Lesson 2
Consumer behavior

- Budget constraint
- Preferences
- Utility

BUDGET CONSTRAINT

Budget Constraints

- A consumption bundle containing x_1 units of commodity 1, x_2 units of commodity 2 and so on up to x_n units of commodity n is denoted by the vector $(x_1, x_2, \ldots , x_n)$.
- Commodity prices are $p_1, p_2, \ldots , p_n$.

Budget Constraints

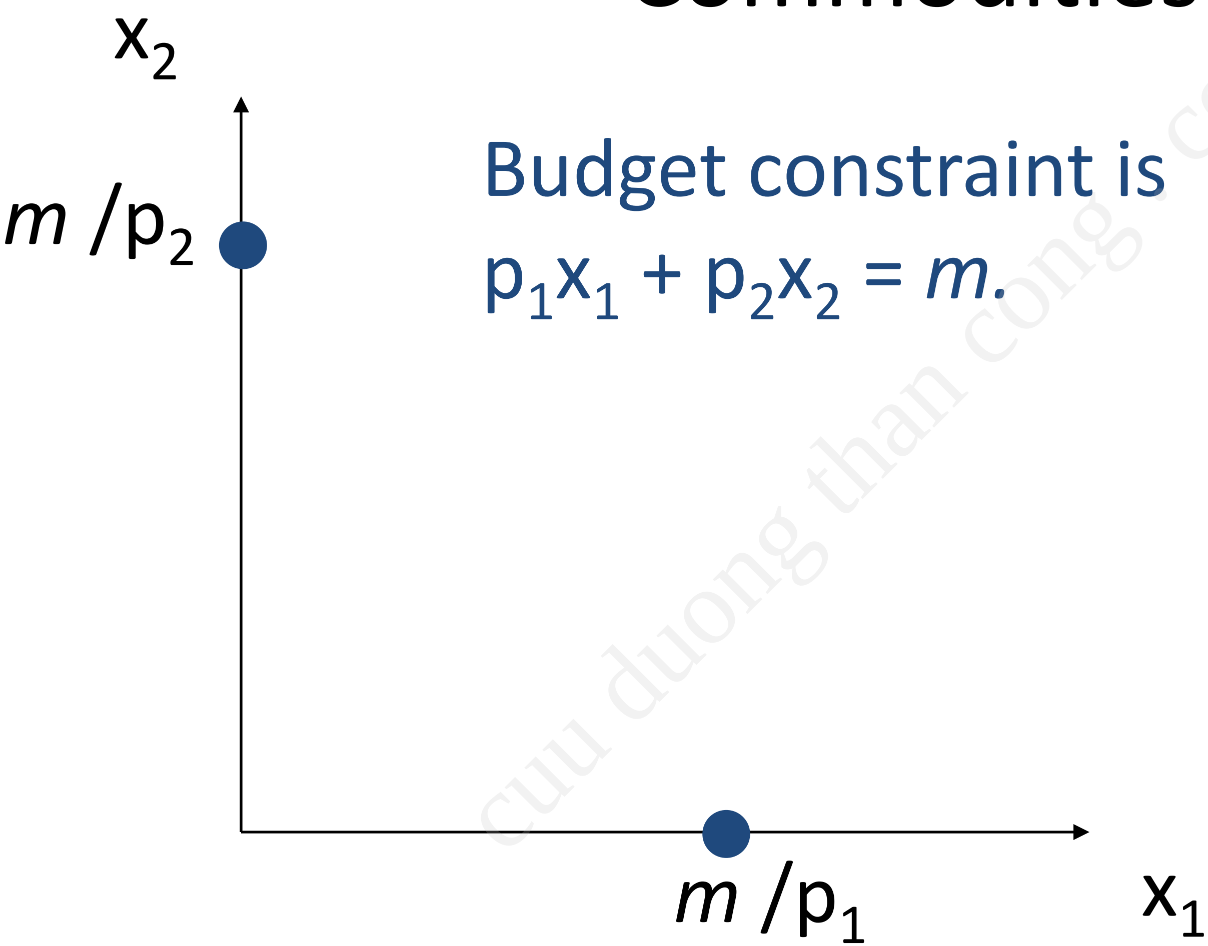
- The bundles that are only just affordable form the consumer's budget constraint. This is the set

$$\{ (x_1,\ldots,x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0 \text{ and } p_1x_1 + \ldots + p_nx_n = m \}.$$

Budget Constraints

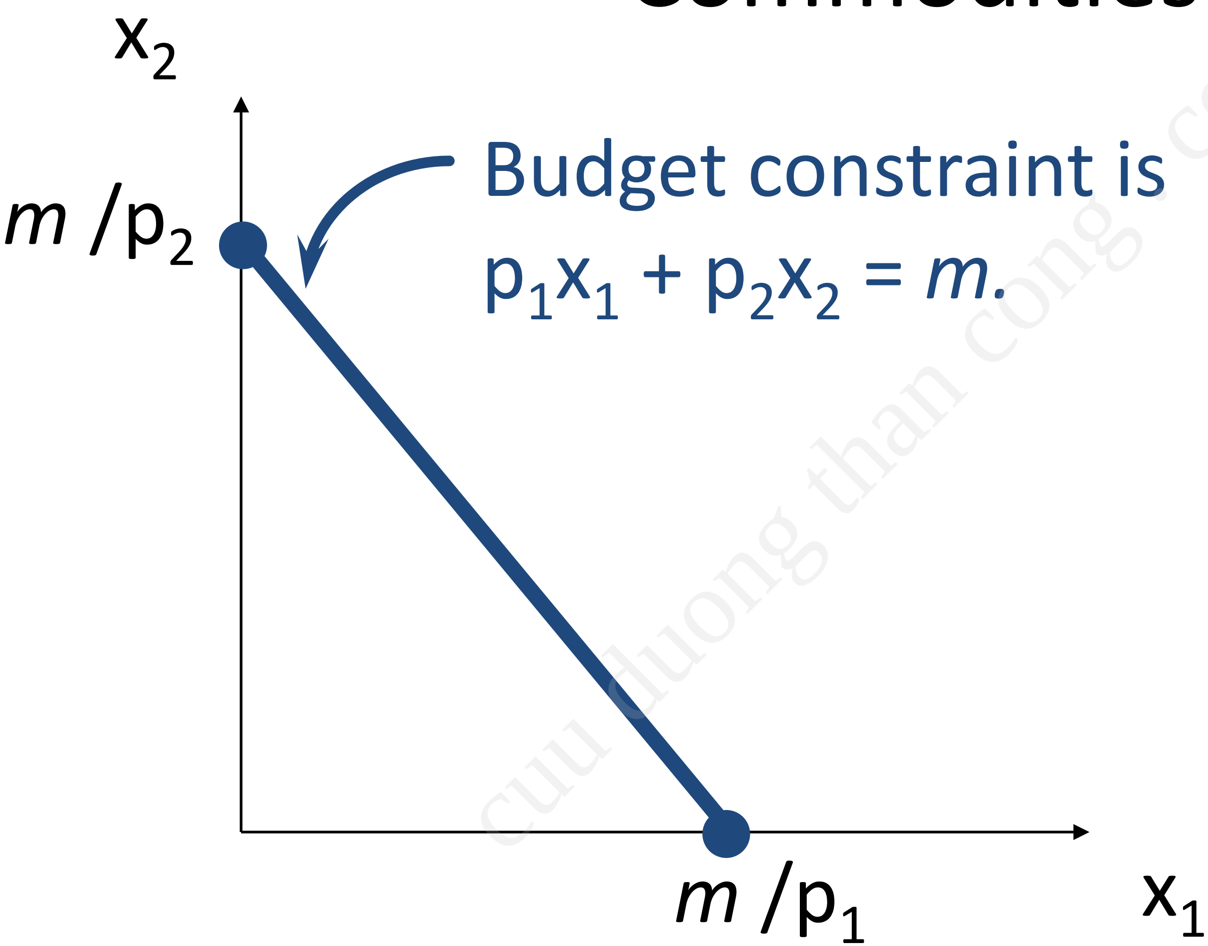
- The consumer's budget set is the set of all affordable bundles;
  $B(p_1, \ldots, p_n, m) =$
  $\{ (x_1, \ldots, x_n) \mid x_1 \geq 0, \ldots, x_n \geq 0$ and
  $p_1x_1 + \ldots + p_nx_n \leq m \}$
- The budget constraint is the upper boundary of the budget set.

Budget Set and Constraint for Two Commodities

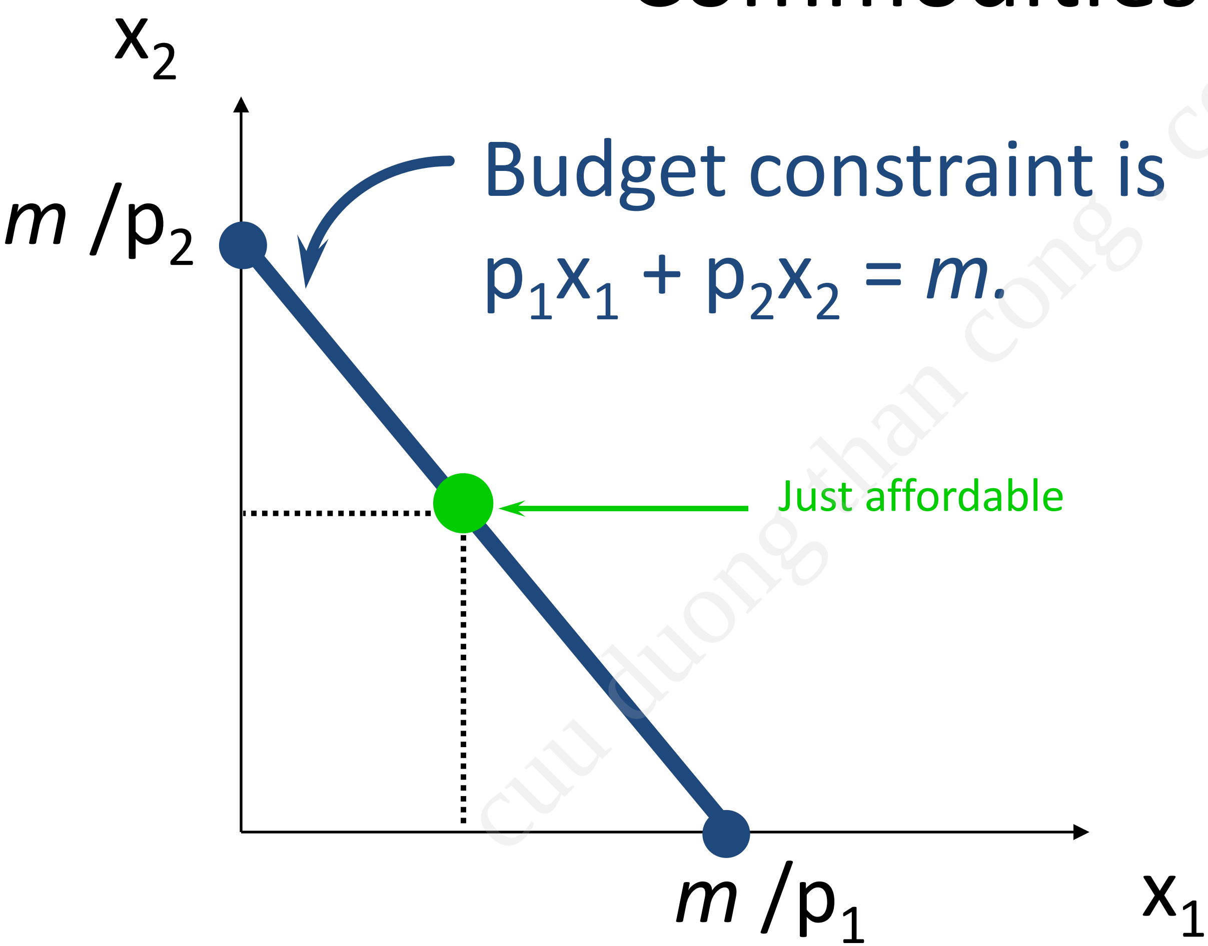


Budget constraint is
$p_1x_1 + p_2x_2 = m$.

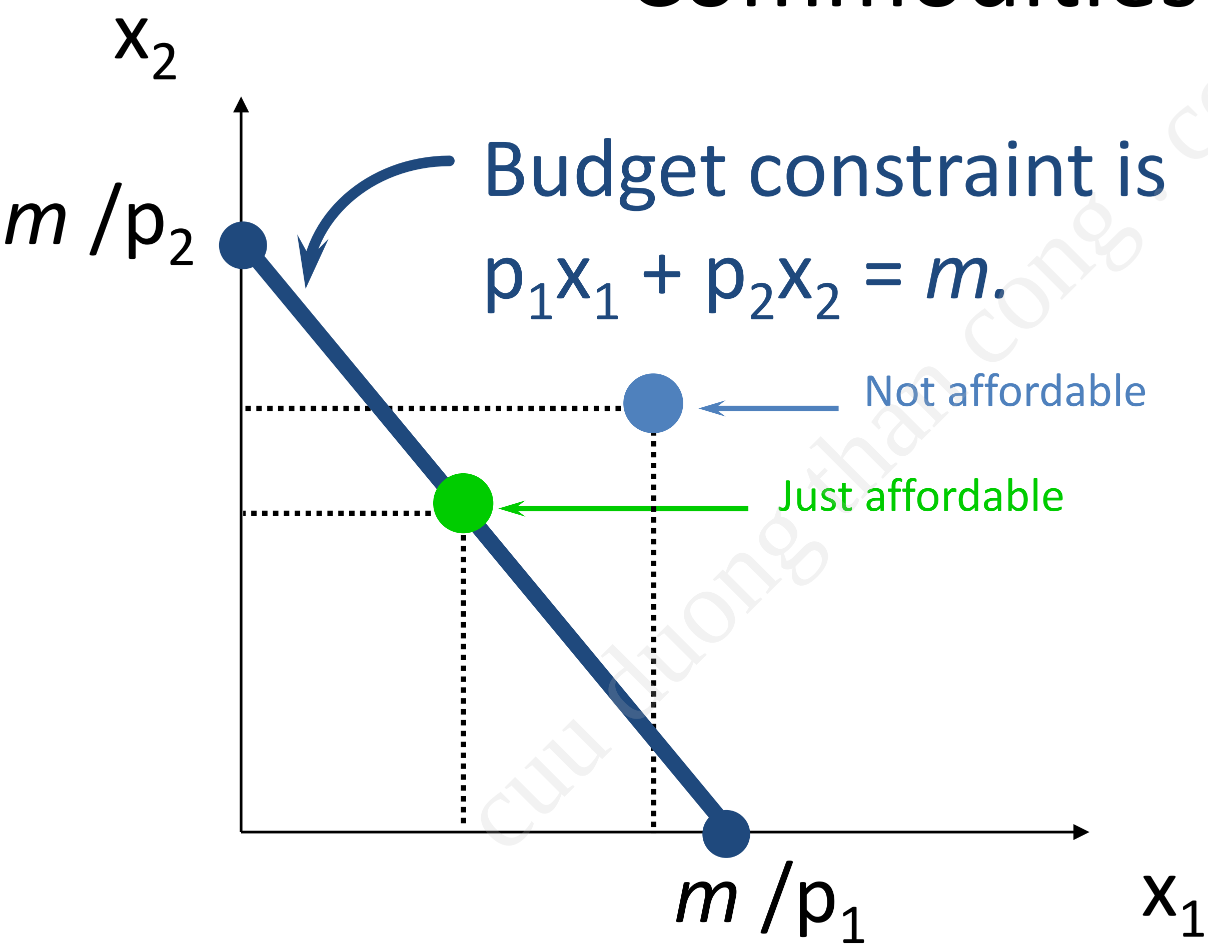
Budget Set and Constraint for Two Commodities



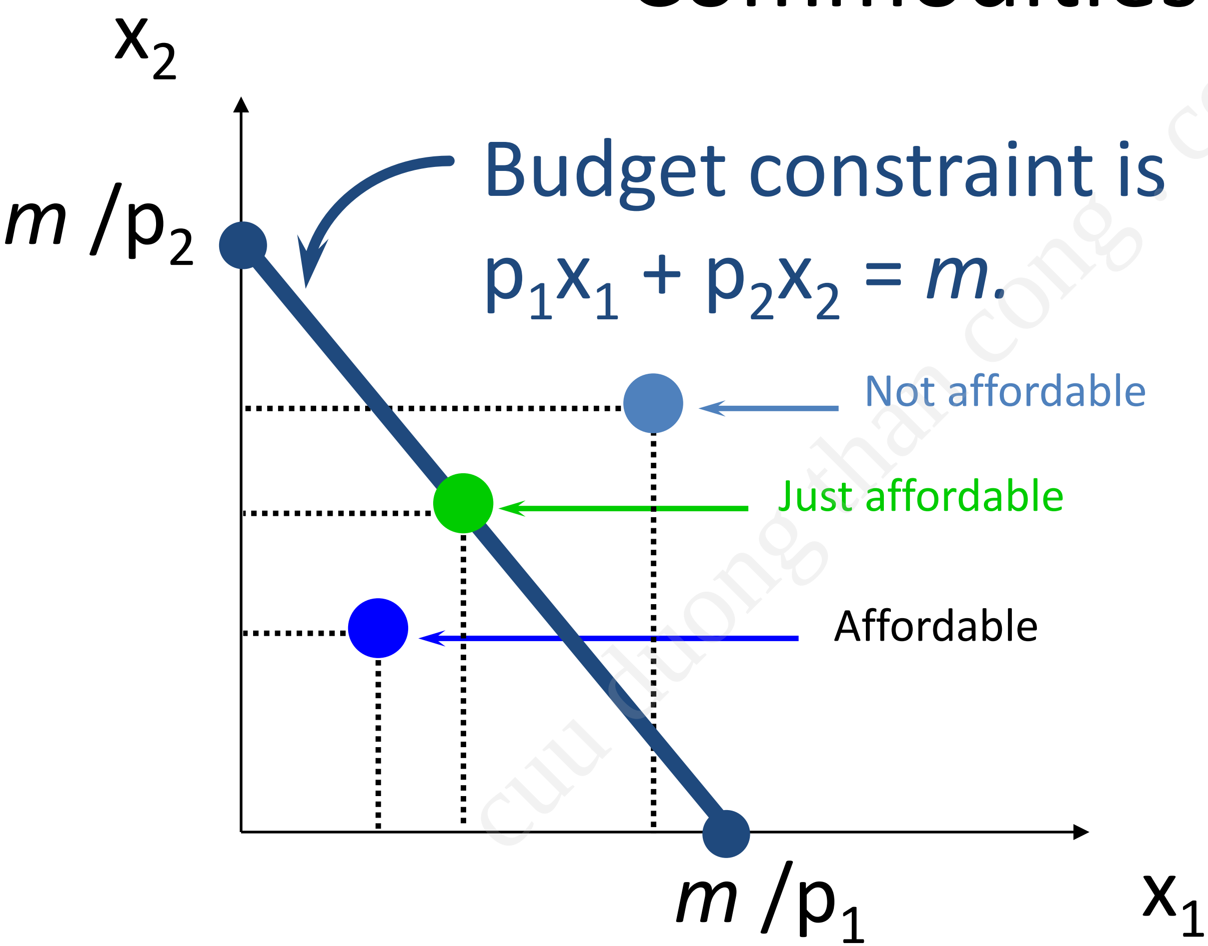
Budget Set and Constraint for Two Commodities



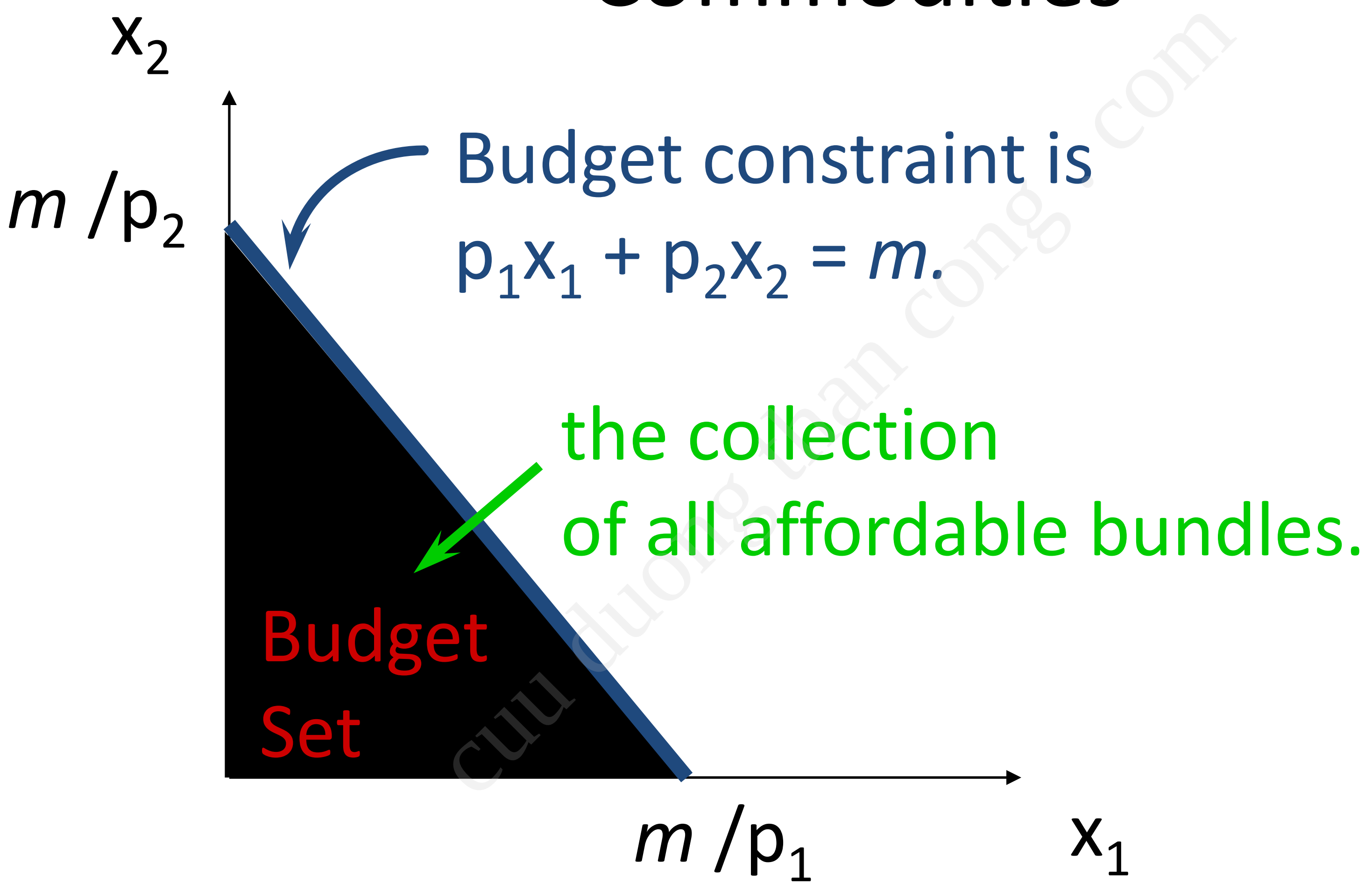
Budget Set and Constraint for Two Commodities



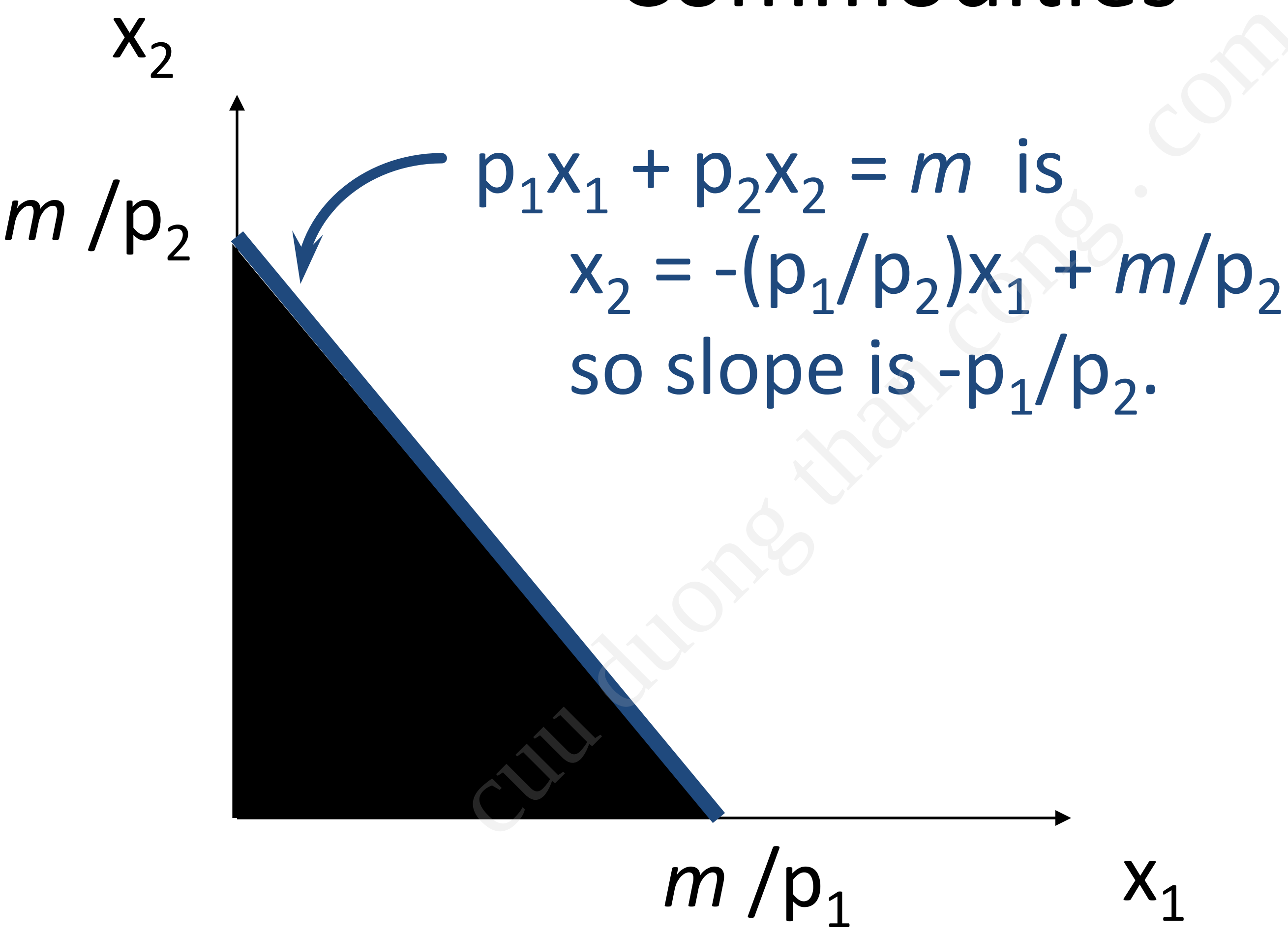
Budget Set and Constraint for Two Commodities



Budget Set and Constraint for Two Commodities



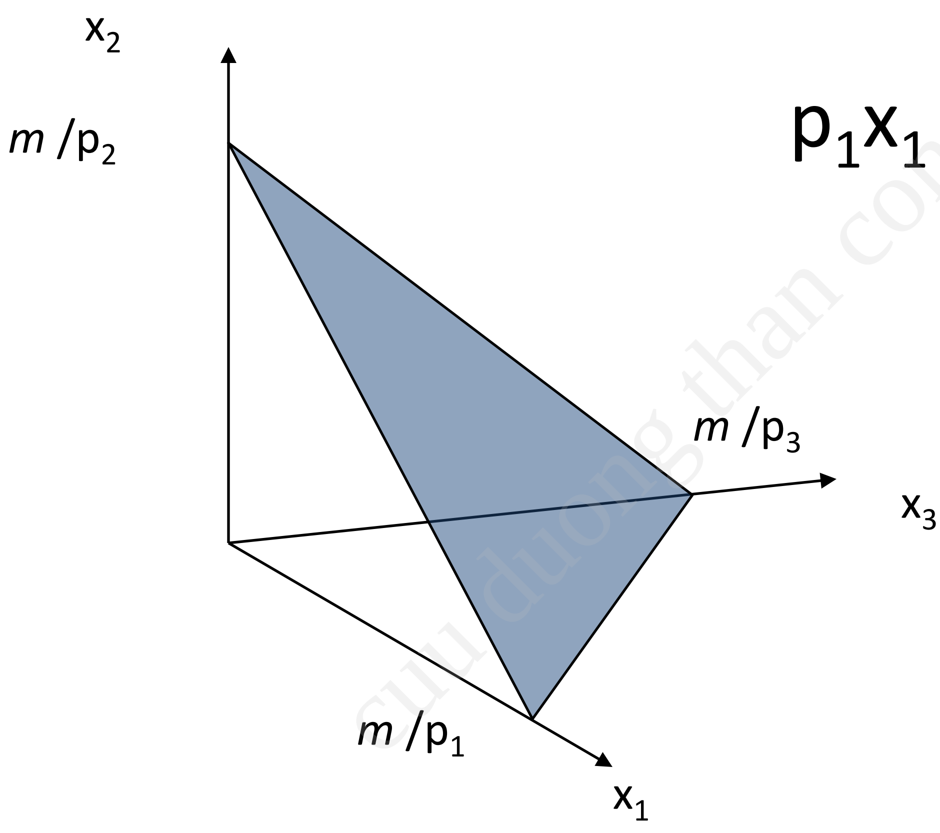
Budget Set and Constraint for Two Commodities



Budget Constraints

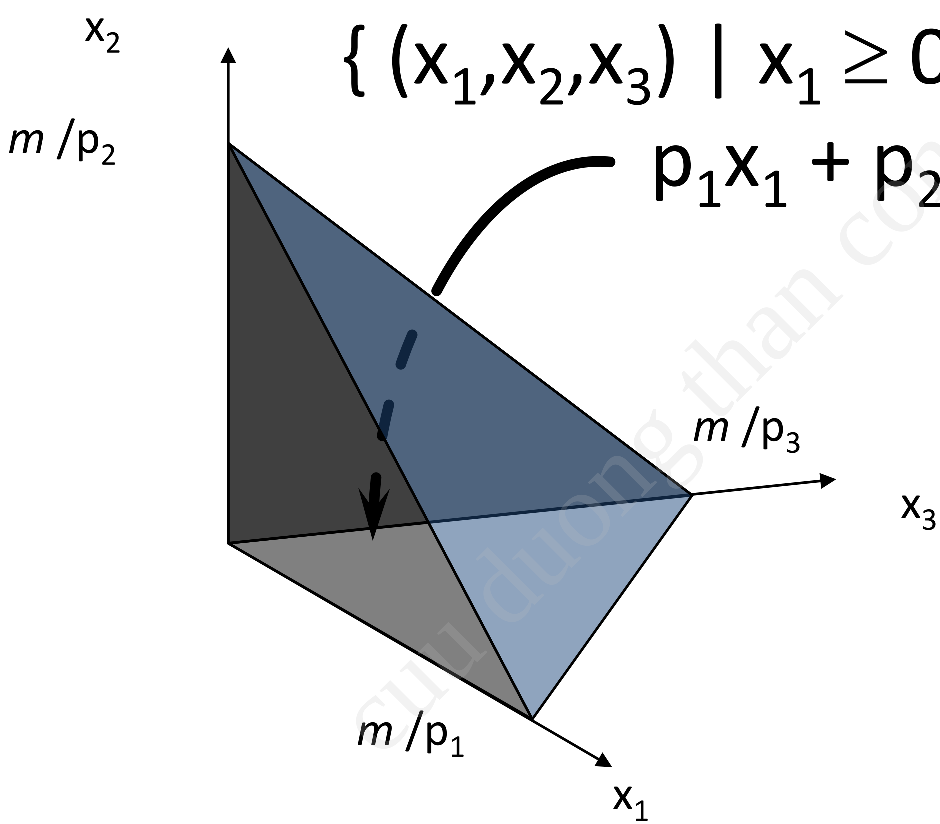
- If n = 3 what do the budget constraint and the budget set look like?

