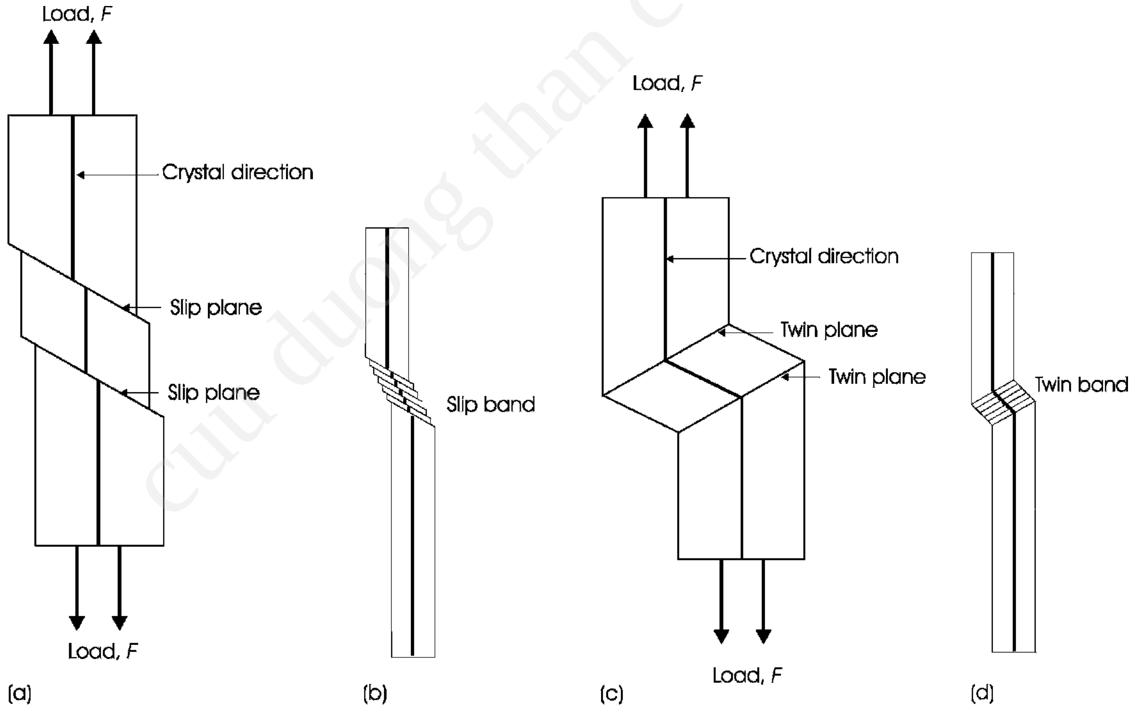

Figure 10.12 (a) Slip in a rod, characterised by diagonal planes across which atoms in the crystal have sheared because of an applied load; (b) slip band, in which slip planes are aggregated into narrow regions; (c) mechanical twin planes, across which the atoms in the crystal are reflected because of an applied load; (d) twin band, in which twin planes are aggregated. Note: slip and twinning are both caused by stress and are difficult to distinguish in macroscopic samples

a number of these translated slices group together to form a slip band (Figure 10.12b), which has the appearance of a line on the crystal face. The crystal direction along the deformed crystal does not change during slip.

The process of twinning gives a crystal in which the atom positions are reflected across the twin plane (Figure 10.12c). Twins also tend to occur in bands (Figure 10.12d). The distinction between slip and twinning is best determined by means of X-ray crystallography or transmission electron microscopy, as these techniques are able to reveal the crystallographic relations on each side of a planar boundary.

Slip is generally easier than twinning in metals, but the reverse is true for many ceramic materials. In polycrystalline solids, both twinning and slip may operate.

The amount of energy required to move a plane of atoms from one stable position to another, from A to B in Figure 10.13, can be calculated from the strength of the chemical bonds in the solid, in much the same way as the modulus of elasticity and strength of a solid is estimated (Section S4.2). It is found that the calculated energy is greater than that observed in practice. The discrepancy was resolved by a realisation that slip is achieved by the movement of dislocations through the structure. The motion of an edge dislocation through a metallic crystal is illustrated in Figure 10.14. In fact, slip is the process by which plastic deformation is produced by dislocation motion. The plane on which the dislocation moves is called the slip plane. The result of the dislocation movement is slip across a single atom plane (Figures 10.15a and 10.15b). A similar situation occurs with screw dislocation movement (Figures 10.15c and 10.15d). Mixed dislocations move by a combination of both mechanisms.

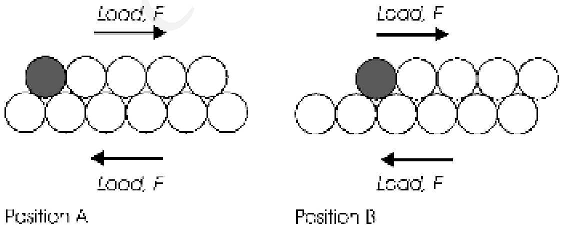

Figure 10.13 Slip, caused by a shear stress displacing atoms planes with respect to each other; it becomes increasingly difficult to move the atoms as the length of the atom rows increases

The movement of dislocations is constrained by crystallography. Some planes allow movement to take place more easily than do others, and these preferred planes are called slip planes. Similarly, dislocation movement is easier in some directions than it is in others. These preferred directions are called slip directions. The combination of a slip plane and a slip direction is called a slip system.

In general, slip planes are planes with the highest area density of atoms, and slip directions tend to be directions corresponding to the direction in the slip plane with the highest linear density of atoms. For example, the planes containing most atoms in metals that adopt the face-centred cubic (A1) structure are $\{1\,1\,1\}$ planes (Figure 10.16). In these planes, the greatest linear density of atoms occurs along the $\langle 1\,1\,0\rangle$ directions. In such a metal there are four different $\{1\,1\,1\}$ planes and three different $\langle 1\,1\,0\rangle$ directions. The number of slip systems is thus equal to 12. The possibilities are conveniently written as $\{1\,1\,1\}\langle 1\,\bar{1}\,0\rangle$.

Slip in hexagonal metal crystals occurs mainly parallel to the basal plane of the unit cell, normal to the c axis. The slip systems can be described as $\{0\,0\,0\,1\}\langle 1\,1\,\bar{2}\,0\rangle$, of which there are three. Body-centred cubic metals have slip described by $\{1\,1\,0\}\langle \bar{1}\,1\,1\rangle$, giving 12 combinations in all. Other slip systems also occur in metals, but those described operate at lowest energies.

10.1.10 Brittle and ductile materials

Although ceramics are regarded as being predominantly brittle, many show ductility at high temperatures. This is a general phenomenon, not limited to ceramics, and these ductile-to-brittle transitions are of great importance in engineering. It is generally found that materials are brittle at low temperatures and ductile at high temperatures, although the definition of 'high' and 'low' is very sensitive to the material under consideration. For an elastomer

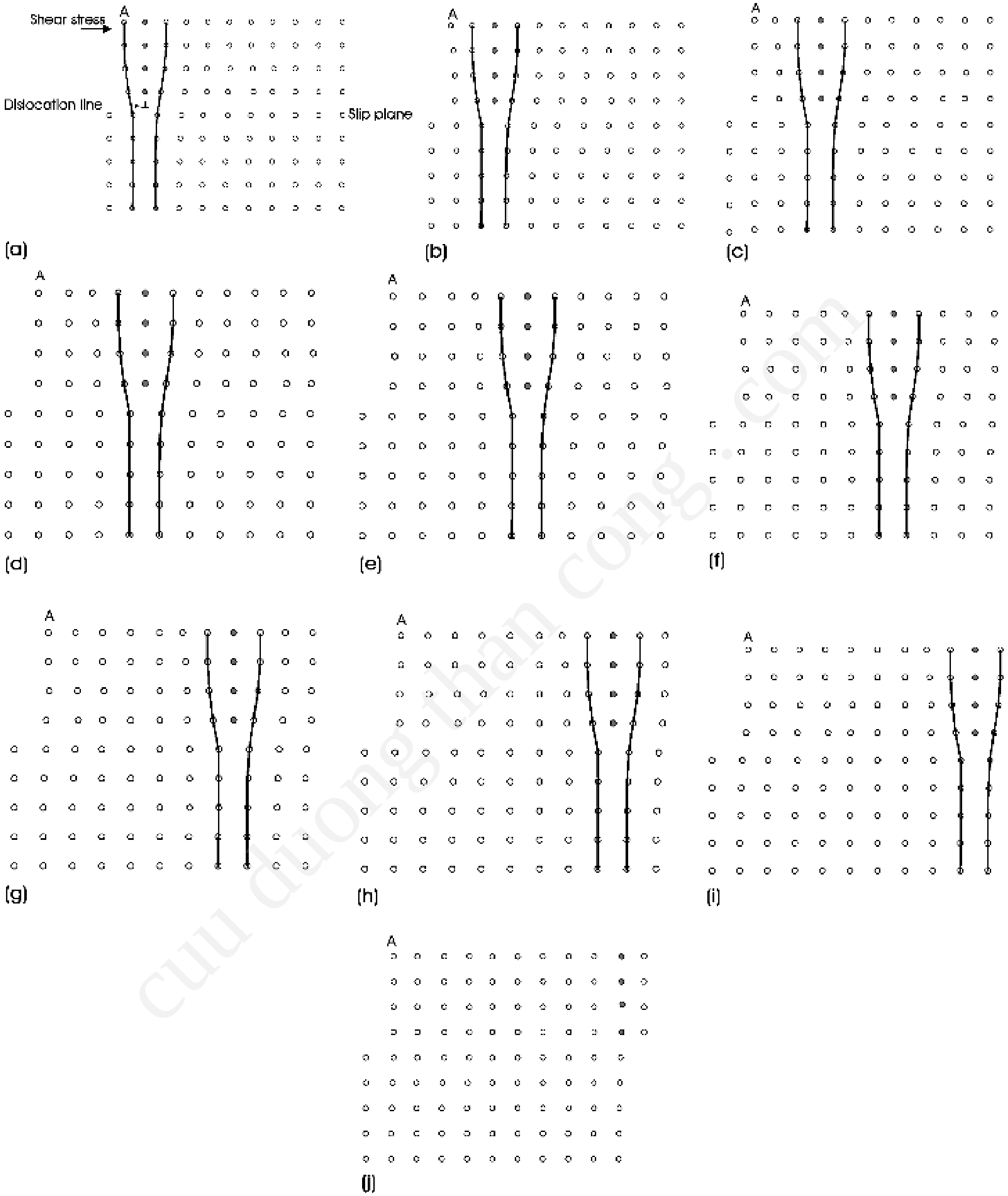

Figure 10.14 Slip caused by the movement of an edge dislocation under a shear stress. At each step (from part a to part b, part b to part c, etc.) only a small number of atomic bonds must be broken, and the process occurs at much lower shear stress levels than are needed in perfect crystals (Figure 10.13)

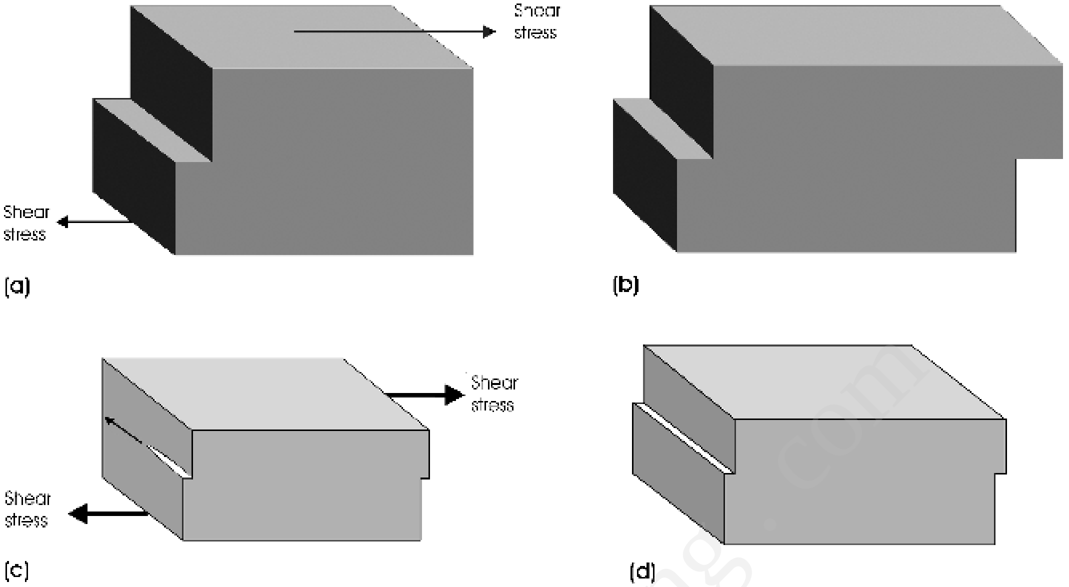

Figure 10.15 (a) Initial configuration in a crystal containing an edge dislocation; (b) final configuration after the dislocation has moved through the solid. (c) Initial configuration of a crystal containing a screw dislocation; (d) final configuration after the dislocation has moved through the solid

such as rubber, room temperature is already 'high'. In the case of polymeric materials, the glass transition temperature, T_g, gives a good indication of the boundary between the brittle and ductile regions.

