National Income: Where it Comes From and Where it Goes

(chapter 3)

macro

macroeconomics

fifth edition

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PowerPoint® Slides
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Introduction

- In the last lecture we defined and measured some key macroeconomic variables.
- Now we start building theories about what determines these key variables.
- In the next couple lectures we will build up theories that we think hold in the long run, when prices are flexible and markets clear.
- Called Classical theory or Neoclassical.

The Neoclassical model

Is a <u>general equilibrium model</u>:

- Involves multiple markets
- each with own supply and demand
- Price in each market adjusts to make quantity demanded equal quantity supplied.

Neoclassical model

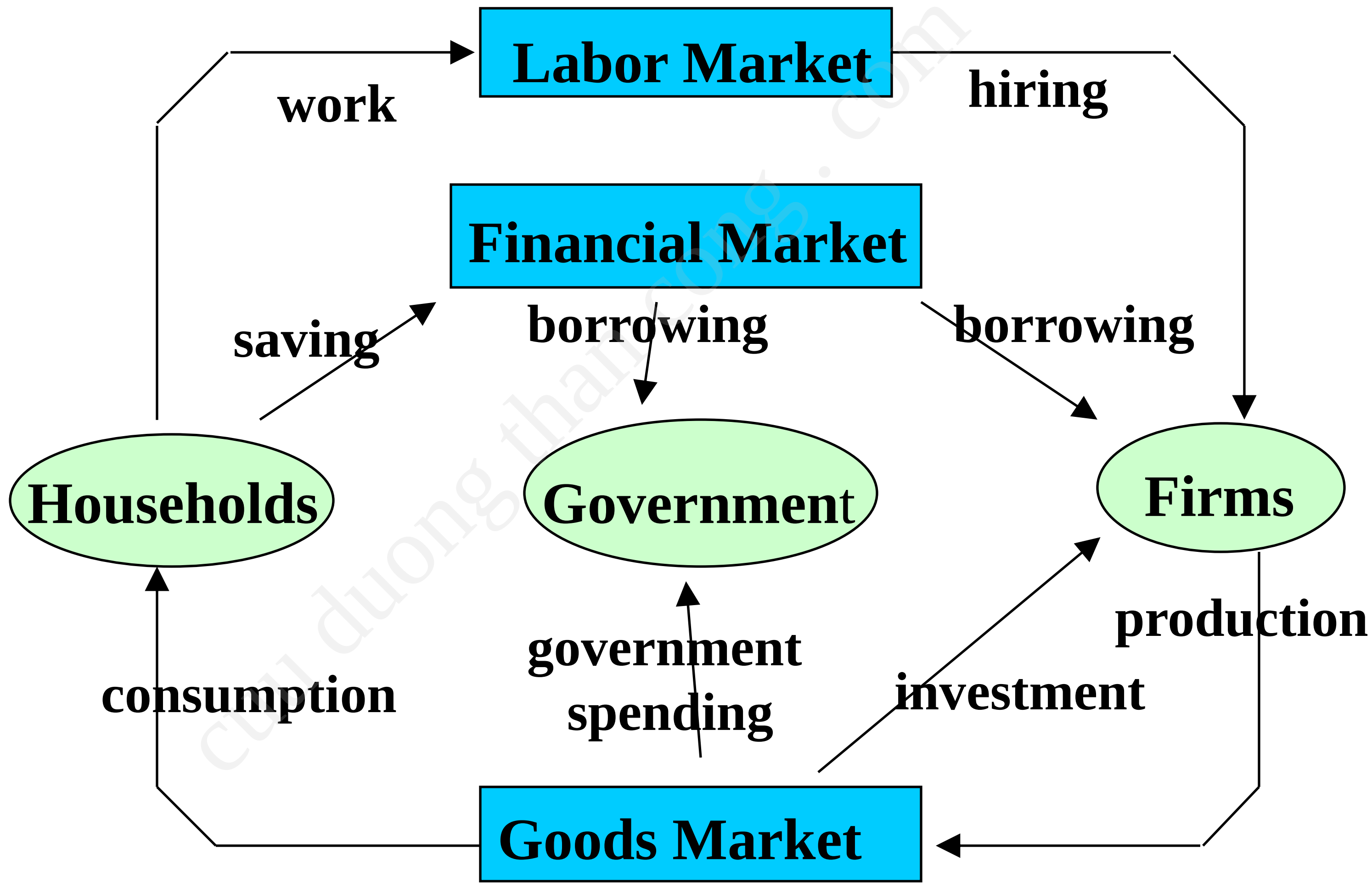
The macroeconomy involves three types of markets:

1. Goods (and services) Market
2. Factors Market or Labor market , needed to produce goods and services
3. Financial market

Are also three types of agents in an economy:

1. Households
2. Firms
3. Government

Three Markets – Three agents



Neoclassical model

Agents interact in markets, where they may be demander in one market and supplier in another

1) Goods market:

Supply: firms produce the goods

Demand: by households for consumption, government spending, and other firms demand them for investment

Neoclassical model

2) Labor market (factors of production)

Supply: Households sell their labor services.

Demand: Firms need to hire labor to produce the goods.

3) Financial market

Supply: households supply private savings: income less consumption

Demand: firms borrow funds for investment; government borrows funds to finance expenditures.

Neoclassical model

- We will develop a set of equations to characterize supply and demand in these markets
- Then use algebra to solve these equations together, and see how they interact to establish a general equilibrium.
- Start with production…

Part 1: Supply in goods market: Production

Supply in the goods market depends on a production function:

denoted $\boldsymbol{Y} = \boldsymbol{F}(\boldsymbol{K}, \boldsymbol{L})$

Where

K = capital: tools, machines, and structures used in production

L = labor: the physical and mental efforts of workers

The production function

- shows how much output (***Y***) the economy can produce from ***K*** units of capital and ***L*** units of labor.
- reflects the economy's level of technology.
- Generally, we will assume it exhibits ***constant returns to scale***.

Returns to scale

Initially $Y_1 = F(K_1, L_1)$

Scale all inputs by the same factor *z:*

$K_2 = zK_1$ and $L_2 = zL_1$ for z>1

(If $z = 1.25$, then all inputs increase by 25%)

What happens to output, $Y_2 = F(K_2, L_2)$?

- If *constant returns to scale*, $Y_2 = zY_1$
- If *increasing returns to scale*, $Y_2 > zY_1$
- If *decreasing returns to scale*, $Y_2 < zY_1$

Exercise: *determine returns to scale*

Determine whether each of the following production functions has constant, increasing, or decreasing returns to scale:

a) $F(K, L) = 2K + 15L$

b) $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

Exercise: *determine returns to scale*

Does $F(zK, zL) = zF(K, L)$?

a) Suppose $F(K, L) = 2K + 15L$

$$F(zK, zL) = 2zK + 15zL$$

$$= z(2K + 15L)$$

$$= zF(K, L)$$

Yes, constant returns to scale

Exercise: *determine returns to scale*

b) Suppose $F(K,L) = 2\sqrt{K} + 15\sqrt{L}$

$$F(zK, zL) = 2\sqrt{zK} + 15\sqrt{zL}$$

$$= 2\sqrt{z}\sqrt{K} + 15\sqrt{z}\sqrt{L}$$

$$= \sqrt{z}\left[2\sqrt{K} + 15\sqrt{L}\right]$$

$$= \sqrt{z}F(K,L)$$

$$< zF(K,L)$$

No, decreasing returns to scale

Assumptions of the model

1. Technology is fixed.
2. The economy's supplies of capital and labor are fixed at

$$K = \bar{K} \quad \text{and} \quad L = \bar{L}$$

Determining GDP

Output is determined by the fixed factor supplies and the fixed state of technology:

So we have a simple initial theory of supply in the goods market:

$$\bar{Y} = F(\bar{K}, \bar{L})$$

Part 2: Equilibrium in the factors market

- Equilibrium is where factor supply equals factor demand.
- Recall: Supply of factors is fixed.
- Demand for factors comes from firms.

Demand in factors market

Analyze the decision of a typical firm.

- It buys labor in the labor market, where price is wage, W.
- It rents capital in the factors market, at rate R.
- It uses labor and capital to produce the good, which it sells in the goods market, at price P.

Demand in factors market

Assume the market is competitive:

Each firm is small relative to the market, so its actions do not affect the market prices.

It takes prices in markets as given - W,R, P.

Demand in factors market

It then chooses the optimal quantity of Labor and capital to maximize its profit.

