SEMICONDUCTOR MATERIALS & DEVICES

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CHAPTER 3: PN JUNCTION ELECTROSTATICS - PN JUNCTION DIODE: I-V CHARACTERISTICS

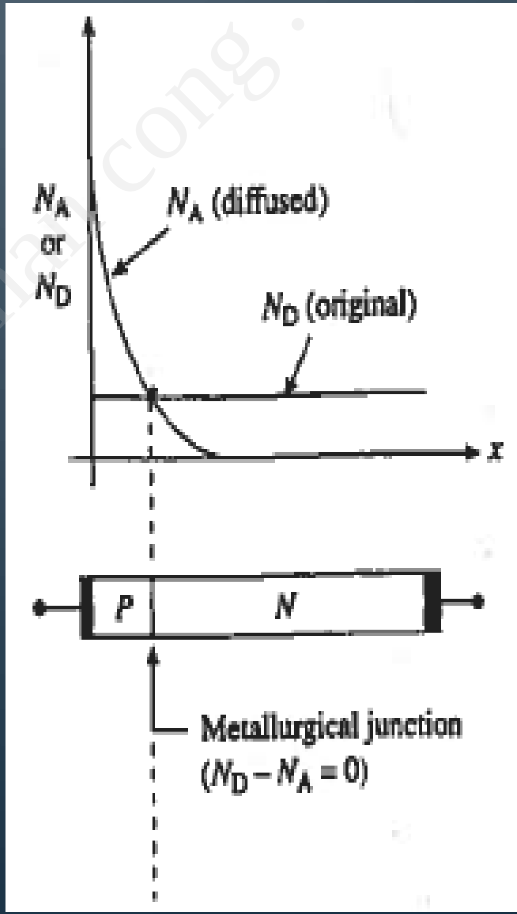
PN JUNCTION ELECTROSTATICS

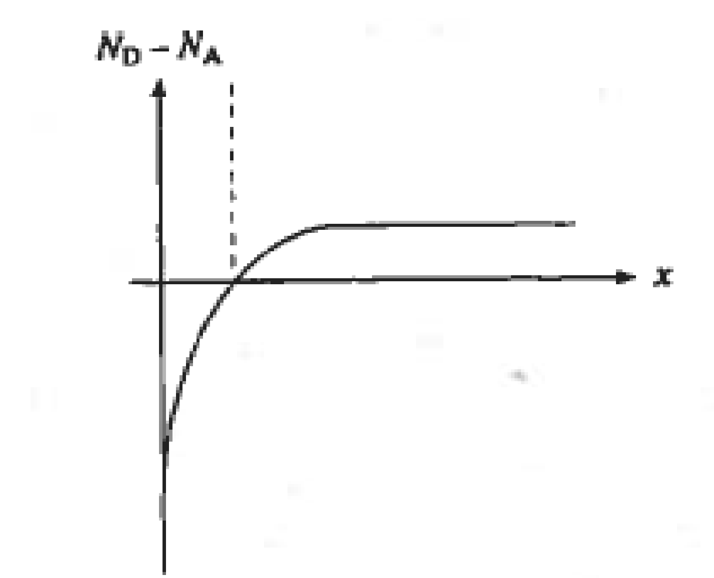
- **Suppose a pn junction formed by diffusing p-dopant into n-doped wafer**

near surface: $N_A > N_D$

deeper inside semi: $N_A < N_D$

metallurgical junction: $N_A = N_D$





Doping profile – net doping vs. position

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- **Poisson's equation: quantitative solutions for electrostatic variables**

$$\nabla \cdot \mathscr{E} = \frac{\rho}{K_S \varepsilon_0}$$

3-D case

$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_S \varepsilon_0}$$

1-D case

K_s : semiconductor dielectric constant

ε_o : permittivity of free space

ρ: charge density

$$\rho = q(p - n + N_D - N_A)$$

Assume dopants to be totally ionized

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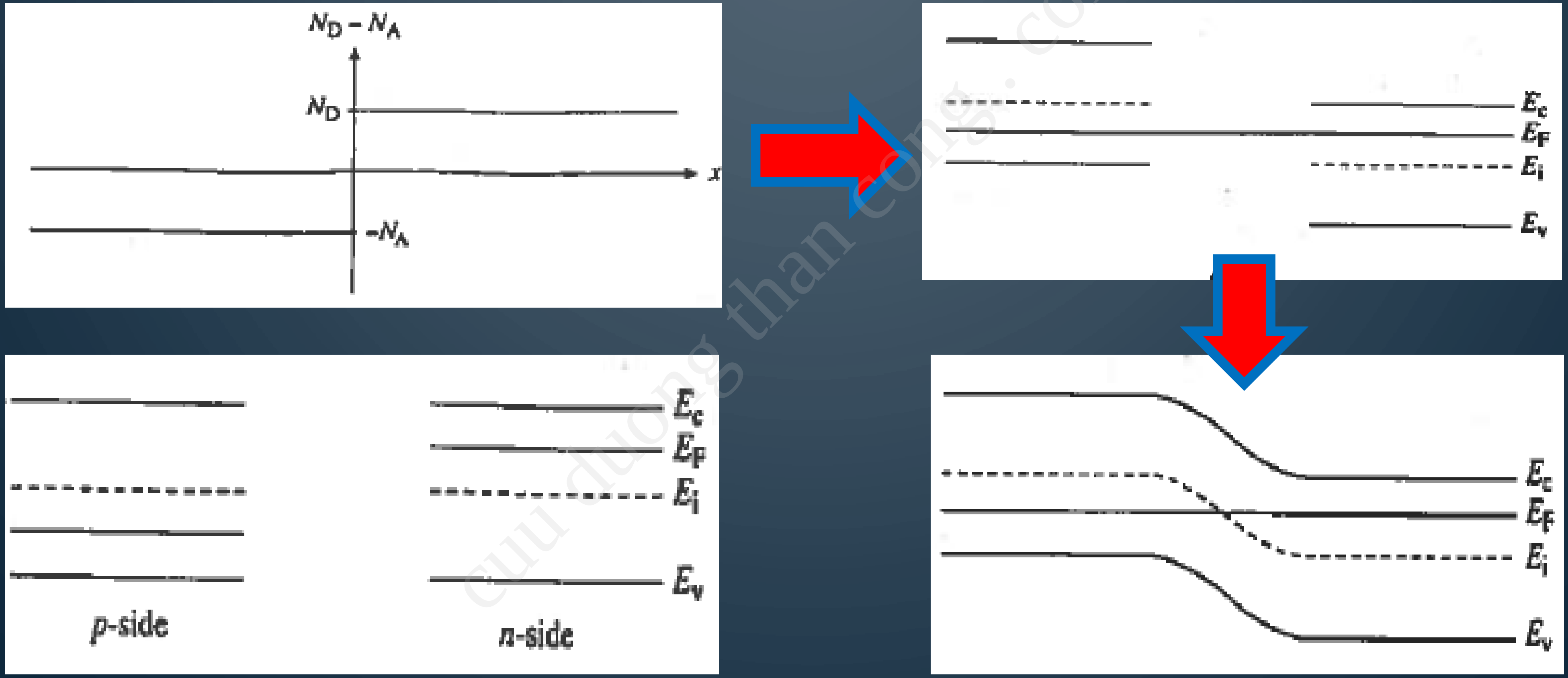
- **Quanlitative solution for 1D junction, equilibrium conditions**



Step-by-step construction of the equilibrium energy band diagram for a *pn* junction diode. (a) Assumed step junction profile and energy band diagrams for the semiconductor regions far removed from the metallurgical junction. (b) Alignment of the part (a) diagrams to the position-independent Fermi level. (c) The completed energy band diagram.

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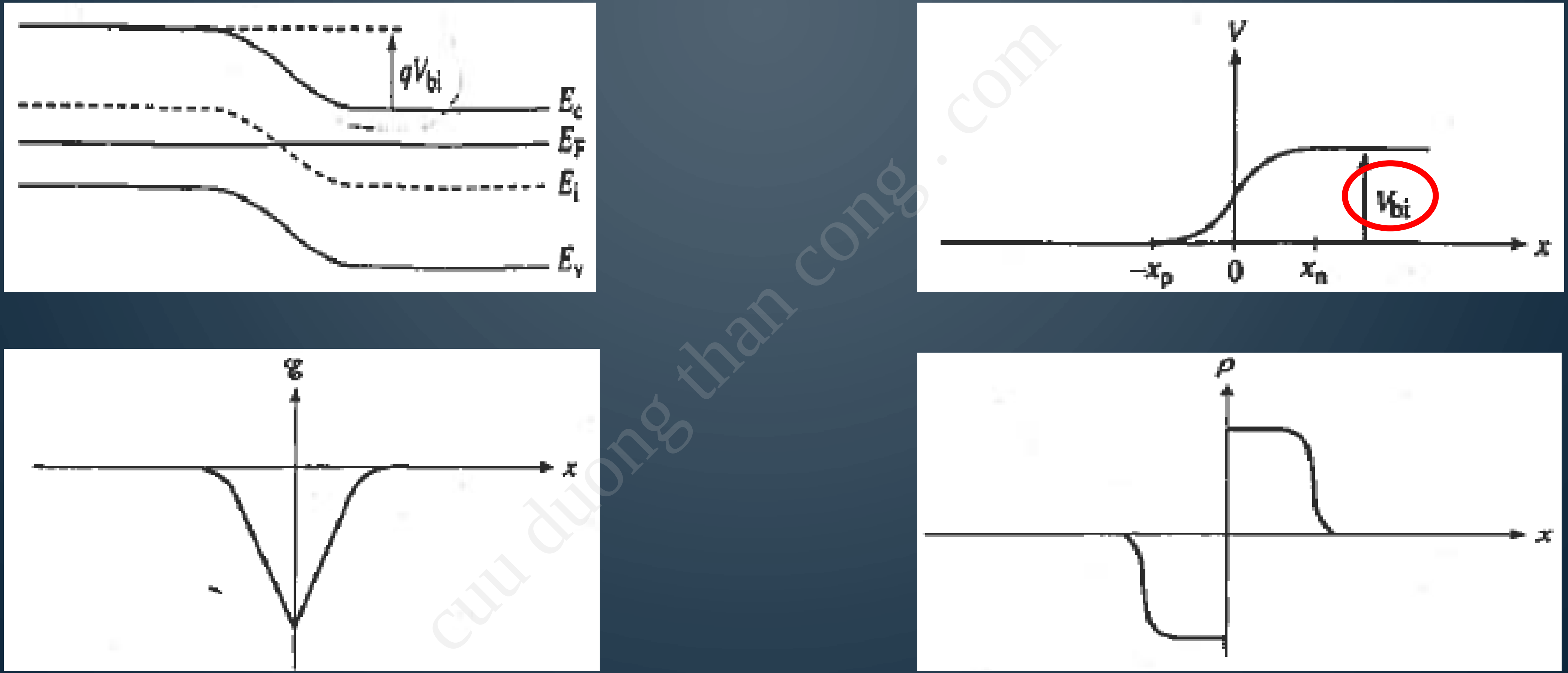

General functional form of the electrostatic variables in a *pn* junction under equilibrium conditions. (a) Equilibrium energy band diagram. (b) Electrostatic potential, (c) electric field, and (d) charge density as a function of position.

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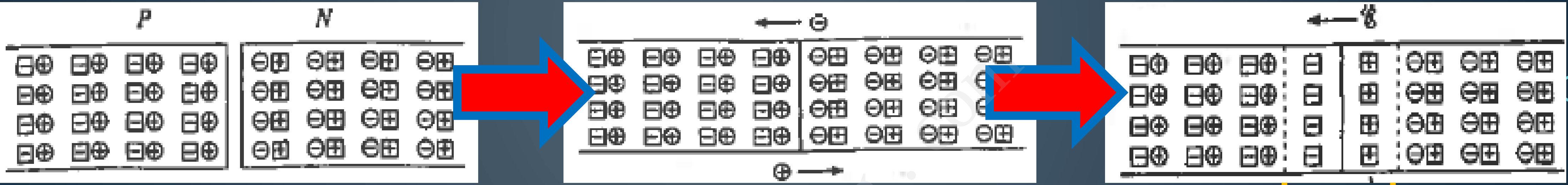
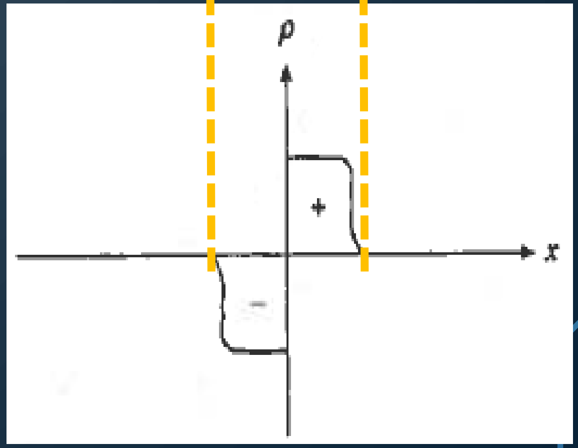


Space charge region/depletion region
Diffusion current = Drift current



Conceptual *pn* junction formation and associated charge redistribution. (a) Isolated *p* and *n* regions. (b) Electrons and holes diffuse to the opposite side of the junction moments after joining the *p* and *n* regions. (c) Charge redistribution completed and equilibrium conditions re-established. (d) Previously deduced charge density versus position. (⊕–holes, ⊟–ionized acceptors, ⊖–electrons, and ⊞–ionized donors.)

