<u>Additive Weighting</u>

☆ **Additive weighting** method involves the sums

$$V(A_i) \quad = \quad \sum_{j=1,J} w_j \, v_j(x_{ij}) \qquad i=1,\ldots,I$$

where

$V(A_i)$	=	aggregate score for project A_i
w_j	=	weight for criterion C_j
x_{ij}	=	outcome of project A_i with respect to criterion C_j
$v_j(\ )$	=	value transformation for criterion C_j

☆ In additive weighting, $v_j(x_{ij})$ often calculated as

$$v_j(x_{ij}) = \frac{(x_{ij} - x_{j*})}{(x_j^* - x_{j*})} \qquad j \in C^+$$

$$v_j(x_{ij}) = \frac{(x_j^* - x_{ij})}{(x_j^* - x_{j*})} \qquad j \in C^-$$

$$= 1 - \frac{(x_{ij} - x_{j*})}{(x_j^* - x_{j*})} \qquad j \in C^-$$

C^+ = set of positive criteria

C^- = set of negative criteria

C = $C^+ \cup C^-$ = set of criteria

$$x_j^* = \max_{i=1,I}\{x_{ij}\}$$

$$x_{j*} = \min_{i=1,I}\{x_{ij}\}$$

Other Normalisation

$$v_j(x_{ij}) = \frac{x_{ij}}{x_j^*} \qquad j \in C^+$$

$$v_j(x_{ij}) = \frac{x_{j*}}{x_{ij}} \qquad j \in C^-$$

☆ Vector normalisation

$$v_j(x_{ij}) = \frac{x_{ij}}{\{\sum_{i=1,I} x_{ij}^2\}^{1/2}} \qquad j \in C$$

Outcome Matrix

	Congestion (V/C ratio at worst case intersection) dimensionless	**Bus Speed** (Both routes during peak period) mph	**Air Pollution** (CO concentration at worst reception location over 1 hour) ppm	**Fuel Consumption** (All vehicles, one mile length) gallons	**Capital Cost** $
A_1	**0.62**	**9.5**	**7.58**	**287**	**0**
A_2	**0.62**	**9.5**	**7.42**	**235**	**1050**
A_3	**0.53**	**13.3**	**8.22**	**284**	**2800**
A_4	**0.91**	**9.6**	**7.49**	**310**	**1050**
A_5	**0.43**	**10.7**	**7.73**	**266**	**400**

A_1 **Present situation**

A_2 **No parking in either direction**

A_3 **Exclusive bus-lane northbound**

A_4 **Parking permitted in both directions**

A_5 **Traffic engineering improvements**

Source: **R. Kuner (1989), Alternatives Analysis for Arterial Streets, Traffic Quarterly, 33, 459-472**