In the case of ceramics, ductility begins to become important at higher temperatures. As an empirical guide, this is above the Tamman temperature, which is about half of the melting temperature in kelvin. In this regime, ceramics can deform via slip. However, dislocation motion is much harder in ceramics than it is in metals, partly because of the stronger bonding but also because of electrostatic

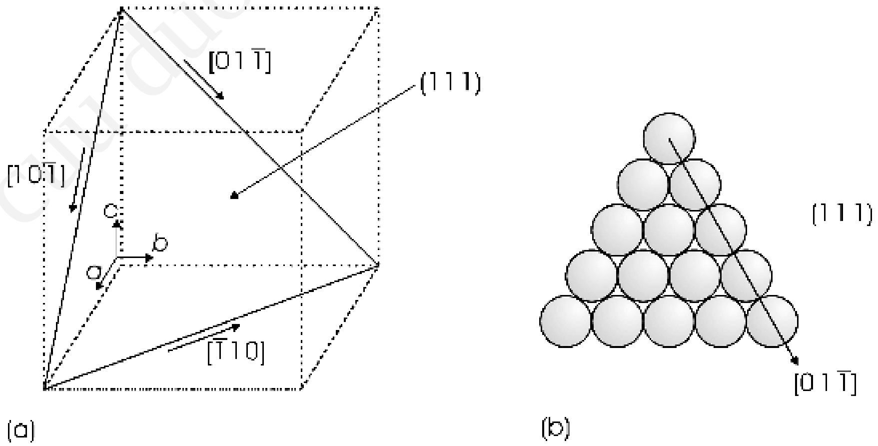

Figure 10.16 (a) The (1 1 1) plane in a crystal of a metal with the face-centred cubic (A1, copper) structure (shaded). The directions (arrows) along each of the edges are given. (b) The same (1 1 1) plane represented as a packing of atoms. One direction, $[0\,1\,\bar{1}]$, is marked

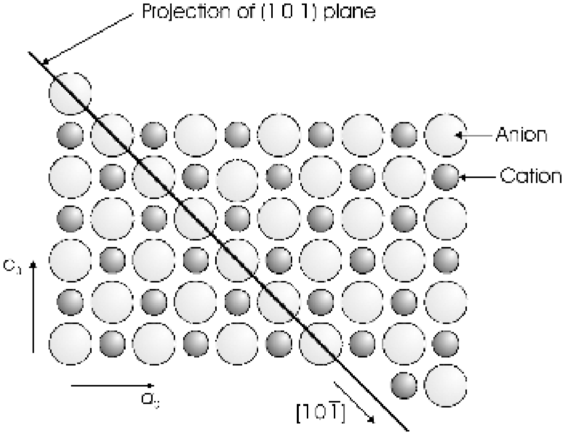

Figure 10.17 A $\{1\,1\,0\}$ slip plane in the *halite* (NaCl, B1) structure

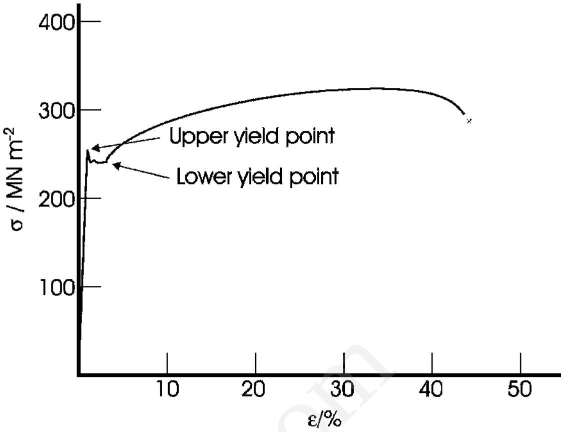

Figure 10.18 Engineering stress–engineering strain (σ–ε) curve for a typical carbon steel

repulsion between similarly charged ions, and twinning is often the favoured mechanism of releasing stress. The reason why ceramics tend to be brittle and metals ductile is therefore not due to the presence of dislocations in metals and their absence in ceramics but because of the greater difficulty of slip in ceramics at normal temperatures.

The preferred slip plane in ionic crystals with the *halite* (NaCl) structure, such as NaCl or LiF, is $\{1\,1\,0\}$, and the slip direction used is $\langle 1\,\bar{1}\,0\rangle$. This slip system is sketched in Figure 10.17. For the more metallic *halite* structure solids such as titanium carbide (TiC), the slip system is similar to that in face-centred cubic metals, $\{1\,1\,1\}\langle 1\,\bar{1}\,0\rangle$.

For some metals, notably steels, there is an abrupt break in the stress–strain plot at the upper yield point (Figure 10.18). This is followed by continued deformation at a lower yield stress, at the lower yield point, before the curve rises again. Between the upper and lower yield points, deformation occurs in localised regions that have the form of bands, rather than across the specimen in a uniform manner. The reason for this is that the dislocations, which would move during plastic deformation, are pinned in the steel, mainly by the interstitial carbon atoms present. At the upper yield stress, these become mobile and, once released, they can move and multiply at a lower stress value. This is analogous to sticking and slipping when a body overcomes friction. The region in which dislocation movement starts forms a band. Ultimately, the whole of the dislocation network is mobile, at which point the stress–strain curve begins to rise again, as in a ductile material.

10.1.11 Plastic deformation of polymers

Polymers that show considerable amounts of plastic deformation are usually partly crystalline thermoplastics or elastomers. The yield strength is taken as the initial maximum of the curve, and the tensile strength as the fracture point (Figure 10.4c). The tensile strength can be less than the yield point, especially at higher temperatures. In partly crystalline polymers, the stress is imposed on both the amorphous region and the crystalline region. The weakest links are those between the coiled chains in the amorphous regions, and these slip past each other as the sample elongates (Figure 10.19). Ultimately, these molecules become more or less aligned, and the stress now acts on the molecular chains, in which strong covalent bonding occurs. At this point, the crystalline regions can begin to slip, and, ultimately, fracture will occur. Elastomers behave differently in that the coiled regions predominate and can be uncoiled to give enormous extension before the molecules are all more or less

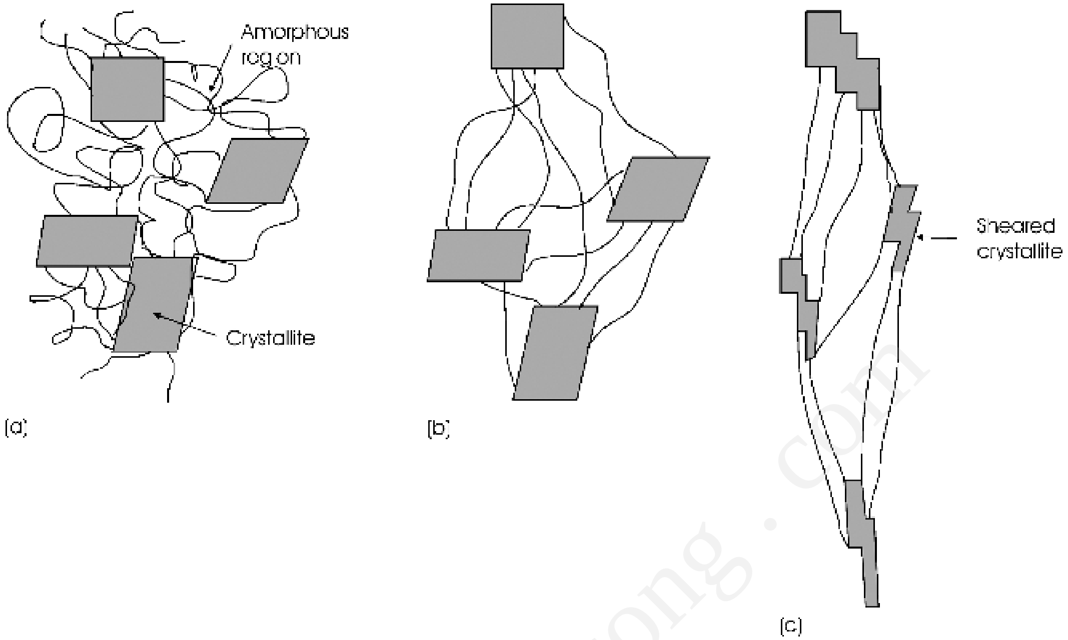

Figure 10.19 Deformation of a semicrystalline polymer (the crystalline regions are represented by shaded quadrilaterals). (a) The unstressed state. (b) On loading the polymer the molecules in the amorphous regions elongate initially. (c) When the polymer chains can no longer accommodate the stress, the crystalline regions deform

parallel and before the strong covalent bonds of the molecules are stressed (see Section 6.4.4).

10.1.12 Fracture following plastic deformation

Solids that show considerable plastic deformation before fracture generally fail in a different way from brittle solids, discussed above. An important parameter that characterises this type of failure is the ductility of the material. Ductility is a measure of the degree of plastic deformation that can be sustained by a material at fracture. The ductility of metals can be estimated by measuring the percentage elongation of a sample after fracture, where:

$$\text{elongation} = \frac{\text{final length } (l) - \text{initial length } (l_o)}{\text{initial length } (l_o)}$$

$$\text{percentage elongation} = \frac{l - l_o}{l_o} \times 100$$

During the early stages of deformation of a metal, the test sample retains its original shape, although when the stress is released the original dimensions are not recovered. When a metal fractures after considerable plastic deformation, the central part of the sample is found to have formed a neck. This occurs after the highest point in the stress–strain curve (Figure 10.20). Because a large amount of the final deformation occurs in a rather small neck region, the percentage elongation measured will depend on the value taken for l_0. It is for this reason that standard gauge lengths, usually 5 cm for metal samples, are used for specimens that are to undergo tensile testing.

The ductility can also be estimated by a measurement of the reduction in the cross-sectional area of the sample at the break. The percentage reduction in area of a sample after fracture is:

$$\text{reduction in area} = \frac{\text{initial area } (A_o) - \text{final area } (A_f)}{\text{initial area } (A_o)}$$

$$\text{percentage reduction in area} = \frac{A_o - A_f}{A_o} \times 100$$

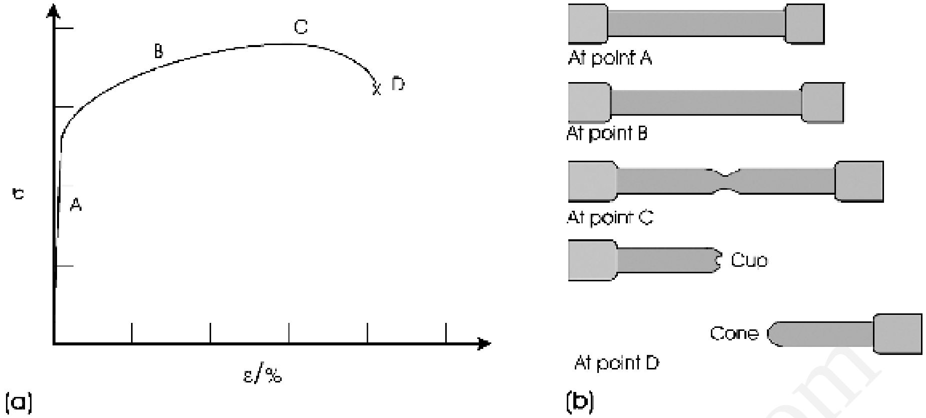

Figure 10.20 (a) Tensile engineering stress–engineering strain (σ–ε) curve for a ductile metal. (b) The corresponding shape of the test piece during the test

The fracture surface after ductile fracture has a characteristic shape, one side resembling a cup and the other a cone (Figure 10.20b, for point D; Figure 10.21).

Although most metals are ductile, some metals are brittle. Tungsten, a particularly brittle metal, is of considerable importance in daily life. It is the best material for use in incandescent lightbulbs, the normal lightbulbs that contain a white-hot filament. In the earliest years of the electric light industry, carbon filaments were used, but these were clearly seen to be a poor choice from the outset. Tungsten, an ideal choice, was too brittle to be formed into filaments or even coiled as wire. The process for making tungsten ductile involves drawing the wire at temperatures above the ductile-to-brittle transformation temperature, but at a temperature too low for the metal to recrystallise, and adding alloying elements, notably potassium, to modify the resulting grain structure of the wire.

