Lesson 3: Consumer behavior

1. Choice
2. Demand
3. Intertemporal choice

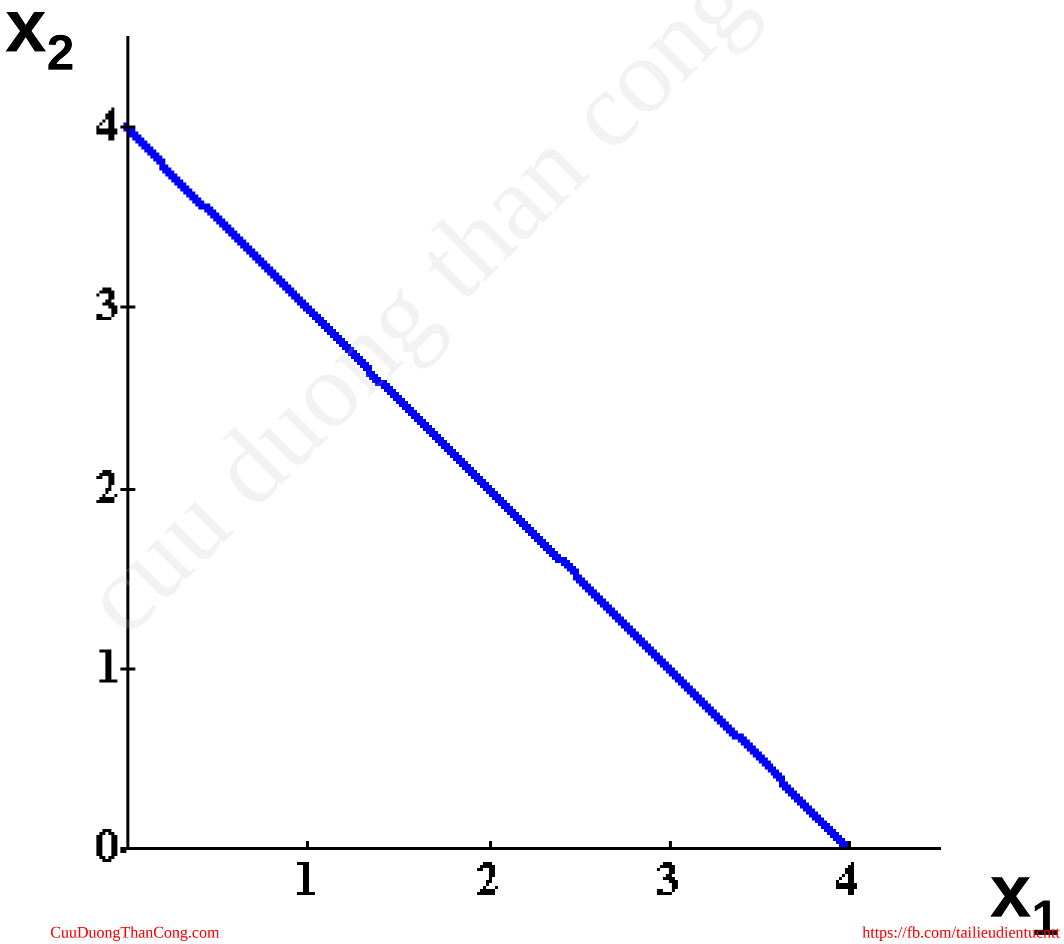
1. Choice

- Optimal choice
- Consumer demand
- Implication of MRS conditions

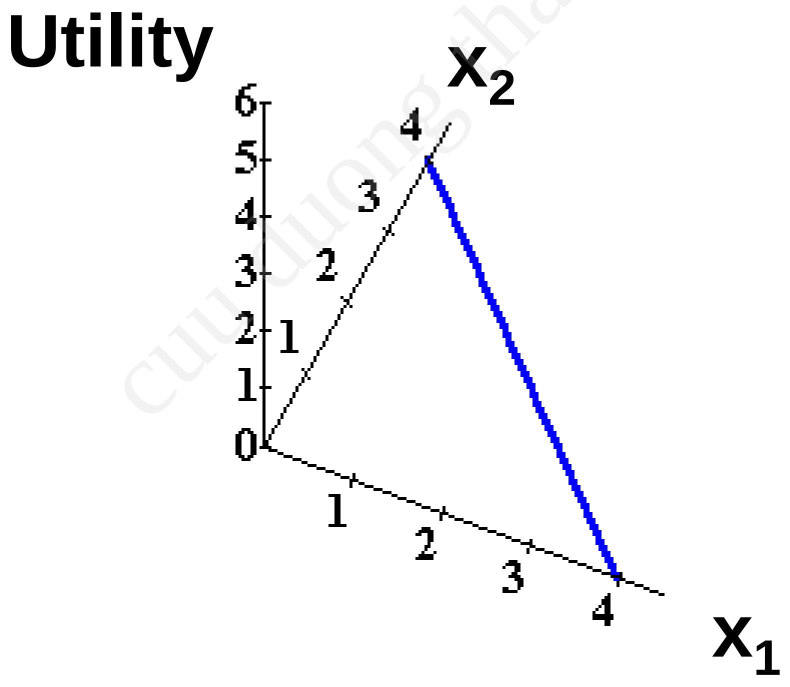
Economic Rationality

- The principal behavioral postulate is that a decisionmaker chooses its most preferred alternative from those available to it.
- The available choices constitute the choice set.
- How is the most preferred bundle in the choice set located?

Rational Constrained Choice



Rational Constrained Choice



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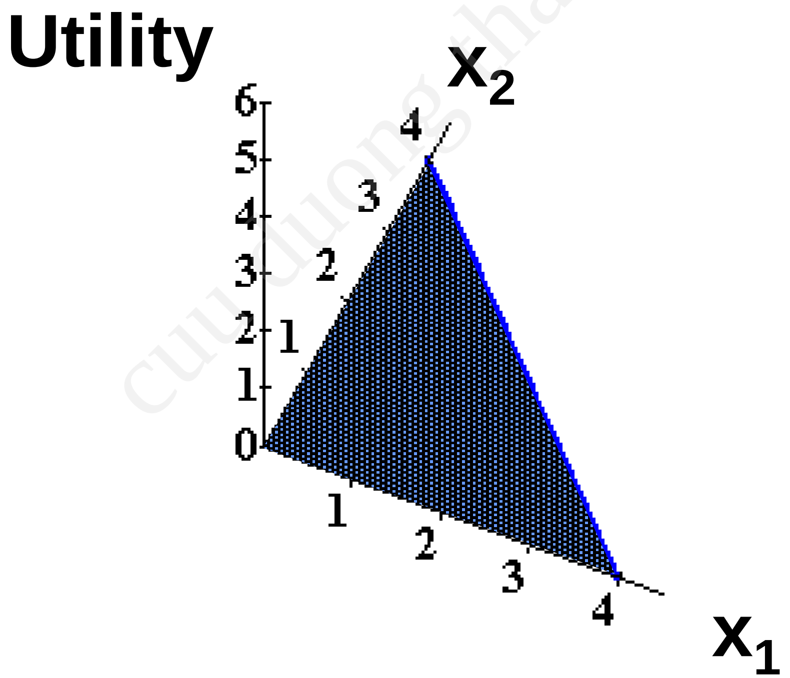

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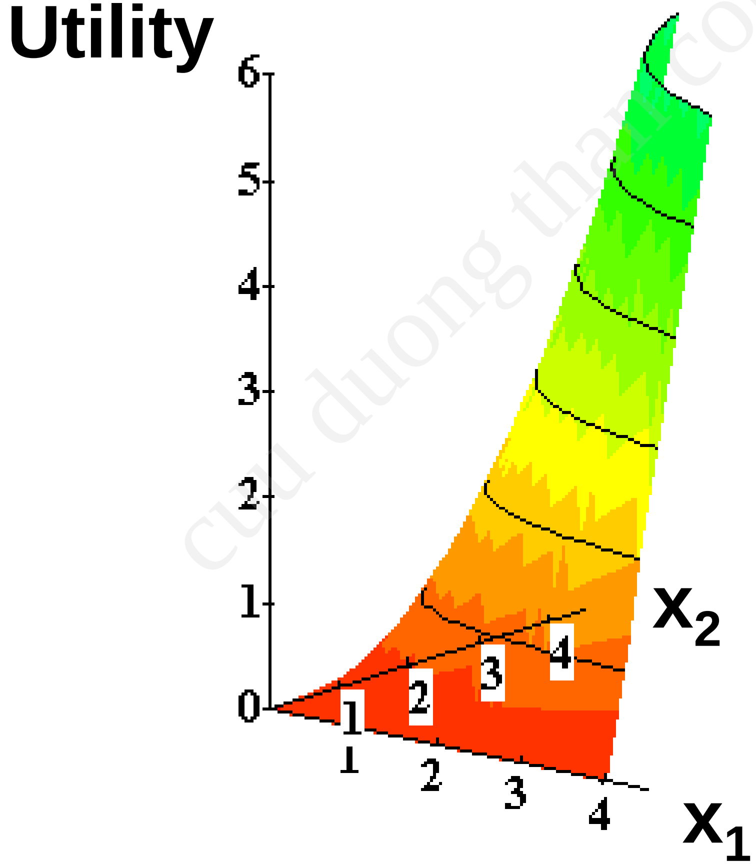
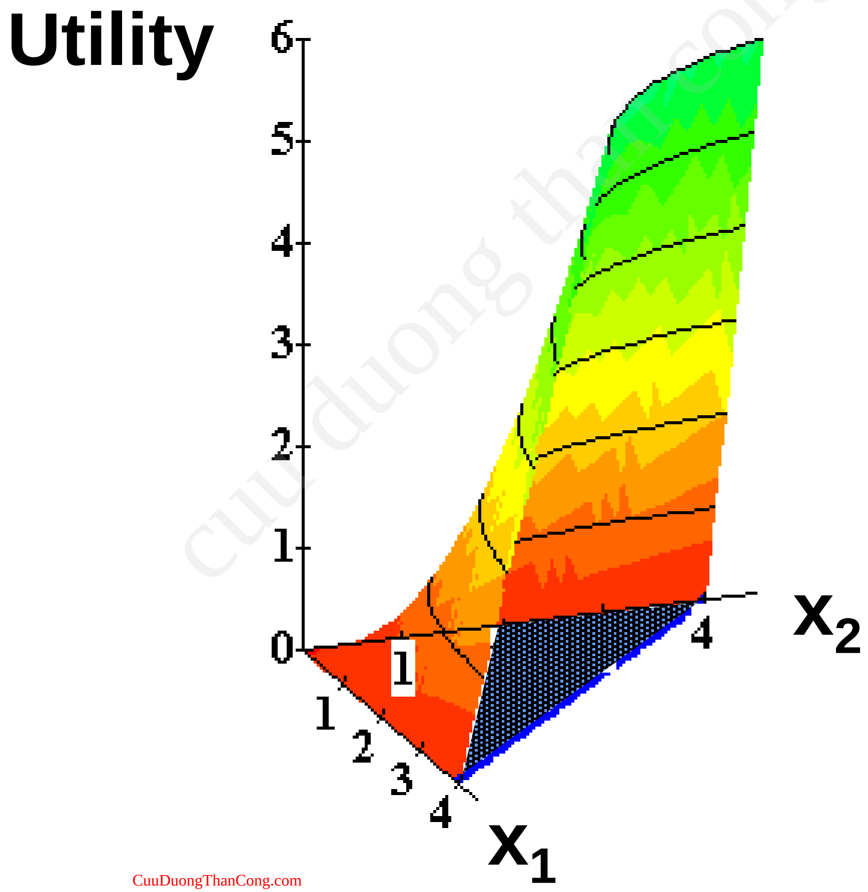
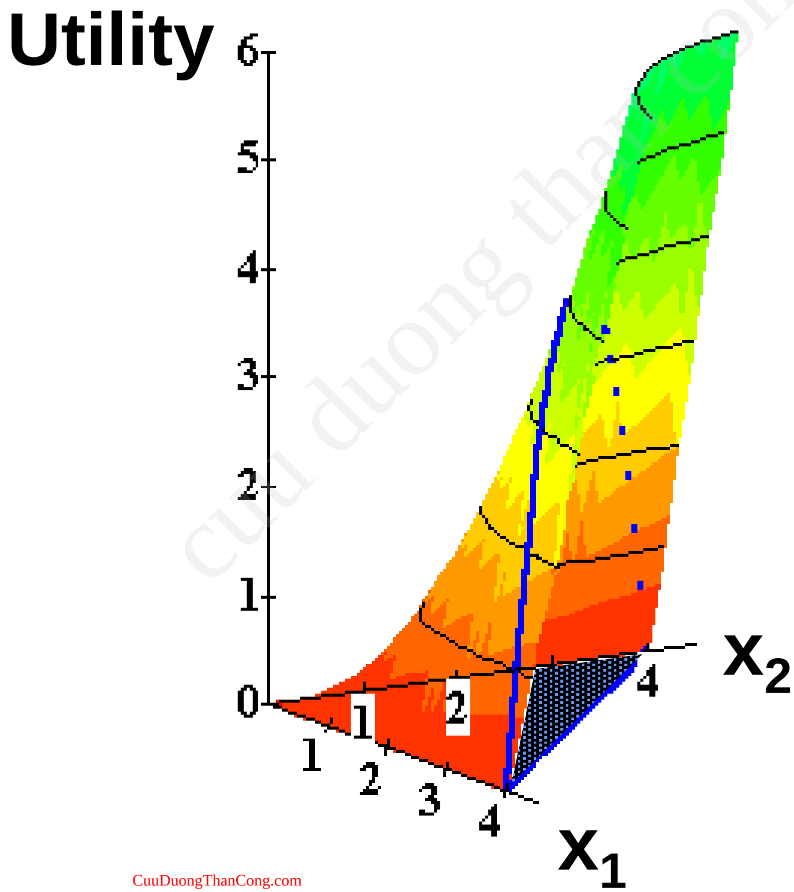


Rational Constrained Choice



Rational Constrained Choice



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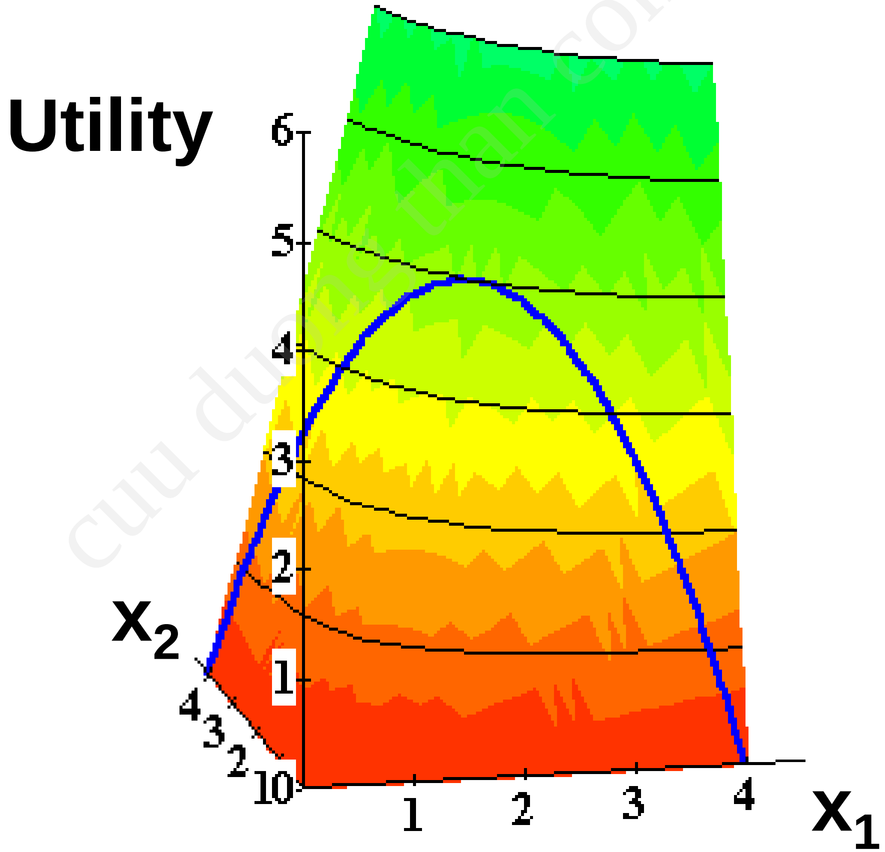

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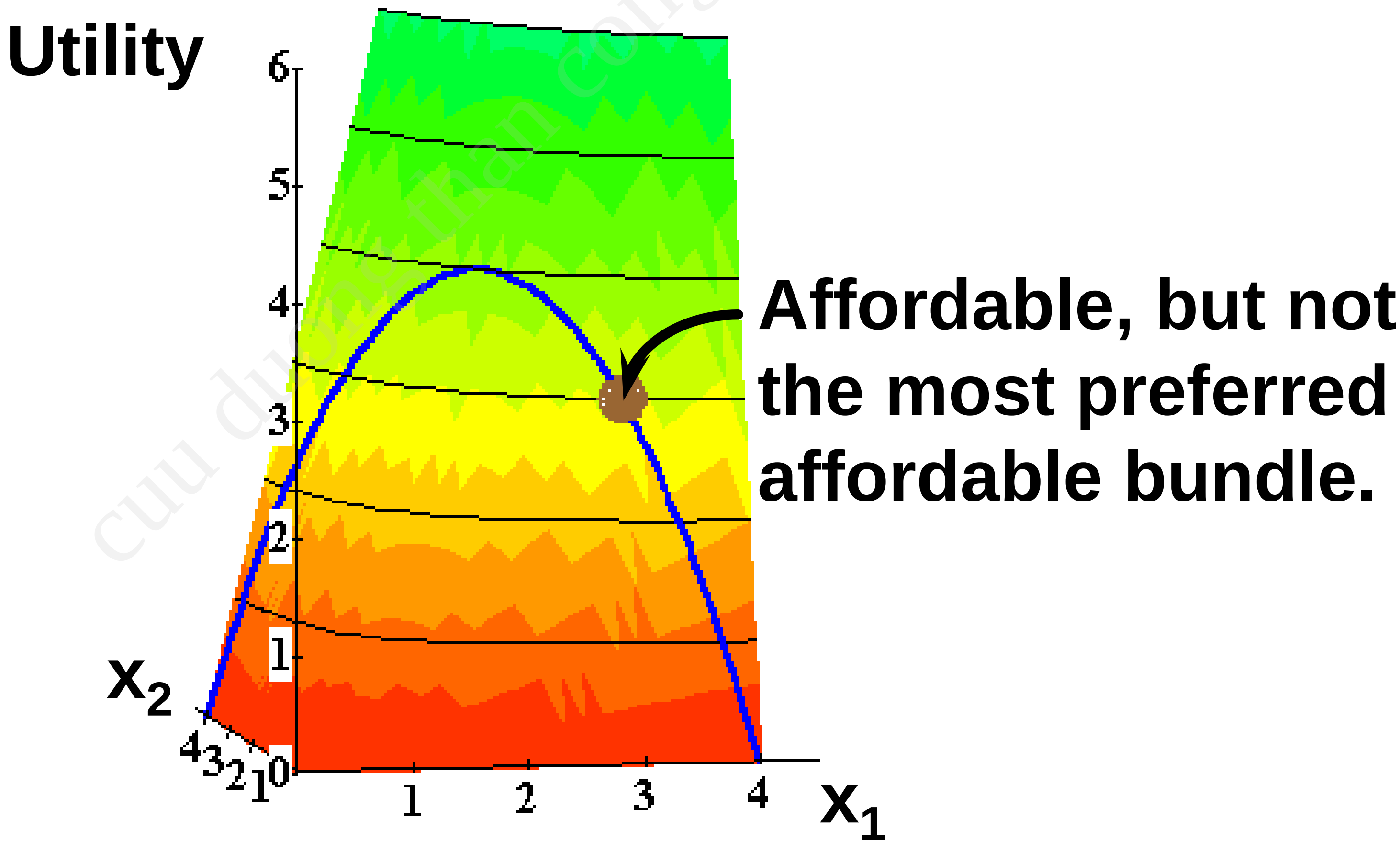

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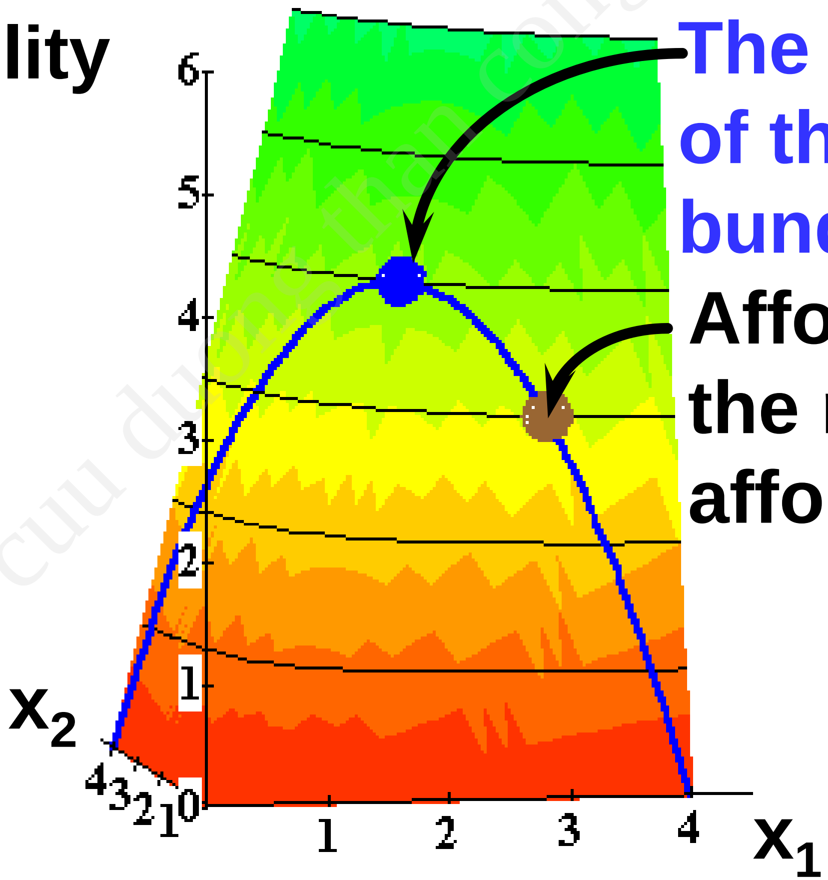
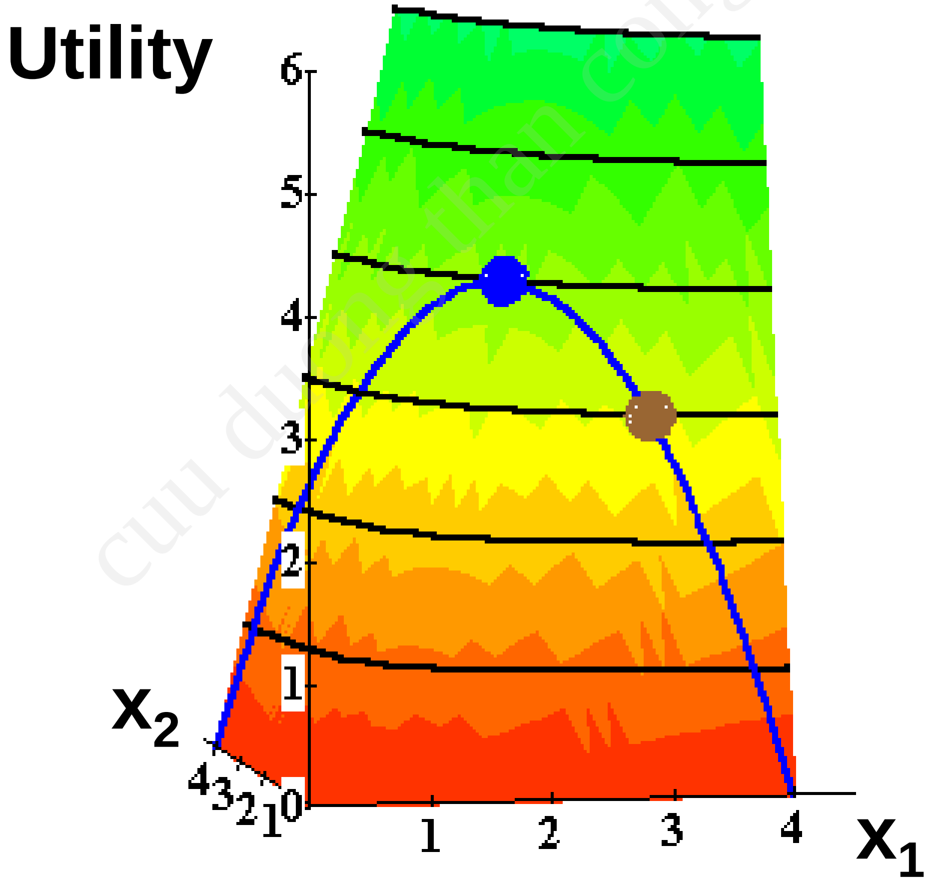


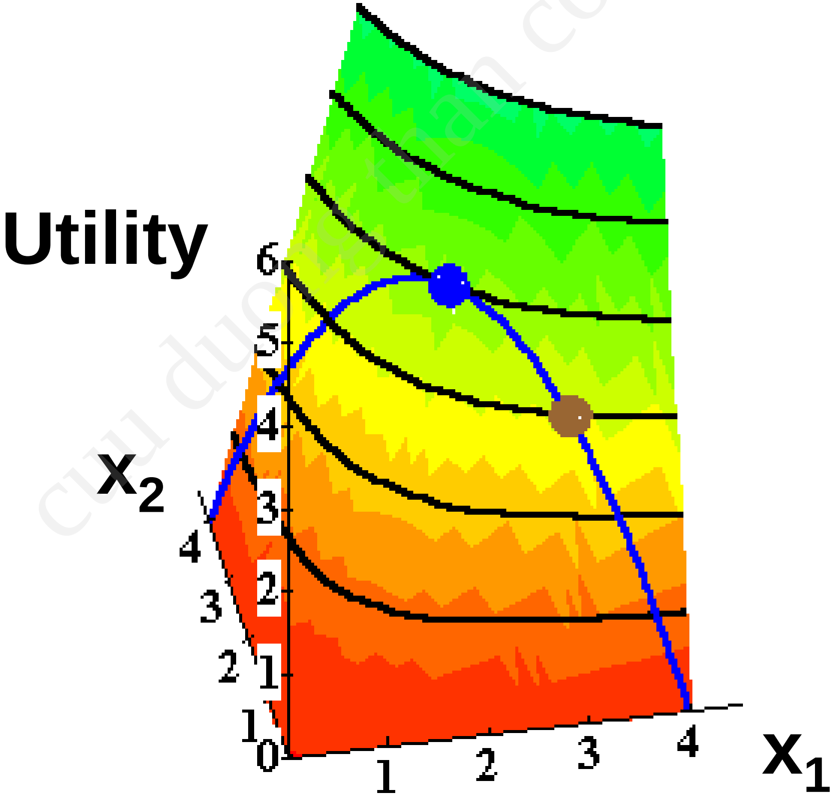
The most preferred of the affordable bundles.

Affordable, but not the most preferred affordable bundle.

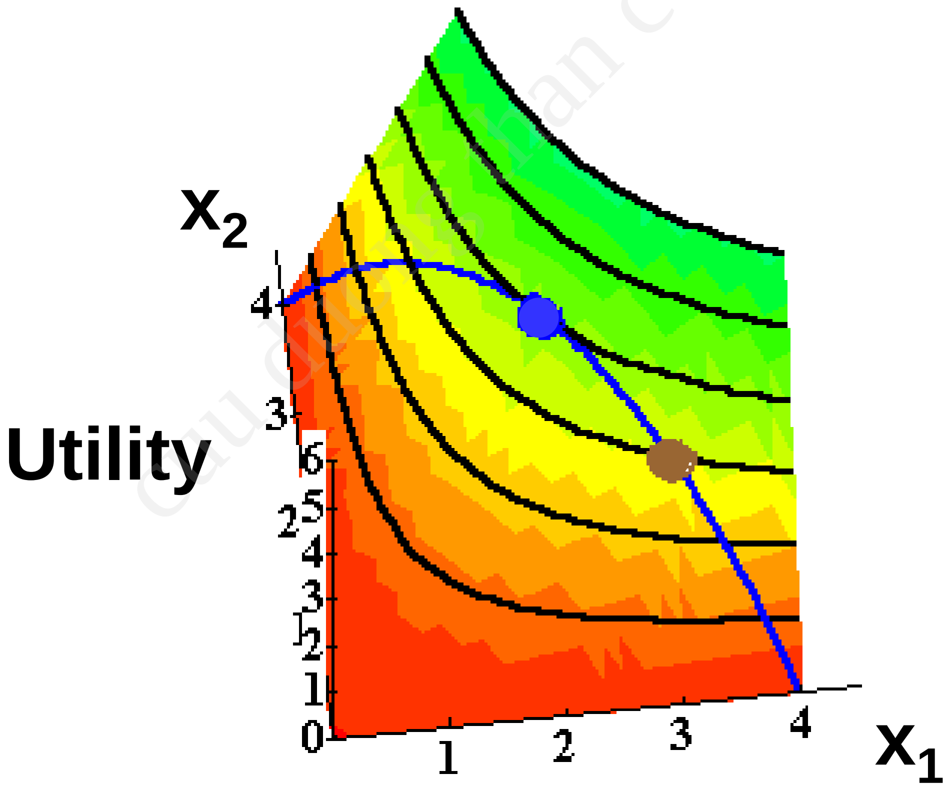
Rational Constrained Choice



Rational Constrained Choice



Rational Constrained Choice



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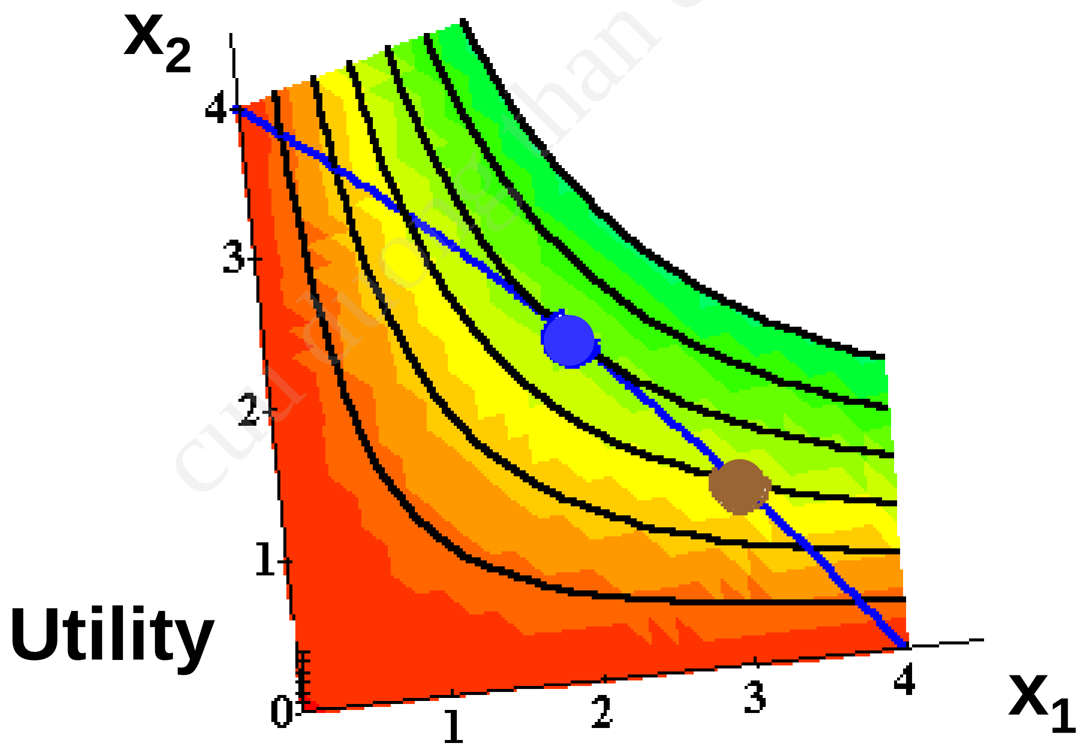

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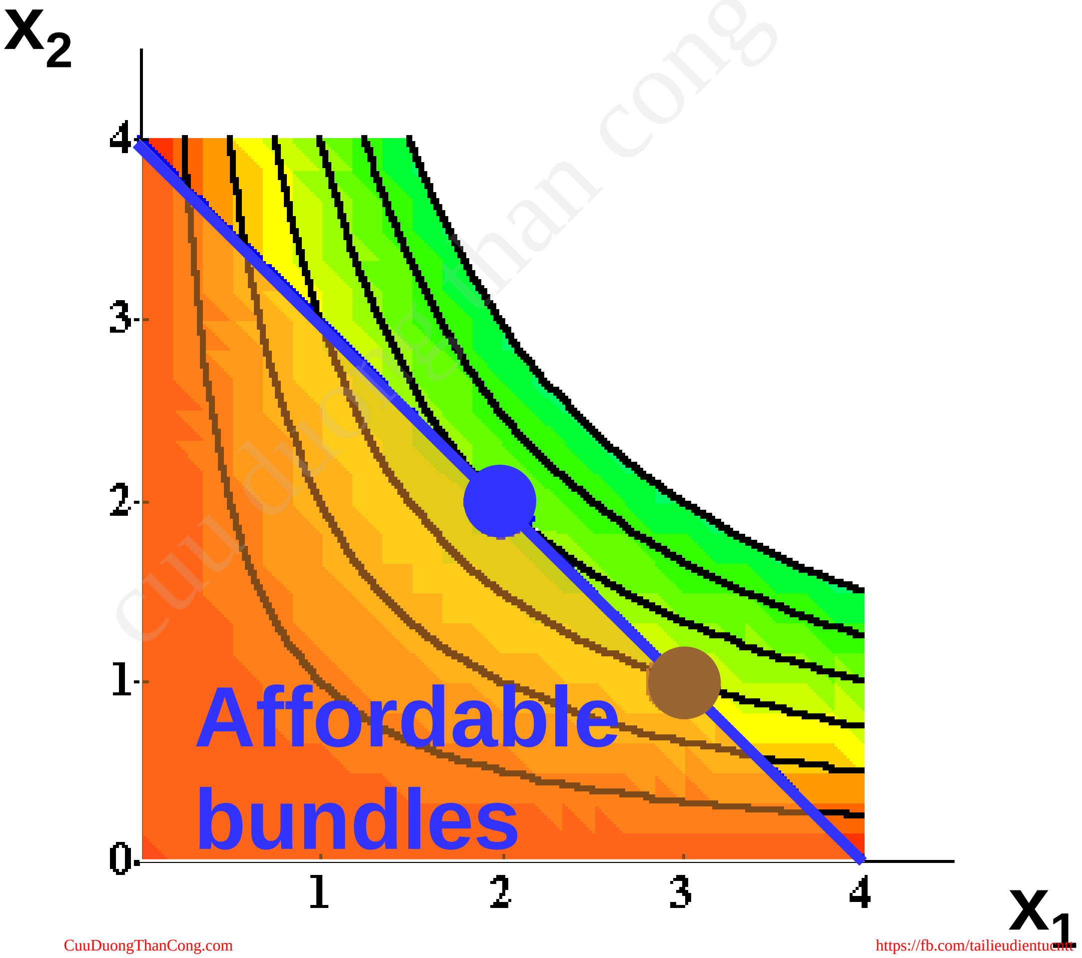