Transformation of Outcomes

$$v_j(x_{ij}) = \frac{(x_{ij} - x_{j*})}{(x_j^* - x_{j*})} \qquad j \in C^+$$

$$v_j(x_{ij}) = \frac{(x_j^* - x_{ij})}{(x_j^* - x_{j*})} \qquad j \in C^-$$

$$= 1 - \frac{(x_{ij} - x_{j*})}{(x_j^* - x_{j*})} \qquad j \in C^-$$

x_{ij} = outcome of alternative i with respect to criterion j

$v_j(\bullet)$ = value or transformation function

C^+ = set of positive criteria

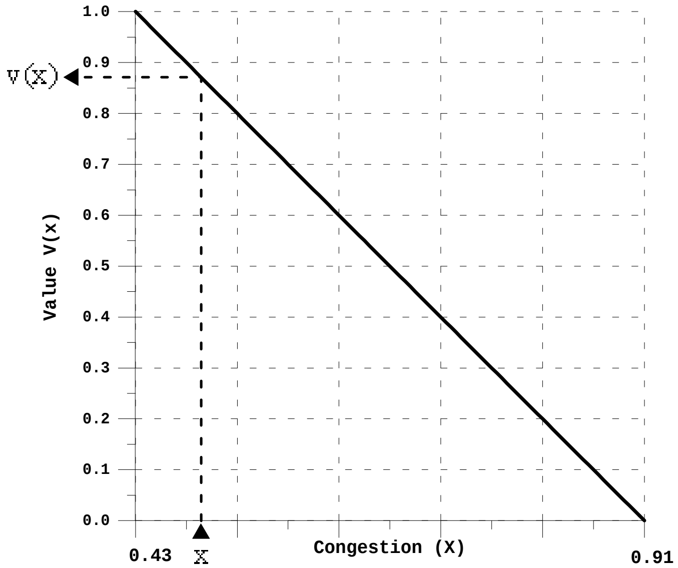
C^- = set of negative criteria

C = $C^+ \cup C^-$ = set of criteria

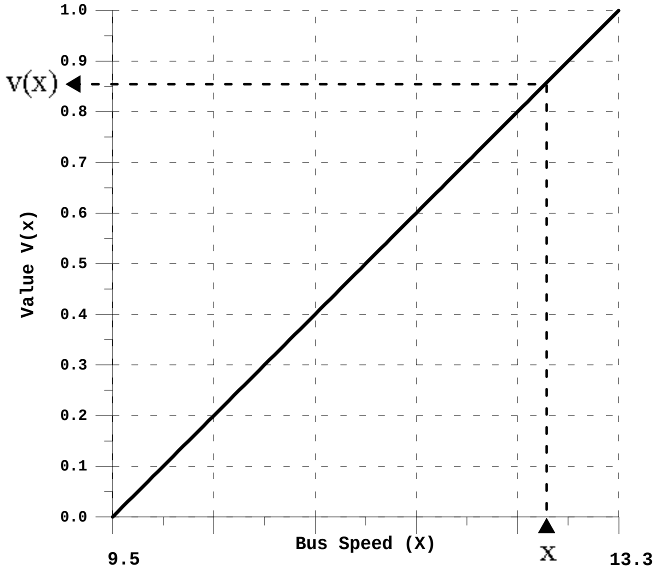
$$x_j^* = \max_{i=1,I}\{x_{ij}\}$$

$$x_{j*} = \min_{i=1,I}\{x_{ij}\}$$

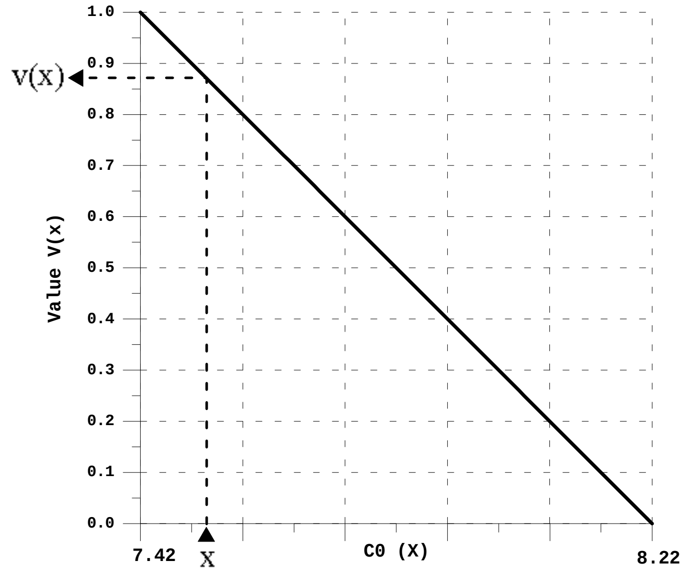
Congestion (V/C ratio)