Budget Constraint for Three Commodities



$$p_1x_1 + p_2x_2 + p_3x_3 = m$$

Budget Set for Three Commodities

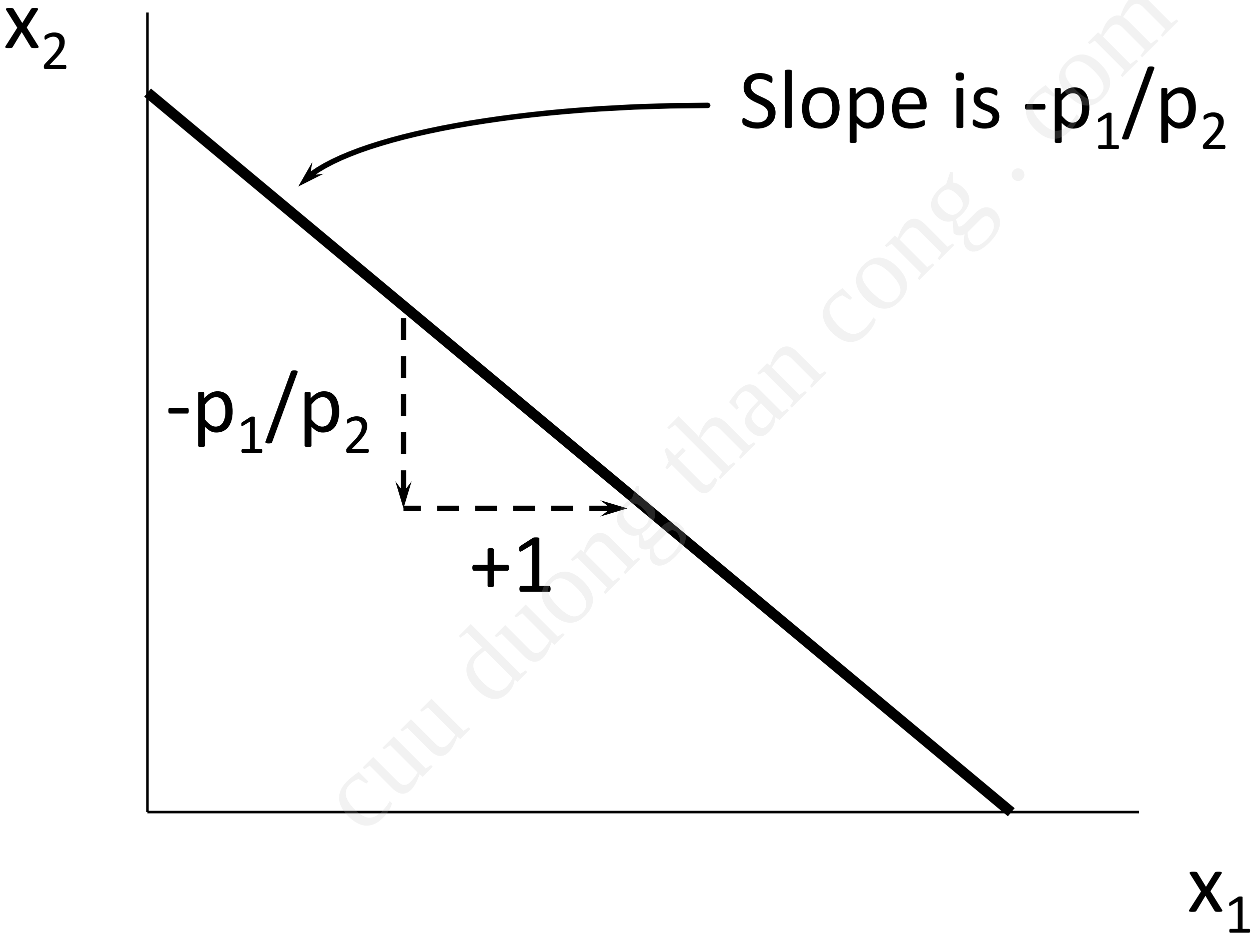


$\{ (x_1,x_2,x_3) \mid x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$ and $p_1x_1 + p_2x_2 + p_3x_3 \leq m\}$

Budget Constraints

- For n = 2 and x_1 on the horizontal axis, the constraint's slope is $-p_1/p_2$. What does it mean?

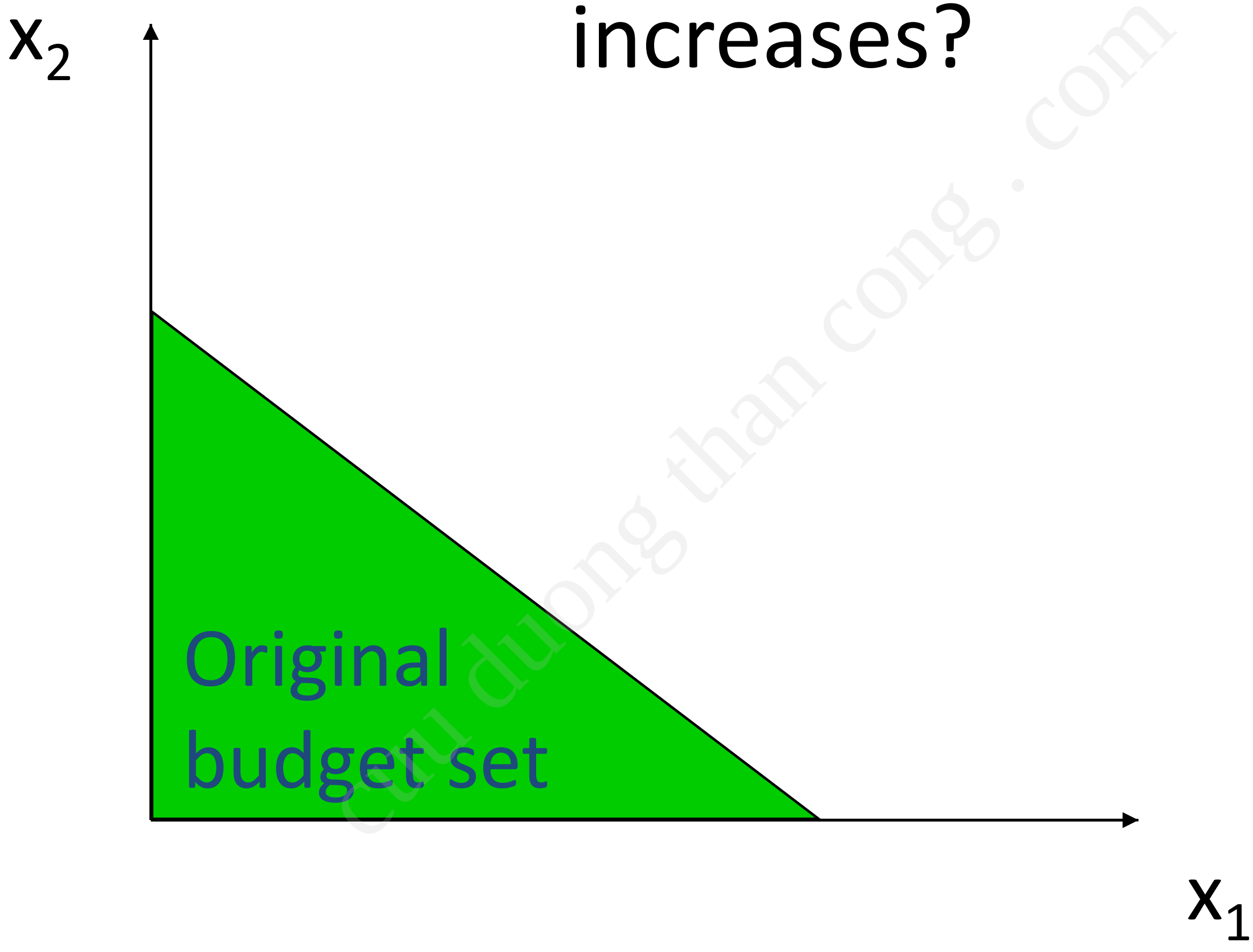
Budget Constraints



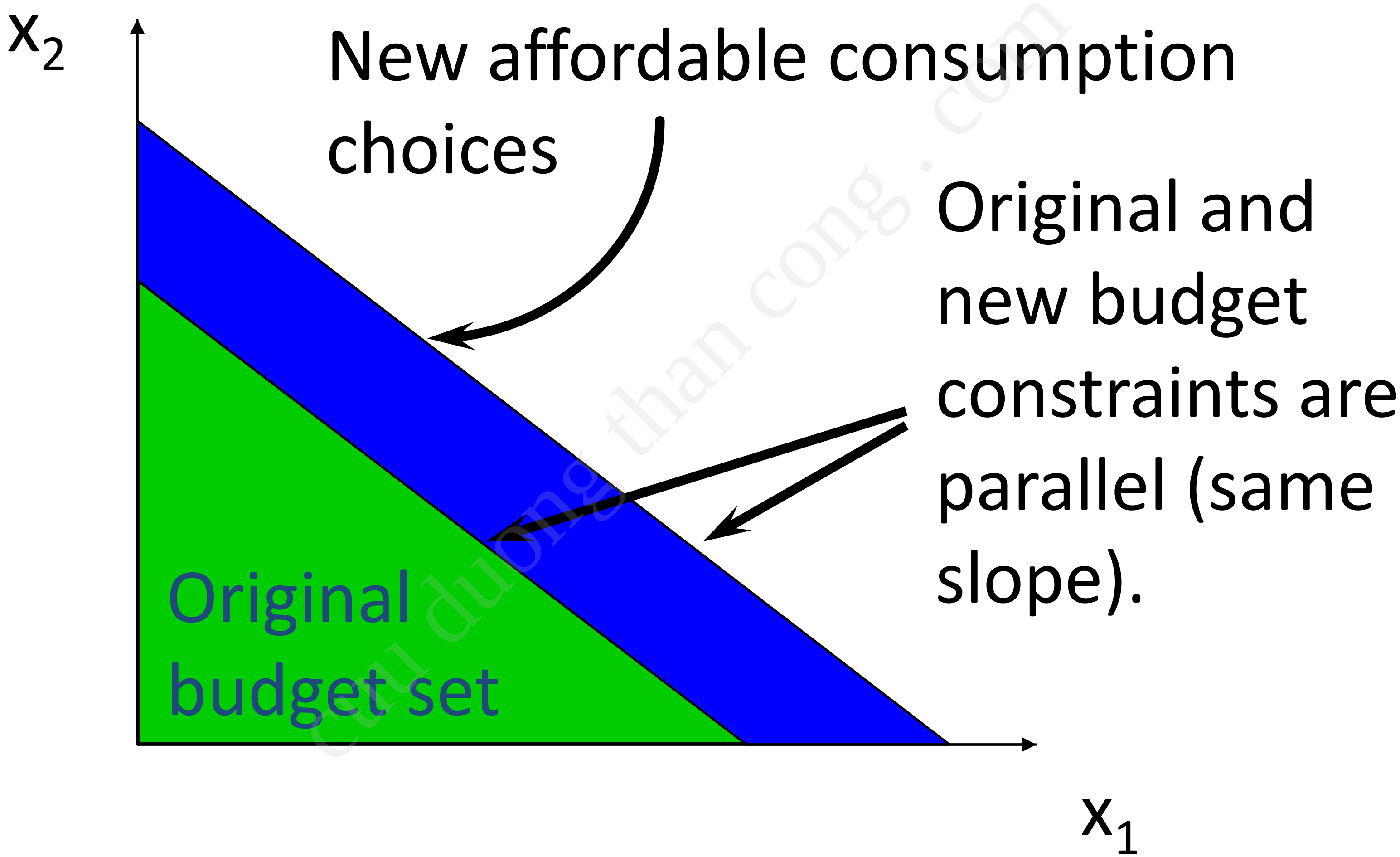
Budget Sets & Constraints; Income and Price Changes

- The budget constraint and budget set depend upon prices and income. What happens as prices or income change?

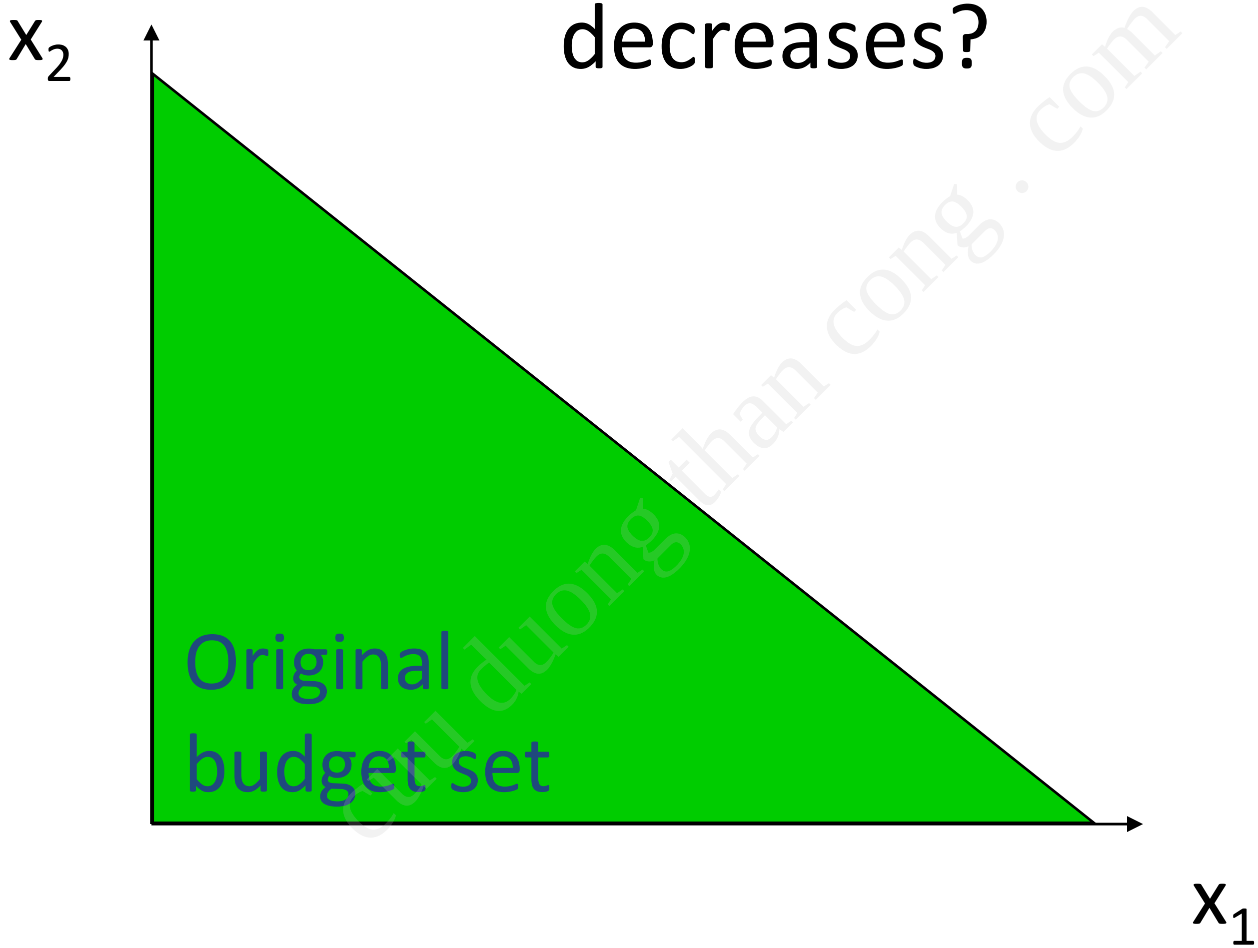
How do the budget set and budget constraint change as income *m* increases?



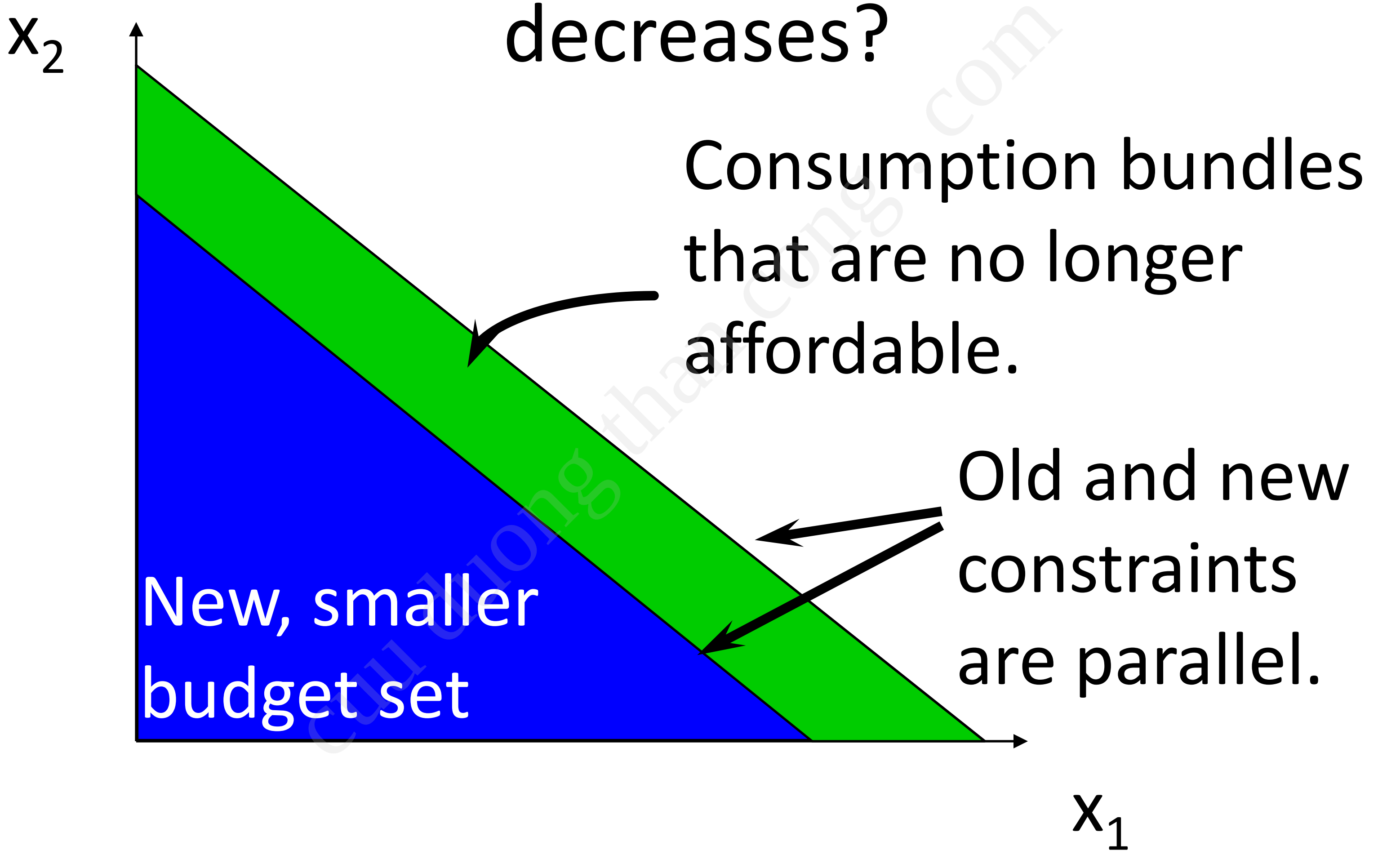
Higher income gives more choice



How do the budget set and budget constraint change as income *m* decreases?



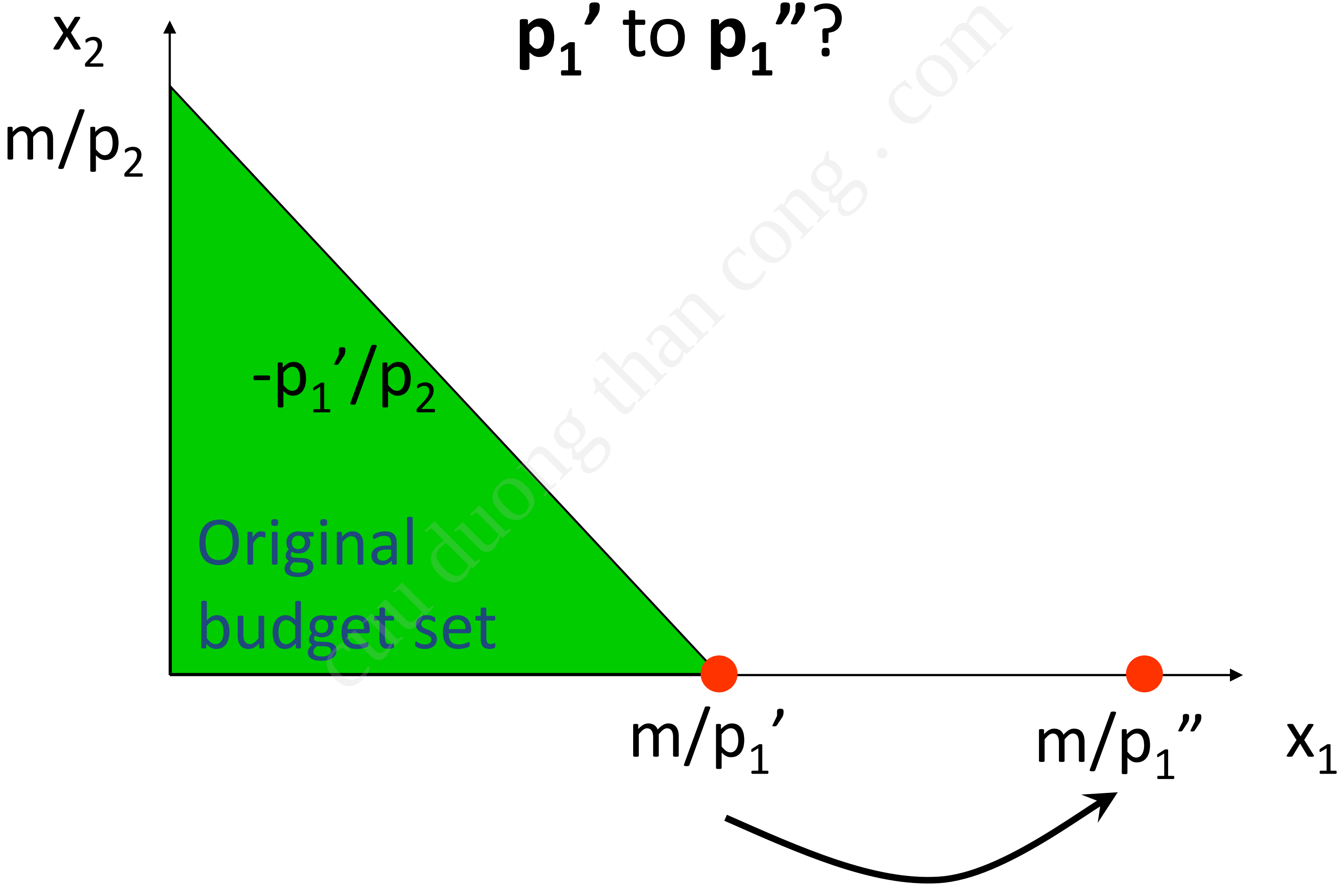
How do the budget set and budget constraint change as income *m* decreases?



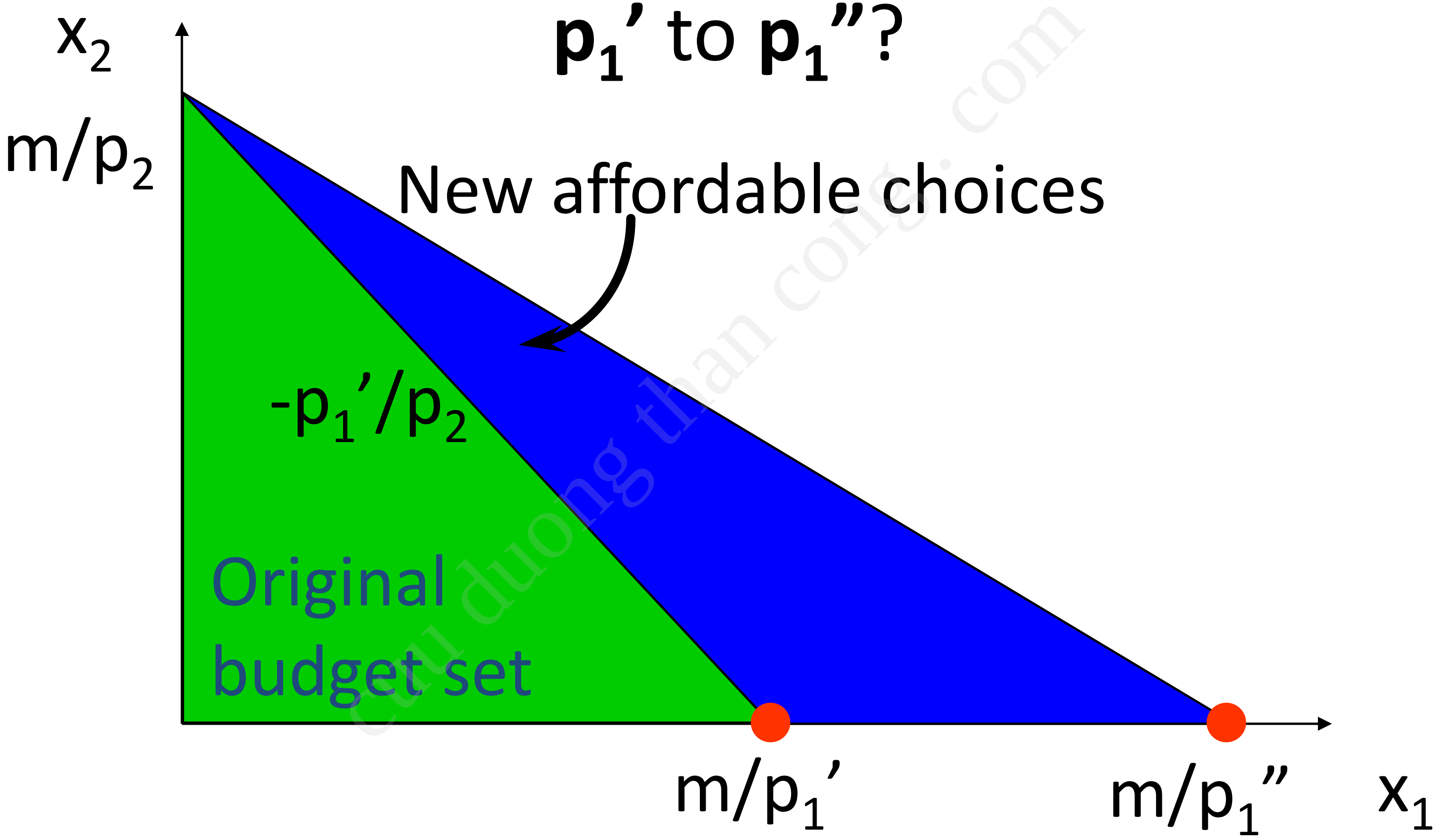
Budget Constraints - Price Changes

- What happens if just one price decreases?
- Suppose p_1 decreases.

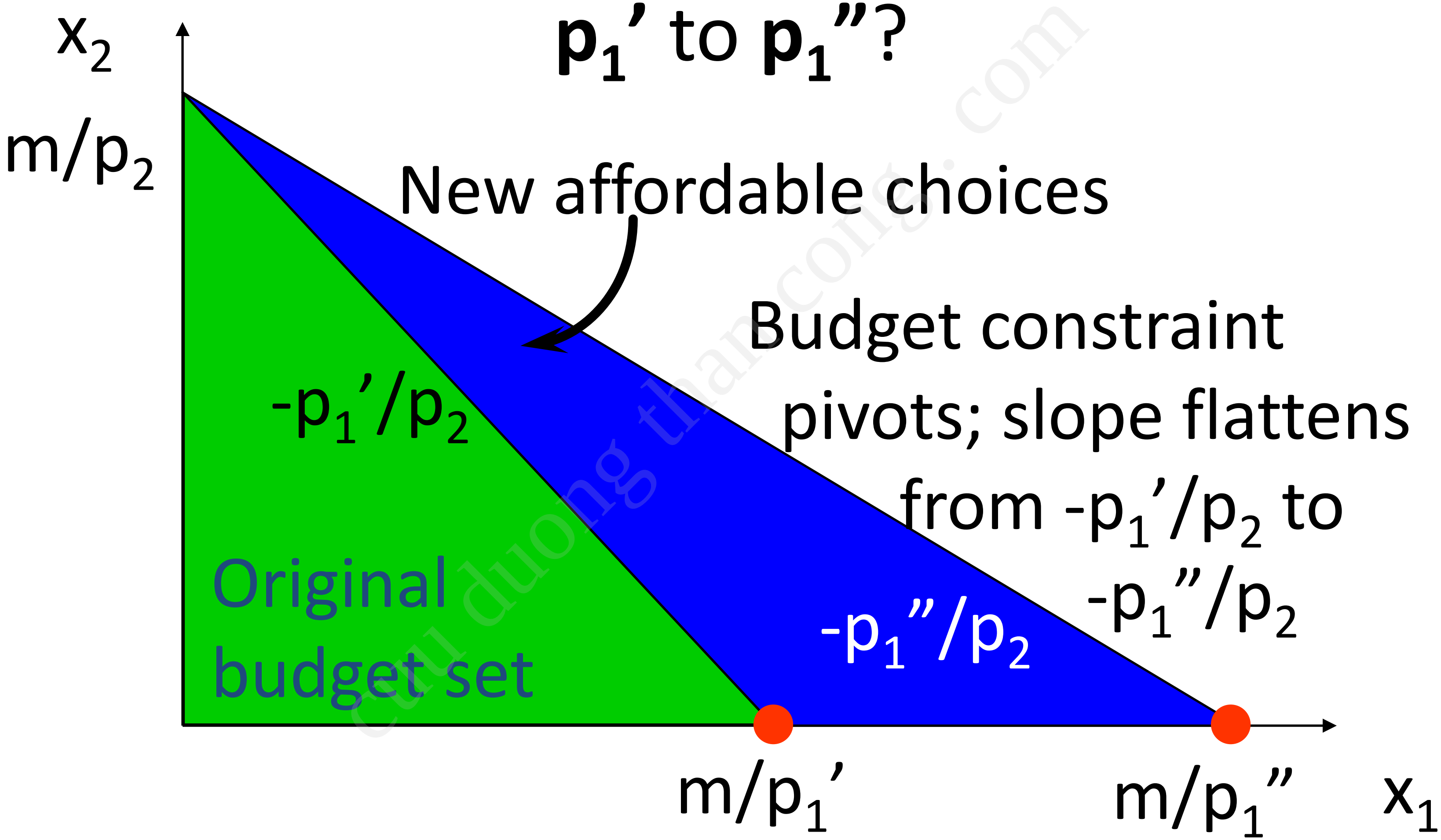
How do the budget set and budget constraint change as p_1 decreases from p_1' to p_1''?



How do the budget set and budget constraint change as $\mathbf{p_1}$ decreases from $\mathbf{p_1}$' to $\mathbf{p_1}$"?



How do the budget set and budget constraint change as $\mathbf{p_1}$ decreases from $\mathbf{p_1}$’ to $\mathbf{p_1}$”?



The Food Stamp Program

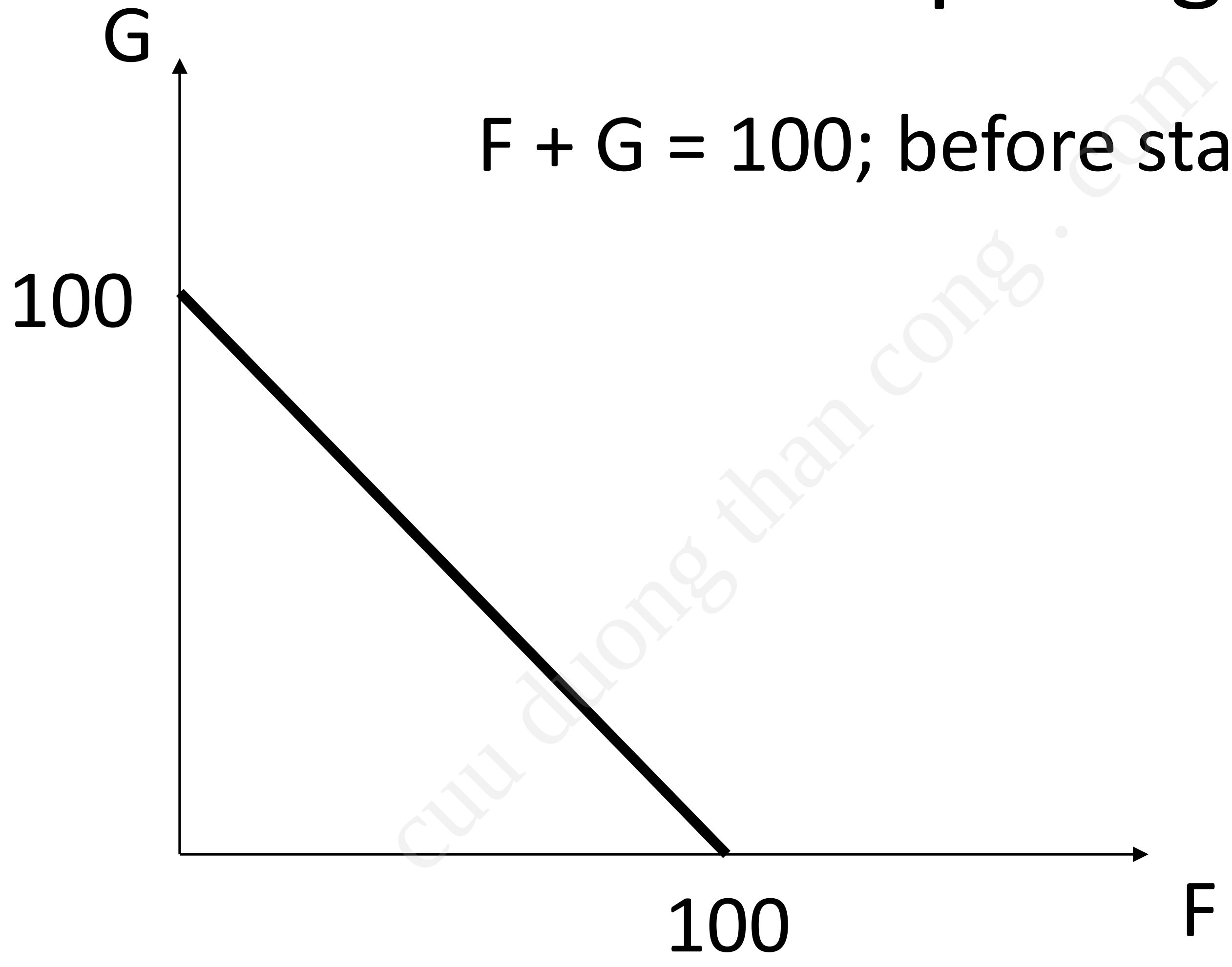
- Food stamps are coupons that can be legally exchanged only for food.
- How does a commodity-specific gift such as a food stamp alter a family's budget constraint?