PN JUNCTION ELECTROSTATICS

- **Built-in Potential (Vbi)**

Built-in potential = voltage drop across depletion region under equilibrium conditions

Assume nondegenerated doped p-n junction

We know

$$\mathscr{E} = -\frac{dV}{dx}$$

Integrating across the depletion region:

$$\int_{-x_p}^{x_n} \mathscr{E}\, dx = \int_{V(-x_p)}^{V(x_n)} dV = V(x_n) - V(-x_p) = V_{bi}$$

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- **Under equilibrium conditions**

$$J_N = q\mu_n n\mathscr{E} + qD_N \frac{dn}{dx} = 0$$

Combine with Einstein relationship, we have

$$\mathscr{E} = -\frac{D_N}{\mu_n}\frac{dn/dx}{n} = -\frac{kT}{q}\frac{dn/dx}{n}$$

$$V_{bi} = -\int_{-x_p}^{x_n} \mathscr{E}\, dx = \frac{kT}{q}\int_{n(-x_p)}^{n(x_n)} \frac{dn}{n} = \frac{kT}{q}\ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

if

$$n(x_n) = N_D$$

$$n(-x_p) = \frac{n_i^2}{N_A}$$

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

(*) Common values for non degeneratedly doped Ge, Si and GaAs diodes, at 300K: 0.66V, 1.12V and 1.42V, respectively

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- Another way:

$$V_{bi} = V(x_n) - V(-x_p)$$

$$= \frac{1}{q}[E_c(-x_p) - E_c(x_n)] = \frac{1}{q}[E_i(-x_p) - E_i(x_n)]$$

or

$$V_{bi} = \frac{1}{q}[(E_i - E_F)_{p\text{-side}} + (E_F - E_i)_{n\text{-side}}]$$

$< Eg/2$ $< Eg/2$

Nondegenerately doped junction under equilibrium conditions

$$V_{bi} < E_G/q$$

$$(E_i - E_F)_{p\text{-side}} = kT \ln(N_A/n_i)$$

$$(E_F - E_i)_{n\text{-side}} = kT \ln(N_D/n_i)$$

(*)

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P: Most real diodes are very heavily doped on one side of the junction. In computing the built-in voltage of p^+-n and n^+-p step junctions, it is common practice to assume that the Fermi level on the heavily doped side is positioned at the band edge; i.e., $E_F = E_v$ in a p^+ material and $E_F = E_c$ in an n^+ material. Making the cited assumption, compute and plot V_{bi} as a function of the doping (N_A or N_D) on the lightly doped side of Si p^+-n and n^+-p step junctions maintained at 300 K. The plot is to cover the range $10^{14}/\text{cm}^3 \le N_A$ or $N_D \le 10^{17}/\text{cm}^3$.

S: Specifically considering a p^+-n junction, we can write

$$(E_i - E_F)_{p\text{-side}} \text{ assumed} = E_i - E_v = E_G/2$$

$$(E_F - E_i)_{n\text{-side}} = kT \ln(N_D/n_i)$$

Substituting into Eq. (5.12), which is valid for arbitrary doping levels, we rapidly conclude

$$V_{bi} = \frac{E_G}{2q} + \frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right)$$

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For n^+-p junctions, N_A simply replaces N_D, yielding a computationally equivalent relationship. The V_{bi} computational program and resultant plot (Fig. E5.1) follow:

MATLAB program script...

```
%Vbi Computation (p+/n and n+/p junctions)

%Constants
EG=1.12;
kT=0.0259;
ni=1.0e10;

%Computation
ND=logspace(14,17);
Vbi=EG/2+kT.*log(ND./ni);

%Plotting
close
semilogx(ND,Vbi); grid
axis([1.0e14 1.0e17 0.75 1])
xlabel('NA or ND (cm-3)'); ylabel('Vbi (volts)')
text(1e16,0.8,'Si, 300K')
text(1e16,0.78,'p+/n and n+/p diodes')
```

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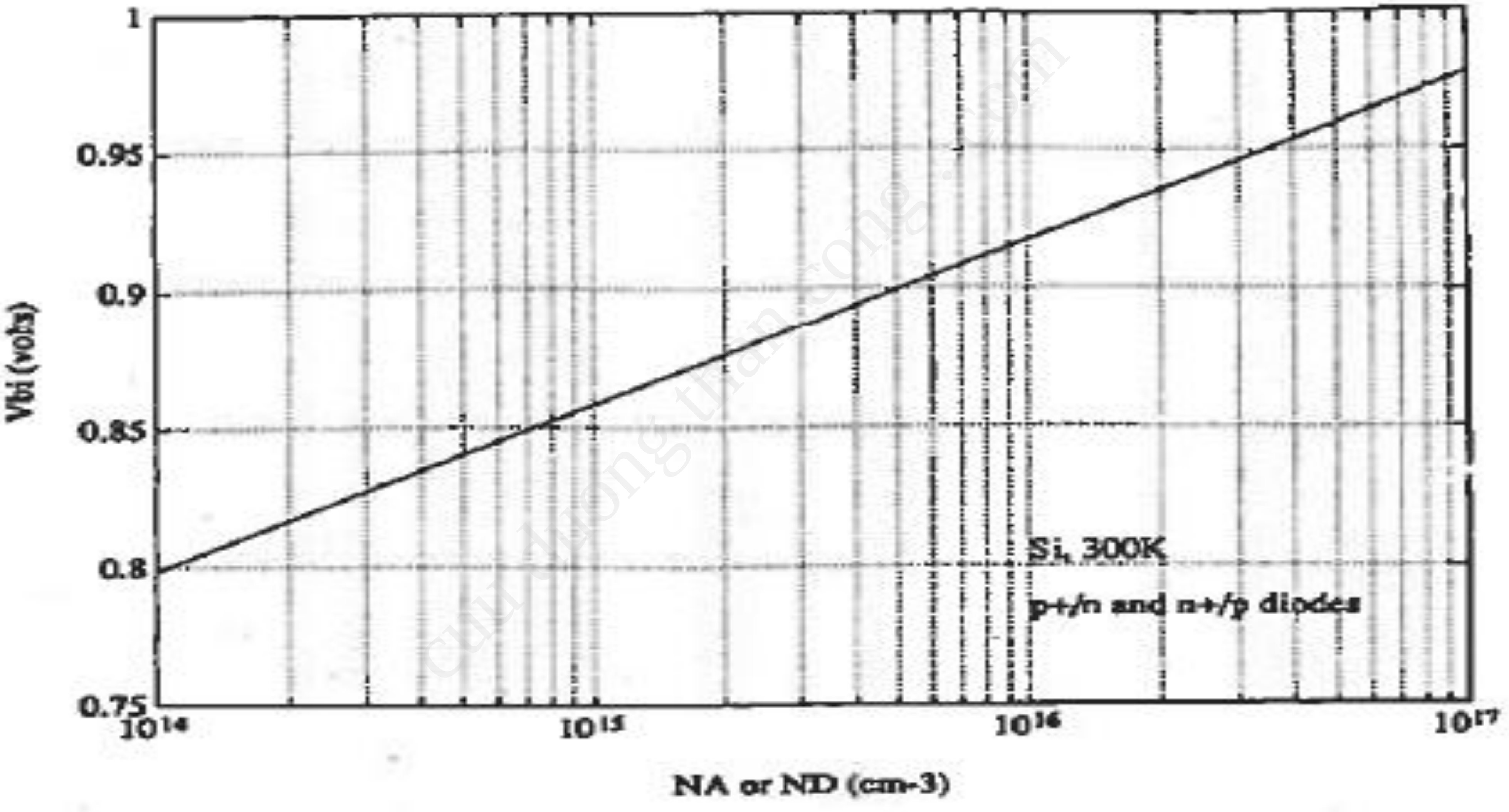

Built-in voltage in Si p^+-n and n^+-p step junction diodes at 300 K.

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- **The depletion approximation**

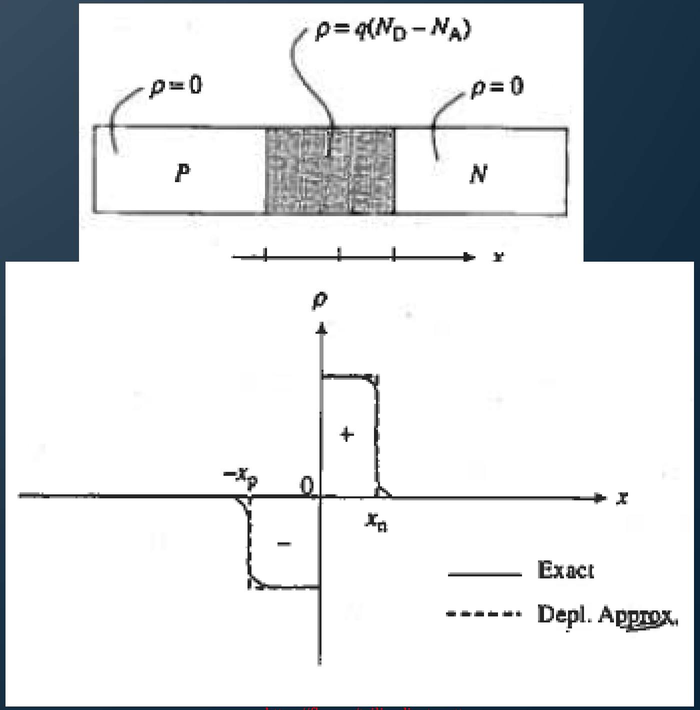
  Solve Poisson's equation → quantitative solution for electrostatic variables

  Depletion approximation → enable to obtain closed-form solutions because of no information of carrier cncentrations

$$\frac{d\mathscr{E}}{dx} = \frac{\rho}{K_S \varepsilon_0} = \frac{q}{K_S \varepsilon_0}(p - n + N_D - N_A)$$

1D Poisson's equation simplifies to

$$\frac{d\mathscr{E}}{dx} \cong \begin{cases} \frac{q}{K_S \varepsilon_0}(N_D - N_A) & \ldots -x_p \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$



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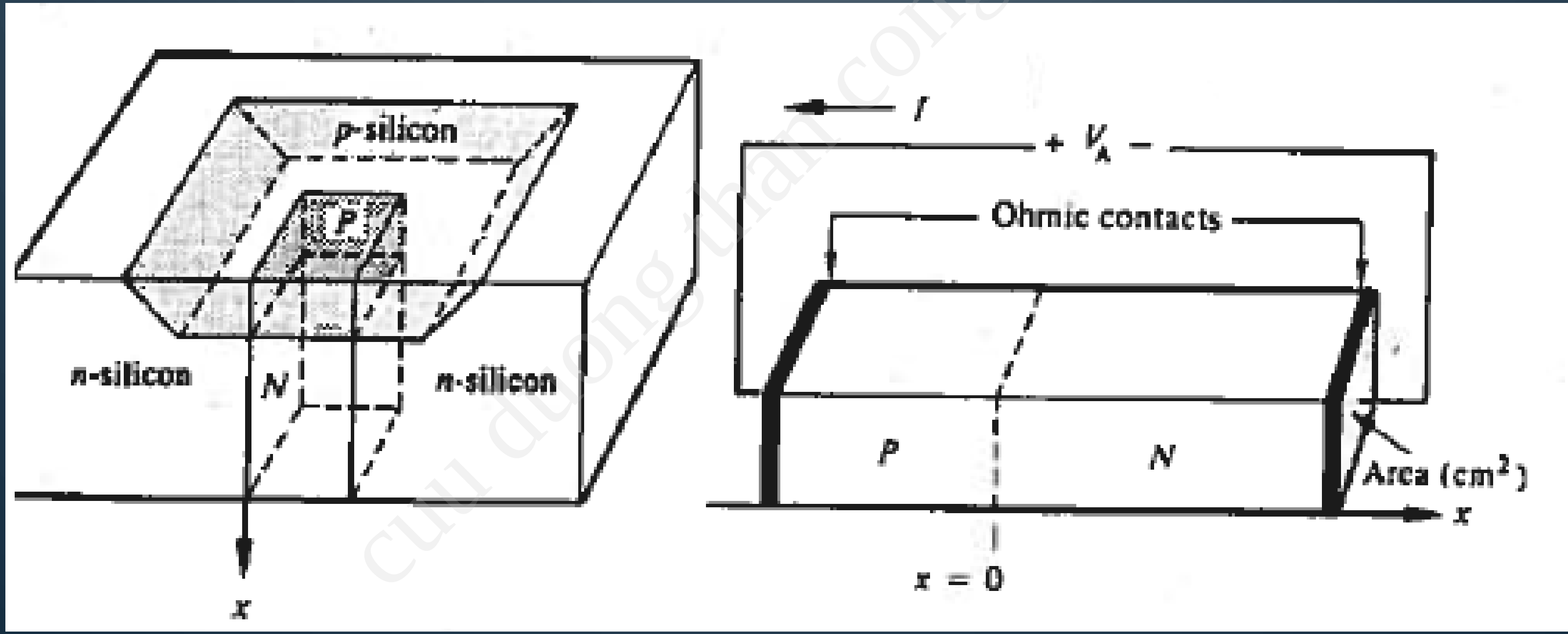
- **Quantitative electrostatic relationships**