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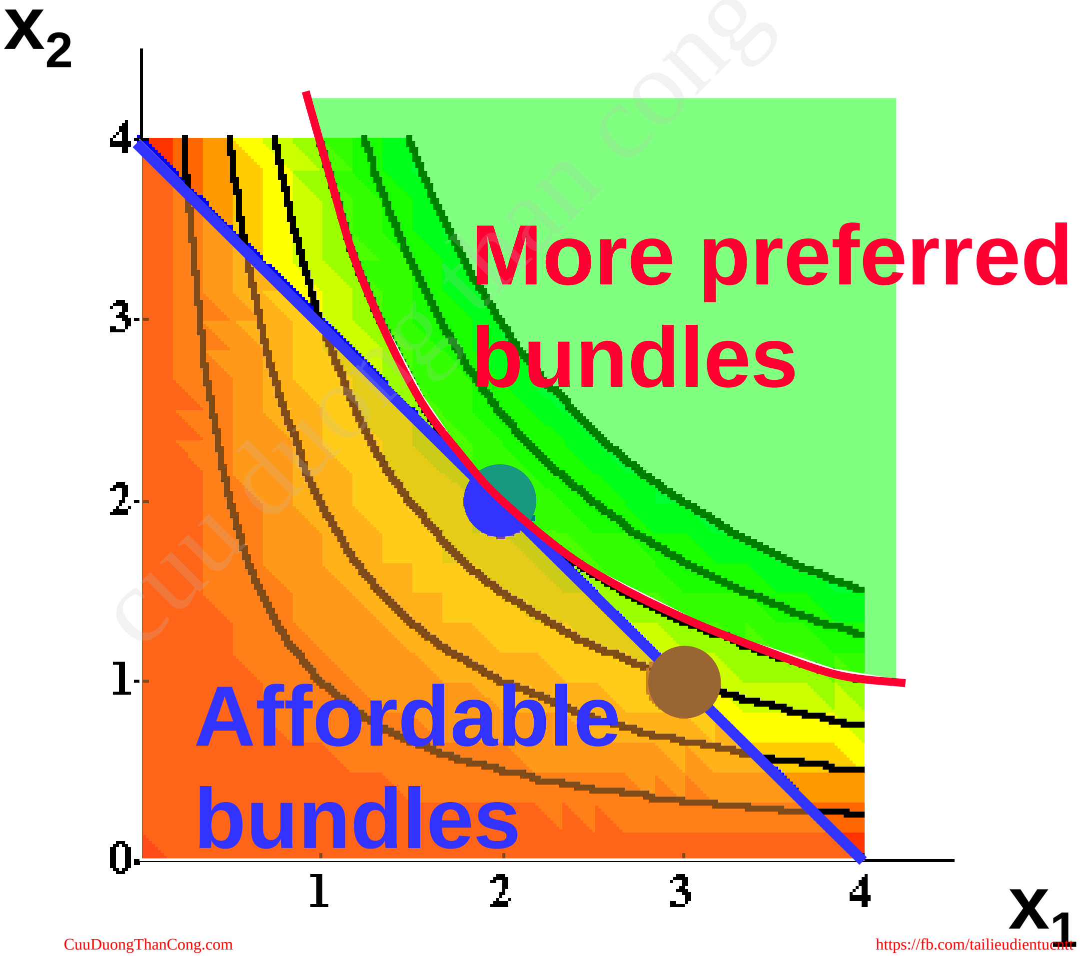

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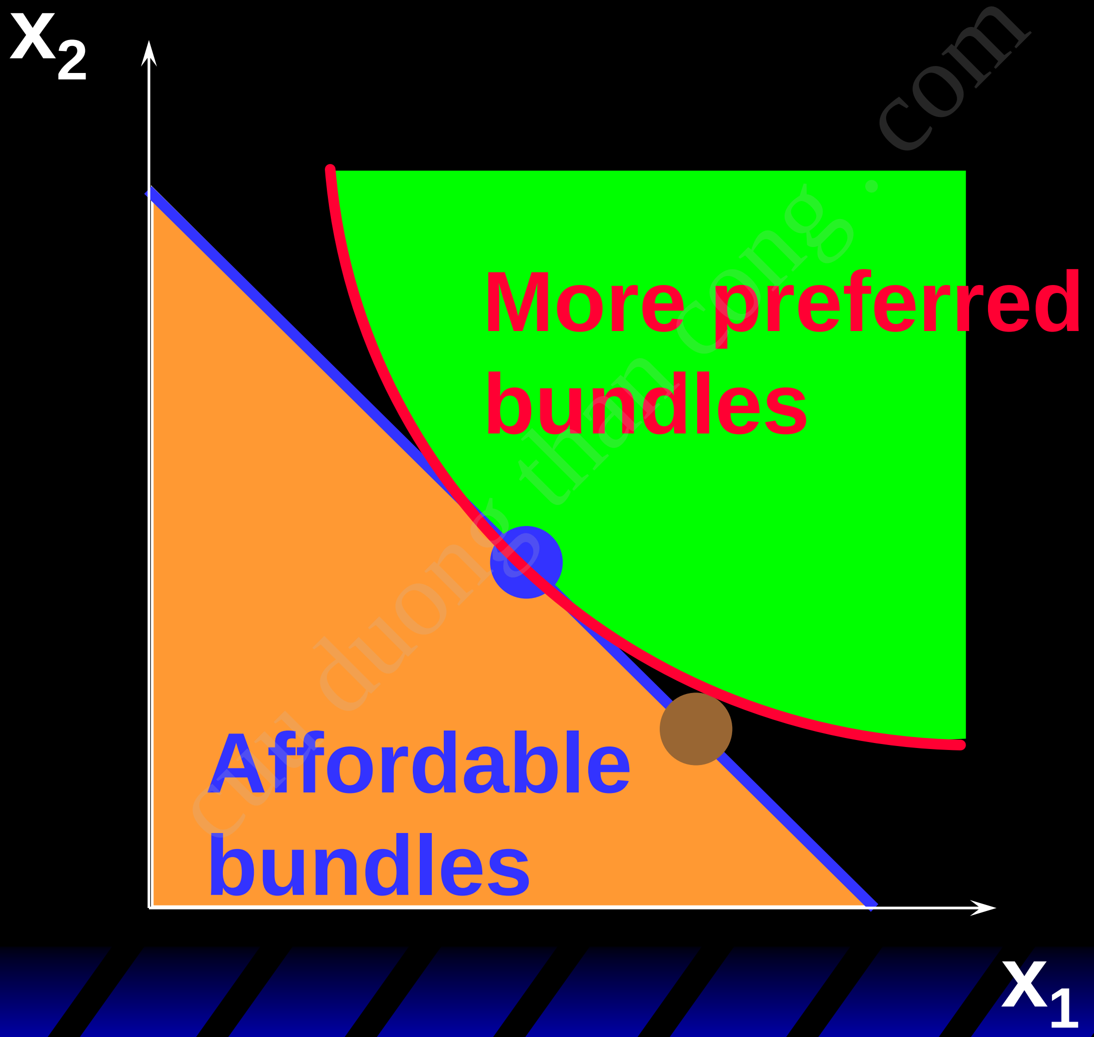
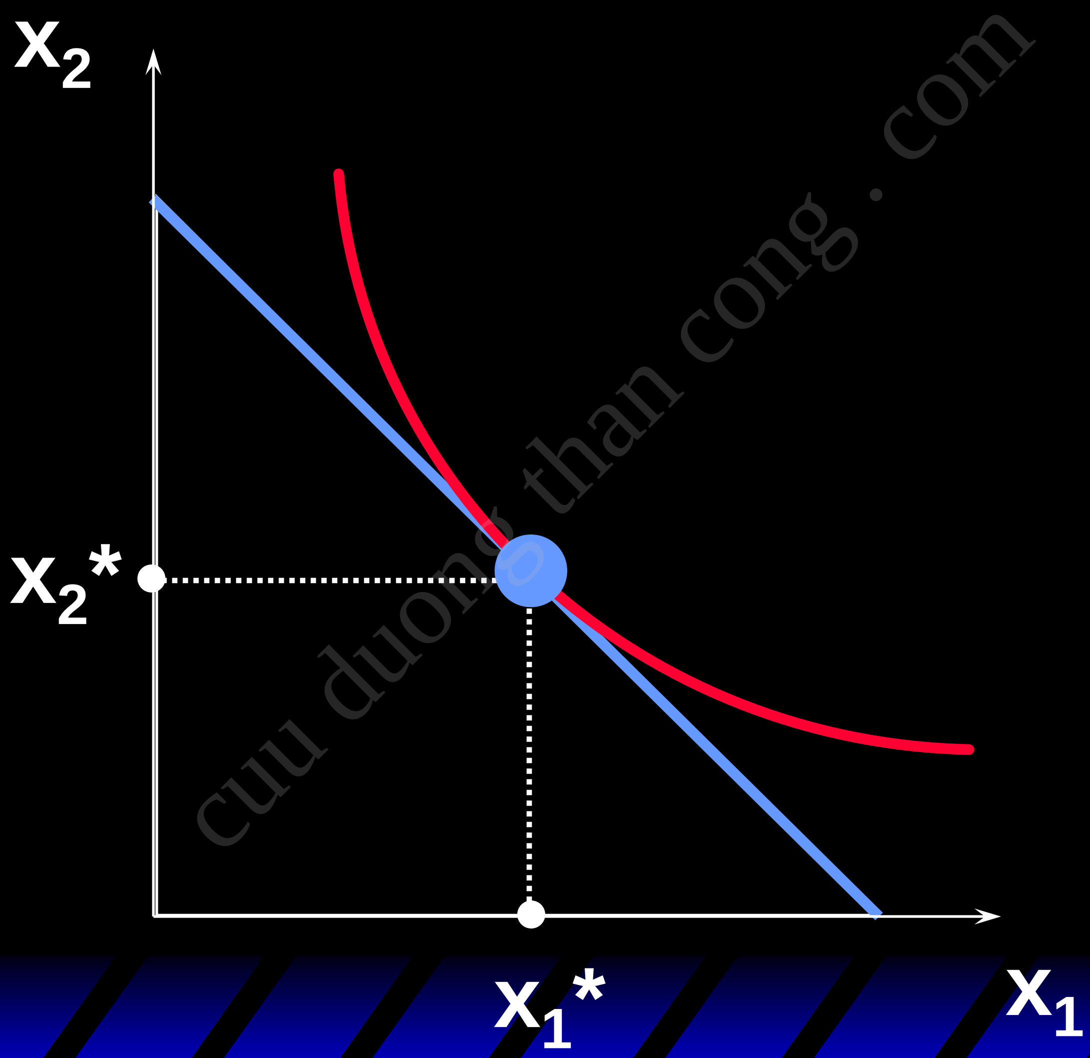
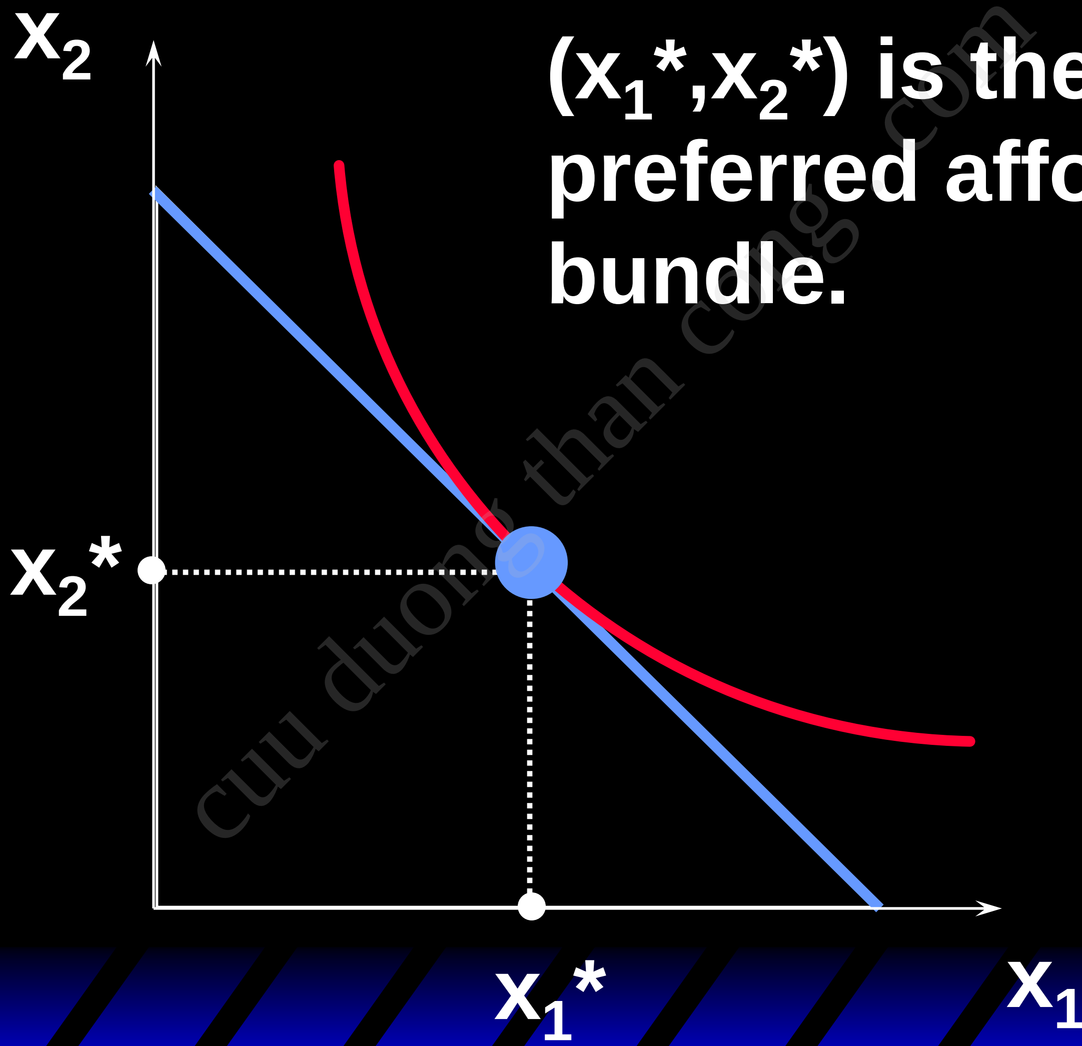


Rational Constrained Choice



Rational Constrained Choice

(x_1^*, x_2^*) is the most preferred affordable bundle.



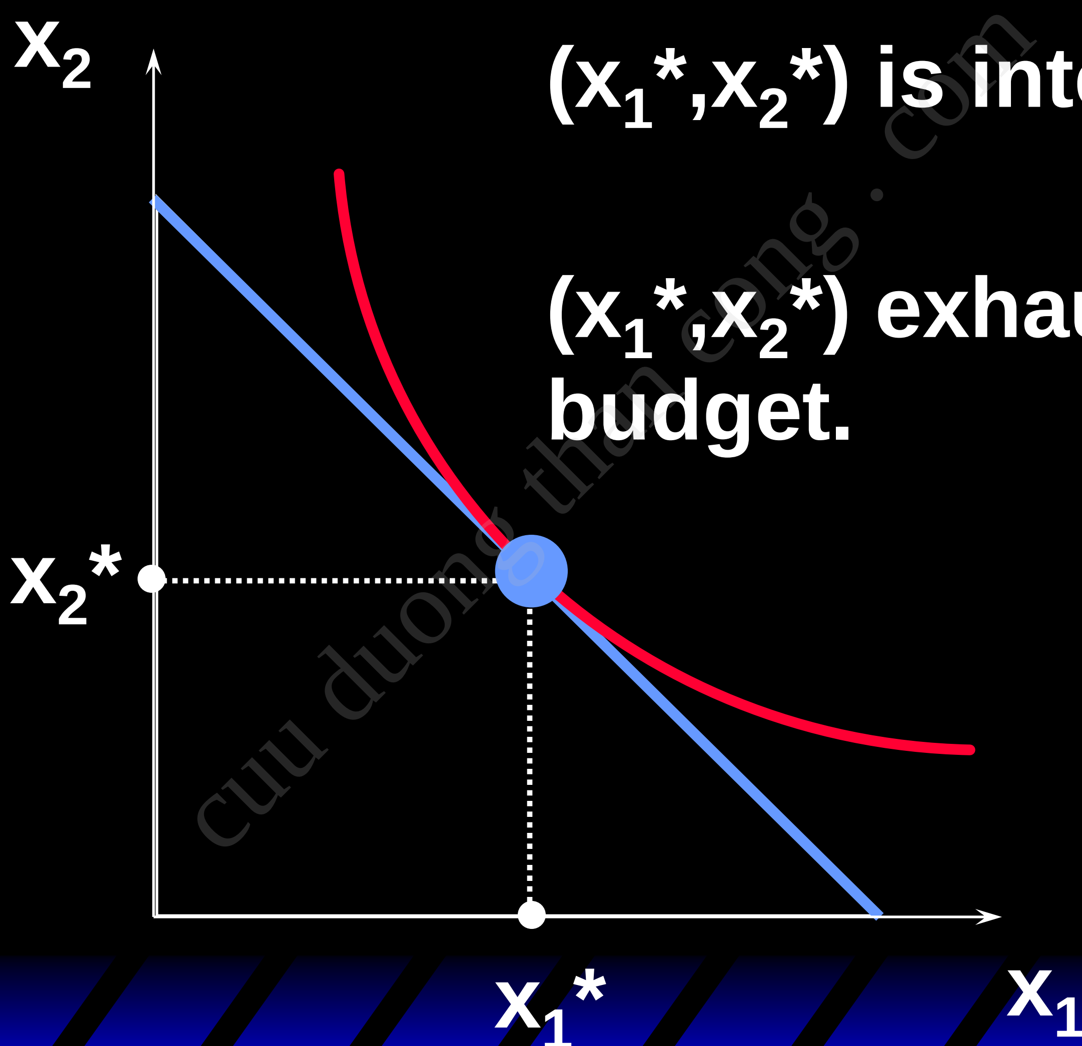
Rational Constrained Choice

- The most preferred affordable bundle is called the consumer's **ORDINARY DEMAND** at the given prices and budget.
- Ordinary demands will be denoted by $x_1*(p_1,p_2,m)$ and $x_2*(p_1,p_2,m)$.

Rational Constrained Choice

- When $x_1^* > 0$ and $x_2^* > 0$ the demanded bundle is **INTERIOR**.
- If buying (x_1^*, x_2^*) costs $m then the budget is exhausted.

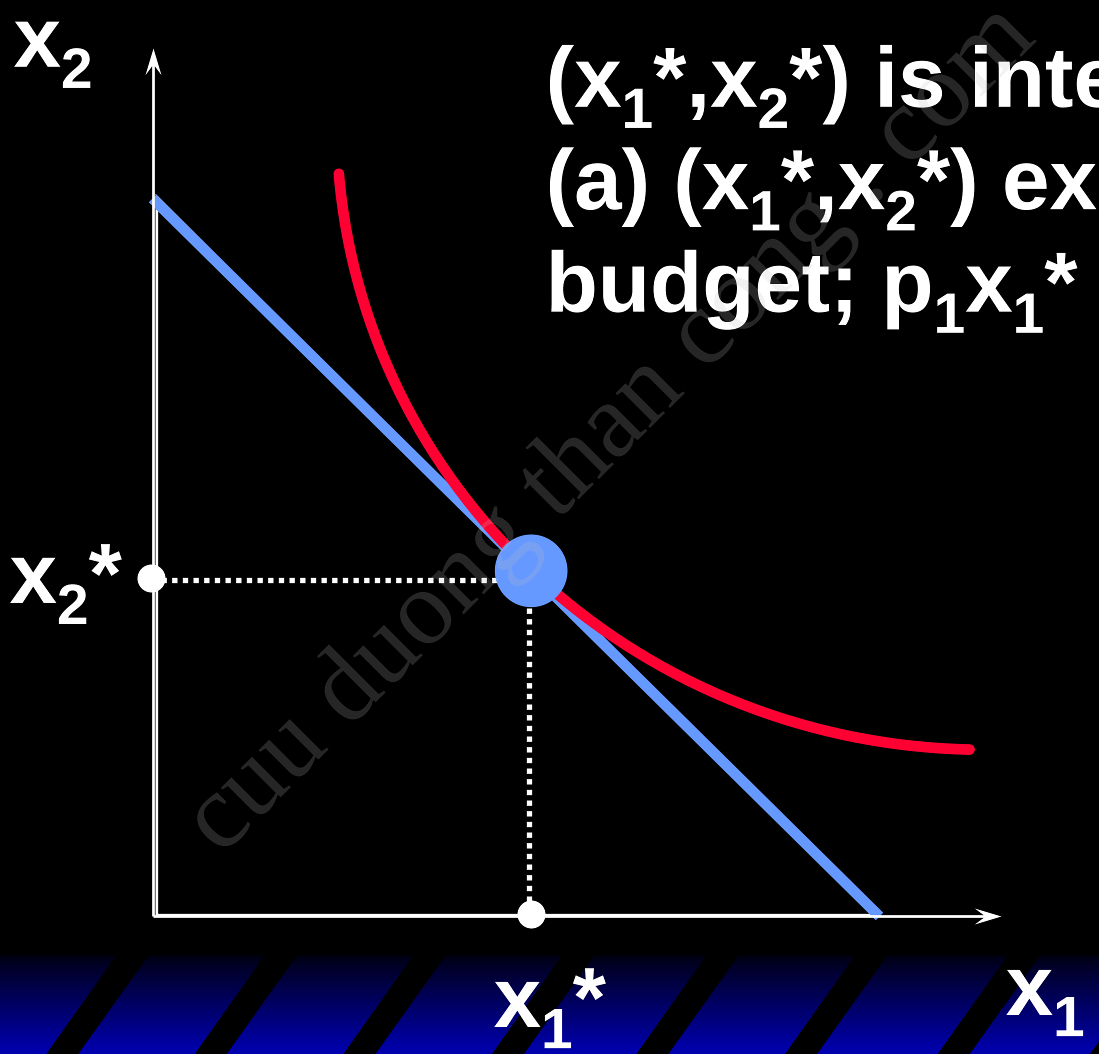
Rational Constrained Choice



(x_1^*, x_2^*) is interior.

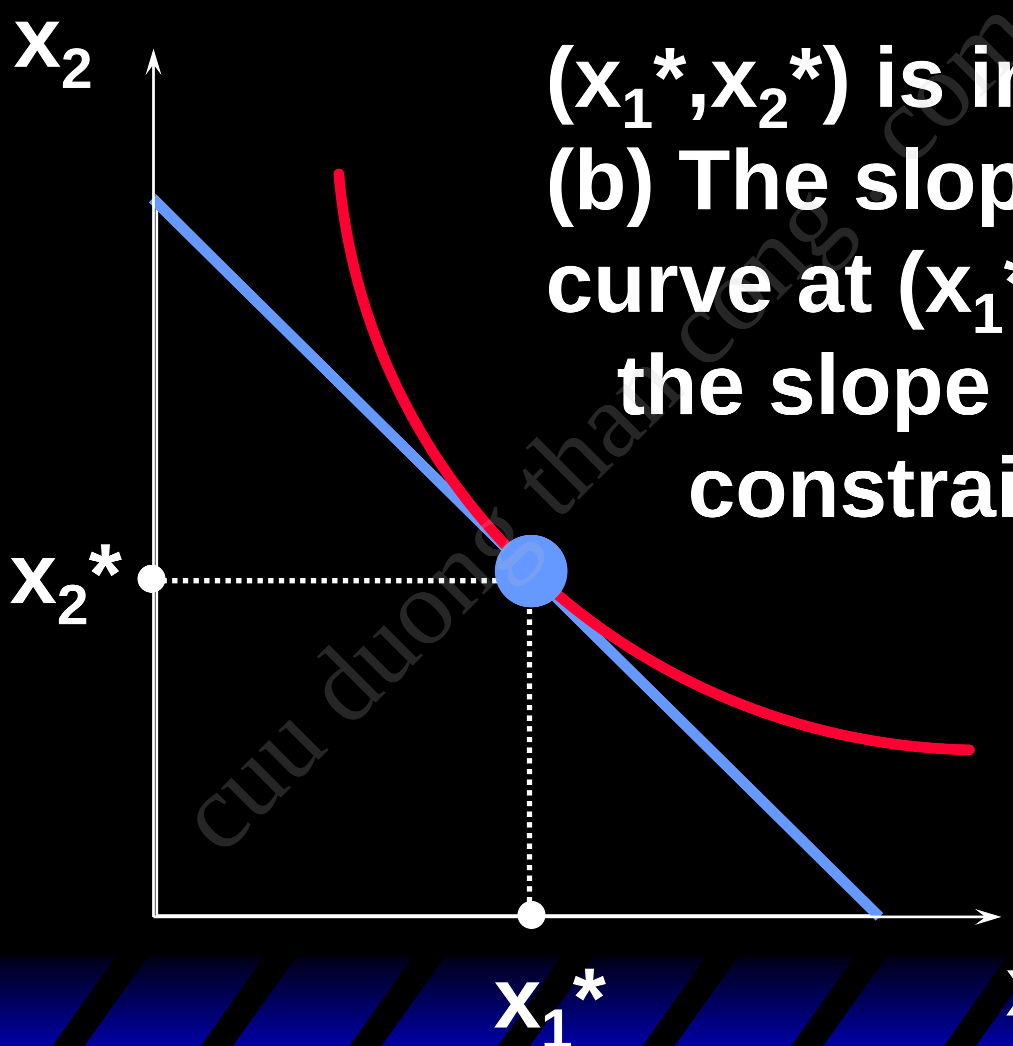
(x_1^*, x_2^*) exhausts the budget.

Rational Constrained Choice



(x_1^*, x_2^*) is interior.
(a) (x_1^*, x_2^*) exhausts the budget; $p_1x_1^* + p_2x_2^* = m$.

Rational Constrained Choice



(x_1^*, x_2^*) is interior .
(b) The slope of the indiff. curve at (x_1^*, x_2^*) equals the slope of the budget constraint.

Rational Constrained Choice

- (x_1^*, x_2^*) satisfies two conditions:
- (a) the budget is exhausted;

$$p_1x_1^* + p_2x_2^* = m$$

- (b) the slope of the budget constraint, $-p_1/p_2$, and the slope of the indifference curve containing (x_1^*, x_2^*) are equal at (x_1^*, x_2^*).

Computing Ordinary Demands - a Cobb-Douglas Example.

- **Suppose that the consumer has Cobb-Douglas preferences.**

$$U(x_1, x_2) = x_1^a x_2^b$$

- **Then** $$MU_1 = \frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^b$$

$$MU_2 = \frac{\partial U}{\partial x_2} = b x_1^a x_2^{b-1}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So the MRS is

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^{b}}{b x_1^{a} x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So the MRS is

$$MRS = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{ax_1^{a-1}x_2^{b}}{bx_1^{a}x_2^{b-1}} = -\frac{ax_2}{bx_1}.$$

- At (x_1^*, x_2^*), MRS = $-p_1/p_2$ so

Computing Ordinary Demands - a Cobb-Douglas Example.

- So the MRS is

$$\text{MRS} = \frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} = -\frac{a x_1^{a-1} x_2^b}{b x_1^a x_2^{b-1}} = -\frac{a x_2}{b x_1}.$$

- At (x_1*,x_2*), MRS = -p_1/p_2 so

$$-\frac{a x_2^*}{b x_1^*} = -\frac{p_1}{p_2} \quad \Rightarrow \quad x_2^* = \frac{b p_1}{a p_2} x_1^*. \qquad \text{(A)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- (x_1^*, x_2^*) also exhausts the budget so

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \textbf{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So now we know that

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \qquad \text{(A)}$$

$$p_1 x_1^* + p_2 x_2^* = m. \qquad \text{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So now we know that

$$x_2^* = \frac{bp_1}{ap_2}x_1^* \qquad \text{(A)}$$

Substitute

$$p_1x_1^* + p_2x_2^* = m. \qquad \text{(B)}$$

Computing Ordinary Demands - a Cobb-Douglas Example.

- So now we know that

$$x_2^* = \frac{bp_1}{ap_2} x_1^* \quad \text{(A)}$$

Substitute

$$p_1 x_1^* + p_2 x_2^* = m. \quad \text{(B)}$$

and get

$$p_1 x_1^* + p_2 \frac{bp_1}{ap_2} x_1^* = m.$$

This simplifies to ….

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

$$x_1^* = \frac{am}{(a+b)p_1}.$$

Substituting for x_1^* in

$$p_1 x_1^* + p_2 x_2^* = m$$

then gives

$$x_2^* = \frac{bm}{(a+b)p_2}.$$

Computing Ordinary Demands - a Cobb-Douglas Example.

So we have discovered that the most preferred affordable bundle for a consumer with Cobb-Douglas preferences

$$U(x_1, x_2) = x_1^a x_2^b$$

is

$$(x_1^*, x_2^*) = \left(\frac{am}{(a+b)p_1}, \frac{bm}{(a+b)p_2}\right).$$

Rational Constrained Choice

- When $x_1^* > 0$ and $x_2^* > 0$
  and (x_1^*,x_2^*) exhausts the budget,
  and indifference curves have no 'kinks', the ordinary demands are obtained by solving:
- (a) $p_1x_1^* + p_2x_2^* = y$
- (b) the slopes of the budget constraint, $-p_1/p_2$, and of the indifference curve containing (x_1^*,x_2^*) are equal at (x_1^*,x_2^*).

Rational Constrained Choice

- But what if $x_1^* = 0$?
- Or if $x_2^* = 0$?
- If either $x_1^* = 0$ or $x_2^* = 0$ then the ordinary demand (x_1^*, x_2^*) is at a **corner solution** to the problem of maximizing utility subject to a budget constraint.

2. Demand

- Income offer curve and Engel curve

- Price offer curve and demand curve

- Inverse demand function

Properties of Demand Functions

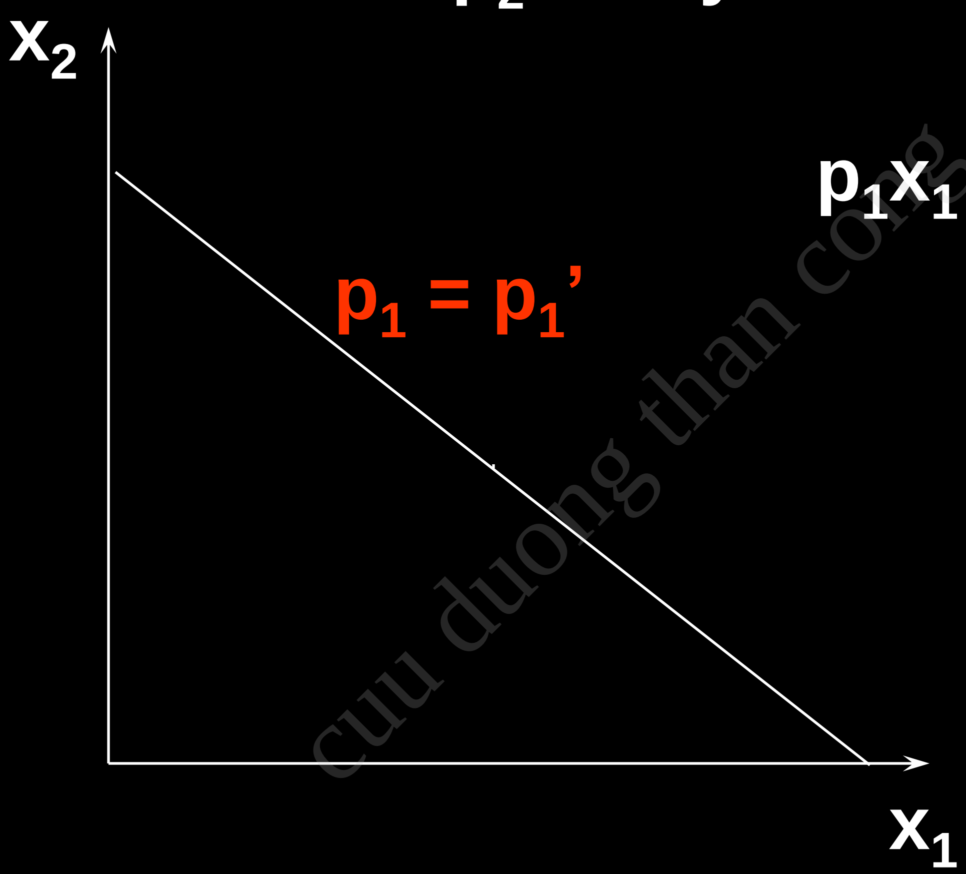
- **Comparative statics analysis** of ordinary demand functions -- the study of how ordinary demands $x_1^*(p_1,p_2,y)$ and $x_2^*(p_1,p_2,y)$ change as prices p_1, p_2 and income y change.

Own-Price Changes

- How does $x_1^*(p_1,p_2,y)$ change as p_1 changes, holding p_2 and y constant?
- Suppose only p_1 increases, from p_1' to p_1'' and then to p_1'''.

Own-Price Changes

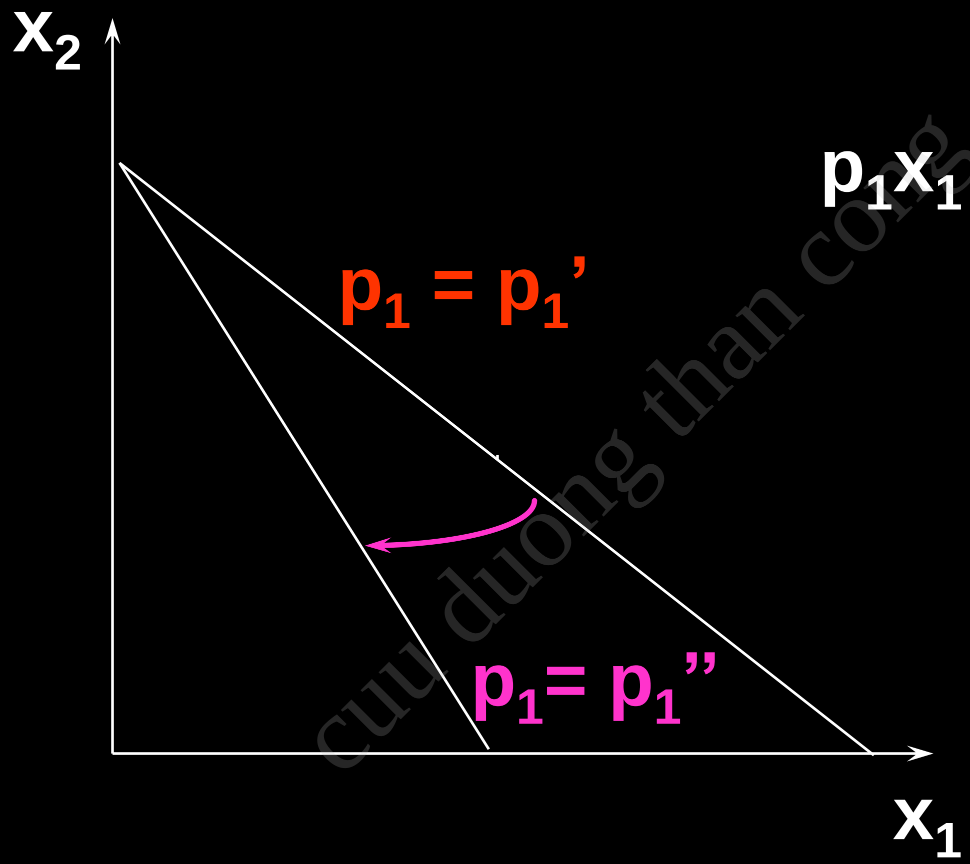
Fixed p_2 and y.



$$p_1x_1 + p_2x_2 = y$$

Own-Price Changes

Fixed p_2 and y.