The Food Stamp Program

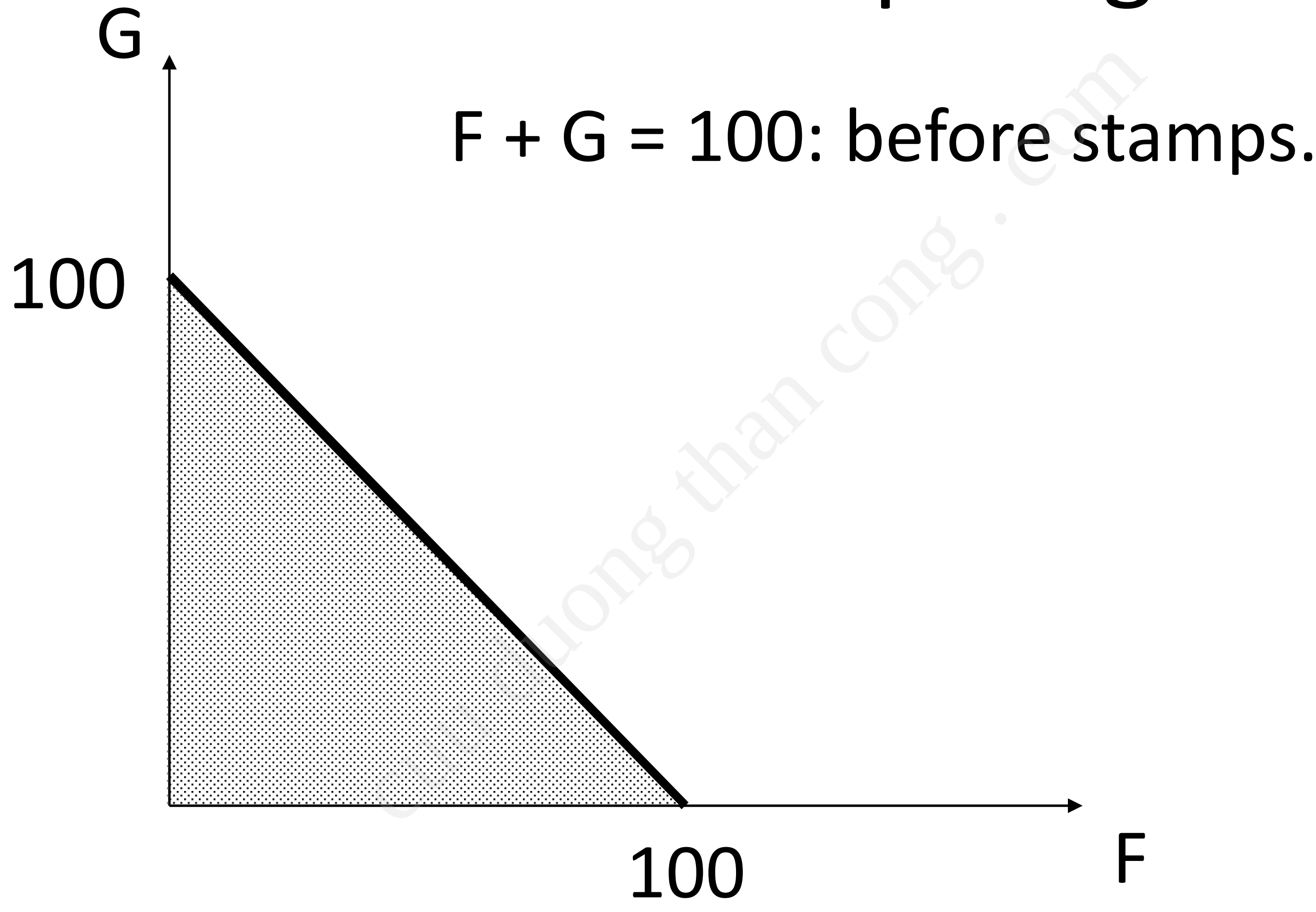
- Suppose m = \$100, p_F = \$1 and the price of "other goods" is p_G = \$1.
- The budget constraint is then

$$F + G = 100.$$

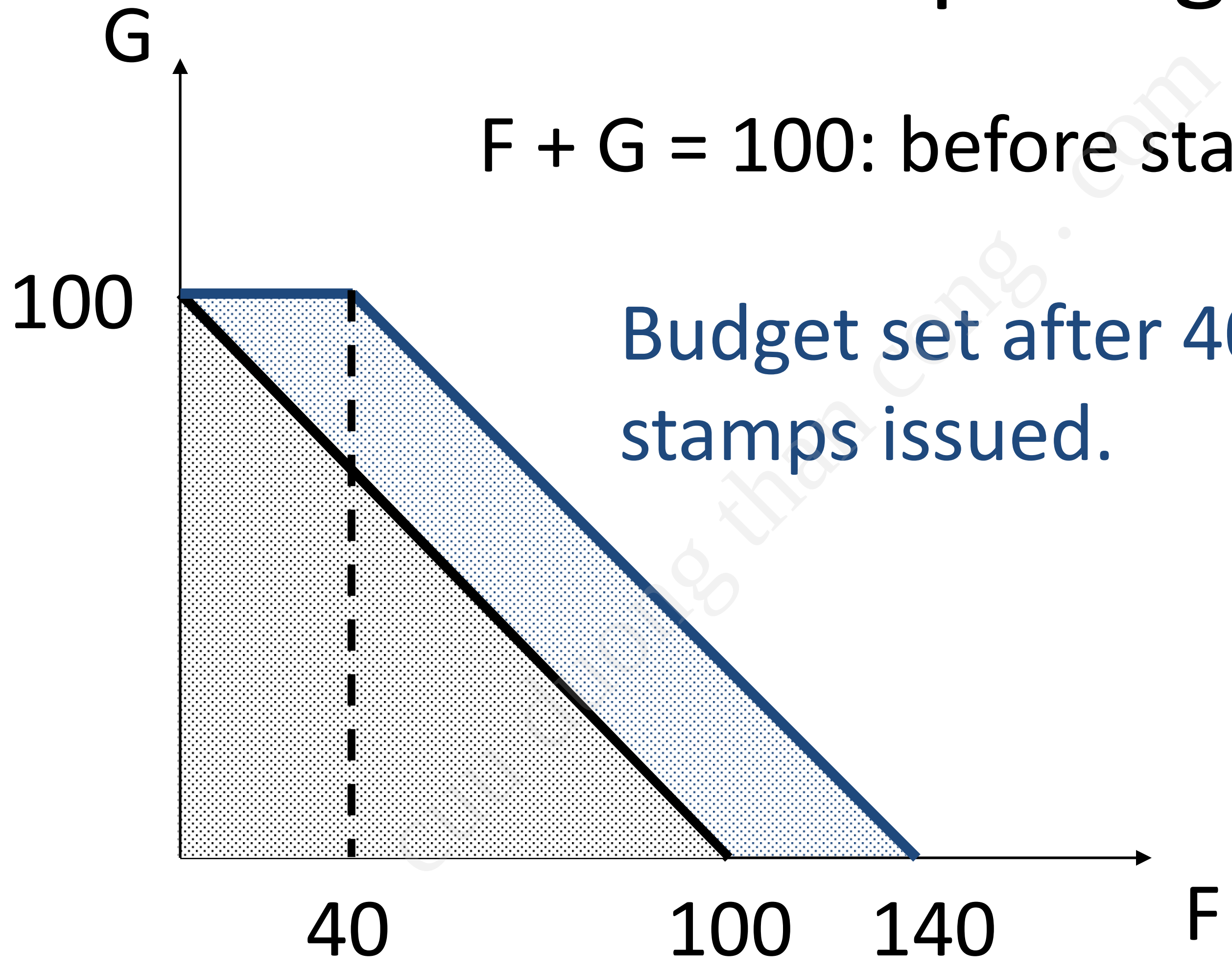
The Food Stamp Program



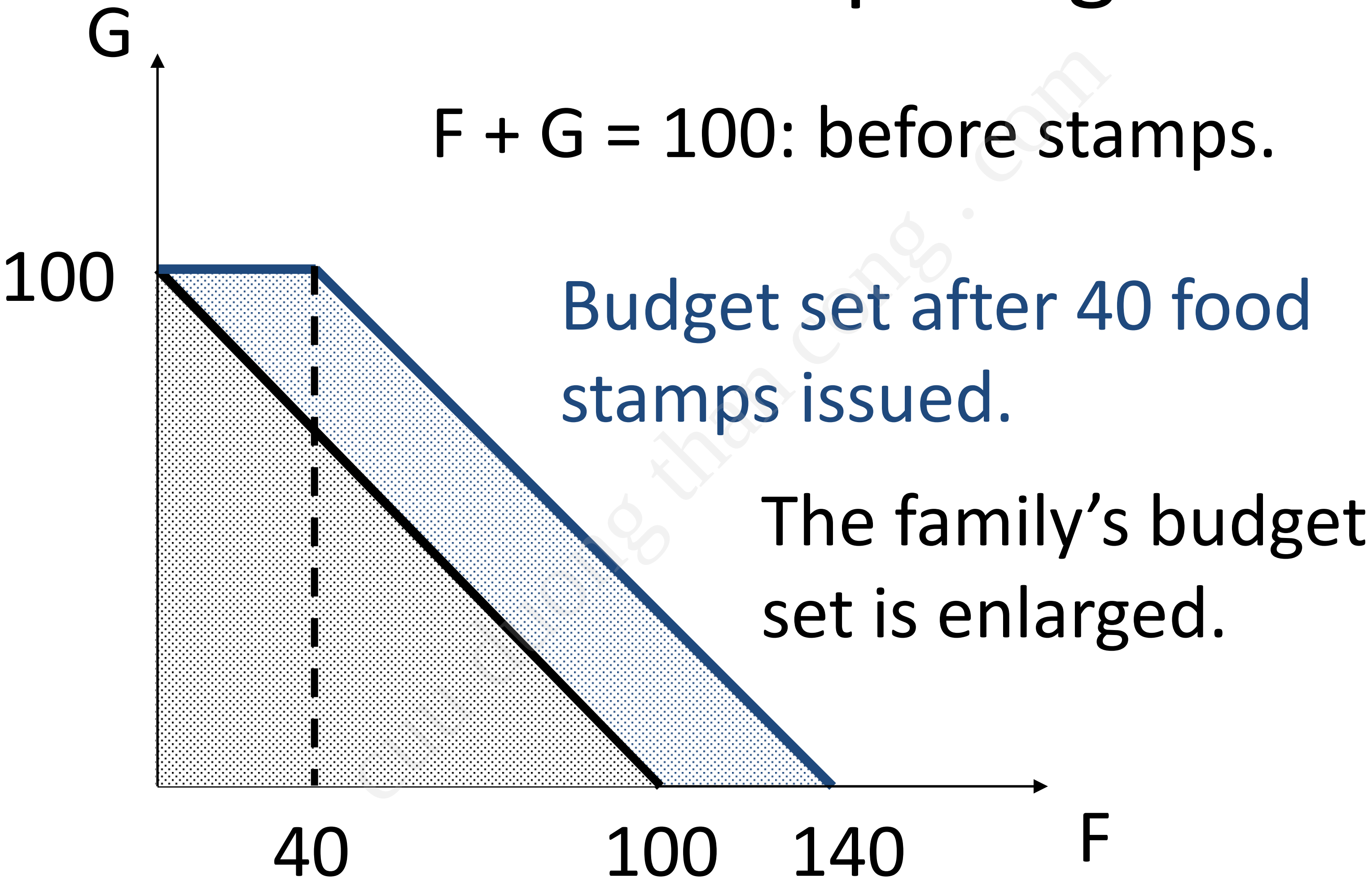
The Food Stamp Program



The Food Stamp Program



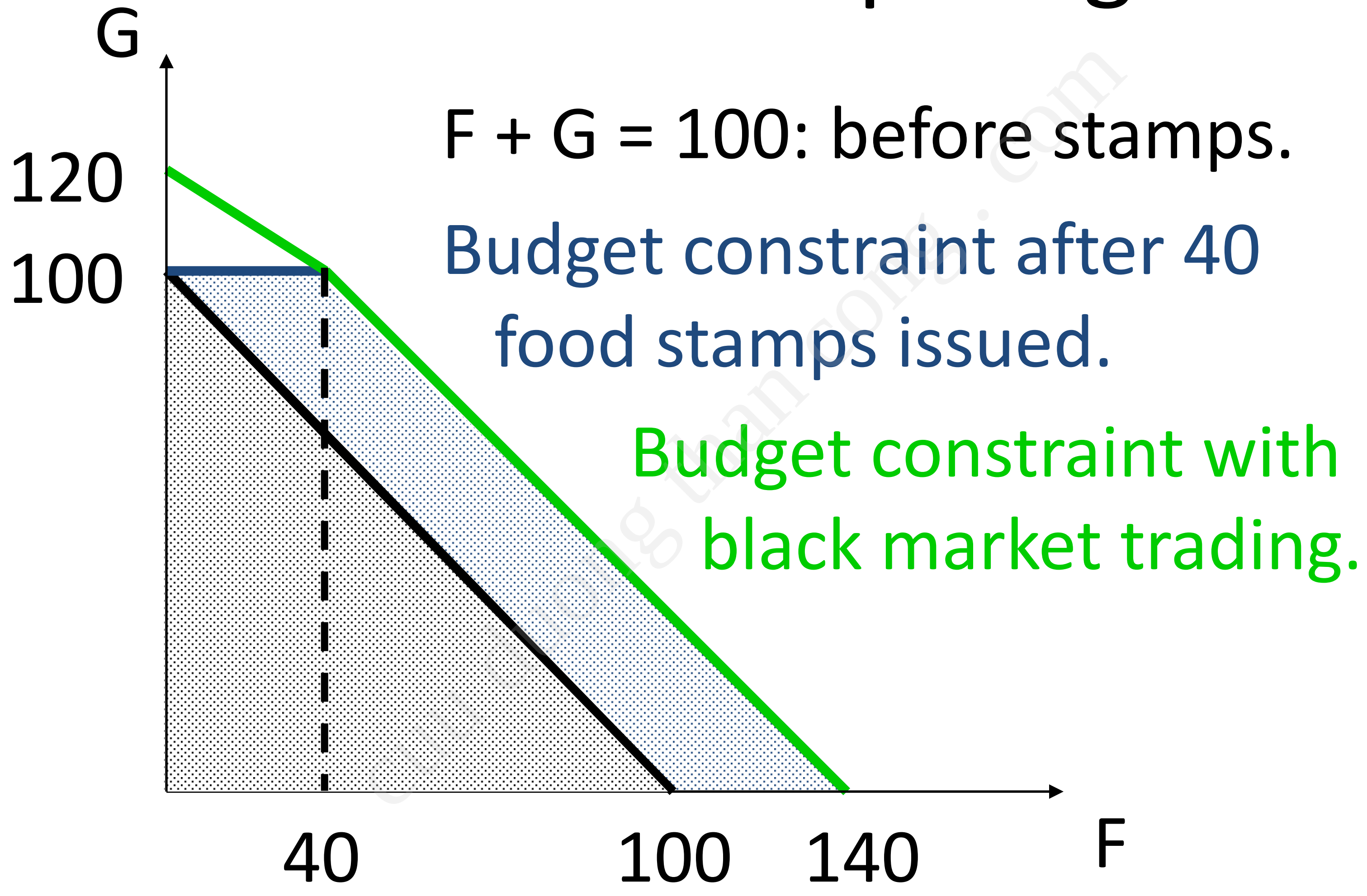
The Food Stamp Program



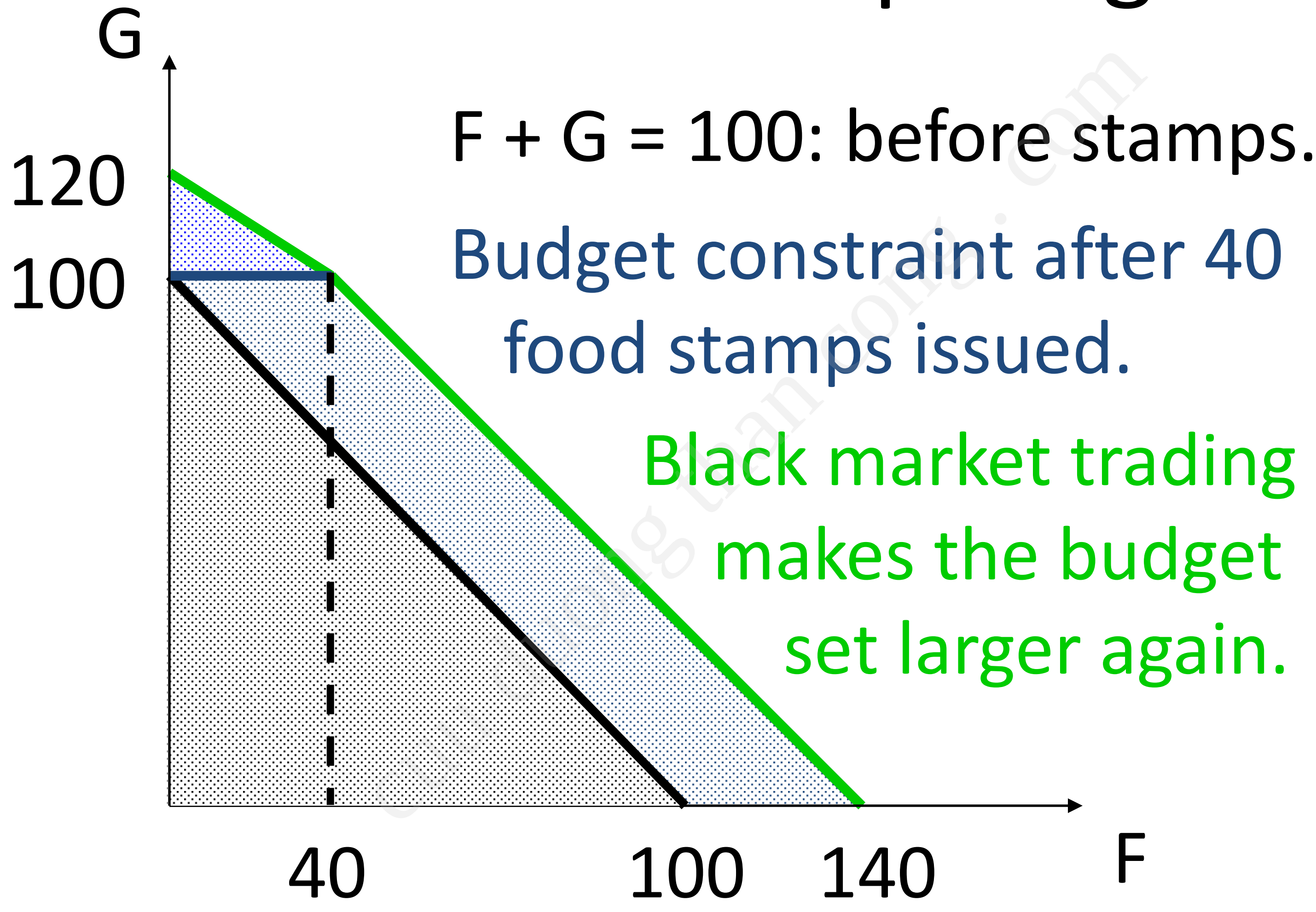
The Food Stamp Program

- What if food stamps can be traded on a black market for $0.50 each?

The Food Stamp Program



The Food Stamp Program



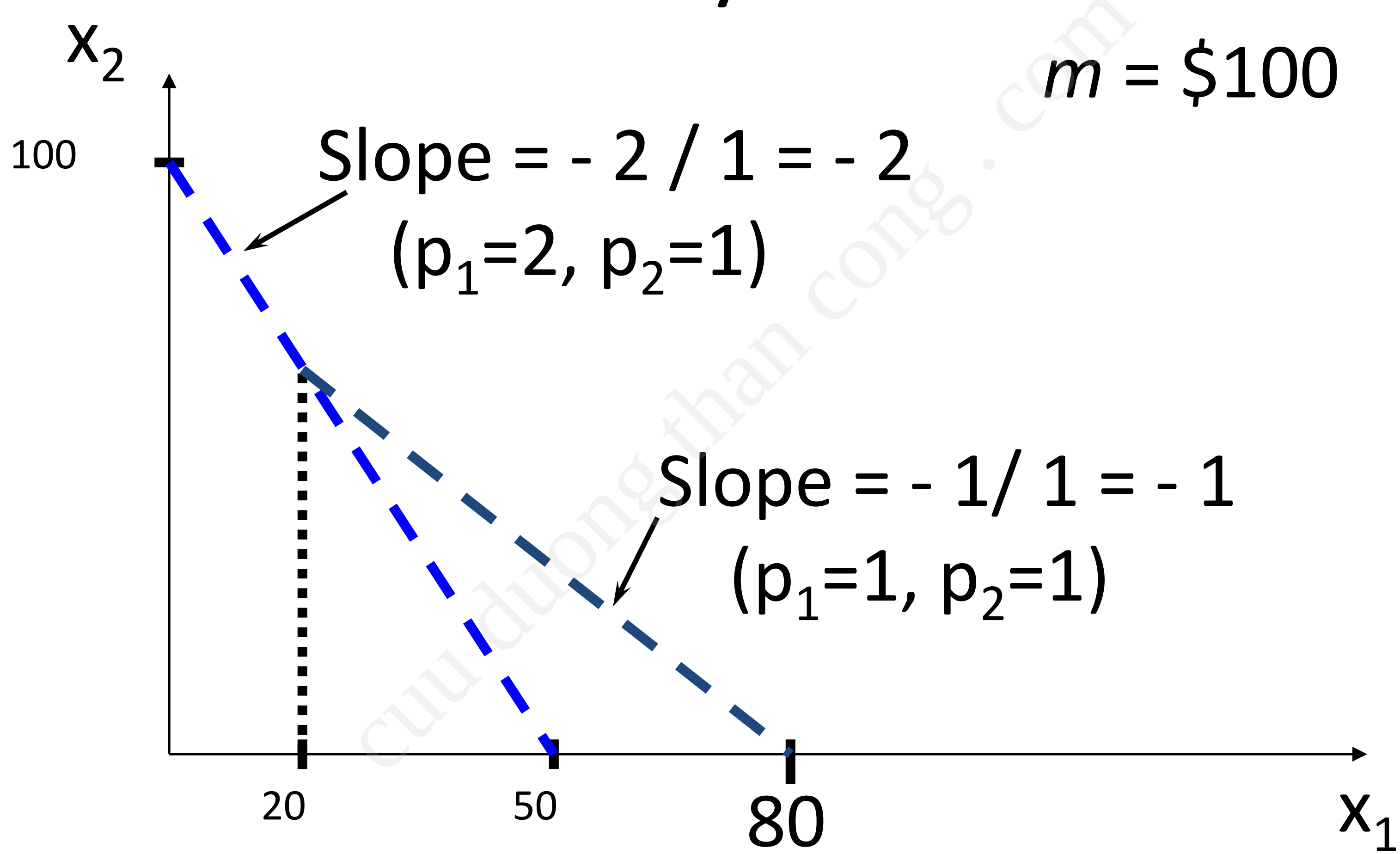
Shapes of Budget Constraints - Quantity Discounts

- Suppose p_2 is constant at \$1 but that p_1=\$2 for $0 \leq x_1 \leq 20$ and p_1=\$1 for x_1>20. Then the constraint's slope is

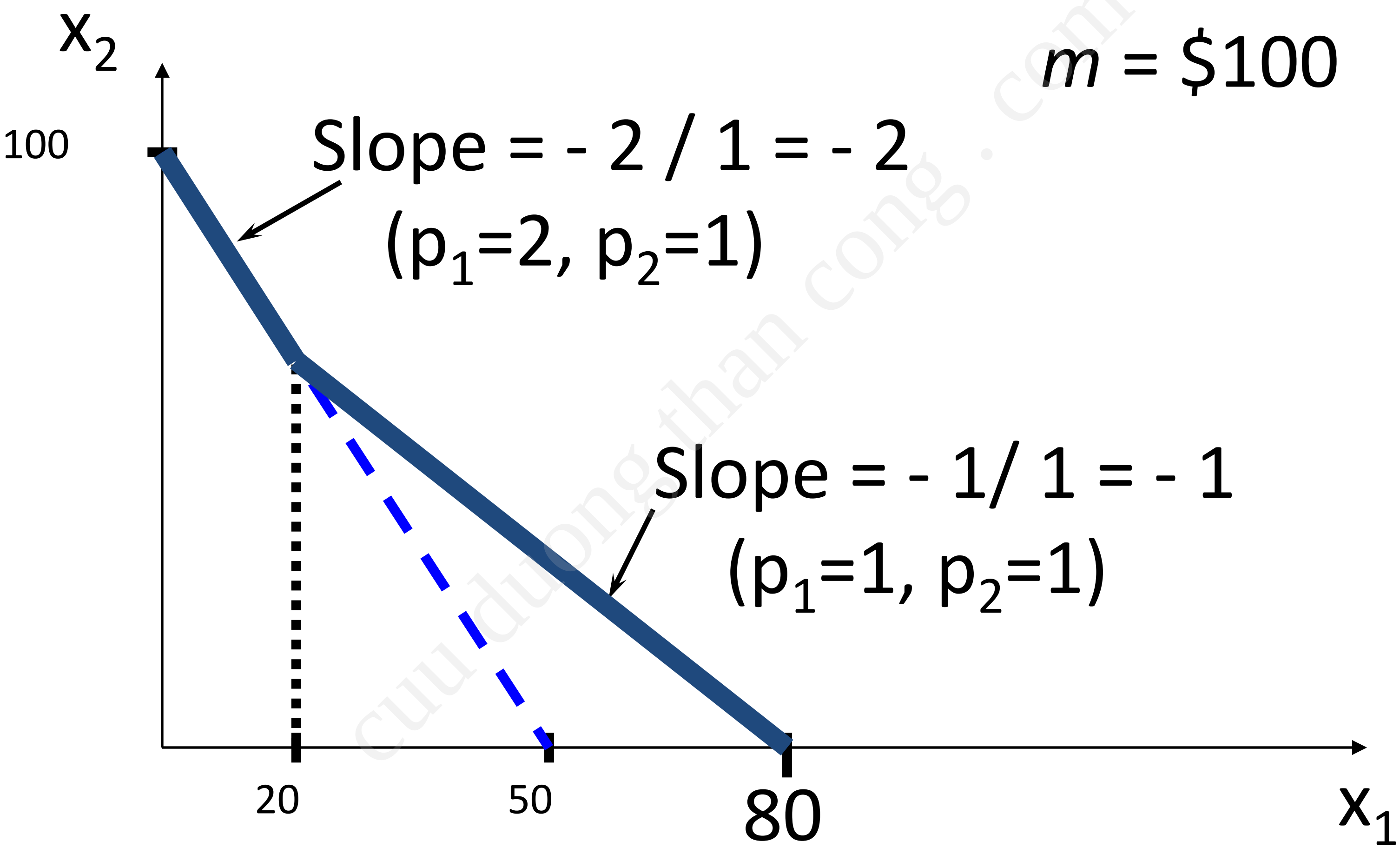
$$-p_1/p_2 = \begin{cases} -2, & \text{for } 0 \leq x_1 \leq 20 \\ -1, & \text{for } x_1 > 20 \end{cases}$$

and the constraint is

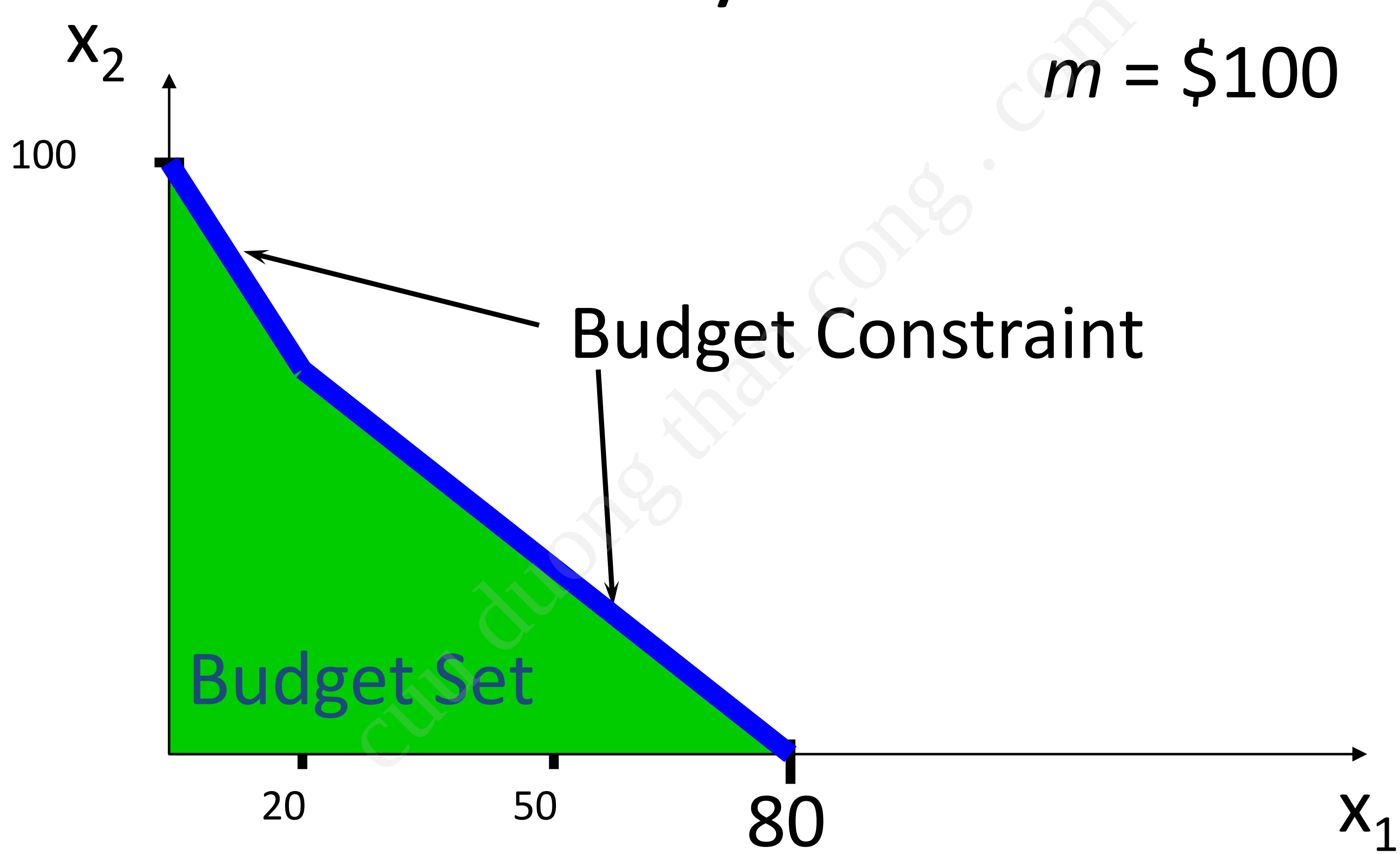
Shapes of Budget Constraints with a Quantity Discount