  Assumptions/Definitions: 1D, VA applied voltage



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- **Junction with $V_A = 0$**

  Solution for ρ from depletion approximation

$$\rho = \begin{cases} -qN_A & \ldots -x_p \le x \le 0 \\ qN_D & \ldots 0 \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$

  Solution for ξ by substitute ρ into Poisson's equation

$$\frac{d\mathscr{E}}{dx} = \begin{cases} -qN_A/K_S\varepsilon_0 & \ldots -x_p \le x \le 0 \\ qN_D/K_S\varepsilon_0 & \ldots 0 \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$

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- For p-side of the depletion region

$$\int_0^{\mathscr{E}(x)} d\mathscr{E}' = -\int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0} dx'$$

$$\mathscr{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x) \quad \ldots -x_p \le x \le 0$$

- For n-side of the depletion region

$$\int_{\mathscr{E}(x)}^{0} d\mathscr{E}' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_0} dx'$$

$$\mathscr{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x) \quad \ldots 0 \le x \le x_n$$

- Note: total charge in depletion region → sum to zero

$$qN_A x_p = qN_D x_n$$

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- **Solution for V**

Since $\mathscr{E} = -dV/dx,$

$$\frac{dV}{dx} = \begin{cases} \frac{qN_A}{K_S\varepsilon_0}(x_p + x) & \ldots -x_p \le x \le 0 \\ \frac{qN_D}{K_S\varepsilon_0}(x_n - x) & \ldots 0 \le x \le x_n \end{cases}$$

boundary conditions

$$V = 0 \quad \text{at} \quad x = -x_p$$

$$V = V_{bi} \quad \text{at} \quad x = x_n$$

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- For p-side of the depletion region

$$\int_0^{V(x)} dV' = \int_{-x_p}^{x} \frac{qN_A}{K_S\varepsilon_0}(x_p + x')dx'$$

$$V(x) = \frac{qN_A}{2K_S\varepsilon_0}(x_p + x)^2 \quad \ldots -x_p \le x \le 0$$

- For n-side of the depletion region

$$\int_{V(x)}^{V_{bi}} dV' = \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_0}(x_n - x')dx'$$

$$V(x) = V_{bi} - \frac{qN_D}{2K_S\varepsilon_0}(x_n - x)^2 \quad \ldots 0 \le x \le x_n$$

- At x = 0

$$\frac{qN_A}{2K_S\varepsilon_0}x_p^2 = V_{bi} - \frac{qN_D}{2K_S\varepsilon_0}x_n^2$$

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- **Solution for x_n and x_p**

Combine

$$\frac{qN_A}{2K_S\varepsilon_0}x_p^2 = V_{bi} - \frac{qN_D}{2K_S\varepsilon_0}x_n^2$$

and

$$qN_Ax_p = qN_Dx_n$$

$$x_n = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_A}{N_D(N_A+N_D)}V_{bi}\right]^{1/2}$$

$$x_p = \frac{N_Dx_n}{N_A} = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}V_{bi}\right]^{1/2}$$

$$W \equiv x_n + x_p = \left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A+N_D}{N_AN_D}\right)V_{bi}\right]^{1/2}$$

Depletion width

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- Example

P: Perform a sample computation to gauge the size of W and $|\mathscr{E}|_{max}$ under equilibrium conditions. Specifically assume a Si step junction operated at 300 K with $N_A = 10^{17}/cm^3$ and $N_D = 10^{14}/cm^3$.

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S: For the given junction,

$$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = (0.0259)\ \ln\left[\frac{(10^{17})(10^{14})}{(10^{20})}\right] = 0.656\ \text{V}$$

Making use of Eqs. (5.30), one computes

$$x_n \cong \left[\frac{2K_S\varepsilon_0}{qN_D}V_{bi}\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\ \times\ 10^{-14})(0.656)}{(1.6\ \times\ 10^{-19})(10^{14})}\right]^{1/2}$$

$$= 2.93\ \times\ 10^{-4}\ \text{cm} = 2.93\ \mu\text{m}$$

$$x_p = \left(\frac{N_D}{N_A}\right)x_n = (10^{-3})x_n = 2.93\ \times\ 10^{-7}\ \text{cm}$$

and

$$W = x_n + x_p \cong x_n = \boxed{2.93\ \mu\text{m}}$$

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Also

$$|\mathscr{E}|_{max} = |\mathscr{E}(0)| = \frac{qN_D}{K_S\varepsilon_0}x_n = \frac{(1.6\times10^{-19})(10^{14})(2.93\times10^{-4})}{(11.8)(8.85\times10^{-14})}$$

$$= \boxed{4.49\times10^{3}\ \text{V/cm}}$$

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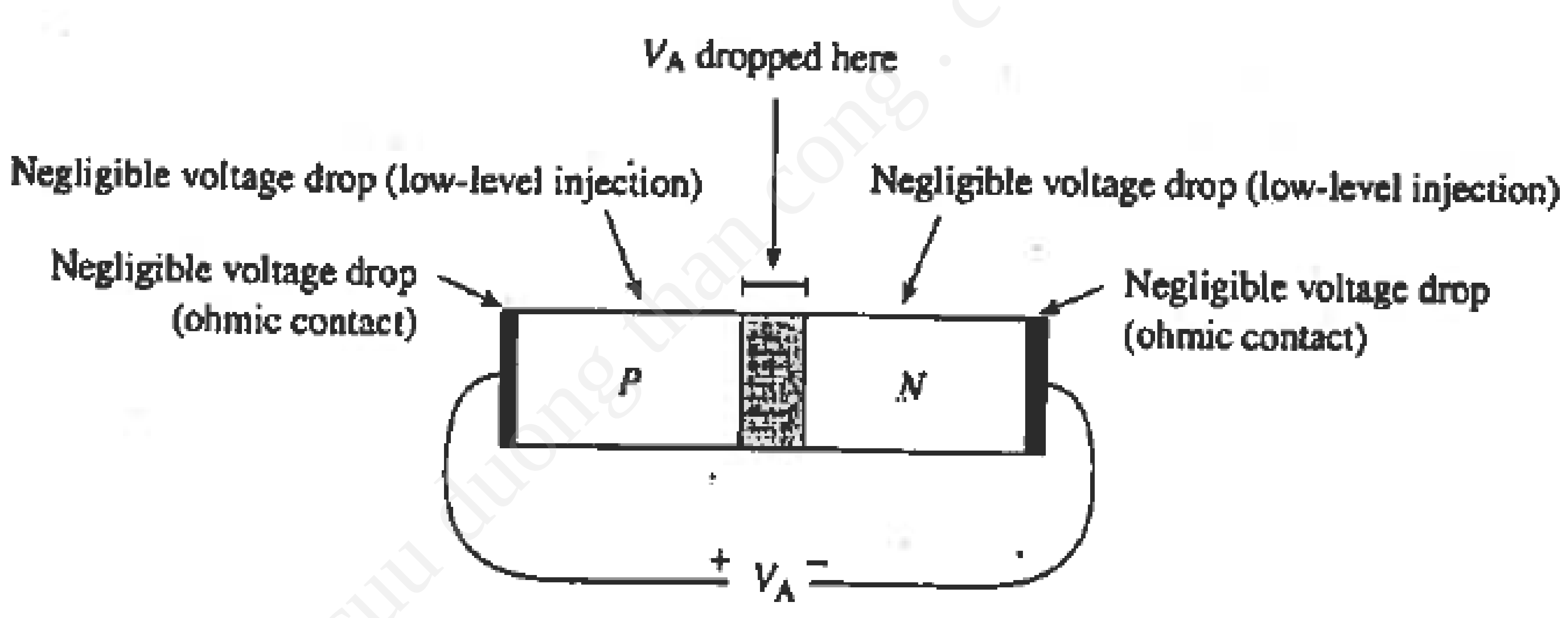
- Junction with $V_A \neq 0$



In depletion region: Vbi - VA

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Simply replace all V_{bi} by $V_{bi} - V_A$

For $-x_p \le x \le 0 \ldots$

$$\mathscr{E}(x) = -\frac{qN_A}{K_S\varepsilon_0}(x_p + x)$$

$$V(x) = \frac{qN_A}{2K_S\varepsilon_0}(x_p + x)^2$$

$$x_p = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A + N_D)}(V_{bi} - V_A)\right]^{1/2}$$

For $0 \le x \le x_n \ldots$

$$\mathscr{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x)$$

$$V(x) = V_{bi} - V_A - \frac{qN_D}{2K_S\varepsilon_0}(x_n - x)^2$$

$$x_n = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_A}{N_D(N_A + N_D)}(V_{bi} - V_A)\right]^{1/2}$$

and

$$W = \left[\frac{2K_S\varepsilon_0}{q}\left(\frac{N_A + N_D}{N_AN_D}\right)(V_{bi} - V_A)\right]^{1/2}$$

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P: Construct a log-log plot of the depletion width (W) versus the impurity concentration (N_A or N_D) on the lightly doped side of Si p^+-n and n^+-p step junctions maintained at 300 K. Include curves for $V_A = 0.5$ V, 0 V, and -10 V covering the range $10^{14}/\text{cm}^3 \leq N_A$ or $N_D \leq 10^{17}/\text{cm}^3$.