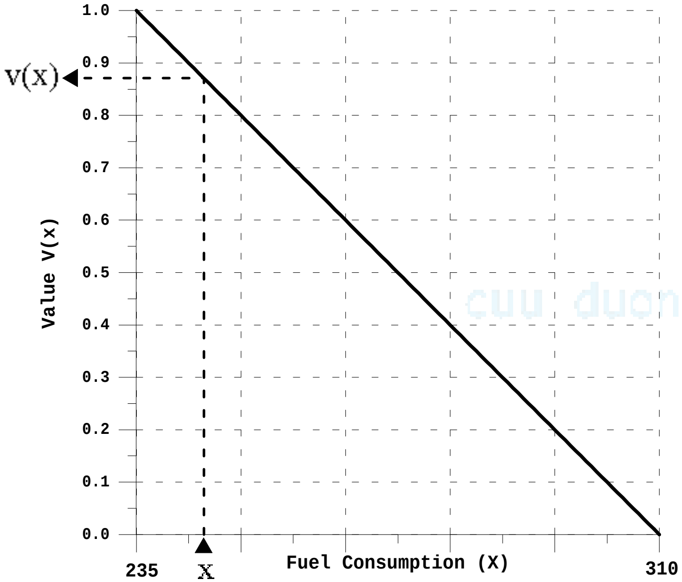
Bus Speed (mph)



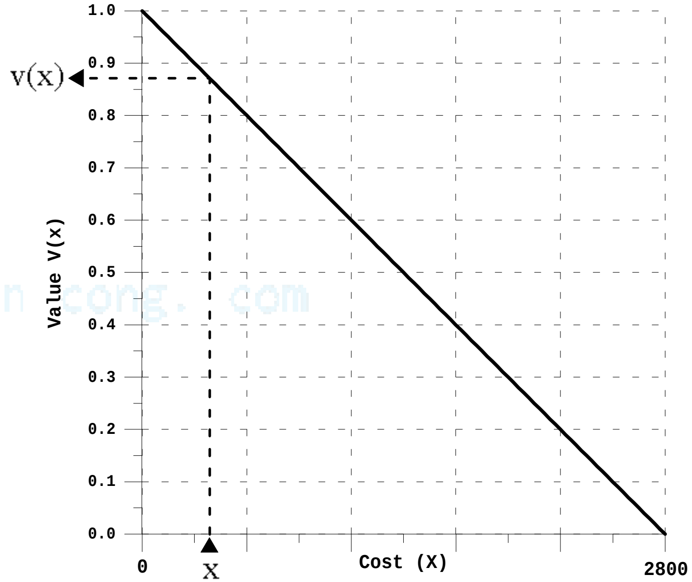
Air Pollution (C0 ppm)



Fuel Consumption (gallons)