How write profit:

Profit = revenue - labor costs - capital costs

$$= PY \quad - WL \quad - RK$$

$$= P\,F(K,L) \quad - WL \quad - RK$$

Demand in the factors market

- Increasing hiring of L will have two effects:

1) Benefit: raise output by some amount

2) Cost: raise labor costs at rate W

- To see how much output rises, we need the marginal product of labor (MPL)

Marginal product of labor (*MPL*)

An approximate definition (used in text) :

The extra output the firm can produce using one additional labor (holding other inputs fixed):

$$MPL = F(K, L+1) - F(K, L)$$

Exercise: *compute & graph MPL*

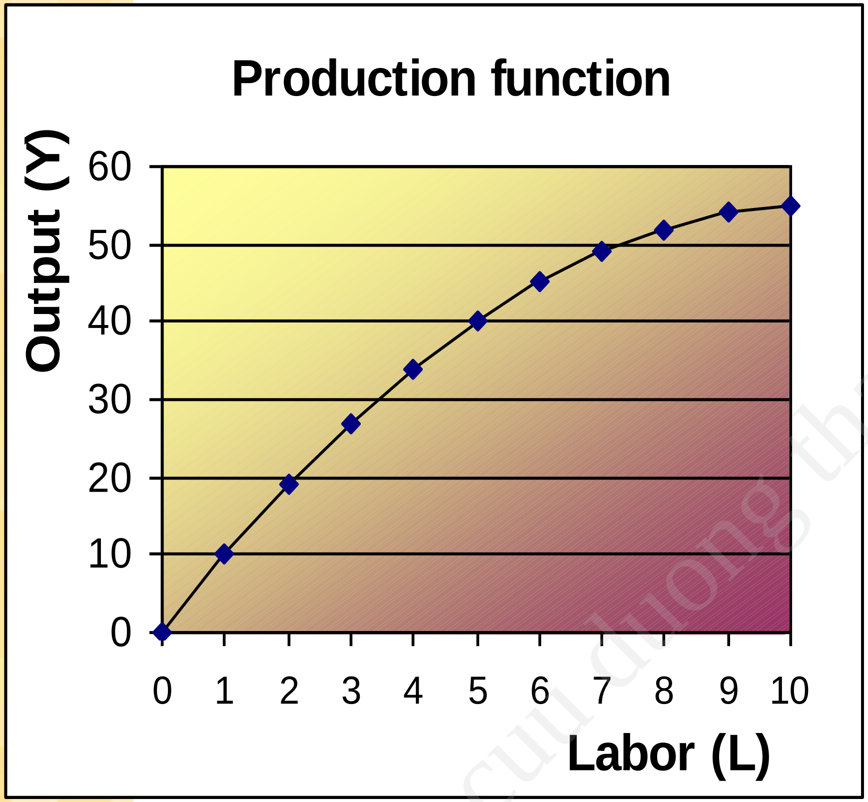
a. Determine ***MPL*** at each value of ***L***

b. Graph the production function

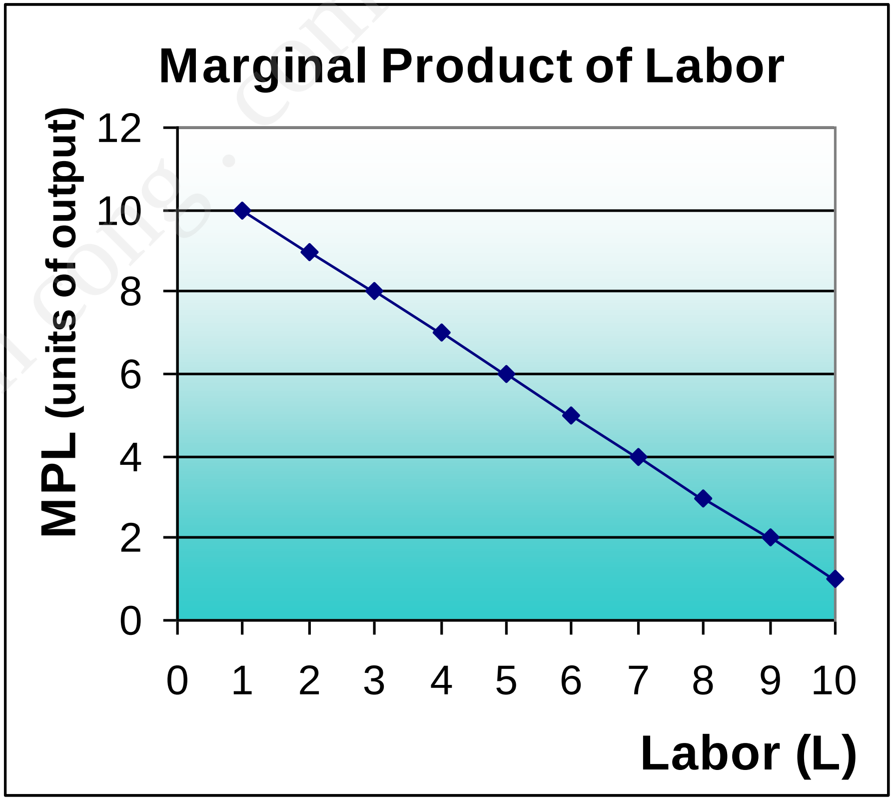
c. Graph the ***MPL*** curve with ***MPL*** on the vertical axis and ***L*** on the horizontal axis

L	***Y***	***MPL***
0	0	n.a.
1	10	?
2	19	?
3	27	8
4	34	?
5	40	?
6	45	?
7	49	?
8	52	?
9	54	?
10	55	?

answers:

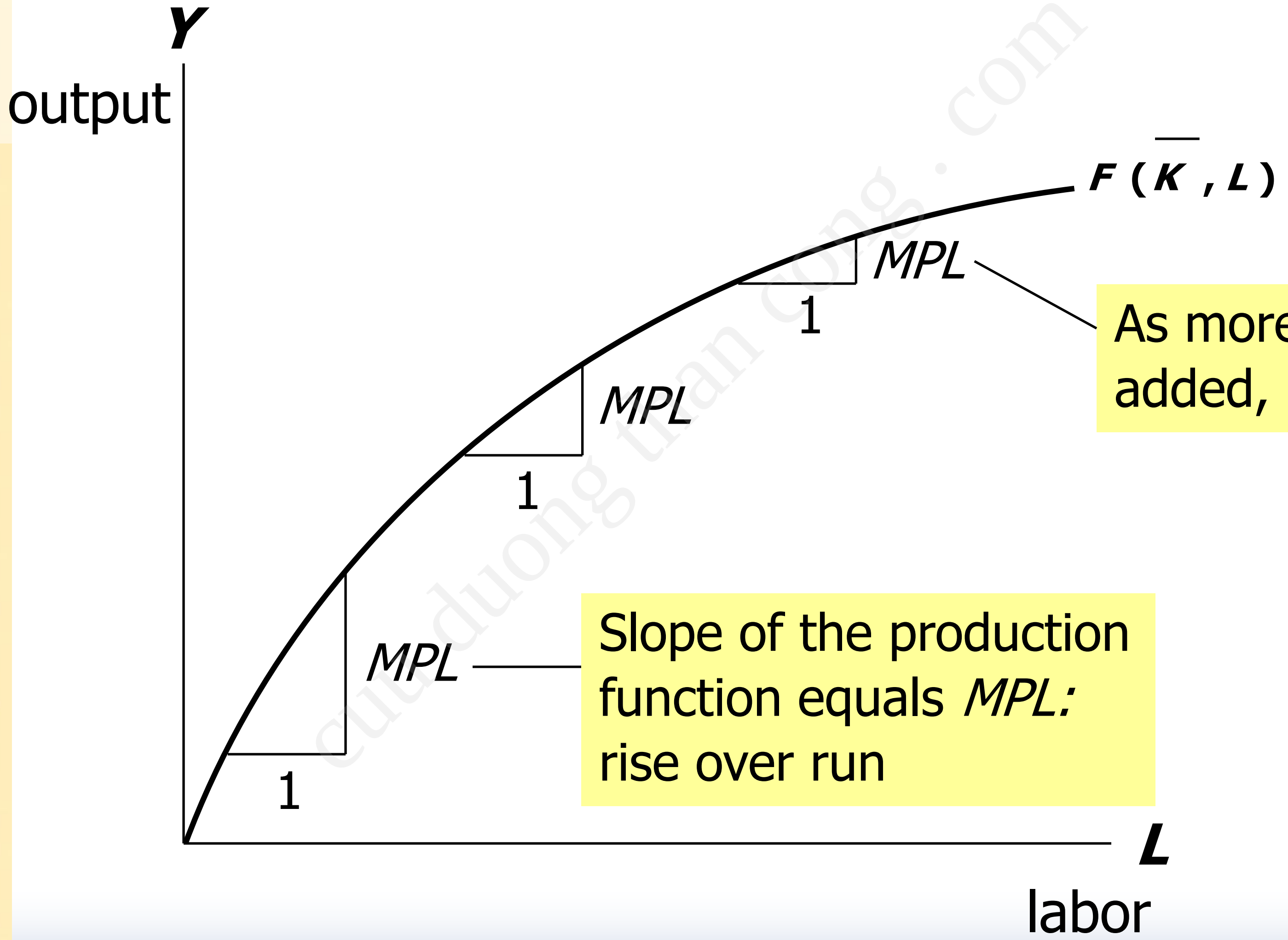


Production function
Output (Y)
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9 10
Labor (L)



Marginal Product of Labor
MPL (units of output)
12
10
8
6
4
2
0
0 1 2 3 4 5 6 7 8 9 10
Labor (L)

The MPL and the production function



As more labor is added, *MPL* ↓

Slope of the production function equals *MPL:* rise over run

Diminishing marginal returns

- As a factor input is increased, its marginal product falls (other things equal).

- Intuition:
  ↑***L*** while holding ***K*** fixed
  ⇒ fewer machines per worker
  ⇒ lower productivity

MPL with calculus

We can give a more precise definition of MPL:

The rate at which output rises for a small amount of additional labor (holding other inputs fixed):

$$MPL = [F(K, L + \Delta L) - F(K, L)] / \Delta L$$

where Δ is 'delta' and represents change

- Earlier definition assumed that $\Delta L = 1$.

$$F(K, L + 1) - F(K, L)$$

- We can consider smaller change in labor.