The term ductility is generally reserved for metals and in the case of polymers ductility is replaced by elongation. The elongation is simply measured as the length of the polymer after stretching, l, divided by the original length, l_0, often given as a percentage:

$$\text{percentage elongation} = \frac{l}{l_0} \times 100$$

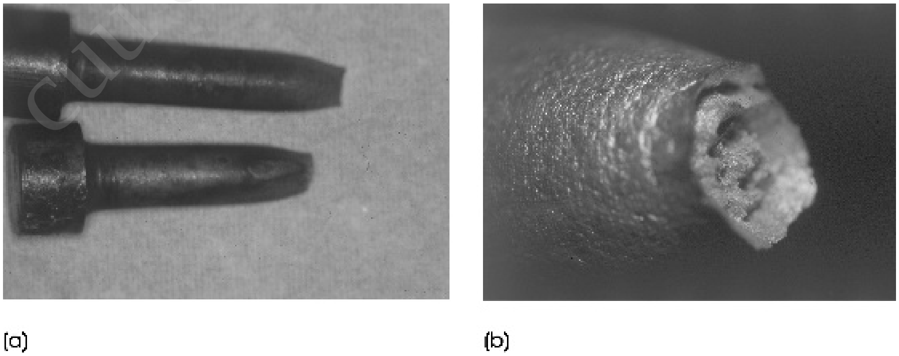

Figure 10.21 (a) A fractured semi-ductile steel tensile test specimen, showing necking and cup and cone fracture. (b) Detail of cup and cone fracture surface from a copper tensile test specimen. The diameter of the neck is approximately 2 mm

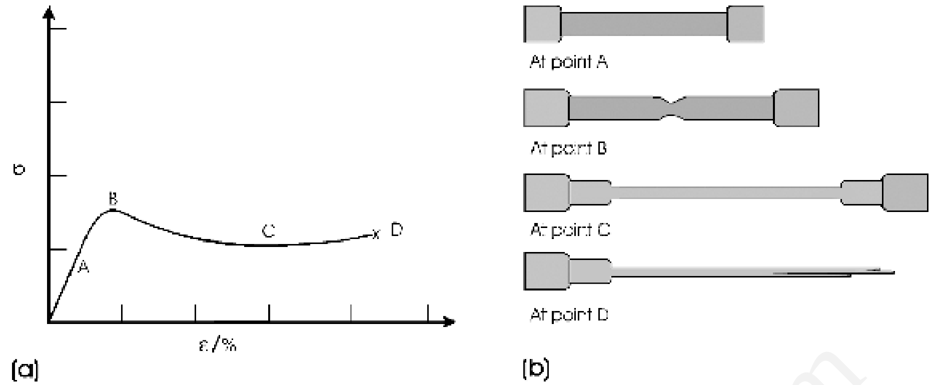

Figure 10.22 (a) Tensile engineering stress–engineering strain (σ–ε) curve for a polymer. (b) The corresponding shape of the test piece during the test

The deformation of a polymer tensile specimen is rather different from that of a metal. Polymers neck, and the neck region itself will extend in a ribbon until the material ultimately tears (Figure 10.22). The term 'percentage elongation at fracture' used for metals is generally replaced by 'percentage elongation at break' for polymers.

In both metals and polymers, fracture is preceded by the formation of voids or holes. In a metal, these voids tend to segregate at the central region of the neck, and coalesce into a large crack. This crack ultimately spreads to the edge of the neck. The fracture surface has a pulled-out, fibrous texture exhibited in the 'cup' and the 'cone' regions of the break. In the case of a polymer sheet, the holes are formed by the continual alignment of the polymer chains. Ultimately, the separate holes coalesce to form a long tear. As with metals, the edges of the tear have a pulled-out, fibrous texture.

10.1.13 Strengthening

Knowledge of the ways in which solids fail can be exploited to improve the mechanical properties, particularly the strength, of solids. Pure metals tend to be soft and relatively weak. Ductility can be reduced and the metal strengthened by restricting dislocation movement. However, if this is continued too far the metal will become brittle. A compromise is often required.

Historically, three methods have been used to strengthen metals. Two of these, reduction in grain size and work hardening, are brought about by heat treatment and mechanical deformation, for example hammering. The third method, alloying, was for most of historical times based on empirical observation. As technology advanced, these methods have been supplemented by precipitation strengthening, in which careful heat treatment is used to initiate the formation of precipitates within the metal matrix. In all cases, the metal is strengthened because dislocation movement is restricted.

Alloying, first widely used to transform soft copper into much stronger bronze, relies on the strain set up in the crystal structure by the impurity or dopant atoms. This strain field impedes dislocation movement because dislocations also generate a strain in the structure. The two strain components mutually repel each other to hinder slip. If sufficient of the second component is added, precipitates of a second phase can form in the crystal matrix. These hinder dislocation movement simply by blocking slip planes. Reduction of grain size has a similar effect. Dislocations can cross from one grain to another only by expending energy, and often grain boundaries block almost all dislocation movement. Work hardening, which is also called cold working or strain hardening, is the result of repeated deformation. This causes dislocation numbers to increase. Initially, this can lower the strength but beyond a certain dislocation density dislocations become tangled and movement becomes impeded. Again, this is due to interaction of the strain fields around the dislocations.

Ceramics normally fail in a brittle mode. They can be strengthened by the removal of the Griffith flaws. Although this is not always possible, careful protection of the surface from contamination, chemical reaction or mechanical damage will help. Optical fibres, when freshly drawn, are extremely strong, but reaction of the surface with water vapour present in the air rapidly degrades the fibre strength. For this reason, freshly drawn fibres are immediately coated with a polymer to maintain strength. Similarly, single-crystal ceramics can have surfaces carefully polished to increase strength. Polycrystalline ceramics need to be fabricated in such a way as to minimise the amount of internal porosity.

Polymers are somewhat different. The strength of a polymer is generally greatly enhanced by increasing the amount of crystalline phase present with respect to the amount of amorphous phase present. Similarly, cross-linking will transform a soft thermoplastic into a hard and brittle solid. However, a strengthening route available to polymer science that is not available for metals and ceramics is the formation of block copolymers. Copolymers of an elastomer and a fibre will give a strong but elastic fibre, and copolymers of elastomers with brittle polymers will give a tough and flexible solid.

For many engineering (and biological) applications, solids are strengthened by mixing two materials together. In the case of polymers, these can be blended, but for metals and ceramics the two phases remain separate, and the material is called a composite (see Section 10.4).

10.1.14 Hardness

Hardness is a property that is intuitively understood but difficult to define. It is usually taken to be a measure of the resistance of a material to permanent (plastic) local deformation and is often measured by forcing a chosen solid into the surface of the material to be tested. Hardness therefore measures a compressive property rather than a tensile property.

The first use of hardness was in the characterisation of minerals. The hardness of a mineral was determined by observation of whether it could scratch or be scratched by another mineral. This is sometimes called the scratch hardness of a solid. The *ad hoc* system in use in medieval times was quantified by Mohs, who listed 10 minerals that, as far as possible, were equally spaced on a hardness scale of 1 to 10. Even today, the commonest description of hardness is still in terms of this scale, called the Mohs scale of hardness. The minerals chosen, and their hardness are: 1, talc [$Mg_3Si_4O_{10}(OH)_2$]; 2, gypsum ($CaSO_4 \cdot 2H_2O$); 3, calcite ($CaCO_3$); 4, fluorite (CaF_2); 5, apatite [$Ca_5(PO_4)_3(OH)$]; 6, orthoclase ($KAlSi_3O_8$); 7, quartz (SiO_2); 8, topaz [$Al_2SiO_4(OH)_2$]; 9, corundum (Al_2O_3); and 10, diamond (C). In addition, Mohs suggested the following values: fingernail, less than 2; copper coin, 3; pocket knife blade, just over 5; window glass, 5.5; and a steel file, 6.5. Each of these materials will scratch the surface of the solid immediately below it in the list. The softest material is talc, which will not scratch any of the others. Diamond is the hardest material known.

To measure hardness more precisely a known load is applied slowly to a hard indenter that is placed onto the smooth surface to be tested. The surface is deformed plastically, and the indent size or depth after the indenter is removed is taken as the measure of the indentation hardness of the material. The hardness is often recorded as an empirical hardness number, related to the size of the indentation.

There are four major indentation hardness tests, which differ from each other in the shape of the indenter (Figure 10.23). The first of these, described in 1900, was the Brinell test, using a 10 mm steel ball indenter (Figure 10.23a), giving the Brinell hardness number, BHN. This was suitable only for metals softer than steel. In 1920 Rockwell developed a number of tests, including the B, E, F and G scales, in which the indenter is steel, and the A, C and D scales, using a conical diamond indenter with a spherical tip (Figures 10.23b and 10.23c). In the Rockwell test the difference in size between the

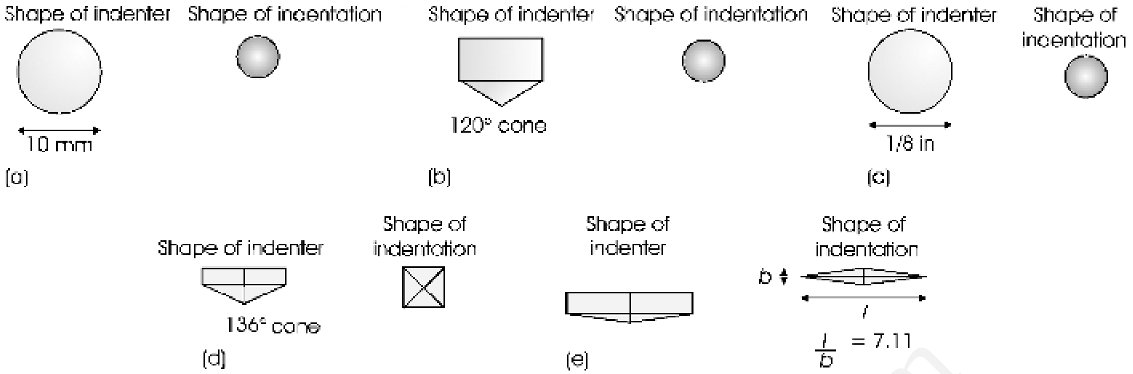

Figure 10.23 Hardness indenters: (a) Brinell steel or tungsten carbide sphere; (b) Rockwell A, C, D diamond cone; (c) Rockwell B, F, G steel sphere; (d) Vickers diamond pyramid; and (e) Knoop diamond pyramid

indentations caused by a small and a large load are compared to give the Rockwell hardness number, RHN. The Vickers test, introduced in 1924, uses a pyramidal diamond indenter (Figure 10.23d) and a standard load time of 15 s, to give the Vickers hardness number, VHN. The Knoop test, widely used for brittle materials such as minerals and glass, was introduced in 1939. This method uses an elongated pyramidal diamond indenter (Figure 10.23e) and gives the Knoop hardness number, KHN. Both the Vickers test and the Knoop test use small loads, of the order of grams, and measure what is generally called microhardness.

When hardness was first measured, it was reported simply as a series of internally consistent 'hardness numbers', as Mohs hardness still is. Vickers hardness can readily be converted into conventional units. The Vickers hardness is given by the applied load (kg) divided by the projected area of the indentation (mm^2). The units of kg mm^{-2} is readily converted into SI units by converting the load in kg to N, and the area into m^2, so that 1 kg mm^{-2} is equal to 9.81 MPa.

The development of thin-film engineering and the increasing use of nanomaterials have led to the development of nano-indenters. These are described in more detail in Section 10.3.3.

The hardness of a material is related to the strength of the chemical bonds within the solid. As with other mechanical properties, however, the direct correlation is masked by the defect structure of the solid. The deformation, that acts as a measure of hardness, is produced by compression and subsequent plastic deformation. The compression is a temporary elastic displacement operating while the load is imposed. Plastic deformation is due to dislocation slip, initiated by shear stress. It has recently been found there is a good correlation between shear modulus and hardness (Figure 10.24). This allows the hardness of new materials

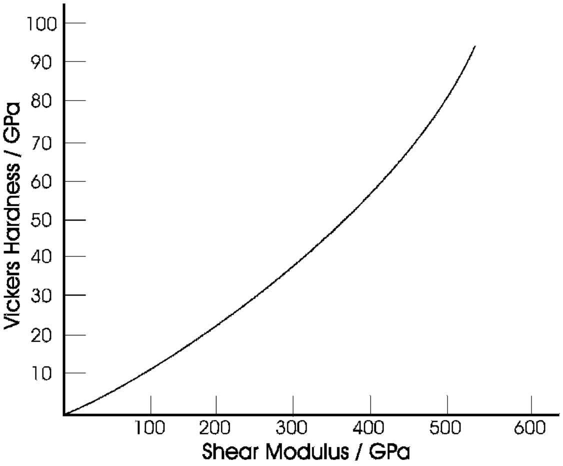

Figure 10.24 Schematic relationship between Vickers hardness and shear modulus. Adapted from D.M. Teter, 1998, 'Computational Alchemy: The Search for New Superhard Materials', *Materials Research Society Bulletin* **23** (January) 22

to be predicted from knowledge of the shear modulus.

In recent years, efforts have been made to try to make a harder material than diamond. The hardness of diamond is attributed to the strong sp^3 hybrid bonds linking the crystal into one giant molecule, and most attempts at a synthetic alternative have focused on iso-electronic crystals. These are crystals with the same average number of bonding electrons available as in diamond, four per atom. The idea is to force the atoms in the new material to bond via strong sp^3 tetrahedral hybrids. The first such material made was cubic boron nitride, BN. It is seen from the periodic table that boron lies one place to the left of carbon, and has three valence electrons per atom, whereas nitrogen, one place to the right, has five. The combination of equal numbers of boron and nitrogen atoms gives an average of four valence electrons per atom. Cubic boron nitride is the second hardest material, after diamond. [A form of boron nitride resembling graphite, with a low hardness, is also known (Section 5.3.8).]

Compounds intermediate in composition between diamond and cubic boron nitride, such as BC_2N, have recently been synthesised, as well as other iso-electronic compounds such as B_2CO and B_5NO_2, but none, so far, has the hardness of diamond.

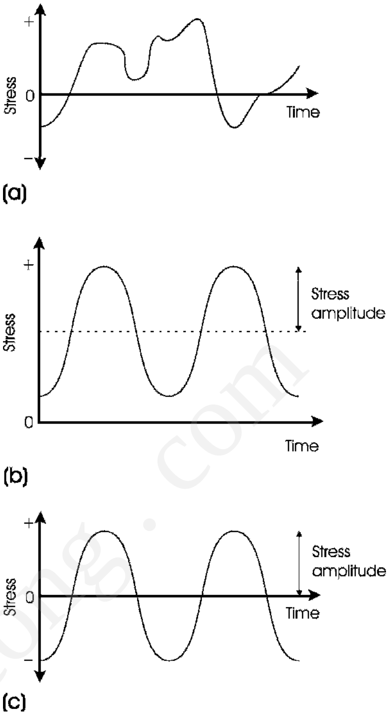

Figure 10.25 Stress cycles: (a) irregular, (b) positive sinusoidal cycle and (c) sinusoidal with alternating positive and negative stress

10.2 Time-dependent properties

A number of mechanical properties are a function of time. Although these are not so amenable to theoretical analysis as the elastic properties of solids, they are of enormous practical importance. Two of these properties, fatigue and creep, will be described in this section.