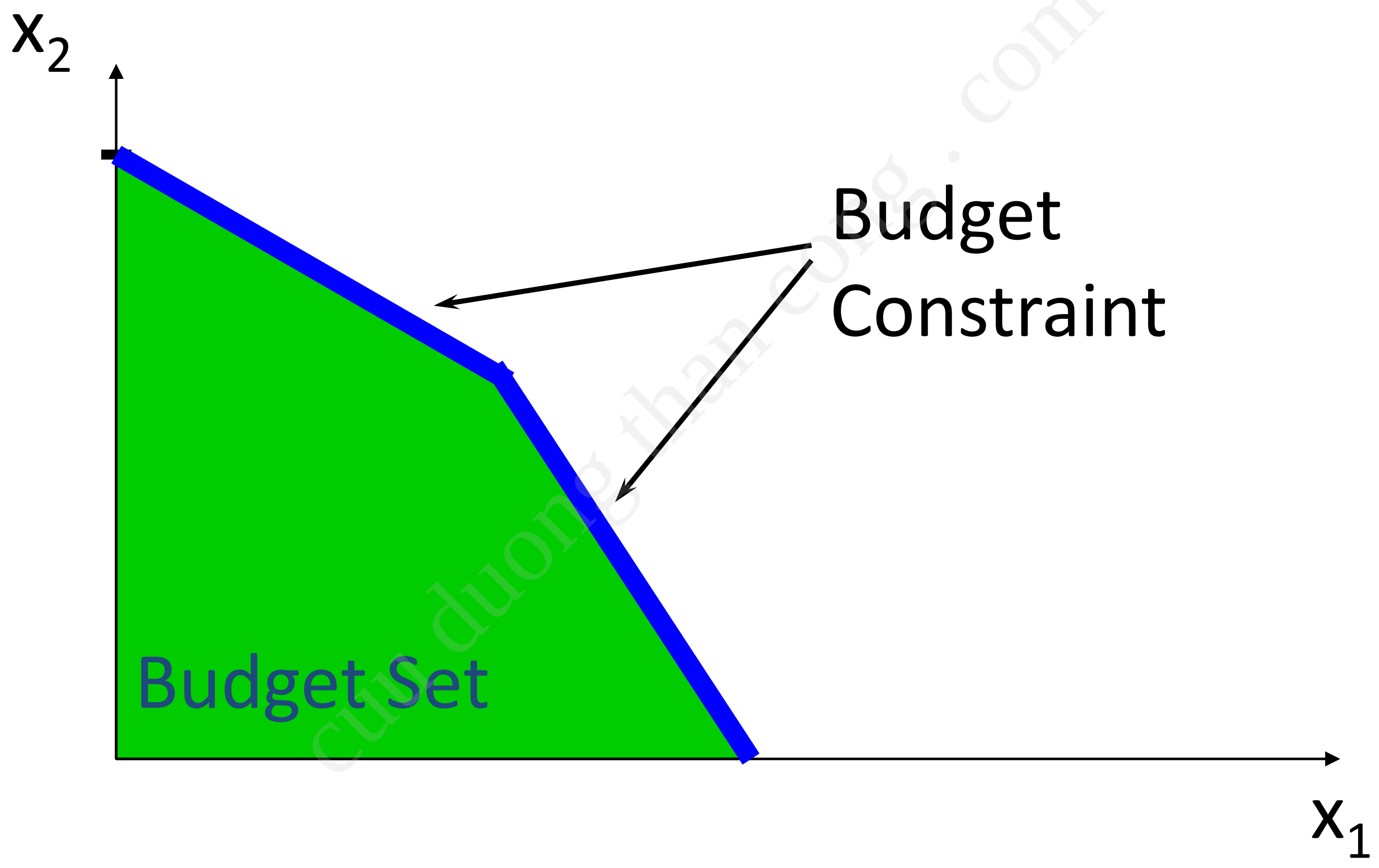
Shapes of Budget Constraints with a Quantity Discount



Shapes of Budget Constraints with a Quantity Discount



Shapes of Budget Constraints with a Quantity Penalty

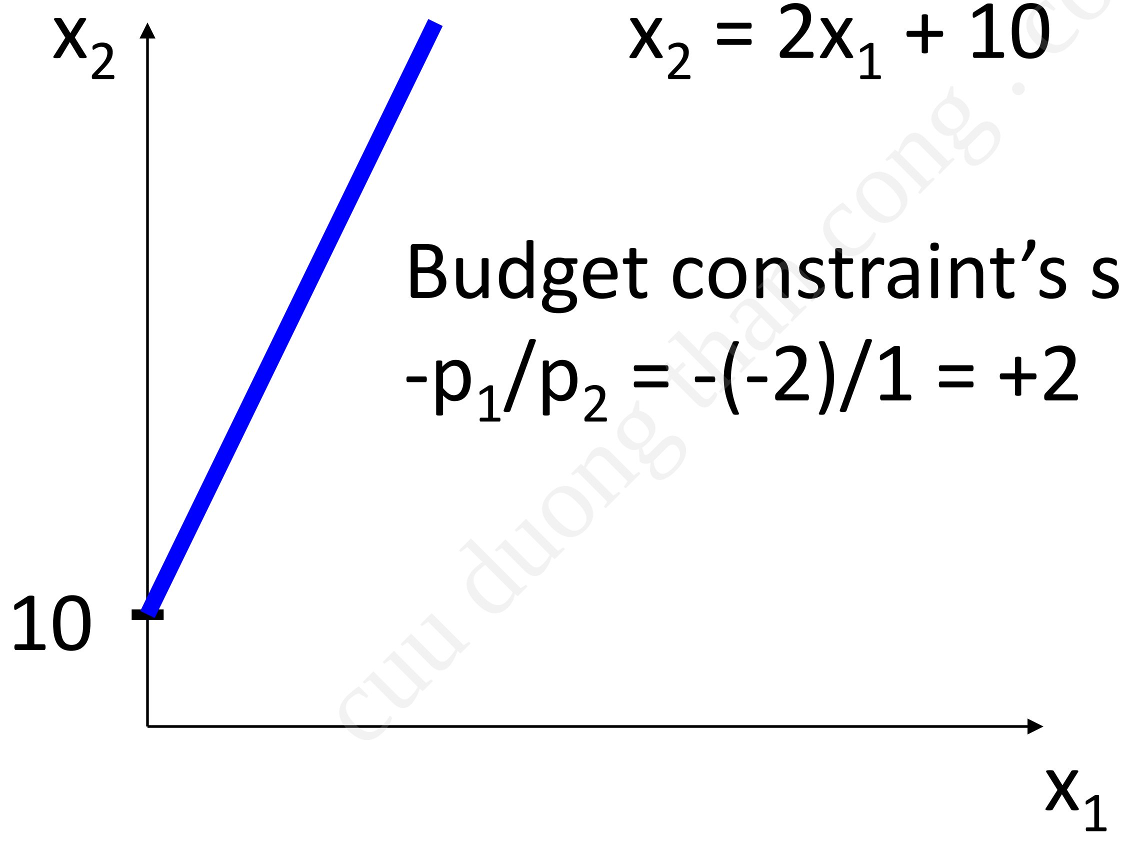


Shapes of Budget Constraints - One Price Negative

- Commodity 1 is stinky garbage. You are paid \$2 per unit to accept it; *i.e.* p_1 = - \$2. p_2 = \$1. Income, other than from accepting commodity 1, is *m* = \$10.
- Then the constraint is

  $- 2x_1 + x_2 = 10$ or $x_2 = 2x_1 + 10$.

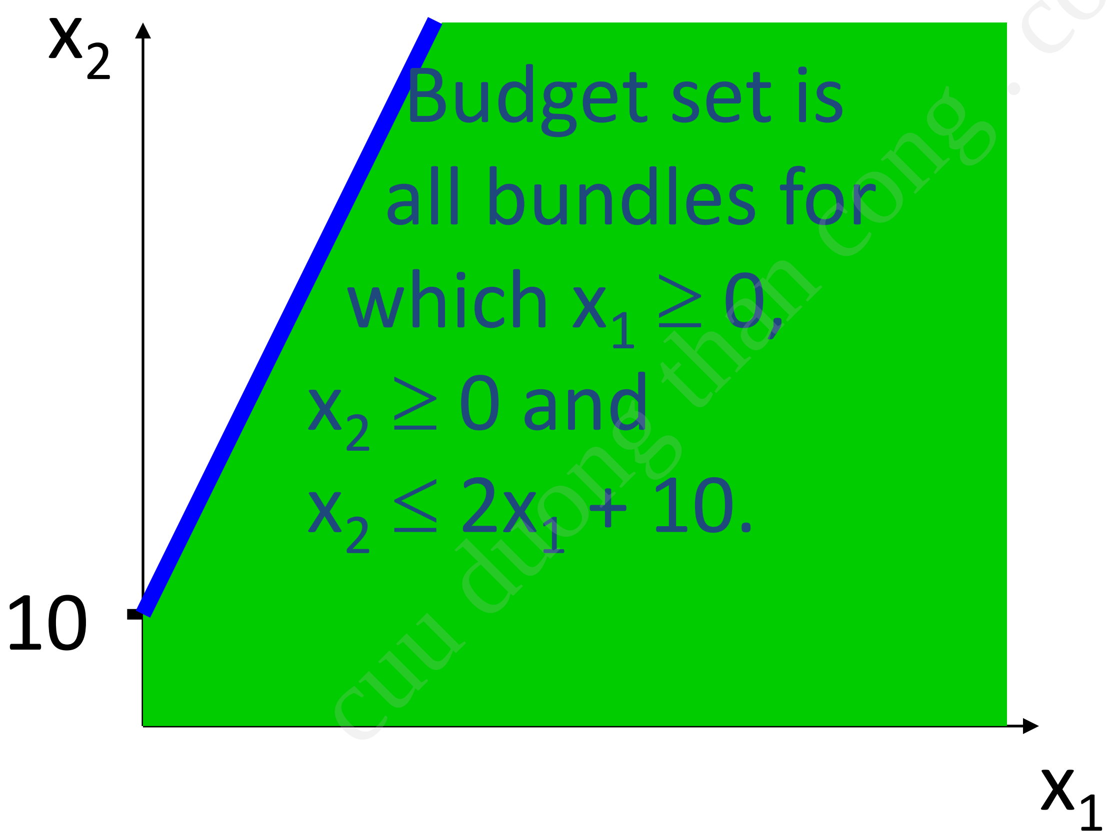
Shapes of Budget Constraints - One Price Negative



$x_2 = 2x_1 + 10$

Budget constraint's slope is $-p_1/p_2 = -(-2)/1 = +2$

Shapes of Budget Constraints - One Price Negative



Numeraire

- "Numeraire" means "unit of account".
- Suppose prices and income are measured in dollars. Say p_1=\$2, p_2=\$3, m = \$12. Then the constraint is

$$2x_1 + 3x_2 = 12.$$

Numeraire

- If prices and income are measured in cents, then p_1=200, p_2=300, m=1200 and the constraint is

$$200x_1 + 300x_2 = 1200,$$

the same as

$$2x_1 + 3x_2 = 12.$$

- Changing the numeraire changes neither the budget constraint nor the budget set.

Relative Prices

- p_1=2, p_2=3 and p_3=6 $\Rightarrow$
- price of commodity 2 relative to commodity 1 is 3/2,
- price of commodity 3 relative to commodity 1 is 3.
- Relative prices are the rates of exchange of commodities 2 and 3 for units of commodity 1.

PREFERENCES

Preference Relations

- Comparing two different consumption bundles, x and y:
  - strict preference: x is more preferred than is y.
  - weak preference: x is as at least as preferred as is y.
  - indifference: x is exactly as preferred as is y.

Preference Relations

- $\succ$ denotes strict preference so $x \succ y$ means that bundle x is preferred strictly to bundle y.
- $\sim$ denotes indifference; $x \sim y$ means x and y are equally preferred.
- $\succsim$ denotes weak preference; $x \succsim y$ means x is preferred at least as much as is y.

Assumptions about Preference Relations

- Completeness: For any two bundles x and y it is always possible to make the statement that either

$$x \succsim y$$

or

$$y \succsim x.$$

Assumptions about Preference Relations

- Reflexivity: Any bundle x is always at least as preferred as itself; *i.e.*

$$x \succsim x.$$

Assumptions about Preference Relations

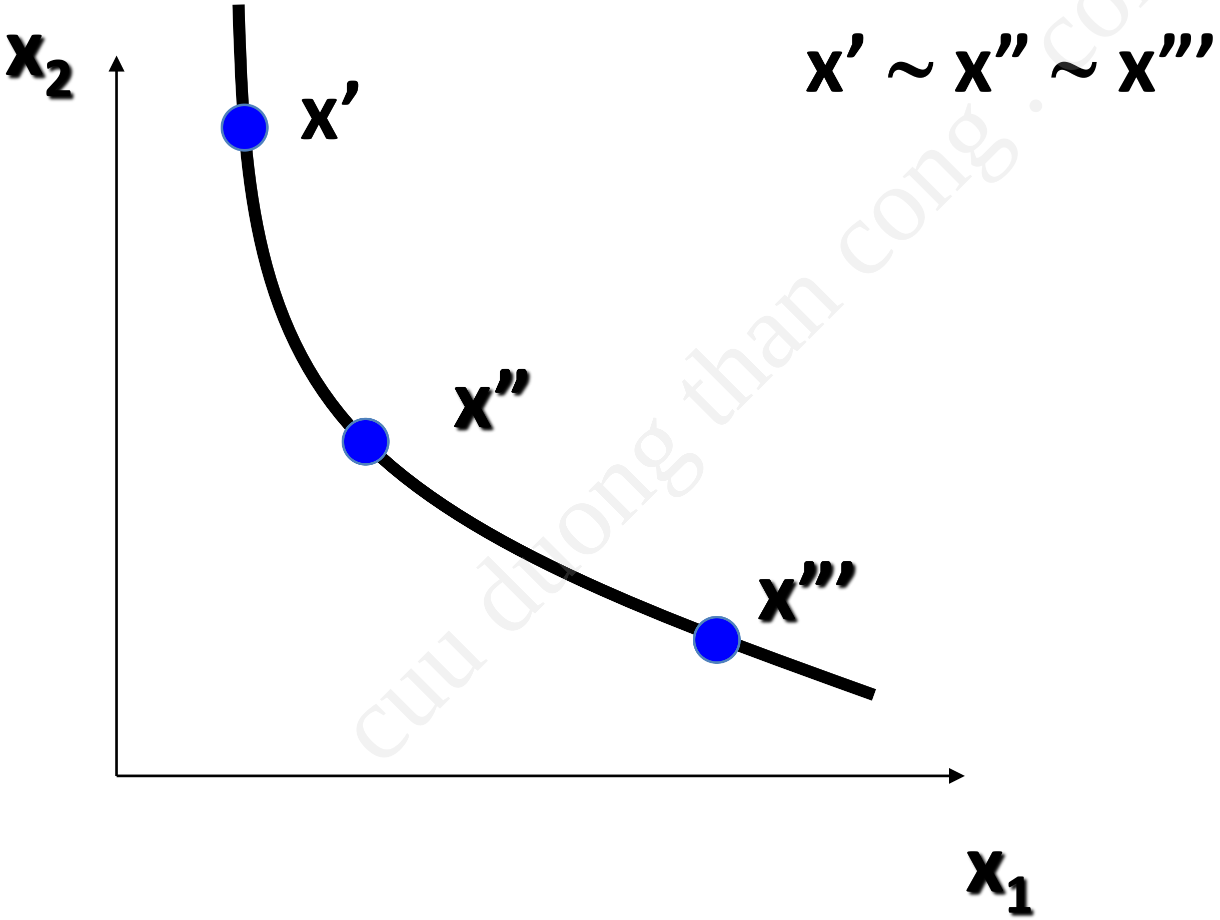
- Transitivity: If
  x is at least as preferred as y, and
  y is at least as preferred as z, then
  x is at least as preferred as z; *i.e.*

$$x \succsim y \text{ and } y \succsim z \Rightarrow x \succsim z.$$

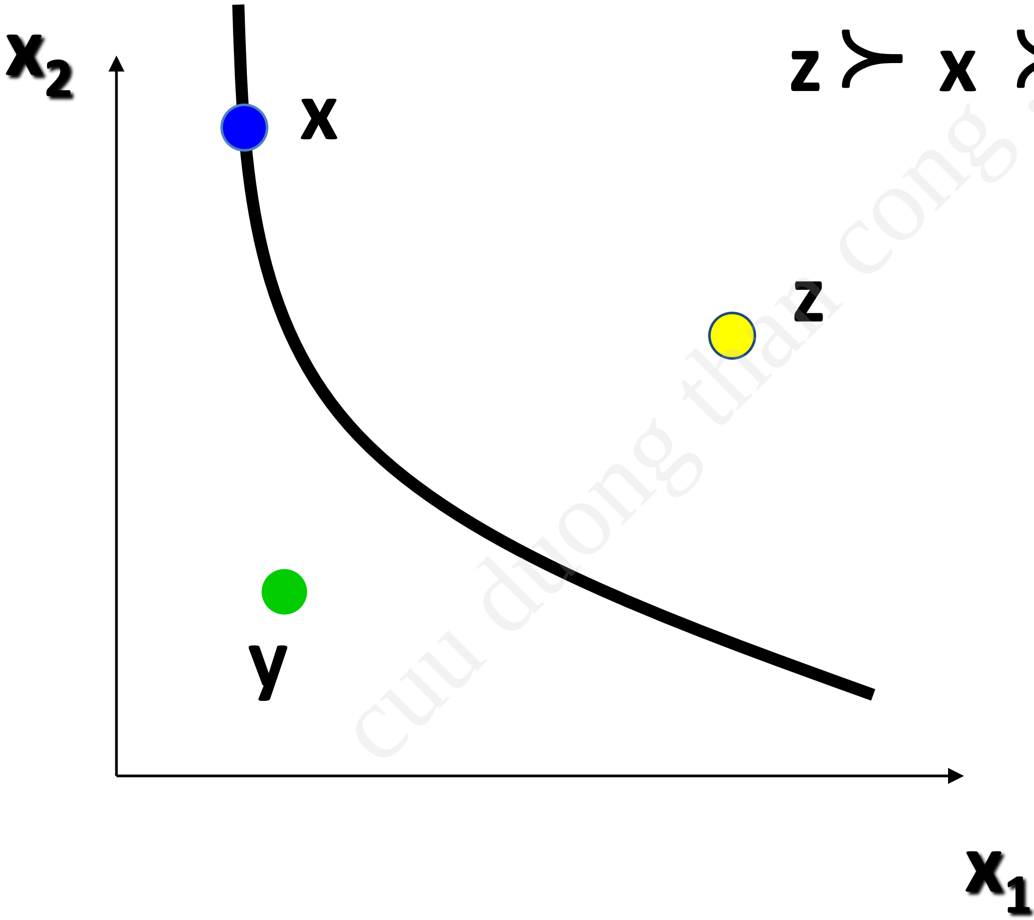
Indifference Curves

- Take a reference bundle x'. The set of all bundles equally preferred to x' is the indifference curve containing x'; the set of all bundles y $\sim$ x'.
- Since an indifference "curve" is not always a curve a better name might be an indifference "set".

Indifference Curves



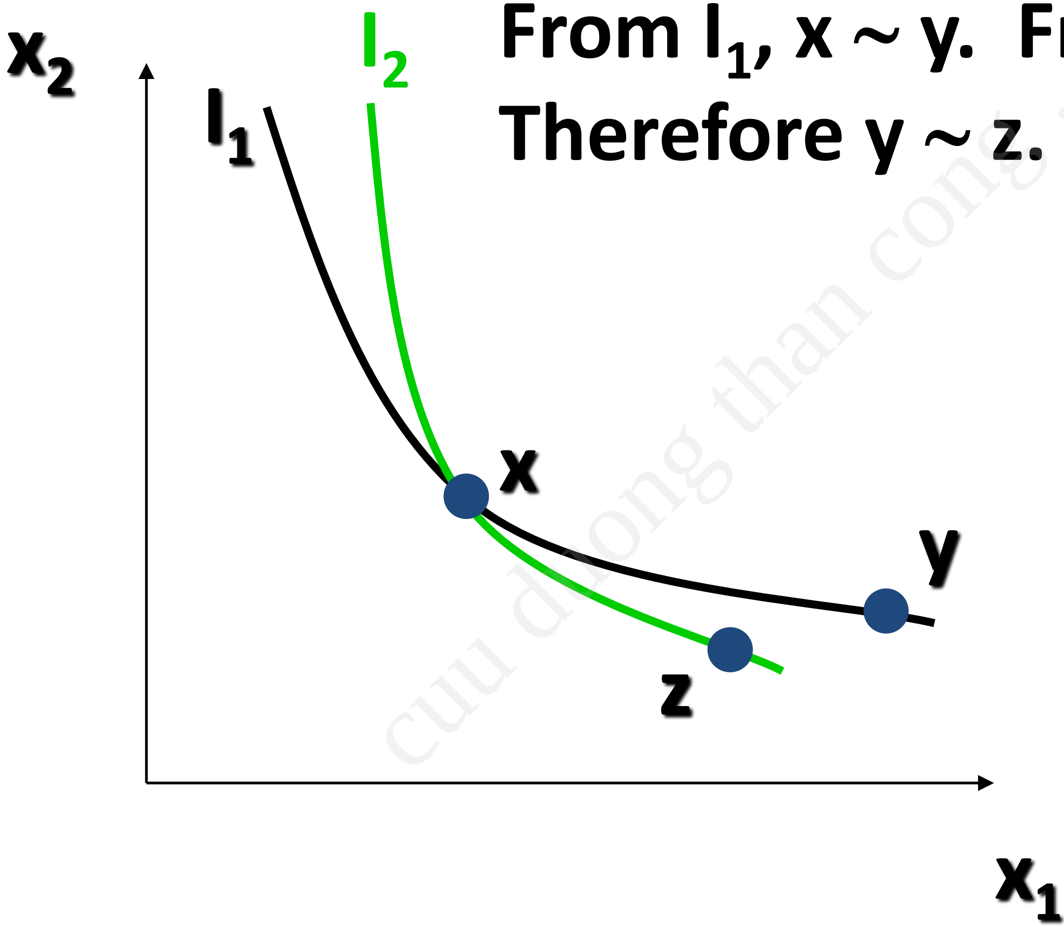
Indifference Curves



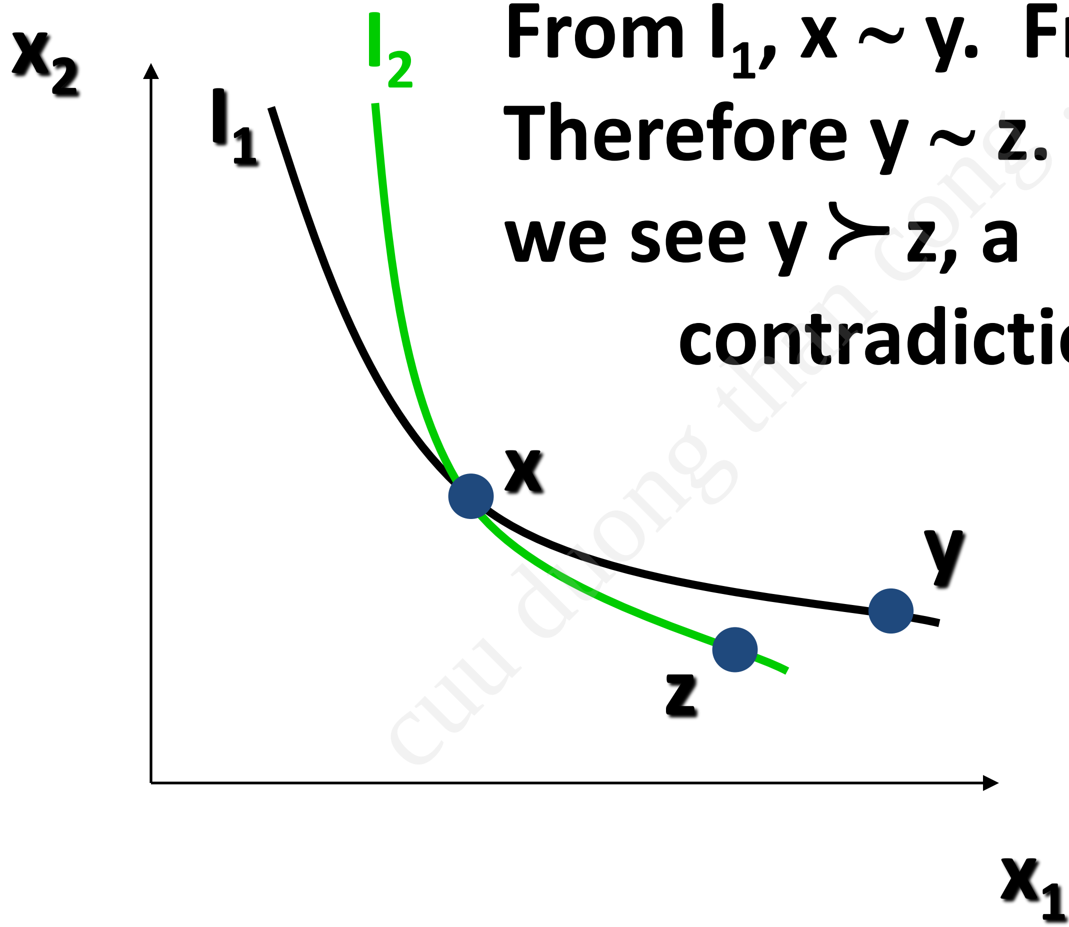
$z \succ x \succ y$

Indifference Curves Cannot Intersect

From I_1, $x \sim y$. From I_2, $x \sim z$.
Therefore $y \sim z$.



Indifference Curves Cannot Intersect

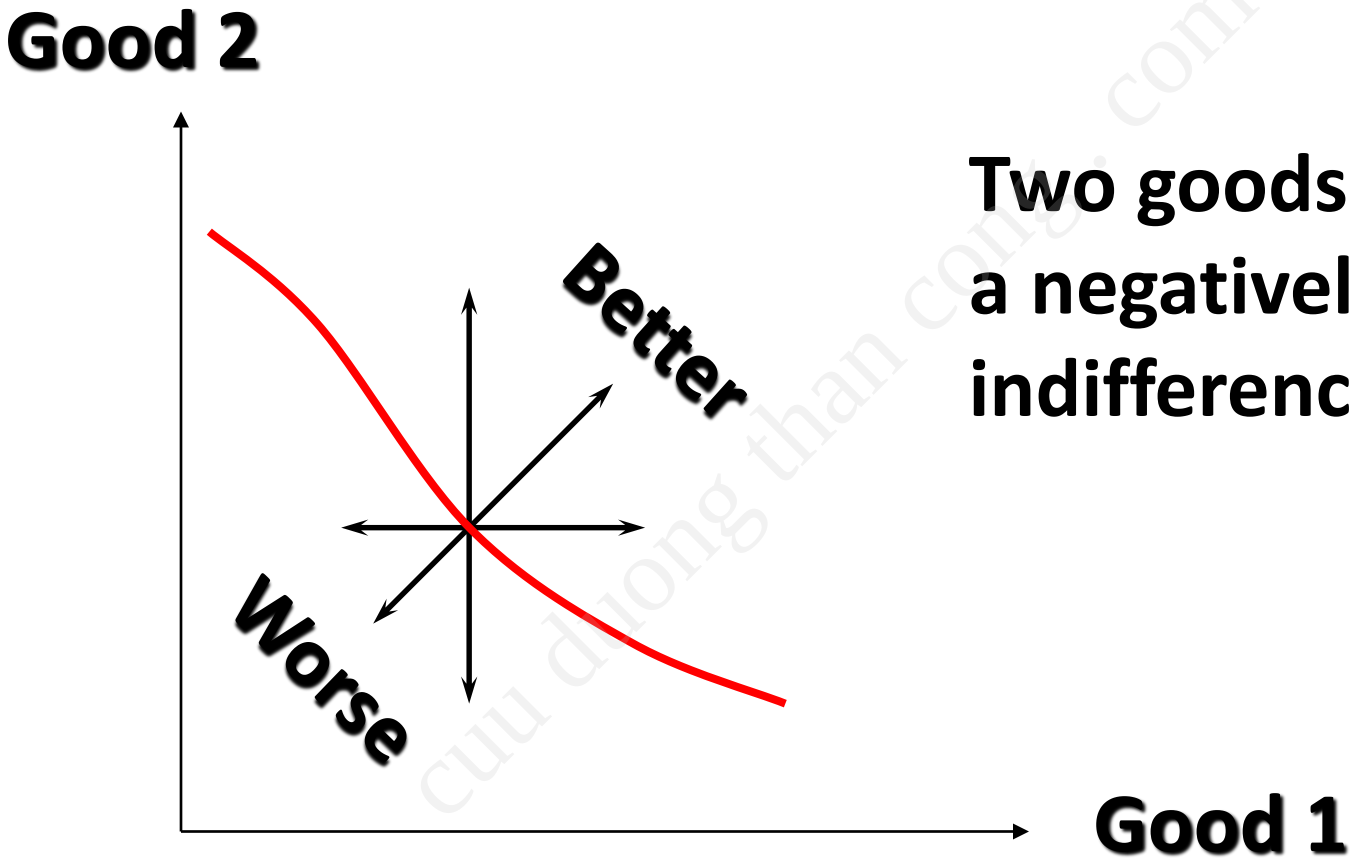


From I_1, $x \sim y$. From I_2, $x \sim z$. Therefore $y \sim z$. But from I_1 and I_2 we see $y \succ z$, a contradiction.

Slopes of Indifference Curves

- When more of a commodity is always preferred, the commodity is a good.
- If every commodity is a good then indifference curves are negatively sloped.

Slopes of Indifference Curves

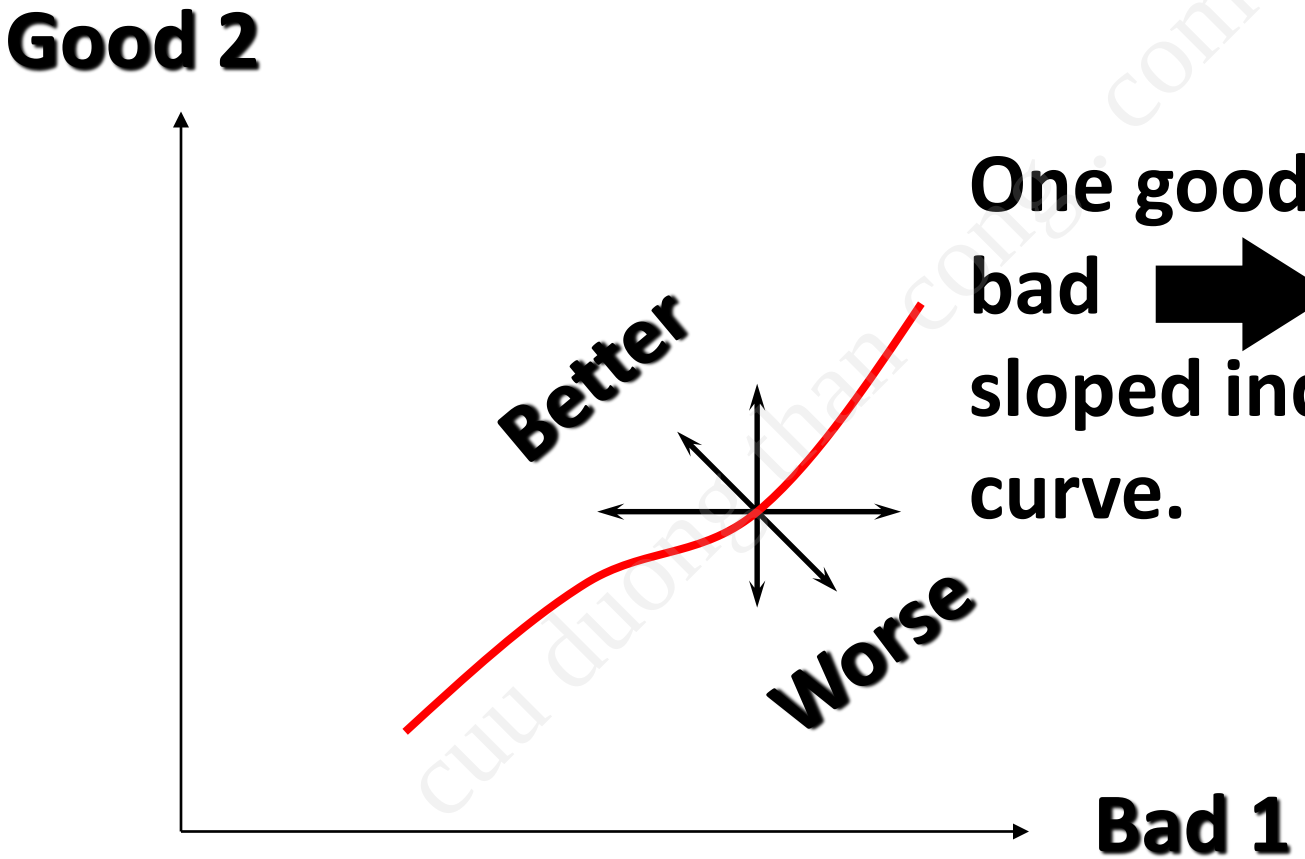


Two goods ➡ a negatively sloped indifference curve.

Slopes of Indifference Curves

- If less of a commodity is always preferred then the commodity is a bad.