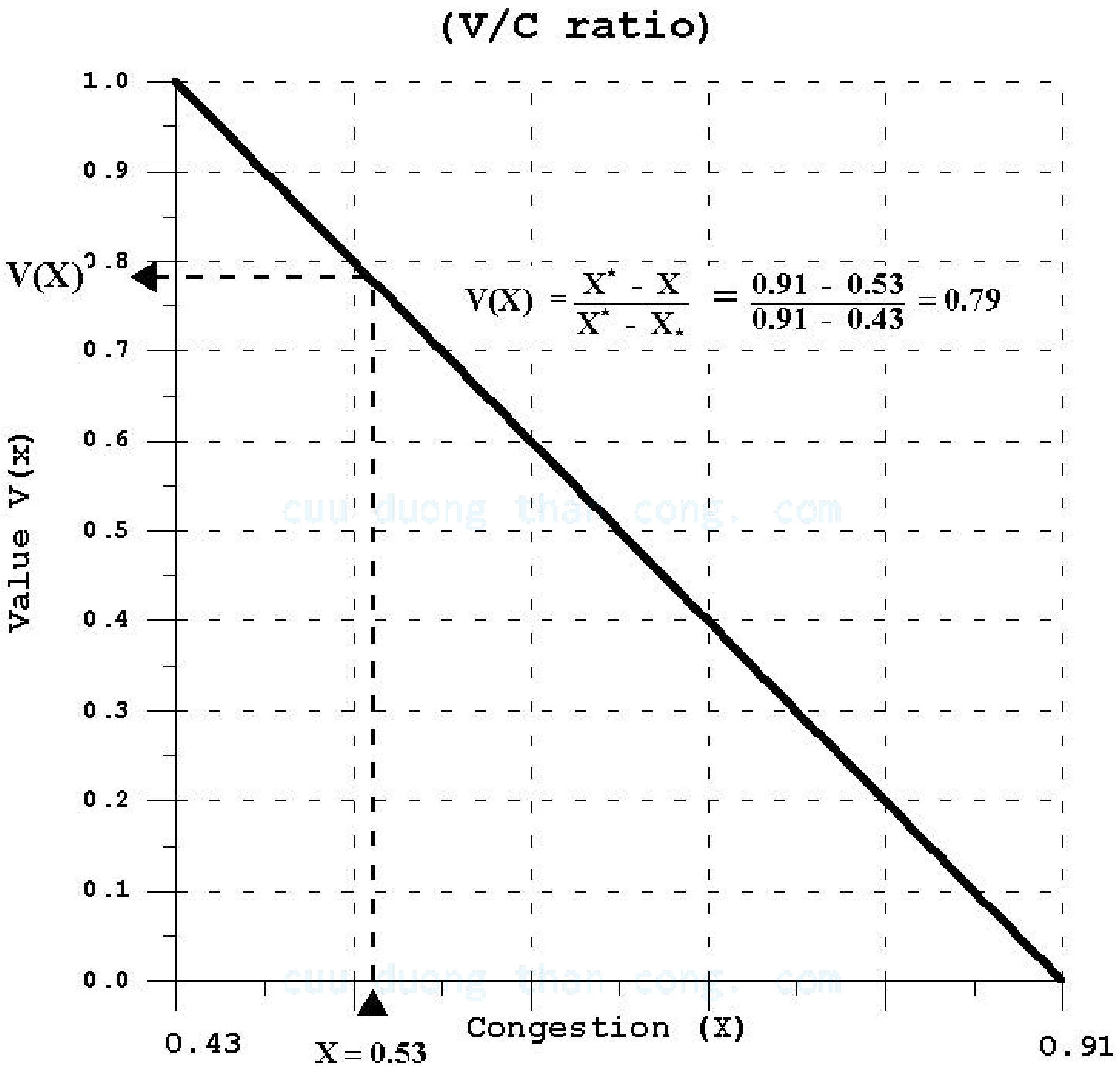


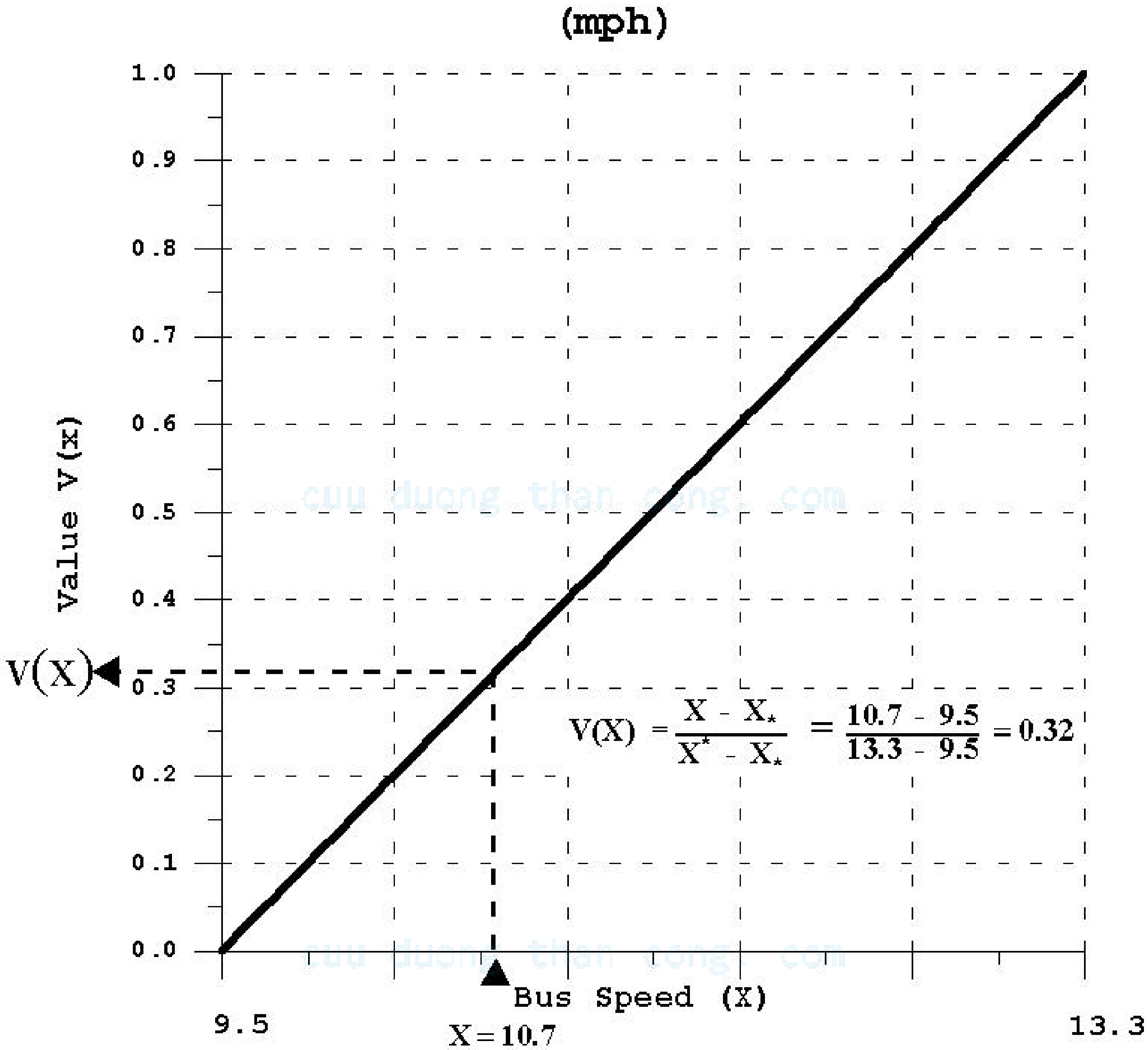
Cost ($)



	A	B	C	D	E	F	G
1	**Addditive Weighting**						
2		**C1**	**C2**	**C3**	**C4**	**C5**	
3	**A1**	0.62	9.5	7.58	287	0	
4	**A2**	0.62	9.5	7.42	235	1050	
5	**A3**	0.53	13.3	8.22	284	2800	
6	**A4**	0.91	9.6	7.49	310	1050	
7	**A5**	0.43	10.7	7.73	266	400	
8							
9	**MAX**	0.91	13.3	8.22	310	2800	
10	**MIN**	0.43	9.5	7.42	235	0	
11							
12	**Weight**	0.1	0.4	0.3	0.05	0.15	
13							
14		**C1**	**C2**	**C3**	**C4**	**C5**	
15	**A1**	0.60417	0	0.8	0.30667	1	
16	**A2**	0.60417	0	1	1	0.625	
17	**A3**	0.79167	1	0	0.34667	0	
18	**A4**	0	0.02632	0.9125	0	0.625	
19	**A5**	1	0.31579	0.6125	0.58667	0.85714	
20							
21	**A1**	0.46575					
22	**A2**	0.50417					
23	**A3**	0.4965					
24	**A4**	0.37803					
25	**A5**	0.56797					
26							