S: The V_{bi} associated with the p^+-n and n^+-p step junctions is computed using the relationship established in Exercise 5.1. Also, with the junction asymmetrically doped, the doping factor in the Eq. (5.38) expression for W simplifies to

$$\frac{N_A + N_D}{N_A N_D} \cong \frac{1}{N_B}$$

where N_B is the doping (N_A or N_D) on the lightly doped side of the junction. The MATLAB program for the W versus doping computation and the program results (Fig. E5.3) follow:

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MATLAB program script...

```
% This program calculates and plots the depletion width vs impurity
% concentration in Silicon p+/n and n+/p step junctions at 300K.
%
% Three plots are generated corresponding to VA = 0.5V, 0.0V, and -10V
%
%    The Vbi relationship employed is Vbi=(EG/2q)+(kT/q)ln(NB/ni)
%    where NB is the impurity concentration on the lightly doped side.

%Constants and Parameters
T=300;              % Temperature in Kelvin
k=8.617e-5;         % Boltzmann constant (eV/K)
e0=8.85e-14;        % permittivity of free space (F/cm)
q=1.602e-19;        % charge on an electron (coul)
KS=11.8;            % dielectric constant of Si at 300K
ni=1e10;            % intrinsic carrier conc. in Silicon at 300K (cm^-3)
EG=1.12;            % band gap of Silicon (eV)

%Choose variable values
NB=logspace(14,17);        % doping ranges from 1e14 to 1e17
VA=[0.5 0 -10];            % VA values set
```

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```
%Depletion width calculation
Vbi=EG/2+k*T.*log(NB./ni);
W(1,:)=1.0e4*sqrt(2*KS*e0/q.*(Vbi-VA(1))./NB);
W(2,:)=1.0e4*sqrt(2*KS*e0/q.*(Vbi-VA(2))./NB);
W(3,:)=1.0e4*sqrt(2*KS*e0/q.*(Vbi-VA(3))./NB);

%Plot
close
loglog(NB, W,'-'); grid
axis([1.0e14 1.0e17 1.0e-1 1.0e1])
xlabel('NA or ND (cm^-3)')
ylabel('W (micrometers)')
set(gca,'DefaultTextUnits','normalized')
text(.38,.26,'VA=0.5V')
text(.38,.50,'VA=0')
text(.38,.76,'VA=-10V')
text(.77,.82,'Si,300K')
text(.77,.79,'p+/n and n+/p')
set(gca,'DefaultTextUnits','data')
```

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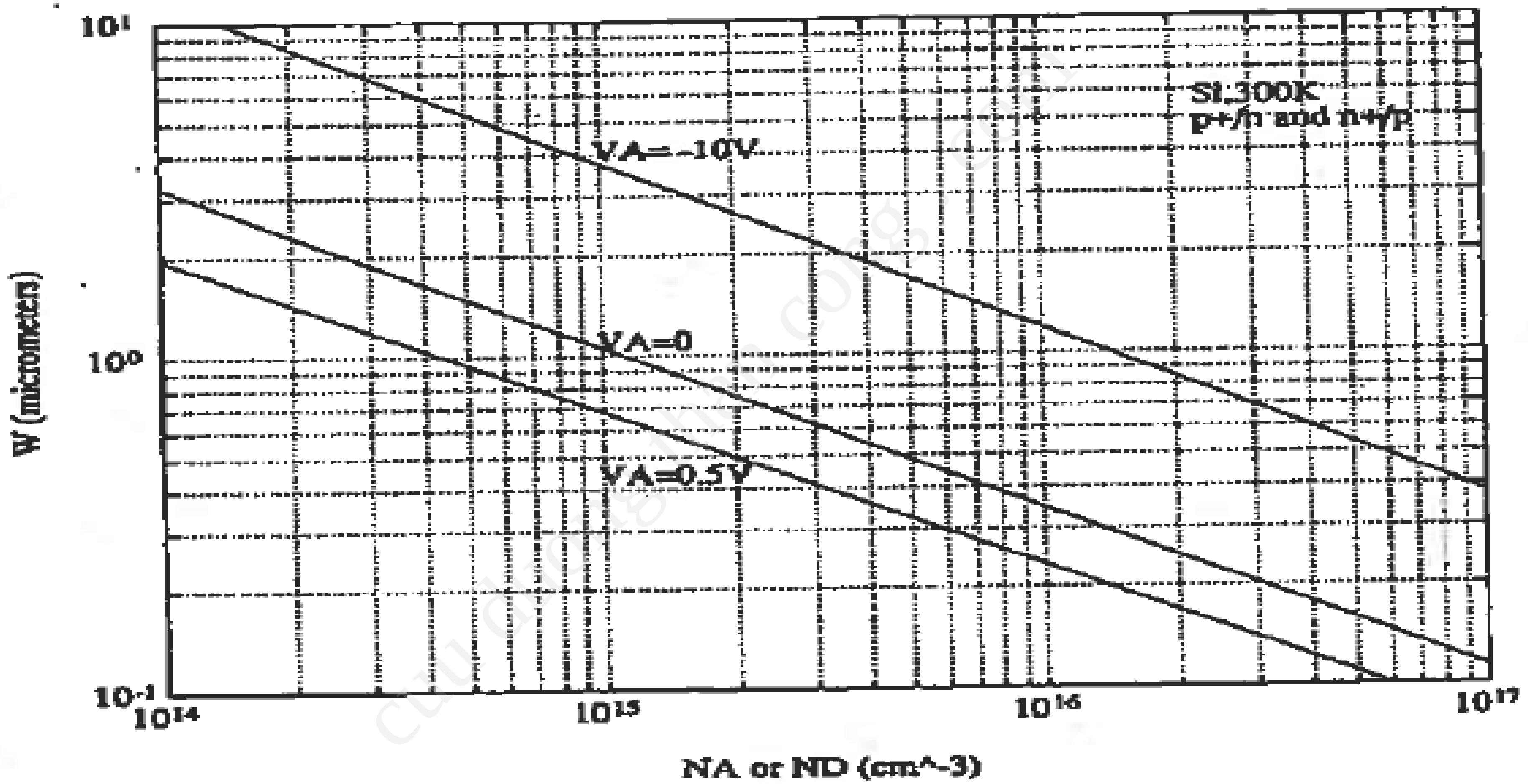

Figure E5.3 Depletion width at select applied biases as a function of doping (Si, p^+-n and n^+-p step junctions, 300 K).

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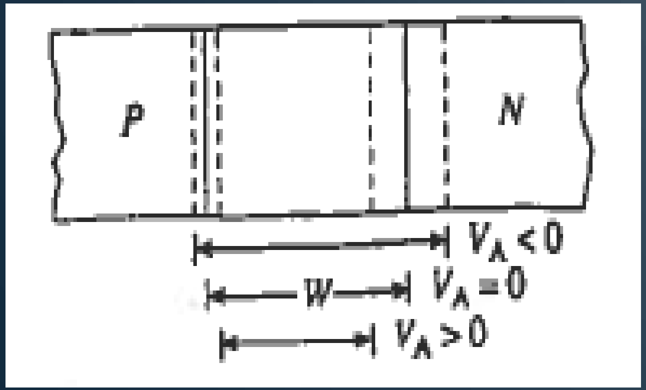
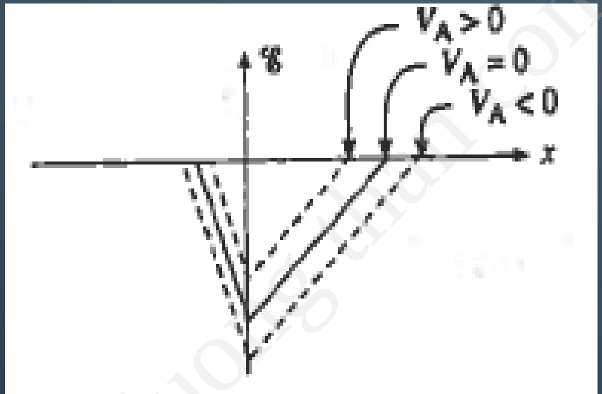
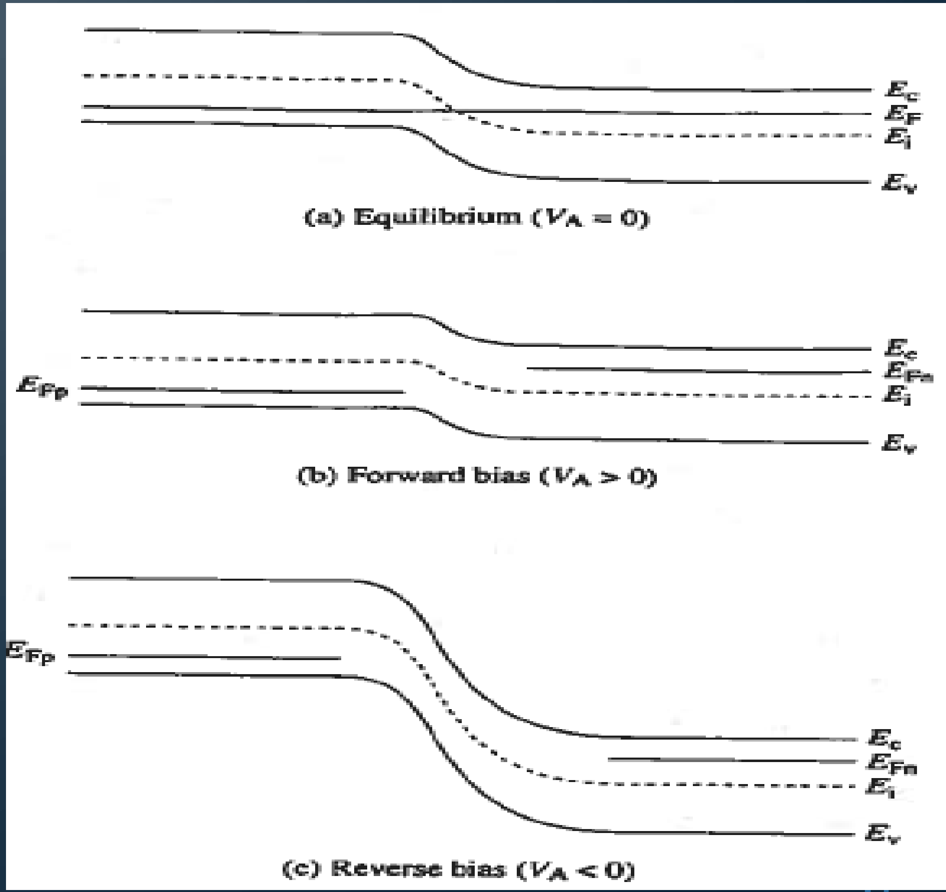
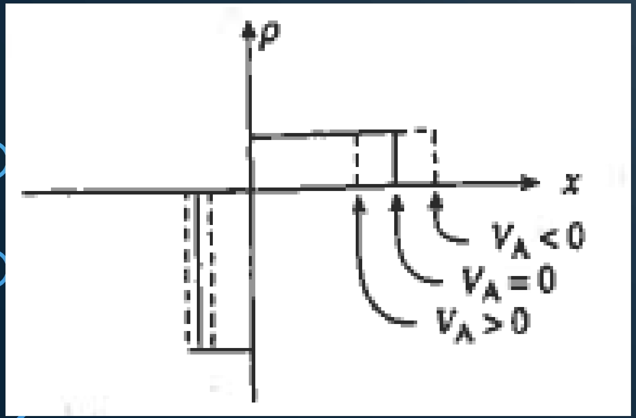
- **Examination/Extrapolation of results**

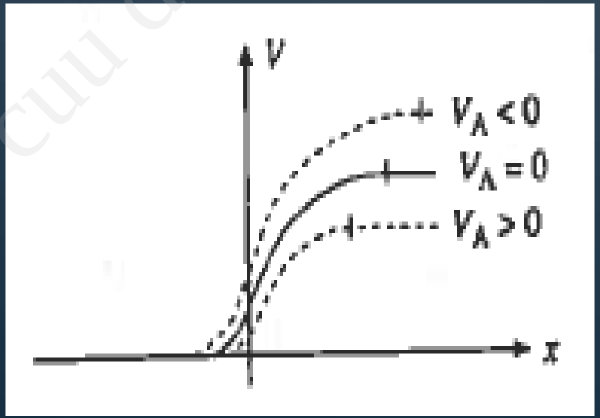
  The width W decrease under forward bias (V_{bi} –V_A) and increase under reverse bias (V_{bi} + V_A). Electric field changes respectively











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Once a quantitative relationship has been established for the electrostatic potential, it becomes possible to construct a fully dimensioned energy band diagram. The "Diagram Generator" program that follows draws the equilibrium energy band diagram

for a nondegenerately doped Si step junction maintained at room temperature. The user is prompted to input the *p*- and *n*-side doping concentrations. Run the program trying different N_A and N_D combinations. It is informative to include at least one combination each where $N_A \gg N_D$, $N_A \cong N_D$, and $N_A \ll N_D$. The asymmetrical junctions are of particular interest because the resultant "one-sided" diagrams differ from those normally included in textbooks. The user might also consider modifying the program so that it draws the energy band diagram for an arbitrary applied bias.

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MATLAB program script...