MPL as a derivative

As we take the limit for small change in L:

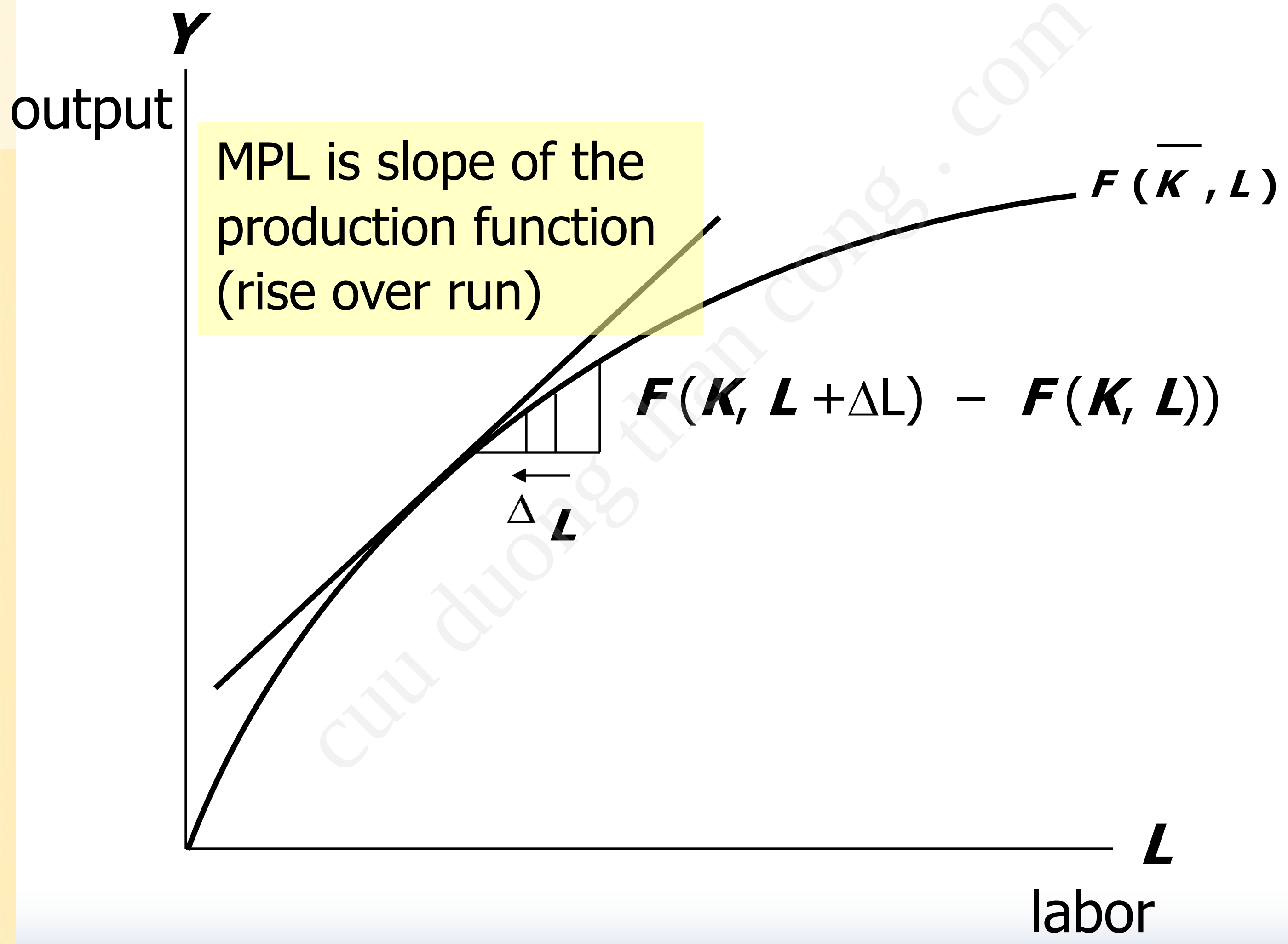
$$MPL = \lim_{\Delta L \to 0} \frac{F(K, L + \Delta L) - F(K, L)}{\Delta L}$$

$$= f_L(K, L)$$

Which is the definition of the (partial) derivative of the production function with respect to ***L***, treating ***K*** as a constant.

This shows the slope of the production function at any particular point, which is what we want.

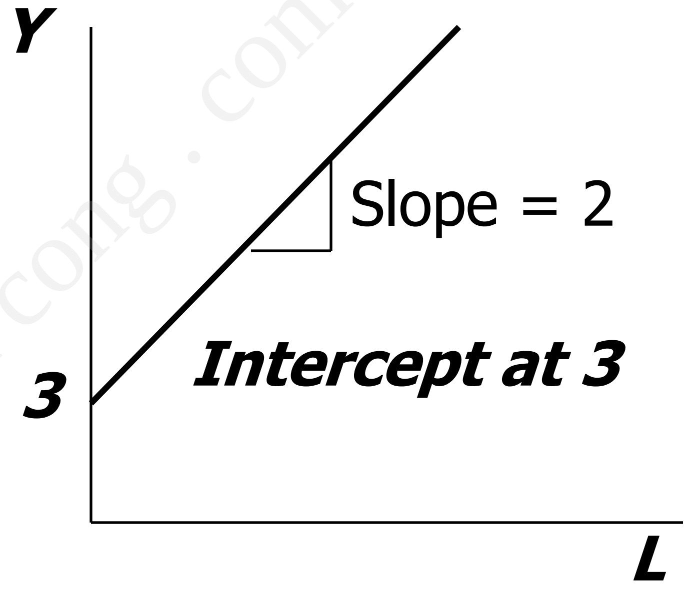
The MPL and the production function



A brief calculus review: Derivatives

1) $Y = F(L) = 2L + 3$

$$\frac{\partial Y}{\partial L} = f_L = 2$$

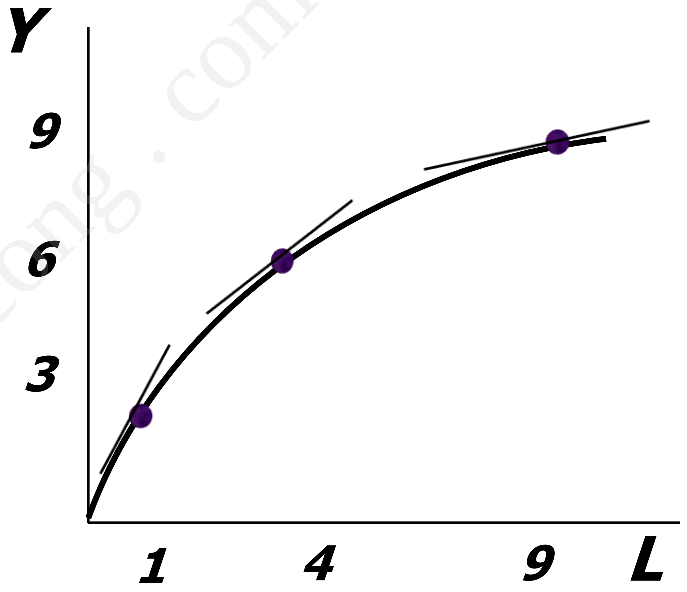


Calculus: Rules of derivatives

$$2)\ Y = F(L) = 3L^{\frac{1}{2}} = 3\sqrt{L}$$

$$\frac{\partial Y}{\partial L} = f_L = 3 \cdot \frac{1}{2} L^{\frac{1}{2}-1}$$

$$= \frac{3}{2} L^{-\frac{1}{2}} = \frac{3}{2\sqrt{L}}$$



L:	1	4	9
F(L):	3	6	9
f_L:	1.5	0.75	0.5

Calculus: More rules of derivatives

3) Sums:

$$Y = 3L^{\frac{1}{2}} + 2L + 3$$

$$\frac{\partial Y}{\partial L} = \frac{3}{2}L^{-\frac{1}{2}} + 2$$

4) Chain rule:

$$Y = 3x + 1^{\frac{3}{4}}$$

$$\frac{\partial Y}{\partial L} = \frac{3}{4}\; 3x + 1^{\frac{3}{4}-1} \cdot 3 = \frac{9}{4}\; 3x + 1^{-\frac{1}{4}}$$

Calculus: Yet more rules of derivatives

5) Product rule:

$$Y = 2L + 3 \cdot 3L + 1^{\frac{3}{4}}$$

$$\frac{\partial Y}{\partial L} = \left[2L + 3 \cdot \frac{3}{4}\, 3L + 1^{\frac{3}{4}-1} \cdot 3 \right]$$

$$+ \left[3L + 1^{\frac{3}{4}} \cdot 2 \right]$$

Calculus: Logarithms

if $e^{Y} = L$, where $e = 2.17...$

then $Y = \ln L$

and $\frac{\partial Y}{\partial L} = \frac{1}{L}$

So if $Y = \ln 2L + 3$

$$\frac{\partial Y}{\partial L} = \left(\frac{1}{2L + 3} \right) \cdot 2$$

note: $\ln L \cdot K = \ln L + \ln K$

Calculus: Partial derivatives

Suppose a function of two variables:

$$Y = 3L^2 + 4K^{\frac{1}{2}}$$

Partial derivative: allow one variable to change, 'holding all else constant'

$$\frac{\partial Y}{\partial L} = 6L \qquad \frac{\partial Y}{\partial K} = 2K^{-\frac{1}{2}}$$

Calculus: Total differentials

if $Y = K^2 L^3$

$$\frac{\partial Y}{\partial K} = 2KL^3 \qquad \frac{\partial Y}{\partial L} = 3K^2L^2$$

Can write 'total differential' to show the combined effect of changes in both variables.

$$\partial Y = 2KL^3 \ \partial K + 3K^2L^2 \ \partial L$$

Return to firm problem: hiring L

Firm chooses ***L*** to maximize its profit.

How will increasing ***L*** change profit?

Δ profit = Δ revenue - Δ cost

= *P* * *MPL* - *W*

If this is:

	> 0	should hire more
	< 0	should hire less
	= 0	hiring right amount

Firm problem continued

So the firm's demand for labor is determined by the condition:

$$P * MPL = W$$

Hires more and more ***L***, until MPL falls enough to satisfy the condition.