10.2.1 Fatigue

When placed under a cyclical or varying load a material may fail as a result of fatigue. This is invariably at a much lower stress than the part could withstand during a single application of the stress. Failure under repeated cycling is called fatigue failure. Fatigue affects moving parts in machinery but is also of importance in components that are slightly flexed in a repetitive fashion, such as an aircraft fuselage under the varying pressure regimes in the atmosphere.

The stress pattern imposed on a solid can vary greatly, from sinusoidal changes, often used in testing laboratories, to completely irregular patterns (Figure 10.25). In the laboratory, a sample is tested by imposing a cyclic strain and testing until the part fails. The results are displayed as the stress amplitude, *S*, plotted against the number of cycles that the sample can tolerate, *N*, before failure (Figure 10.26). Alloys of iron (ferrous alloys) have a behaviour represented by curve A, and most nonferrous alloys and pure metals such as copper or aluminium have a behaviour represented by curve B. In curves of type A, the abrupt change in slope is called a knee, and the part of the curve parallel to the *N* axis is called the endurance limit or fatigue limit. At stress levels

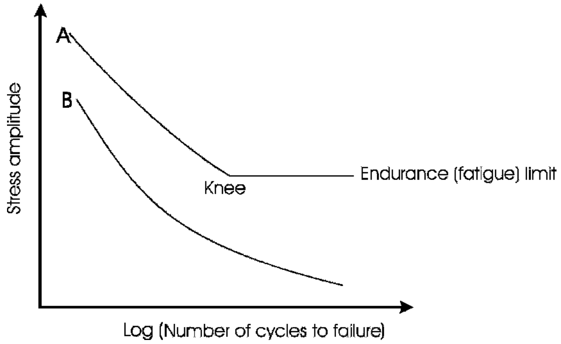

Figure 10.26 S–N curves (stress amplitude plotted against the logarithm of the number of cycles to failure): curve A, typical steel; curve B, many nonferrous metals and alloys, such as copper, aluminium, brasses, etc.

below the endurance limit, fatigue failure will never occur.

The process of fatigue is empirically divided into a number of stages. The initial damage occurs in stage 1. An originally perfect specimen will crack at an angle of approximately 45° to the direction of the cyclic stress (Figure 10.27). The surface also becomes distorted at the crack, giving rise to intrusions and extrusions, called persistent slip bands. The crack propagates across a small number of grains at this time. Stage 2 of the failure is indicated by a change in crack direction to approximately normal to the cyclic stress direction. During this stage, the crack opens a little during each cycle, gradually spreading across much of the specimen. In practice, it is not easy to separate stages 1 and 2 of the process. Stage 3 represents the final failure of the component. This occurs when the crack in stage 2 has grown to such an extent that the whole part fails catastrophically, by ductile or brittle failure, depending on the nature of the material.

As might be anticipated, an important factor in fatigue is the state of the surface. Surface flaws and roughness can increase the stress locally by a large amount, thus initiating stage 1 of the process. Additionally, notches, sharp edges, holes or changes in cross-sectional area, called stress raisers, should also be avoided. (Aircraft have rounded windows, rather than square ones, for this reason!) The local environment is also relevant, and fatigue initiated by corrosion, called corrosion fatigue, is often important in chemical plants.

10.2.2 Creep

Creep is the gradual elongation of a material under a constant load or stress. That is, under a constant stress, a gradual increase in strain (creep) can be

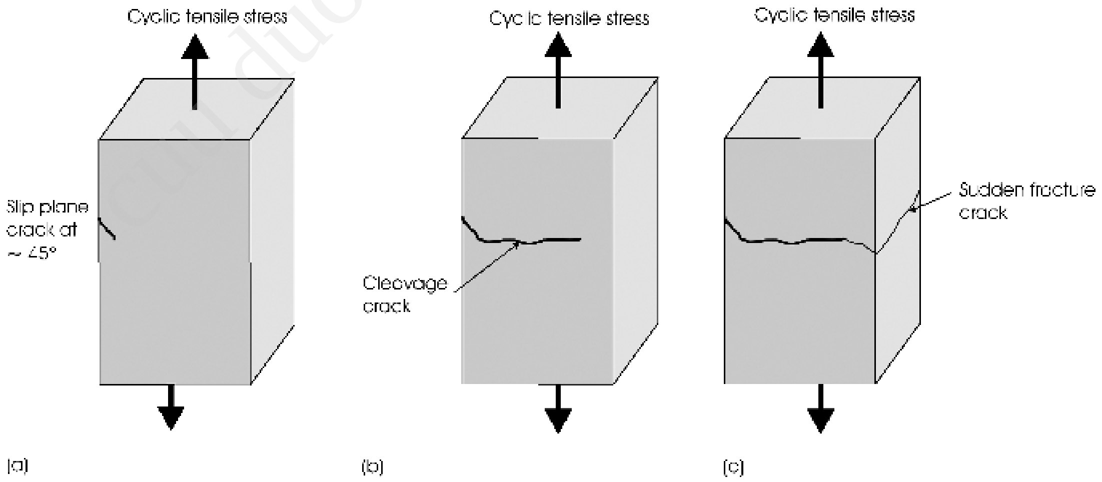

Figure 10.27 Fatigue failure: (a) stage 1, consisting of initial crack formation on a slip plane; (b) stage 2, consisting of crack growth along cleavage planes; and (c) stage 3, consisting of sudden failure as a result of rapid crack propagation

observed. Although creep is not very important at normal temperatures for most metals and ceramics, creep of polymers can be extensive. It is especially important in high-temperature applications such as in turbine blades, where even the smallest change in dimensions can lead to catastrophic failure. Creep is mainly due to progressive plastic deformation of the solid under the constant load. Because the strain in the material increases, creep must be caused by the movement of material. It can be movement of crystallites at grain boundaries, movement of dislocations or diffusion of atoms. The mechanism of creep will often depend on the temperature as well as the solid in question. For example, diffusion is more likely at high temperatures in metals and ceramics. Low-temperature creep in plastics is due to movement between the coiled polymer chains in the material.

Creep is usually displayed as a graph of strain against time, in a creep curve (Figure 10.28). The section of the curve OA is the extension that occurs when the specimen is first stressed. This is elastic deformation for most ordinary loads and corresponds to the straight-line part of a stress–strain curve. Creep as such is indicated by the remaining parts of the curve.

10.2.2.1 Primary creep

The initial section is the primary stage (Figure 10.28), also called primary creep. In this regime, the creep rate is continually decreasing. The strain is largely believed to be by dislocation movement in the case of metals and ceramics, and by polymer chains sliding past each other in plastics. This part of the creep curve is described by one of two equations:

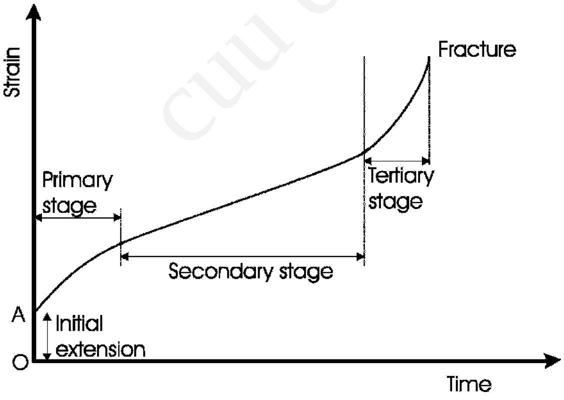

Figure 10.28 A normal creep curve for a solid subjected to a constant load at constant temperature

$$\varepsilon = \alpha \log t$$

where ε is the strain, t is the time, and α is a constant. This equation is applicable at lower temperatures, fitting well with polymers such as rubber. At higher temperatures the equation

$$\varepsilon = \beta t^m$$

is applicable, where ε is the strain, β and m are constants, and t is the time. The value of m can vary between about 0.03 to 1.0, a range of about 10^2.

The decreasing rate of creep is attributed to cold working in the case of metals. Here, the dislocations move until they begin to impede each other. In the case of polymers, uncoiling of tangled polymer chains proceeds at slower rates as the energetically easiest rearrangements give way to rearrangement involving cross-linking or crystal movement.

10.2.2.2 Secondary creep

The middle, secondary, stage of the curve (Figure 10.28) is linear and is called secondary or steady-state creep. In this case, the internal relaxation in the solid is balanced by the strain induced by the load. This linear portion of the curve is described approximately by an equation

$$\varepsilon = K t$$

where ε is the strain, and K is a constant, the slope of the linear portion of the curve.

A number of mechanisms have been proposed to explain secondary creep. At lower temperatures, the rearrangements of dislocations into lower-energy configurations, by dislocation glide, is thought to be the most important process. As the temperature increases, dislocation climb is thought to become increasingly important. These processes lead to the reduction in the number of dislocations present, a process called recovery. This mechanism of creep, in which dislocation movement controls the creep rate, is called dislocation creep or power law creep, as the slope of the creep curve, K, is given by an equation of the general type

$$K = \frac{d\,\varepsilon}{d\,t} = \frac{\pi^2 D \sigma^n}{(bN)^{1/2} G^n k T} \tag{10.7}$$

where D is the diffusion coefficient of the rate-limiting species, G is the shear modulus of the solid, b is the Burgers vector of the rate-limiting dislocations, N is the number of dislocations, k is Boltzmann's constant, T is the absolute temperature, and σ is the stress. The exponent n has a magnitude of about 5–6 for metals and ceramics and about 2 for plastics.

At higher temperatures, diffusion is widely believed to control the rate of creep in ceramics and metals, and this process is called diffusion creep. Two mechanisms have been suggested. At relatively lower temperatures in the diffusion creep regime, creep rate is limited by diffusion along the grain boundaries, that is, short-circuit diffusion is the rate-limiting step. This is referred to as Coble creep (Figure 10.29a). The slope of the secondary creep curve for Coble creep is given by:

$$\frac{d\,\varepsilon}{d\,t} = \frac{47\,\sigma\,\Omega\,D_b\delta}{k\,T\,d^3} \tag{10.8}$$

where σ is the stress, Ω is the atomic volume, D_b is the short-circuit diffusion coefficient, δ is the grain boundary width, d is the grain diameter, k is the Boltzmann constant, and T is the absolute temperature.

At relatively higher temperatures, diffusion of atoms within the grains, that is, bulk diffusion, becomes more important in metals and ceramics. This is called Herring–Nabarro creep (Figure 10.29b). In this model, the slope of the secondary creep curve is given by:

$$\frac{d\,\varepsilon}{d\,t} = \frac{13.3\,\sigma\,\Omega\,D}{k\,T\,d^2} \tag{10.9}$$

where Ω is the volume of the vacancy, D is the vacancy diffusion coefficient for diffusion within the grains, d is the grain diameter, k is the Boltzmann constant, T is the absolute temperature, and σ is the stress.

In both of these processes, atom diffusion is away from boundaries under compression towards boundaries under tension, leading to relief of the stress at the grain boundaries generated by the load on the sample. This causes the grains to elongate along the direction of the stress. [Note that diagrams often show the flow of vacancies rather than the flow of atoms. Vacancy flow is opposite to atom flow, and is from boundaries more or less parallel to the compressive stress to boundaries more or less perpendicular to the compressive stress.]

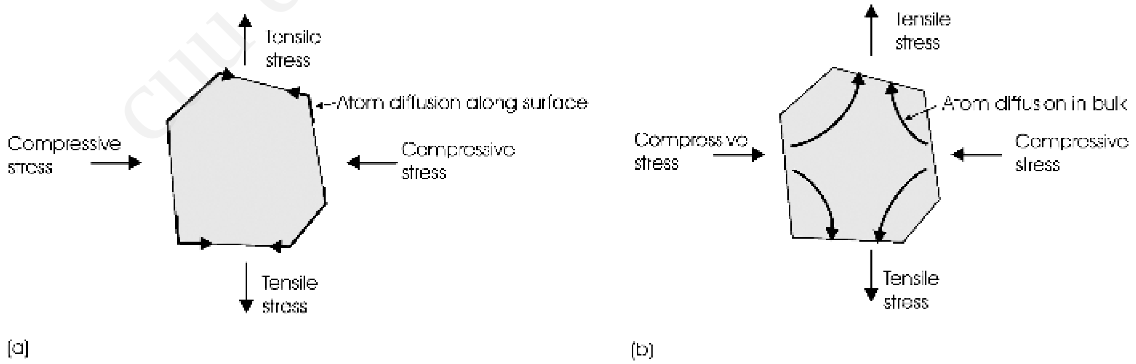

Figure 10.29 (a) Coble creep, in which atoms diffuse along grain boundary surfaces to grain boundaries parallel to the compressive stress; (b) Herring–Nabarro creep, in which atoms diffuse within grains towards boundaries parallel to the compressive stress. Note that vacancy diffusion will be in the opposite direction from that of atom diffusion (shown)

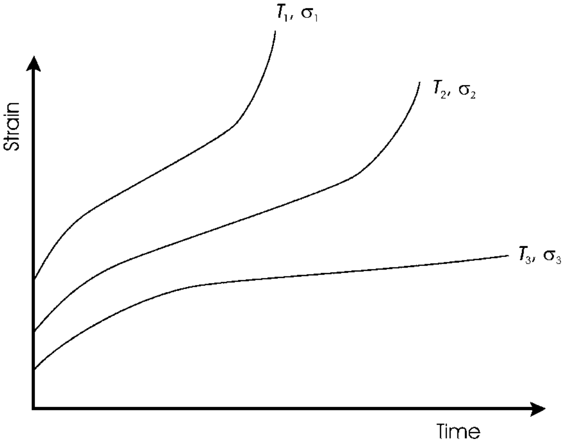

Figure 10.30 The effect of temperature on creep curves. When the stresses σ_1, σ_2 and σ_3 are equal, the temperature values T_1, T_2 and T_3 lie in the sequence $T_1 > T_2 > T_3$. When the temperatures T_1, T_2 and T_3 are equal, the stress values σ_1, σ_2 and σ_3 lie in the sequence $\sigma_1 > \sigma_2 > \sigma_3$

Equations (10.8) and (10.9) reveal that the rate of diffusion creep increases rapidly as the grain size decreases. This has implications for the creep of thin films and nanomaterials. In such fine-grained solids, creep is often surprisingly large, compared with the bulk phase, because of the grain boundary effect.