```
% Equilibrium Energy Band Diagram Generator
%(Si, 300K, nondegenerately doped step junction)

%Constants
T=300;              % Temperature in Kelvin
k=8.617e-5;         % Boltzmann constant (eV/K)
e0=8.85e-14;        % permittivity of free space (F/cm)
q=1.602e-19;        % charge on an electron (coul)
KS=11.8;            % Dielectric constant of Si
ni=1.0e10;          % intrinsic carrier conc. in Silicon at 300K (cm^-3)
EG=1.12;            % Silicon band gap (eV)

%Control constants
xleft = -3.5e-4;    % Leftmost x position
xright = -xleft;    % Rightmost x position
NA=input ('Please enter p-side doping (cm^-3), NA = ');
ND=input ('Please enter n-side doping (cm^-3), ND = ');
```

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```
%Computations
Vbi=k*T*log((NA*ND)/ni^2);
xN=sqrt(2*KS*e0/q*NA*Vbi/(ND*(NA+ND)));      % Depletion width n-side
xP=sqrt(2*KS*e0/q*ND*Vbi/(NA*(NA+ND)));      % Depletion width p-side
x = linspace(xleft, xright, 200);
Vx1=(Vbi-q*ND.*(xN-x).^2/(2*KS*e0).*(x<=xN)).*(x>=0);
Vx2=0.5*q*NA.*(xP+x).^2/(KS*e0).*(x>=-xP & x<0);
Vx=Vx1+Vx2;                                  % V as a function of x
VMAX = 3;                                    % Maximum Plot Voltage
EF=Vx(1)+VMAX/2-k*T*log(NA/ni);              % Fermi level

%Plot Diagram
close
plot (x, -Vx+EG/2+VMAX/2);
axis ([xleft xright 0 VMAX]);
axis ('off'); hold on
plot (x, -Vx-EG/2+VMAX/2);
```

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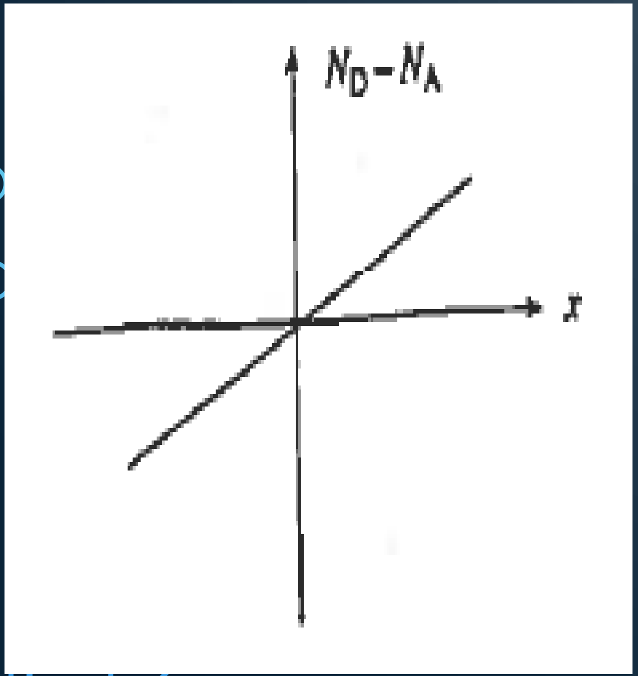
```
plot (x, -Vx+VMAX/2,'w:');
plot ([xleft xright], [EF EF], 'w');
plot ([0 0], [0.15 VMAX-0.5], 'w--');
text(xleft*1.08,(-Vx(1)+EG/2+VMAX/2-.05),'Ec');
text(xright*1.02,(-Vx(200)+EG/2+VMAX/2-.05),'Ec');
text(xleft*1.08,(-Vx(1)-EG/2+VMAX/2-.05),'Ev');
text(xright*1.02,(-Vx(200)-EG/2+VMAX/2-.05),'Ev');
text(xleft*1.08,(-Vx(1)+VMAX/2-.05),'Ei');
text(xright*1.02, EF-.05,'EF');
set(gca,'DefaultTextUnits','normalized')
text(.18, 0,'p-side');
text(.47, 0, 'x=0');
text(.75, 0,'n-side');
set(gca,'DefaultTextUnits','data')
hold off
```

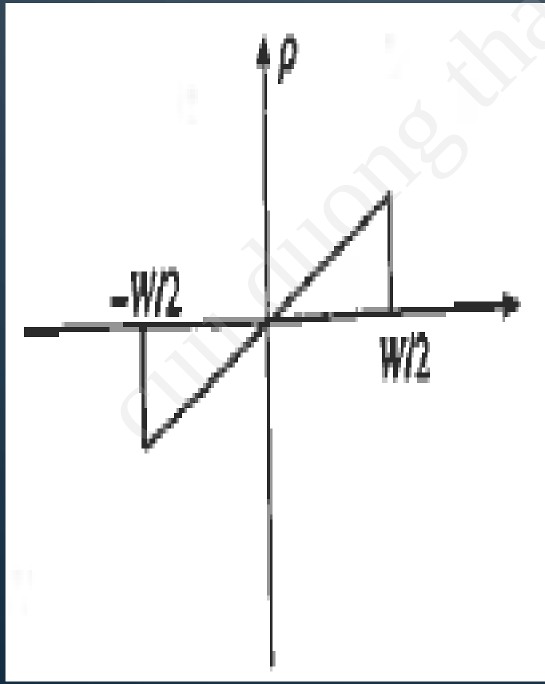
PN JUNCTION ELECTROSTATICS

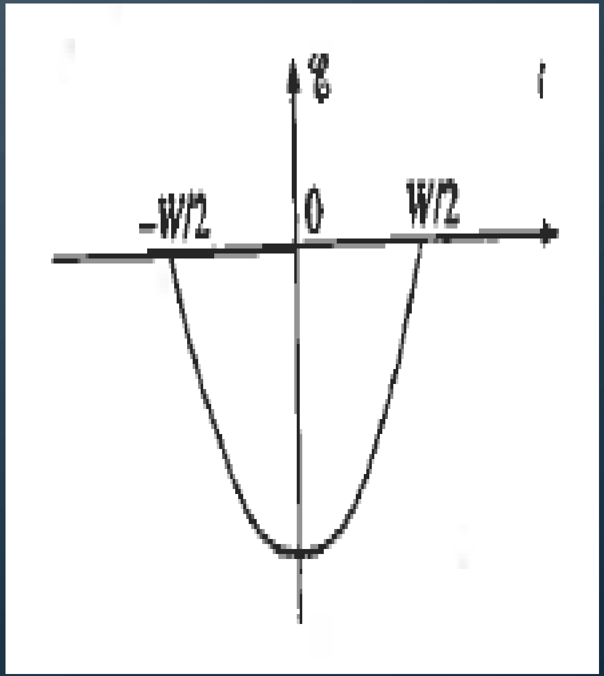
- **Linearly graded junctions**

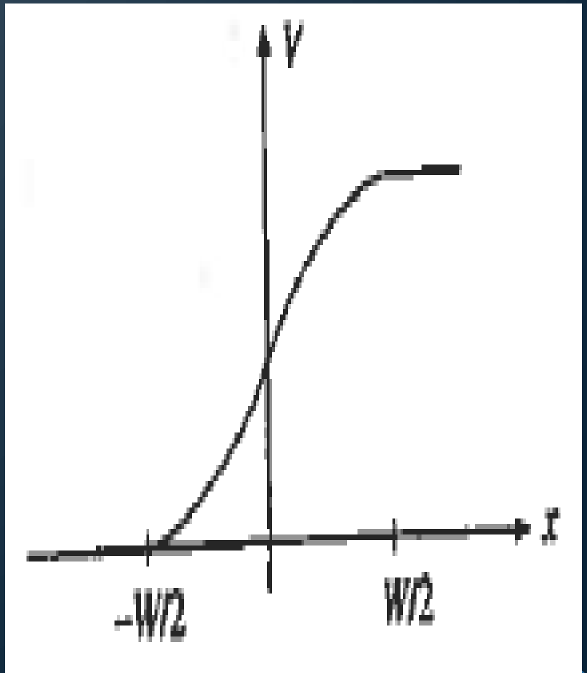
$$N_D - N_A = ax$$

where a has units of cm^{-4} and is called the *grading constant.*









PN JUNCTION ELECTROSTATICS

Depletion approximation

$$\rho(x) = \begin{cases} qax & \ldots -W/2 \le x \le W/2 \\ 0 & \ldots x \le -W/2 \text{ and } x \ge W/2 \end{cases}$$

Note

$$x_p = x_n = \frac{W}{2}$$

$$\mathscr{E}(x) = \frac{qa}{2K_S\varepsilon_0}\left[x^2 - \left(\frac{W}{2}\right)^2\right] \quad \ldots -\frac{W}{2} \le x \le \frac{W}{2}$$

$$V(x) = \frac{qa}{6K_S\varepsilon_0}\left[2\left(\frac{W}{2}\right)^3 + 3\left(\frac{W}{2}\right)^2 x - x^3\right] \quad \ldots -\frac{W}{2} \le x \le \frac{W}{2}$$

$$W = \left[\frac{12K_S\varepsilon_0}{qa}(V_{bi} - V_A)\right]^{1/3}$$

PN JUNCTION ELECTROSTATICS

For linearly graded junction

$$n(x_n)_{\text{equilibrium}} \cong (N_D - N_A)|_{W_0/2} = aW_0/2$$

$$n(-x_p)_{\text{equilibrium}} = \frac{n_i^2}{p(-x_p)_{\text{equilibrium}}} \cong \frac{n_i^2}{-(N_D - N_A)|_{-W_0/2}} = \frac{n_i^2}{aW_0/2}$$

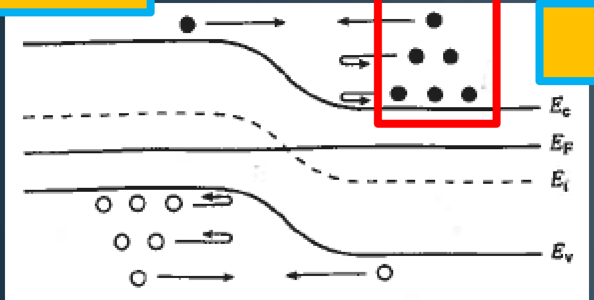
$$V_{bi} = \frac{kT}{q}\ln\left(\frac{aW_0}{2n_i}\right)^2 = \frac{2kT}{q}\ln\left(\frac{aW_0}{2n_i}\right)$$

$$V_{bi} = \frac{2kT}{q}\ln\left[\frac{a}{2n_i}\left(\frac{12K_S\varepsilon_0}{qa}V_{bi}\right)^{1/3}\right]$$

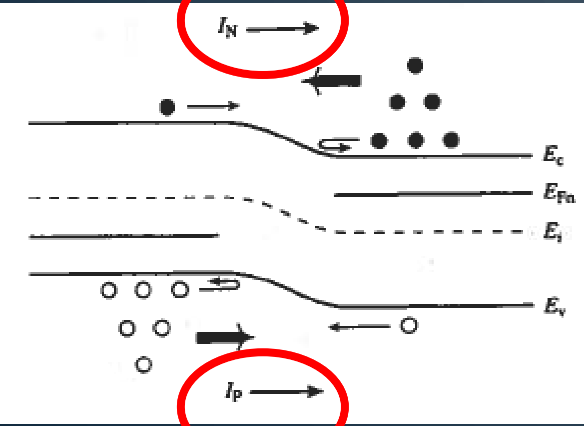
PN JUNCTION DIODE: I-V CHARACTERISTICS

Drift

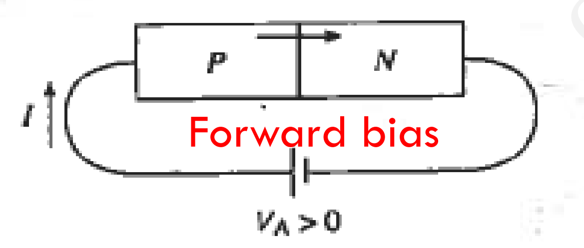
Diffusion



(a) Equilibrium ($V_A = 0$)



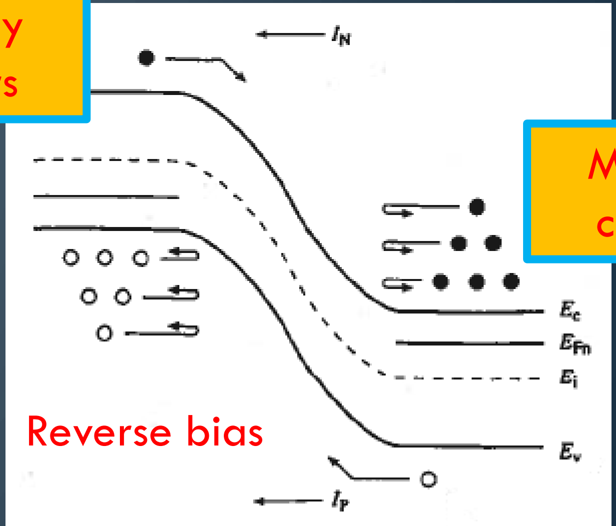
$$I = I_n + I_p$$



(b) Forward bias ($V_A > 0$)

Minority carriers

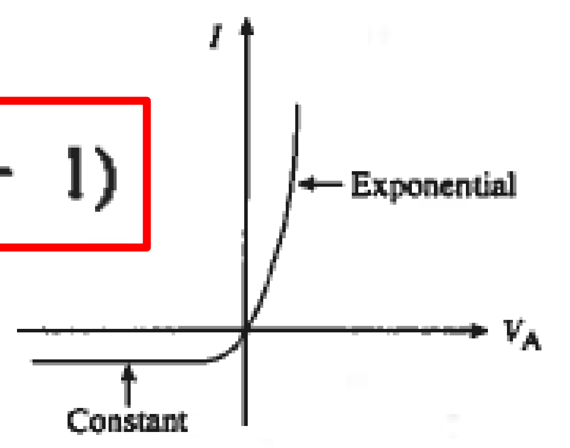
Majority carriers



(c) Reverse bias ($V_A < 0$)

$$I = I_0(e^{V_A/V_{ref}} - 1)$$

V_{ref} is set equal to kT/q.