Slopes of Indifference Curves

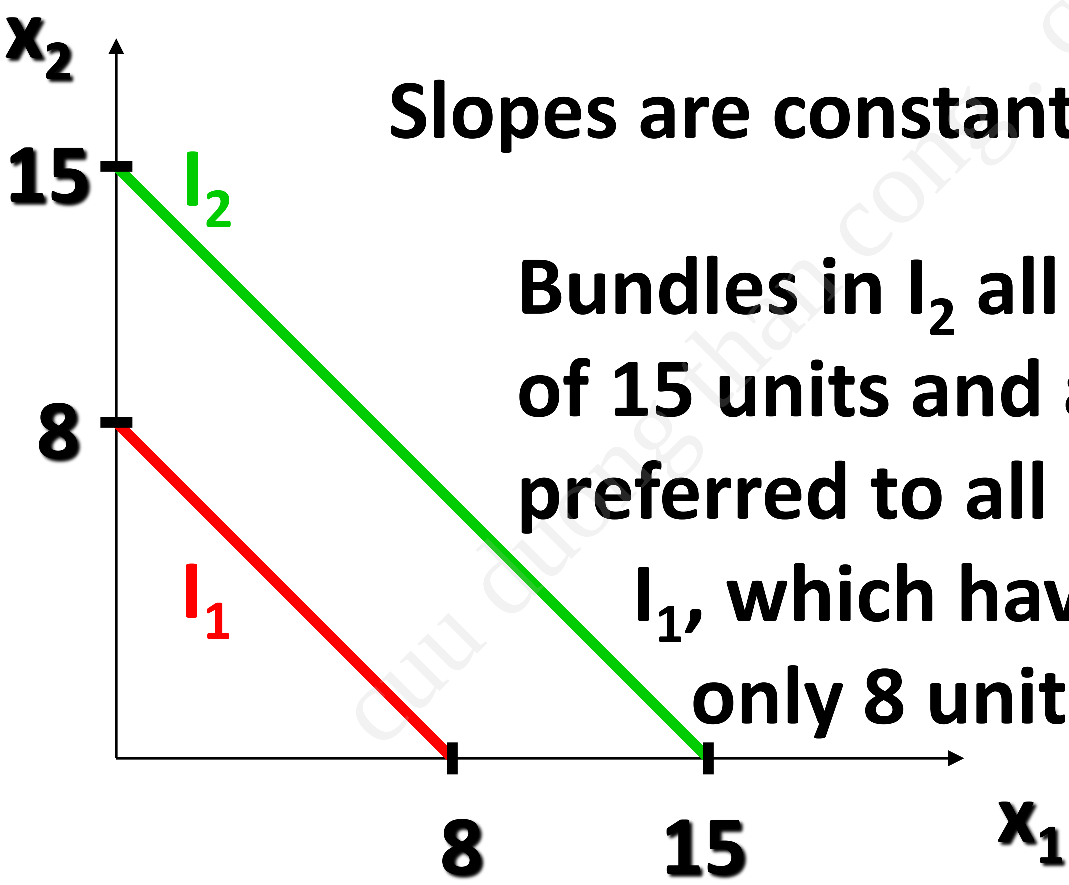


One good and one bad ➡ a positively sloped indifference curve.

Extreme Cases of Indifference Curves; Perfect Substitutes

- If a consumer always regards units of commodities 1 and 2 as equivalent, then the commodities are perfect substitutes and only the total amount of the two commodities in bundles determines their preference rank-order.

Extreme Cases of Indifference Curves; Perfect Substitutes



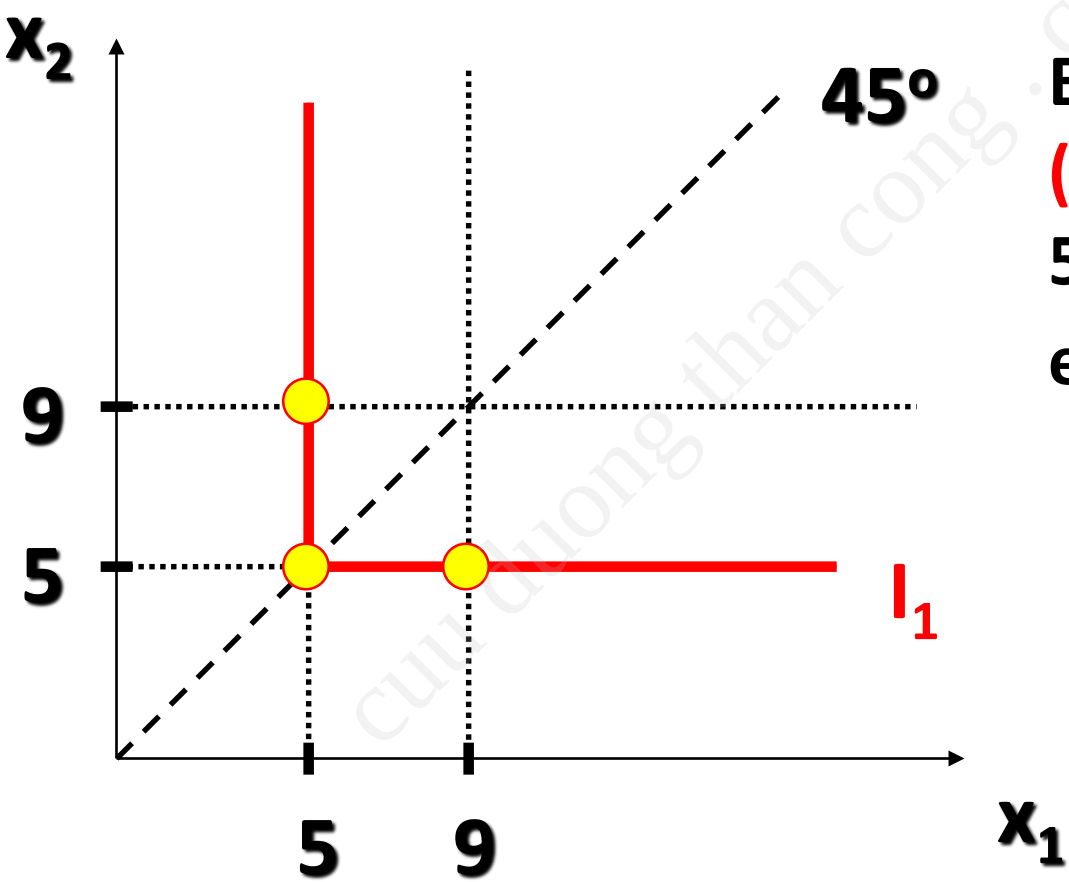
Slopes are constant at - 1.

Bundles in I_2 all have a total of 15 units and are strictly preferred to all bundles in I_1, which have a total of only 8 units in them.

Extreme Cases of Indifference Curves; Perfect Complements

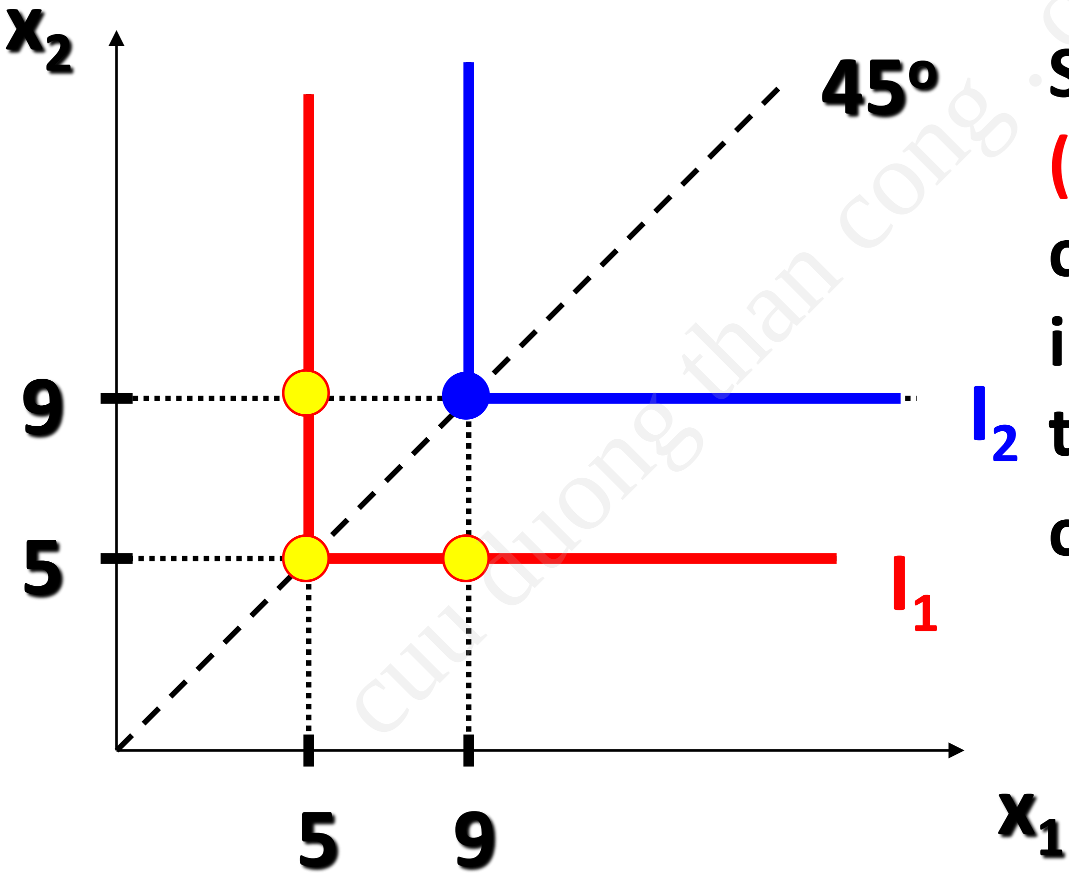
- If a consumer always consumes commodities 1 and 2 in fixed proportion (e.g. one-to-one), then the commodities are perfect complements and only the number of pairs of units of the two commodities determines the preference rank-order of bundles.

Extreme Cases of Indifference Curves; Perfect Complements



Each of (5,5), (5,9) and (9,5) contains 5 pairs so each is equally preferred.

Extreme Cases of Indifference Curves; Perfect Complements

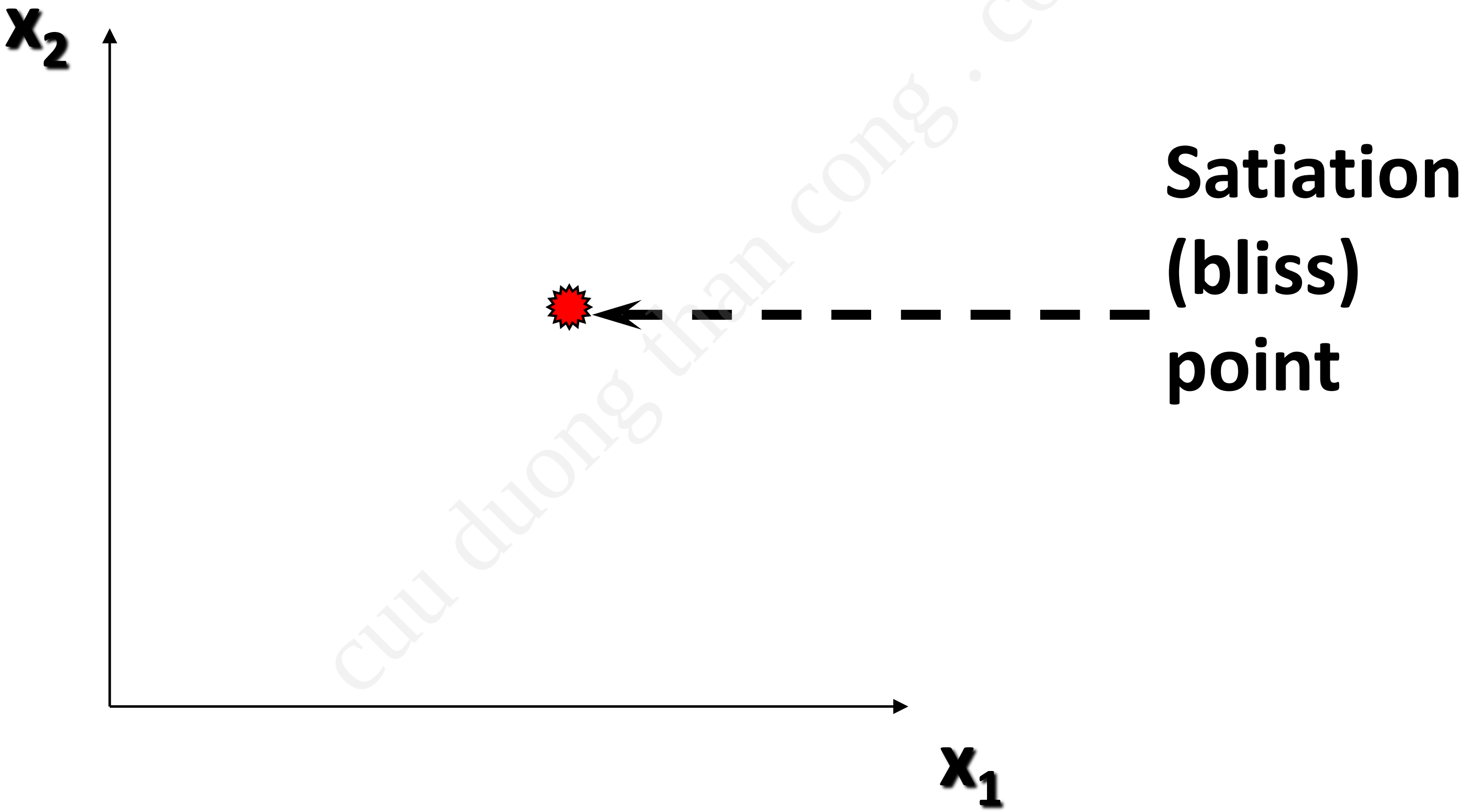


Since each of (5,5), (5,9) and (9,5) contains 5 pairs, each is less preferred than the bundle (9,9) which contains 9 pairs.

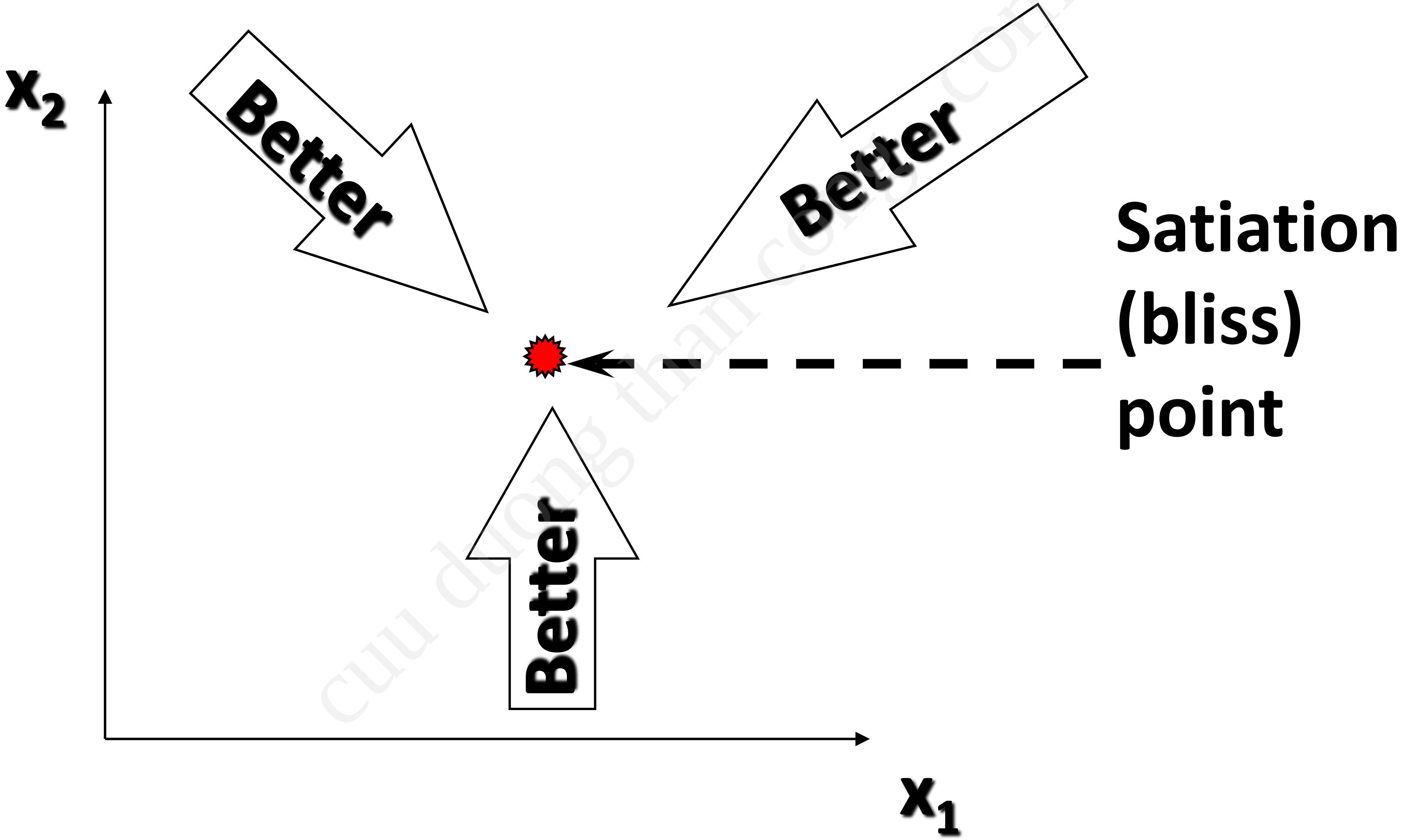
Preferences Exhibiting Satiation

- A bundle strictly preferred to any other is a satiation point or a bliss point.
- What do indifference curves look like for preferences exhibiting satiation?

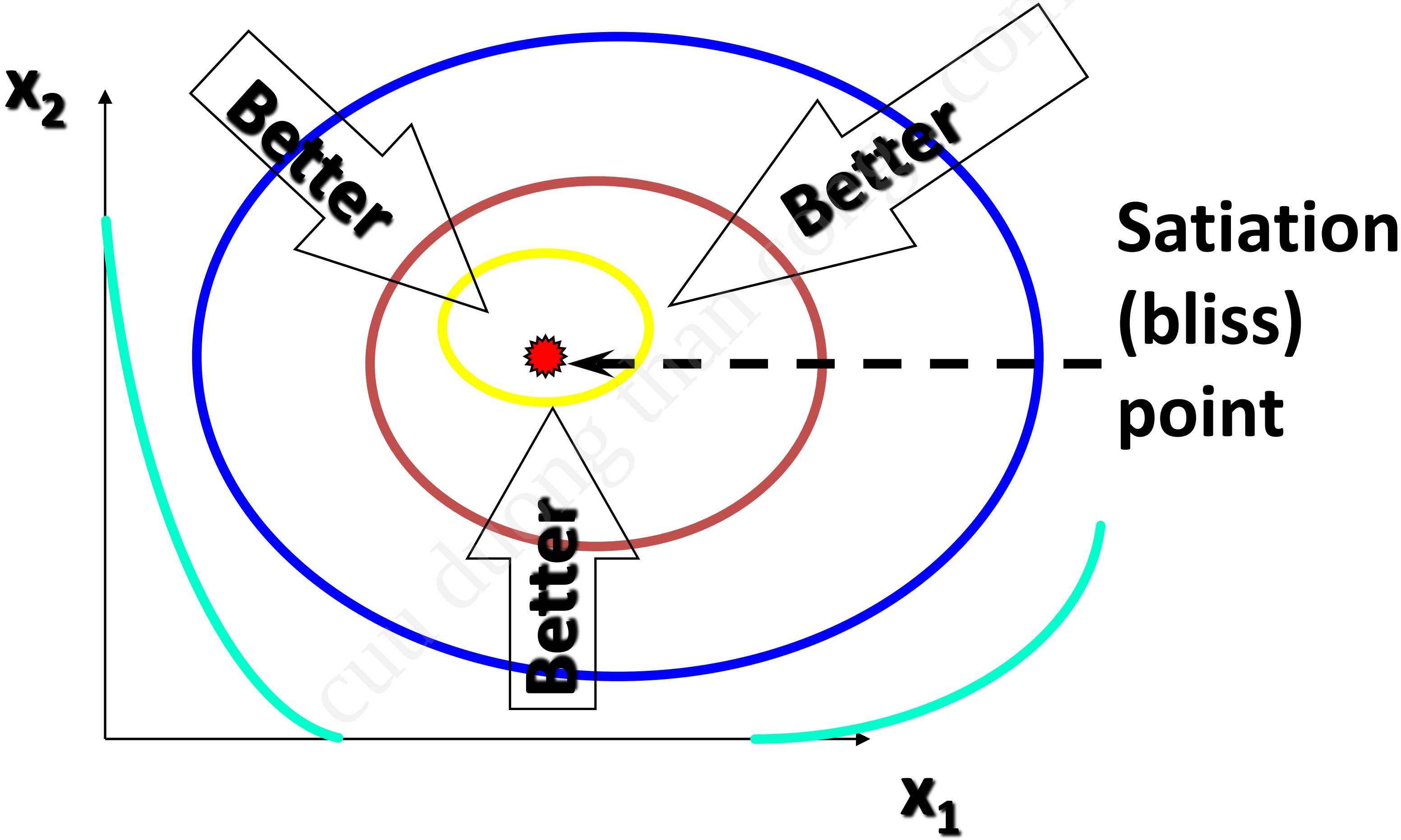
Indifference Curves Exhibiting Satiation



Indifference Curves Exhibiting Satiation



Indifference Curves Exhibiting Satiation



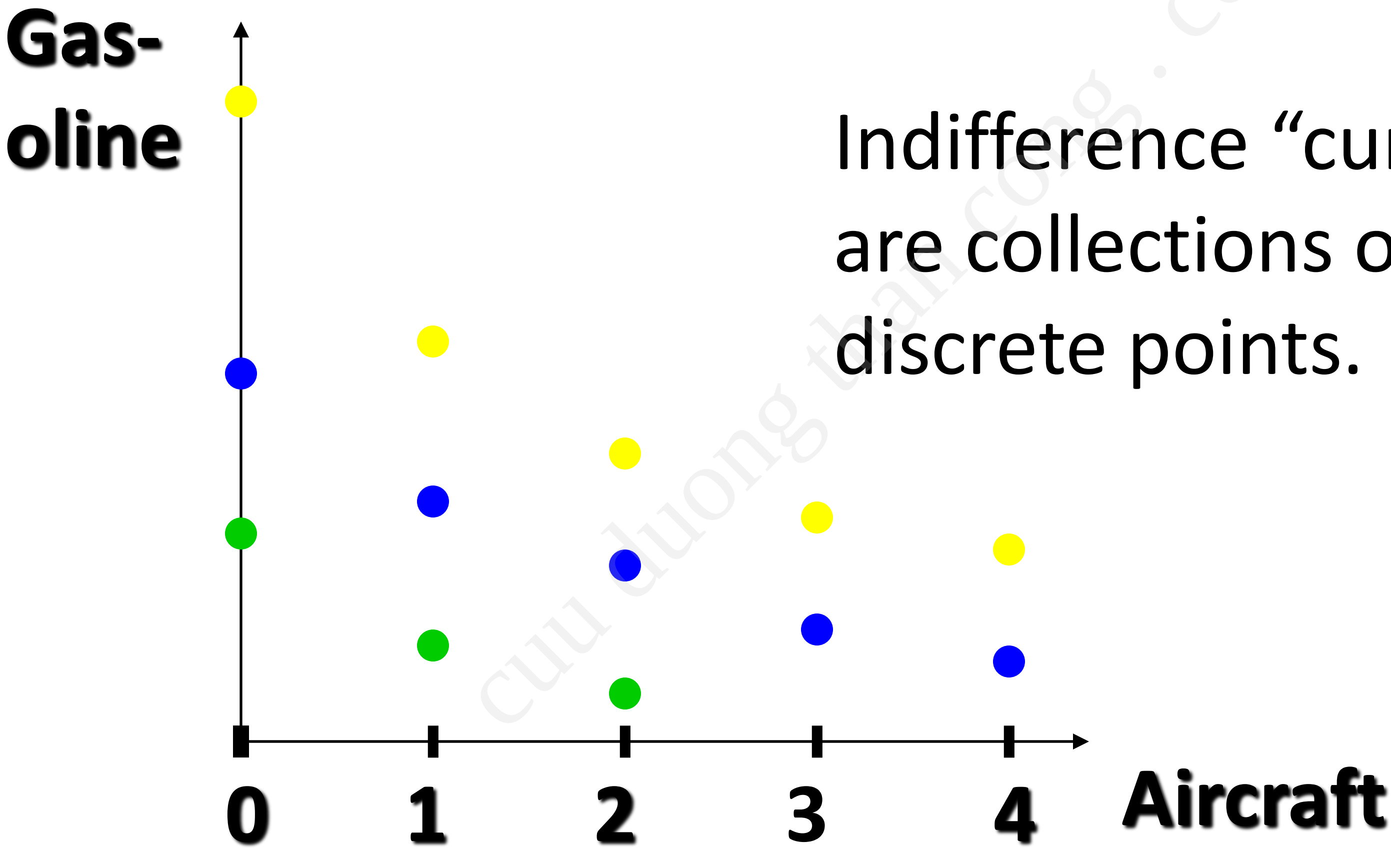
Indifference Curves for Discrete Commodities

- A commodity is infinitely divisible if it can be acquired in any quantity; *e.g.* water or cheese.
- A commodity is discrete if it comes in unit lumps of 1, 2, 3, … and so on; *e.g.* aircraft, ships and refrigerators.

Indifference Curves for Discrete Commodities

- Suppose commodity 2 is an infinitely divisible good (gasoline) while commodity 1 is a discrete good (aircraft). What do indifference "curves" look like?

Indifference Curves With a Discrete Good



Indifference "curves" are collections of discrete points.

Well-Behaved Preferences

- A preference relation is "well-behaved" if it is
  - monotonic and convex.
- Monotonicity: More of any commodity is always preferred (*i.e.* no satiation and every commodity is a good).

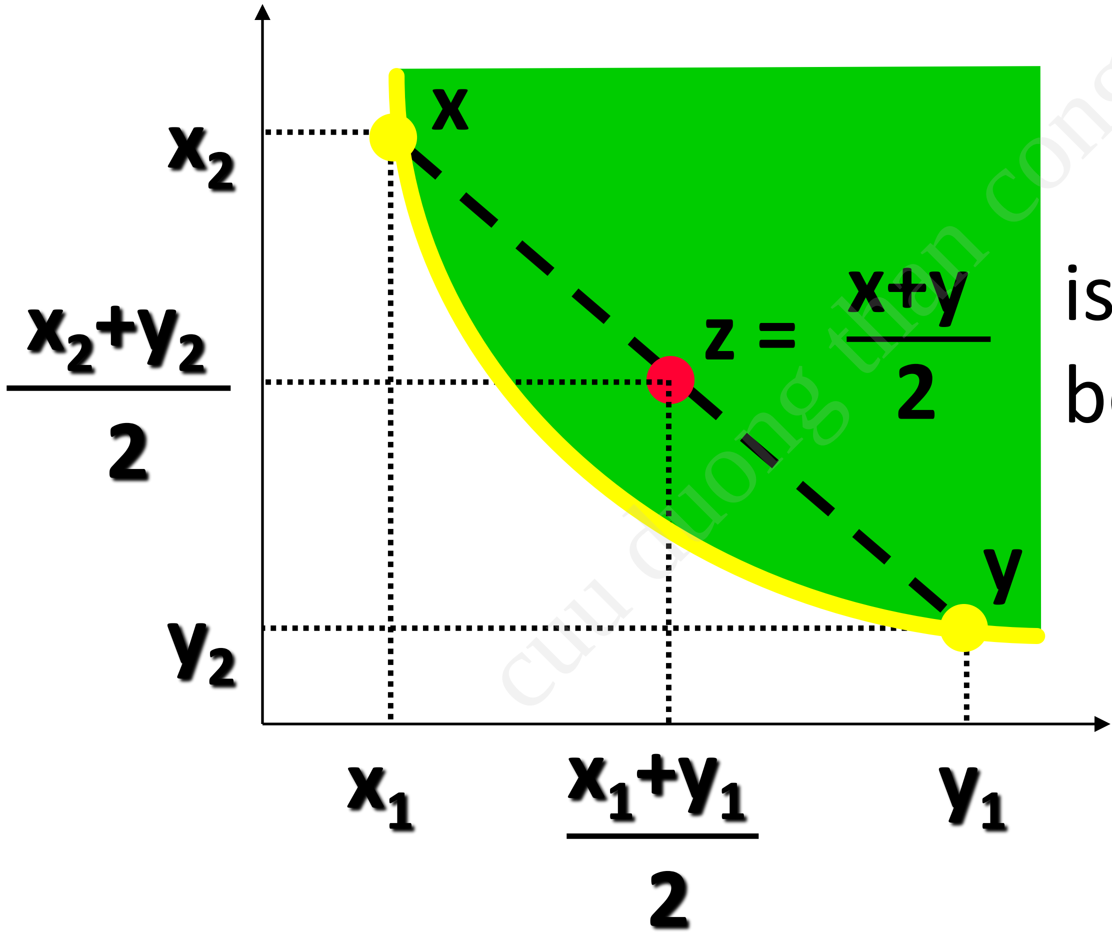
Well-Behaved Preferences

- Convexity: Mixtures of bundles are (at least weakly) preferred to the bundles themselves. E.g., the 50-50 mixture of the bundles x and y is

$$z = (0.5)x + (0.5)y.$$

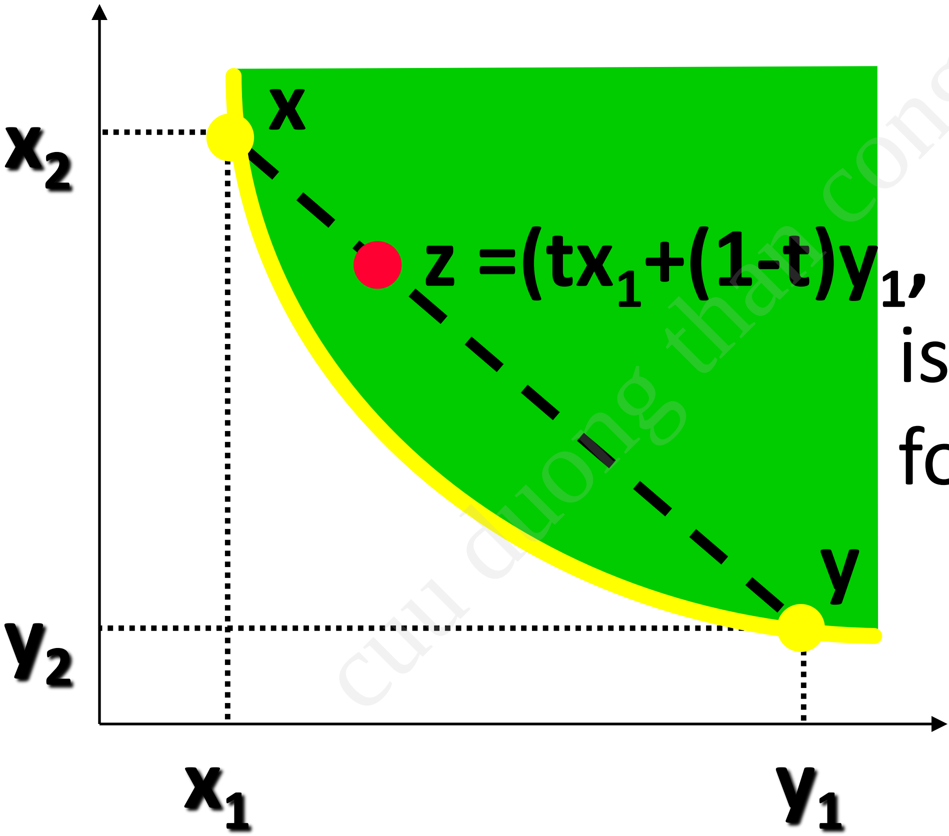
z is at least as preferred as x or y.

Well-Behaved Preferences -- Convexity.



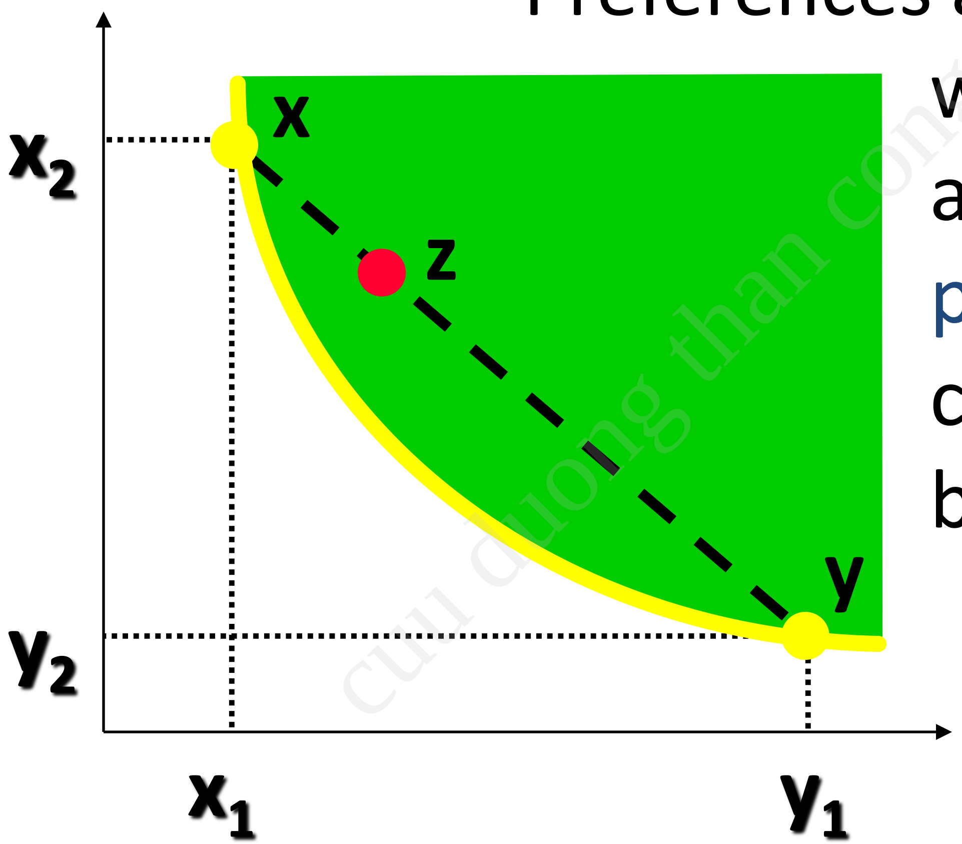
is strictly preferred to both x and y.