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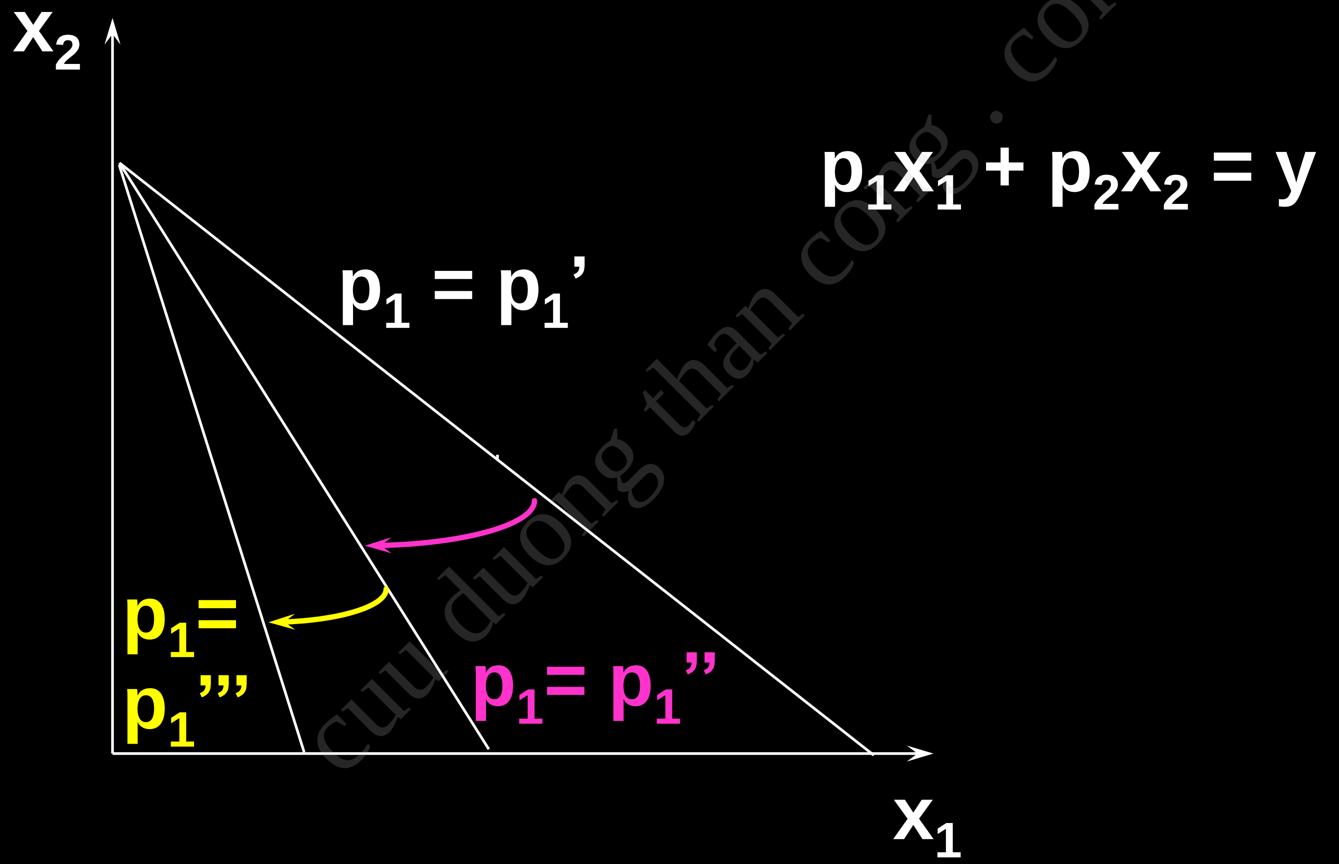

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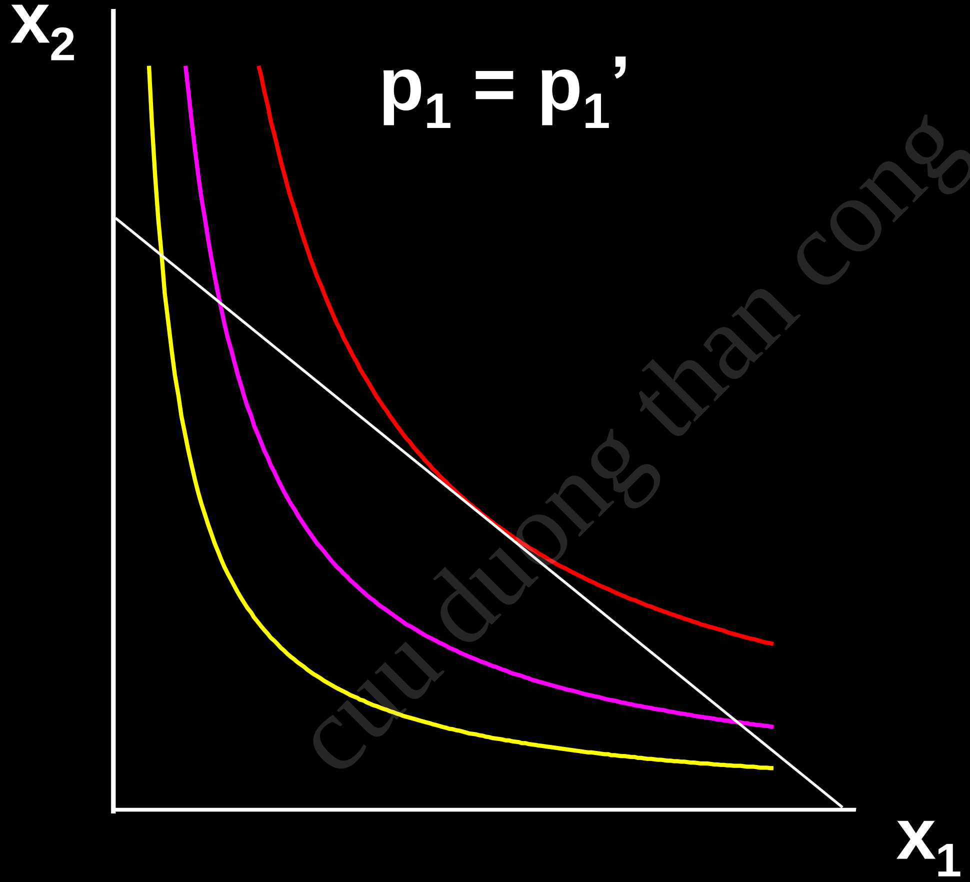

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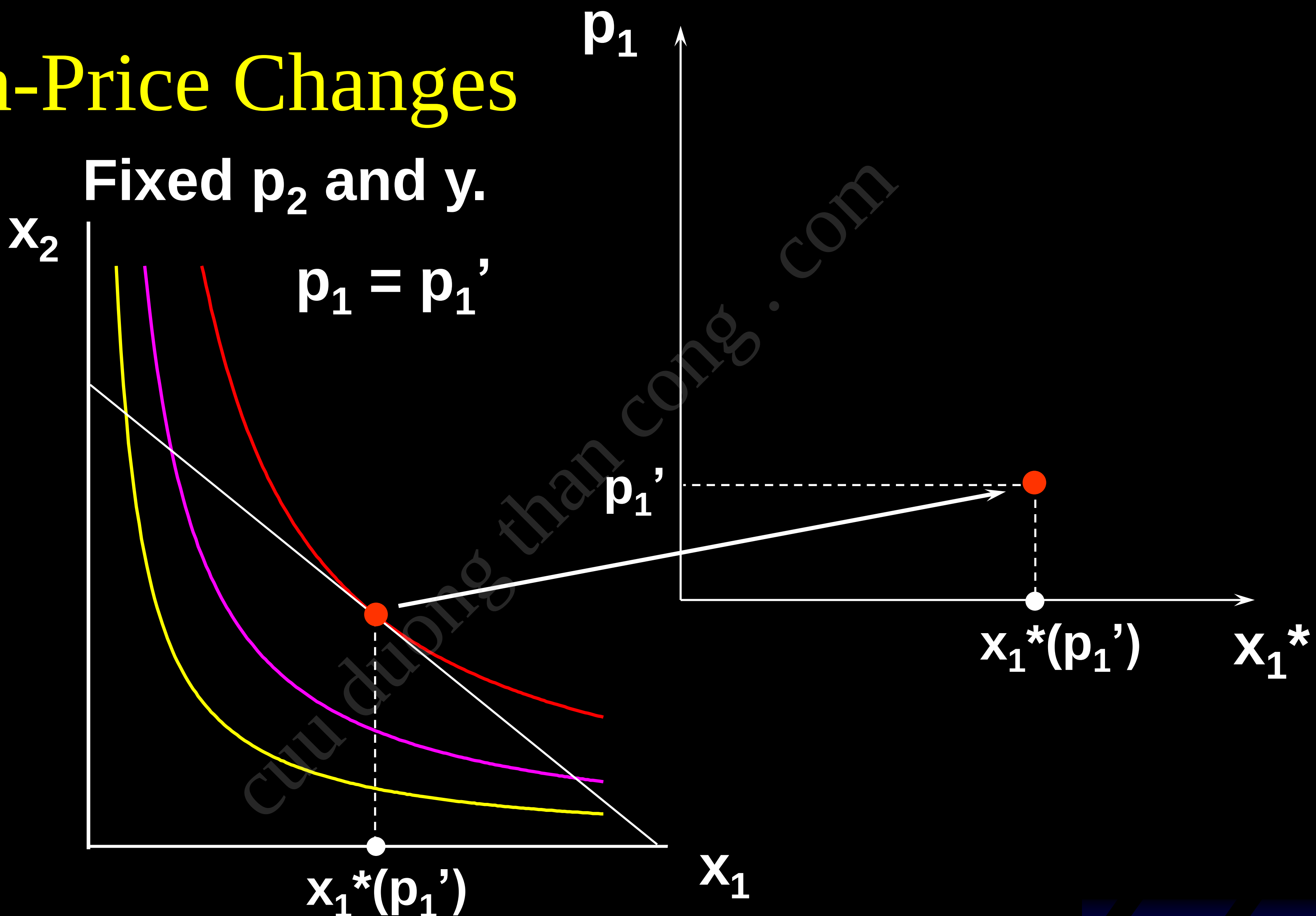

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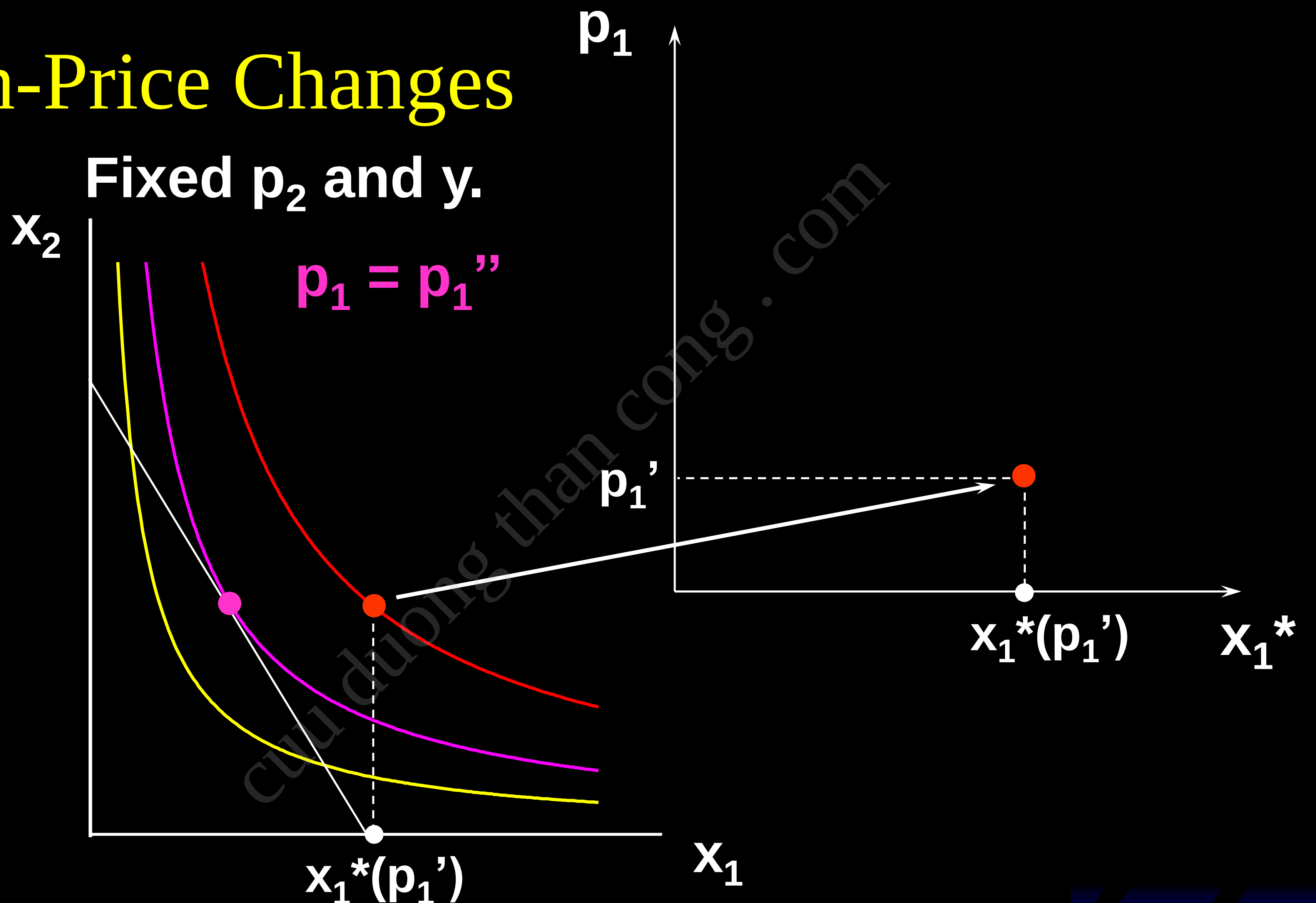

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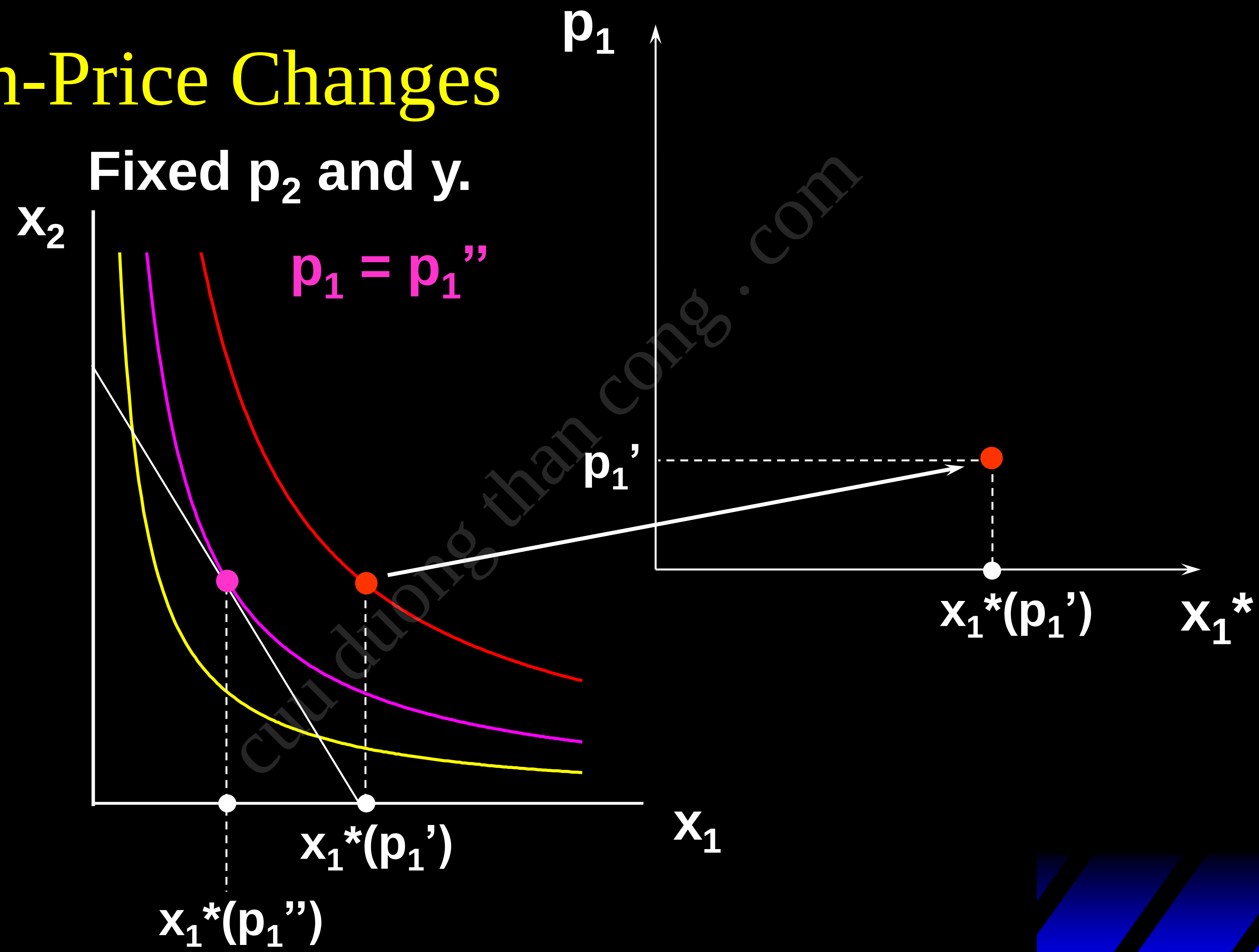

Own-Price Changes

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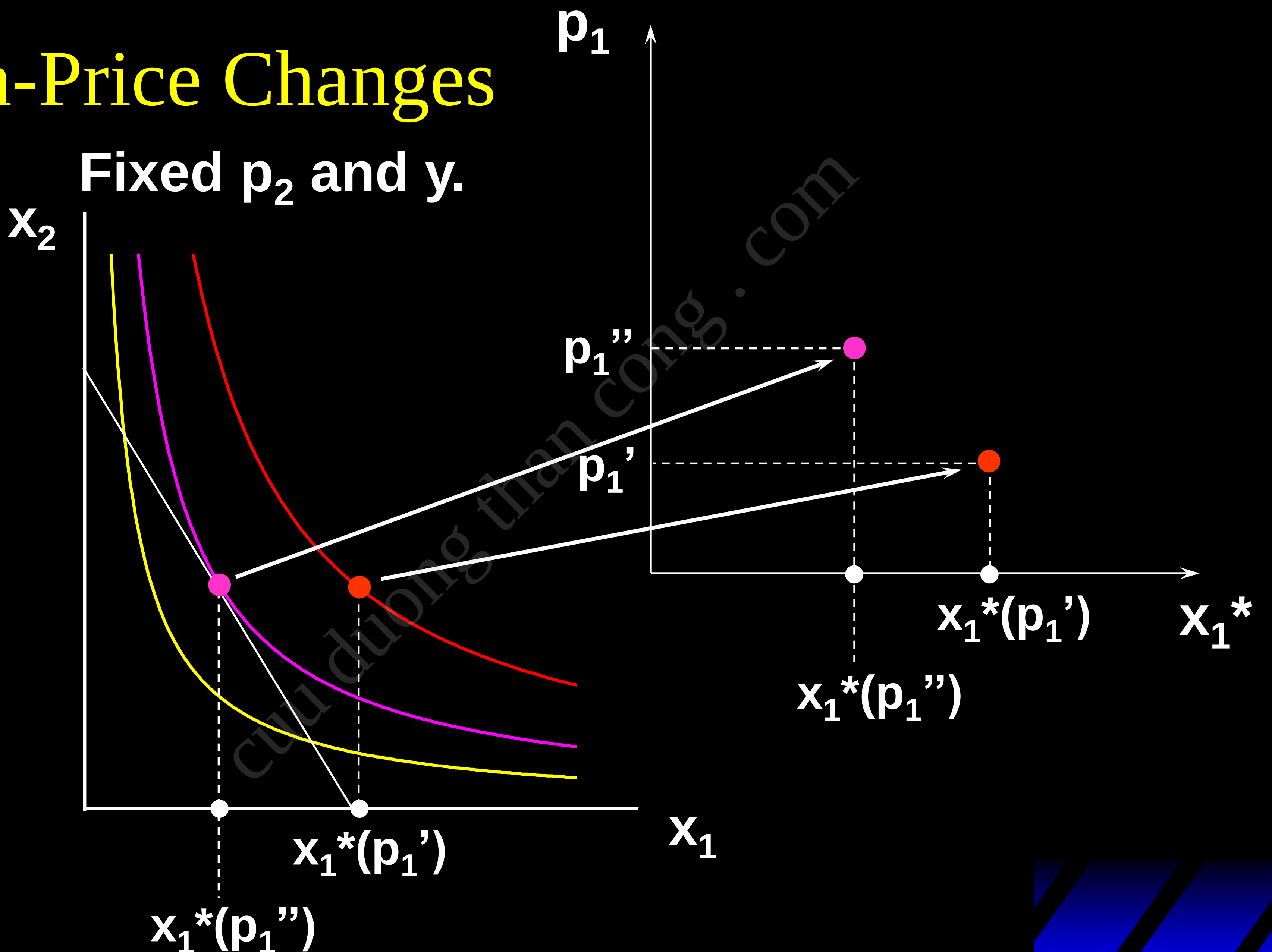

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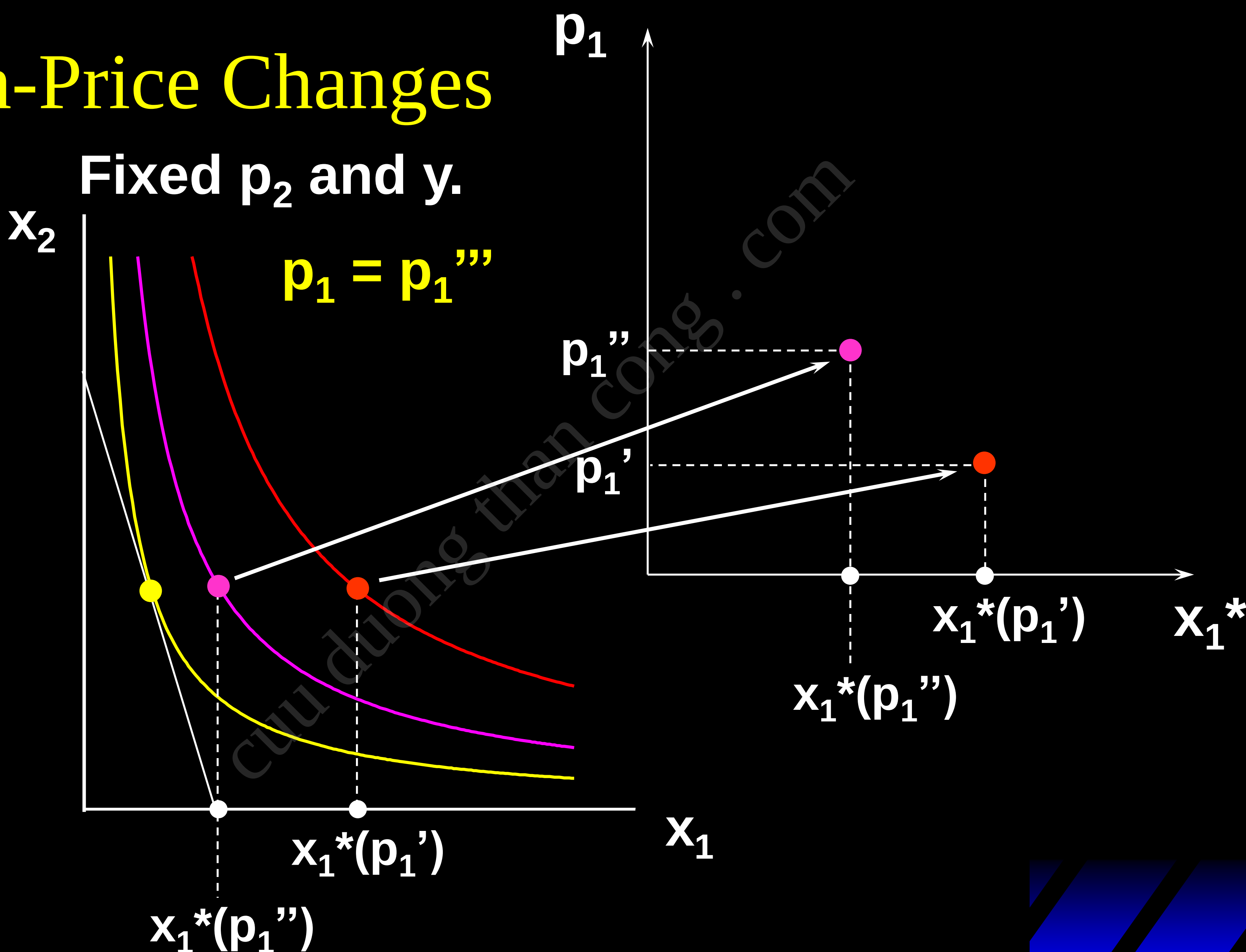

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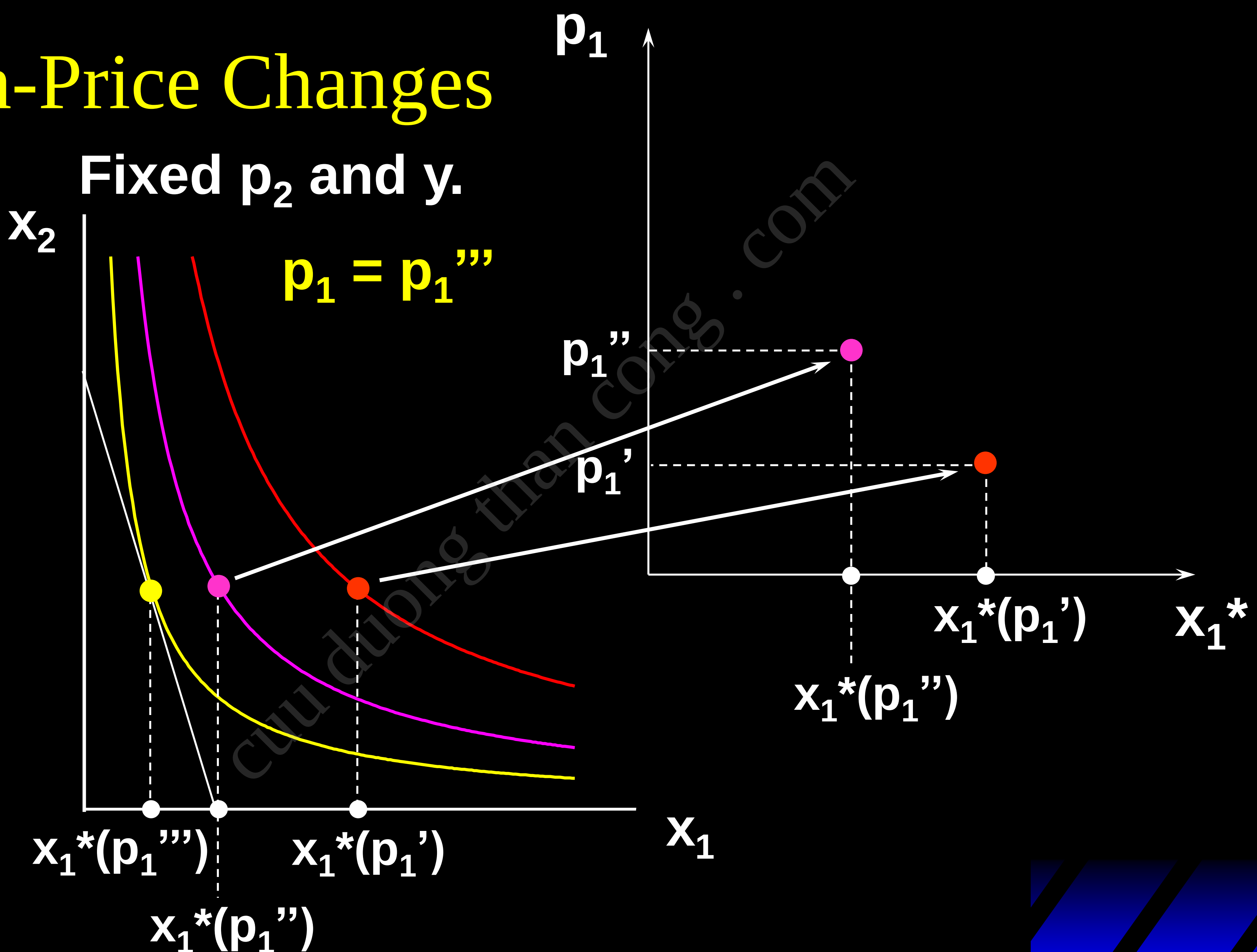

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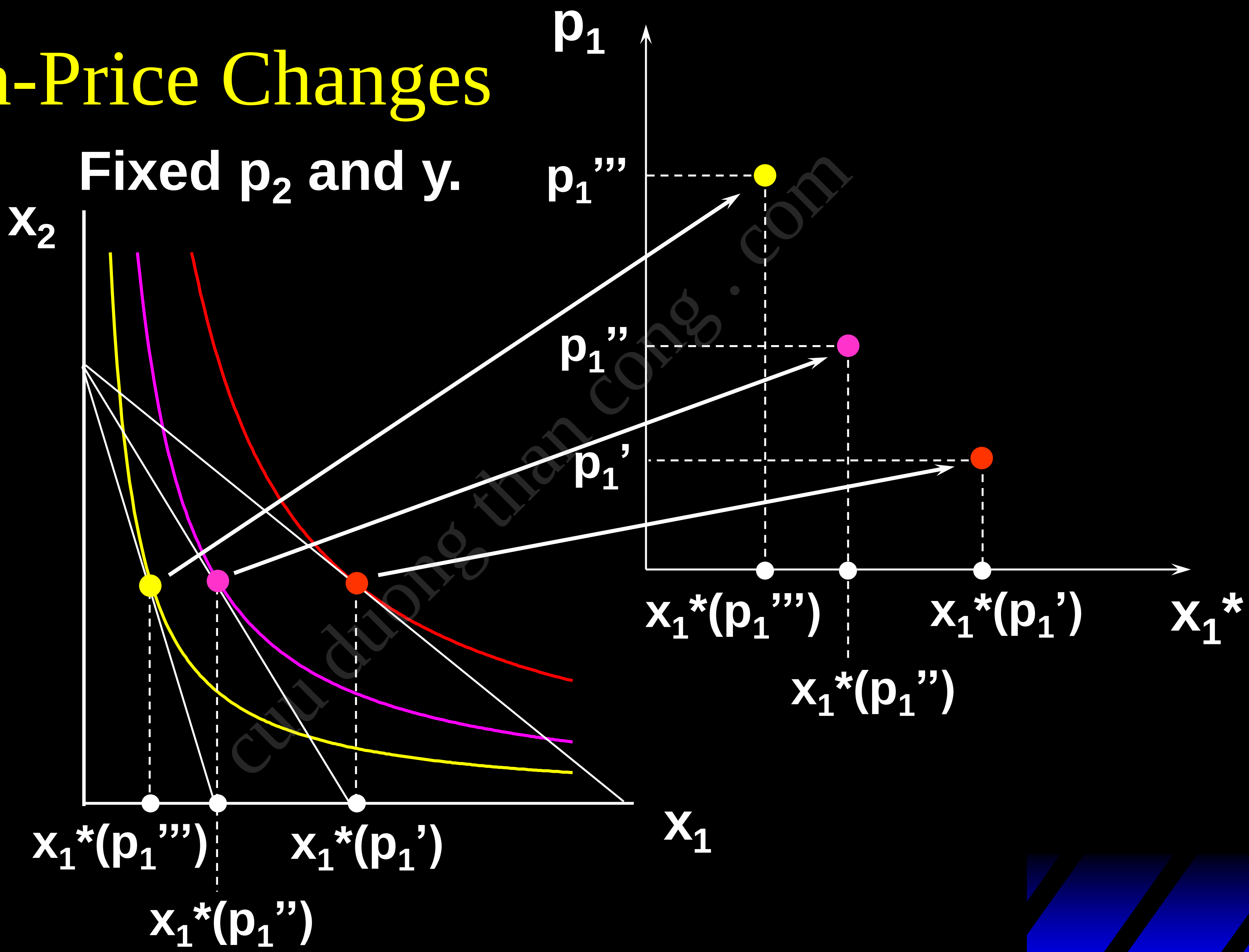

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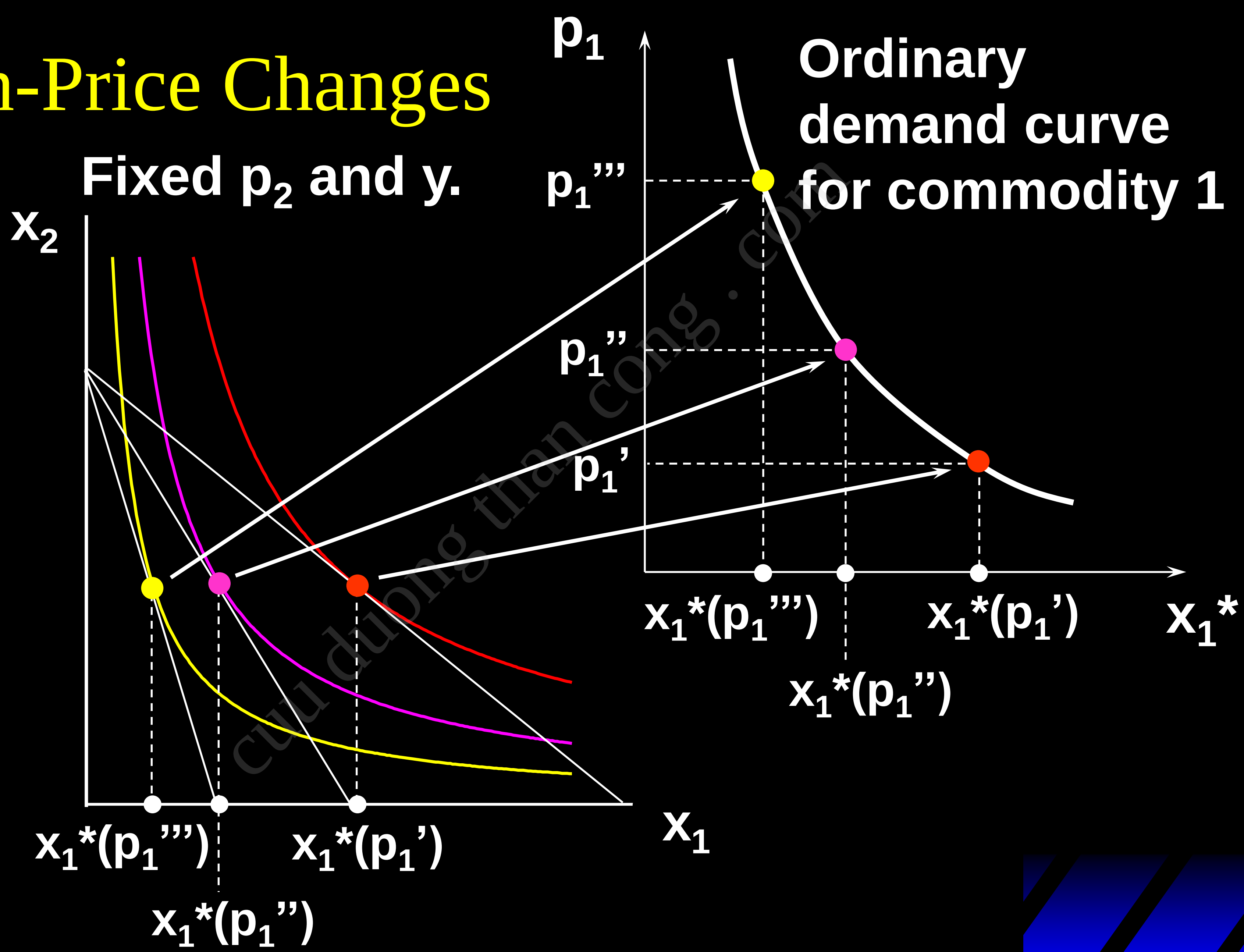