	A	B	C	D	E	F
1						
2		**C1**	**C2**	**C3**	**C4**	**C5**
3	**A1**	0.62	9.5	7.58	287	0
4	**A2**	0.62	9.5	7.42	235	1050
5	**A3**	0.53	13.3	8.22	284	2800
6	**A4**	0.91	9.6	7.49	310	1050
7	**A5**	0.43	10.7	7.73	266	400
8						
9	**MAX**	=MAX(B3:B7)	=MAX(C3:C7)	=MAX(D3:D7)	=MAX(E3:E7)	=MAX(F3:F7)
10	**MIN**	=MIN(B3:B7)	=MIN(C3:C7)	=MIN(D3:D7)	=MIN(E3:E7)	=MIN(F3:F7)
11						
12	**Weight**	0.1	0.4	0.3	0.05	0.15
13						
14		**C1**	**C2**	**C3**	**C4**	**C5**
15	**A1**	=1- (B3 -$B$10)/($B$9 - $B$10)	=(C3 -$C$10)/($C$9 - $C$10)	=1- (D3 -$D$10)/($D$9 - $D$10)	=1- (E3 -$E$10)/($E$9 - $E$10)	=1- (F3 -$F$10)/($F$9 - $F$10)
16	**A2**	=1- (B4 -$B$10)/($B$9 - $B$10)	=(C4 -$C$10)/($C$9 - $C$10)	=1- (D4 -$D$10)/($D$9 - $D$10)	=1- (E4 -$E$10)/($E$9 - $E$10)	=1- (F4 -$F$10)/($F$9 - $F$10)
17	**A3**	=1- (B5 -$B$10)/($B$9 - $B$10)	=(C5 -$C$10)/($C$9 - $C$10)	=1- (D5 -$D$10)/($D$9 - $D$10)	=1- (E5 -$E$10)/($E$9 - $E$10)	=1- (F5 -$F$10)/($F$9 - $F$10)
18	**A4**	=1- (B6 -$B$10)/($B$9 - $B$10)	=(C6 -$C$10)/($C$9 - $C$10)	=1- (D6 -$D$10)/($D$9 - $D$10)	=1- (E6 -$E$10)/($E$9 - $E$10)	=1- (F6 -$F$10)/($F$9 - $F$10)
19	**A5**	=1- (B7 -$B$10)/($B$9 - $B$10)	=(C7 -$C$10)/($C$9 - $C$10)	=1- (D7 -$D$10)/($D$9 - $D$10)	=1- (E7 -$E$10)/($E$9 - $E$10)	=1- (F7 -$F$10)/($F$9 - $F$10)
20						
21						
22	**A1**	=SUMPRODUCT($B$12:$F$12,B15:F15)				
23	**A2**	=SUMPRODUCT($B$12:$F$12,B16:F16)				
24	**A3**	=SUMPRODUCT($B$12:$F$12,B17:F17)				
25	**A4**	=SUMPRODUCT($B$12:$F$12,B18:F18)				
26	**A5**	=SUMPRODUCT($B$12:$F$12,B19:F19)				
27						


(V/C ratio)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
V(X)
Value V(x)
$V(X) = \frac{X^* - X}{X^* - X_*} = \frac{0.91 - 0.53}{0.91 - 0.43} = 0.79$
0.43
X = 0.53
Congestion (X)
0.91
cuu duong than cong. com


(mph)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Value V(x)
V(X)
$V(X) = \frac{X - X_*}{X^* - X_*} = \frac{10.7 - 9.5}{13.3 - 9.5} = 0.32$
Bus Speed (X)
9.5
X = 10.7
13.3

Transformed Outcome Matrix

	Congestion (V/C ratio at worst case intersection) dimensionless	**Bus Speed** (Both routes during peak period) mph	**Air Pollution** (CO concentration at worst reception location over 1 hour) ppm	**Fuel Consumption** (All vehicles, one mile length) gallons	**Capital Cost** $
A_1	**0.60**	**0.00**	**0.80**	**0.31**	**1.00**
A_2	**0.60**	**0.00**	**1.00**	**1.00**	**0.63**
A_3	**0.79**	**1.00**	**0.00**	**0.35**	**0.00**
A_4	**0.00**	**0.03**	**0.91**	**0.0**	**0.63**
A_5	**1.00**	**0.32**	**0.61**	**0.59**	**0.86**