(d)

PN JUNCTION DIODE: I-V CHARACTERISTICS

- **The ideal diode equation**

  Qualitative derivation

- **Forward bias:**

  reduce potential hill between two sides of junction

  permit majority carriers to be injected across depletion region

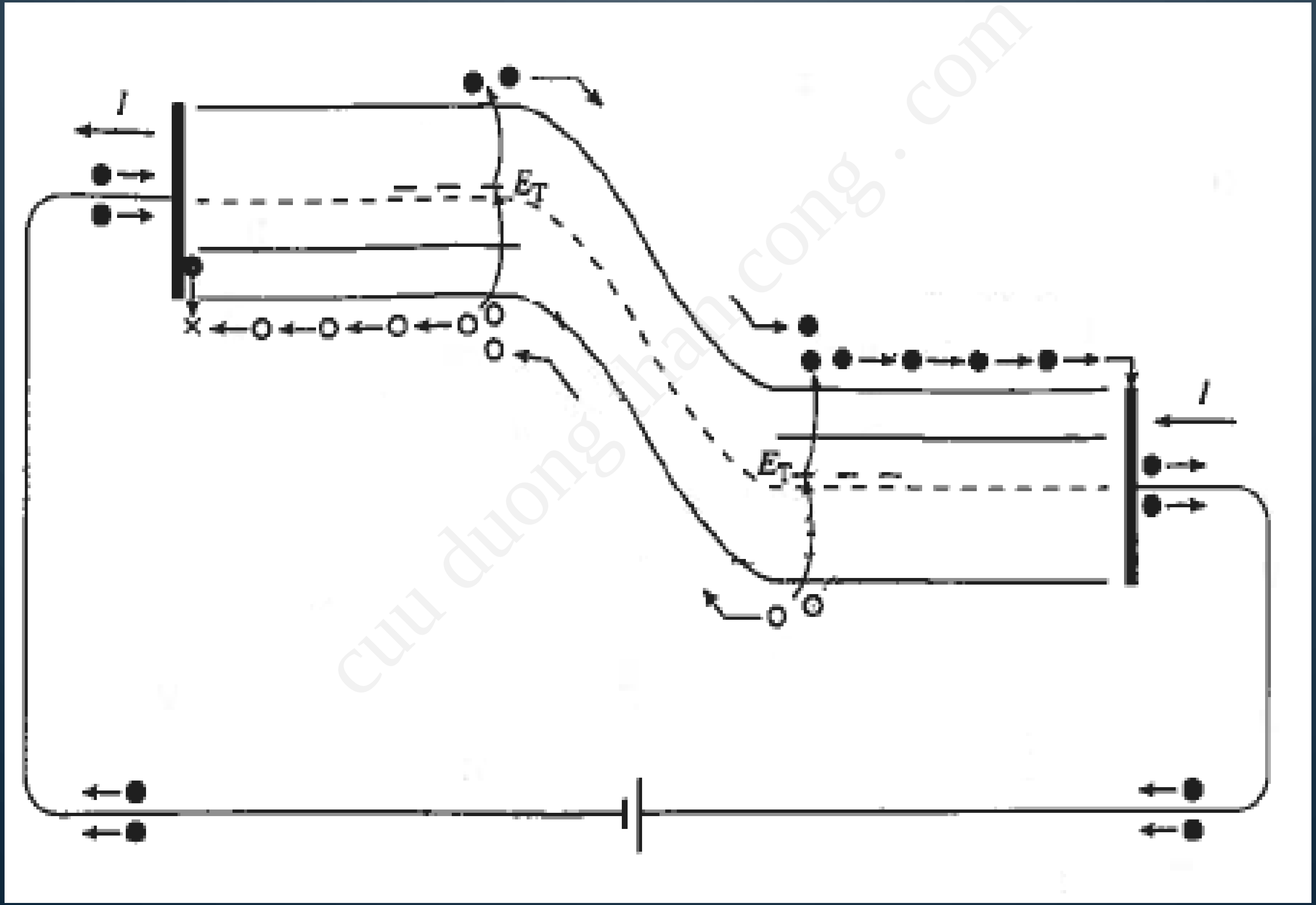
- **Reverse bias:**

  increase potential hill

  cutting off majority carriers injection

  leaving only a small current supplied by minority carriers

PN JUNCTION DIODE: I-V CHARACTERISTICS



PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Quantitative Solution strategy**

Assumptions

1. The diode is operating under steady state conditions
2. A nondegenerately doped step junction models the doping profile
3. The diode is one-dimentional
4. Low-level injection prevails in quasineutral regions
5. No process other than drift, diffusion, and thermal recombination-generation inside the diode ($G_L = 0$)

PN JUNCTION DIODE: I-V CHARACTERISTICS

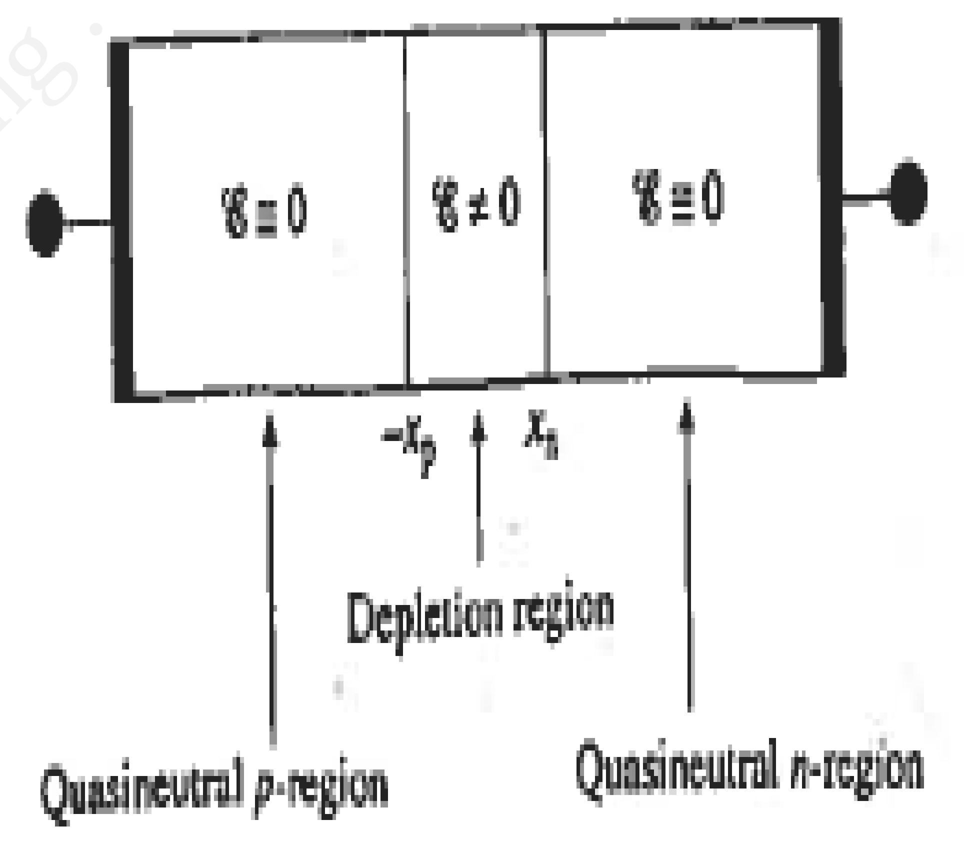
General relationships available for computing the current:

$$I = AJ \qquad (A = \text{cross-sectional area})$$

$$J = J_N(x) + J_P(x)$$

$$J_N = q\mu_n n\mathscr{E} + qD_N\frac{dn}{dx}$$

$$J_P = q\mu_p p\mathscr{E} - qD_P\frac{dp}{dx}$$



PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Quasineutral region considerations:**

Minority carrier diffusion equations for p and n quasineutral regions are:

$$0 = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} \quad \ldots x \leq -x_p$$

$$0 = D_P \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} \quad \ldots x \geq x_n$$

(*)

n_p: electron in p-region

p_n: hole in n-region

PN JUNCTION DIODE: I-V CHARACTERISTICS

Since $\mathscr{E} \cong 0$ and $dn_0/dx = dp_0/dx = 0$

carrier current density in quasineutral regions simplify to

$$J_N = qD_N \frac{d\Delta n_p}{dx} \qquad \ldots x \leq -x_p$$

$$J_P = -qD_P \frac{d\Delta p_n}{dx} \qquad \ldots x \geq x_n$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Depletion Region Considerations:** $\mathscr{E} \neq 0$

Continuity equations simplify to

$$0 = \frac{1}{q}\frac{dJ_N}{dx} + \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}}$$

$$0 = -\frac{1}{q}\frac{dJ_P}{dx} + \left.\frac{\partial p}{\partial t}\right|_{\substack{\text{thermal}\\ \text{R-G}}}$$

Additional assumption: thermal recombination-generation is negligible throughout the depletion region

$$\left.\partial n/\partial t\right|_{\text{thermal R-G}},\ \left.\partial p/\partial t\right|_{\text{thermal R-G}} \Big\} \sim 0 \qquad \frac{dJ_N}{dx} = 0 \qquad \frac{dJ_P}{dx} = 0$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

The constancy of the carrier currents throughout the depletion region

$$J_N(-x_p \le x \le x_n) = J_N(-x_p)$$

$$J_P(-x_p \le x \le x_n) = J_P(x_n)$$

$$J = J_N(-x_p) + J_P(x_n)$$

- **Solution strategy:**
  - → Solve for the minority carrier current densities in the quasineutral regions
  - → Evaluate the current densities at the depletion region edges
  - → Add the edge current densities together
  - → Finally, multiply by A to obtain the current

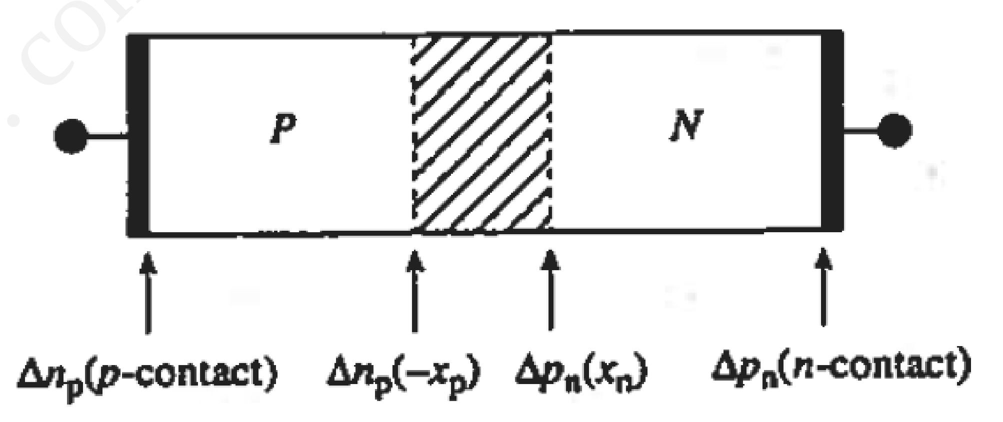
PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Boundary conditions in solving** (*)