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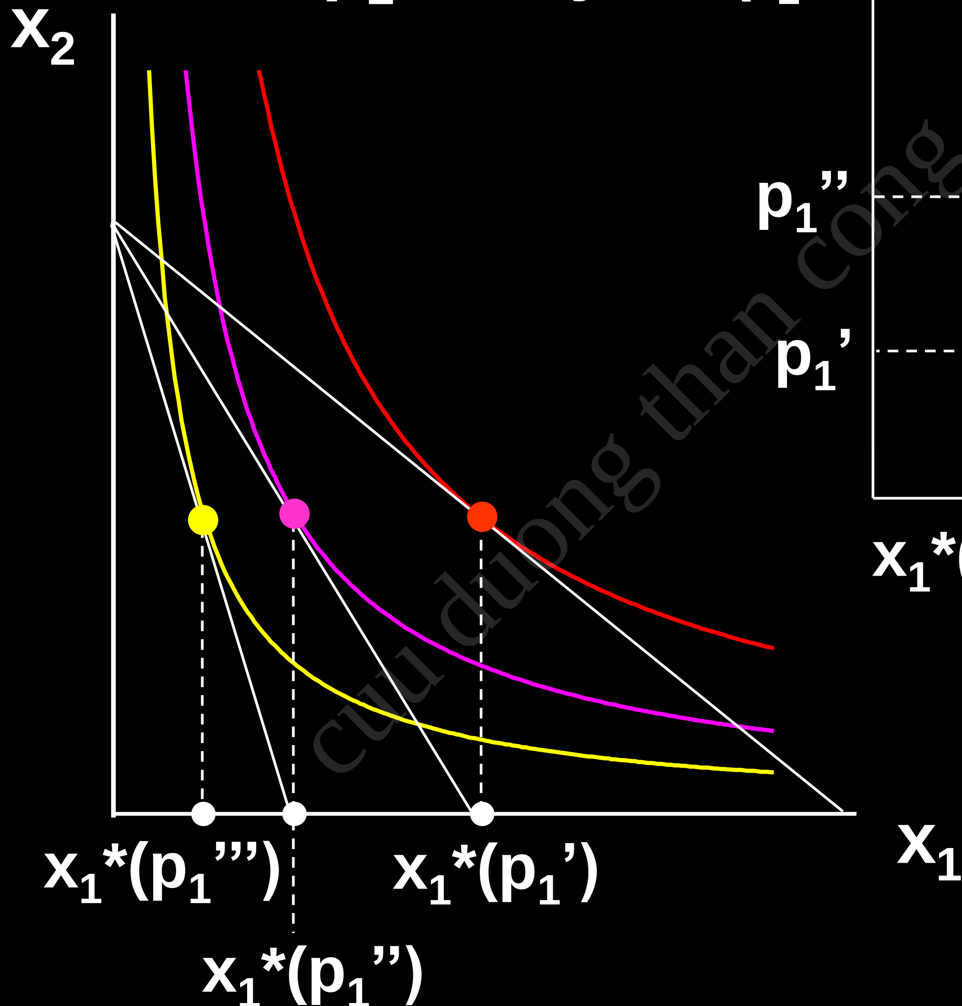

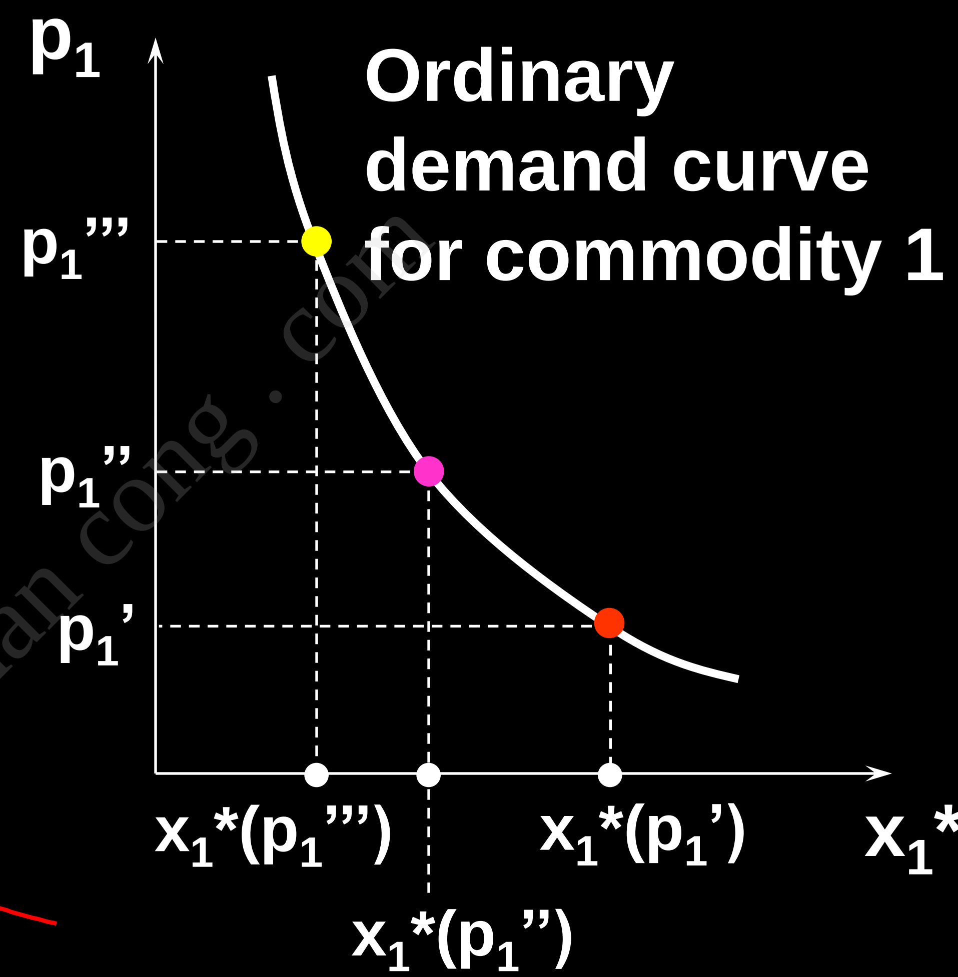

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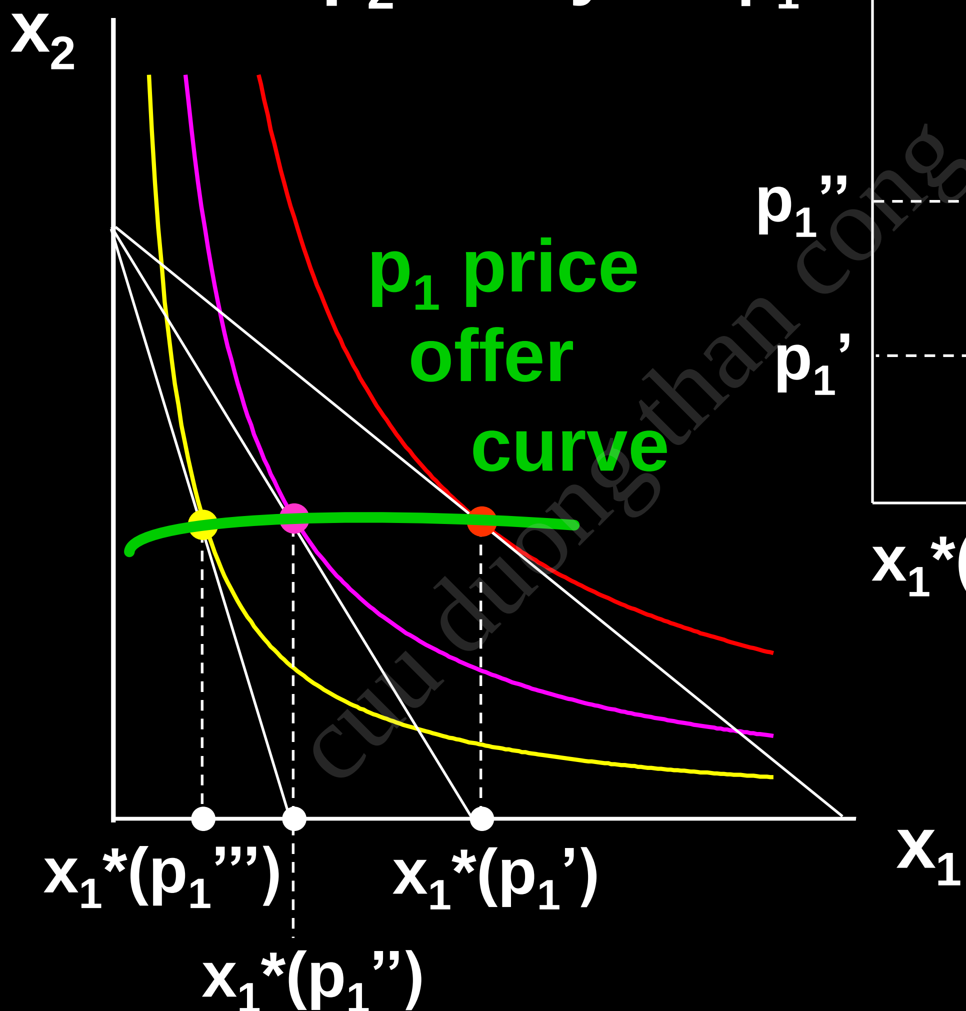
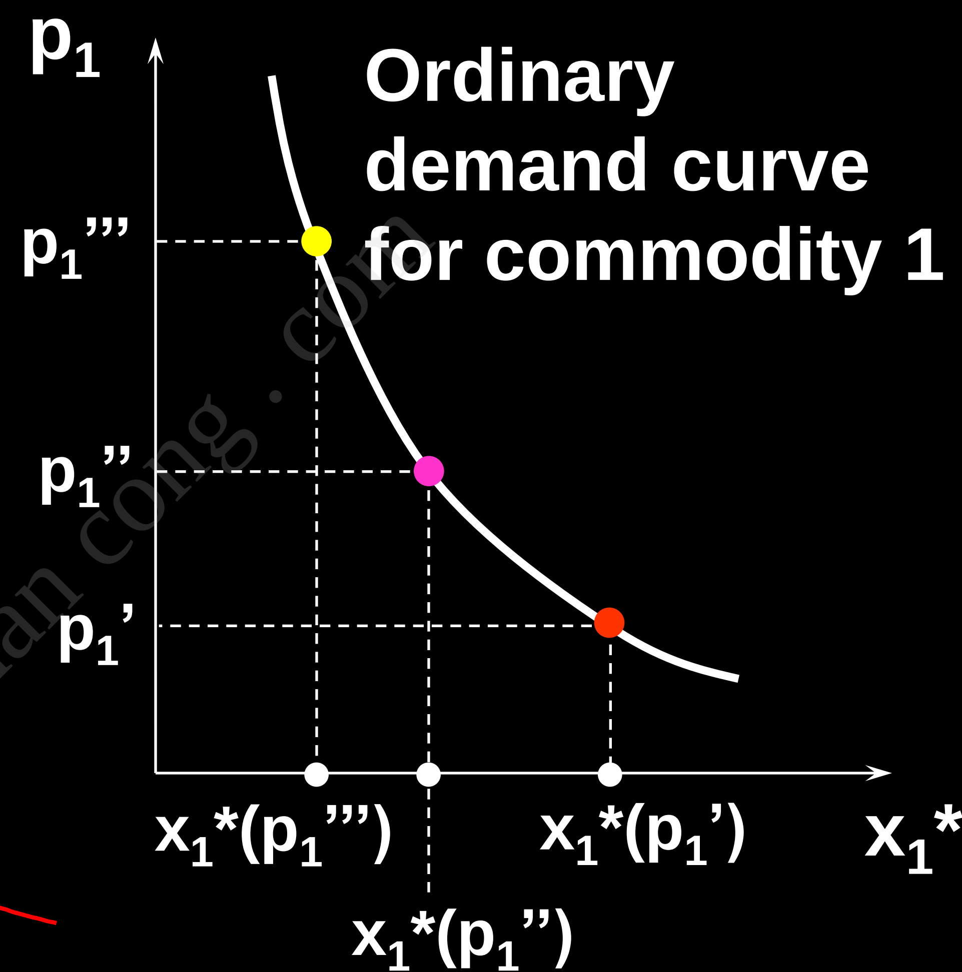




Own-Price Changes

- The curve containing all the utility-maximizing bundles traced out as p_1 changes, with p_2 and y constant, is the p_1- price offer curve.
- The plot of the x_1-coordinate of the p_1- price offer curve against p_1 is the ordinary demand curve for commodity 1.

Own-Price Changes

- What does a p_1 price-offer curve look like for Cobb-Douglas preferences?
- Take

$$U(x_1, x_2) = x_1^a x_2^b.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$x_1^*(p_1, p_2, y) = \frac{a}{a+b} \times \frac{y}{p_1}$$

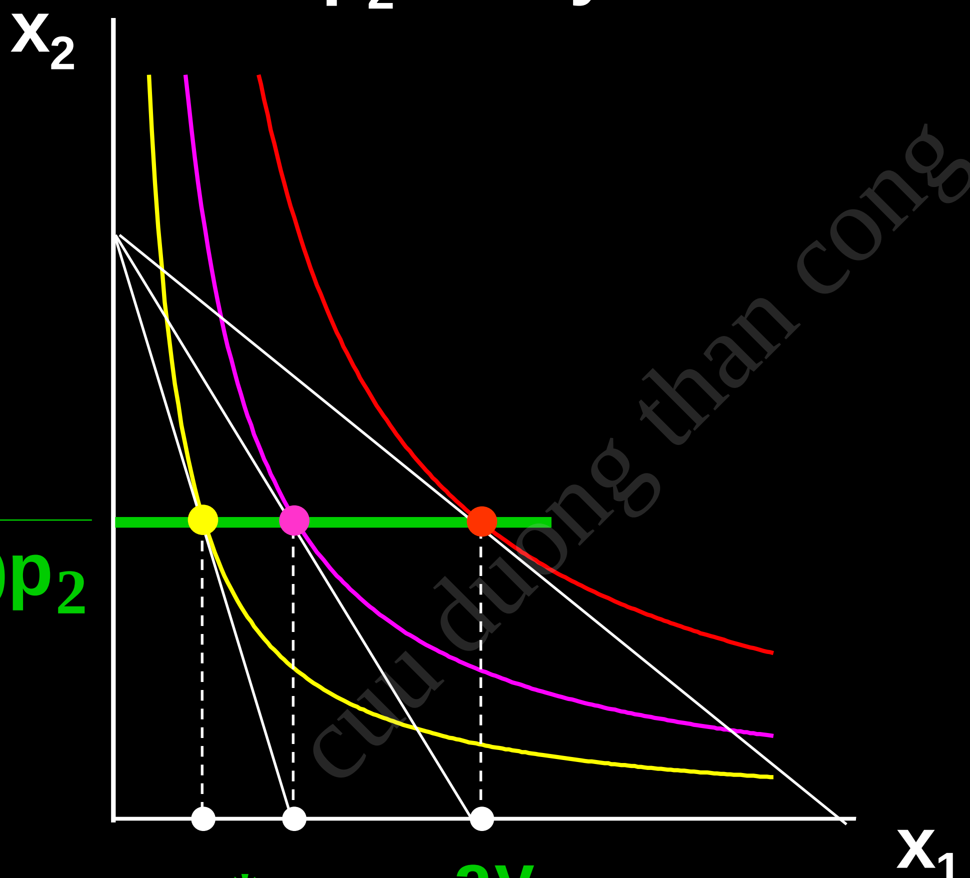
and

$$x_2^*(p_1, p_2, y) = \frac{b}{a+b} \times \frac{y}{p_2}.$$

Notice that x_2* does not vary with p_1 so the p_1 price offer curve is flat and the ordinary demand curve for commodity 1 is a rectangular hyperbola.

Own-Price Changes

Fixed p_2 and y.

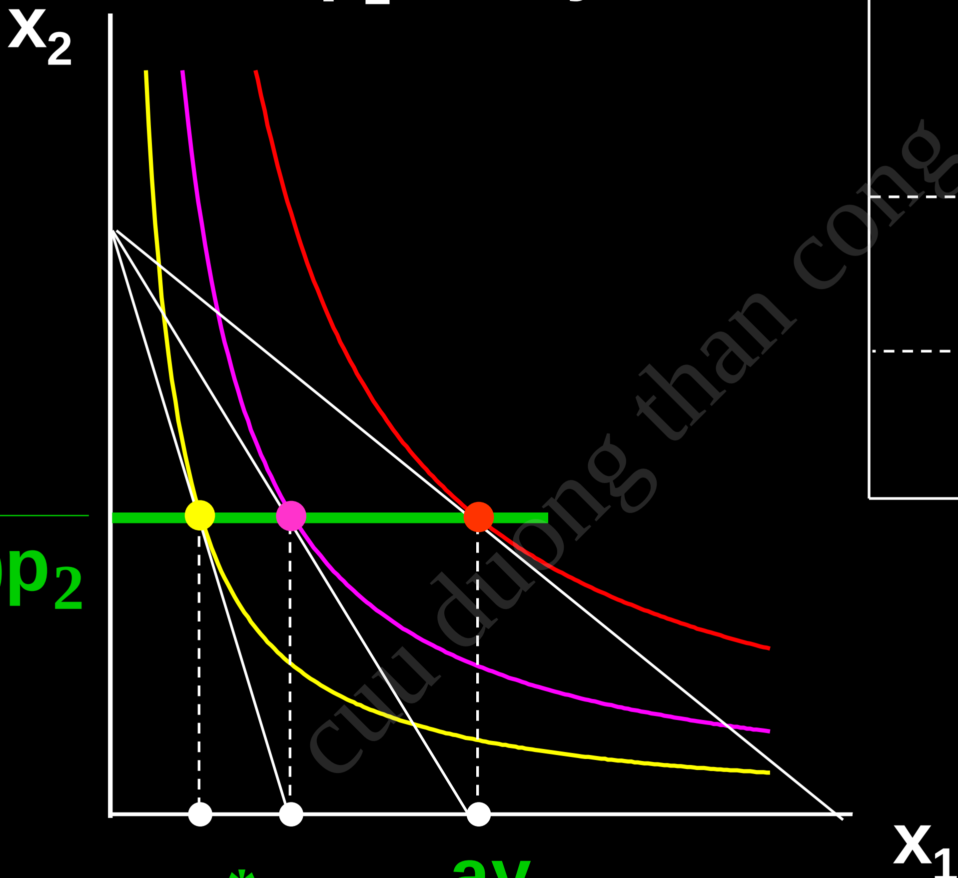


$$x_2^* = \frac{by}{(a+b)p_2}$$

$$x_1^* = \frac{ay}{(a+b)p_1}$$

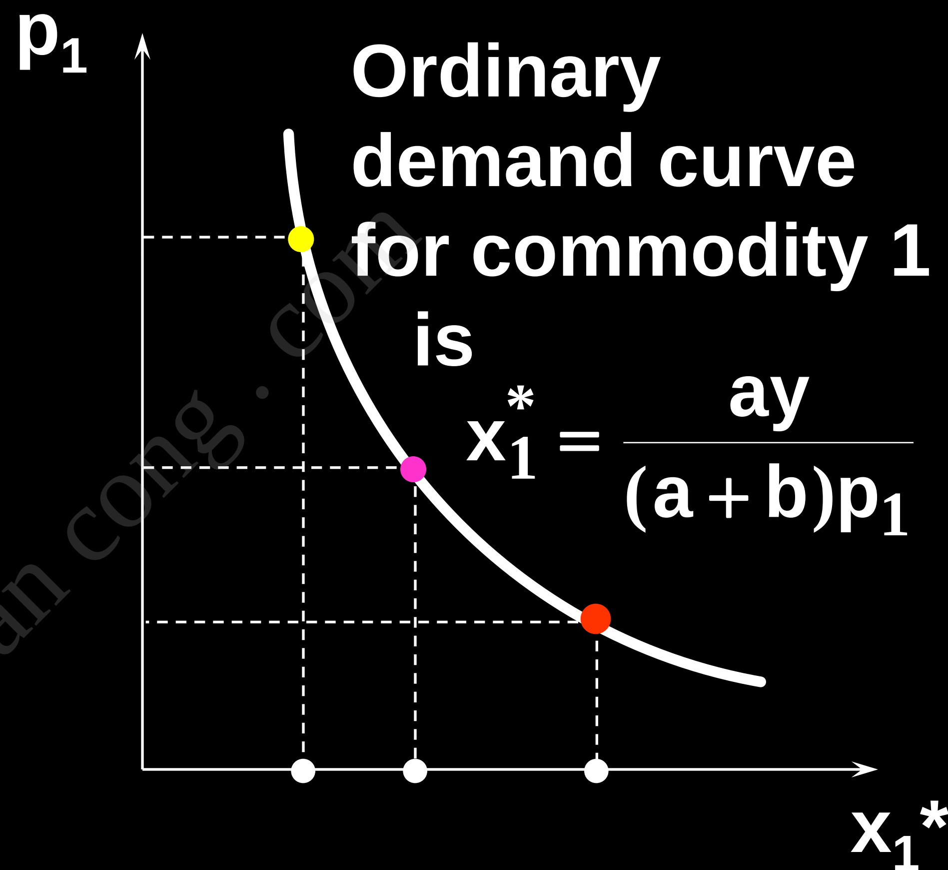
Own-Price Changes

Fixed p_2 and y.



$$x_2^* = \frac{by}{(a+b)p_2}$$

$$x_1^* = \frac{ay}{(a+b)p_1}$$



Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{ay}{(a+b)p_1}$$

Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-complements utility function?

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

Own-Price Changes

$$x_1^*(p_1, p_2, y) = x_2^*(p_1, p_2, y) = \frac{y}{p_1 + p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \frac{y}{p_2}.$

Own-Price Changes

$$x_1^*(p_1,p_2,y) = x_2^*(p_1,p_2,y) = \frac{y}{p_1+p_2}.$$

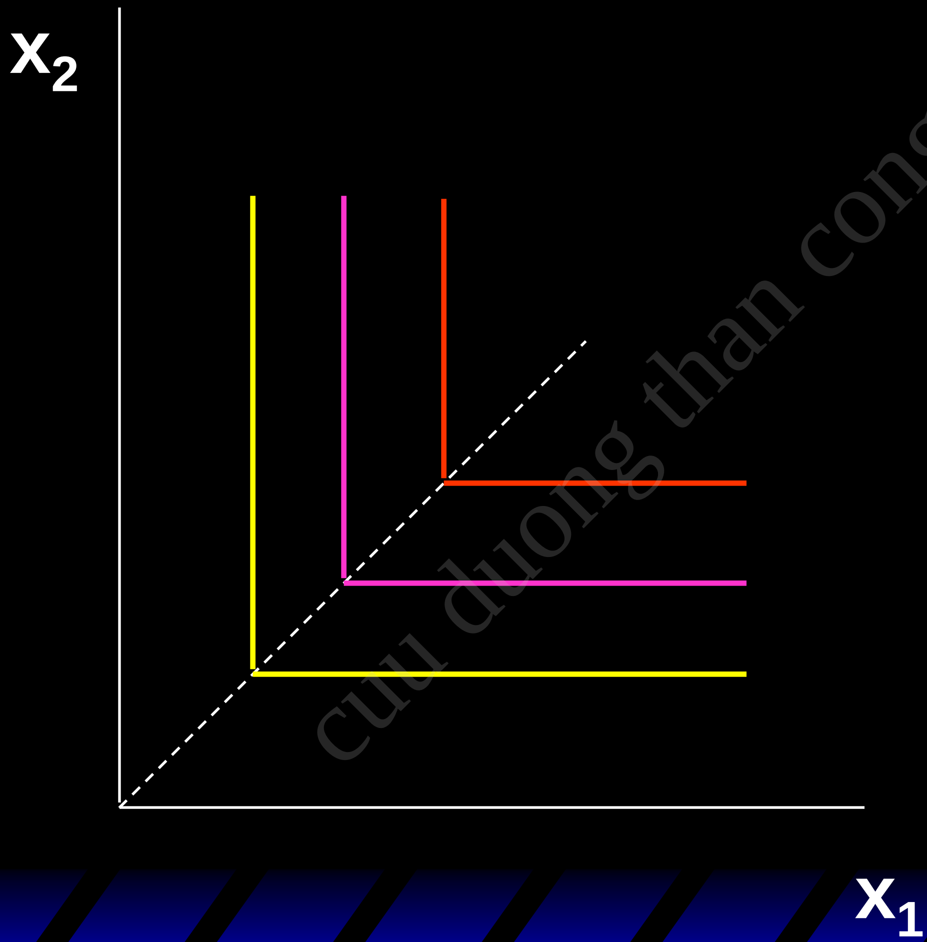
With p_2 and y fixed, higher p_1 causes smaller x_1* and x_2*.

As $p_1 \to 0, \quad x_1^* = x_2^* \to \frac{y}{p_2}.$

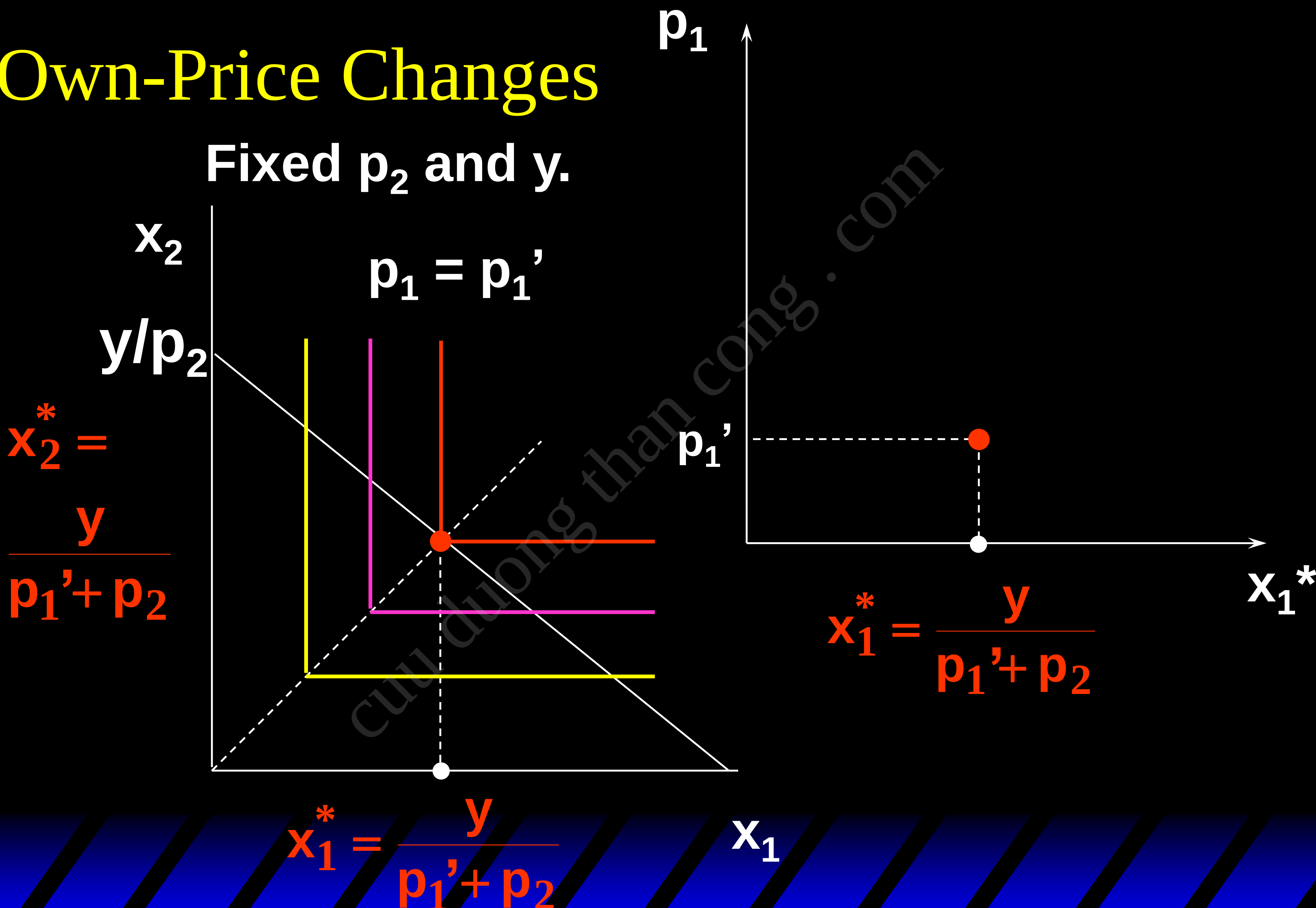
As $p_1 \to \infty, \quad x_1^* = x_2^* \to 0.$

Own-Price Changes

Fixed p_2 and y.



Own-Price Changes



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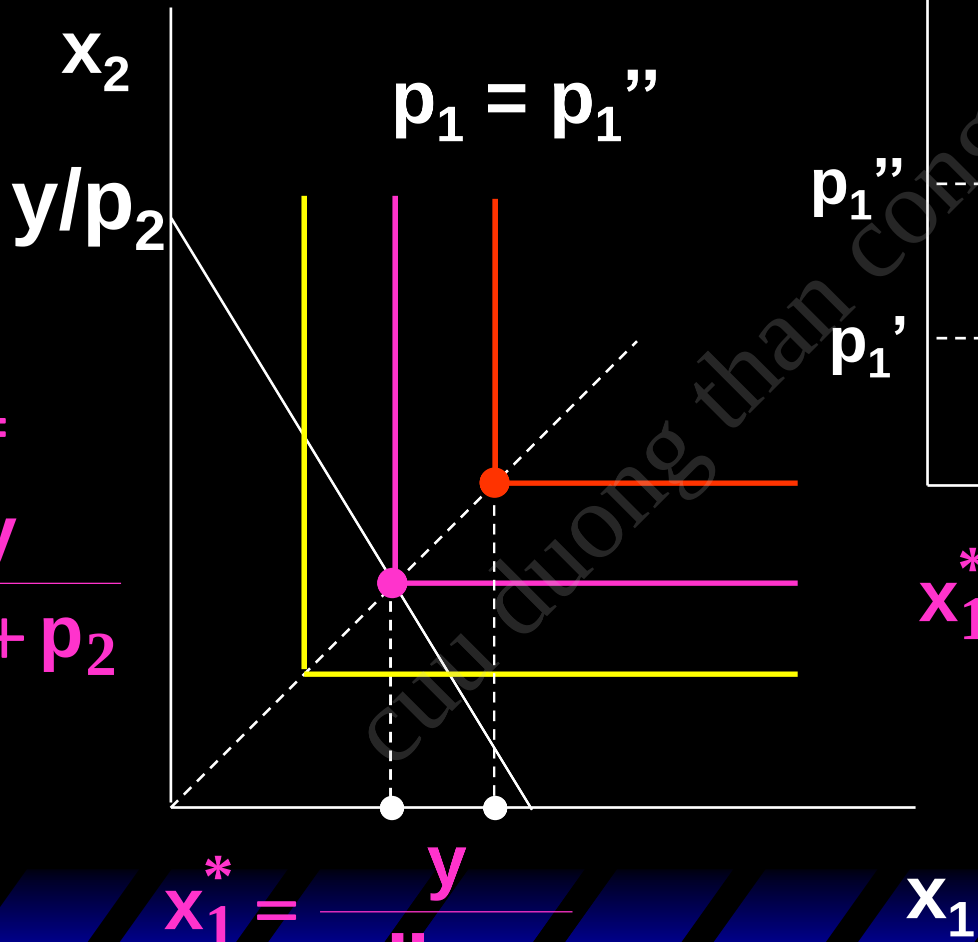

$$x_2^* = \frac{y}{p_1'' + p_2}$$

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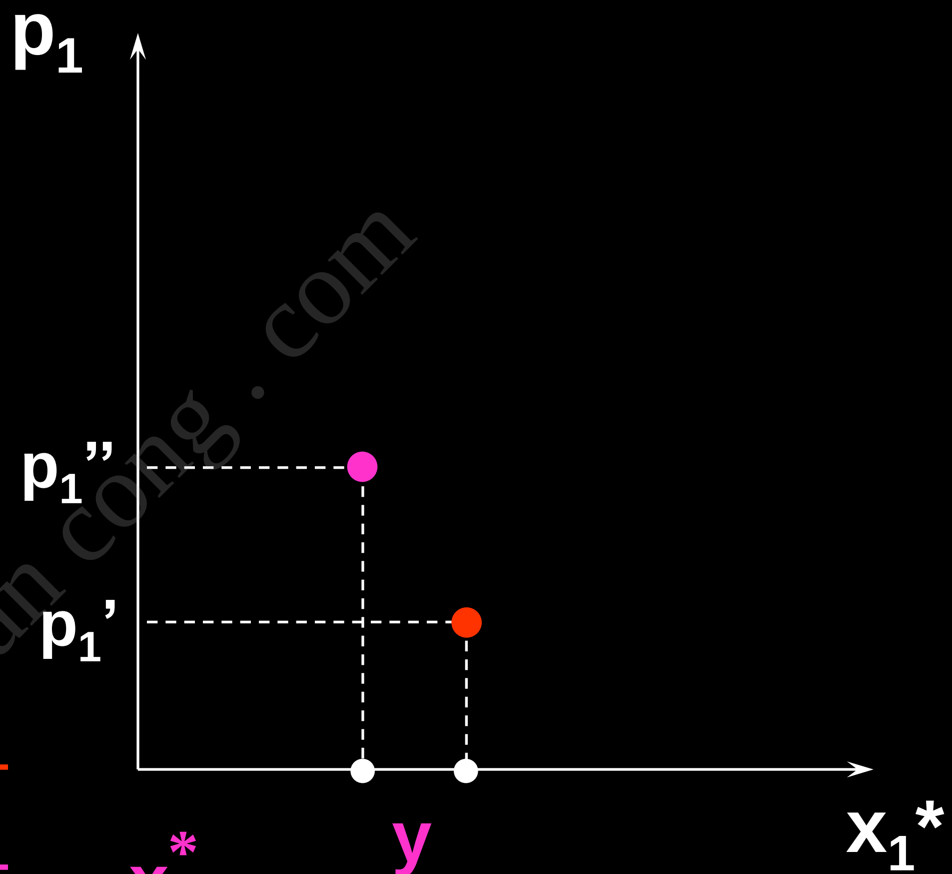
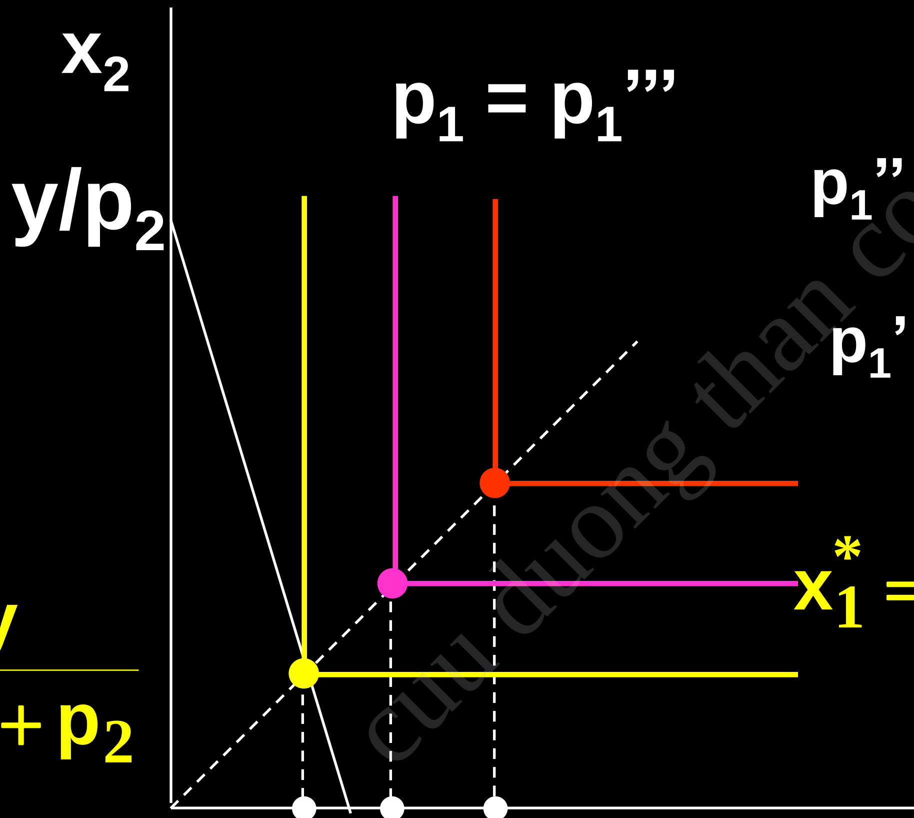
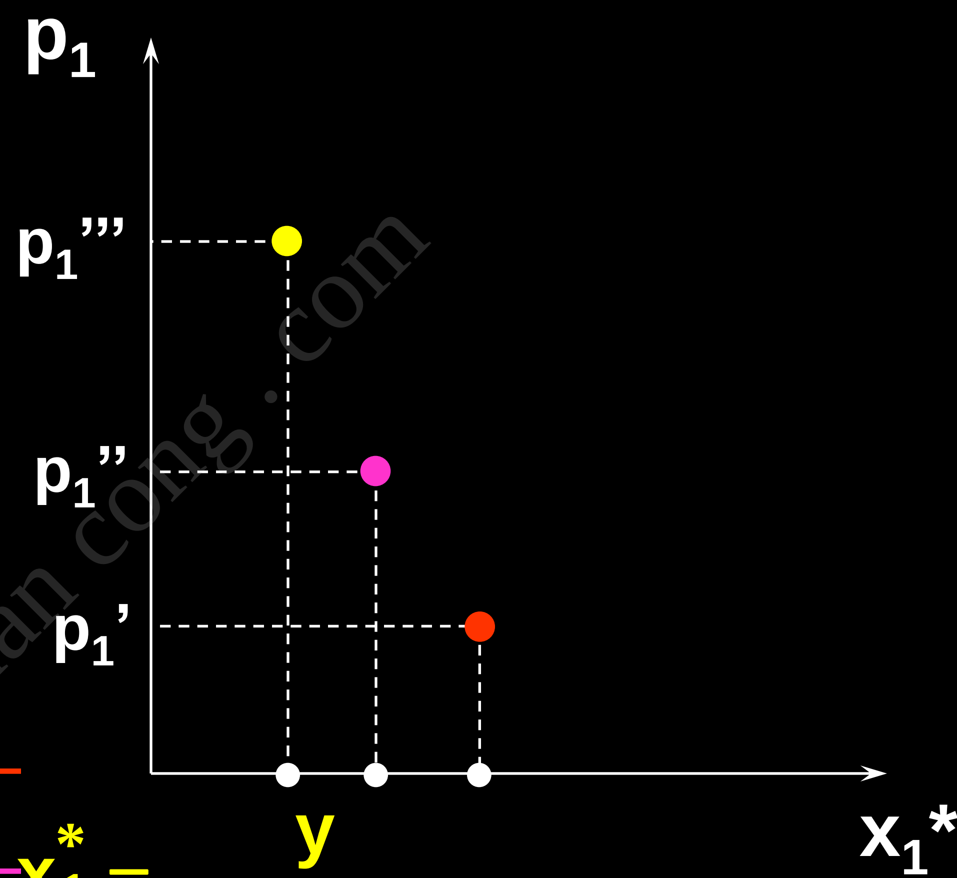


$$x_1^* = \frac{y}{p_1'' + p_2}$$

Own-Price Changes

Fixed p_2 and y.