Well-Behaved Preferences -- Convexity.



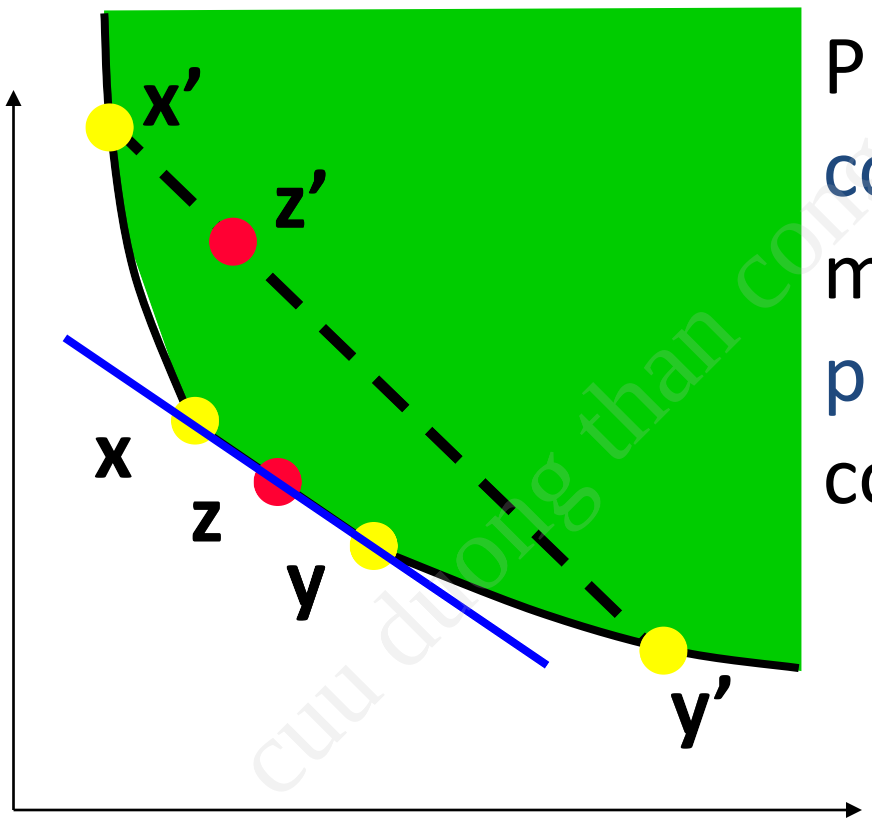
$z = (tx_1+(1-t)y_1, tx_2+(1-t)y_2)$ is preferred to x and y for all $0 < t < 1$.

Well-Behaved Preferences -- Convexity.



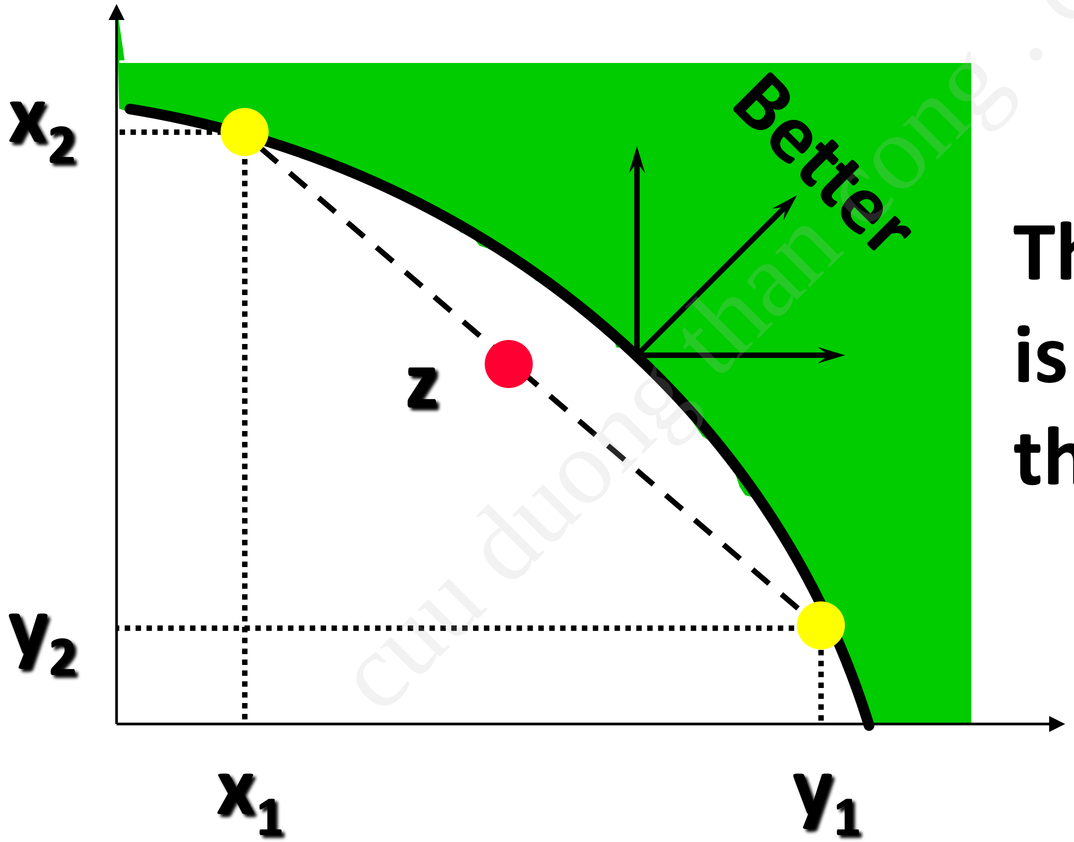
Preferences are strictly convex when all mixtures z are strictly preferred to their component bundles x and y.

Well-Behaved Preferences -- Weak Convexity.



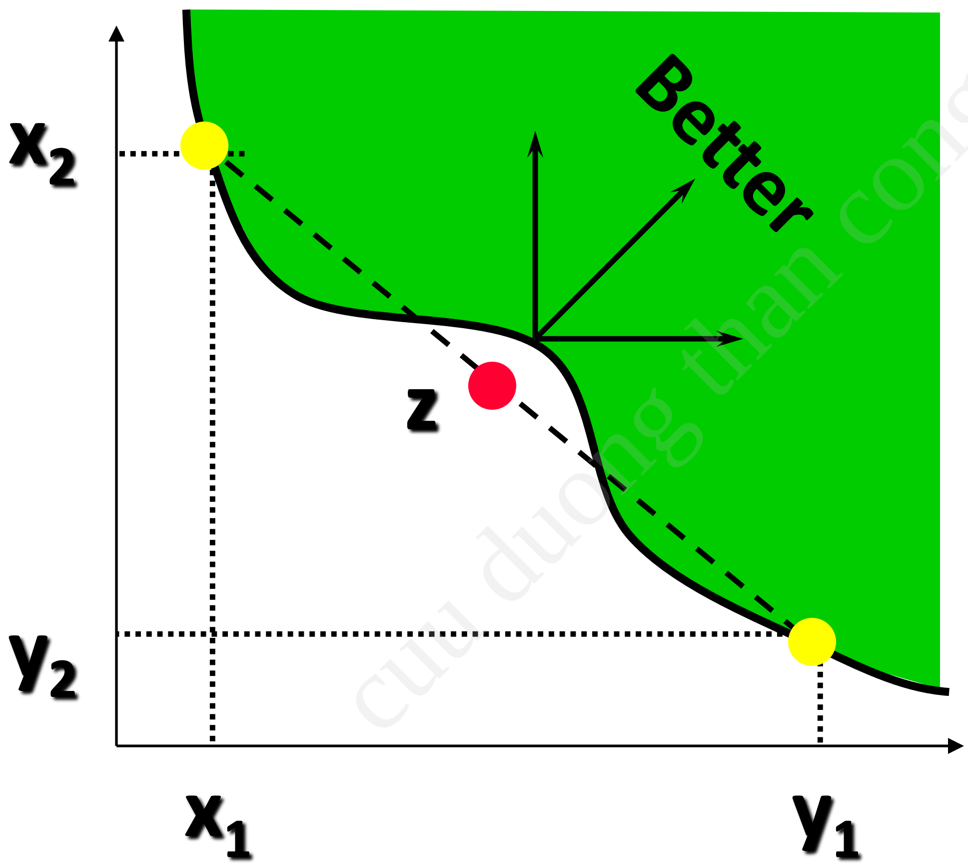
Preferences are weakly convex if at least one mixture z is equally preferred to a component bundle.

Non-Convex Preferences



The mixture z is less preferred than x or y.

More Non-Convex Preferences

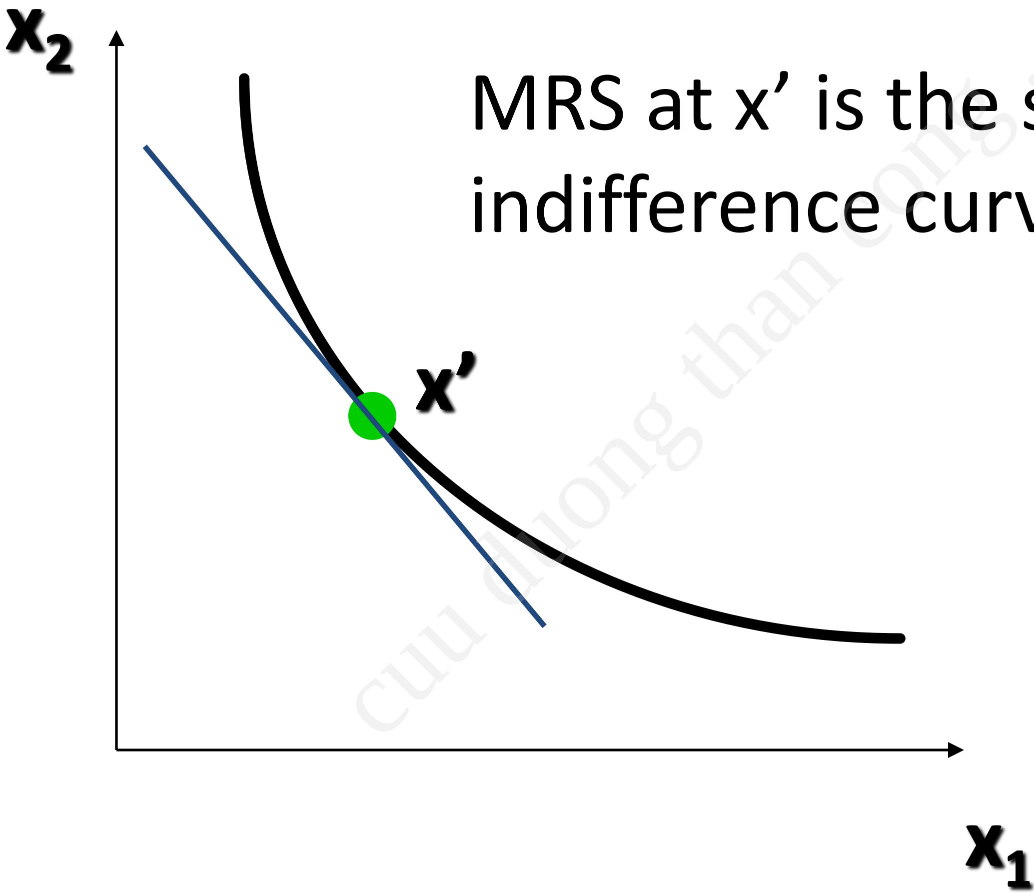


The mixture z is less preferred than x or y.

Slopes of Indifference Curves

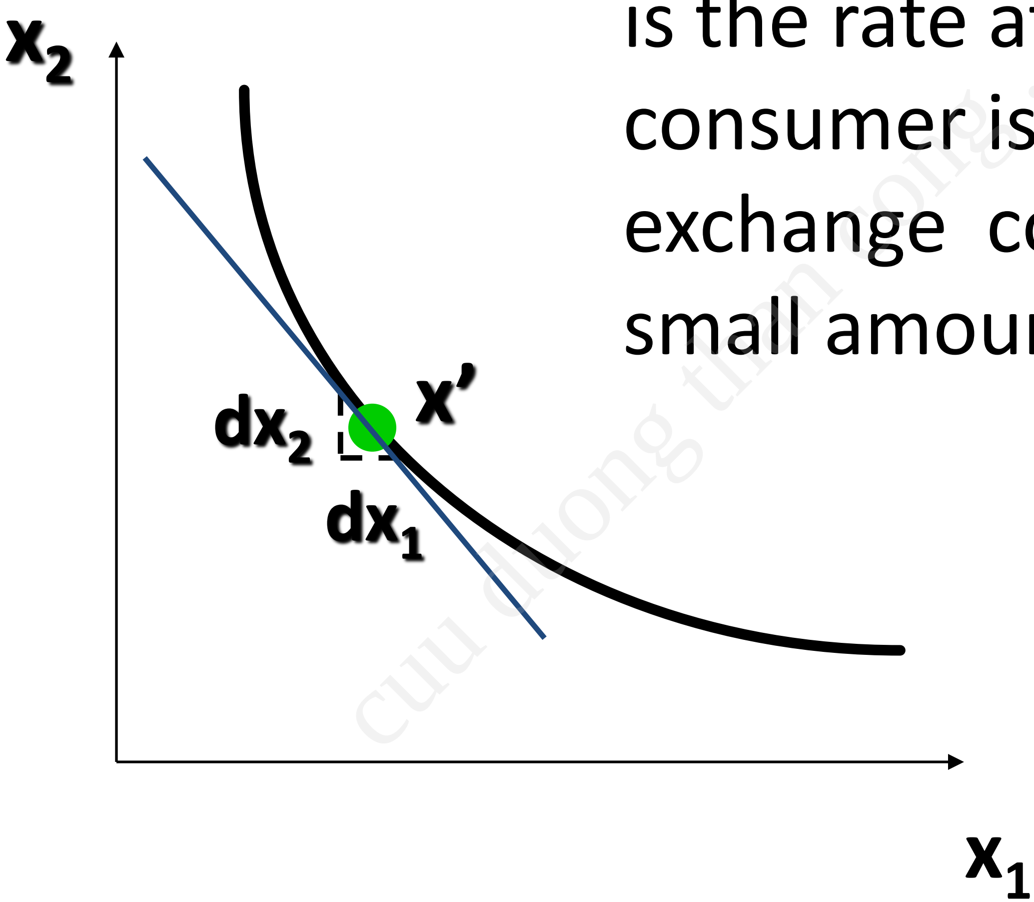
- The slope of an indifference curve is its marginal rate-of-substitution (MRS).
- How can a MRS be calculated?

Marginal Rate of Substitution



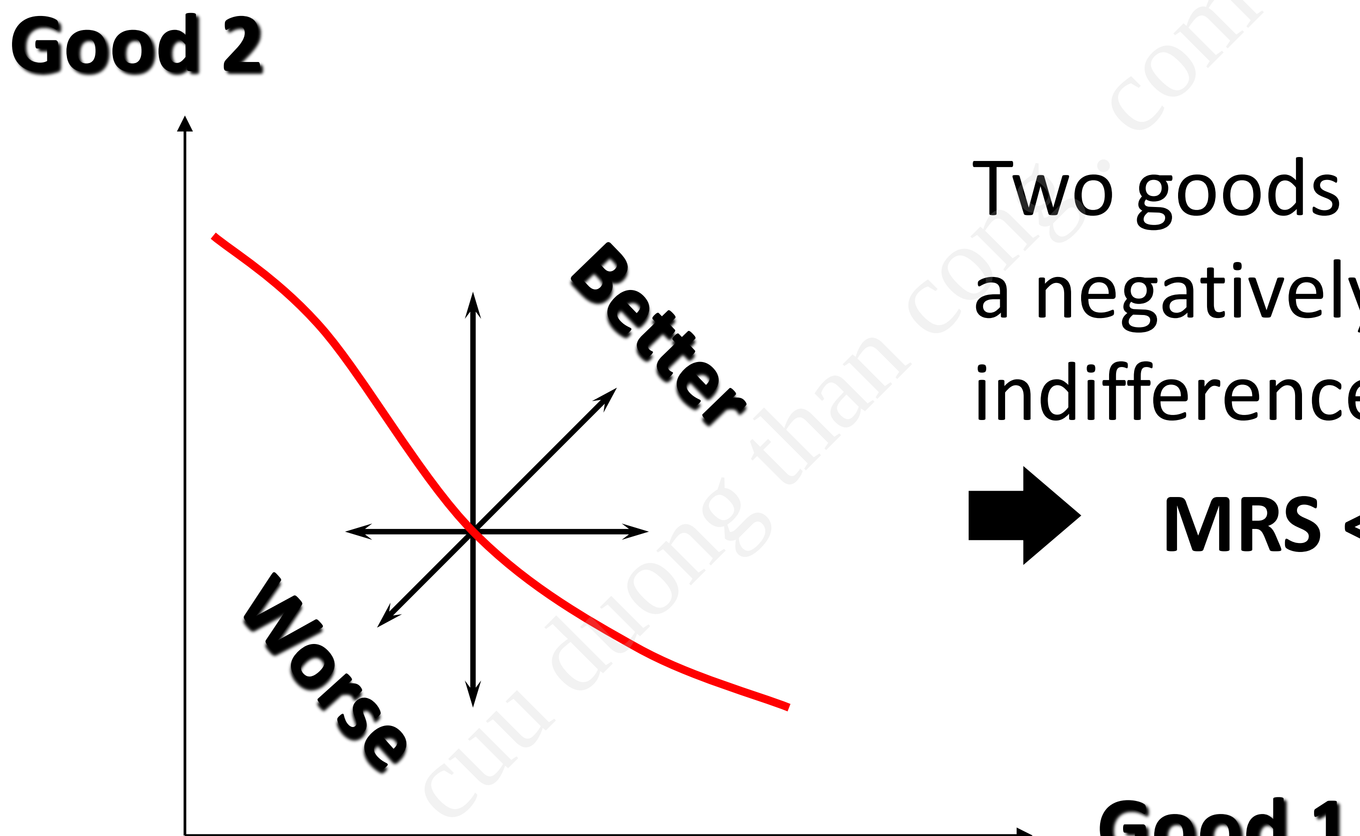
MRS at x’ is the slope of the indifference curve at x’

Marginal Rate of Substitution



$dx_2 = MRS \times dx_1$ so, at x', MRS is the rate at which the consumer is only just willing to exchange commodity 2 for a small amount of commodity 1.

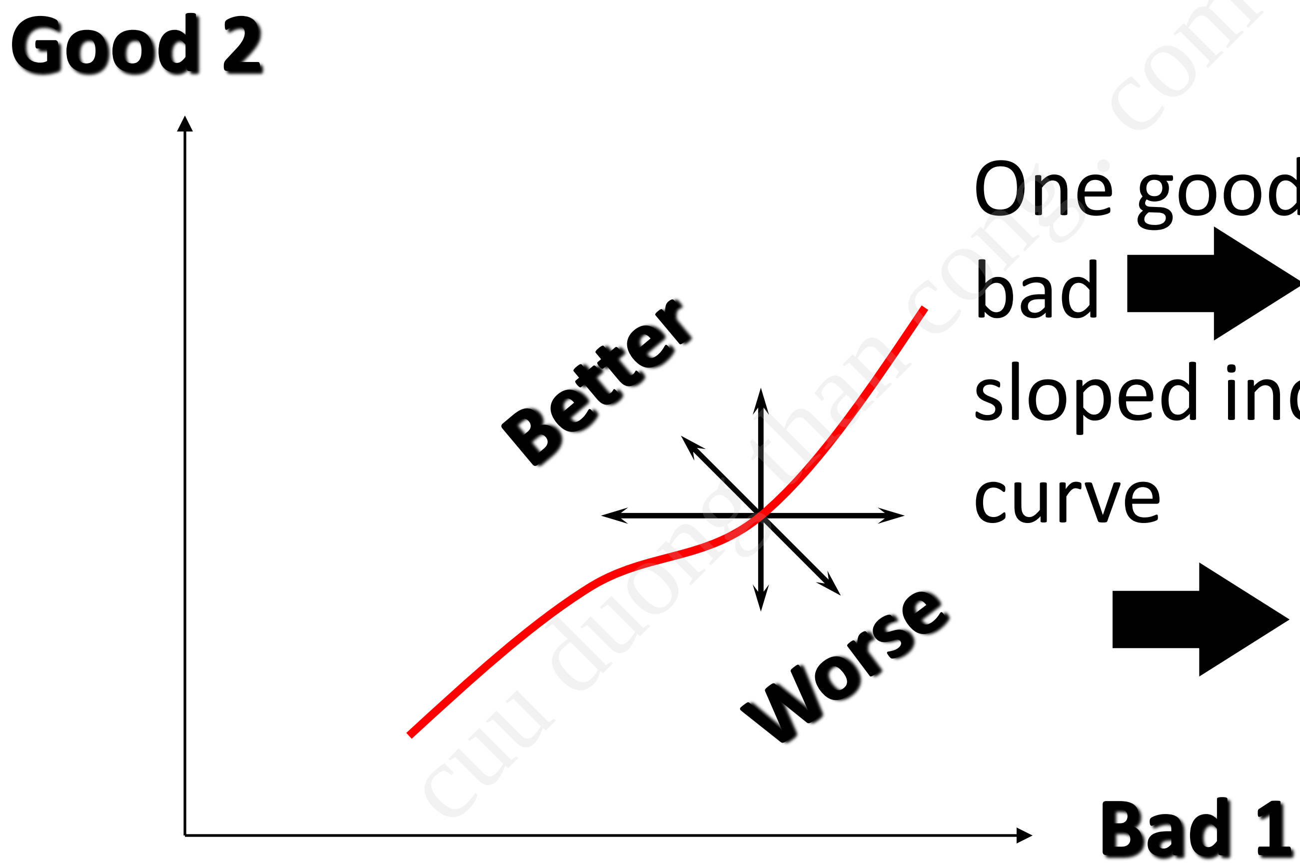
MRS & Ind. Curve Properties



Two goods ➡ a negatively sloped indifference curve

➡ **MRS < 0.**

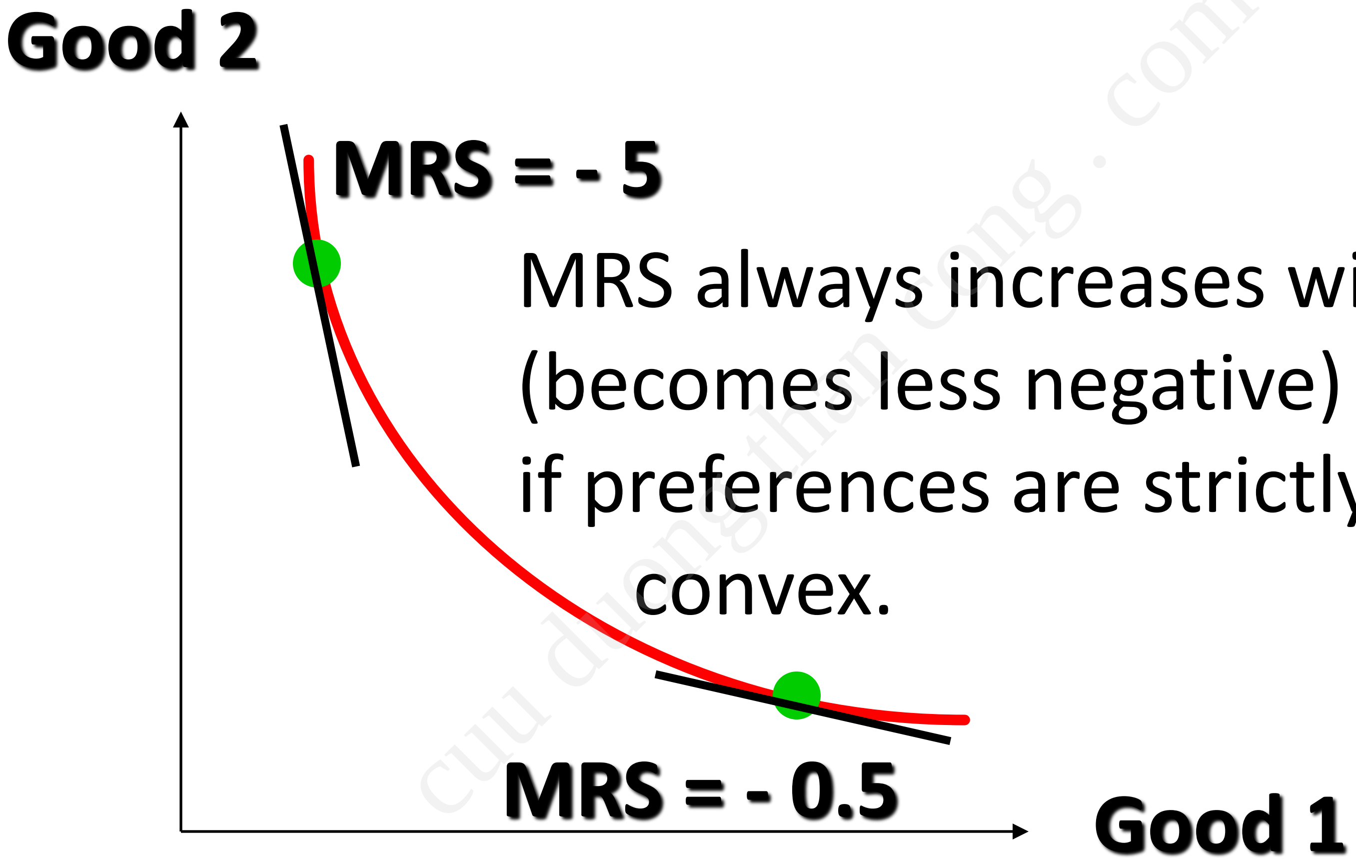
MRS & Ind. Curve Properties



One good and one bad ➡ a positively sloped indifference curve

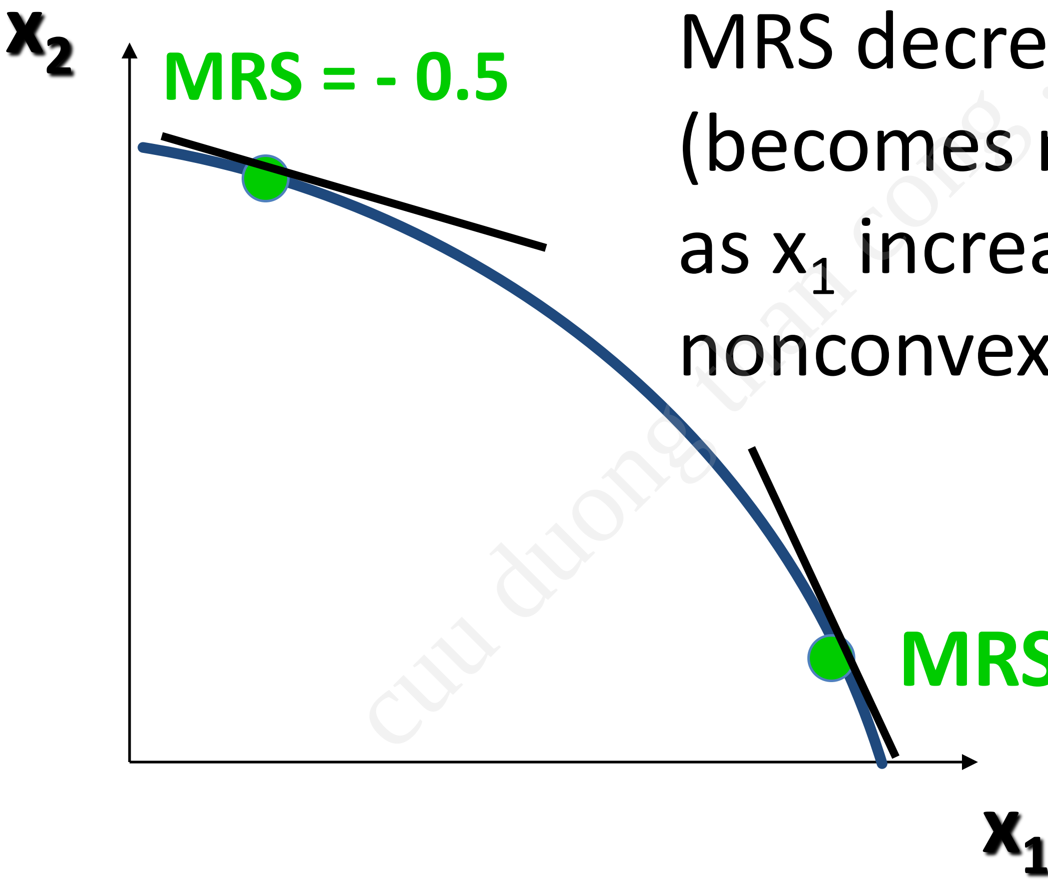
➡ **MRS > 0.**

MRS & Ind. Curve Properties



MRS always increases with x_1 (becomes less negative) if and only if preferences are strictly convex.

MRS & Ind. Curve Properties

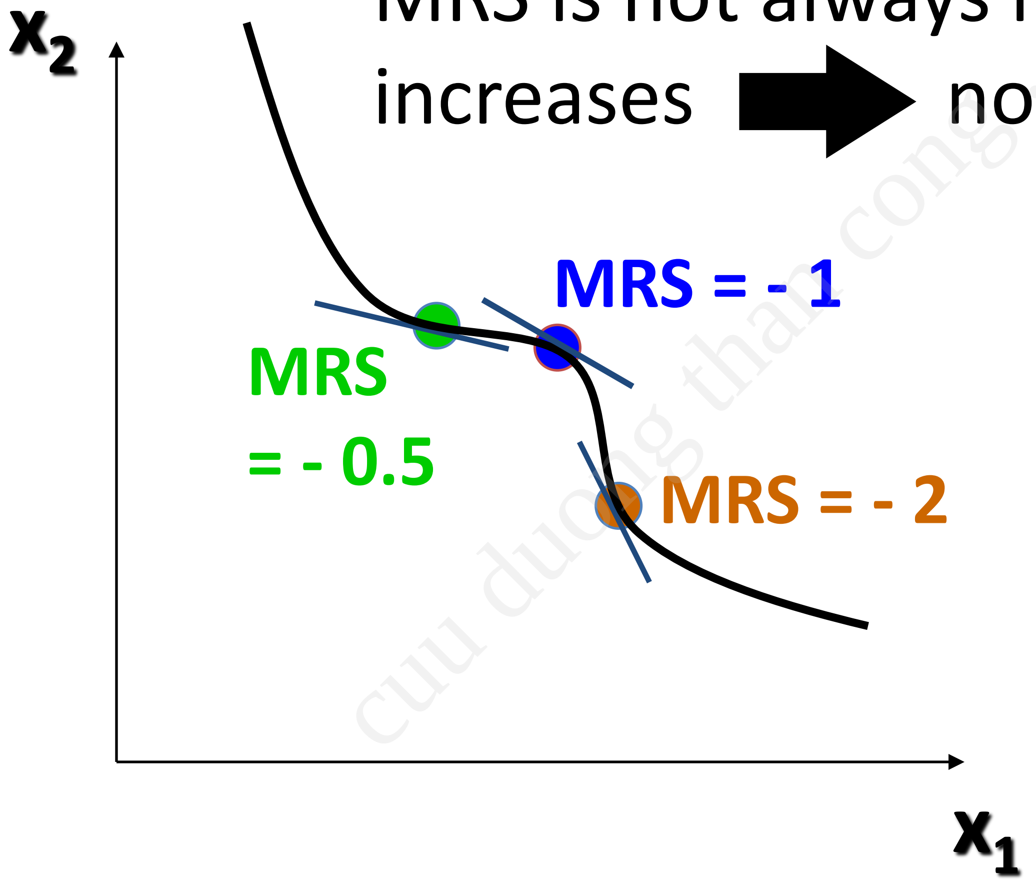


MRS decreases (becomes more negative) as x_1 increases

nonconvex preferences

MRS & Ind. Curve Properties

MRS is not always increasing as x_1 increases ➡ nonconvex preferences.



UTILITY

Utility Functions

- A preference relation that is complete, reflexive, transitive and continuous can be represented by a continuous utility function.
- Continuity means that small changes to a consumption bundle cause only small changes to the preference level.

Utility Functions

- A utility function U(x) represents a preference relation $\succsim$ if and only if:

$$x' \succ x'' \iff U(x') > U(x'')$$

$$x' \prec x'' \iff U(x') < U(x'')$$

$$x' \sim x'' \iff U(x') = U(x'').$$

Utility Functions

- Utility is an ordinal (i.e. ordering) concept.
- *E.g*. if U(x) = 6 and U(y) = 2 then bundle x is strictly preferred to bundle y. But x is not preferred three times as much as is y.

Utility Functions & Indiff. Curves

- Consider the bundles (4,1), (2,3) and (2,2).
- Suppose (2,3) $\succ$ (4,1) $\sim$ (2,2).
- Assign to these bundles any numbers that preserve the preference ordering;
  e.g. U(2,3) = 6 > U(4,1) = U(2,2) = 4.
- Call these numbers utility levels.