Also may be written:

$MPL = W/P$, where W/P is the 'real wage'

Real wage

Think about units:

- ***W*** = $/hour
- ***P*** = $/good
- ***W/P*** = ($/hour) / ($/good) = goods/hour

The amount of purchasing power, measured in units of goods, that firms pay per unit of work

Example: deriving labor demand

- Suppose a production function for all firms in the economy:

$$Y = K^{0.5} L^{0.5}$$

$$MPL = 0.5 K^{0.5} L^{-0.5}$$

Labor demand is where this equals real wage:

$$0.5 K^{0.5} L^{-0.5} = \frac{W}{P}$$

Labor demand continued

or rewrite with L as a function of real wage

$$0.5K^{0.5}L^{-0.5} = \frac{W}{P}$$

$$\left(0.5K^{0.5}L^{-0.5}\right)^{-2} = \left(\frac{W}{P}\right)^{-2}$$

$$\frac{1}{0.25}K^{-1}L = \left(\frac{P}{W}\right)^{2}$$

$$L^{demand} = 0.25K\left(\frac{P}{W}\right)^{2}$$

So a rise in wage $\rightarrow$ want to hire less labor;
rise in capital stock $\rightarrow$ want to hire more labor

Labor market equilibrium

Take this firm as representative, and sum over all firms to derive aggregate labor demand. Combine with labor supply to find equilibrium wage:

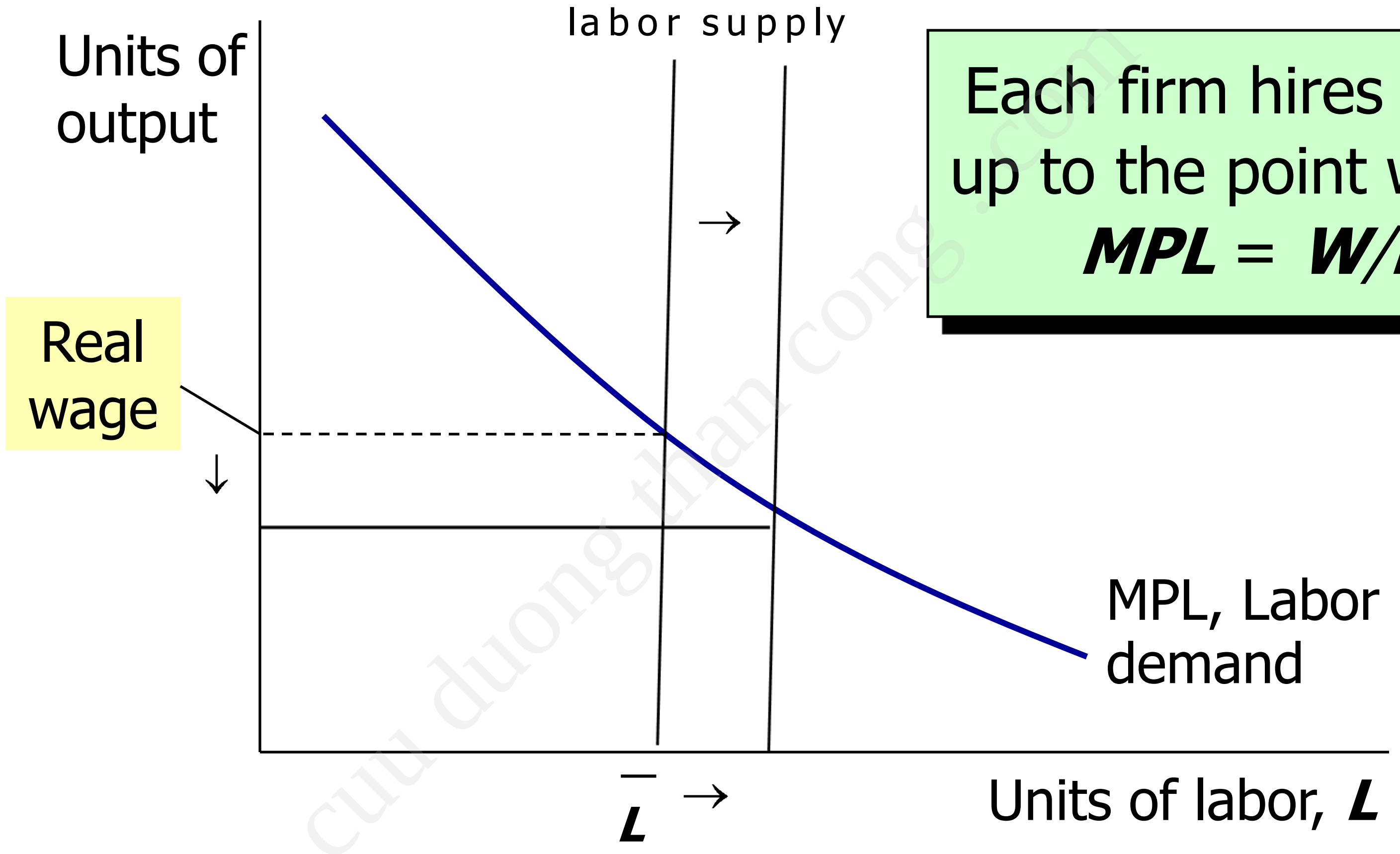
$$\text{demand: } 0.5\boldsymbol{K}^{0.5}\boldsymbol{L}^{\boldsymbol{demand}^{-0.5}} = \frac{\boldsymbol{W}}{\boldsymbol{P}}$$

$$\text{supply: } \boldsymbol{L}^{\text{supply}} = \overline{\boldsymbol{L}}$$

$$\text{equilibrium: } \frac{\boldsymbol{W}}{\boldsymbol{P}} = 0.5\boldsymbol{K}^{0.5}\overline{\boldsymbol{L}}^{-0.5}$$

So rise in labor supply $\rightarrow$ fall in equlibrium real wage

MPL and the demand for labor



Each firm hires labor up to the point where
MPL* = *W/P

Determining the rental rate

We have just seen that ***MPL*** = ***W/P***

The same logic shows that ***MPK*** = ***R/P***:

- diminishing returns to capital: ***MPK***↓ as ***K***↑
- The ***MPK*** curve is the firm's demand curve for renting capital.
- Firms maximize profits by choosing ***K*** such that ***MPK*** = ***R/P***.

How income is distributed:

We found that if markets are competitive, then factors of production will be paid their marginal contribution to the production process.

total labor income $= \dfrac{W}{P}\bar{L} = MPL \times \bar{L}$

total capital income $= \dfrac{R}{P}\bar{K} = MPK \times \bar{K}$

Euler's theorem:

Under our assumptions (constant returns to scale, profit maximization, and competitive markets)…

total output is divided between the payments to capital and labor, depending on their marginal productivities, with no extra profit left over.

$$\bar{Y} = MPL \times \bar{L} + MPK \times \bar{K}$$

national income — labor income ($MPL \times \bar{L}$) — capital income ($MPK \times \bar{K}$)

Mathematical example

Consider a production function with Cobb-Douglas form:

$$Y = AK^{\alpha}L^{1-\alpha}$$

where A is a constant, representing technology

Show this has constant returns to scale:

multiply factors by Z:

$$F(ZK,ZY) = A\,(ZK)^{\alpha}\,(ZL)^{1-\alpha}$$

$$= A\,Z^{\alpha}\,K^{\alpha}\,Z^{1-\alpha}\,L^{1-\alpha}$$

$$= A\,Z^{\alpha}\,Z^{1-\alpha}\,K^{\alpha}\,L^{1-\alpha}$$

$$= Z \times A\,K^{\alpha}\,L^{1-\alpha}$$

$$= Z \times F(K,L)$$

Mathematical example continued

- Compute marginal products:

$$MPL = (1-\alpha)\, A\, K^{\alpha}\, L^{-\alpha}$$

$$MPK = \alpha\, A\, K^{\alpha-1} L^{1-\alpha}$$

- Compute total factor payments:

$$MPL \times L + MPK \times K$$

$$= (1-\alpha)\, A\, K^{\alpha}\, L^{-\alpha} \times L \;+\; \alpha\, A\, K^{\alpha-1} L^{1-\alpha} \times K$$

$$= (1-\alpha)\, A\, K^{\alpha}\, L^{1-\alpha} \;+\; \alpha\, A\, K^{\alpha}\, L^{1-\alpha}$$

$$= A\, K^{\alpha}\, L^{1-\alpha} = Y$$

So total factor payments equals total production.