The various mechanisms proposed for secondary creep do not operate in totally separate temperature regimes. All tend to overlap, and the rate of creep can often best be generalised as a complex function of the many material parameters used in Equations (10.8) and (10.9).

Despite this complexity, the temperature dependence of the secondary creep rate can often be given by an Arrhenius-type equation (see Section 7.4):

$$K = \frac{d\varepsilon}{dt} = A\,\sigma^n \exp\left[\frac{-Q}{RT}\right]$$

where A, Q and n are constants. Q is called the activation energy for creep and is found to have a similar magnitude to the activation energy for self-diffusion in the material. The constant n varies from about 2 to about 6 and is dependent on the material in question.

The whole of the creep curve is enhanced at higher temperatures or at greater stress loading (Figure 10.30).

10.2.7.3 Tertiary Creep

The tertiary stage, or tertiary creep, is characterised by a rapid increase in the strain, leading to failure or creep rupture (Figure 10.28). At this stage, voids form in the region of the fracture and there is considerable grain boundary movement. As would be anticipated, this part of the curve is difficult to analyse theoretically.

10.3 Nanoscale properties

In general, one should not suppose that the properties of bulk materials will apply to materials at the nanoscale level. With respect to the mechanical properties of small-scale solids, it is known that the elastic behaviour, due to bond stretching and twisting, does not vary significantly in nanoparticles compared with that in the bulk. Other properties are more sensitive. For example, the rate of diffusion creep (Nabarro–Herring and Coble creep) is dependent on grain size. Hence, creep will be enhanced in compacts of nanoparticles and in thin films.

In this section, three illustrative examples of the impact of scale on mechanical properties are outlined. First, solid lubricants are discussed, underlining the connection between crystal structure and the observed mechanical properties. Second, auxetic materials, in which crystal structure and microstructure combine to produce materials with negative values of Poisson’s ratio, are discussed. Last, thin films, in which mechanical properties are measured by methods similar to that used in the bulk, are considered.

10.3.1 Solid lubricants

Solid lubricants embody the opposite properties to those of hardness. Solid lubricants are soft and feel greasy to the touch. As with all lubricants, solid lubricants reduce friction and wear and prevent damage between surfaces in relative motion. Solid lubricants have advantages over normal liquid lubricants in certain conditions, including: at high temperatures, where liquids can decompose or boil; at low temperatures, where liquids can freeze; or in a vacuum, where liquids can evaporate or contaminate the vacuum. With respect to use in a vacuum, space applications are becoming more important. Originally, space programmes were frequently limited by electronic reliability. Now that these problems are largely solved, the mechanical wear of moving parts has become of prime importance, and solid lubricants that can operate under the harsh conditions imposed by space exploration are being sought continually.

Solid lubricants need low shear strength in at least one dimension. Solid lubricants fall into three main classes – inorganic solids with a lamellar (layer-like) crystal structure, solids that suffer plastic deformation easily, and polymers in which the constituent chains can slip past each other in an unrestricted way. Although soft metals such as tin and lead have been used for many years as bearings, and polymers, especially Teflon® (PTFE, polytetrafluoroethylene) are used as a coating to create 'nonstick' surfaces, the two categories of most importance are layer structures and soft inorganic compounds.

The most familiar solid lubricant is graphite. This has a layer structure (Figure 10.31a). The layers are about 0.335 nm apart and are linked by weak van der Waals bonds. However, dry graphite is not a perfect lubricant, and much of the lubricating action seems to stem from adsorbed water vapour on and between the layers. This prevents pure graphite from being used for vacuum applications. The lubrication properties are greatly enhanced by forcing the layers apart by inserting atoms into the van der Waals gap. Of these materials, fluorinated graphite, CF_x, in which x can take values between 0.3 and 1.1, is one of the most successful. This material, originally developed for use in lithium batteries (see Section 9.3.6), has a structure in which fluorine atoms covalently bond to the carbon atoms. This destroys the sp^2-bonded skeleton of the planar carbon layers, which become puckered as in diamond. (Figure 10.31b). The increase in spacing between the layers, from 0.335 nm in graphite to 0.68 nm in CF_x, weakens the interlayer interaction so much that lubricating properties are greatly enhanced.

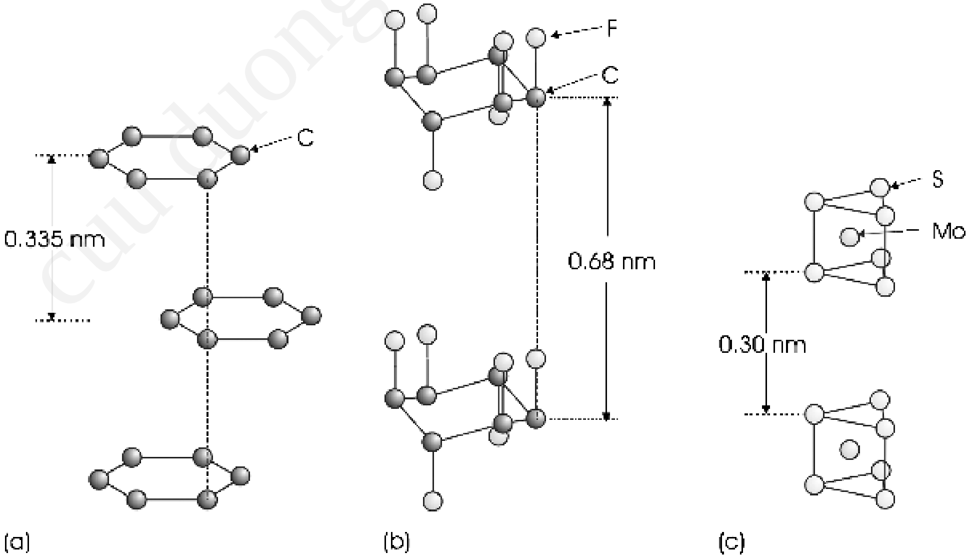

Figure 10.31 The structures of solid lubricants: (a) graphite; (b) graphite fluoride, CF_x; and (c) molybdenum disulphide, MoS_2

Molybdenum disulphide, MoS_2, is another layer structure widely used as a solid lubricant (Figure 10.31c). It is composed of MoS_2 layers, weakly linked by van der Waals bonds. The Mo atoms are surrounded by six sulphur atoms in the form of a trigonal prism. Although the MoS_2 layers are closer together than are the carbon layers in graphite, MoS_2 acts as a much better lubricant for most purposes.

Both graphite and molybdenum disulphide oxidise at higher temperatures in air, and the main alternatives used are soft inorganic fluorides. Like many ceramics, although they show brittleness at room temperature, they display ductility at temperatures of about half the melting point. A solid lubricant that is widely used in the temperature range from approximately 540 °C to 900 °C is the eutectic mixture of calcium fluoride, CaF_2, and barium fluoride, BaF_2. The melting point of the eutectic is 1022 °C.

At present, there is much research on solid lubricants, driven by the requirements of space exploration, and of high temperature engines as well as of a large range of devices in between.

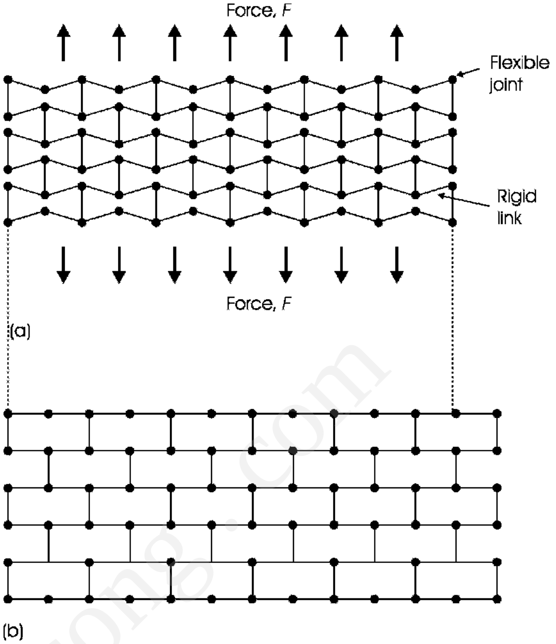

Figure 10.32 An auxetic network. The bonds (drawn as lines) between elements of the structure are of fixed length, but the links (circles) are flexible. (a) Initial state. (b) Configuration under an applied force, *F*; the material expands both parallel and perpendicular to the direction of the force

10.3.2 Auxetic materials

Auxetic materials expand laterally when subjected to a tensile strain. That is, unlike elastic, which gets thinner when pulled, auxetic substances get fatter when pulled. They have a negative Poisson's ratio. This counterintuitive mechanical property was first noticed in foam-like structures (Figure 10.32). Since then the property has been found in other materials

One method of producing auxetic structures is to utilise the microstructures of semicrystalline polymers. These solids are tailored so that rigid blocks of structure are linked by strong bonds. In the usual geometry (left-hand side of Figure 10.33a) the solid behaves normally and becomes thinner as the material is stretched (right-hand side of Figure 10.33a). However, if the bonds have a re-entrant geometry (left-hand side of Figure 10.33b), a tensile force will cause the blocks to move apart, and the material will become fatter (right-hand side of Figure 10.33b). In semicrystalline polymers, the crystallites form the rigid blocks, and the disordered polymer chains between them form the bonds. This type of structure has been made in highly crystalline polyethylene [ultra high-molecular-weight polyethylene (UHMWPE)] and polypropylene fibres. The transformation from a normal polymer to an auxetic solid is achieved by making the crystalline portion the major component and reducing the polymer chains linking the crystallites to such an extent that they become short noncoiled links that act as the rigid bonds depicted in Figure 10.33b.

Surprisingly, many cubic metals behave in a similar fashion, although the effect has been masked by the fact that the mechanical properties are most often measured on polycrystalline solids. For example, the commonplace metallic alloy β-brass, CuZn, which has the CsCl structure, is noticeably auxetic. A tensile stress applied along the [0 0 1] direction

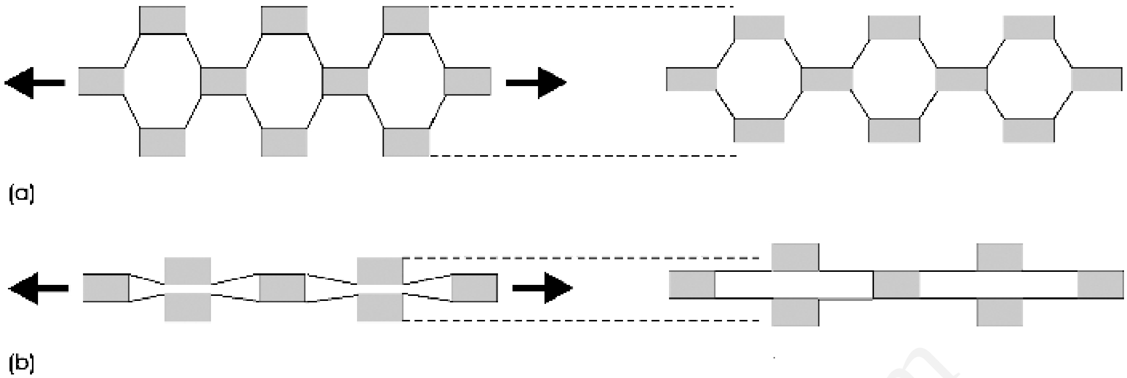

Figure 10.33 A solid composed of rigid blocks of crystal linked by bonds of constant length, but with flexible links to the crystallites: (a) A normal solid becomes thinner under tension (right-hand side) (c) A material with re-entrant bonds (left-hand side) is auxetic and becomes thicker under tension (right-hand side)

will result in a contraction along the lateral direction, perpendicular to the (1 0 0) and (0 1 0) planes. Poisson's ratio has a normal value of +0.39. However, a tensile stress applied along the [1 1 1] direction will result in an expansion in the lateral direction, with a Poisson's ratio of −0.39.

The reason for this behaviour can be anticipated from the bonding and structure in the material. The main bonds in β-brass are between the copper and zinc atoms and are directed along the ⟨1 1 1⟩ directions, towards the corners of the unit cell from the central atom. Weaker bonds lying along the cell edges link the copper atoms (Figure 10.34a). In mechanical terms, it is as if the central atom is linked to the atoms at the unit cell corners by strong springs, and the atoms at the unit cell corners are linked to each other by weaker springs aligned along the unit cell edges. Tension along the [0 0 1] direction will have a minimal effect on the strong bonds and be taken up by the weaker bonds (Figure 10.34b). In this case, the ⟨1 1 1⟩ strong bonds are not stretched or compressed but simply bend a little. As a result, the atoms at the cell corners move in slightly and the unit cell contracts perpendicular to the (1 0 0) and (0 1 0) faces, similar to a slight folding of a four-spoke umbrella (Figure 10.34c).