$$x_2^* = \frac{y}{p_1''' + p_2}$$

$$x_1^* = \frac{y}{p_1''' + p_2}$$

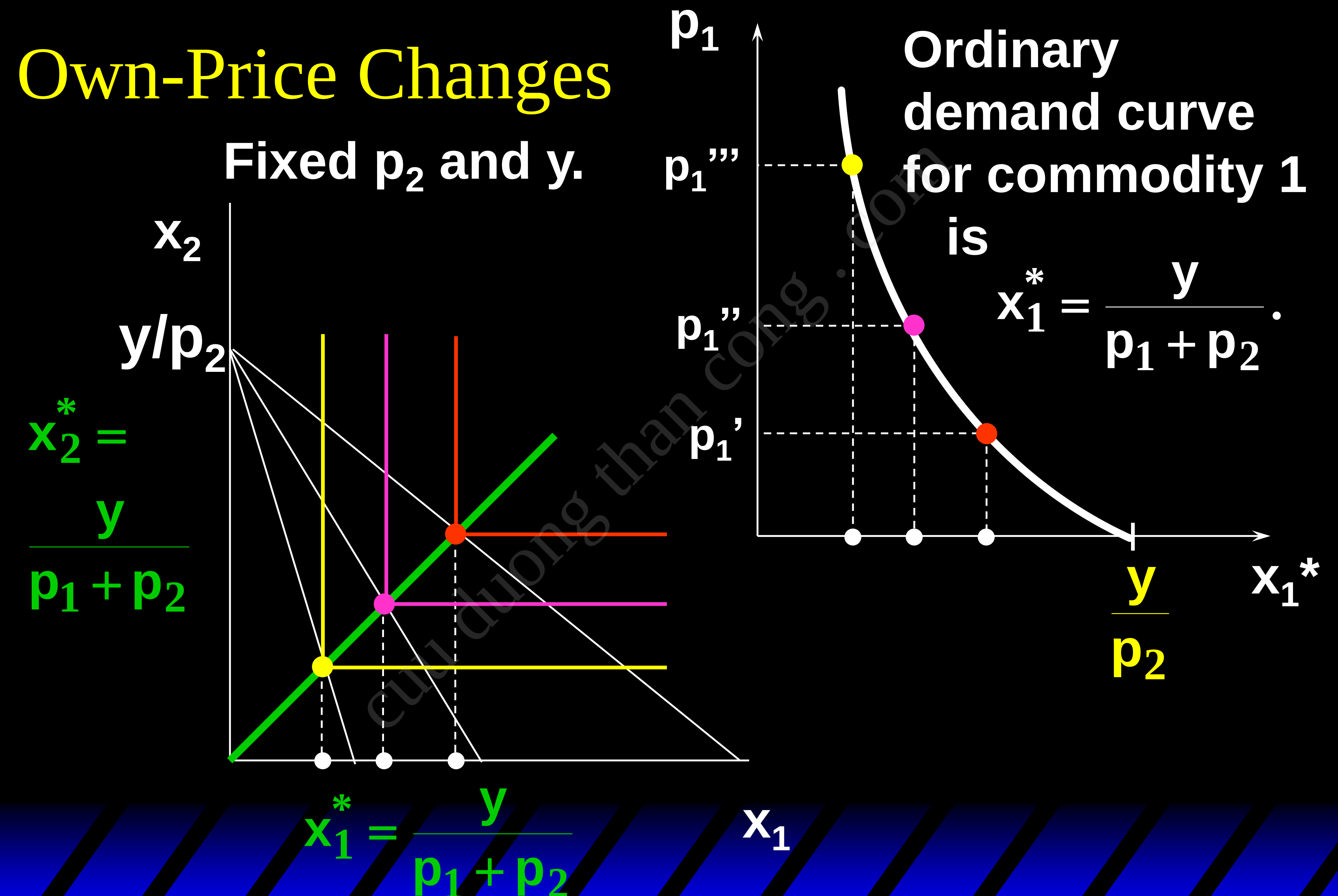
$$x_1^* = \frac{y}{p_1''' + p_2}$$

Own-Price Changes

Fixed p_2 and y.

Ordinary demand curve for commodity 1 is

$$x_1^* = \frac{y}{p_1 + p_2}.$$



Own-Price Changes

- What does a p_1 price-offer curve look like for a perfect-substitutes utility function?

$$U(x_1, x_2) = x_1 + x_2.$$

Then the ordinary demand functions for commodities 1 and 2 are

Own-Price Changes

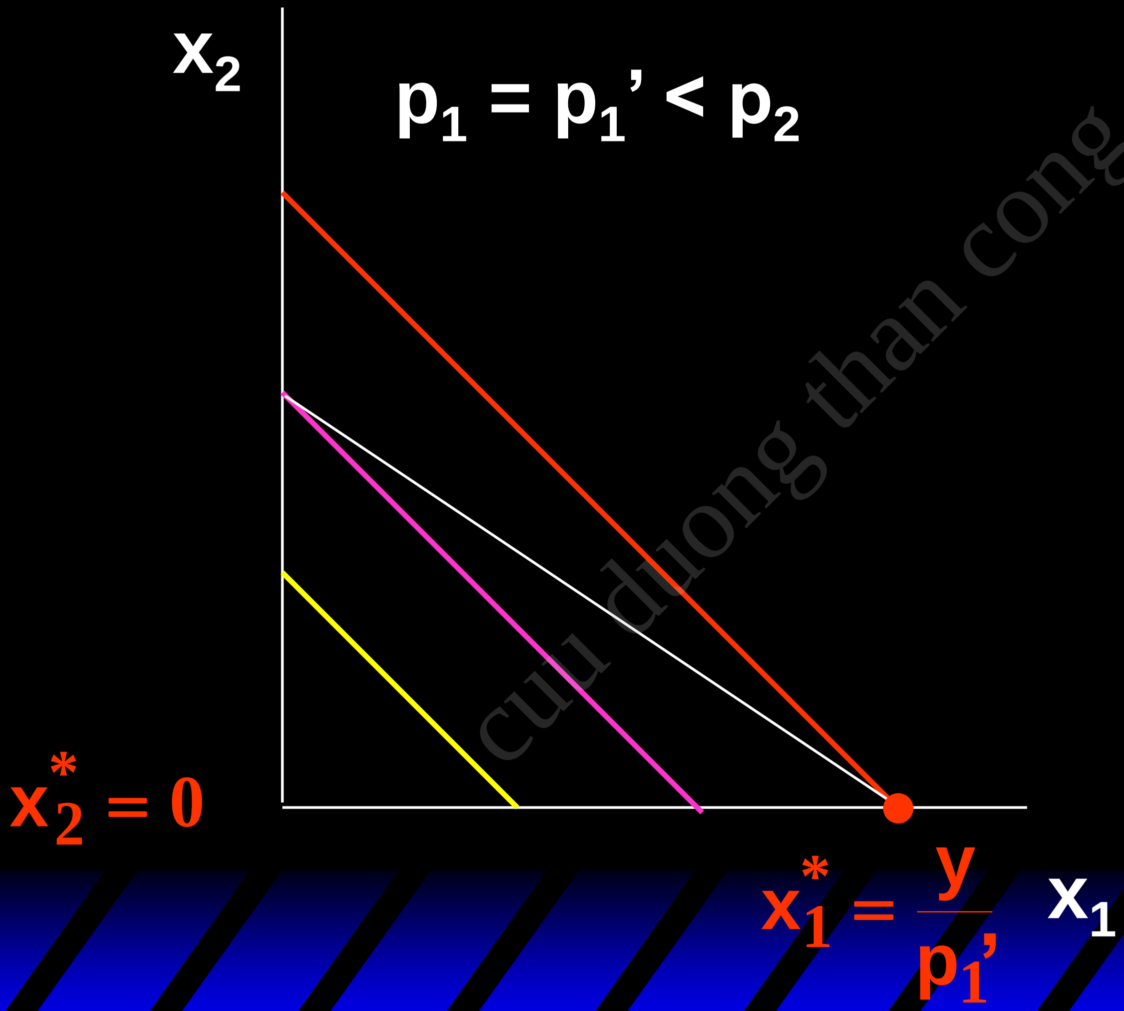
$$x_1^*(p_1,p_2,y) = \begin{cases} 0 & ,\text{if } p_1 > p_2 \\ y/p_1 & ,\text{if } p_1 < p_2 \end{cases}$$

and

$$x_2^*(p_1,p_2,y) = \begin{cases} 0 & ,\text{if } p_1 < p_2 \\ y/p_2 & ,\text{if } p_1 > p_2. \end{cases}$$

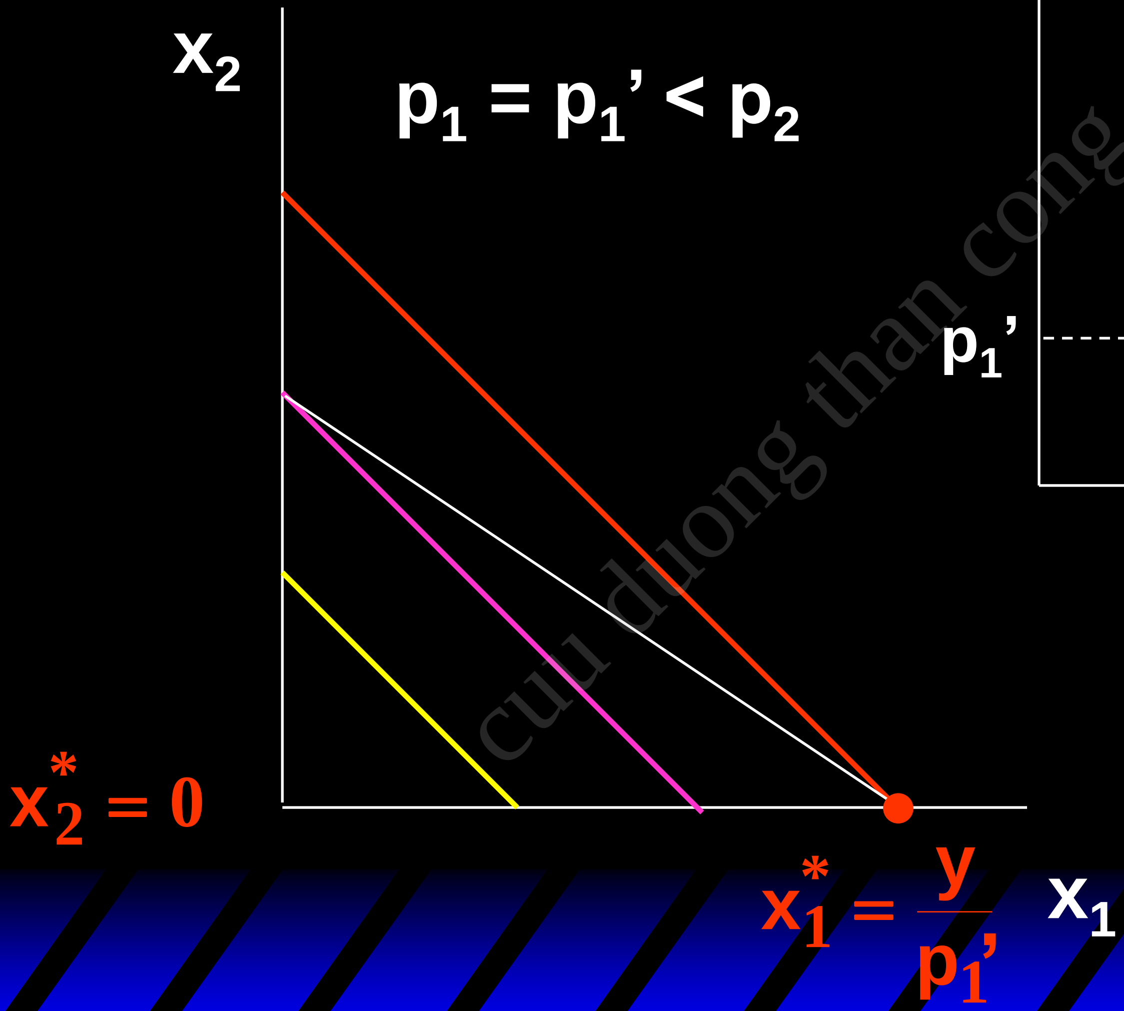
Own-Price Changes

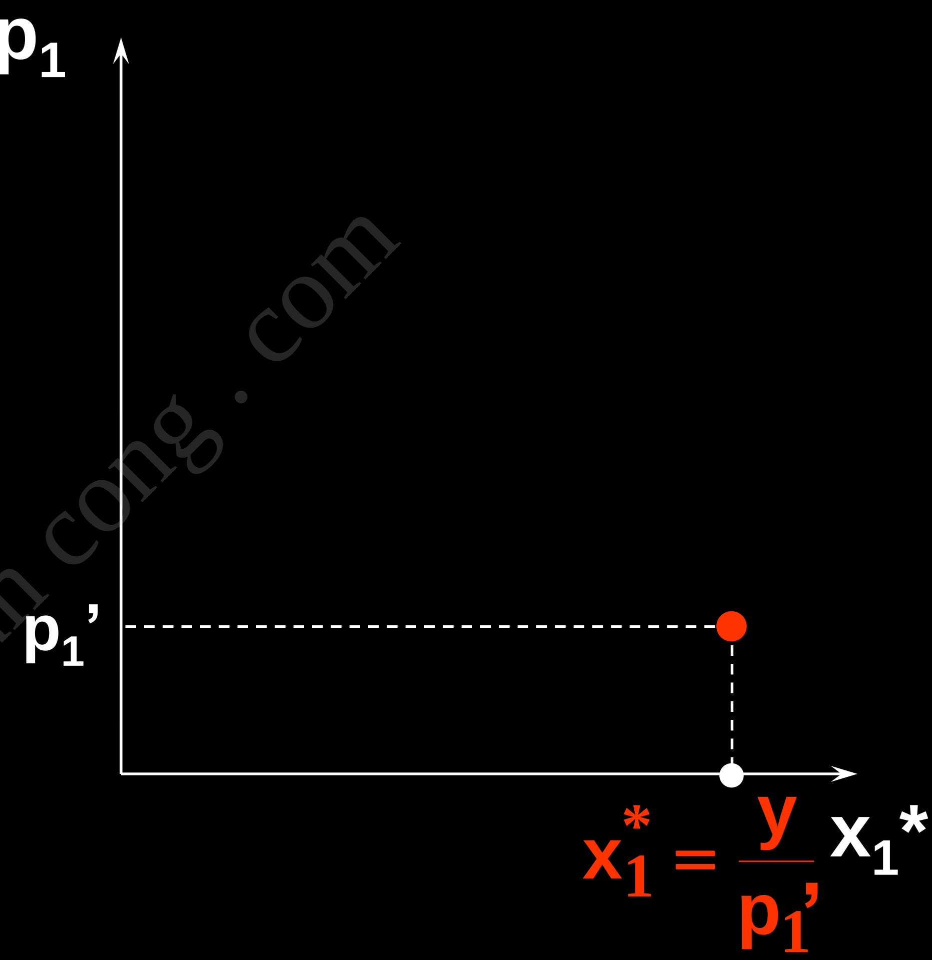
Fixed p_2 and y.



Own-Price Changes

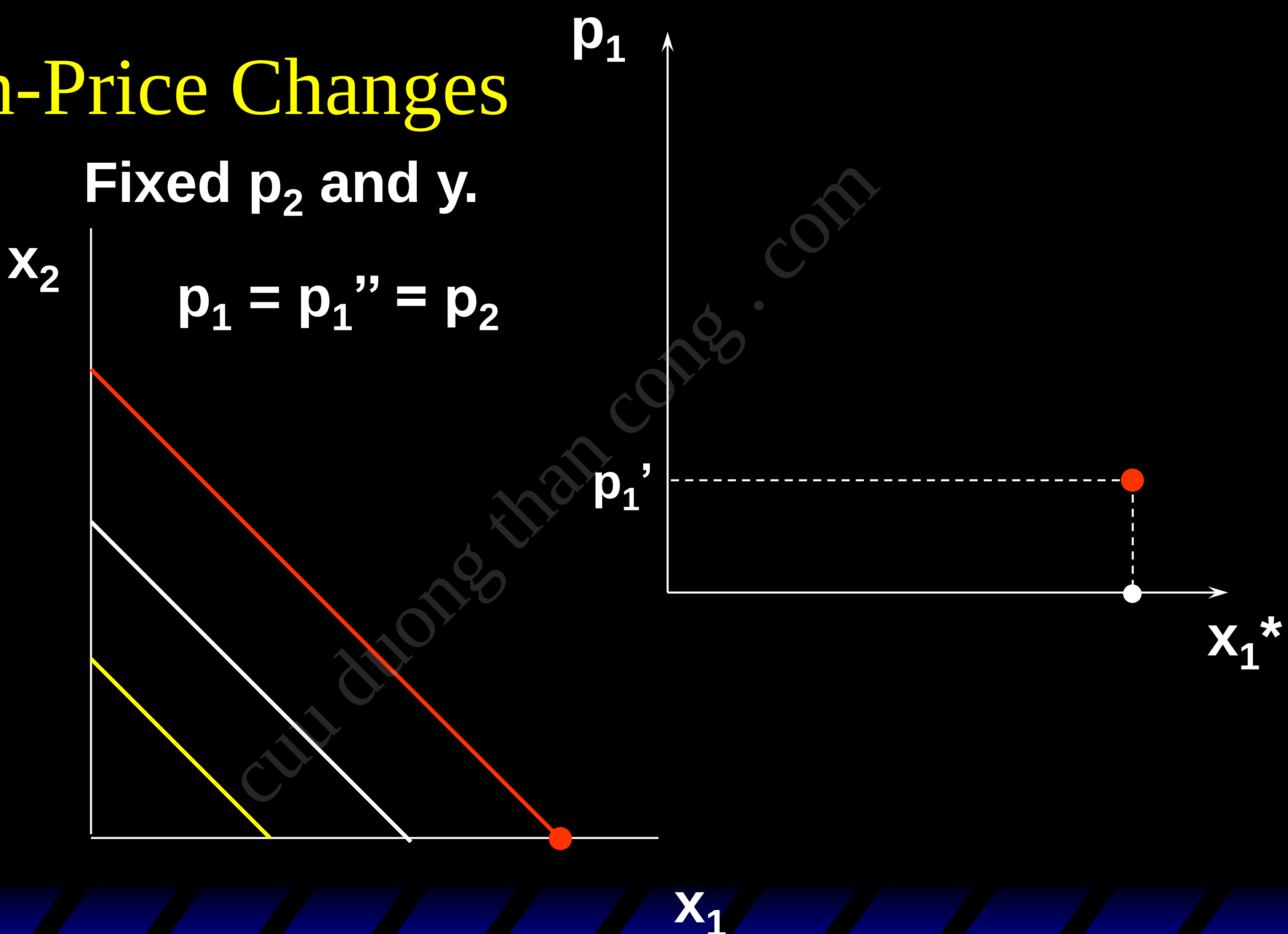
Fixed p_2 and y.



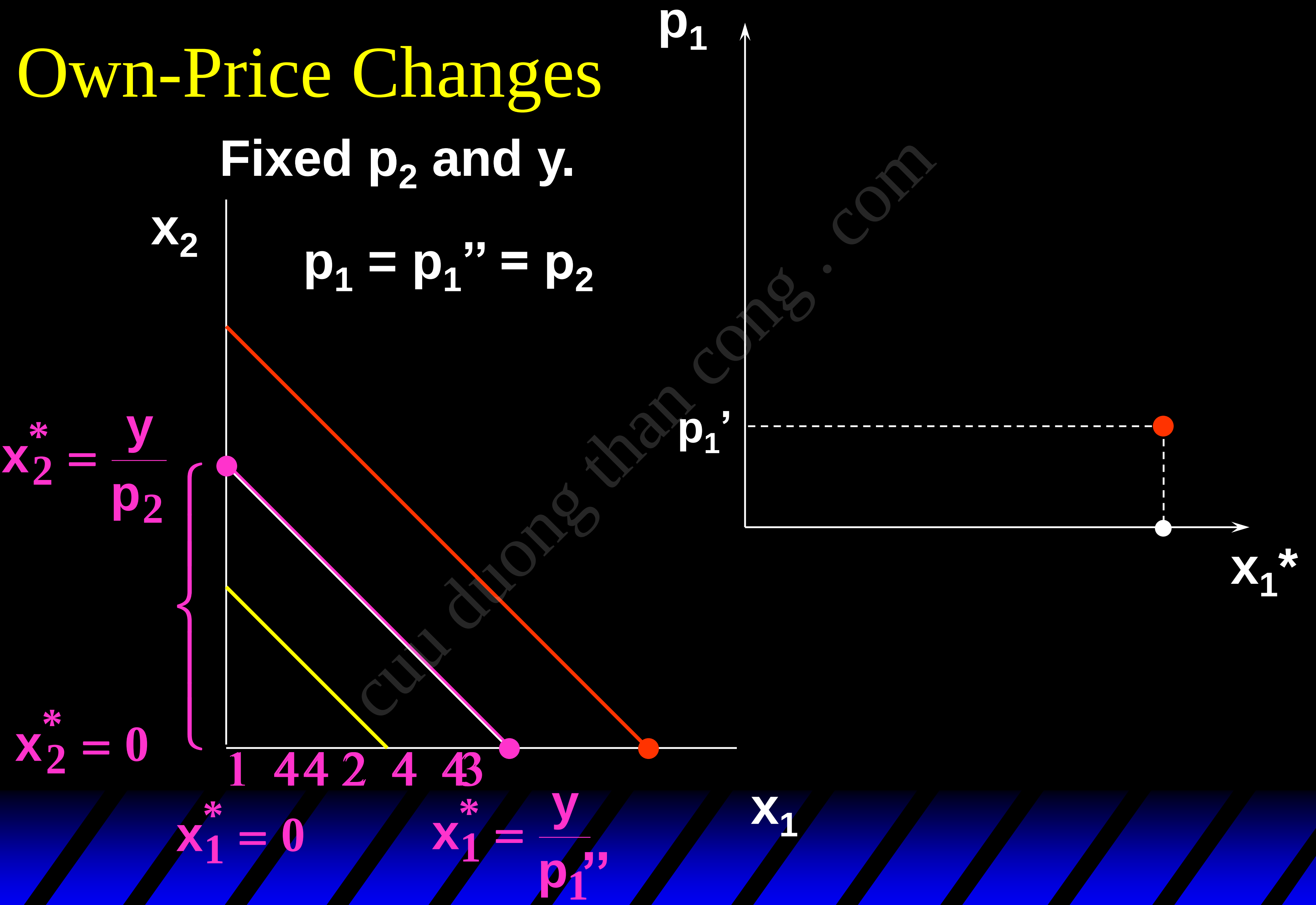


Own-Price Changes

Fixed p_2 and y.

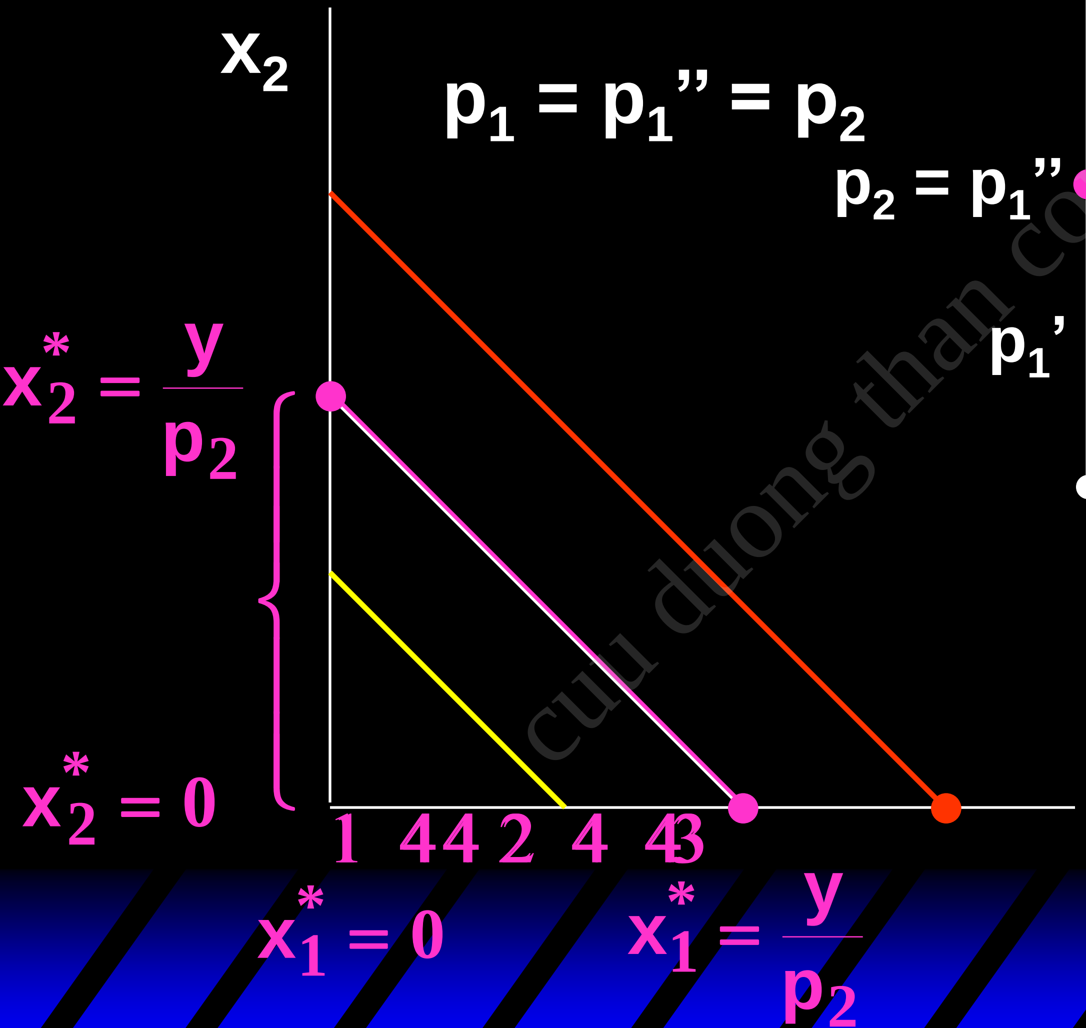


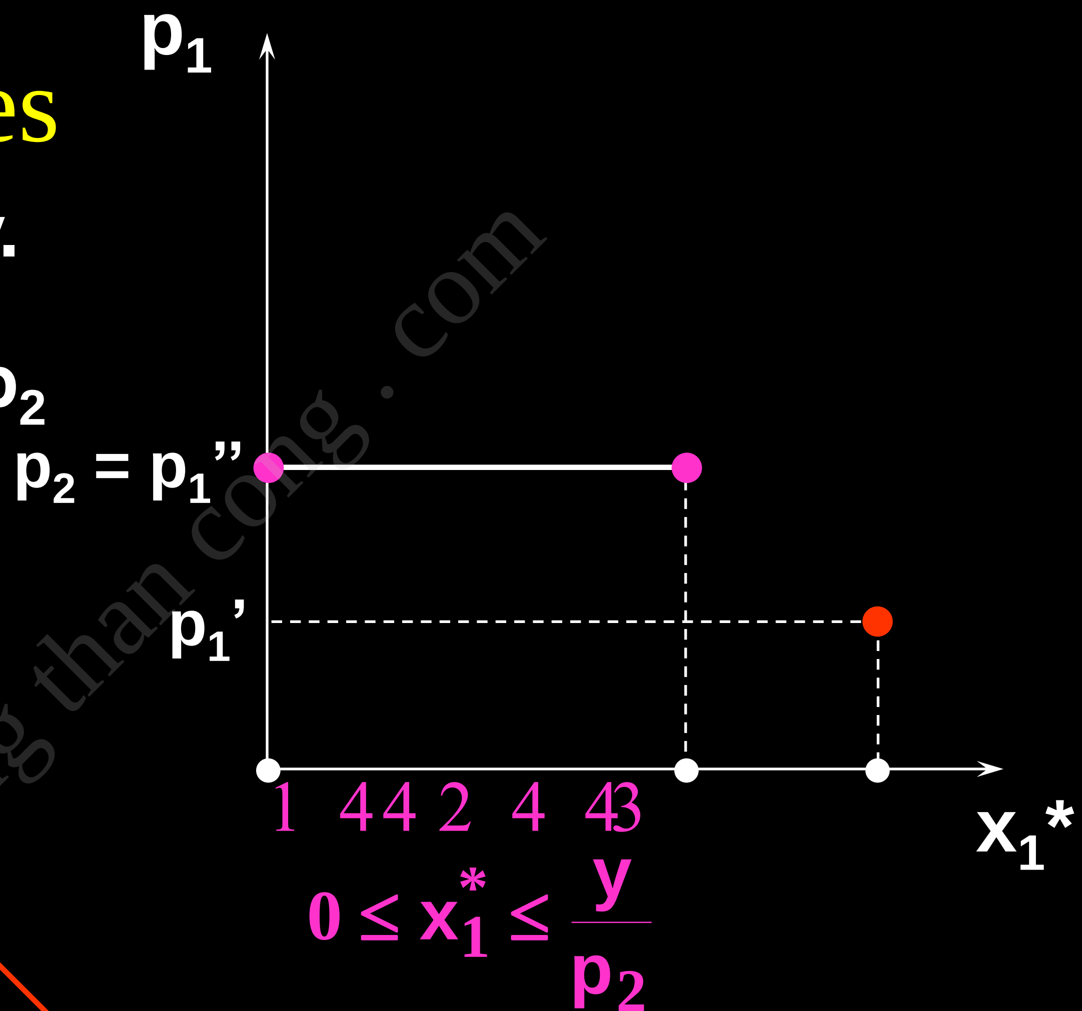
Own-Price Changes



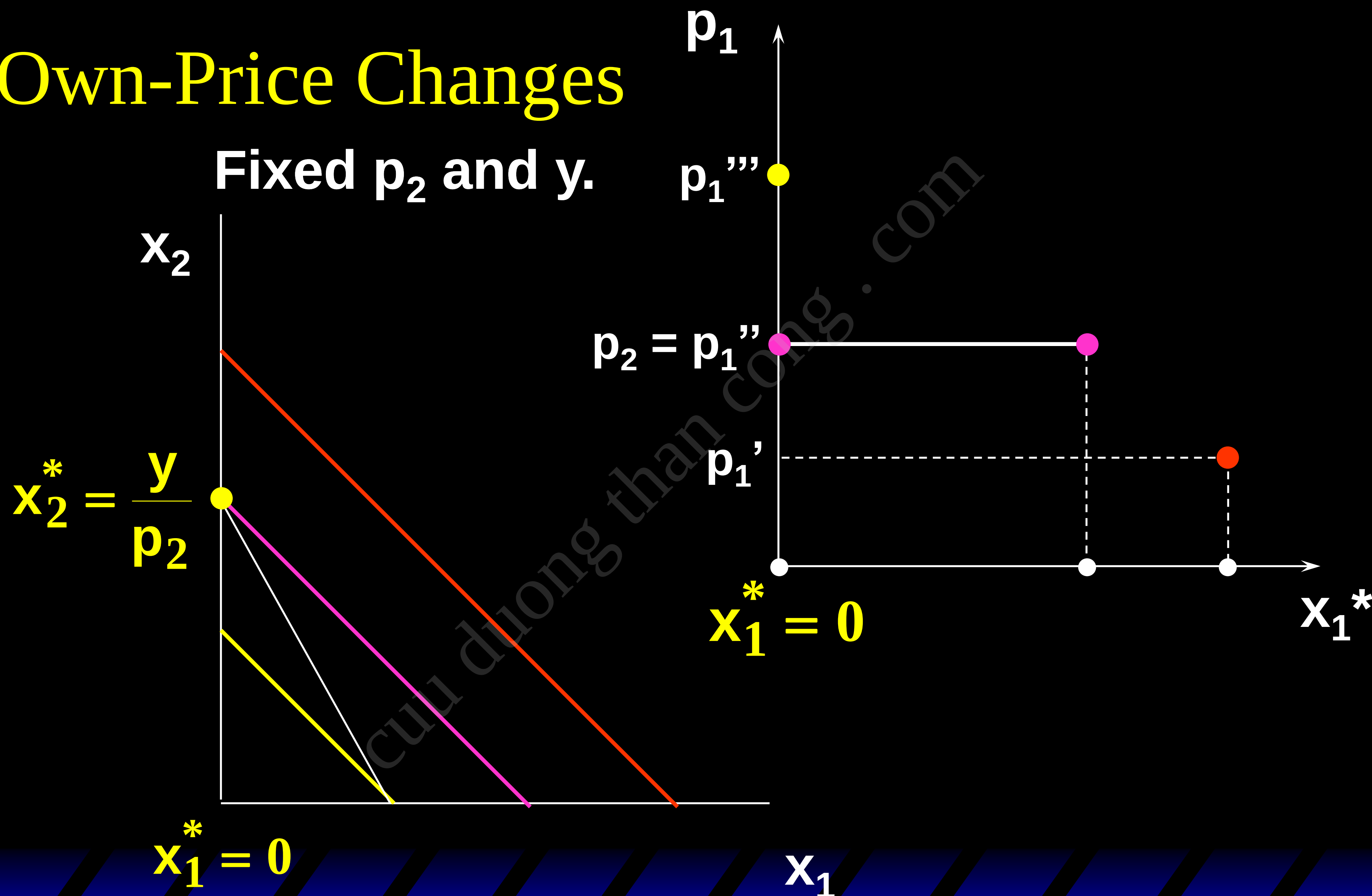
Own-Price Changes

Fixed p_2 and y.