Outline of model

A closed economy, market-clearing model

Goods market:

DONE ☑ Supply side: production

Next → ❑ Demand side: C, I, and G

Factors market

DONE ☑ Supply side

DONE ☑ Demand side

Loanable funds market

❑ Supply side: saving

❑ Demand side: borrowing

Demand for goods & services

Components of aggregate demand:

C = consumer demand for g & s

I = demand for investment goods

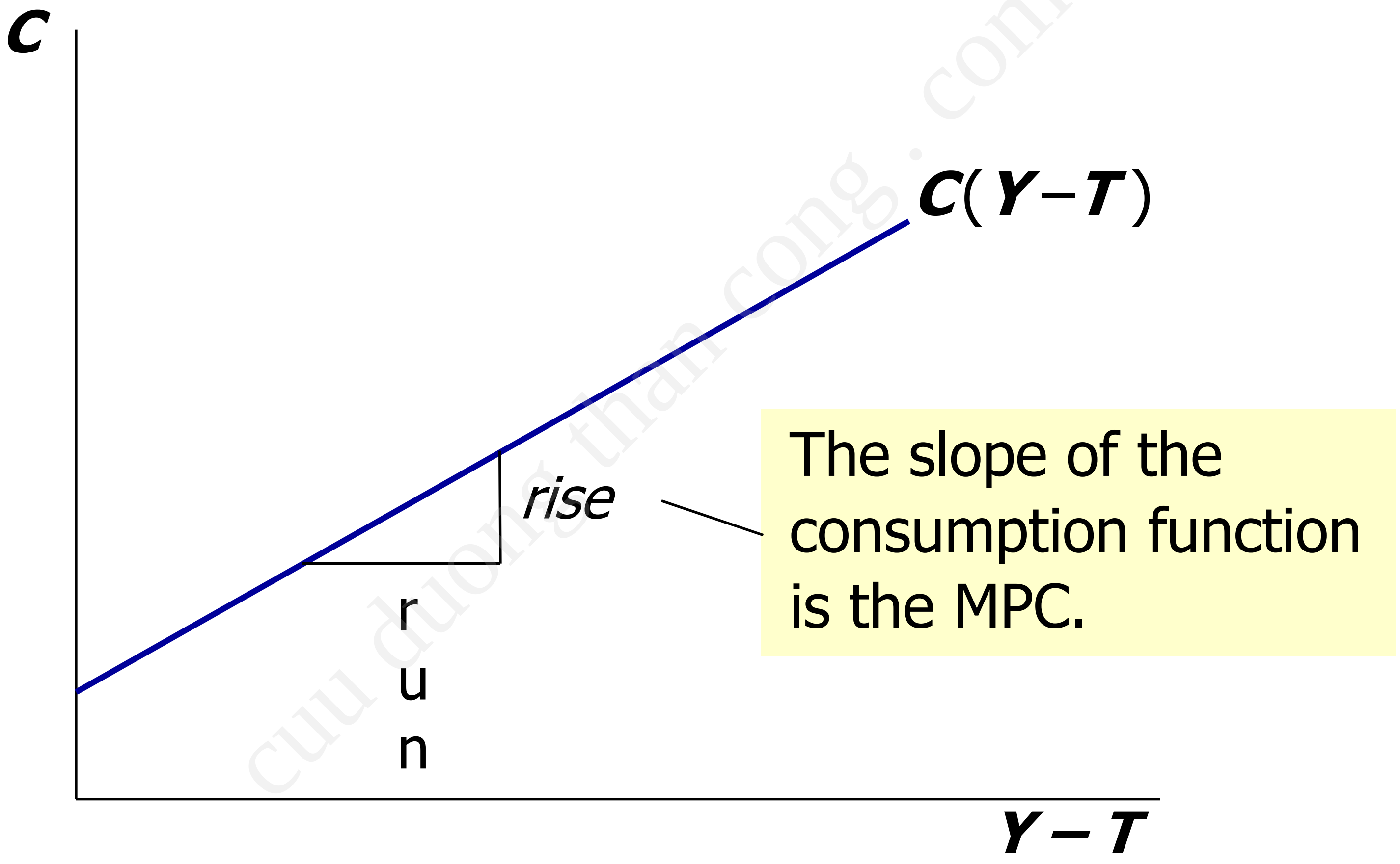
G = government demand for g & s

(closed economy: no ***NX***)

Consumption, *C*

- def: **disposable income** is total income minus total taxes: $Y - T$
- Consumption function: $C = C(Y - T)$
  Shows that $\uparrow(Y - T) \Rightarrow \uparrow C$
- def: The **marginal propensity to consume (MPC)** is the increase in C caused by an increase in disposable income.
- So MPC = derivative of the consumption function with respect to disposable income.
- MPC must be between 0 and 1.

The consumption function



Consumption function cont.

Suppose consumption function:

C=10 + 0.75Y

MPC ***= 0.75***

For extra dollar of income, spend 0.75 dollars consumption

Marginal propensity to save = 1-MPC

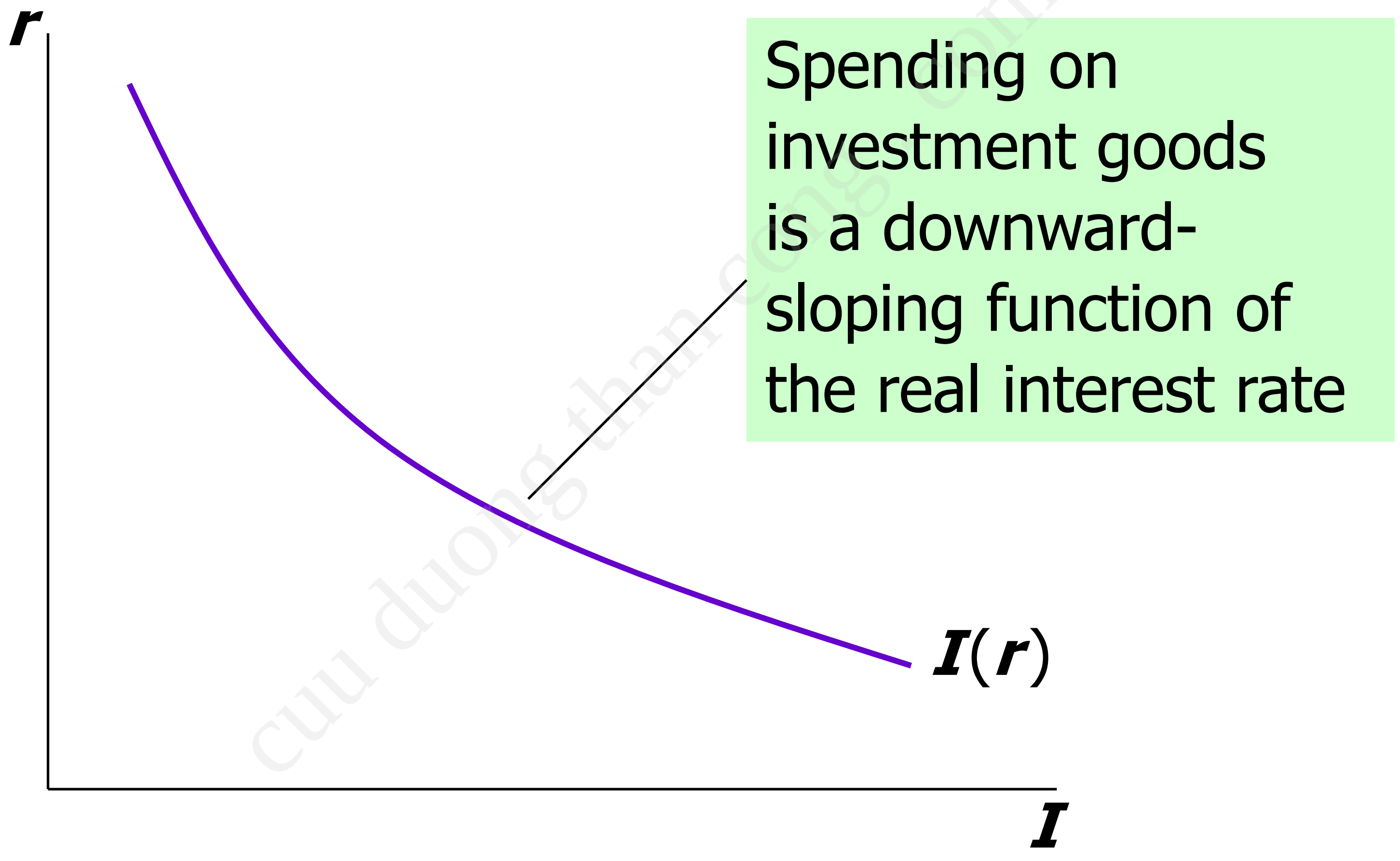
Investment, I

- The investment function is $I = I(r)$, where r denotes the **real interest rate**, the nominal interest rate corrected for inflation.
- The real interest rate is
  - the cost of borrowing
  - the opportunity cost of using one's own funds

  to finance investment spending.

So, $\uparrow r \Rightarrow \downarrow I$

The investment function



Government spending, *G*

- ***G*** includes government spending on goods and services.
- ***G*** excludes *transfer payments*
- Assume government spending and total taxes are exogenous:

$$G = \bar{G} \quad \text{and} \quad T = \bar{T}$$

The market for goods & services

- Agg. demand: $C(\bar{Y}-\bar{T})+I(r)+\bar{G}$
- Agg. supply: $\bar{Y}=F(\bar{K},\bar{L})$
- Equilibrium: $\bar{Y}=C(\bar{Y}-\bar{T})+I(r)+\bar{G}$

The real interest rate adjusts to equate demand with supply.

We can get more intuition for how this works by looking at the loanable funds market

The loanable funds market

A simple supply-demand model of the financial system.