The arrangement of the atoms perpendicular to the (1 1 1) plane is drawn in Figure 10.34d. Between the two apical zinc atoms, A and B, are two triangles of zinc atoms, with the copper atom sandwiched in the centre. Tension along the [1 1 1] direction is along a line AB, and hence directly along a strong bond and strongly resisted. As the atoms A and B move apart, the triangles of zinc atoms move slightly closer together, rather like a three-spoke umbrella opening slightly. This causes an expansion of the structure in the direction perpendicular to line AB (Figure 10.34e).

It has been found that many cubic metals with structures related to the body-centred cubic structure are, in fact, auxetic. A number of silicates also show this unusual property. In each case, the relationship between the bonding and structure controls the response to the tensile stress.

10.3.3 Thin films

Films with a thickness of the order of nanometres are at the heart of microelectronics, and it has become apparent that the mechanical properties of these films must be known in order to ensure complete control over device manufacture and operation. There are a number of microscopic methods available for the determination of the mechanical properties of such films (Figure 10.35). The technique most widely used is the small-scale equivalent to the hardness test, called

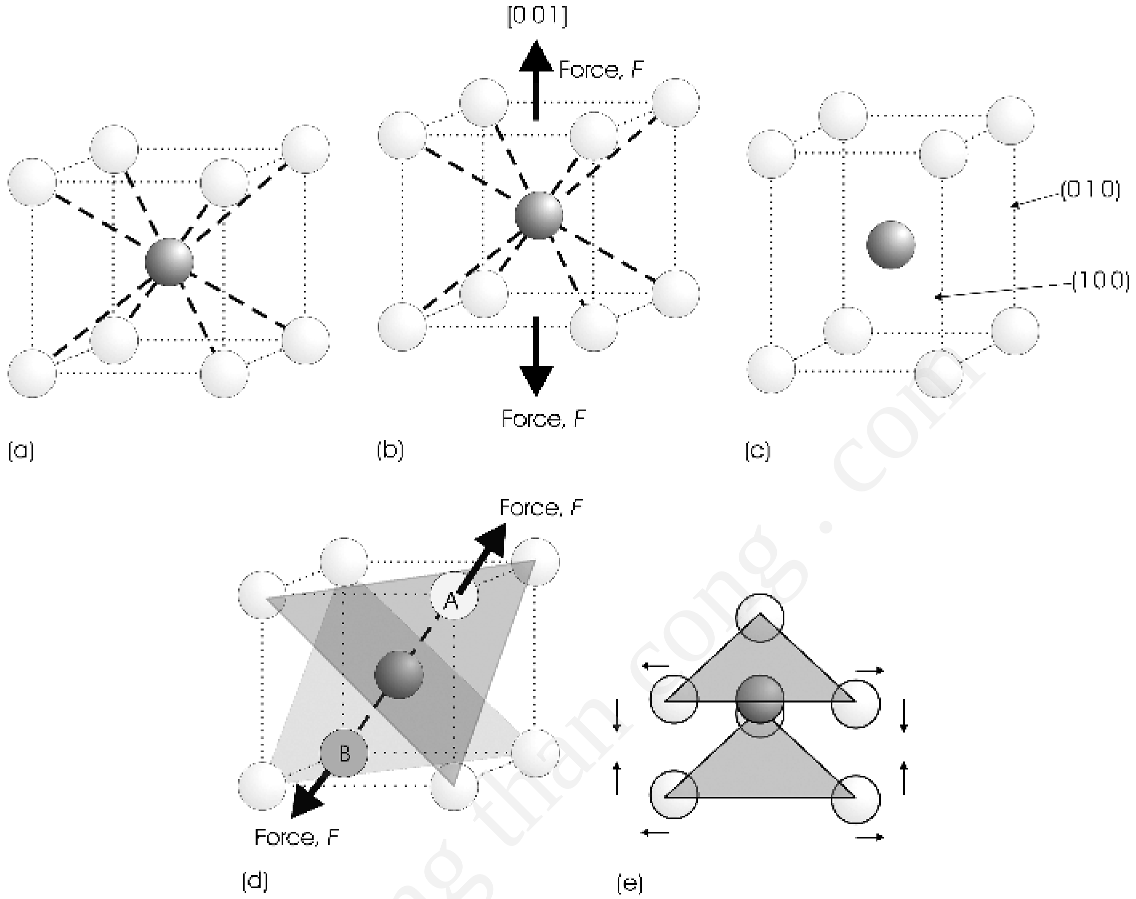

Figure 10.34 (a) The CsCl structure of β-brass (strong bonds are drawn as heavy broken lines, and weak bonds as light broken lines). The application of a force along the [0 0 1] direction to the unit cell of β-brass (part b), results in contraction perpendicular to the (1 0 0) and (0 1 0) faces (part c). (d) The (1 1 1) planes in β-brass; a force along the [1 1 1] direction is directed along strong bonds, causing the triangles of zinc atoms above and below the central copper atom to come together slightly, which causes the structure to expand perpendicular to the [1 1 1] direction (part e). Note: A and B are apical zinc atoms

nano-indentation. It has the advantage that the properties can be measured while the film is attached to a substrate and the surface can be tested in a large number of different places. It is also useful for measuring the properties of surfaces that have been modified by, for example, laser irradiation or optical coatings.

In a nano-indentation test, the load applied to the indenter and the resultant displacement are measured as a function of time. In this sense, the test mirrors the conventional tensile or compression test. The parameters obtained are the elastic modulus, Poisson's ratio and the hardness of the film. However, there are considerable differences between nano-indentation and conventional tests. For example, there is no clearly distinguished elastic region, and the deformation produced during nano-indentation involves both elastic and plastic deformation. Additionally, the area over which the load is applied changes continually during indentation and, because

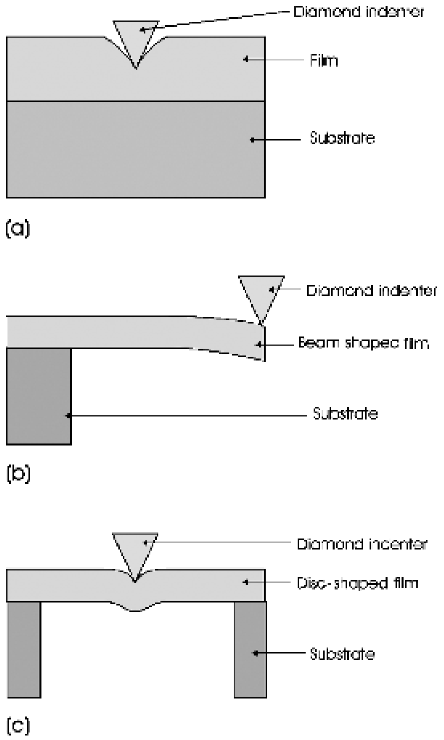

Figure 10.35 Methods of measuring the mechanical properties of thin films: (a) Nano-indentation, (b) beam deflection and (c) disc deflection. (Adapted from G.M. Pharr and W.C. Oliver, 1992, 'Measurement of Thin Film Mechanical Properties Using Nanoindentation', *Materials Research Society Bulletin* **XVII** (July) 28

of the small scale of the measurements, the interaction between the indenter, the film and the substrate cannot be ignored.

The indenter geometry is different from those used in large-scale testing (Figure 10.23). A diamond indenter with a triangular pyramidal shape, called a Berkovich indenter, is used (Figure 10.36a). The loads used can be as small as 0.01 μN, and displacements as small as 0.1 nm can be measured. The displacement of the indenter tip follows a different path on unloading compared with loading, and considerable hysteresis is found. This means that interpretation of the results is less obvious than for large-scale experiments.

A representative load–displacement curve for unloading is drawn in Figure 10.36b. The maximum load applied is $F_{\max}$ and the corresponding maximum displacement is $h_{\max}$. The initial slope of the unloading curve, $\mathrm{d}F/\mathrm{d}h$, is a measure of the stiffness, S_{initial}, and is given by:

$$S_{\text{initial}} = \frac{\mathrm{d}F}{\mathrm{d}h} = \frac{F_{\max}}{h_{\max} - h_0}$$

where h_0 is the extrapolation of the initial slope to $F = 0$. The slope is related to the elastic modulus of the indenter, the film and the substrate.

If the indentation does not exceed approximately 20 % of the film thickness the substrate does not

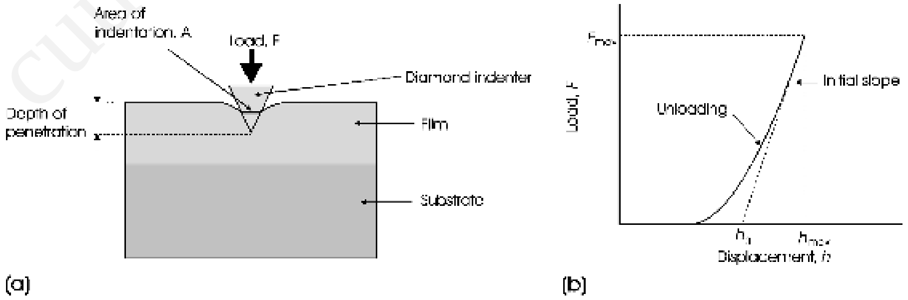

Figure 10.36 (a) The indentation of a thin film. The indentation area and depth will depend on the properties of the indenter and the substrate as well as the film being tested. (b) Schematic unloading curve for a nano-indentation experiment. Adapted from G.M. Pharr and W.C. Oliver, 1992, 'Measurement of Thin Film Mechanical Properties Using Nano-indentation', *Materials Research Society Bulletin* **XVII** (July) 28

need to be taken into account. In this case, the hardness of the film is given by:

$$H = \frac{F_{\max}}{A}$$

where A is the area of the indentation. The value of the contact area, A, is a function of the depth of penetration, and must often be determined experimentally. For an ideally sharp indenter:

$$A = 24.5\, h_0^2$$

The slope is related to the elastic modulus of the indenter, the film and the substrate, and when the substrate is ignored the initial slope is given by:

$$S_{\text{initial}} = \frac{2}{\sqrt{\pi}} E_r \sqrt{A}$$

where A is the area of indentation at maximum load, $F_{\max}$, and E_{r} is the reduced elastic modulus, given by:

$$\frac{1}{E_{\mathrm{r}}} = \frac{1-\nu_{\mathrm{f}}^2}{E_{\mathrm{f}}} - \frac{1-\nu_{\mathrm{i}}^2}{E_{\mathrm{i}}}$$

where the subscripts i and f refer to indenter and film, respectively, E is the relevant elastic modulus, and ν is the relevant value of Poisson's ratio.

10.4 Composite materials

Composites are solids made of more than one material type, designed to have enhanced properties compared with those of the separate materials themselves (see Sections 6.1.5, 6.2.4 and 6.4.5). The mechanical properties of composite materials are often difficult to obtain because of the complex microstructures found, especially in biological structures. However, in simple cases these can be modelled.

10.4.1 Elastic modulus of large-particle composites

The elastic modulus of a composite containing large particles depends on the volume fraction of the constituent phases. The elastic modulus falls between two limits, an upper and a lower limit:

$$E_{\mathrm{c}}(\text{upper limit}) = E_{\mathrm{m}} V_{\mathrm{m}} + E_{\mathrm{p}} V_{\mathrm{p}}$$

$$E_{\mathrm{c}}(\text{lower limit}) = \frac{E_{\mathrm{m}} E_{\mathrm{p}}}{E_{\mathrm{m}} V_{\mathrm{p}} + E_{\mathrm{p}} V_{\mathrm{m}}}$$

where E_{c}, E_{m} and E_{p} are the elastic moduli of the composite, matrix and particles, respectively, and V_{c} (equal to 1.0), V_{m} and V_{p} are the corresponding volume fractions.

The cemented carbides are large-particle composites. These materials are used as cutting tools to machine hard steels. The first cemented carbide made was formed of tungsten carbide particles embedded in a matrix of cobalt metal. The hard carbide cuts the steel but is brittle. The toughness comes from the cobalt matrix.

The large-particle composite in greatest use is concrete.

10.4.2 Elastic modulus of fibre-reinforced composites

The objective of fibre reinforcement is to endow a lightweight matrix with high strength and stiffness. A critical fibre length, l_{c}, is necessary to achieve this:

$$l_{\mathrm{c}} = \frac{\sigma_{\mathrm{f}} d}{2\,\tau_{\mathrm{c}}}$$

where d is the fibre diameter, σ_{f} is the (ultimate) tensile strength of the fibre, and τ_{c} is the shear yield strength of the matrix. The fibre length relative to the critical fibre length gives rise to two terminologies:

- if $l > 15\, l_{\mathrm{c}}$ the material is termed a continuous fibre composite;
- if $l < 15\, l_{\mathrm{c}}$ the material is termed a discontinuous fibre composite.