At the Ohmic contacts:

$$\Delta n_p(x \rightarrow -\infty) = 0$$

$$\Delta p_n(x \rightarrow +\infty) = 0$$



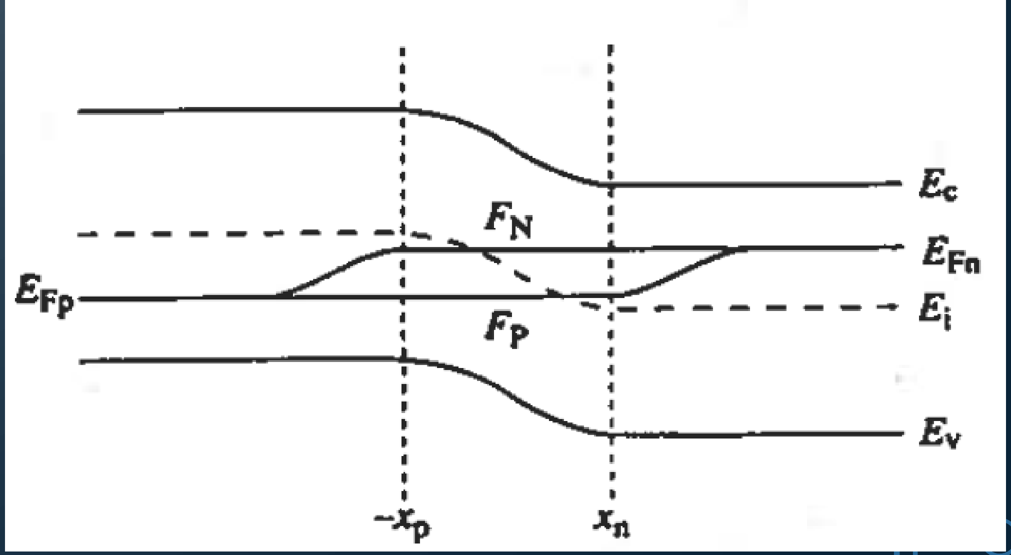
At the depletion region edges:

$$np = n_i^2 e^{(F_N - F_P)/kT}$$

$$F_N - F_P \leq E_{Fn} - E_{Fp} = qV_A$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)$$



PN JUNCTION DIODE: I-V CHARACTERISTICS

To obtain an analytical solution for current flowing in ideal diode as a function of the applied voltage:

1. Solve the minority carrier diffusion equations employing boundary conditions to obtain Δn_p and Δp_n in quasineutral regions

$$0 = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} \quad \ldots x \leq -x_p$$

$$0 = D_P \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} \quad \ldots x \geq x_n$$

$$\Delta n_p(x \to -\infty) = 0$$

$$\Delta p_n(x \to +\infty) = 0$$

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

2. Compute the minority carrier current densities in the quasineutral regions using

$$J_N = qD_N \frac{d\Delta n_p}{dx} \qquad \ldots x \le -x_p$$

$$J_P = -qD_P \frac{d\Delta p_n}{dx} \qquad \ldots x \ge x_n$$

3. Evaluate the quasineutral region solutions for $J_N(x)$ and $J_P(X)$ at the edges of the depletion region and the sum the two edge-current densities.

Finally, multiply the result by the cross section area of the diode

$$J = J_N(-x_p) + J_P(x_n)$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Derivation**

  Let us work with holes on ***quasineutral n-side of the junction***.

  We must solve

$$0 = D_P \frac{d^2 \Delta p_n}{dx'^2} - \frac{\Delta p_n}{\tau_p} \qquad \ldots x' \geq 0$$

subject to the boundary conditions

$$\Delta p_n(x' \rightarrow \infty) = 0$$

$$\Delta p_n(x' = 0) = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

General solution

$$\Delta p_n(x') = A_1 e^{-x'/L_P} + A_2 e^{x'/L_P} \qquad \ldots x' \geq 0$$

where

$$L_P = \sqrt{D_P \tau_p}$$

Because $\exp(x'/L_P) \to \infty$ as $x' \to \infty$ $\Rightarrow$ $A_2 = 0$, $A_1 = \Delta p_n(x' = 0)$

$$\Delta p_n(x') = \frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{-x'/L_P}$$

$$J_P(x') = -qD_P\frac{d\Delta p_n}{dx'} = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)e^{-x'/L_P}$$

$$\ldots x' \geq 0$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

- On *quasineutral p-side of the junction*.

$$\Delta n_p(x'') = \frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)e^{-x''/L_N}$$

$$J_N(x'') = -qD_N\frac{d\Delta n_p}{dx''} = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)e^{-x''/L_N}$$

$$\ldots x' \geq 0$$

- So,

$$J_N(x = -x_p) = J_N(x''=0) = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)$$

$$J_P(x = x_n) = J_P(x'=0) = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

- And then,

$$I = AJ = qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)(e^{qV_A/kT} - 1)$$

$$I = I_0(e^{qV_A/kT} - 1)$$

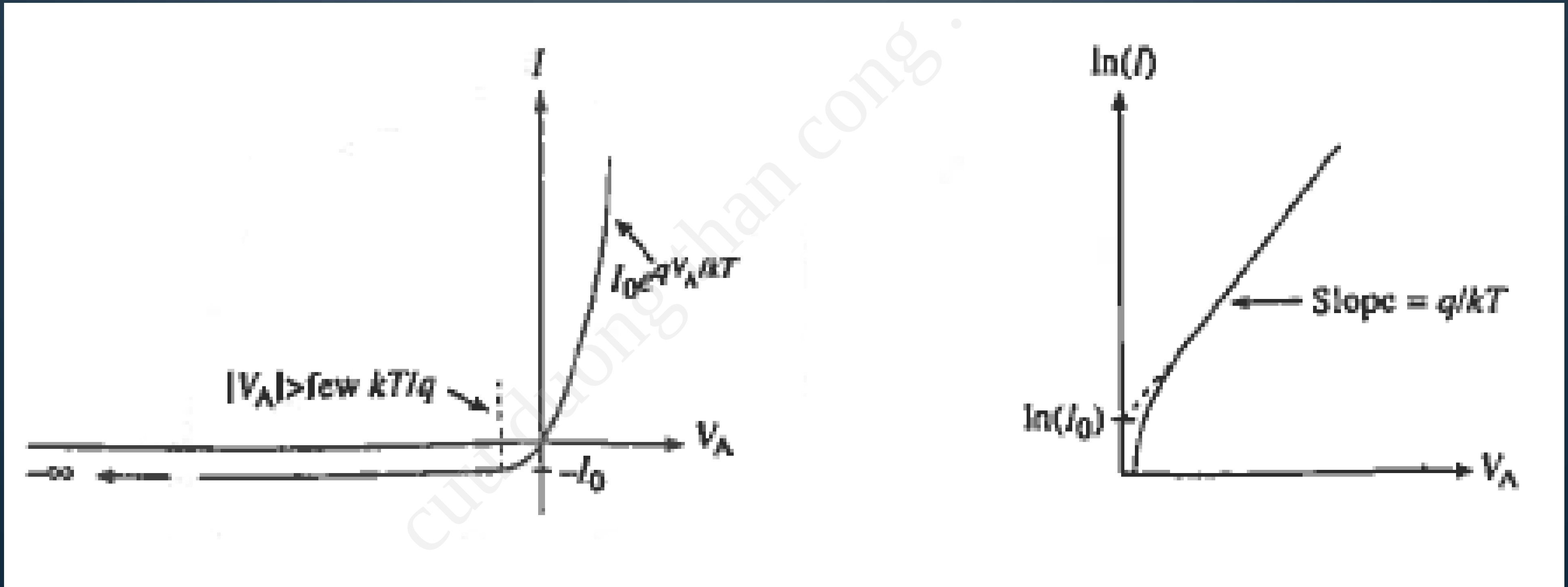
$$I_0 \equiv qA\left(\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right)$$

Shockley equation

PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Examination of results**

  1. Ideal I-V



$$\ln(I) = \ln(I_0) + \frac{q}{kT}V_A \quad \ldots \text{ if } V_A > \text{few } \frac{kT}{q}$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

- **Examination of results**

  2. The saturation current

  Note:

  I_o has strong dependence on materials

  I_o has relation to asymetrycally doped junctions

$$I_0 \cong qA \frac{D_P}{L_P} \frac{n_i^2}{N_D} \quad \ldots p^+\text{-}n \text{ diodes}$$

$$I_0 \cong qA \frac{D_N}{L_N} \frac{n_i^2}{N_A} \quad \ldots n^+\text{-}p \text{ diodes}$$

As a general rule, the heavily doped side of an asymmetrical junction can be ignored in determining the electrical characteristics of the junction

PN JUNCTION DIODE: I-V CHARACTERISTICS

- Exercise

P: Two ideal p^+-n step junction diodes maintained at room temperature are identical except that $N_{D1} = 10^{15}/\text{cm}^3$ and $N_{D2} = 10^{16}/\text{cm}^3$. Compare the I–V characteristics of the two diodes; sketch both characteristics on a single set of axes.

S: For p^+-n diodes

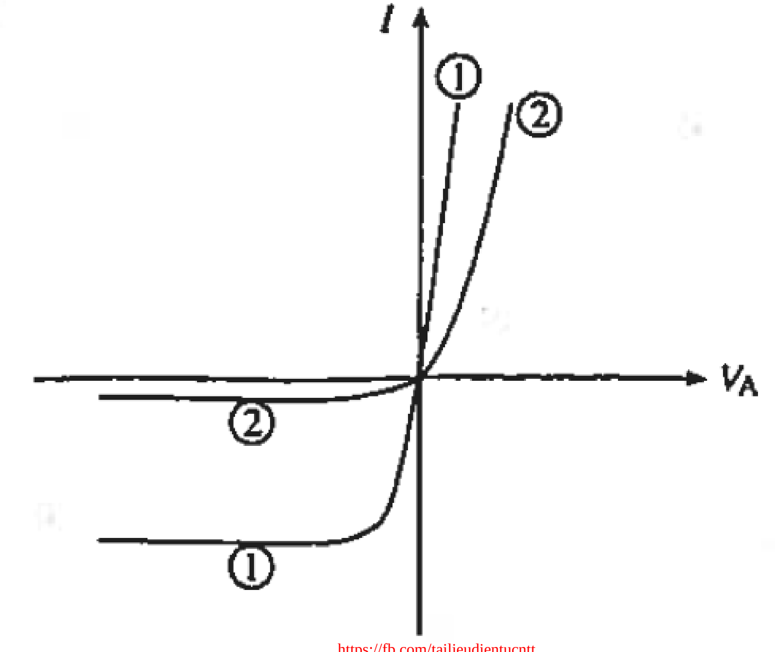
$$I_0 \cong qA \frac{D_P}{L_P} \frac{n_i^2}{N_D}$$

Also

$$\frac{D_P}{L_P} = \sqrt{\frac{D_P}{\tau_p}} = \sqrt{\frac{(kT/q)\mu_p}{\tau_p}}$$

$\tau_{p1} = \tau_{p2}$, since the R–G center concentrations are taken to be identical in the two diodes. Moreover, although the semiconductor material is not specified in the problem statement, there is only a small difference between the $N_D = 10^{15}/cm^3$ and $N_D = 10^{16}/cm^3$ mobilities in most materials (see Fig. 3.5). Thus

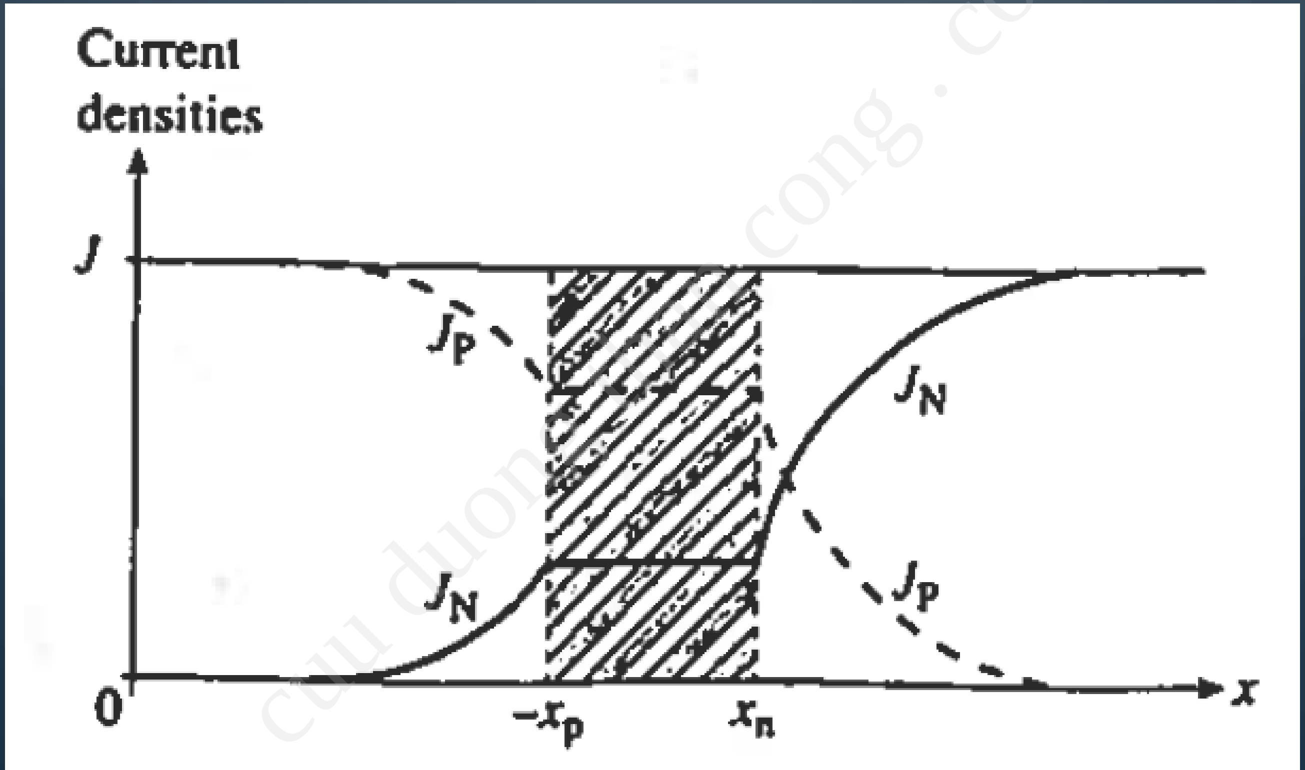
$$\frac{I_{01}}{I_{02}} = \frac{N_{D2}}{N_{D1}} \sqrt{\frac{\mu_{p1}}{\mu_{p2}}} \cong 10$$