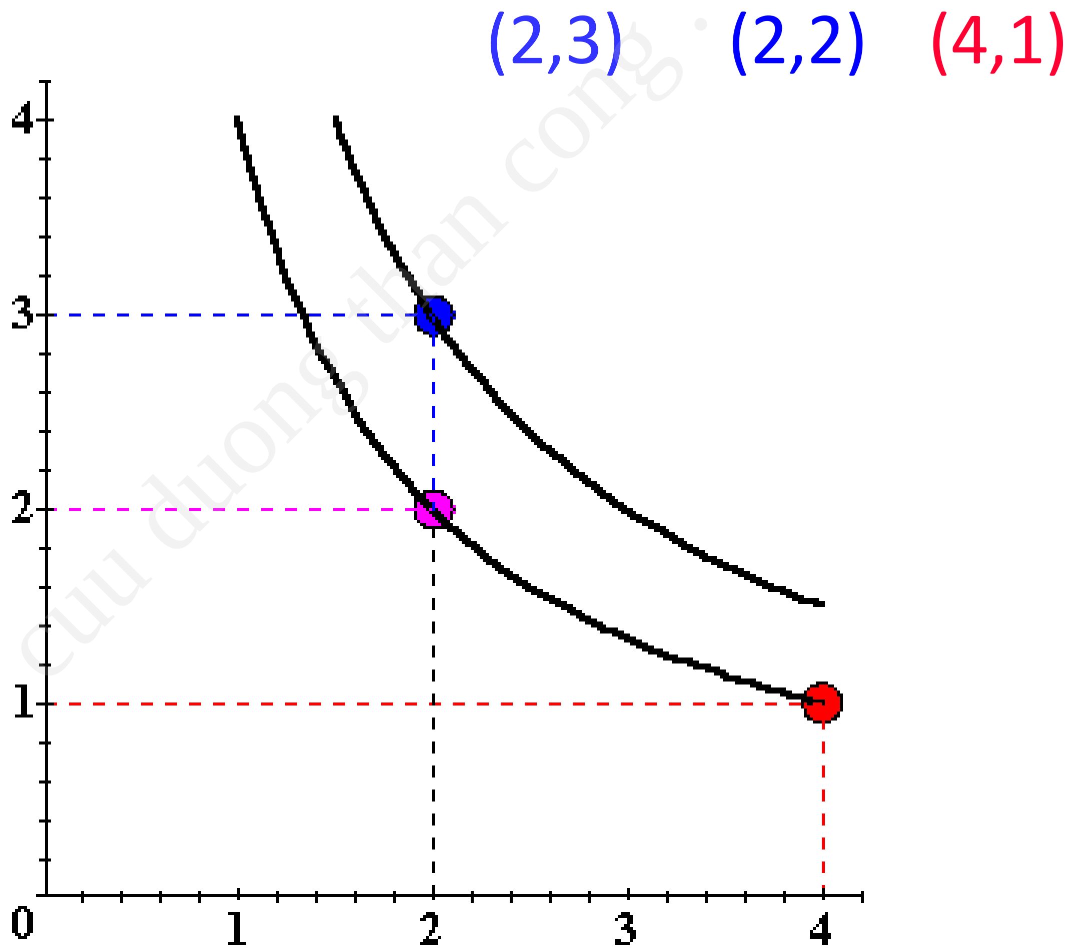
Utility Functions & Indiff. Curves

- An indifference curve contains equally preferred bundles.
- Equal preference $\Rightarrow$ same utility level.
- Therefore, all bundles in an indifference curve have the same utility level.

Utility Functions & Indiff. Curves

- So the bundles (4,1) and (2,2) are in the indiff. curve with utility level $U \equiv 4$
- But the bundle (2,3) is in the indiff. curve with utility level $U \equiv 6$.
- On an indifference curve diagram, this preference information looks as follows:

Utility Functions & Indiff. Curves

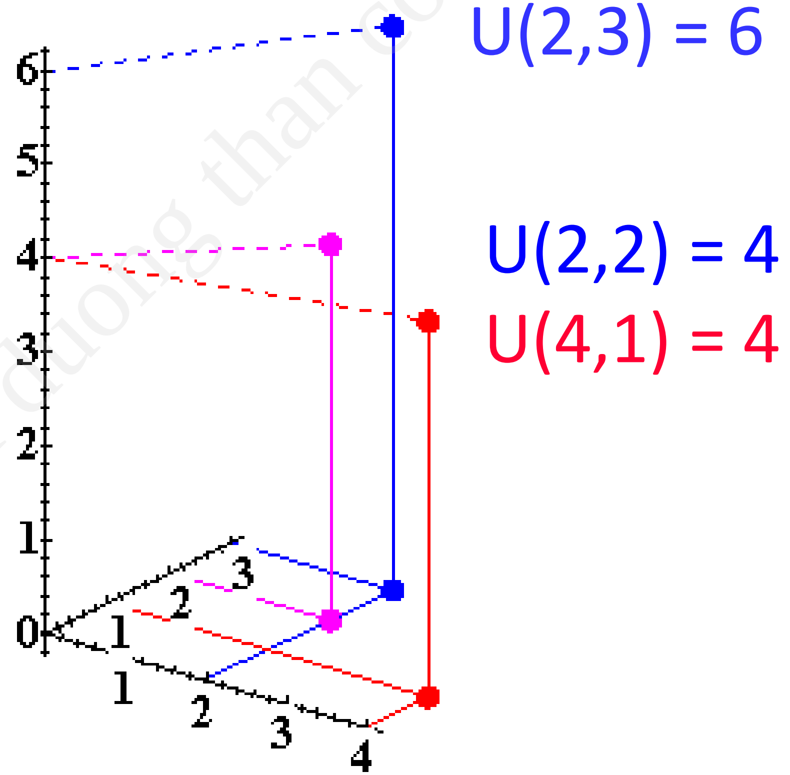


Utility Functions & Indiff. Curves

- Another way to visualize this same information is to plot the utility level on a vertical axis.

Utility Functions & Indiff. Curves

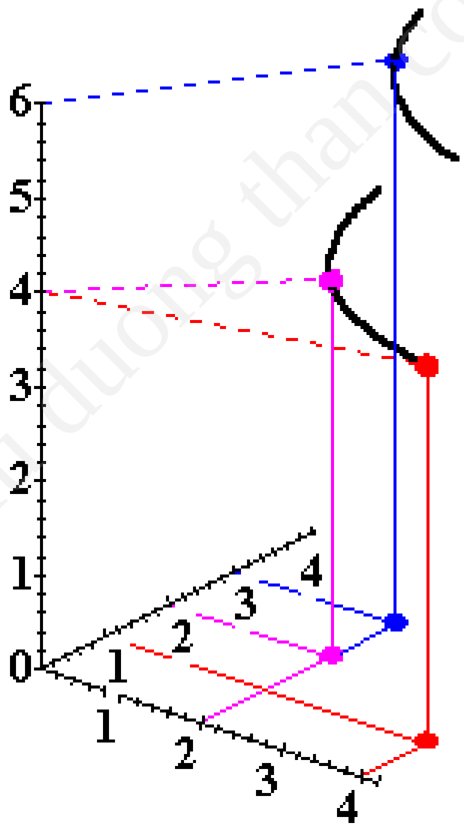
3D plot of consumption & utility levels for 3 bundles



Utility Functions & Indiff. Curves

- This 3D visualization of preferences can be made more informative by adding into it the two indifference curves.

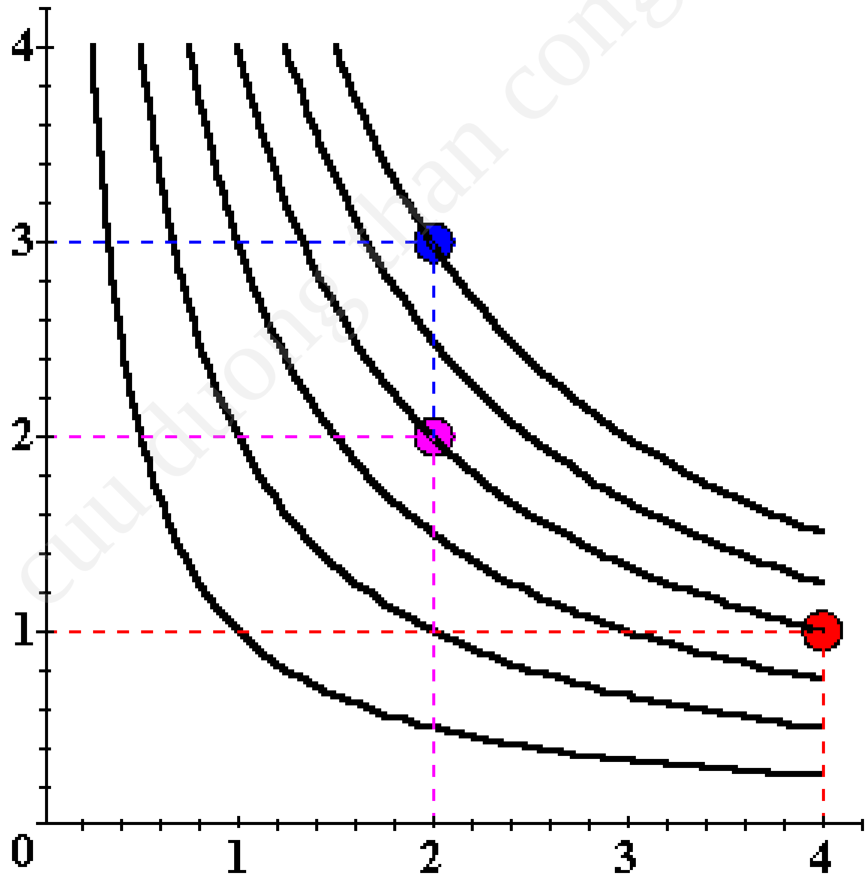
Utility Functions & Indiff. Curves



Utility Functions & Indiff. Curves

- Comparing more bundles will create a larger collection of all indifference curves and a better description of the consumer’s preferences.

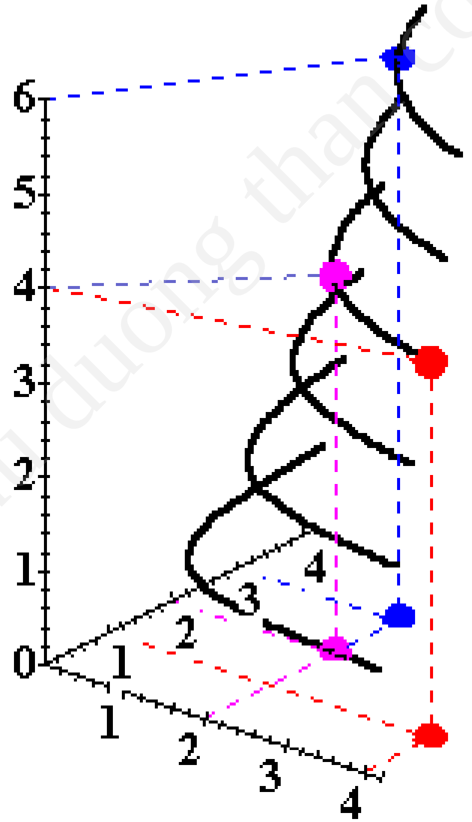
Utility Functions & Indiff. Curves



Utility Functions & Indiff. Curves

- As before, this can be visualized in 3D by plotting each indifference curve at the height of its utility index.

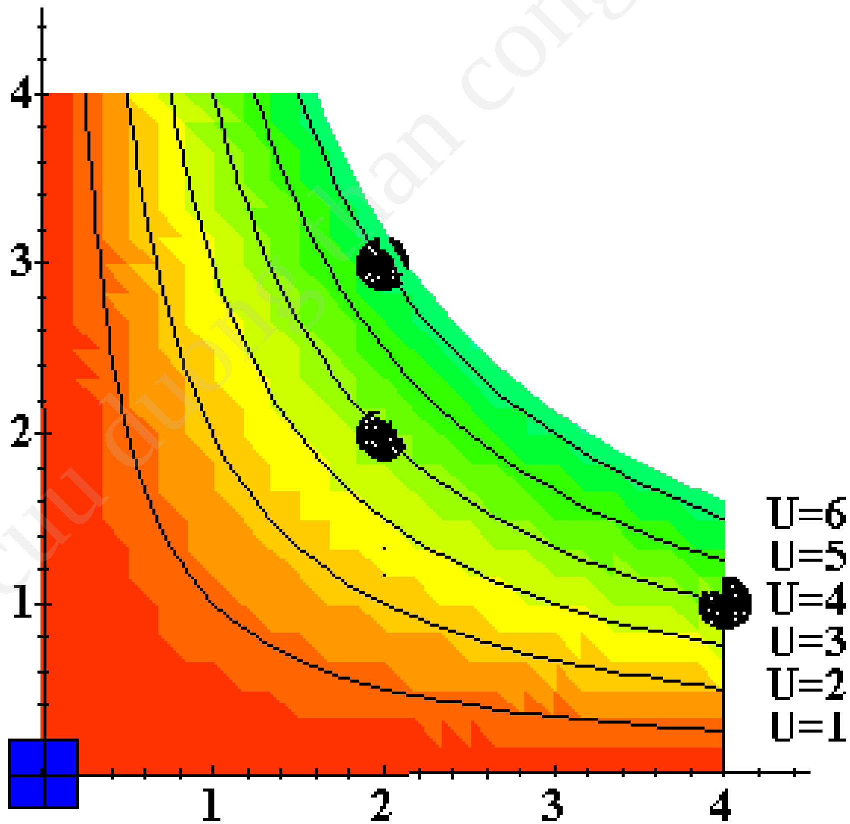
Utility Functions & Indiff. Curves



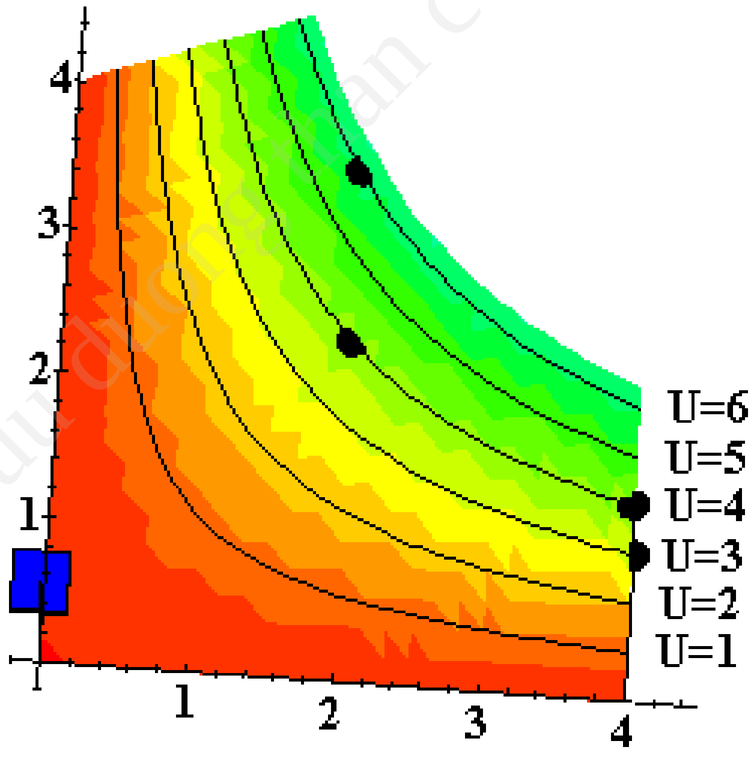
Utility Functions & Indiff. Curves

- Comparing all possible consumption bundles gives the complete collection of the consumer's indifference curves, each with its assigned utility level.
- This complete collection of indifference curves completely represents the consumer's preferences.

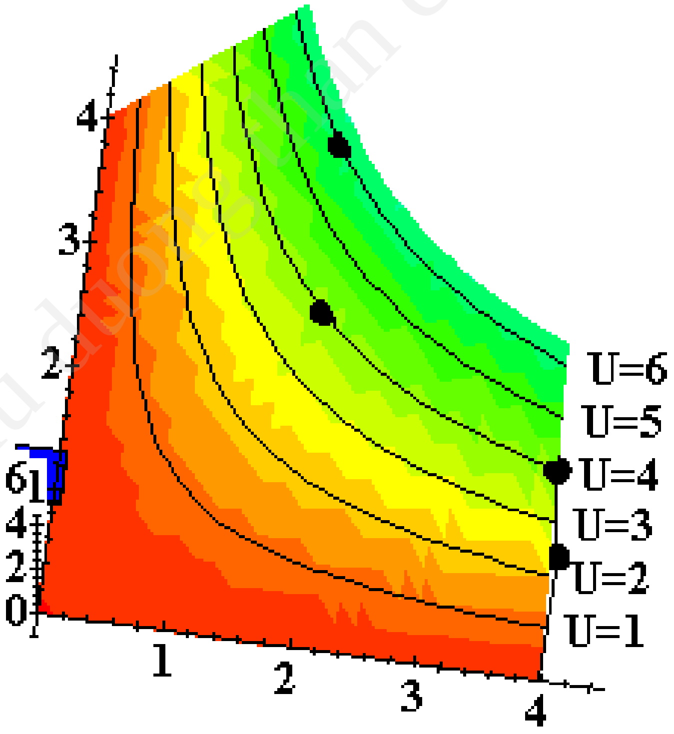
Utility Functions & Indiff. Curves



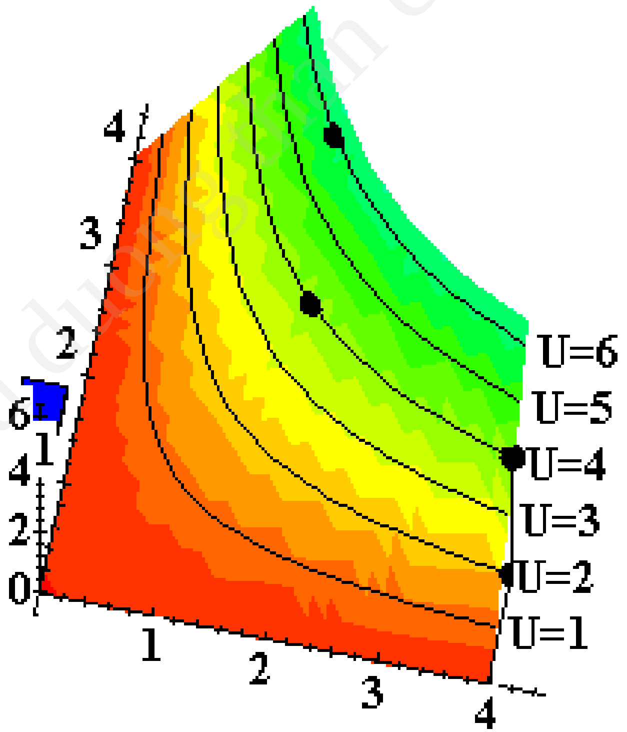
Utility Functions & Indiff. Curves



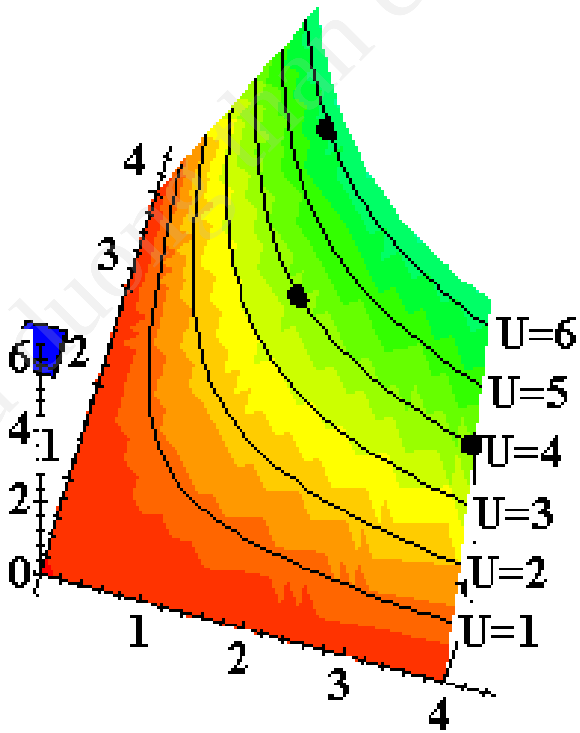
Utility Functions & Indiff. Curves



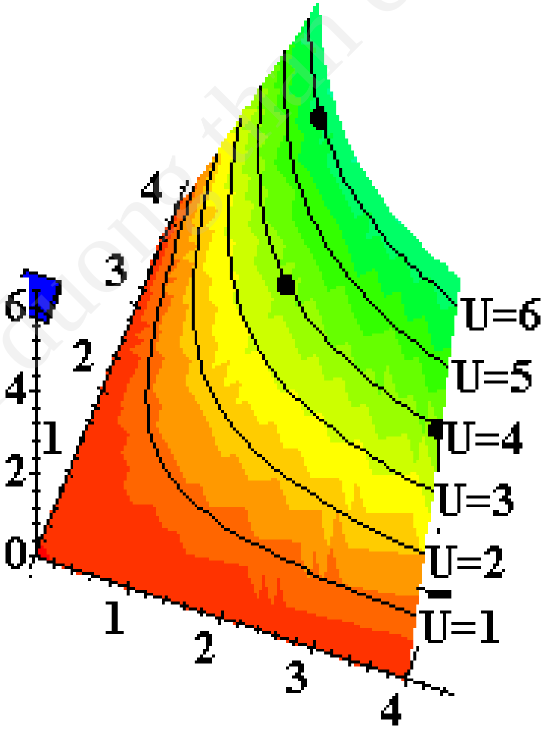
Utility Functions & Indiff. Curves



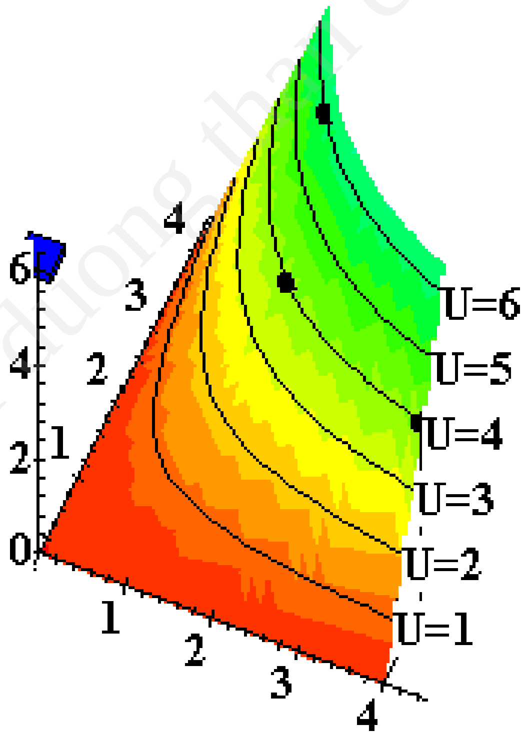
Utility Functions & Indiff. Curves



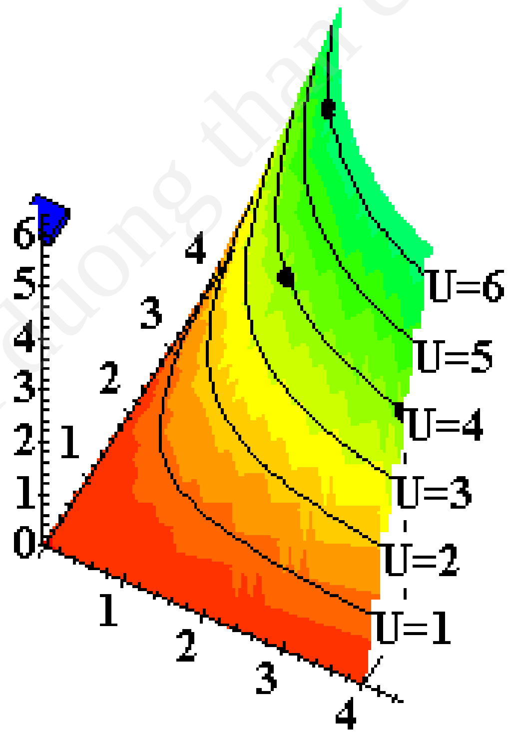
Utility Functions & Indiff. Curves



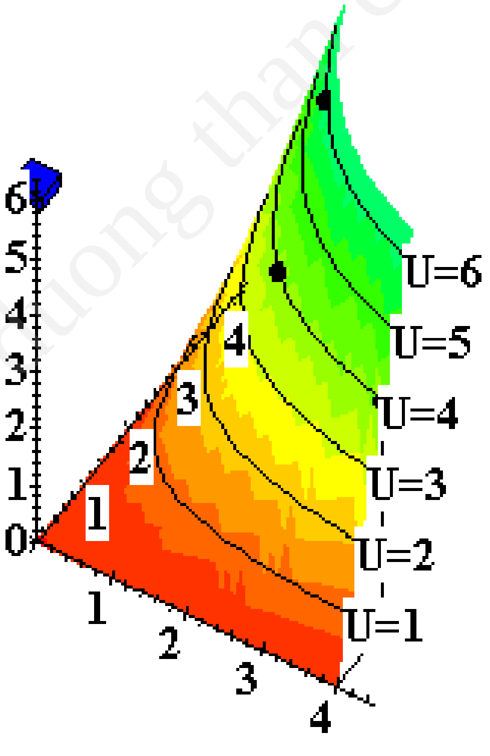
Utility Functions & Indiff. Curves



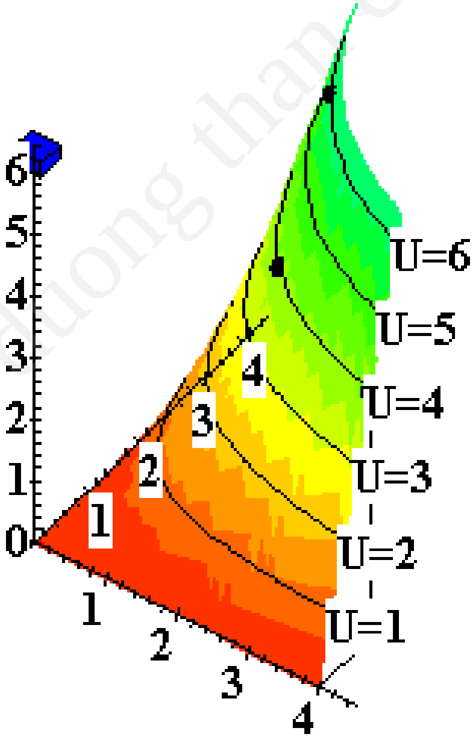
Utility Functions & Indiff. Curves



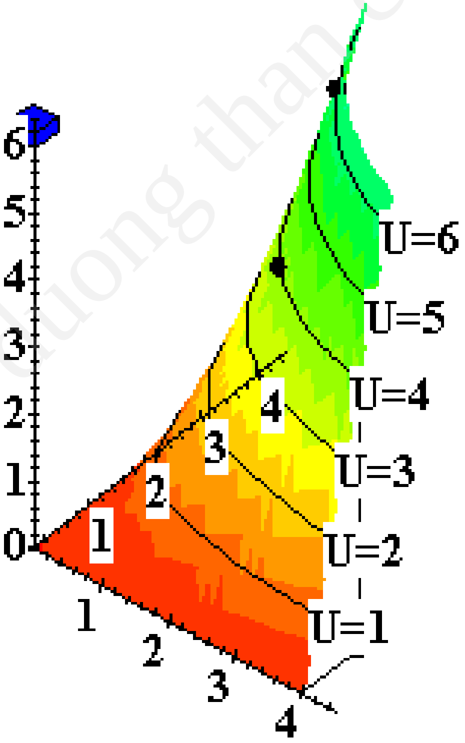
Utility Functions & Indiff. Curves



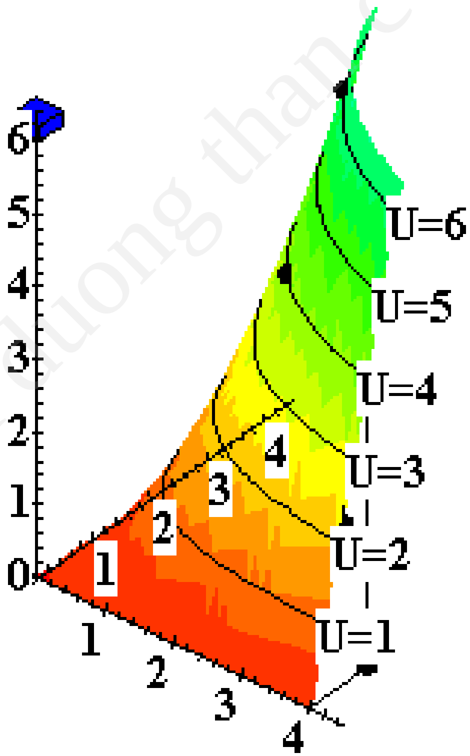
Utility Functions & Indiff. Curves



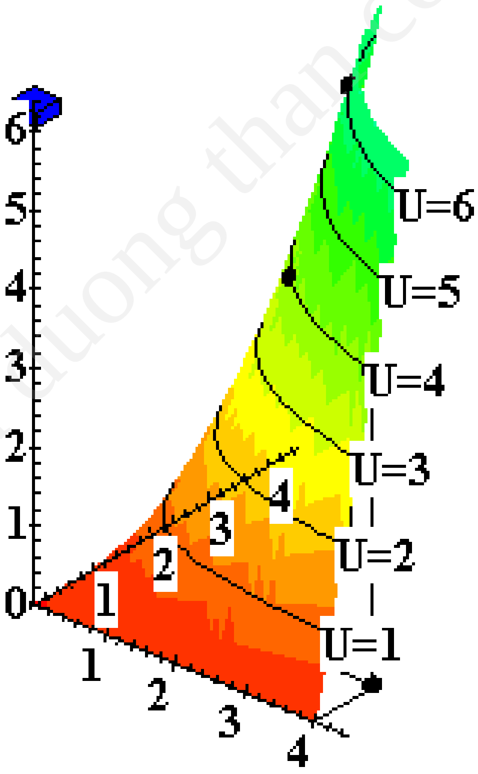
Utility Functions & Indiff. Curves



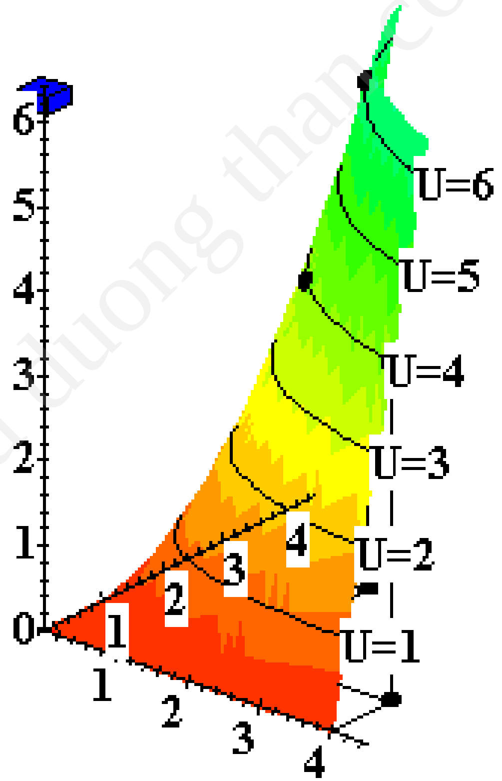
Utility Functions & Indiff. Curves



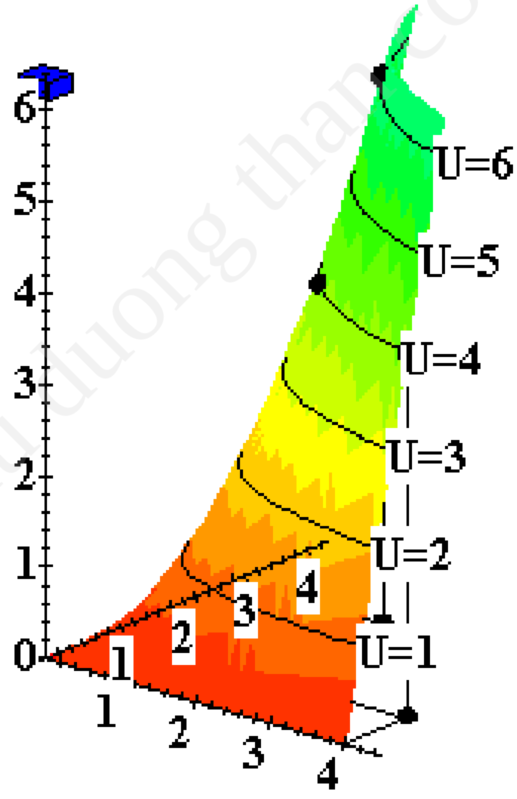
Utility Functions & Indiff. Curves



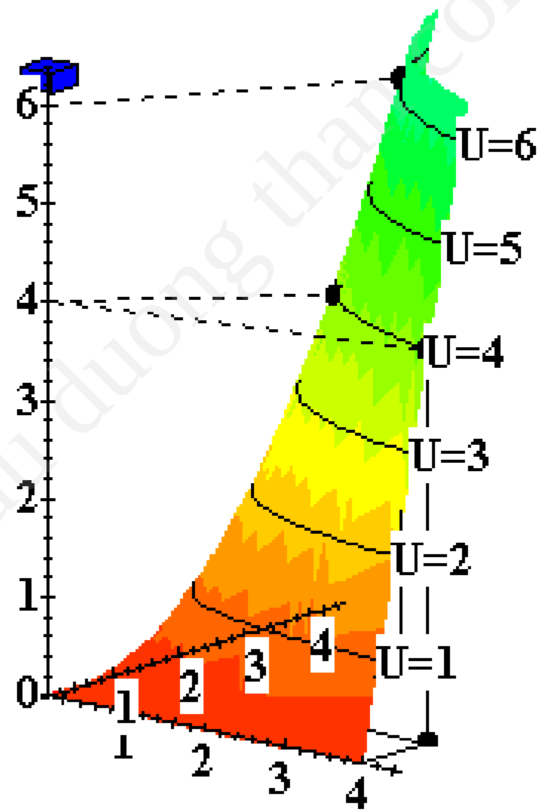
Utility Functions & Indiff. Curves

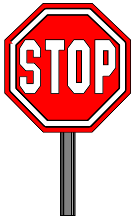


Utility Functions & Indiff. Curves



Utility Functions & Indiff. Curves





Utility Functions & Indiff. Curves

- The collection of all indifference curves for a given preference relation is an indifference map.
- An indifference map is equivalent to a utility function; each is the other.