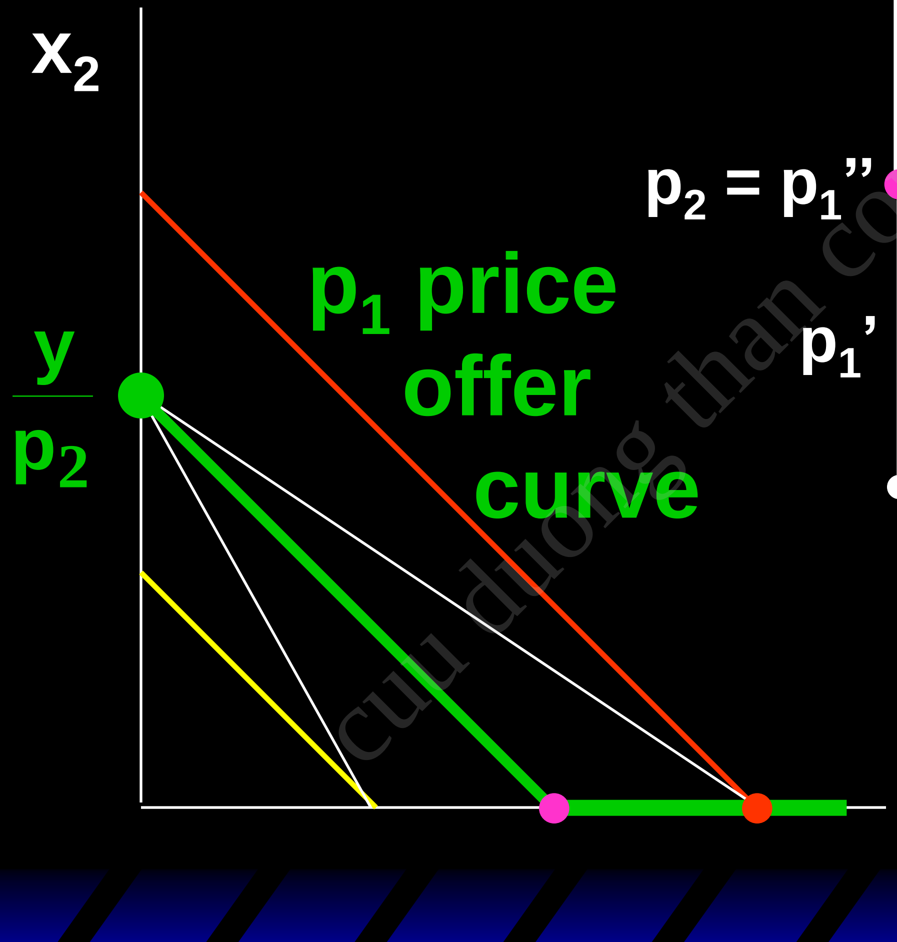


Own-Price Changes

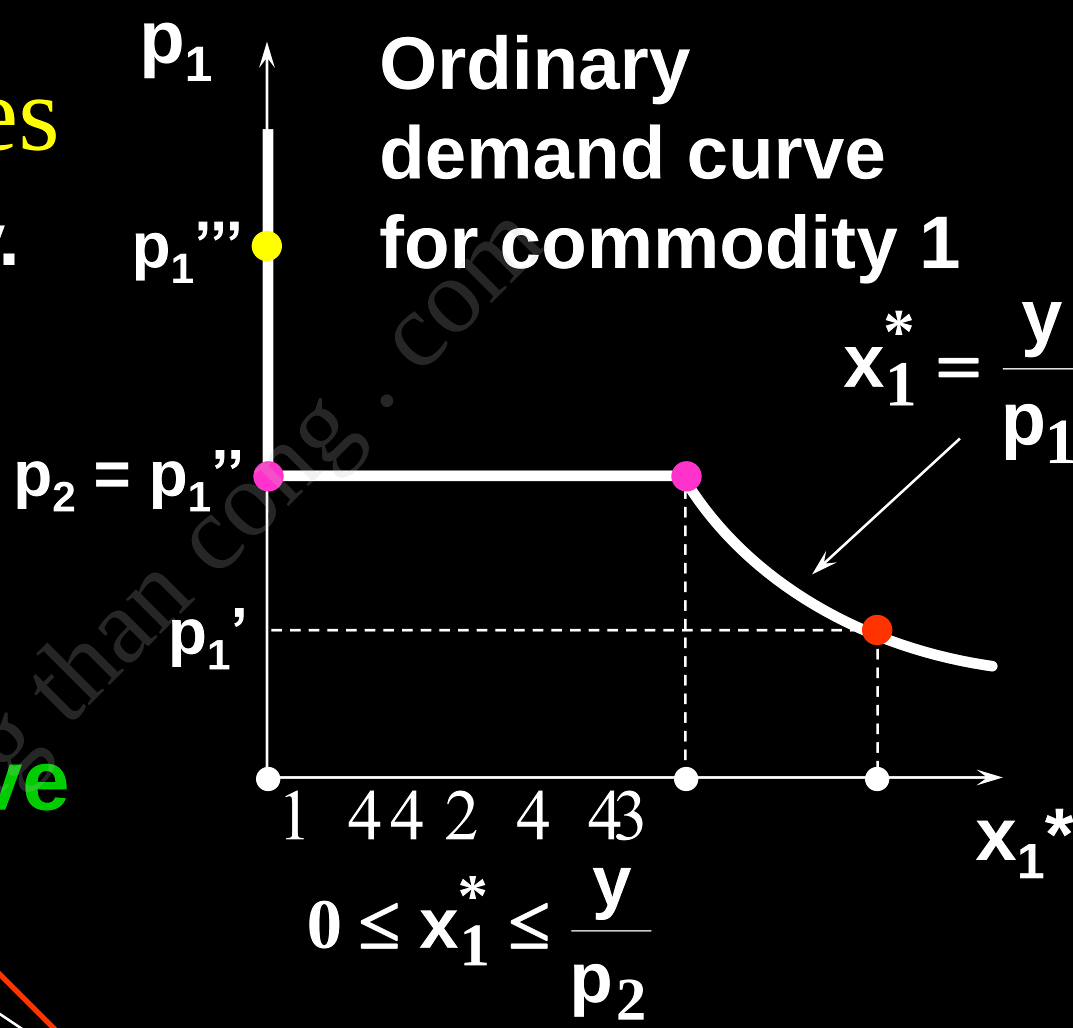


Own-Price Changes

Fixed p_2 and y.



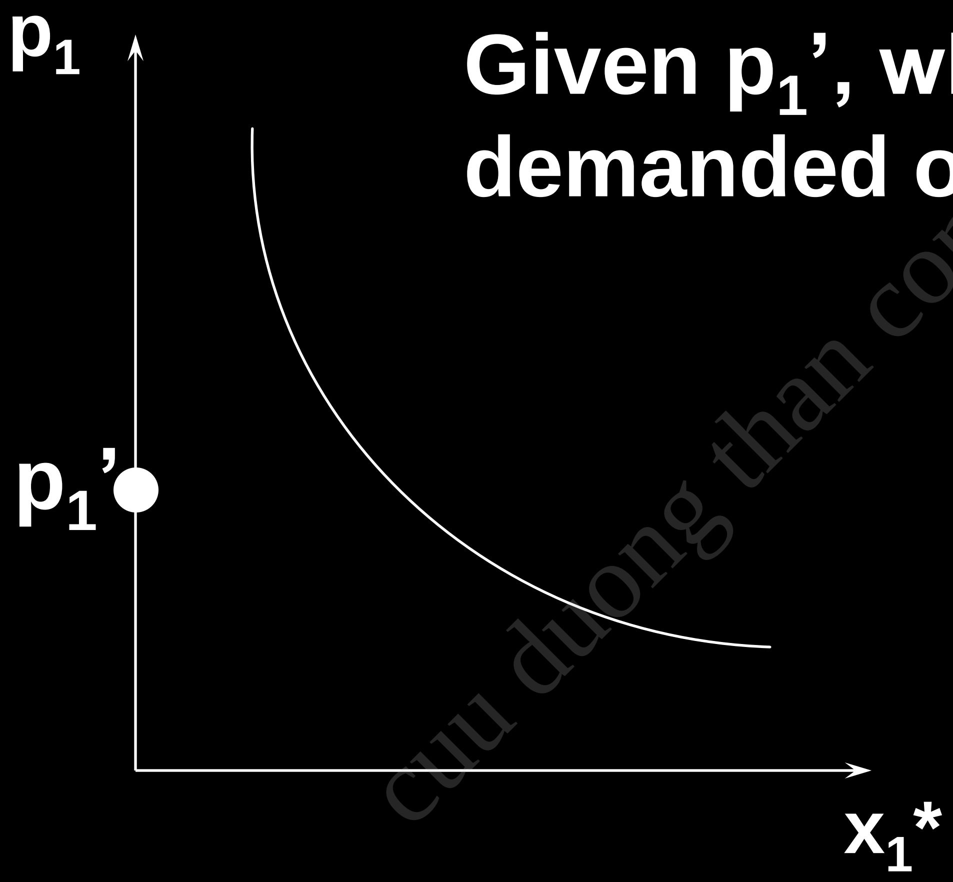
Ordinary demand curve for commodity 1



Own-Price Changes

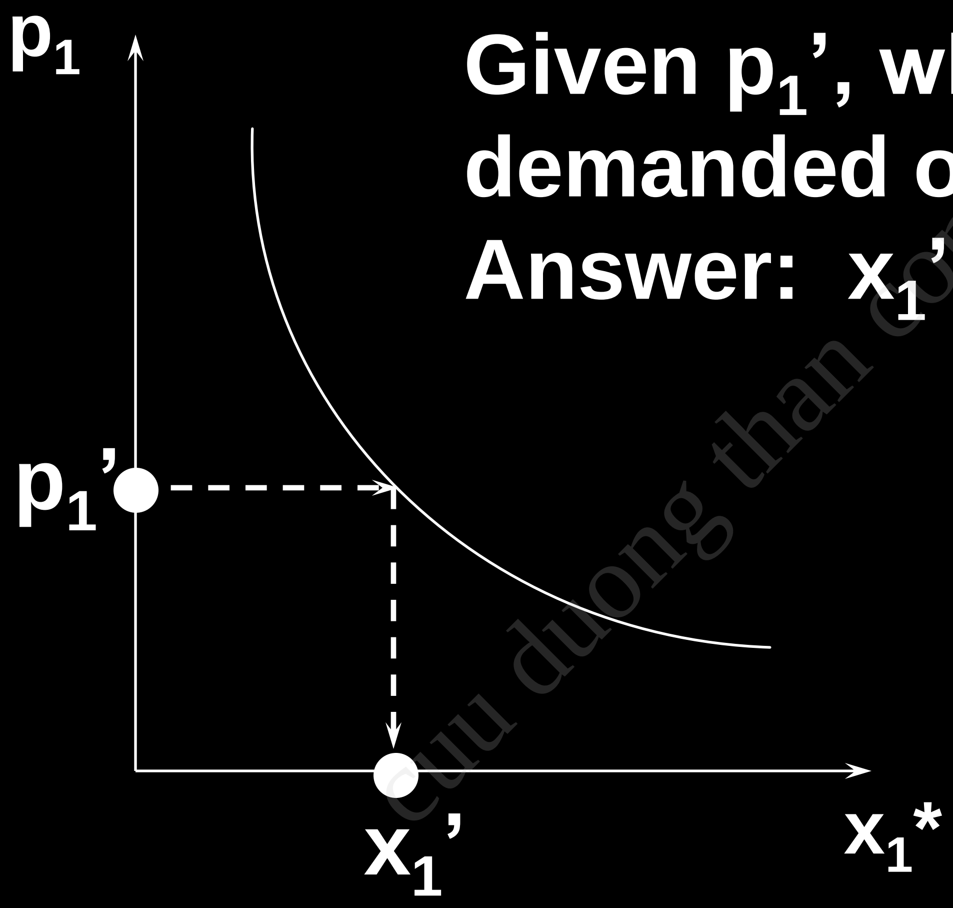
- Usually we ask "Given the price for commodity 1 what is the quantity demanded of commodity 1?"
- But we could also ask the **inverse** question "At what price for commodity 1 would a given quantity of commodity 1 be demanded?"

Own-Price Changes



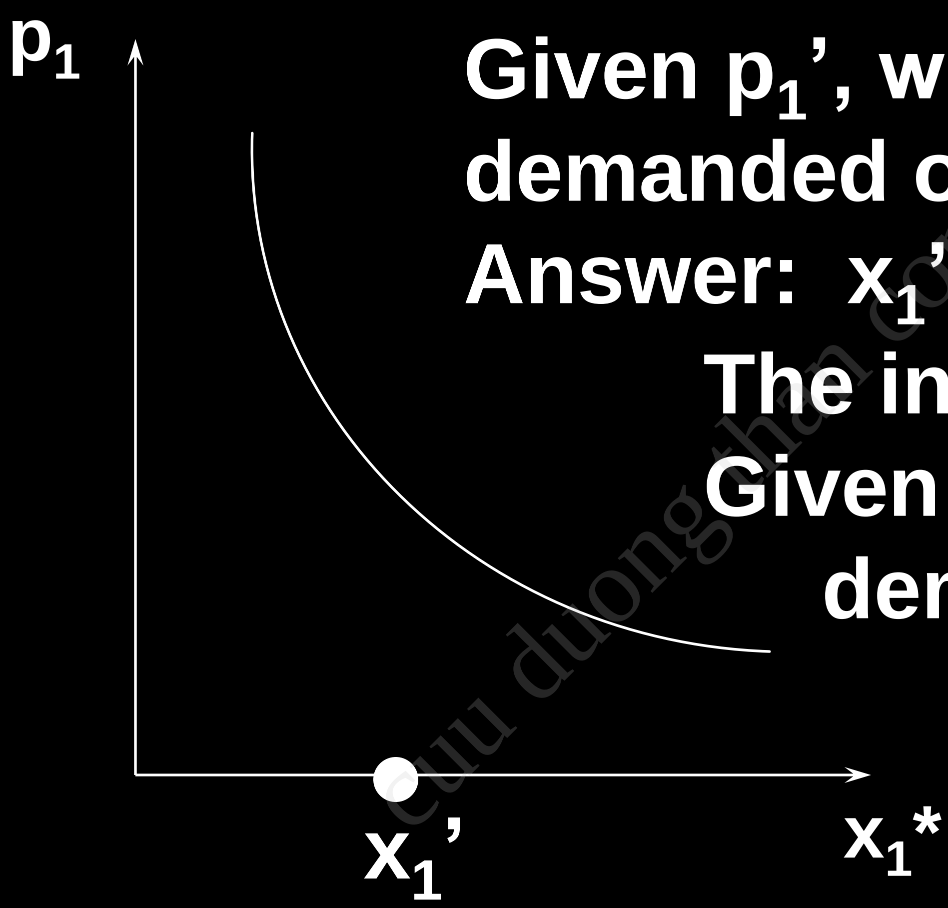
Given p_1', what quantity is demanded of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?
Answer: x_1' units.

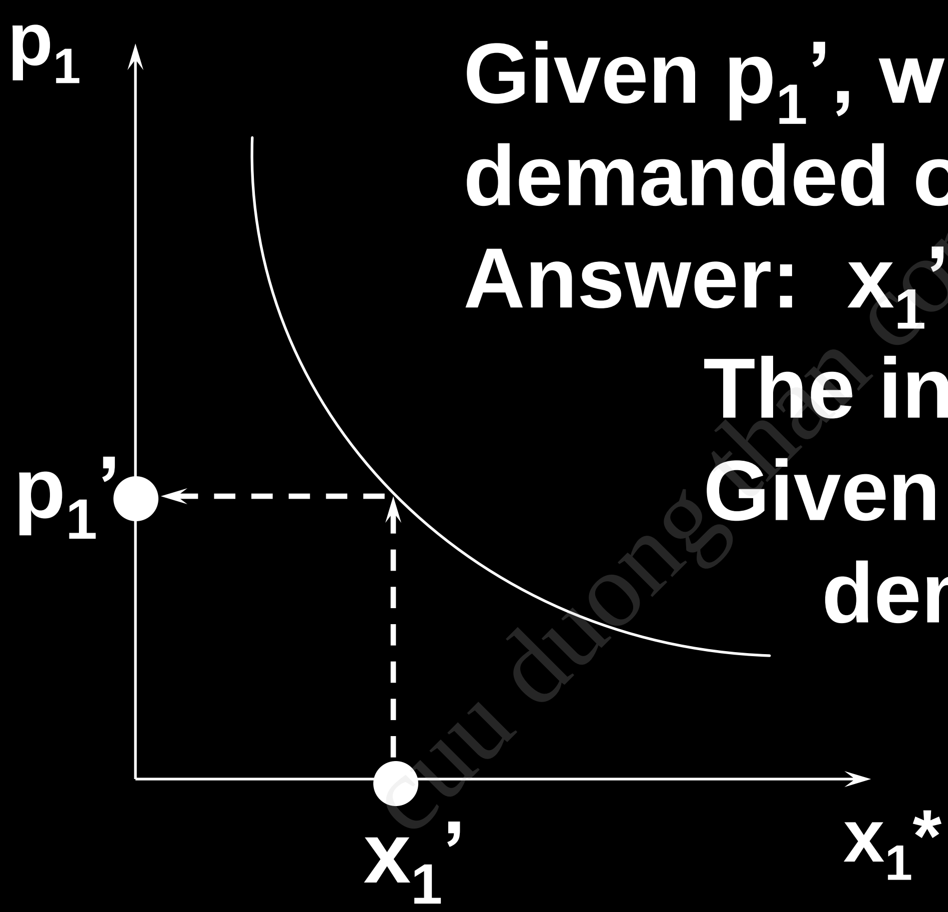
Own-Price Changes



Given p_1', what quantity is demanded of commodity 1? Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?

Own-Price Changes



Given p_1', what quantity is demanded of commodity 1?
Answer: x_1' units.

The inverse question is: Given x_1' units are demanded, what is the price of commodity 1?
Answer: p_1'

Own-Price Changes

- **Taking quantity demanded as given and then asking what must be price describes the inverse demand function of a commodity.**

Own-Price Changes

A Cobb-Douglas example:

$$x_1^* = \frac{ay}{(a+b)p_1}$$

is the ordinary demand function and

$$p_1 = \frac{ay}{(a+b)x_1^*}$$

is the inverse demand function.

Own-Price Changes

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

is the ordinary demand function and

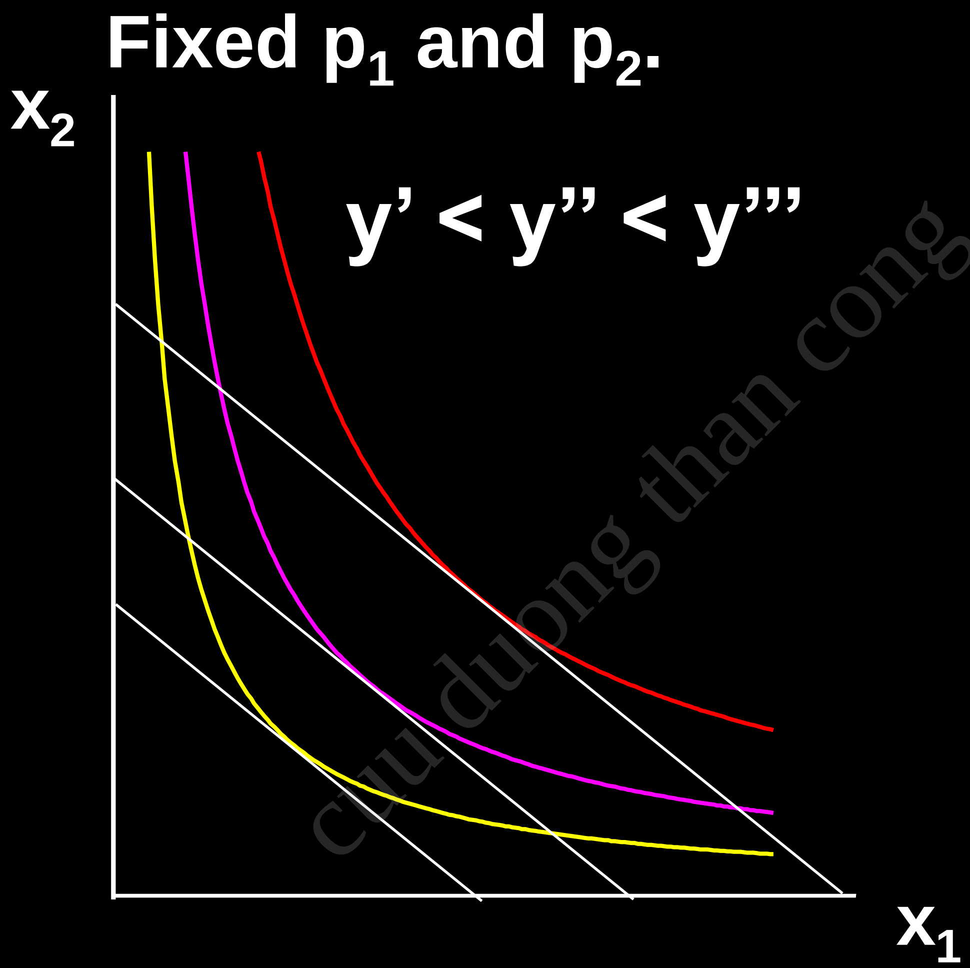
$$p_1 = \frac{y}{x_1^*} - p_2$$

is the inverse demand function.

Income Changes

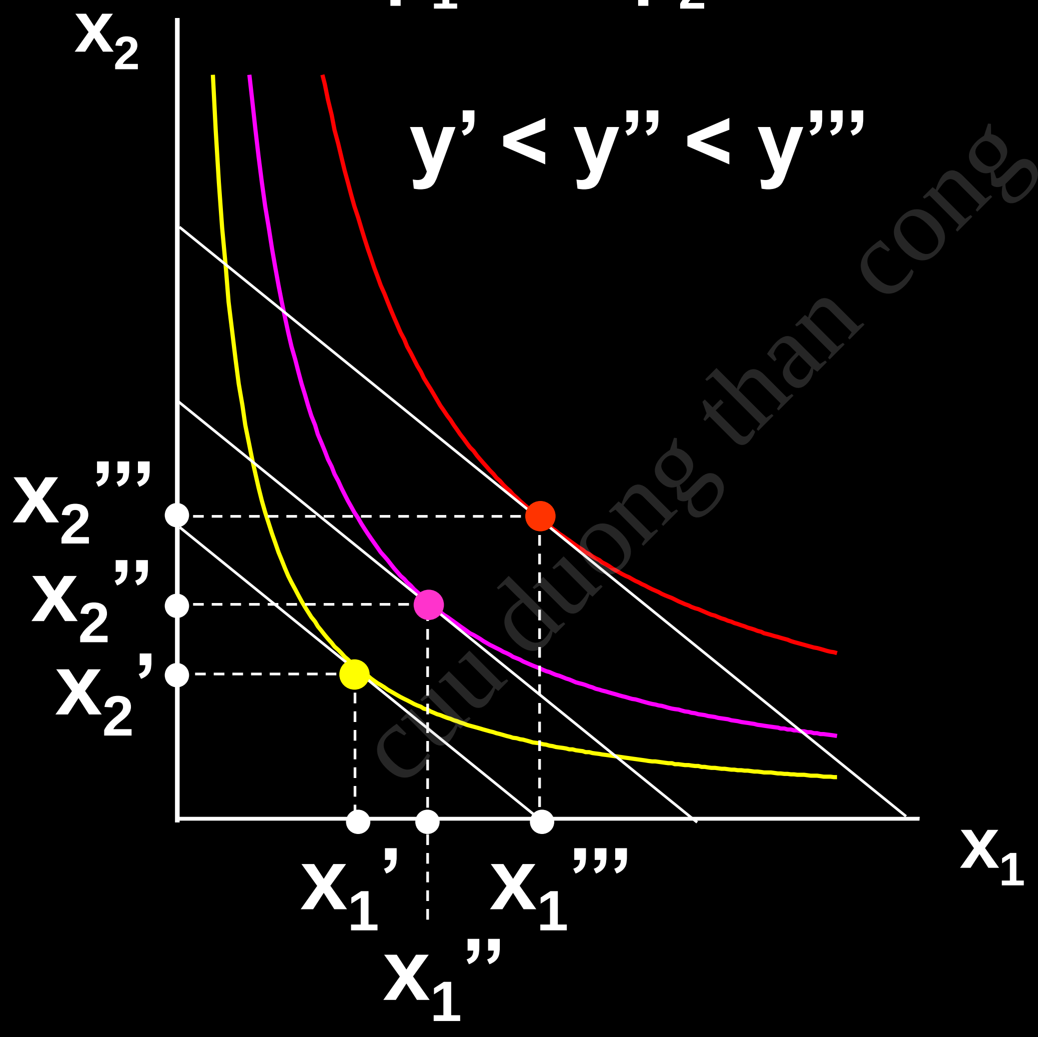
- How does the value of $x_1^*(p_1,p_2,y)$ change as y changes, holding both p_1 and p_2 constant?

Income Changes

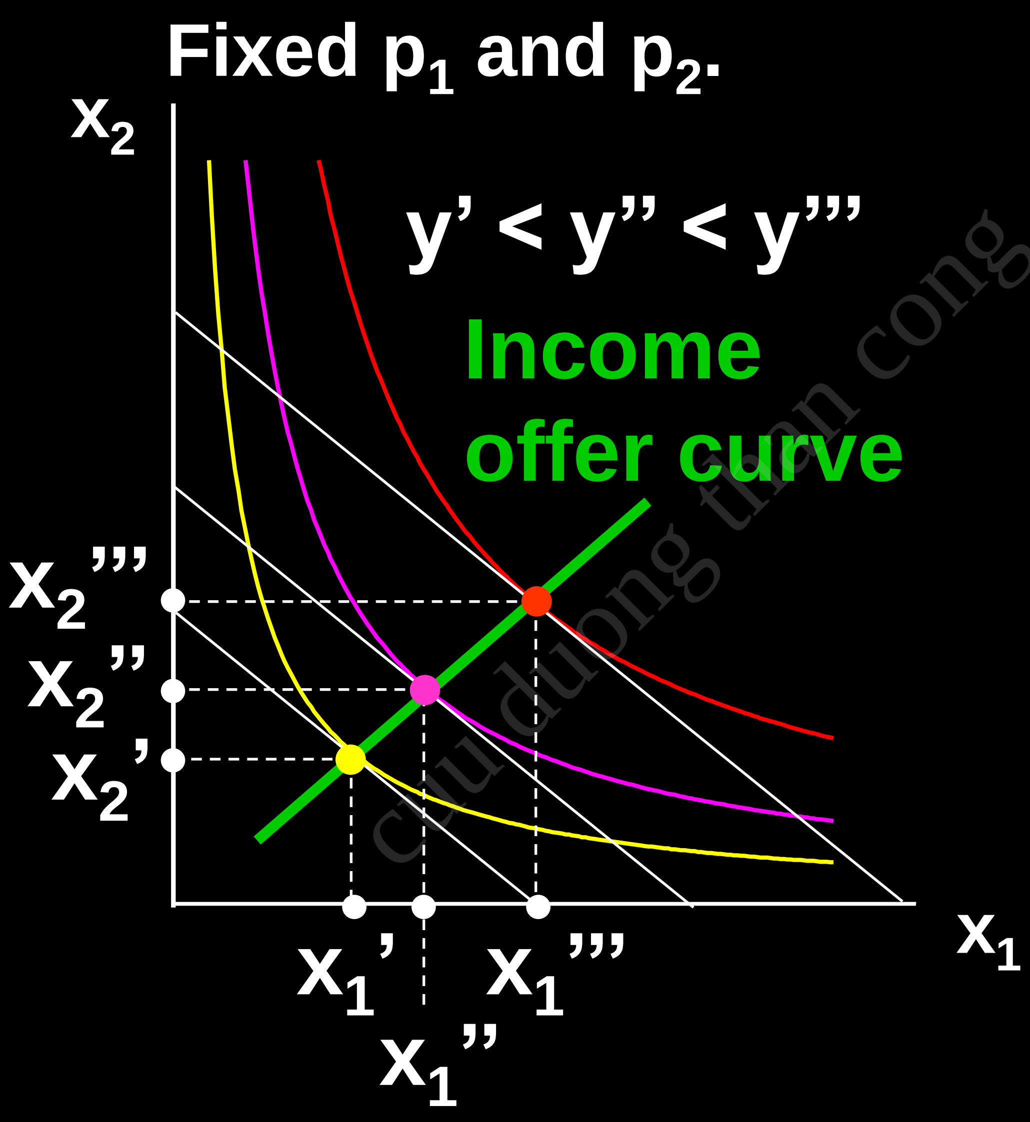


Income Changes

Fixed p_1 and p_2.