The diode 1 current is approximately ten times larger than the diode 2 current for all applied voltages

PN JUNCTION DIODE: I-V CHARACTERISTICS

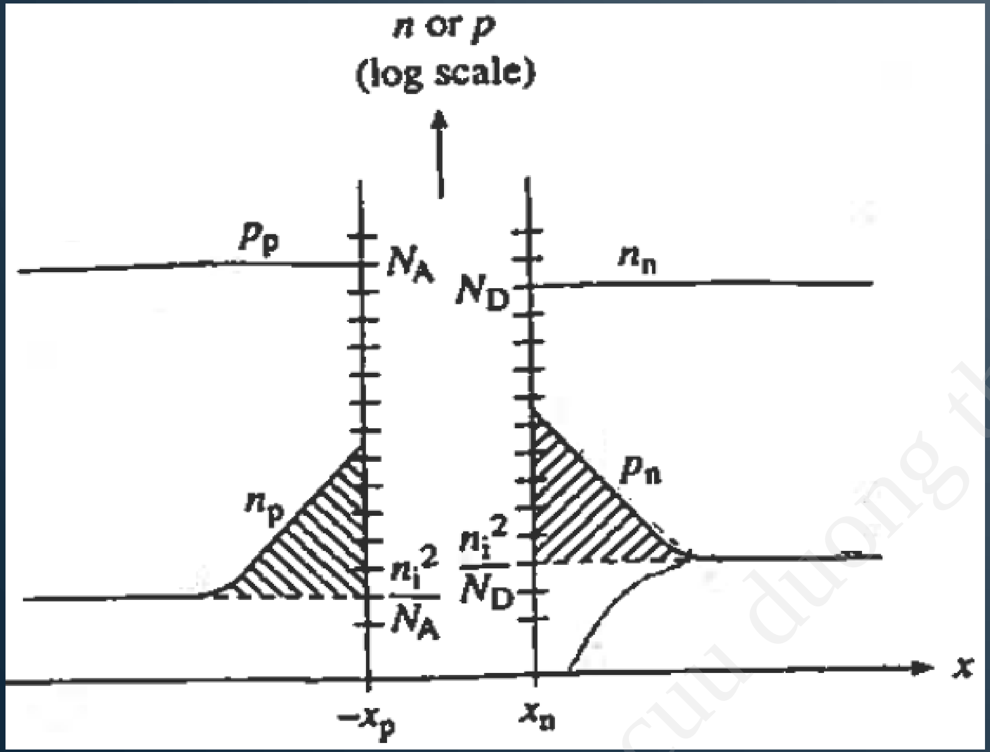
- **Carrier currents:**



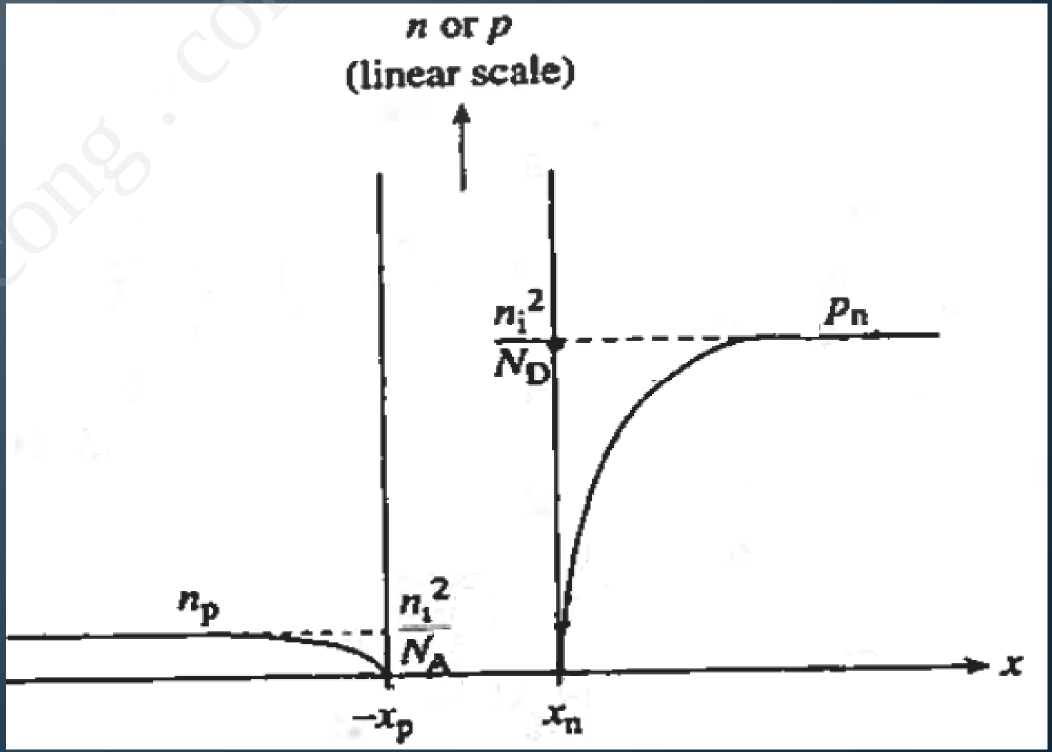
Carrier and total current densities vs. position inside a forward-biased pn junction diode

PN JUNCTION DIODE: I-V CHARACTERISTICS

- Carrier concentrations



Forwad biasing



Reverse biasing

PN JUNCTION DIODE: I-V CHARACTERISTICS

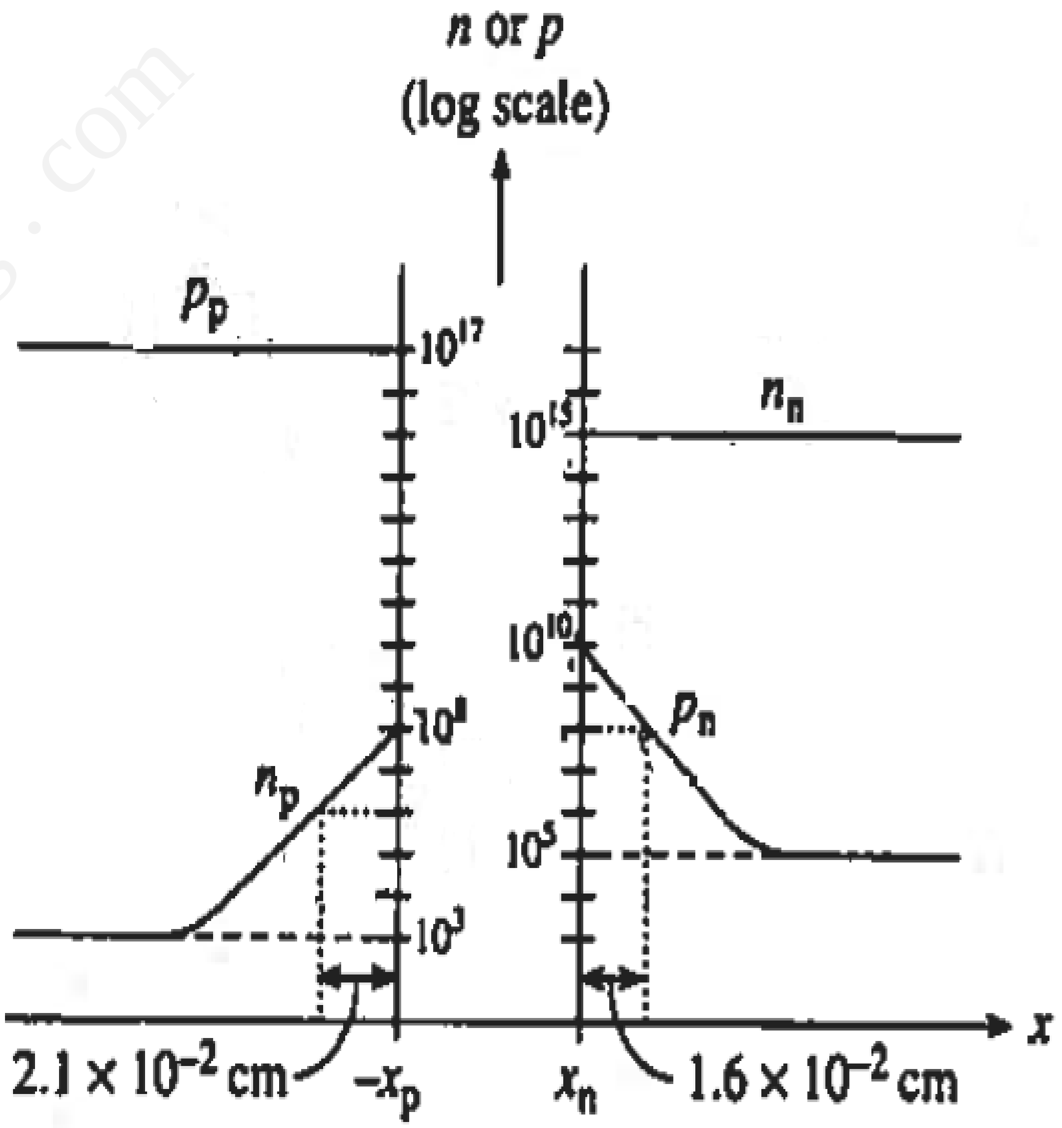
P: Figure E6.3 is a dimensioned plot of the steady state carrier concentrations inside a *pn* junction diode maintained at room temperature.

(a) Is the diode forward or reverse biased? Explain how you arrived at your answer.

(b) Do low-level injection conditions prevail in the quasineutral regions of the diode? Explain how you arrived at your answer.

(c) Determine the applied voltage, V_A.

(d) Determine the hole diffusion length, L_P.



PN JUNCTION DIODE: I-V CHARACTERISTICS

S: (a) The diode is forward biased. There is a pile-up or minority carrier excess ($\Delta n_p > 0$ and $\Delta p_n > 0$) at the edges of the depletion region.

(b) Low-level injection conditions *do* prevail. $\Delta p_n \ll n_n$ and $\Delta n_p \ll p_p$ everywhere inside the quasineutral regions.

(c) We can make use of either the depletion edge boundary conditions or the "law of the junction" to determine V_A. Specifically, solving Eq. (6.12) for V_A gives

$$V_A = \frac{kT}{q} \ln\left(\frac{np}{n_i^2}\right) \quad \ldots -x_p \leq x \leq x_n$$

PN JUNCTION DIODE: I-V CHARACTERISTICS

Evaluating the V_A expression at the n-edge of the depletion region and noting $np \to n_i^2$ for $x \to \pm\infty$, we compute

$$V_A = \frac{kT}{q} \ln\left[\frac{n_n(x_n)p_n(x_n)}{n_n(\infty)p_n(\infty)}\right] = (0.0259)\ \ln\ \frac{10^{25}}{10^{20}} \cong 0.3\ \text{V}$$

(d) Equation (6.23) can be rewritten

$$\Delta p_n(x') = \Delta p_n(x'=0)e^{-x'/L_P}$$

In the near vicinity of the depletion region edge $\Delta p_n \cong p_n$, giving

$$p_n(x') = p_n(0)e^{-x'/L_P}$$

or

$$\ln\left[\frac{p_n(x')}{p_n(0)}\right] = -\frac{x'}{L_P}$$

and

$$L_P = \frac{x'}{\ln\left[\frac{p_n(0)}{p_n(x')}\right]} = \frac{1.6 \times 10^{-2}}{\ln\left(\frac{10^{10}}{10^8}\right)} = 3.47 \times 10^{-3}\ \text{cm}$$