Utility Functions

- There is no unique utility function representation of a preference relation.
- Suppose $U(x_1,x_2) = x_1x_2$ represents a preference relation.
- Again consider the bundles (4,1), (2,3) and (2,2).

Utility Functions

- $U(x_1,x_2) = x_1x_2$, so

  $U(2,3) = 6 > U(4,1) = U(2,2) = 4$;

  that is, $(2,3) \succ (4,1) \sim (2,2)$.

Utility Functions

- $U(x_1,x_2) = x_1x_2$ ➡ $(2,3) \succ (4,1) \sim (2,2)$.
- Define $V = U^2$.

Utility Functions

- $U(x_1,x_2) = x_1x_2$ ➡ $(2,3) \succ (4,1) \sim (2,2)$.
- Define $V = U^2$.
- Then $V(x_1,x_2) = x_1^2x_2^2$ and
  $V(2,3) = 36 > V(4,1) = V(2,2) = 16$
  so again
  $(2,3) \succ (4,1) \sim (2,2)$.
- V preserves the same order as U and so represents the same preferences.

Utility Functions

- $U(x_1,x_2) = x_1x_2$ ⇒ $(2,3) \succ (4,1) \sim (2,2)$.
- Define W = 2U + 10.

Utility Functions

- $U(x_1,x_2) = x_1x_2$ ⇒ $(2,3) \succ (4,1) \sim (2,2)$.
- Define W = 2U + 10.
- Then $W(x_1,x_2) = 2x_1x_2+10$ so $W(2,3) = 22 > W(4,1) = W(2,2) = 18$. Again, $(2,3) \succ (4,1) \sim (2,2)$.
- W preserves the same order as U and V and so represents the same preferences.

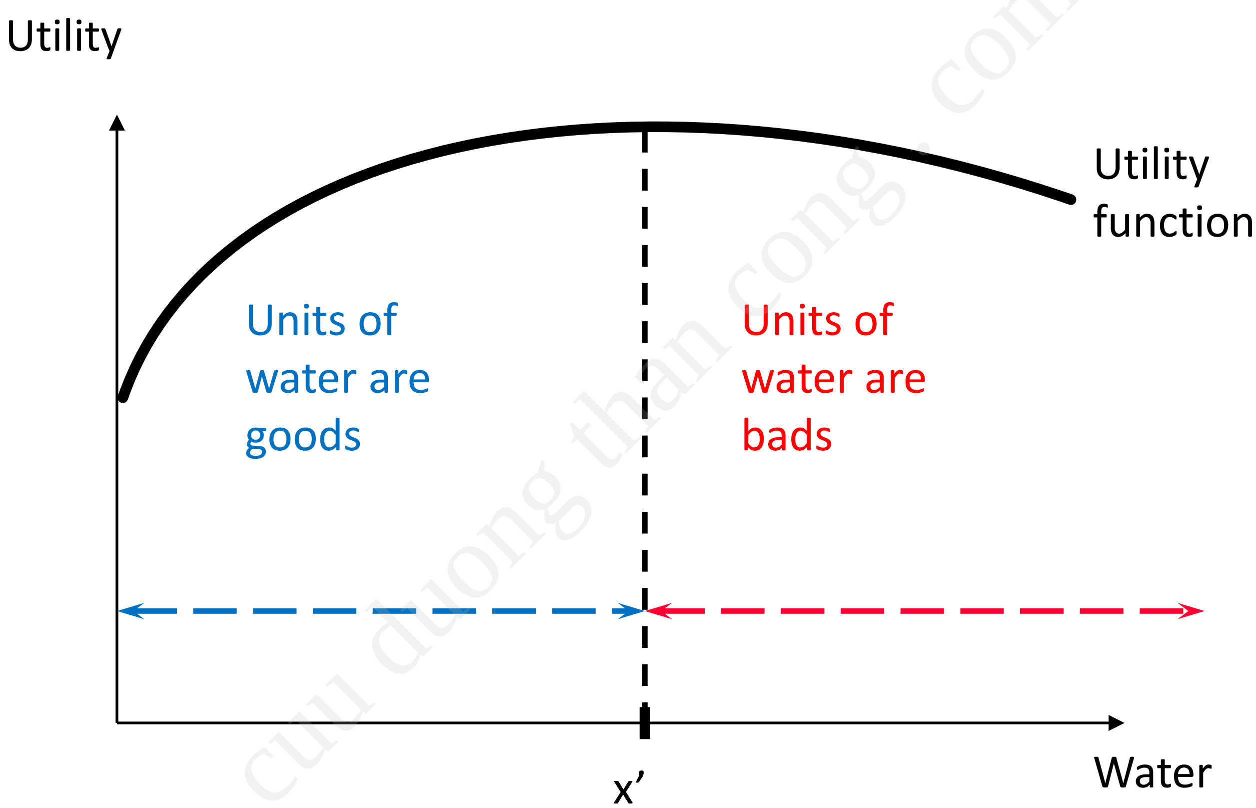
Utility Functions

- If
  - U is a utility function that represents a preference relation $\succsim$ and
  - f is a strictly increasing function,
- then V = f(U) is also a utility function representing $\succsim$.

Goods, Bads and Neutrals

- A good is a commodity unit which increases utility (gives a more preferred bundle).
- A bad is a commodity unit which decreases utility (gives a less preferred bundle).
- A neutral is a commodity unit which does not change utility (gives an equally preferred bundle).

Goods, Bads and Neutrals



Around x' units, a little extra water is a neutral.

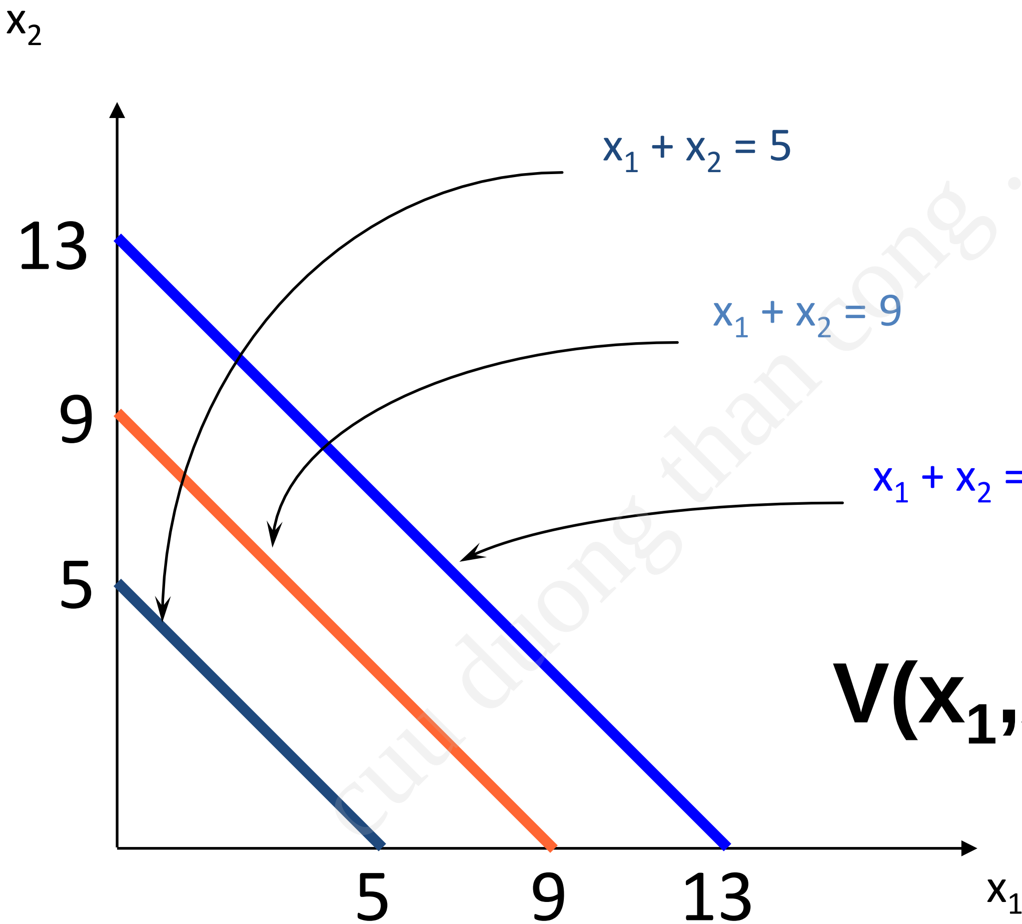
Some Other Utility Functions and Their Indifference Curves

- Instead of $U(x_1,x_2) = x_1x_2$ consider

$$V(x_1,x_2) = x_1 + x_2.$$

What do the indifference curves for this "perfect substitution" utility function look like?

Perfect Substitution Indifference Curves



$$V(x_1,x_2) = x_1 + x_2.$$

All are linear and parallel.

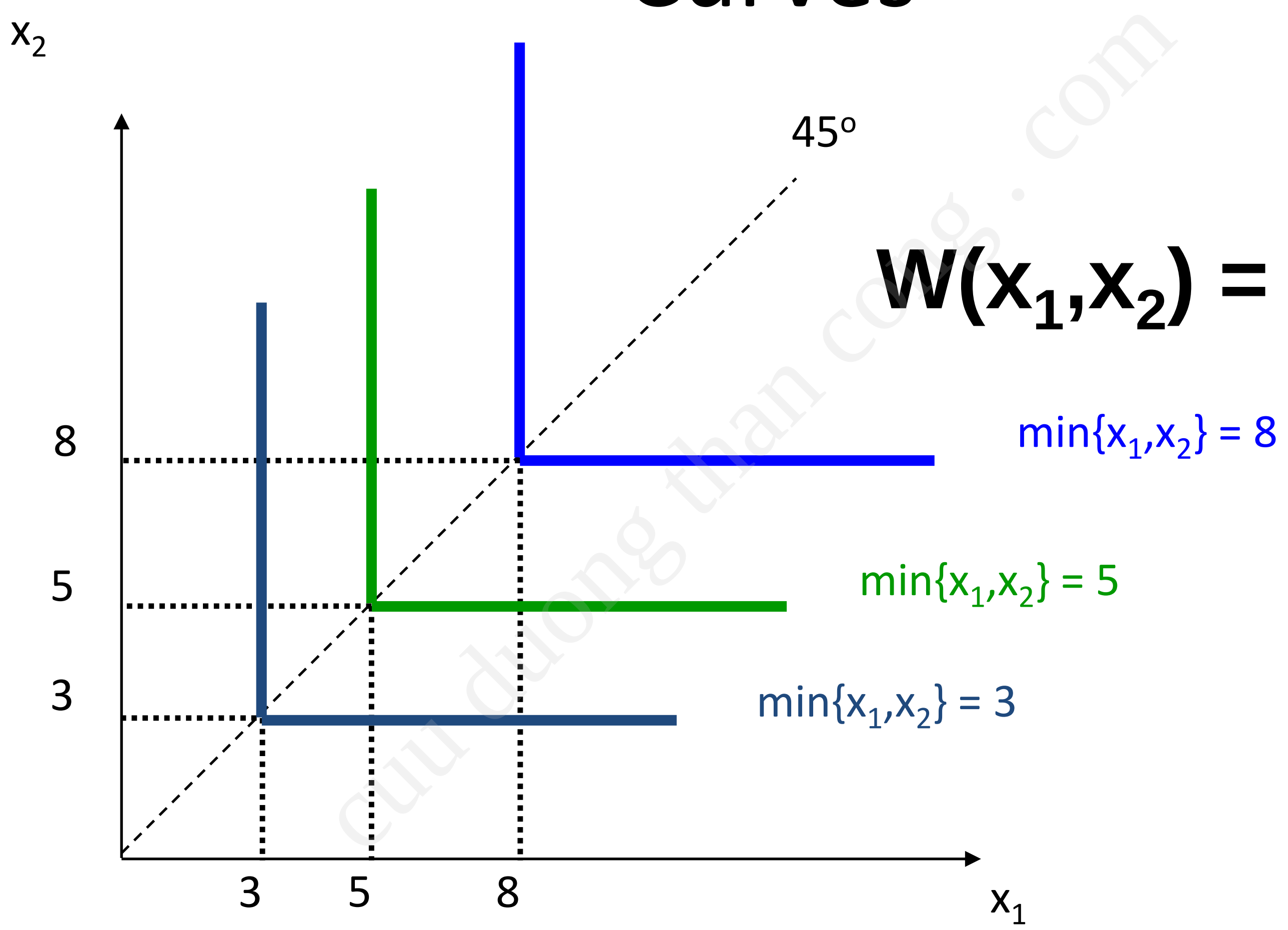
Some Other Utility Functions and Their Indifference Curves

- Instead of $U(x_1,x_2) = x_1x_2$ or $V(x_1,x_2) = x_1 + x_2$, consider

$$W(x_1,x_2) = \min\{x_1,x_2\}.$$

What do the indifference curves for this "perfect complementarity" utility function look like?

Perfect Complementarity Indifference Curves



$$W(x_1,x_2) = \min\{x_1,x_2\}$$

All are right-angled with vertices on a ray from the origin.

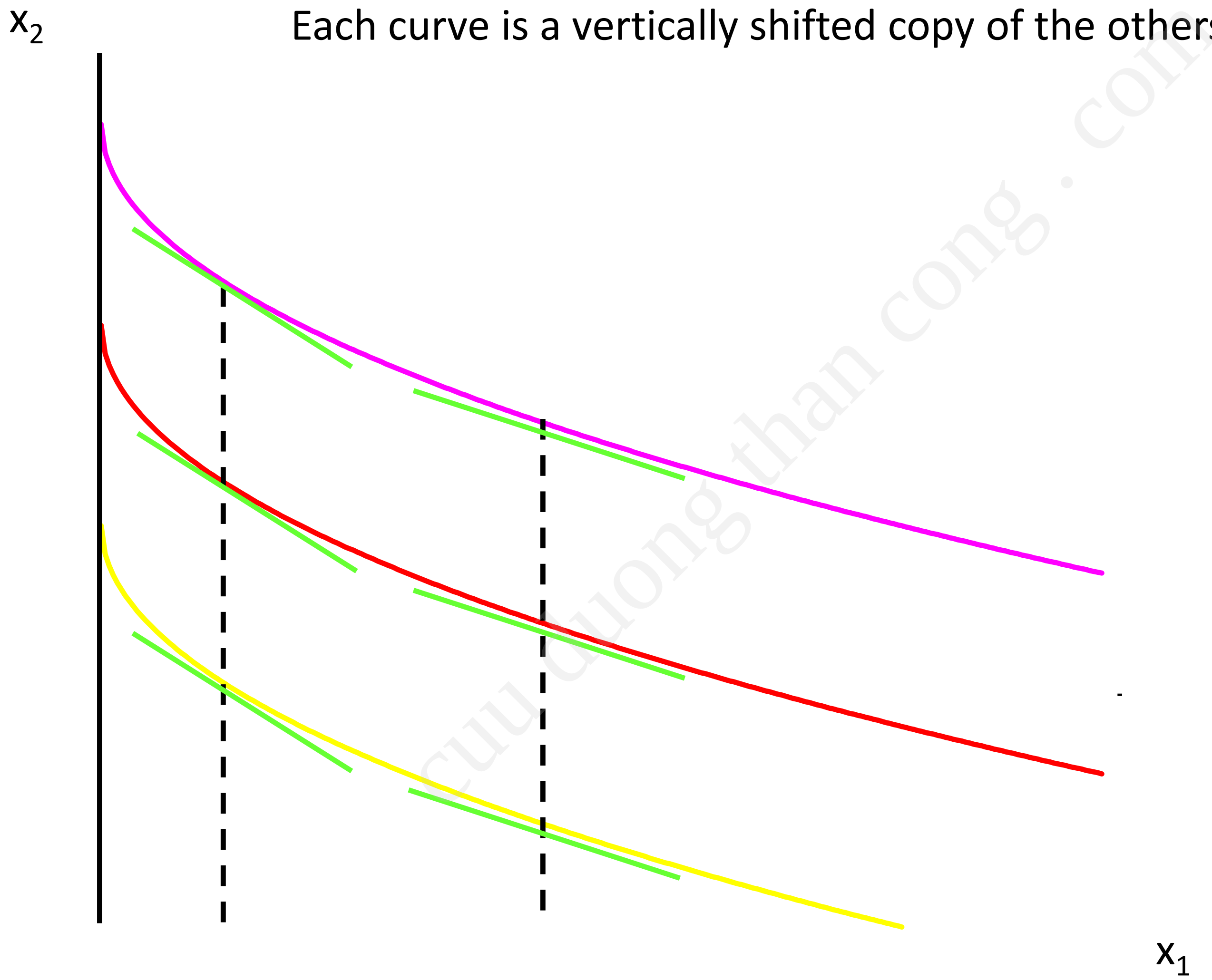
Some Other Utility Functions and Their Indifference Curves

- A utility function of the form

$$U(x_1,x_2) = f(x_1) + x_2$$

  is linear in just x_2 and is called quasi-linear.
- *E.g.* $U(x_1,x_2) = 2x_1^{1/2} + x_2$.

Quasi-linear Indifference Curves



Some Other Utility Functions and Their Indifference Curves

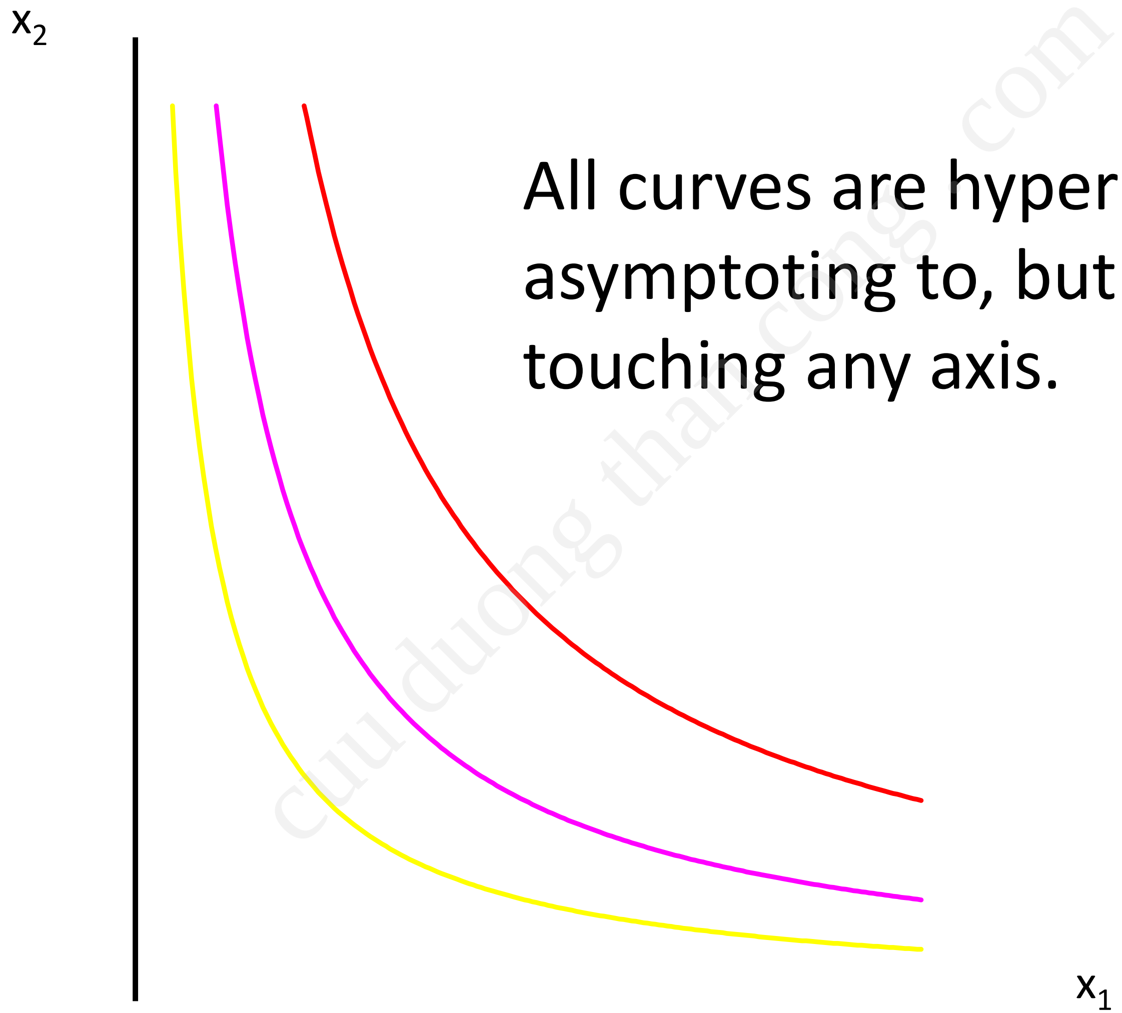
- Any utility function of the form

$$U(x_1,x_2) = x_1^{a} x_2^{b}$$

with a > 0 and b > 0 is called a Cobb-Douglas utility function.

- *E.g.* $U(x_1,x_2) = x_1^{1/2} x_2^{1/2}$ (a = b = 1/2)
  $V(x_1,x_2) = x_1 x_2^{3}$ (a = 1, b = 3)

Cobb-Douglas Indifference Curves



All curves are hyperbolic, asymptoting to, but never touching any axis.

Marginal Utilities

- Marginal means "incremental".
- The marginal utility of commodity i is the rate-of-change of total utility as the quantity of commodity i consumed changes; *i.e.*

$$MU_i = \frac{\partial U}{\partial x_i}$$

Marginal Utilities

- *So,* if $U(x_1,x_2) = x_1^{1/2} x_2^2$ then

$$MU_1 = \frac{\partial U}{\partial x_1} = \frac{1}{2} x_1^{-1/2} x_2^2$$

$$MU_2 = \frac{\partial U}{\partial x_2} = 2 x_1^{1/2} x_2$$

Marginal Utilities and Marginal Rates-of-Substitution

- The general equation for an indifference curve is

  $U(x_1,x_2) \equiv k$, a constant.

  Totally differentiating this identity gives

$$\frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0$$

Marg. Utilities & Marg. Rates-of-Substitution; An example

- Suppose $U(x_1,x_2) = x_1x_2$. Then

$$\frac{\partial U}{\partial x_1} = (1)(x_2) = x_2$$

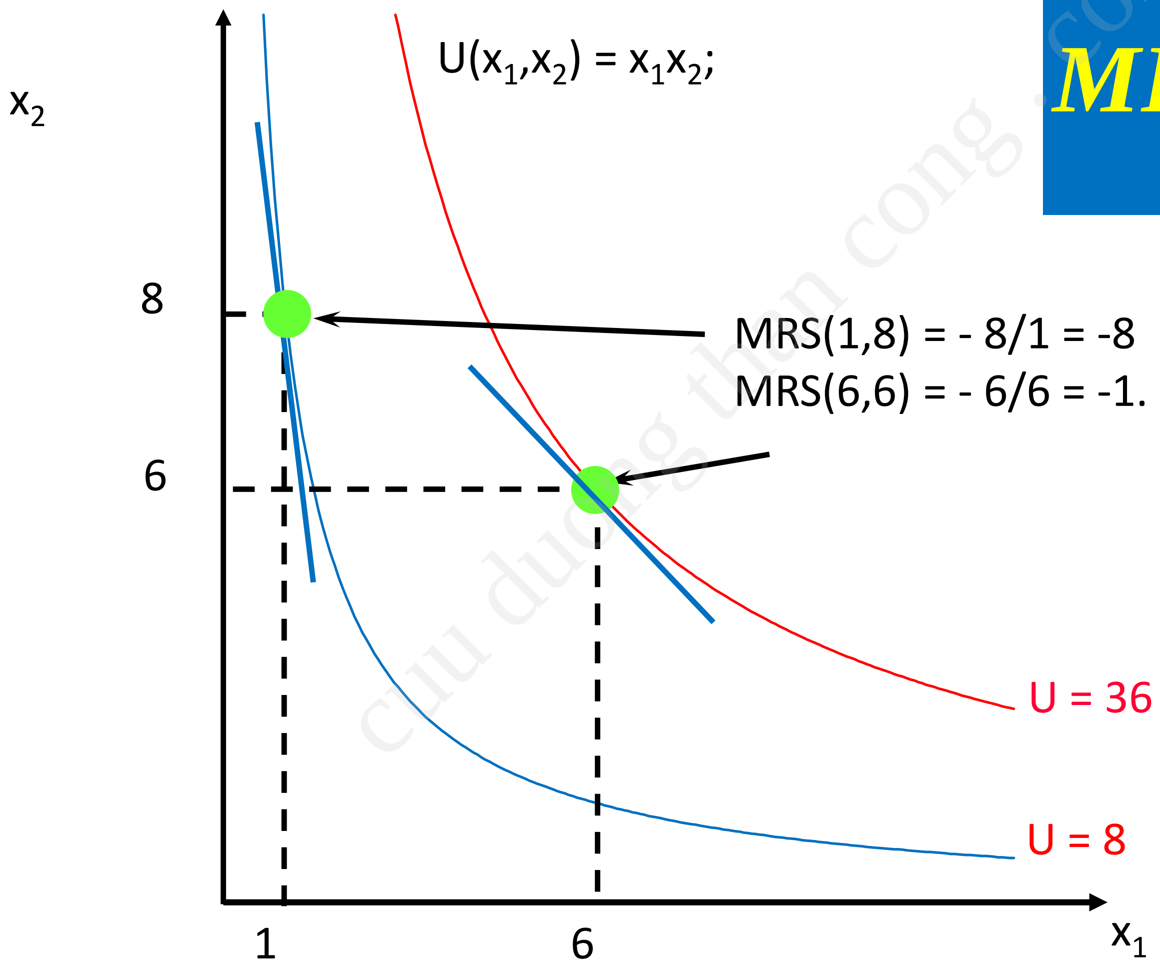
$$\frac{\partial U}{\partial x_2} = (x_1)(1) = x_1$$

so

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{x_2}{x_1}.$$

Marg. Utilities & Marg. Rates-of-Substitution; An example

$$MRS = -\frac{x_2}{x_1}$$



Marg. Rates-of-Substitution for Quasi-linear Utility Functions

- A quasi-linear utility function is of the form $U(x_1,x_2) = f(x_1) + x_2$.

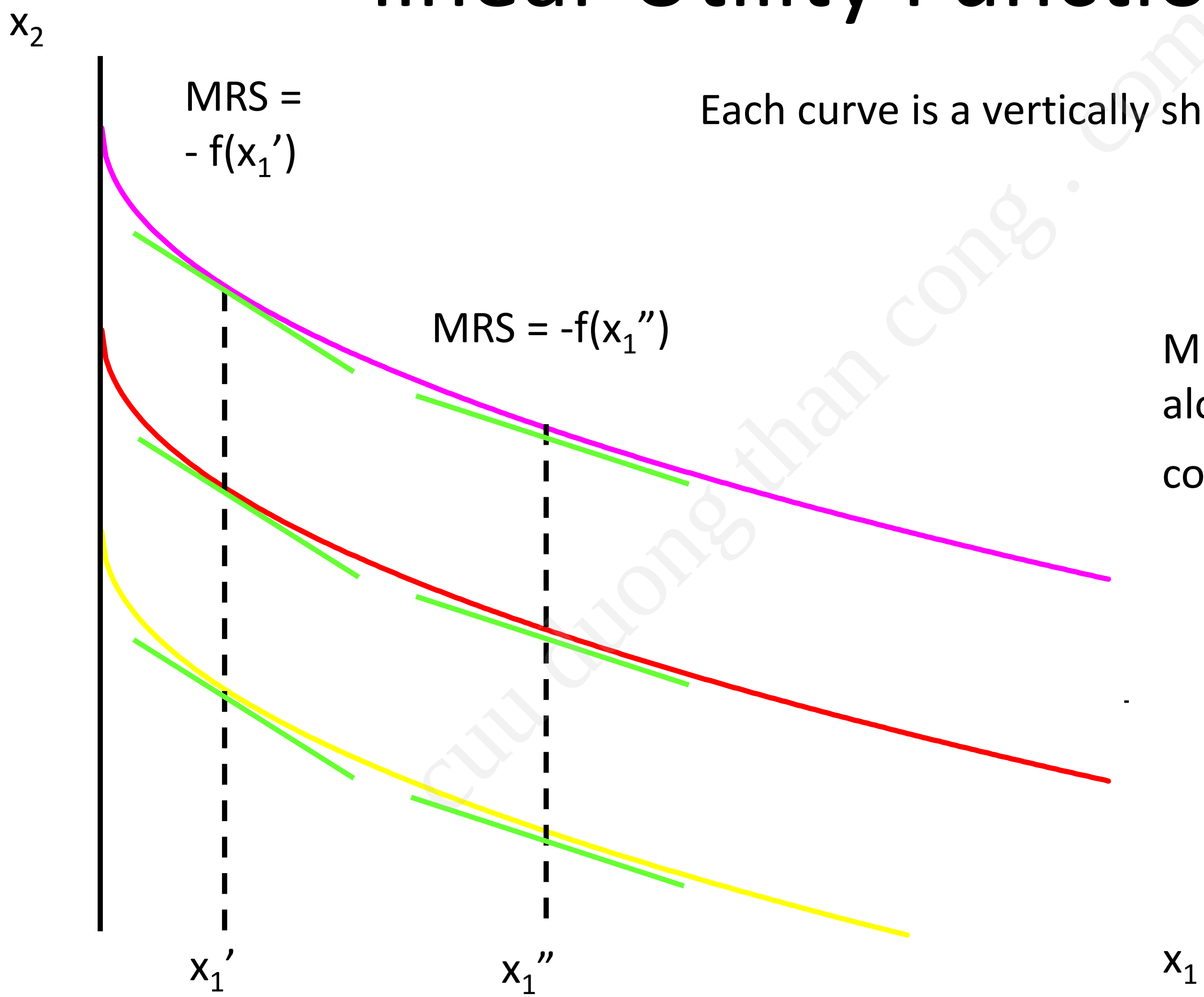
$$\frac{\partial U}{\partial x_1} = f'(x_1) \qquad \frac{\partial U}{\partial x_2} = 1$$

so $$MRS = \frac{d\,x_2}{d\,x_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = -f'(x_1).$$

Marg. Rates-of-Substitution for Quasi-linear Utility Functions

- MRS = - $f'(x_1)$ does not depend upon x_2 so the slope of indifference curves for a quasi-linear utility function is constant along any line for which x_1 is constant. What does that make the indifference map for a quasi-linear utility function look like?

Marg. Rates-of-Substitution for Quasi-linear Utility Functions



Each curve is a vertically shifted copy of the others.

MRS is a constant along any line for which x_1 is constant.

Monotonic Transformations & Marginal Rates-of-Substitution

- Applying a monotonic transformation to a utility function representing a preference relation simply creates another utility function representing the same preference relation.
- What happens to marginal rates-of-substitution when a monotonic transformation is applied?

Monotonic Transformations & Marginal Rates-of-Substitution

- For $U(x_1,x_2) = x_1x_2$ the MRS = $-\, x_2/x_1$.
- Create $V = U^2$; *i.e.* $V(x_1,x_2) = x_1^2x_2^2$. What is the MRS for V?

$$MRS = -\frac{\partial V / \partial x_1}{\partial V / \partial x_2} = -\frac{2x_1x_2^2}{2x_1^2x_2} = -\frac{x_2}{x_1}$$

which is the same as the MRS for U.

Monotonic Transformations & Marginal Rates-of-Substitution

- More generally, if V = f(U) where f is a strictly increasing function, then

$$MRS = -\frac{\partial V / \partial x_1}{\partial V / \partial x_2} = -\frac{f'(U) \times \partial U / \partial x_1}{f'(U) \times \partial U / \partial x_2}$$

$$= -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}.$$

So MRS is unchanged by a positive monotonic transformation.