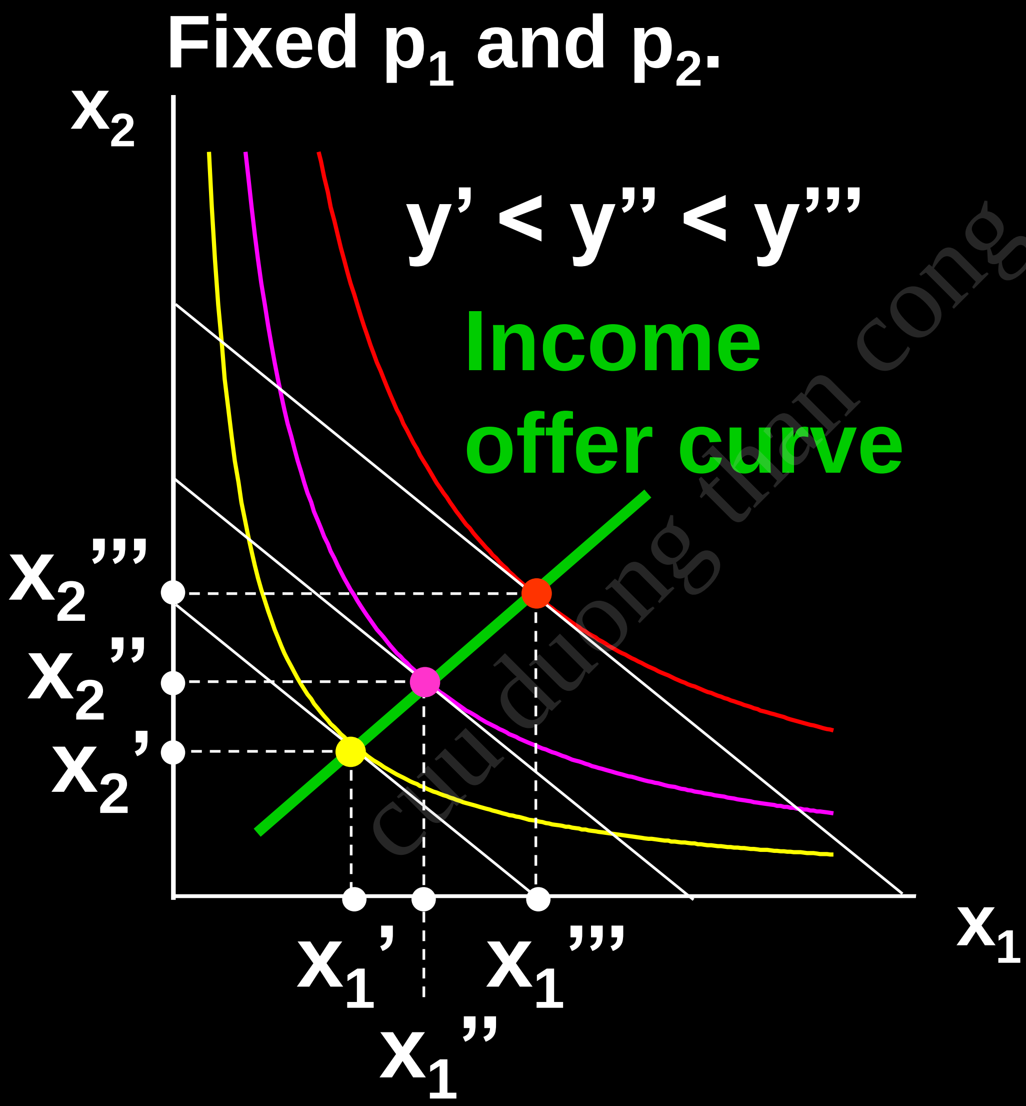
Income Changes



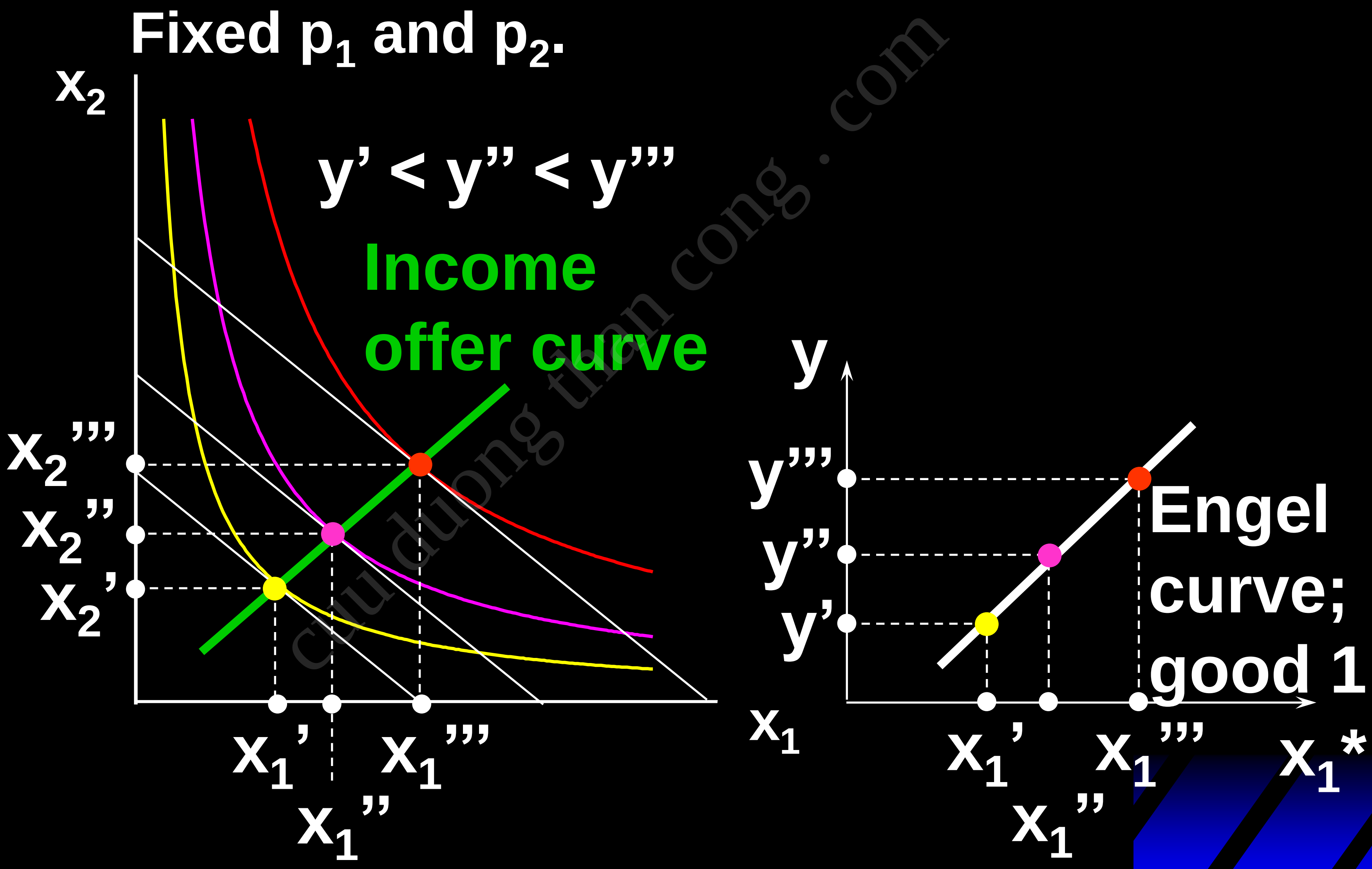
Income Changes

- **A plot of quantity demanded against income is called an Engel curve.**

Income Changes

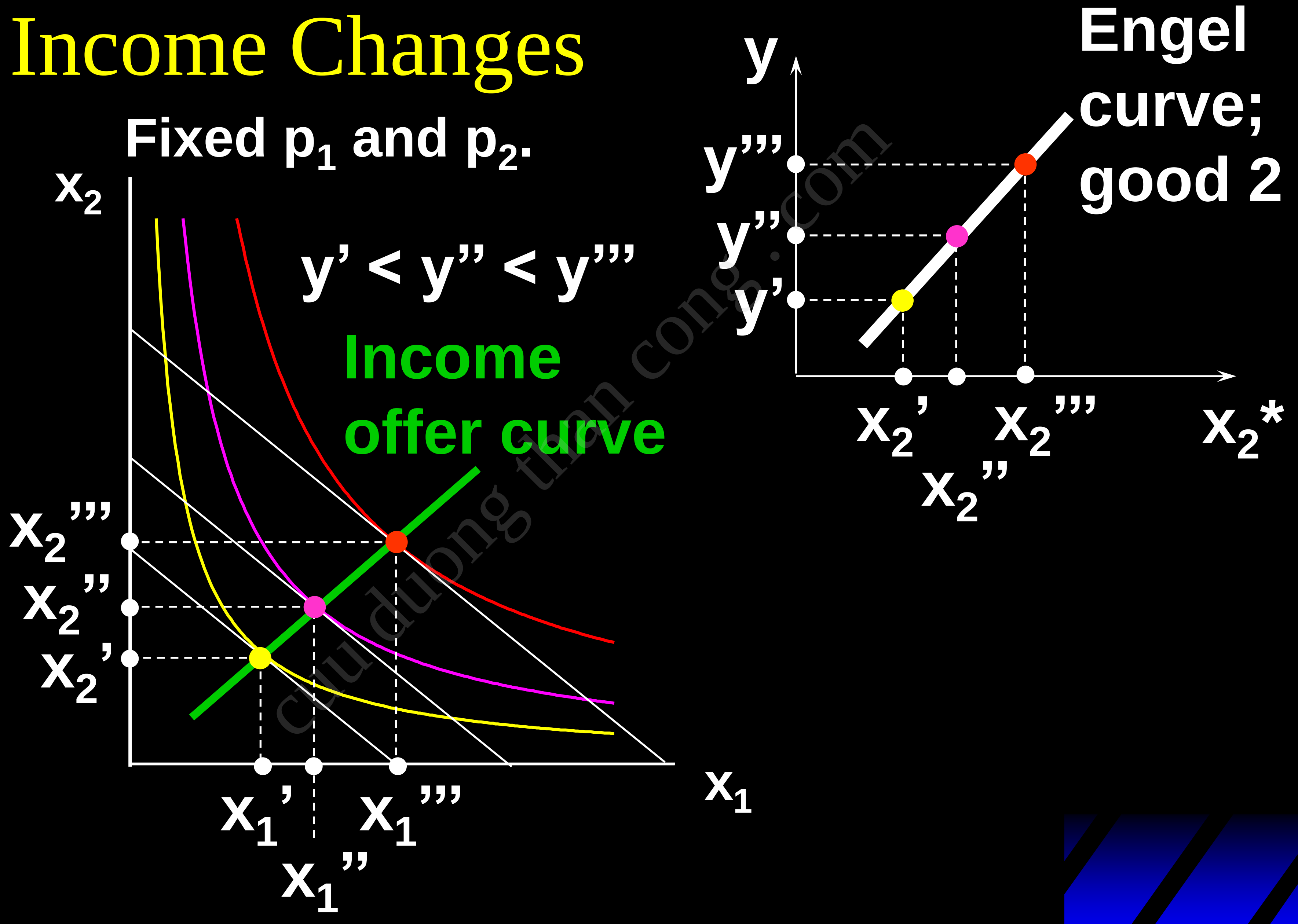


Income Changes

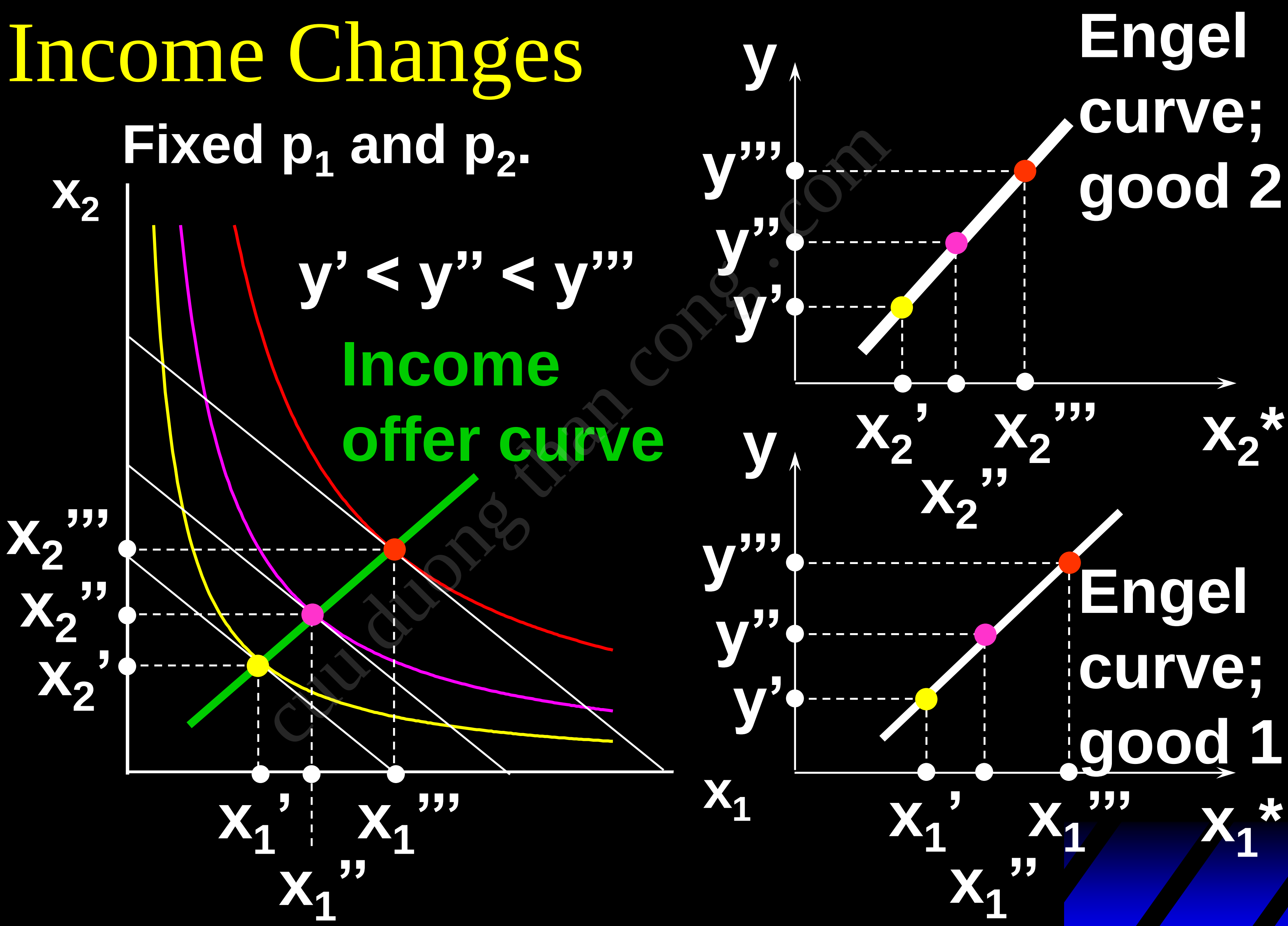


Income Changes

Fixed p_1 and p_2.



Income Changes



Income Changes and Cobb-Douglas Preferences

- An example of computing the equations of Engel curves; the Cobb-Douglas case.

$$U(x_1, x_2) = x_1^a x_2^b.$$

- The ordinary demand equations are

$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Income Changes and Cobb-Douglas Preferences

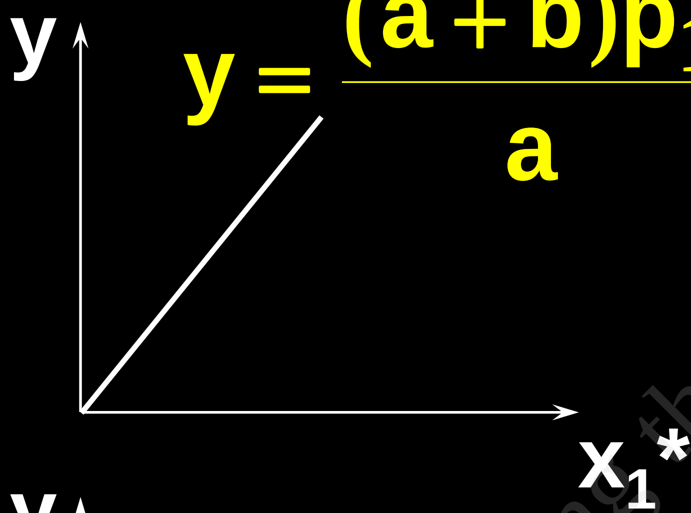
$$x_1^* = \frac{ay}{(a+b)p_1}; \quad x_2^* = \frac{by}{(a+b)p_2}.$$

Rearranged to isolate y, these are:

$$y = \frac{(a+b)p_1}{a} x_1^*$$ Engel curve for good 1

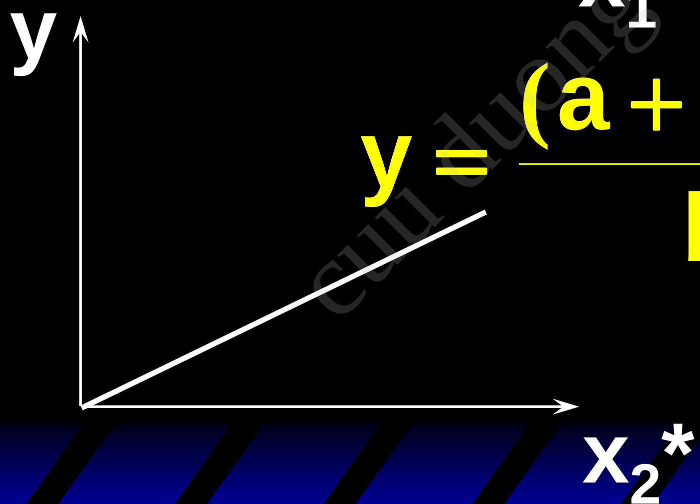
$$y = \frac{(a+b)p_2}{b} x_2^*$$ Engel curve for good 2

Income Changes and Cobb-Douglas Preferences



$$y = \frac{(a+b)p_1}{a} x_1^*$$

Engel curve for good 1



$$y = \frac{(a+b)p_2}{b} x_2^*$$

Engel curve for good 2

Income Changes and Perfectly-Complementary Preferences

- Another example of computing the equations of Engel curves; the perfectly-complementary case.

$$U(x_1, x_2) = \min\{x_1, x_2\}.$$

- The ordinary demand equations are

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

Income Changes and Perfectly-Complementary Preferences

$$x_1^* = x_2^* = \frac{y}{p_1 + p_2}.$$

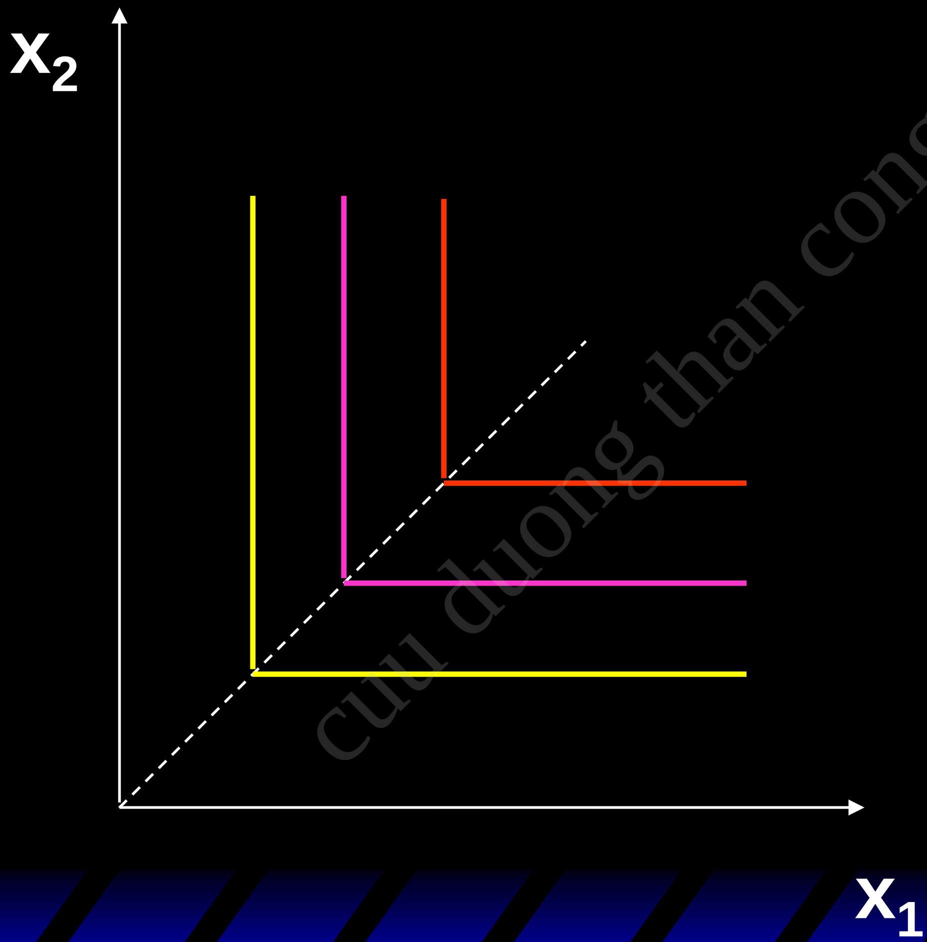
Rearranged to isolate y, these are:

$$y = (p_1 + p_2)x_1^*$$ **Engel curve for good 1**

$$y = (p_1 + p_2)x_2^*$$ **Engel curve for good 2**

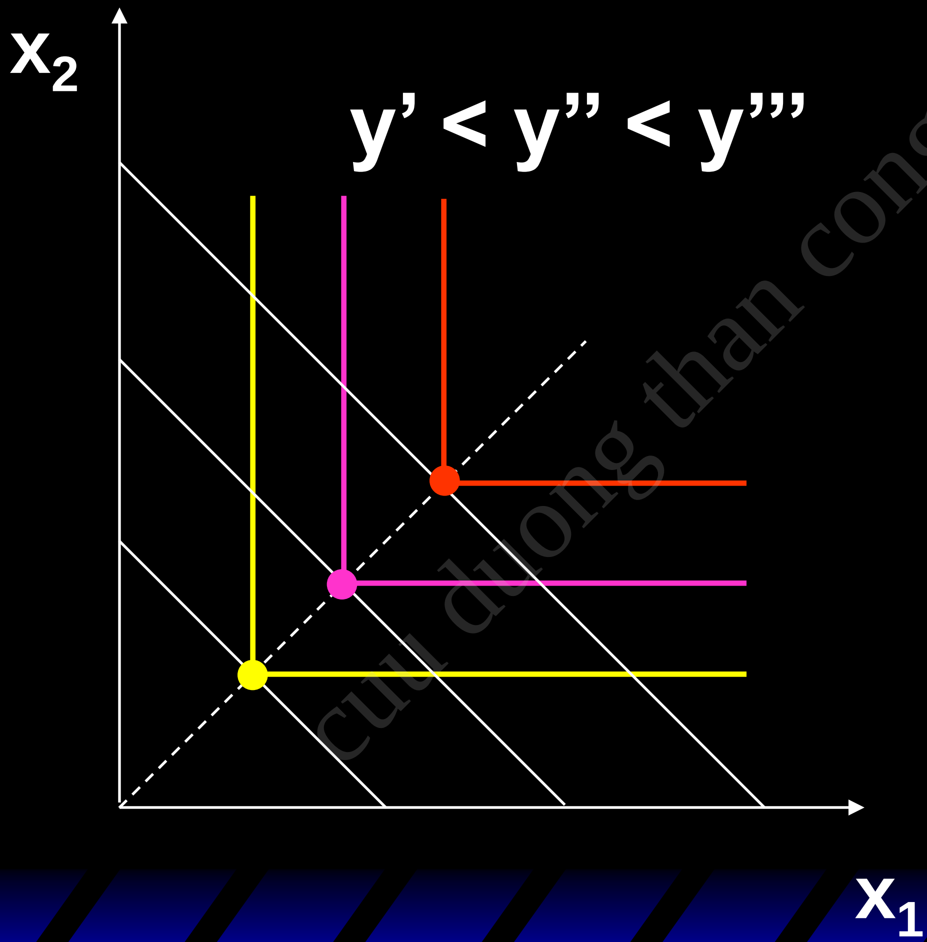
Income Changes

Fixed p_1 and p_2.



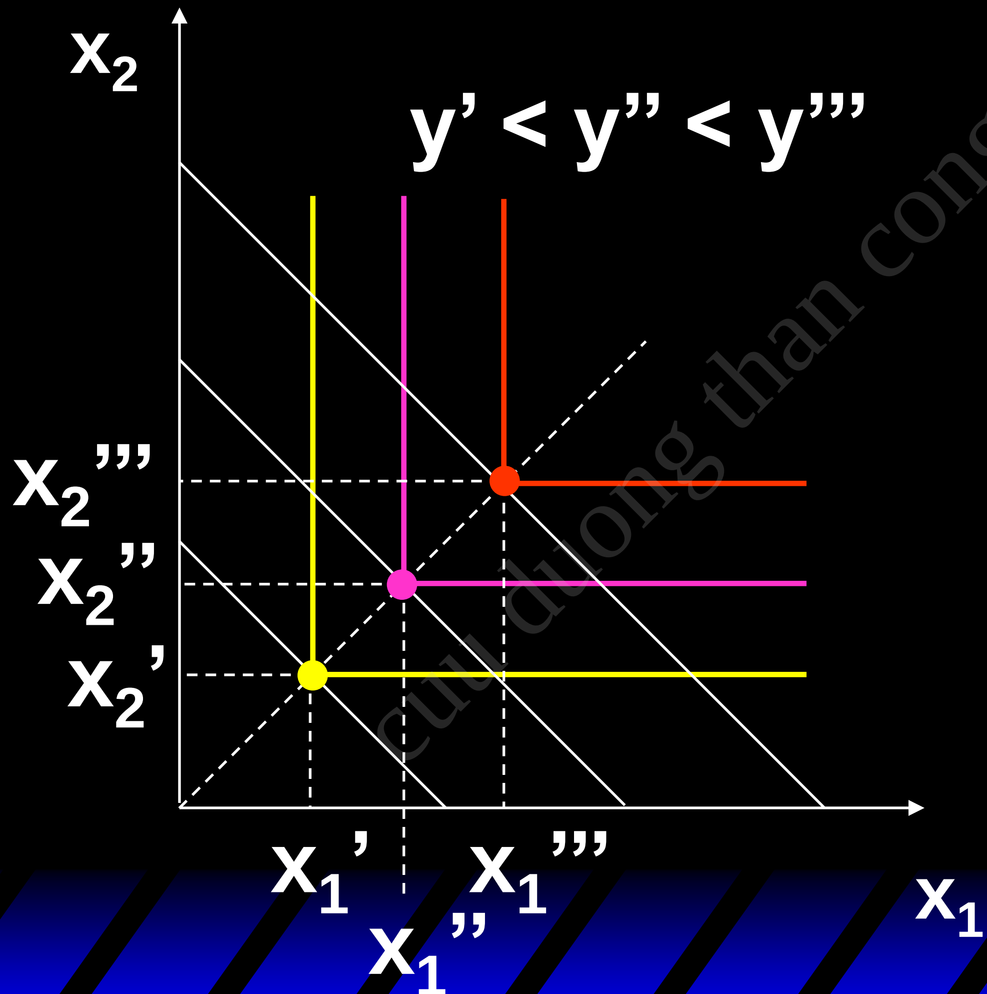
Income Changes

Fixed p_1 and p_2.

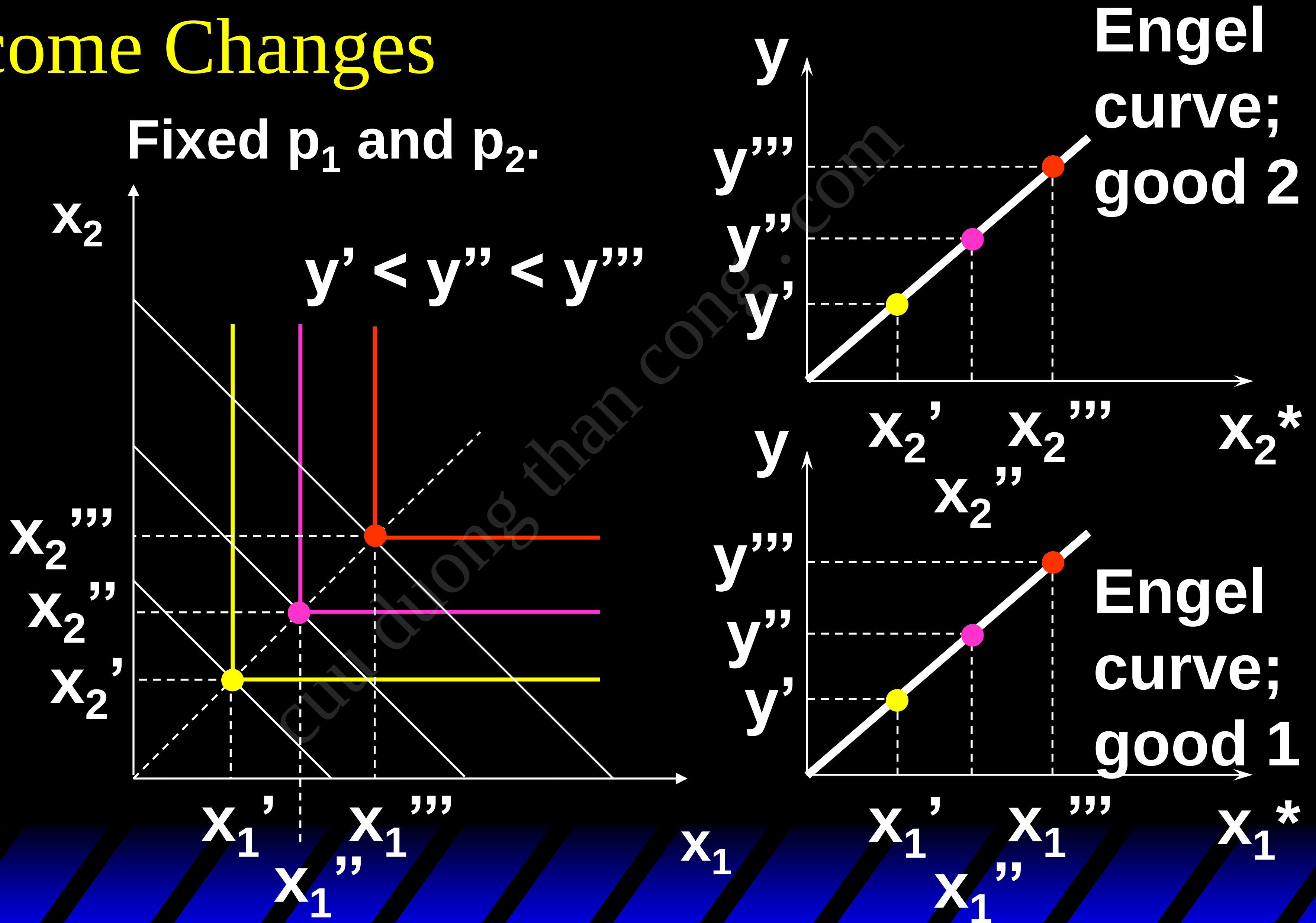


Income Changes

Fixed p_1 and p_2.



Income Changes

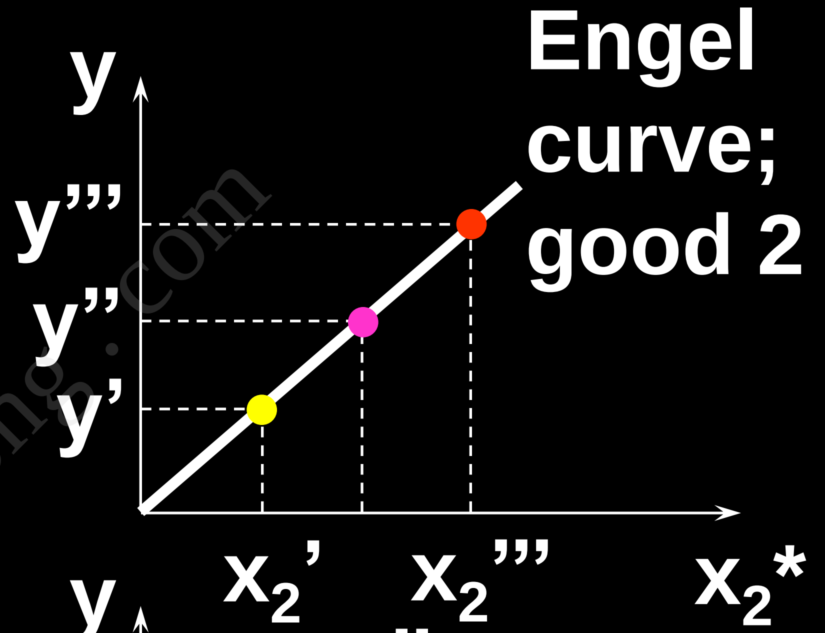


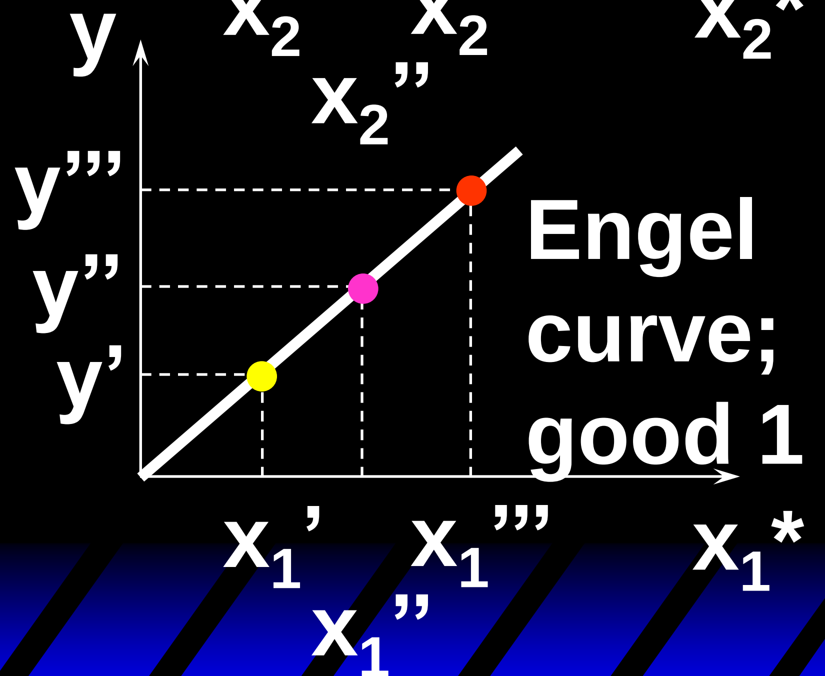
Income Changes

Fixed p_1 and p_2.

$$y = (p_1 + p_2)x_2^*$$

$$y = (p_1 + p_2)x_1^*$$





Income Changes and Perfectly-Substitutable Preferences

- **Another example of computing the equations of Engel curves; the perfectly-substitution case.**

$$U(x_1, x_2) = x_1 + x_2.$$

- **The ordinary demand equations are**

Income Changes and Perfectly-Substitutable Preferences

$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 > p_2 \\ y/p_1 & \text{, if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & \text{, if } p_1 < p_2 \\ y/p_2 & \text{, if } p_1 > p_2. \end{cases}$$

Income Changes and Perfectly-Substitutable Preferences

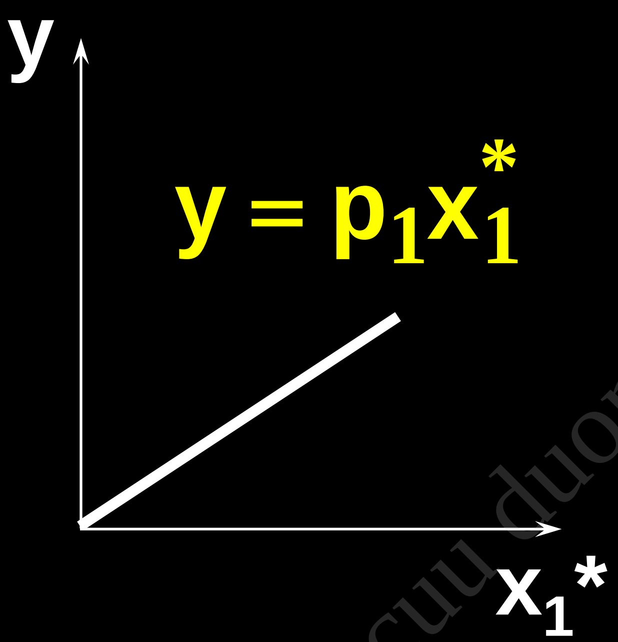
$$x_1^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 > p_2 \\ y/p_1 & , \text{if } p_1 < p_2 \end{cases}$$

$$x_2^*(p_1, p_2, y) = \begin{cases} 0 & , \text{if } p_1 < p_2 \\ y/p_2 & , \text{if } p_1 > p_2. \end{cases}$$

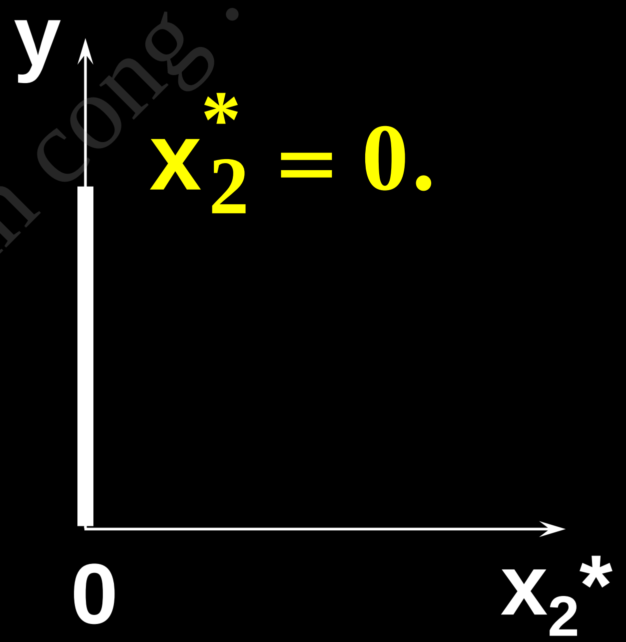
Suppose $p_1 < p_2$. Then $x_1^* = \dfrac{y}{p_1}$ and $x_2^* = 0$

$\Rightarrow$ $y = p_1 x_1^*$ and $x_2^* = 0.$

Income Changes and Perfectly-Substitutable Preferences



Engel curve for good 1



Engel curve for good 2

Income Changes

- In every example so far the Engel curves have all been straight lines? Q: Is this true in general?
- A: No. Engel curves are straight lines if the consumer's preferences are **homothetic**.

Homotheticity

- A consumer's preferences are **homothetic** if and only if

$$(x_1,x_2) \succ (y_1,y_2) \Leftrightarrow (kx_1,kx_2) \succ (ky_1,ky_2)$$

for every k > 0.

- That is, the consumer's MRS is the same anywhere on a straight line drawn from the origin.