A_1 **Present situation**

A_2 **No parking in either direction**

A_3 **Exclusive bus-lane northbound**

A_4 **Parking permitted in both directions**

A_5 **Traffic engineering improvements**

Weights

☆ Let x_j^* be the best level of attribute x_j

☆ Let x_{j*} be the worst level of attribute x_j

☆ Hypothetical best alternative $\{x_1^*, x_2^*, \ldots, x_J^*\}$

☆ Hypothetical worst alternative $\{x_{1*}, x_{2*}, \ldots, x_{J*}\}$

☆ Given $\{x_{1*}, x_{2*}, \ldots, x_{J*}\}$, DM asked which attribute he/she would **move from its worst value to its best value**

☆ Then DM asked which one should be changed second, third, etc.

☆ Order in which DM wants to change attribute levels from worst to best depends on the relative value difference between x_j^* and x_{j*}

☆ Attribute that seems to make the most difference in value should be improved first, etc.

☆ Process establishes a rank order of weights, since it ranks-orders the terms

$$V(x_{1*}, x_{2*}, \ldots, x_j^*, \ldots, x_{J*}) - V(x_{1*}, x_{2*}, \ldots, x_{j*}, \ldots, x_{J*})$$

☆ These terms are the weights w_j.

☆ To show this, assume $v_j(x_j^*) = 1$, $v_j(x_{j*}) = 0$.

☆ Then

$$\begin{aligned} V(x_{1*}, x_{2*}, \ldots, x_j^*, \ldots, x_{J*}) \\ &= w_j v_j(x_j^*) + \sum_{k \neq j} w_k v_k(x_{k*}) \\ &= w_j(1) + \sum_{k \neq j} w_k(0) \\ &= w_j \end{aligned}$$

- ✰ To obtain ratio-scaled weights from these rank orders:
  - ✰ Arbitrarily assign a raw value difference of 100 to an attribute that was selected first choice for improvement from worst to best
  - ✰ Equally arbitrarily assign a raw value difference of 0 to an attribute (not necessarily one of $\{x_1, x_2, \ldots, x_J\}$) that for which it would make absolutely no (value) difference for improvement from worst to best

- ✰ Express all other value differences as percentages of 100
- ✰ E.g. 50 per cent means that the value improvement resulting from moving an attribute from its worst to its best level is half as great as that obtained from moving the attribute chosen first.
- ✰ Renormalise the raw weights thus obtained

✰ These weights called *swing weights*

Weighting in Additive Weighting
Karl Street Example

☆ Assume

Bus Speed (C_2)	≻	Air Pollution (C_3)	≻	Cost (C_5)	≻	Congestion (C_1)	≻	Fuel Consumption (C_4)

☆ Assign 'Bus Speed' (λ_2 =100 for C_2)

☆ 'Air Pollution' considered 75% as important as 'Bus Speed' (λ_3 =0.75 for C_3)

☆ 'Cost' considered 37.5% as important as 'Bus Speed' (λ_5 =0.375 for C_5)

☆ 'Congestion' considered 25% as important as 'Bus Speed' (λ_1 =0.25 for C_1)

☆ 'Fuel Consumption' considered 12.5% as important as 'Bus Speed' (λ_4 =0.125 for C_1)

$$w_2 = \lambda_2/\sum_{j=1,5} \lambda_j = 100/250 = 0.40$$

$$w_3 = \lambda_3/\sum_{j=1,5} \lambda_j = 75/250 = 0.30$$

$$w_5 = \lambda_5/\sum_{j=1,5} \lambda_j = 37.5/250 = 0.15$$

$$w_1 = \lambda_1/\sum_{j=1,5} \lambda_j = 25/250 = 0.10$$

$$w_4 = \lambda_4/\sum_{j=1,5} \lambda_j = 12.5/250 = 0.05$$