In practice, the fibre orientation, concentration and distribution all contribute to the final properties of

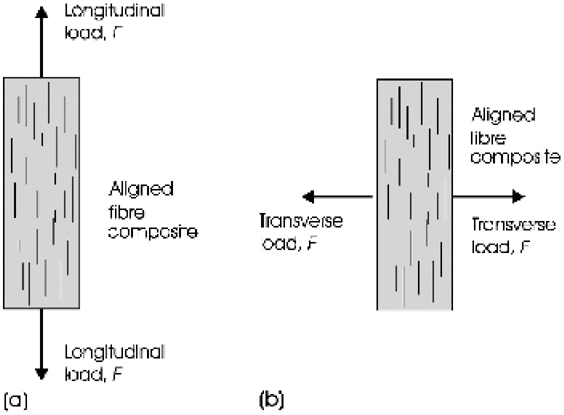

Figure 10.37 (a) A longitudinal load applied to an aligned fibre composite and (b) a transverse load applied to an aligned fibre composite

the composite. Moreover, the mechanical properties of the composite depend on whether the load is applied along the fibre direction or normal to it.

10.4.2.1 Longitudinal load on a continuous and aligned fibre composite

To obtain a value of the elastic modulus of a composite in which the fibres are both continuous and aligned parallel to each other, it is simplest to assume that the deformation of the fibres and the matrix is the same, that is, that they are firmly bonded together (Figure 10.37a). This is termed the iso-strain condition, in which case:

$$\varepsilon_c = \varepsilon_m = \varepsilon_f$$

where ε_c is the strain experienced by the composite as a whole, ε_m is the strain experienced by the matrix, and ε_f is the strain experienced by the fibre. In this case it is reasonable to suppose that the tensile stress, σ, is divided into parts acting on the fibre and on the matrix in proportion to the volume fraction of each. With these assumptions:

$$E_{cl} = E_m V_m + E_f V_f$$

where E_{cl} is the elastic modulus of the composite under a longitudinal load, E_m is the elastic modulus of the matrix, E_f is the elastic modulus of the fibre, V_f is the volume fraction of fibre, given by

$$V_f = \frac{\text{volume of fibre}}{\text{total volume}}$$

and V_m is the volume fraction of the matrix, given by

$$V_m = \frac{\text{volume of matrix}}{\text{total volume}}$$

For two components:

$$V_m = 1 - V_f$$
$$E_{cl} = E_m(1 - V_f) + E_f V_f$$

This is the same equation as the upper bound for large-particle composites.

The fraction of the load carried by each component is given by:

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m}$$

where F_f is the load carried by fibre, and F_m is the load carried by the matrix.

10.4.2.2 Transverse load on a continuous and aligned fibre composite

When a load is applied in a transverse direction to a composite which contains continuous and aligned fibres (Figure 10.37b) the iso-strain condition is unreasonable and it is more appropriate to assume an iso-stress condition applies:

$$\sigma_c = \sigma_m = \sigma_f = \sigma$$

where σ represents the stress and the subscripts have the same meaning as in Sub-section 10.4.2.1. In this case:

$$E_{ct} = \frac{E_m E_f}{E_f V_m + E_m V_f} = \frac{E_m E_f}{(1 - V_f)E_f + E_m V_f}$$

where E_{ct} is the modulus of elasticity in a transverse direction for the composite. This is the same equation as the lower bound for large-particle composites.

10.4.3 Elastic modulus of a two-phase system

Many ordinary solids, such as ceramics, are made up of several phases. Strictly speaking, these are not composite materials, but similar reasoning can be applied to obtain the elastic modulus and other mechanical properties of such systems. Although the equations for a solid composed of several phases with a complex microstructure are frequently unwieldy, simpler equations exist for well-defined geometries.

A ceramic body composed of two phases, one of which is distributed as particles within the matrix of the other, has a modulus of elasticity given by:

$$E_c(\text{upper limit}) = E_m V_m + E_p V_p$$

$$E_c(\text{lower limit}) = \frac{E_m E_p}{E_m V_p + E_p V_m}$$

where E_c, E_m and E_p are the elastic moduli of the ceramic, matrix and particles, respectively, and V_c(equal to 1.0), V_m and V_p are the corresponding volume fractions. These equations are identical to those for large-particle composites given in Section 10.4.1.

A ceramic body composed of layers aligned parallel to a uniaxial stress, in which the strain is shared equally by the two phases, the iso-strain condition, has an elastic modulus:

$$E_c(\text{upper limit or Voigt bound}) = E_m V_m + E_p V_p$$

This approximation is called the Voigt model, and the value of the elastic modulus is often known as the Voigt bound. The expression is identical to that for a continuous aligned fibre composite under a longitudinal load, and gives the elastic modulus when the load is applied parallel to the sheets. Similarly, if the stress is applied perpendicular to the layers, and an iso-stress condition applies (the Reuss model), the elastic modulus is:

$$E_c(\text{lower limit or Reuss bound}) = \frac{E_m E_p}{E_m V_p + E_p V_p}$$

The value of the elastic modulus, often called the Reuss bound, is identical to that for transverse loading on a fibre composite, and gives a value for the elastic modulus normal to the layers. In both of these equations, E_c, E_m and E_p are the elastic moduli of the ceramic, matrix and particles, respectively, and V_c (equal to 1.0), V_m and V_p are the corresponding volume fractions.

The presence of pores in ceramics usually leads to weakness. The elastic modulus of a body with a Poisson's ratio of 0.3, containing isolated closed pores, is described by the equation:

$$E_c = E_0(1 - 1.9\,V_p + 0.9\,V_p^2)$$

where E_c is the elastic modulus of the porous ceramic, E_0 is the elastic modulus of the nonporous ceramic, and V_p is the volume fraction of pores.

Answers to introductory questions

How are stress and strain defined?

Stress is defined with respect to the force applied to an object and is measured as the force applied to a unit area of the specimen. The application of a stress results in a dimensional change, which is called the strain.

For a rod-shaped specimen,

$$\text{stress}(\sigma_T) = \frac{F}{A}$$

where F is the force (or load) applied to the rod, and A is the cross-sectional area subjected to the force.

For practical purposes, it is often adequate to ignore the continuous change in cross-sectional area that occurs when a force is applied. The stress so defined is called the engineering (or nominal) stress.

$$\sigma = \frac{F}{A_0}$$

where F is the average force (or load) applied, and A_0 is the initial cross-sectional area of the sample.

The strain that is found on application of a load to a rod is equal to the elongation of the rod. The increment in tensile strain experienced, $\Delta\varepsilon$, when a rod is extended, is defined as the ratio of the increase in length, Δl, to the total length:

$$\text{strain}(\Delta\varepsilon_T) = \frac{\Delta l}{l}$$

The total strain is then given by:

$$\begin{aligned}\text{strain}\ (\varepsilon_T) &= \int_{l_0}^{l} \frac{\mathrm{d}l}{l_0}\\ &= \ln\left(\frac{l}{l_0}\right)\end{aligned}$$

where l is the final length of the specimen, and l_0 is the original length. As strain is a ratio, it has no units. If the incremental changes are ignored, the engineering (or nominal) strain is:

$$\varepsilon = \frac{(l - l_0)}{l_0}$$

where l is the final length of the specimen, and l_0 is the original length of the specimen.

Many materials are used under compression rather than tension. At low loads, the compressed material behaves in a similar way to materials tested under tension. In compression tests, the value of the force is taken as negative and hence we have negative values of stress and strain compared with those obtained in tension.

Why are alloys stronger than pure metals?

Four principal methods have been used to strengthen metals. Three of these – reduction in grain size, work hardening and precipitation strengthening – are brought about by heat treatment and mechanical deformation (e.g. hammering). The fourth method – alloying – was for most of historical times based on empirical observation. In all cases, the metal is strengthened because dislocation movement is restricted.

Alloying, first widely used to transform soft copper into much stronger bronze, relies on the strain set up in the crystal structure by the impurity or dopant atoms. This strain field impedes dislocation movement because dislocations also generate a strain in the structure. The two strain components mutually repel each other to hinder slip. If sufficient of the second component is added, precipitates of a second phase can form in the crystal matrix. These hinder dislocation movement simply by blocking slip planes.

What are solid lubricants?

Solid lubricants, which are greasy to the touch, are highly anisotropic solids with a low shear strength in at least one dimension. Solid lubricants fall into three main classes – inorganic solids with a lamellar (layer-like) crystal structure, solids that suffer plastic deformation easily and polymers in which the constituent chains can slip past each other in an unrestricted way. The categories of most importance are layer structures and soft inorganic compounds.

The most familiar solid lubricant is graphite, which has a layer structure. The layers are about 0.335 nm apart and are linked by weak van der Waals bonds. Dry graphite is found to be a poor lubricant and, in reality, the lubrication properties are brought about by forcing the layers apart by inserting atoms or molecules into the van der Waals gap. In ordinary circumstances this is water, and much of the lubricating action of graphite seems to stem from adsorbed water vapour on and between the layers. Fluorinated graphite, CF_x, in which fluorine is inserted between the layers, is another very successful lubricant.

Molybdenum disulphide, MoS_2, is another layer structure widely used as a solid lubricant. It is composed of MoS_2 layers, weakly linked by van der Waals bonds. The Mo atoms are surrounded by six sulphur atoms in the form of a trigonal prism. Although the MoS_2 layers are closer together than the carbon layers in graphite, MoS_2 acts as a much better lubricant for most purposes.

Both graphite and molybdenum disulphide oxidise at higher temperatures in air, and the main alternatives used are soft inorganic fluorides. Like many ceramics, although they show brittleness at room temperature, they display ductility at temperatures of about half the melting point. A solid lubricant that is widely used in the temperature range from approximately 540 °C to 900 °C is the eutectic mixture of calcium fluoride, CaF_2, and barium fluoride, BaF_2. The melting point of the eutectic is 1022 °C.

Further reading

L.C. Bryant, B.P. Bewlay, 1995, 'The Coolidge Process for Making Ductile Tungsten', *Materials Research Society Bulletin* **XX** (August) 67.

M.E. Eberhart, 1999, 'Why Things Break', *Scientific American* **281** (October) 44.

S.S. Hecker, A.K. Ghosh, 1976, 'The Forming of Sheet Metal', *Scientific American* **235** (November) 100.

H.J. McQueen, W.J.M. Tegart, 1995, 'The Deformation of Metals at High Temperatures', *Scientific American* **232** (April) 116.

G.M. Pharr, W.C. Oliver, 1992, 'Measurement of Thin Film Mechanical Properties Using Nanoindentation', *Materials Research Society Bulletin* **XVII** (July) 28.

S.L. Semiatin, G.D. Lahoti, 1981, 'The Forging of Metals', *Scientific American* **245** (August) 82.

D.M. Teter, 1998, '*Computational Alchemy: The Search for New Superhard Materials*', *Materials Research Society Bulletin* **23** (January) 22.

Problems and exercises

Quick quiz

1 *A* material under a tensile force is:
  (a) Stretched
  (b) Twisted
  (c) Compressed

2 Torsional forces occur in a material that is:
  (a) Sheared
  (b) Twisted
  (c) Compressed

3 Both tensile and compressive forces occur in a rod that is:
  (a) Bent
  (b) Twisted
  (c) Stretched

4 For a rod-shaped specimen of a metal, the stress is defined as:
  (a) The change in length per unit length
  (b) The change in length per unit force
  (c) Force per unit area

5 For a rod-shaped specimen of a metal, the strain is defined as:
  (a) The change in length per unit length
  (b) The change in length per unit force
  (c) Force per unit area

6 The engineering strain is given by:
  (a) Force divided by the original cross-sectional area
  (b) The change in length divided by the original length
  (c) Force divided by the original length

7 The strength of a ceramic sample is usually tested by:
  (a) Stretching a bar
  (b) Bending a bar
  (c) Twisting a bar

8 The engineering stress versus engineering strain curve for a solid indicates:
  (a) Only elastic behaviour
  (b) Only ductile behaviour
  (c) Both elastic and ductile behaviour

9 The initial (linear) part of an engineering stress–engineering strain curve represents:
  (a) Plastic deformation
  (b) The tensile strength
  (c) Elastic deformation

10 In comparison with a strongly cross-linked polymer, a weakly cross-linked form of the polymer will have:
  (a) A higher Young's modulus
  (b) A lower Young's modulus
  (c) The same Young's modulus

11 Permanent deformation of a solid when all stress is removed is a sign of:
   (a) Plastic deformation
   (b) Elastic deformation
   (c) Tensile deformation

12 The defects mainly held responsible for plastic deformation are:
   (a) Point defects
   (b) Precipitates
   (c) Dislocations

13 The tensile strength of a solid is:
   (a) The maximum load that can be carried before fracture
   (b) The maximum load before plastic deformation occurs
   (c) The load when fracture occurs

14 The yield stress indicates the point at which:
   (a) The solid starts to thin down (neck)
   (b) The elastic behaviour changes to plastic behaviour
   (c) The solid starts to fracture

15 The specific strength of a material is:
   (a) The tensile strength divided by the specific gravity
   (b) The tensile strength divided by the volume
   (c) The tensile strength divided by the weight

16 An engineering stress–engineering strain curve does *not* give information on:
   (a) The modulus of elasticity of the material
   (b) The yield strength of the material
   (c) The hardness of the material

17 Superelasticity is caused by:
   (a) Hysteresis
   (b) Martensite formation
   (c) Deformation

18 The slope of the stress–strain curve in the elastic region is a measure of:
   (a) Young's modulus
   (b) The tangent modulus
   (c) The secant modulus

19 The lateral dimensional change that accompanies a longitudinal strain is:
   (a) Poisson's modulus
   (b) Poisson's ratio
   (c) Poisson's strain

20 Fracture of a polycrystalline solid that takes place between the crystallites is:
   (a) Intergranular fracture
   (b) Transgranular fracture
   (c) Cleavage