One asset: "loanable funds"

demand for funds: investment

supply of funds: saving

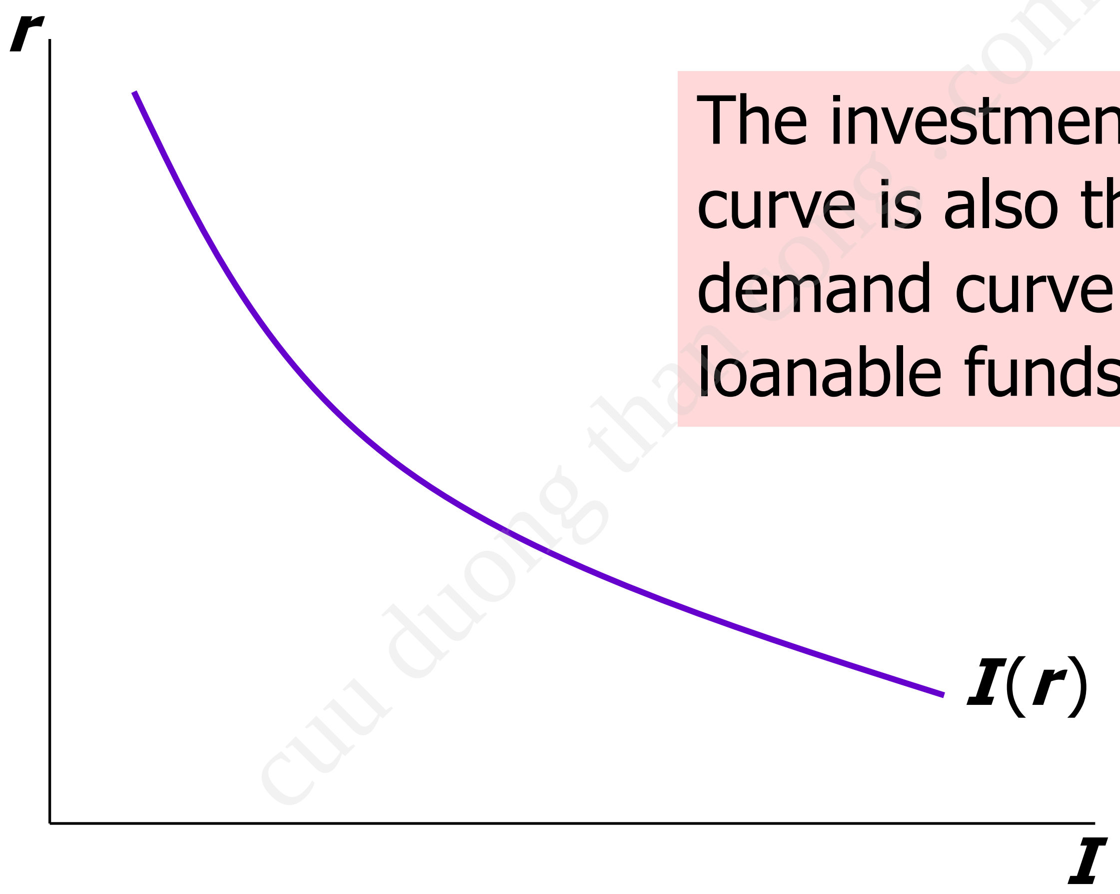
"price" of funds: real interest rate

Demand for funds: Investment

The demand for loanable funds:

- comes from investment:
  Firms borrow to finance spending on plant & equipment, new office buildings, etc. Consumers borrow to buy new houses.
- depends negatively on ***r***, the "price" of loanable funds (the cost of borrowing).

Loanable funds demand curve



The investment curve is also the demand curve for loanable funds.

Supply of funds: Saving

The supply of loanable funds comes from saving:

- Households use their saving to make bank deposits, purchase bonds and other assets. These funds become available to firms to borrow to finance investment spending.
- The government may also contribute to saving if it does not spend all of the tax revenue it receives.

Types of saving

- **private saving** = $(Y - T) - C$
- **public saving** = $T - G$
- **national saving**, S

= private saving + public saving

$$= (Y - T) - C + T - G$$

$$= Y - C - G$$

EXERCISE:
Calculate the change in saving

Suppose MPC = 0.8 and MPL = 20.

For each of the following, compute ΔS :

a. $\Delta G = 100$

b. $\Delta T = 100$

c. $\Delta Y = 100$

d. $\Delta L = 10$

Answers

$$\Delta S = \Delta Y - \Delta C - \Delta G = \Delta Y - 0.8(\Delta Y - \Delta T) - \Delta G$$
$$= 0.2\Delta Y + 0.8\Delta T - \Delta G$$

a. $\Delta S = -100$

b. $\Delta S = 0.8 \times 100 = 80$

c. $\Delta S = 0.2 \times 100 = 20$

d. $\Delta Y = \text{MPL} \times \Delta L = 20 \times 10 = 200,$

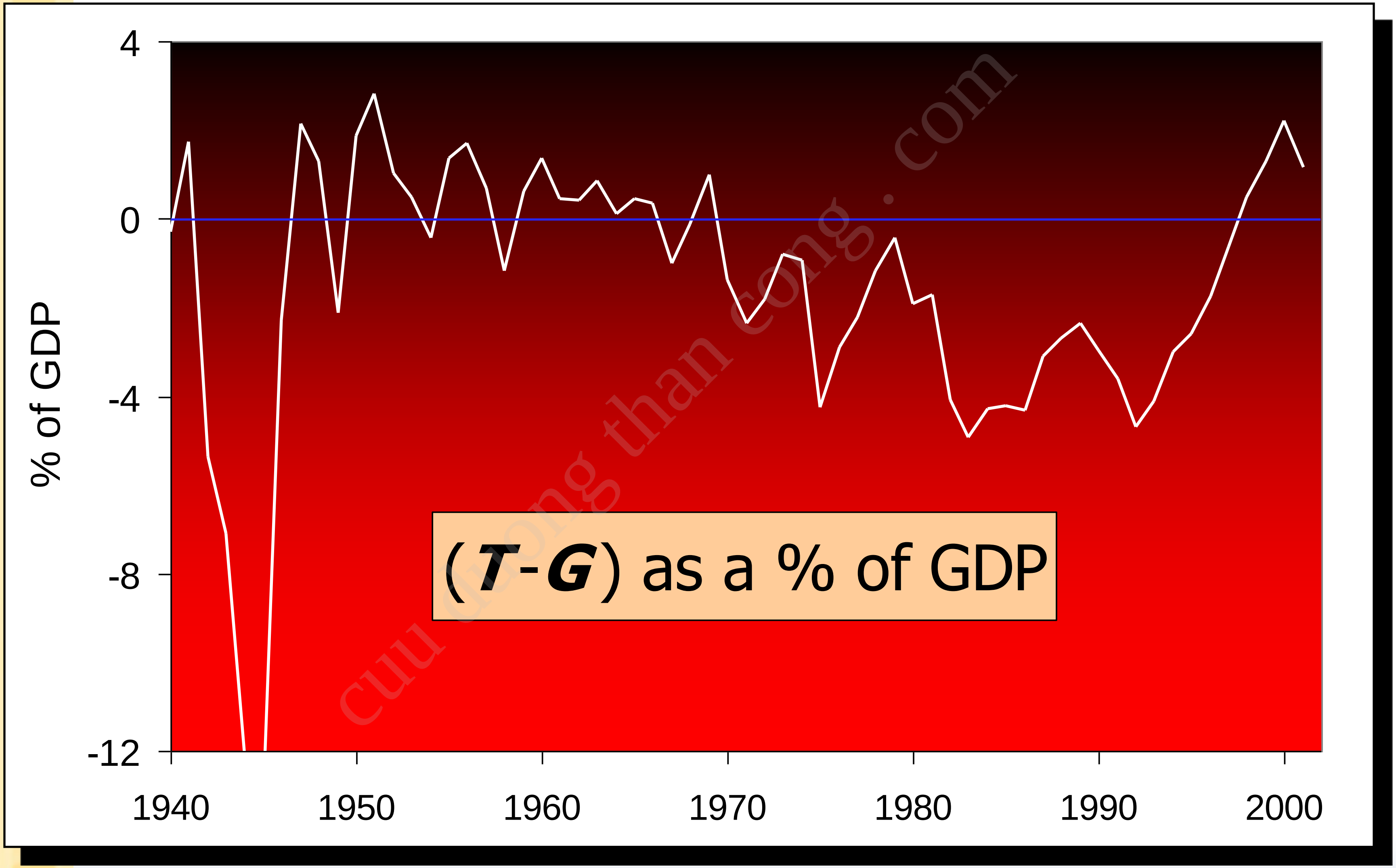
$\Delta S = 0.2 \times \Delta Y = 0.2 \times 200 = 40.$

digression:

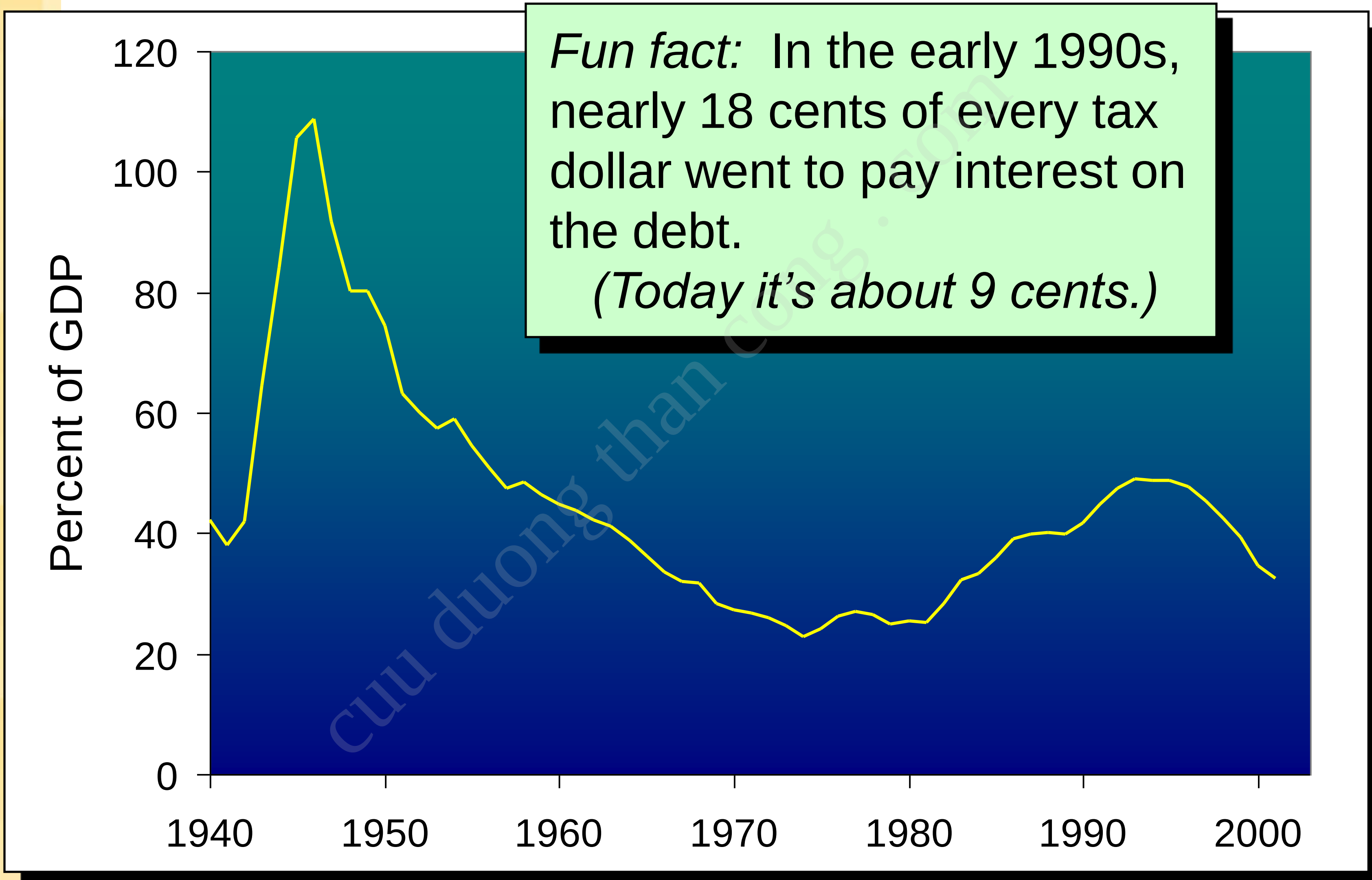
Budget surpluses and deficits

- When $T > G$,
  budget surplus = $(T - G)$ = public saving
- When $T < G$,
  budget deficit = $(G - T)$
  and public saving is negative.
- When $T = G$,
  budget is balanced and public saving = 0.

The U.S. Federal Government Budget



The U.S. Federal Government Debt

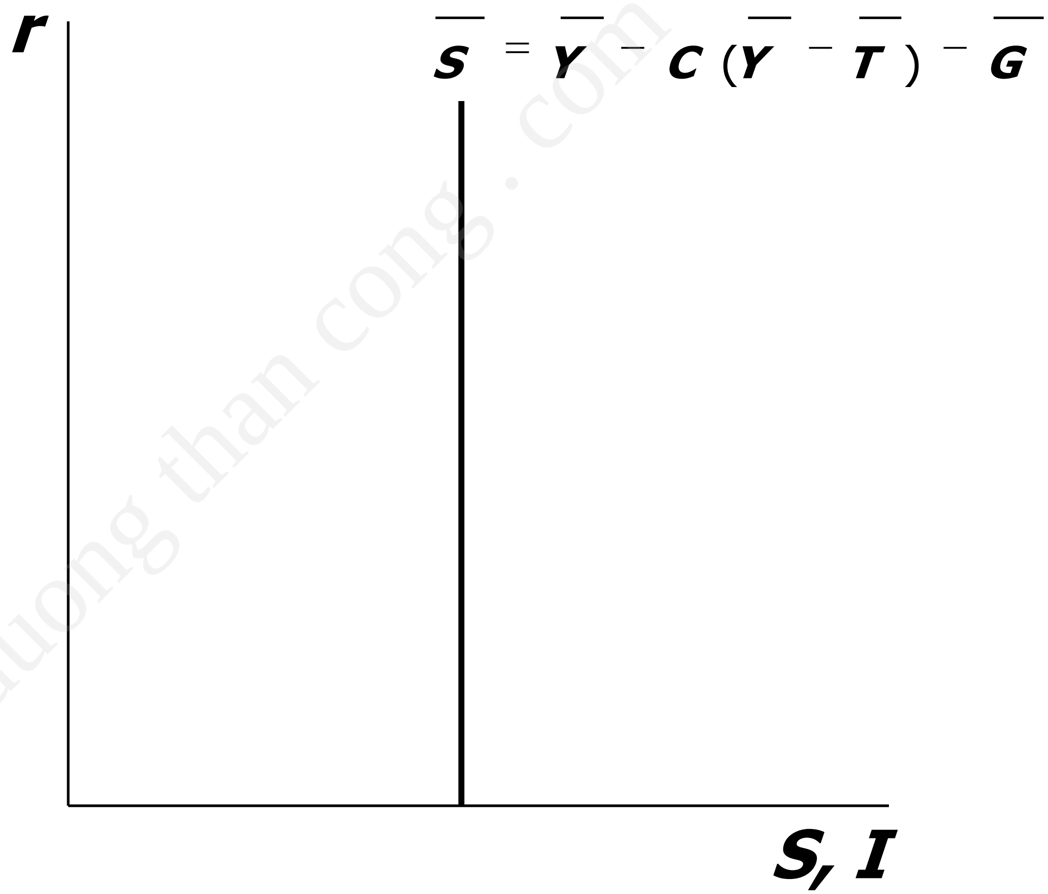


Fun fact: In the early 1990s, nearly 18 cents of every tax dollar went to pay interest on the debt.

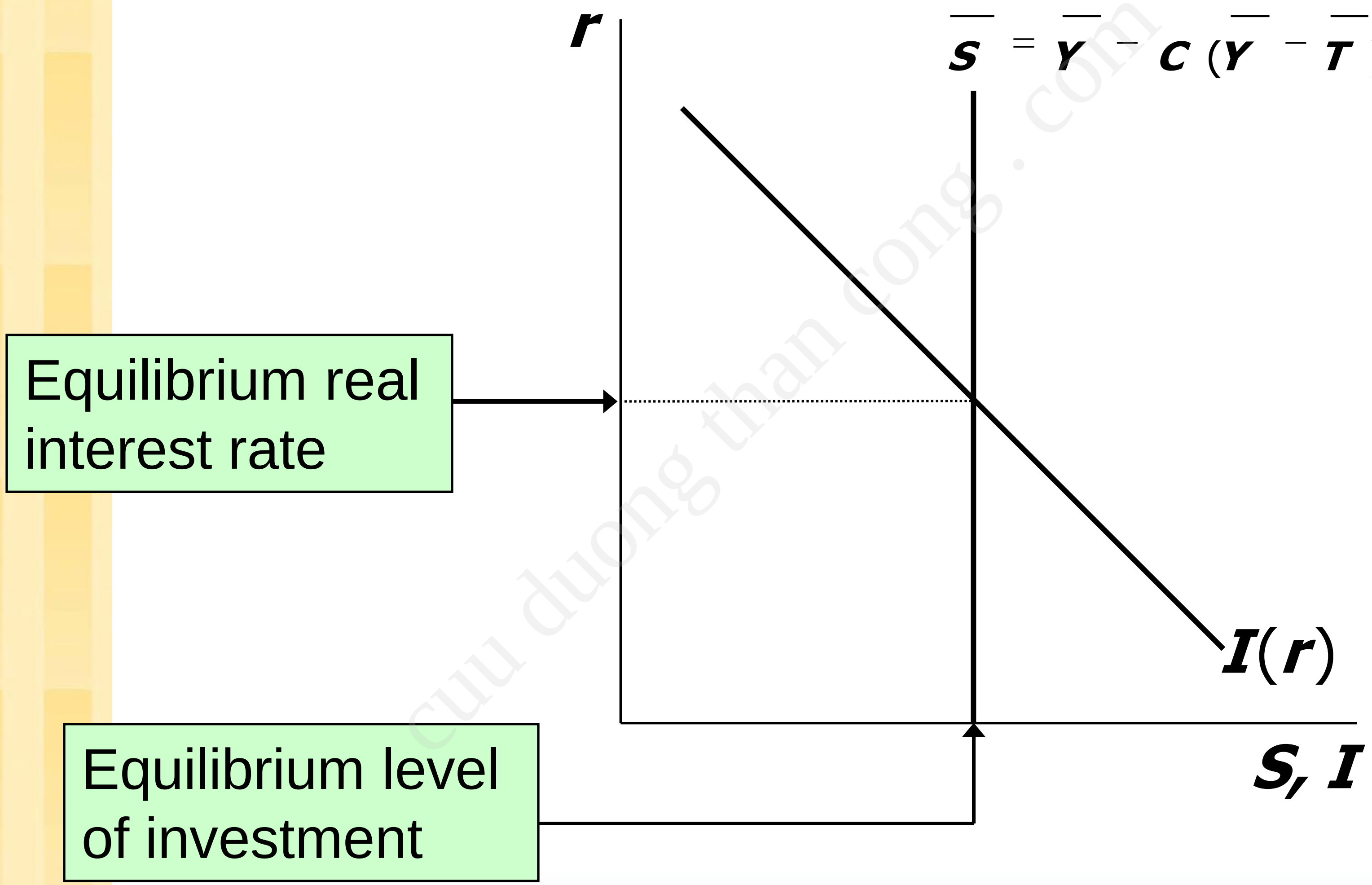
(Today it's about 9 cents.)

Loanable funds supply curve

National saving does not depend on ***r***, so the supply curve is vertical.