21 Conchoidal fracture is displayed by brittle solids that are:
   (a) Polycrystalline
   (b) Amorphous
   (c) Natural (biomaterials)

22 The Griffith theory of brittle fracture postulates that the fracture is due to:
   (a) Dislocations
   (b) Surface defects
   (c) Small cracks

23 During plastic deformation, dislocations move preferentially on:
   (a) Slip planes
   (b) Twin planes
   (c) Slip directions

24 Ceramics are more brittle than metals because:
   (a) They contain fewer dislocations than metals
   (b) Dislocation movement is simpler than in metals
   (c) Dislocation movement is more difficult than in metals

25 The initial process of plastic deformation in semicrystalline polymers is a result of deformation in:
   (a) The crystalline regions
   (b) The amorphous regions
   (c) The crystalline and the amorphous regions

26 The ductility of a metal can be estimated from:
   (a) The elongation at fracture

(b) The stress applied at fracture
(c) The strain at the yield point

27 Ceramics can be strengthened by:
(a) Immobilising dislocations
(b) Removing Griffith flaws
(c) Increasing the degree of crystallinity

28 One of the following does *not* describe hardness:
(a) Knoop
(b) Brinell
(c) Poise

29 Materials that fail after repeated cycles of stress are said to suffer:
(a) Fatigue failure
(b) Creep failure
(c) Catastrophic failure

30 Fatigue failure does not occur at stress levels below:
(a) The persistence limit
(b) The fatigue limit
(c) The endurance limit

31 The gradual elongation of a material under a constant load is called:
(a) Fatigue
(b) Creep
(c) Yield

32 Steady-state creep is defined by:
(a) The initial part of a creep curve
(b) The middle part of the creep curve
(c) The final part of the creep curve

33 Both Coble creep and Herring–Nabarro creep describe:
(a) Power law creep
(b) Creep due to dislocation movement
(c) Creep due to atomic diffusion

34 The temperature dependence of creep is often described by:
(a) An Arrhenius law
(b) A linear law
(c) A parabolic rate law

35 Solid lubricants often have:
(a) Layer structures
(b) Amorphous structures
(c) Liquid crystal structures

36 Materials that expand when under tension are called:
(a) Nanomaterials
(b) Eutactic materials
(c) Auxetic materials

37 The hardness of thin films is measured by using:
(a) A Vickers indenter
(b) A Knoop indenter
(c) A Berkovich indenter

38 Cemented carbides that are used in cutting tools are:
(a) Longitudinally reinforced fibre composites
(b) Large-particle composites
(c) Ceramic composites

Calculations and questions

10.1 A weight of 500 kg is hung from a 2 cm diameter rod of brass. What is the engineering stress?

10.2 A weight of 3500 kg load is applied to a 1.5 cm diameter rod of nickel. What is the engineering stress?

10.3 A steel wire 75 cm long and 1 mm in diameter is subjected to a load of 22 kN. The elastic modulus of the steel 201.9 GPa. Calculate the new length.

10.4 A rod of bronze 150 cm long and 3 mm in diameter is subjected to a load of 30 kN. The elastic modulus of the bronze is 105.3 GPa. Calculate the new length.

10.5 A rod of copper 60 cm long is subjected to a tensile stress of 300 MPa. The elastic modulus of copper is 129.8 GPa. Calculate the new length.

10.6 A rod of aluminium 100 cm long is subjected to a tensile stress of 250 MPa. The elastic modulus of aluminium is 70.3 GPa. Calculate the new length.

10.7 A cast-iron rod of length 200 mm and dimensions 10 mm × 20 mm is subjected to a load of 70 kN. An extension of 0.46 mm is observed. Calculate the Young's modulus of the cast iron.

10.8 A zinc bar of length 125 mm and dimensions 5 mm × 7.5 mm is subjected to a load of 40 kN. An extension of 1.23 mm is observed. Calculate the Young's modulus of zinc.

10.9 Calculate Poisson's ratio for a bar of metal originally 10 mm × 10 mm × 100 mm, which is extended to 101 mm, if there is no change in the overall volume of the sample.

10.10 A copper bar of 10-mm square section is subjected to a tensile load that increases its length from 100 mm to 102 mm. The value of Poisson's ratio for copper is 0.343. Calculate the new dimensions of the bar.

10.11 A brass rod of 12.5 mm diameter is subjected to a tensile load that increases its length from 150 mm to 151.5 mm. The value of Poisson's ratio for brass is 0.350. Calculate the new diameter.

10.12 A cylindrical titanium rod of diameter 15 mm is subjected to a tensile load applied along the long axis. The modulus of elasticity of the metal is 115.7 GPa, and Poisson's ratio is 0.321. Determine the magnitude of the load needed to produce a contraction in diameter of 5×10^{-3} mm if the deformation is elastic.

10.13 A steel rod of diameter 16.2 mm and length 25 cm is subjected to a force of 50 000 N in tension along the long axis. The modulus of elasticity is 210 GPa, and Poisson's ratio is 0.293. Determine (a) the amount that the specimen will elongate in the direction of the applied force and (b) the change in diameter of the rod.

10.14 A niobium bar of dimensions 15-mm square and of length 300 mm is subjected to a tensile force of 25 000 N. The elastic modulus of niobium is 104.9 GPa, and Poisson's ratio is 0.397. Determine (a) the engineering stress, (b) the elongation, (c) the engineering strain and (d) the change in the cross-section of the bar.

10.15 A tungsten rod of 12.5 mm diameter and of length 350 mm is subjected to a tensile force of 90 000 N. The elastic modulus of tungsten is 411.0 GPa, and Poisson's ratio is 0.280. Determine (a) the engineering stress, (b) the elongation, (c) the engineering strain and (d) the change in the diameter of the rod.

10.16 A tensile test specimen of magnesium has a gauge length of 5 cm. The metal is subjected to a tensile loading until the gauge markings are 5.63 cm apart. Calculate (a) the engineering stress and (b) the percentage elongation.

10.17 A tensile test specimen of brass has a gauge length of 5 cm. The metal is subjected to a tensile loading until the gauge markings are 6.05 cm apart. Calculate (a) the engineering stress and (b) the percentage elongation.

10.18 An aluminium alloy specimen of 3 mm diameter with 50 mm gauge length was tested to destruction in a tensile test. The results are given in Table 10.2. The maximum load applied was 8100 N and the final length between the gauge marks was 53.2 mm.

Table 10.2 Data for Question 10.18

Load/N	Extension/mm
1000	0.06
2000	0.180
3000	0.290
4000	0.402
5000	0.504
6000	0.697
7000	0.900
7500	1.297
8000	2.204
7150 (fracture)	3.200

(a) Plot an engineering stress versus engineering strain curve [Note: not shown in the answers at the end of this book.]

(b) Determine the modulus of elasticity of the alloy

(c) Determine the tensile strength of the alloy

(d) Determine the 0.2 % offset yield strength of the alloy

(e) Determine the percentage elongation at fracture

10.19 A steel specimen of 12 mm diameter with 50 mm gauge length was tested to destruction in a tensile test. The results are given in Table 10.3. The maximum load applied was 152 kN.

(a) Plot an engineering stress versus engineering strain curve [Note: not shown in the answers at the end of this book.]

(b) Determine the modulus of elasticity of the alloy

Table 10.3 Data for Question 10.19

Load/kN	extension/mm
10	0.030
20	0.064
30	0.098
40	0.130
50	0.170
60	0.195
70	0.218
80	0.256
90	0.294
100	0.335
110	0.400
120	0.505
130	0.660
140	0.898
150	1.300
152 (maximum)	1.500
150	1.700
140	1.960
133 (fracture)	2.070

(c) Determine the tensile strength

(d) Determine the 0.1 % offset yield stress of the alloy

(e) Determine the percentage elongation at fracture

10.20 Figure 10.38 shows the engineering stress–engineering strain behaviour of a carbon steel. Determine:

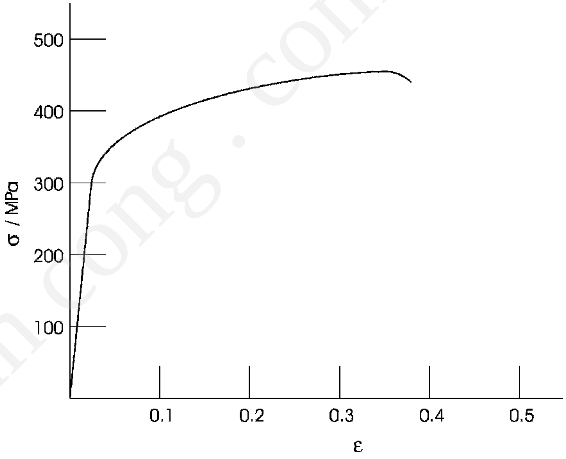

Figure 10.38 Engineering stress–engineering strain (σ–ε) curve of a carbon steel, for Question 10.20

(a) The modulus of elasticity

(b) The stress at 0.2 % offset strain (proof stress)

(c) The maximum load that can be sustained by a rod of diameter 12.5 mm

(d) The change in length of a rod originally 250 mm long subjected to an axial stress of 400 MPa

10.21 A tensile test carried out on a sample of polypropylene of dimensions 12.5 mm width, 3.5 mm thick and gauge length 50 mm gave the data in Table 10.4. Estimate:

Table 10.4 Data for Question 10.21

Force/N	Extension/mm
25	0.018
50	0.042
75	0.071
100	0.115
125	0.145
150	0.187
175	0.230
200	0.285
225	0.345
250	0.387
275	0.460
300	0.543
286 (break)	0.720

(a) the initial modulus;
(b) the secant modulus at 0.2 % strain;
(c) the tangent modulus at 0.2 % strain;
(d) the secant modulus at 0.4% strain;
(e) the tangent modulus at 0.4% strain;
(f) the percentage elongation at break.

10.22 A copper–nickel alloy has a 1 % offset yield strength of 350 MPa and a modulus of elasticity of 130 GPa.

(a) Determine the maximum load that may be applied to a specimen of cross-section 10 mm × 13 mm without significant plastic deformation occurring.

(b) If the original specimen length is 100 mm, what is the maximum length to which it can be stretched elastically?

10.23 Using the Griffith criterion, Equation (10.6), estimate the stress at which a glass plate containing a surface crack of 1.2 μm will fracture as a result of a force applied perpendicular to the length of the crack. The elastic modulus of the glass is 71.3 GPa and the surface energy of the glass is $0.360\ \mathrm{J\ m^{-2}}$.

10.24 A glass plate has to withstand a stress of $10^8\ \mathrm{N\ m^{-2}}$. Using the data Question 10.23, what will be the critical crack size for this to be achieved?

10.25 A plate of high-density polyethylene has a surface crack 7.5 μm in one face. The plate fractures in a brittle fashion when a force of $6 \times 10^6\ \mathrm{N\ m^{-2}}$ is applied in a direction perpendicular to the crack. The elastic modulus of the polyethylene is 0.95 GPa. Estimate the surface energy of the material.

10.26 Determine (a) the upper-bound and (b) the lower-bound elastic modulus of an ingot of magnesium metal containing 30 vol% magnesia (MgO) particles. The modulus of elasticity of magnesium is 44.7 GPa and that of magnesia is 210.3 GPa.

10.27 An aluminium alloy is to be strengthened by the incorporation of beryllium oxide (BeO) particles. Calculate (a) the upper-bound and (b) the lower-bound elastic moduli of a composite consisting of 40 wt% alloy and 60 wt% BeO. The elastic modulus of the alloy is 70.3 GPa and its density is $2698\ \mathrm{kg\ m^{-3}}$. The elastic modulus of BeO is 301.3 GPa and its density is $3010\ \mathrm{kg\ m^{-3}}$.

10.28 Compute the modulus of elasticity of a composite consisting of continuous and aligned glass fibres of 50 % volume fraction in an epoxy resin matrix under (a) longitudinal and (b) transverse loading. The modulus of elasticity of the glass fibres is 76 GPa and that of the resin is 3 GPa.

10.29 Compute the modulus of elasticity of a composite consisting of continuous and aligned carbon fibres of 60 % weight fraction in an epoxy resin matrix under (a) longitudinal and (b) transverse loading. The modulus of elasticity of the carbon fibres is 290 GPa and the density is $1785\ \mathrm{kg\ m^{-3}}$. The elastic modulus of the resin is 3.2 GPa and its density is $1350\ \mathrm{kg\ m^{-3}}$.

10.30 Determine (a) the Voigt and (b) the Reuss bounds to the modulus of elasticity of a ceramic material consisting of layers of alumina and a high-silica glass. The modulus of elasticity of the alumina is 380 GPa and that of the glass is 72.4 GPa, and the glass represents 30 vol% of the solid.

10.31 A mineral with an approximate formula $Mg_7Si_8O_{23}$ has a structure that is made up of alternating layers with compositions of 7MgO and $8SiO_2$. Estimate the elastic modulus of the material when stressed (a) parallel and (b) perpendicular to the layers. The elastic modulus and density of MgO are 210.3 GPa and 3580 kg m^{-3}, respectively, and for silica are 72.4 GPa and 2650 kg m^{-3}m, respectively.

10.32 The elastic modulus of sintered calcia stabilised zirconia with a porosity of 5 % is 151.7 GPa. Estimate the elastic modulus of completely-pore-free material.

10.33 The elastic modulus of sintered silicon carbide with 5 % porosity is 468.9 GPa. What is the porosity of a specimen with an elastic modulus of 350 GPa?