Loanable funds market equilibrium



The special role of *r*

r adjusts to equilibrate the goods market <u>and</u> the loanable funds market simultaneously:

If L.F. market in equilibrium, then

$$Y - C - G = I$$

Add $(C + G)$ to both sides to get

$$Y = C + I + G \quad \textit{(goods market eq'm)}$$

Thus,

Eq'm in L.F. market	$\Leftrightarrow$	Eq'm in goods market

Algebra example

Suppose an economy characterized by:

- Factors market supply:
  - labor supply= ***1000***
  - Capital stock supply=***1000***
- Goods market supply:
  - Production function: $Y = 3K + 2L$
- Goods market demand:
  - Consumption function: $C = 250 + 0.75(Y-T)$
  - Investment function: $I = 1000 – 5000r$
  - $G=1000, T = 1000$

Algebra example continued

Given the exogenous variables (***Y, G, T***), find the equilibrium values of the endogenous variables (***r, C, I***)

Find r using the goods market equilibrium condition:

Y = C + I + G
5000 = 250 + 0.75(5000-1000) +1000
-5000r + 1000
5000 = 5250 – 5000r
-250 = -5000r so ***r = 0.05***

And ***I = 1000 – 5000*(0.05) = 750***
C = 250 + 0.75(5000 - 1000) = 3250

Algebra example continued

Suppose government spending rises by 250 to 1250

Use intuition first to make a conjecture.

Verify by algebra:

$Y = C + I + G$

$5000 = 250 + 0.75(5000-1000) + 1000 - 5000r + 1250$

$-500 = -5000r$, so $r = 0.10$

And $I = 1000 - 5000*(0.10) = 500$.
Investment falls by 250.

$C = 250 + 0.75(5000 - 1000) = 3250$ as before for this consumption function.

Digression: mastering models

To learn a model well, be sure to know:

1. Which of its variables are endogenous and which are exogenous.
2. For each curve in the diagram, know
   a. definition
   b. intuition for slope
   c. all the things that can shift the curve
3. Use the model to analyze the effects of each item in 2c .

Mastering the loanable funds model

1. Things that shift the saving curve
   a. public saving
      i. fiscal policy: changes in ***G*** or ***T***
   b. private saving
      i. preferences
      ii. tax laws that affect saving
         - 401(k)
         - IRA
         - replace income tax with consumption tax

CASE STUDY
The Reagan Deficits

- Reagan policies during early 1980s:
  - increases in defense spending: $\Delta G > 0$
  - big tax cuts: $\Delta T < 0$
- According to our model, both policies reduce national saving:

$$\bar{S} = \bar{Y} - C(\bar{Y} - \bar{T}) - \bar{G}$$

$$\uparrow \bar{G} \Rightarrow \downarrow \bar{S}$$

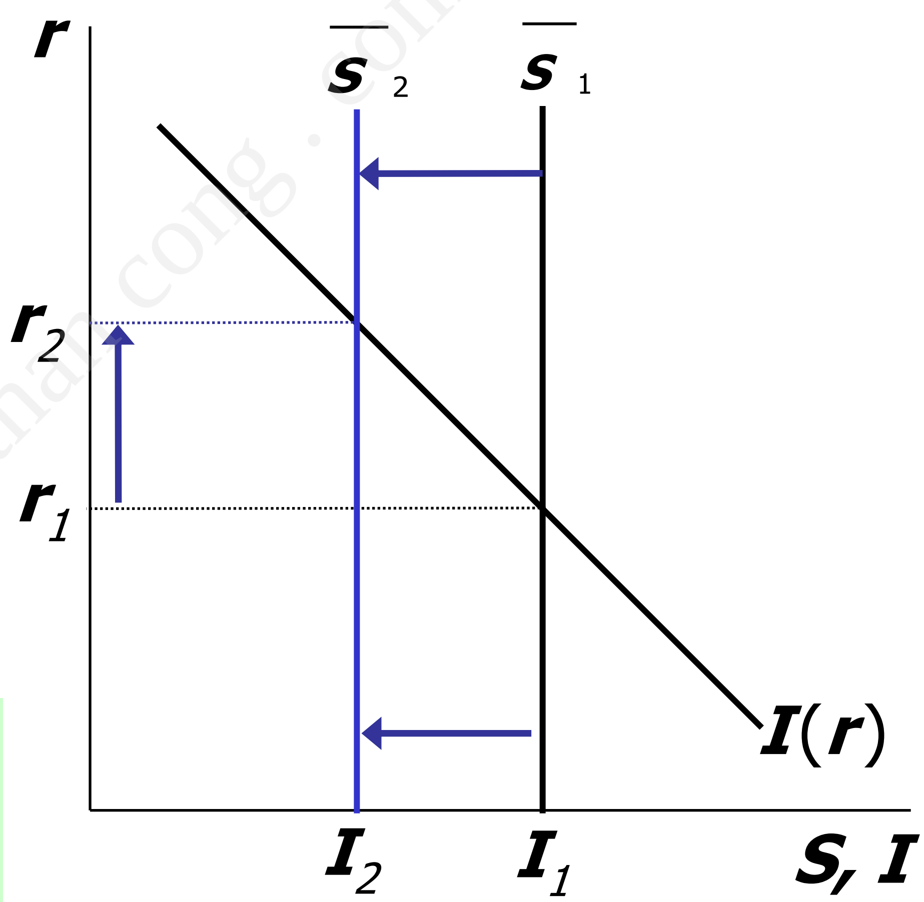
$$\downarrow \bar{T} \Rightarrow \uparrow C \Rightarrow \downarrow \bar{S}$$

1. The Reagan deficits, cont.

1. The increase in the deficit reduces saving…

2. …which causes the real interest rate to rise…

3. …which reduces the level of investment.



Are the data consistent with these results?

variable	1970s	1980s
$T - G$	–2.2	–3.9
S	19.6	17.4
r	1.1	6.3
I	19.9	19.4

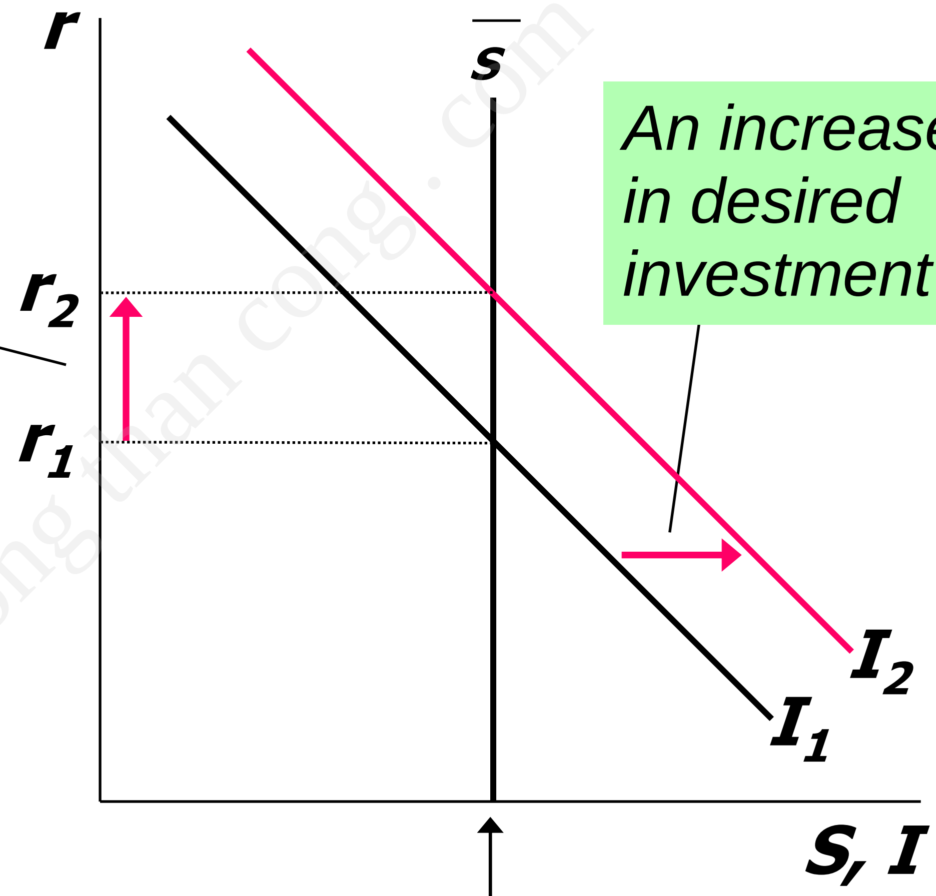
T–G, S, and I are expressed as a percent of GDP

All figures are averages over the decade shown.

Mastering the loanable funds model

2. Things that shift the investment curve
   a. certain technological innovations
      - to take advantage of the innovation, firms must buy new investment goods
   b. tax laws that affect investment
      - investment tax credit

An increase in investment demand



An increase in desired investment…

…raises the interest rate.

But the equilibrium level of investment cannot increase because the supply of loanable funds is fixed.

Chapter summary

1. Total output is determined by
   - how much capital and labor the economy has
   - the level of technology
2. Competitive firms hire each factor until its marginal product equals its price.
3. If the production function has constant returns to scale, then labor income plus capital income equals total income (output).

Chapter summary

4. The economy's output is used for
   - consumption
     (which depends on disposable income)
   - investment
     (depends on the real interest rate)
   - government spending
     (exogenous)

5. The real interest rate adjusts to equate the demand for and supply of
   - goods and services
   - loanable funds

Chapter summary

6. A decrease in national saving causes the interest rate to rise and investment to fall.

   An increase in investment demand causes the interest rate to rise, but does not affect the equilibrium level of investment if the supply of loanable funds is fixed.