Income Effects -- A Nonhomothetic Example

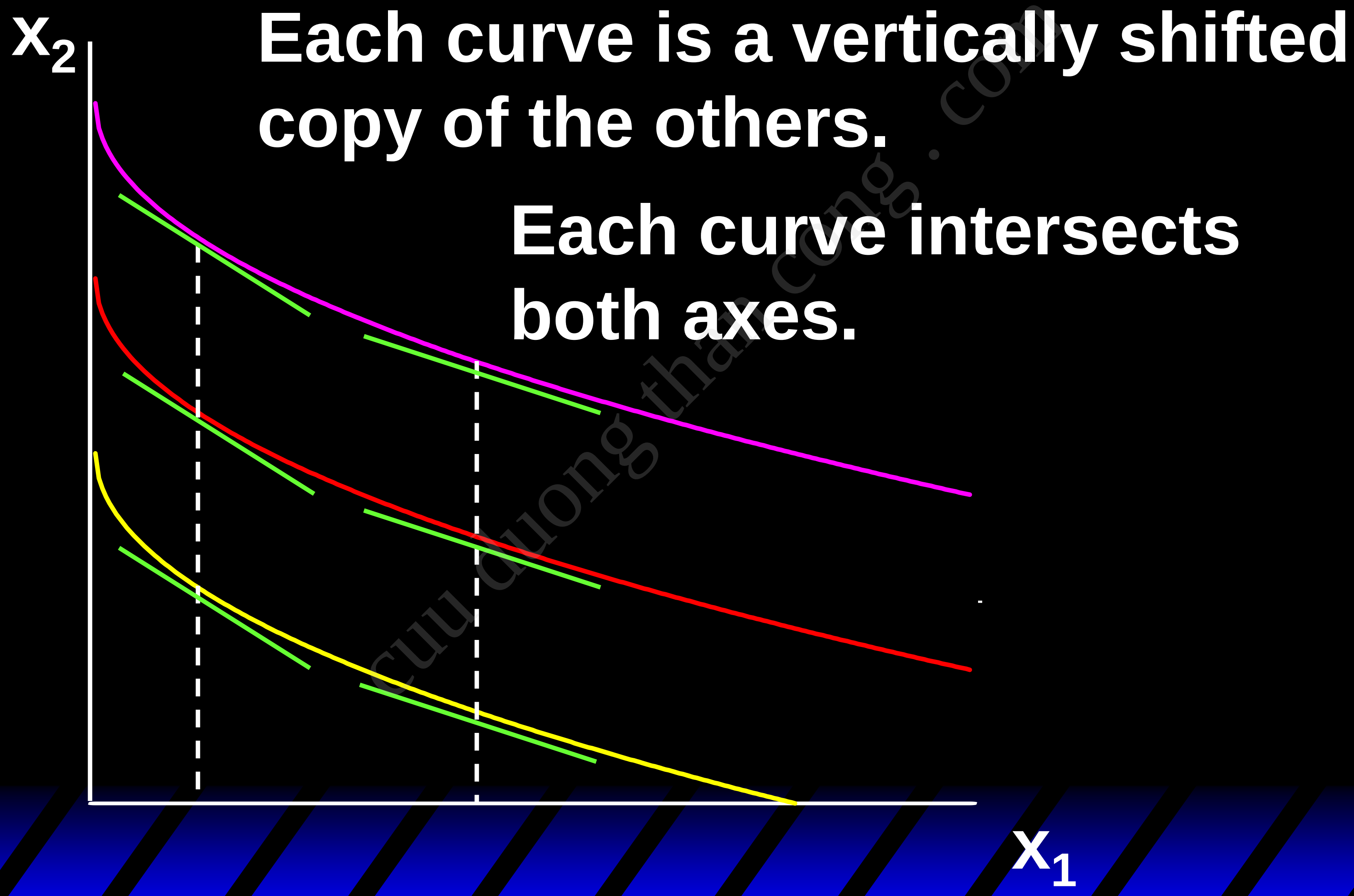
- Quasilinear preferences are not homothetic.

$$U(x_1, x_2) = f(x_1) + x_2.$$

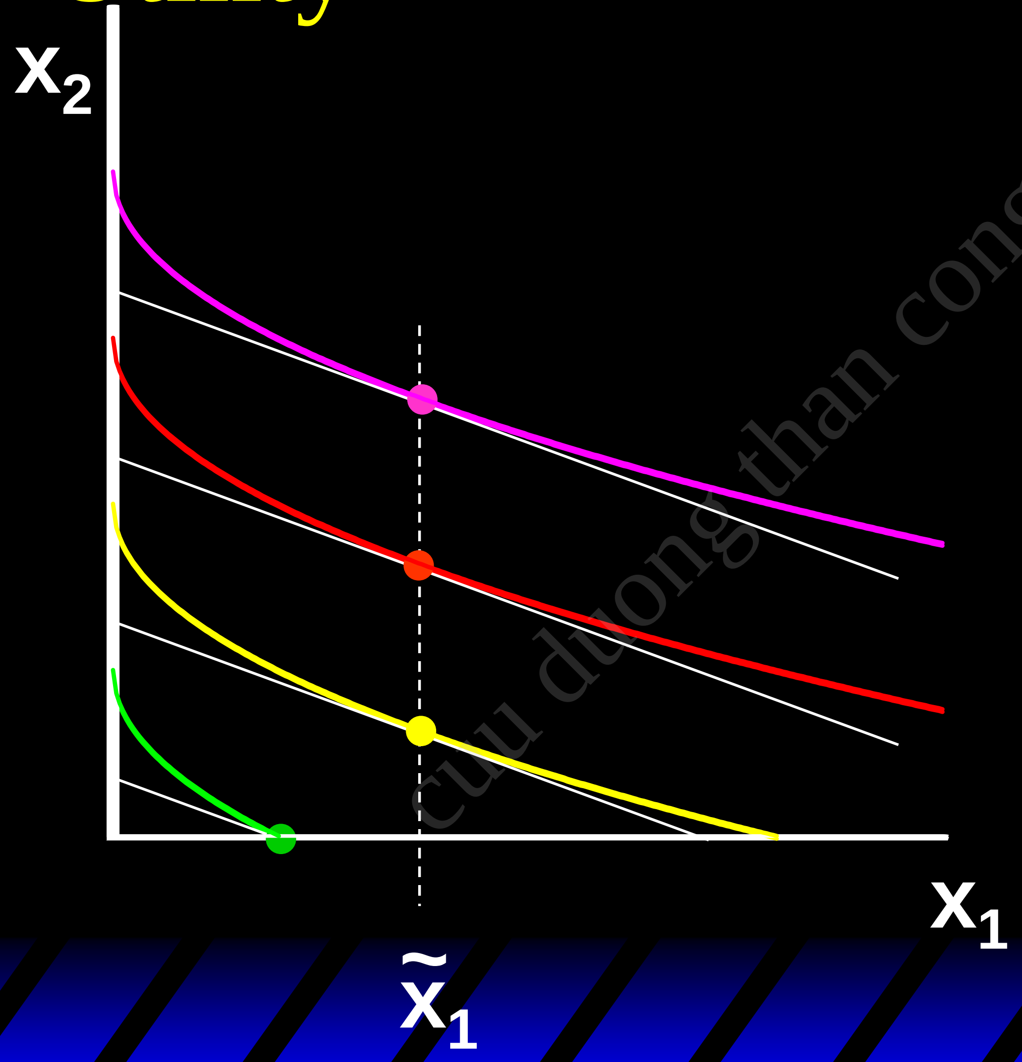
- For example,

$$U(x_1, x_2) = \sqrt{x_1} + x_2.$$

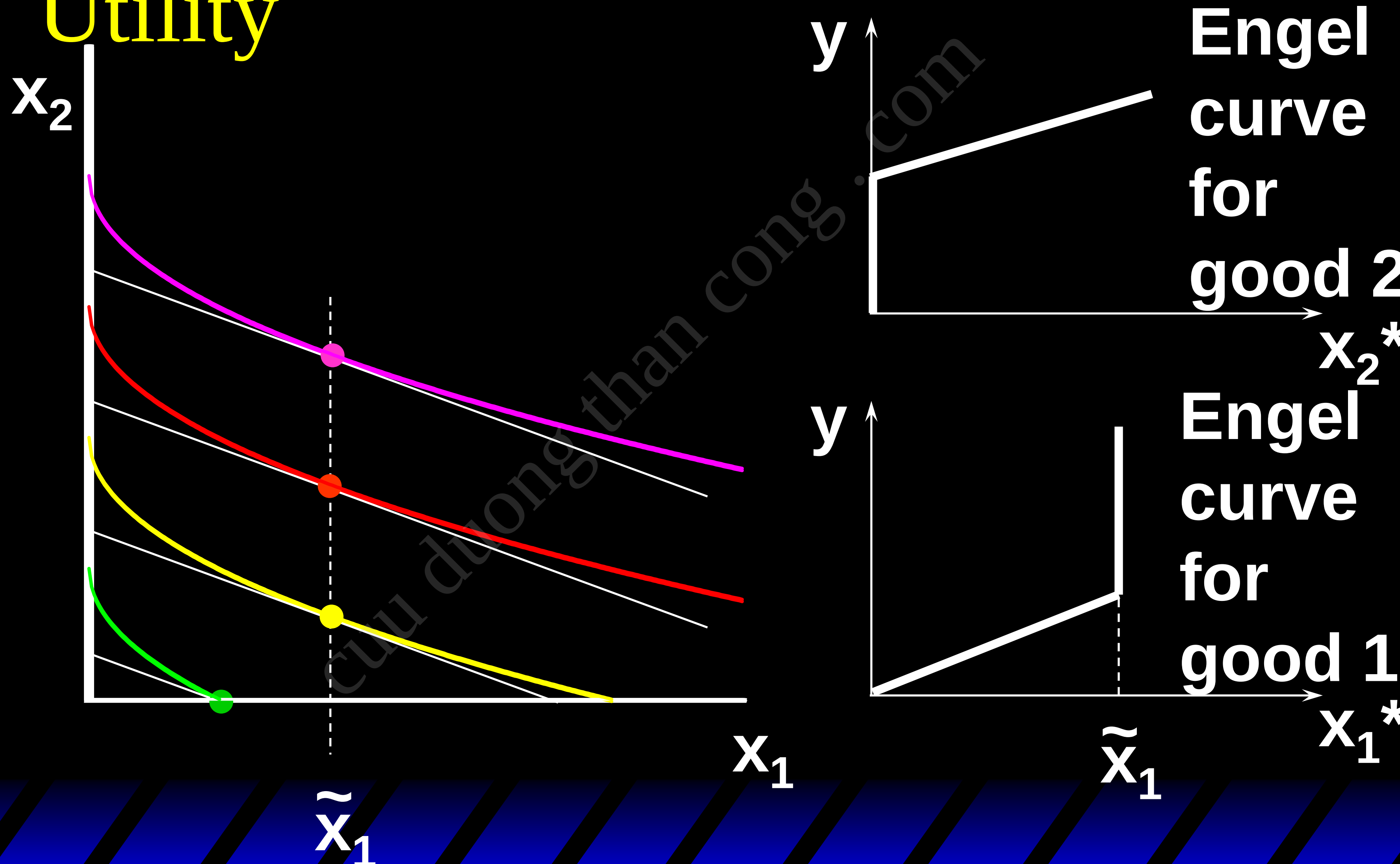
Quasi-linear Indifference Curves



Income Changes; Quasilinear Utility



Income Changes; Quasilinear Utility



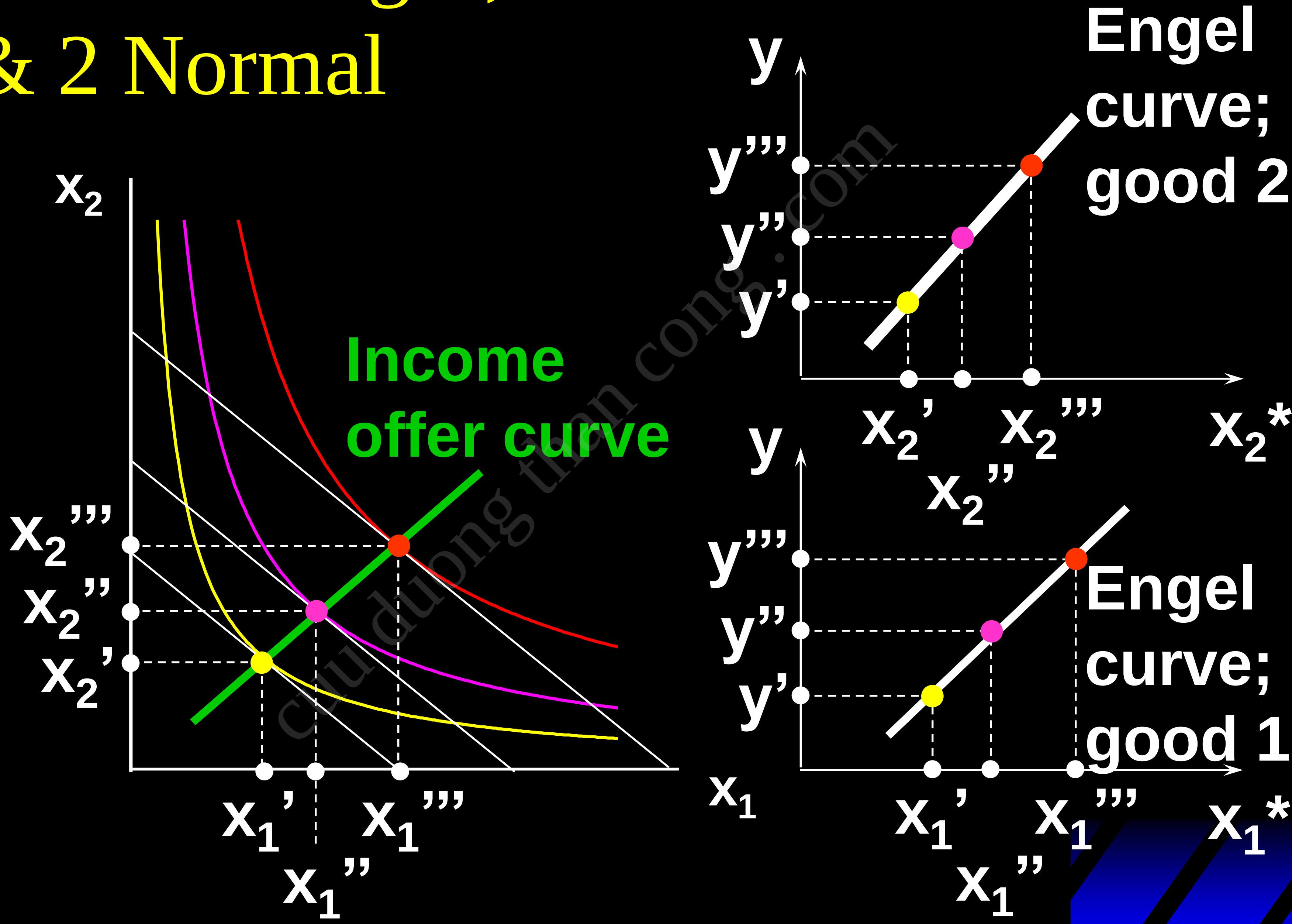
Income Effects

- A good for which quantity demanded rises with income is called **normal**.
- Therefore a normal good's Engel curve is positively sloped.

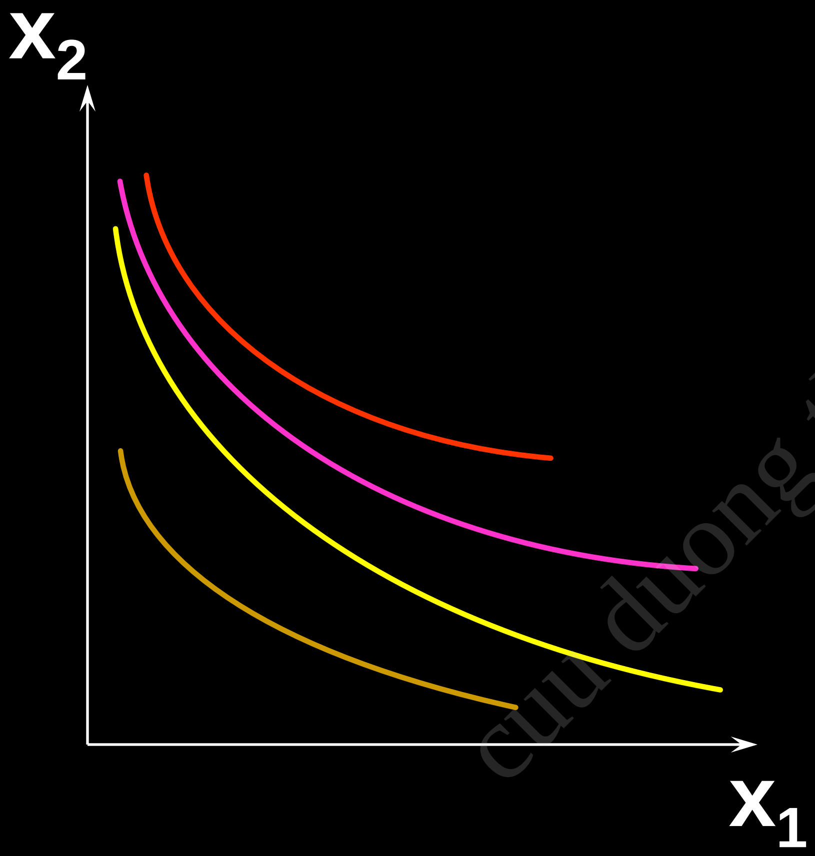
Income Effects

- A good for which quantity demanded falls as income increases is called **income inferior**.
- Therefore an income inferior good's Engel curve is negatively sloped.

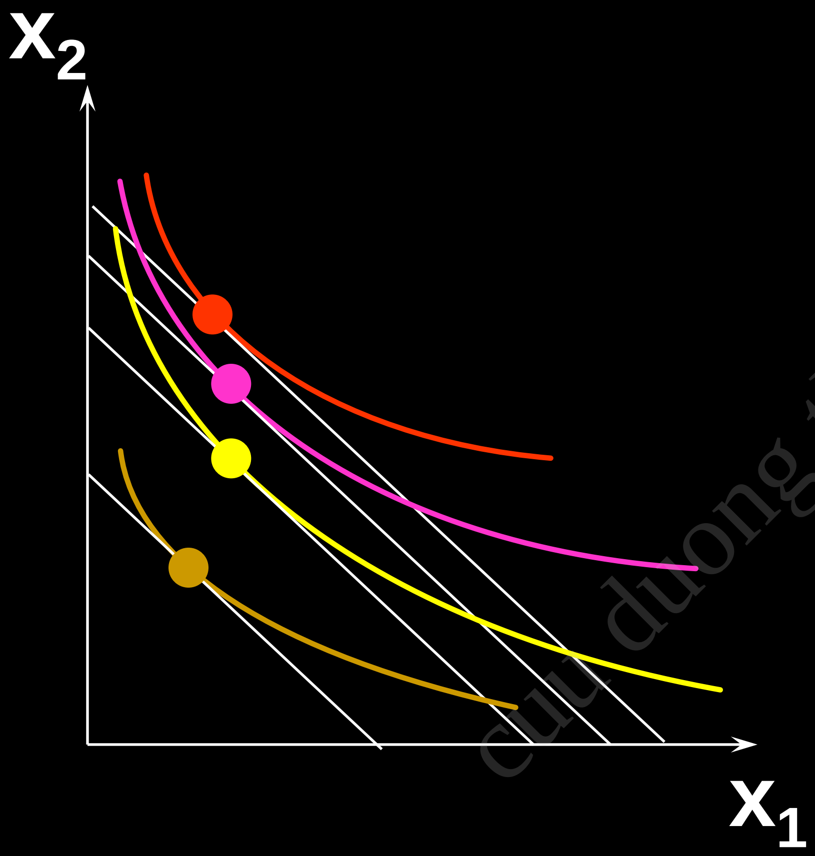
Income Changes; Goods 1 & 2 Normal



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



Income Changes; Good 2 Is Normal, Good 1 Becomes Income Inferior



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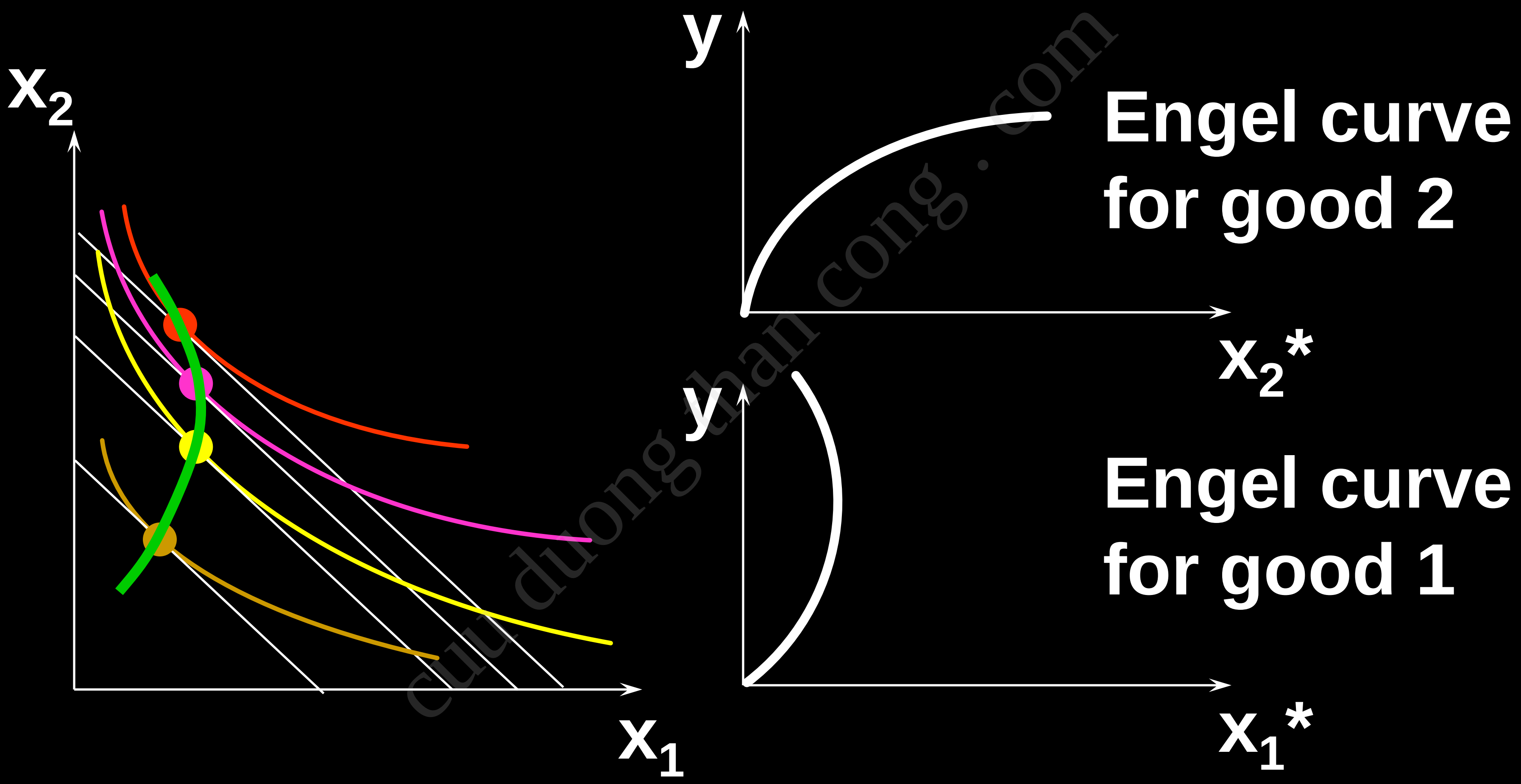
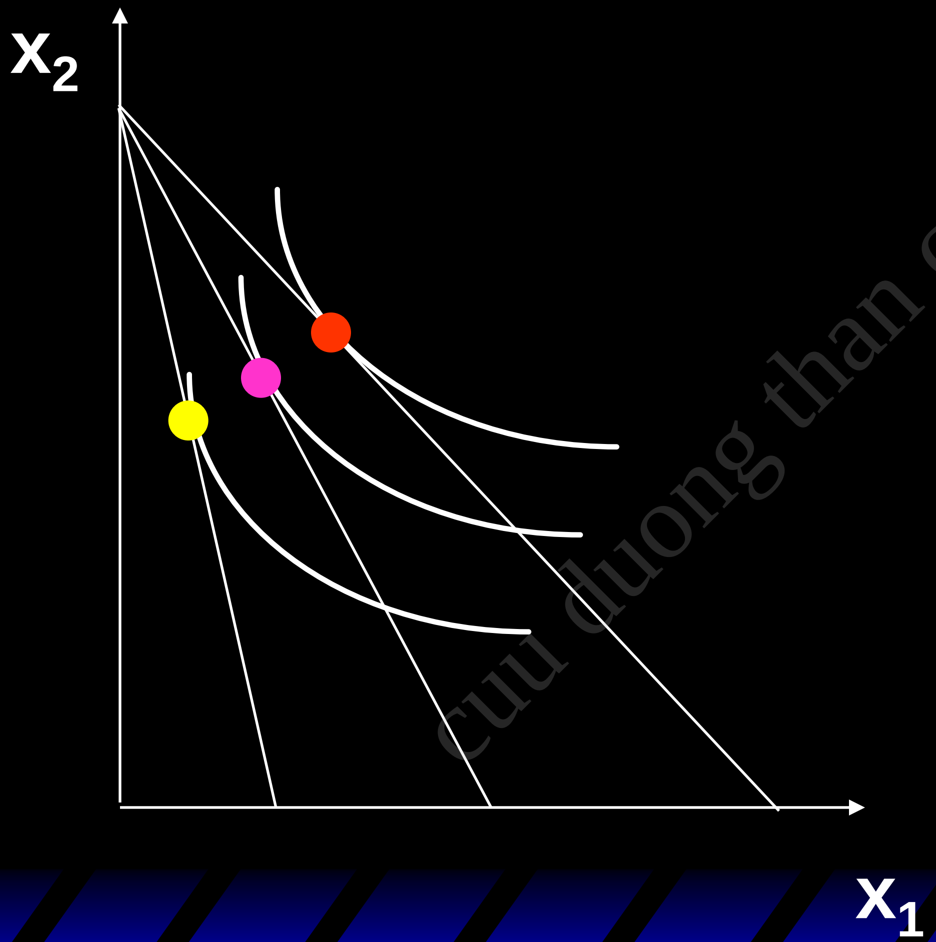


Ordinary Goods

- A good is called **ordinary** if the quantity demanded of it always increases as its own price decreases.

Ordinary Goods

Fixed p_2 and y.



Ordinary Goods

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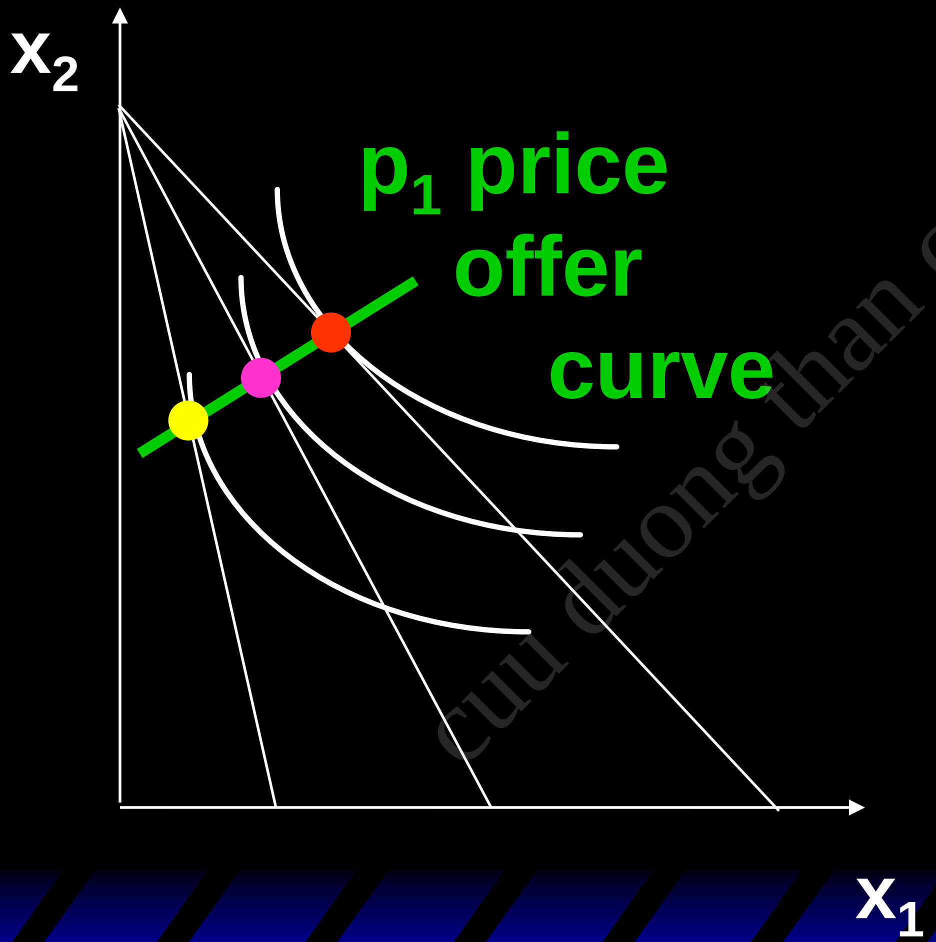
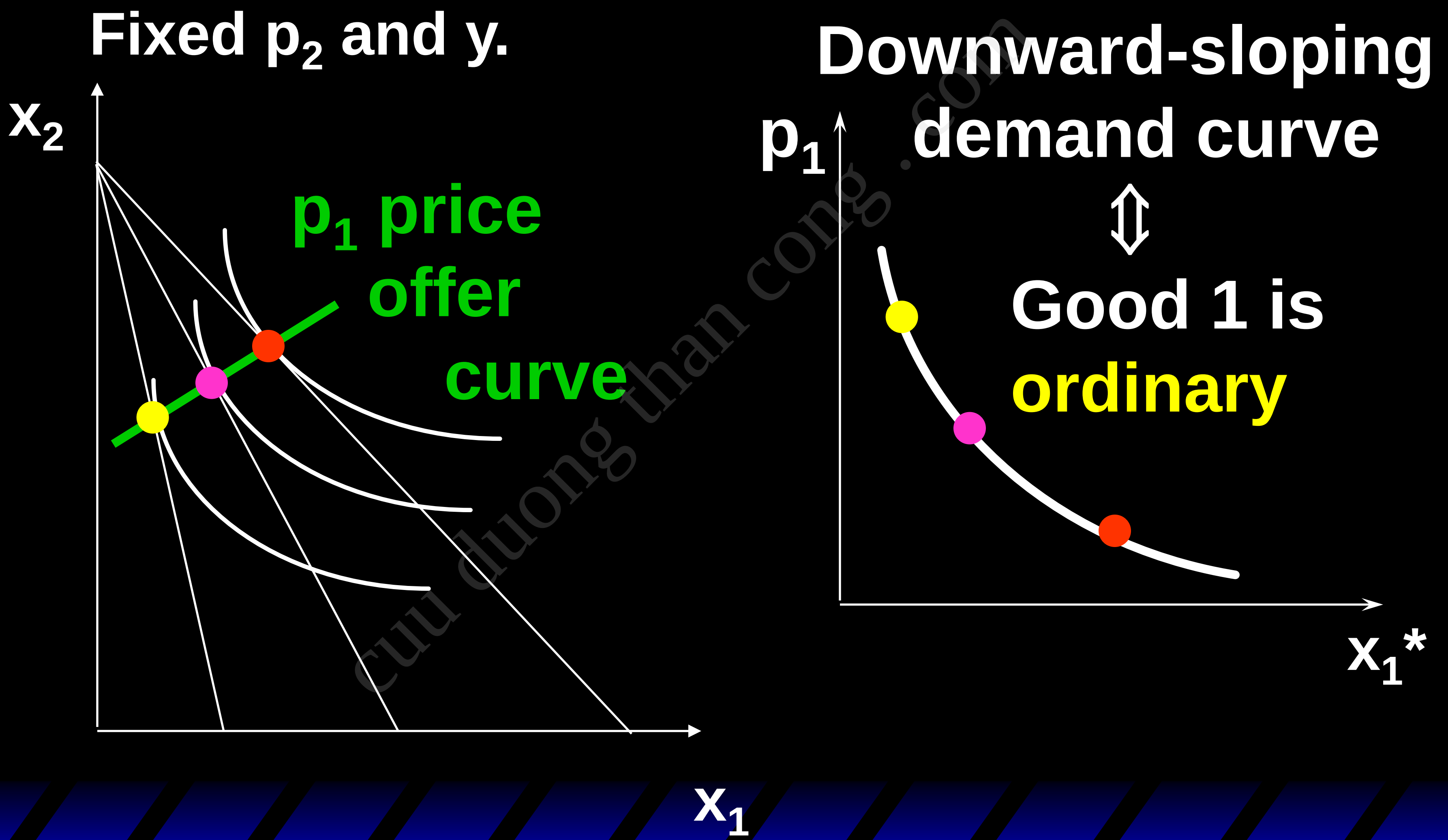


Ordinary Goods

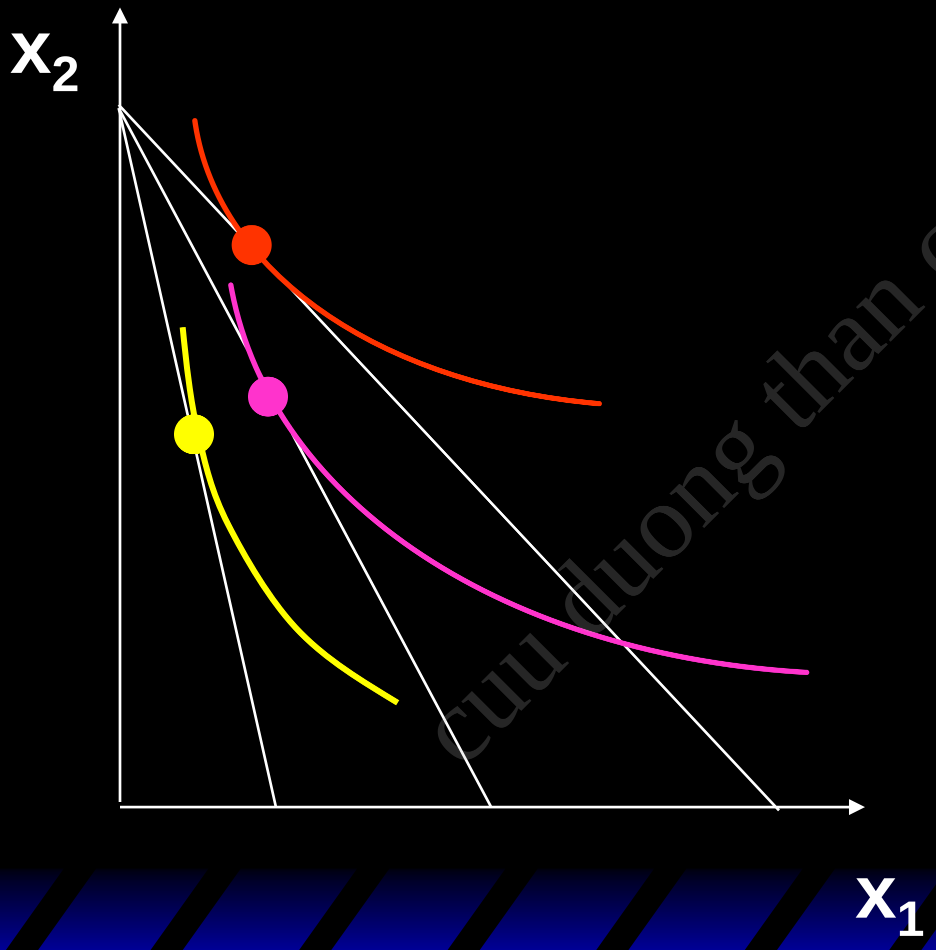


Giffen Goods

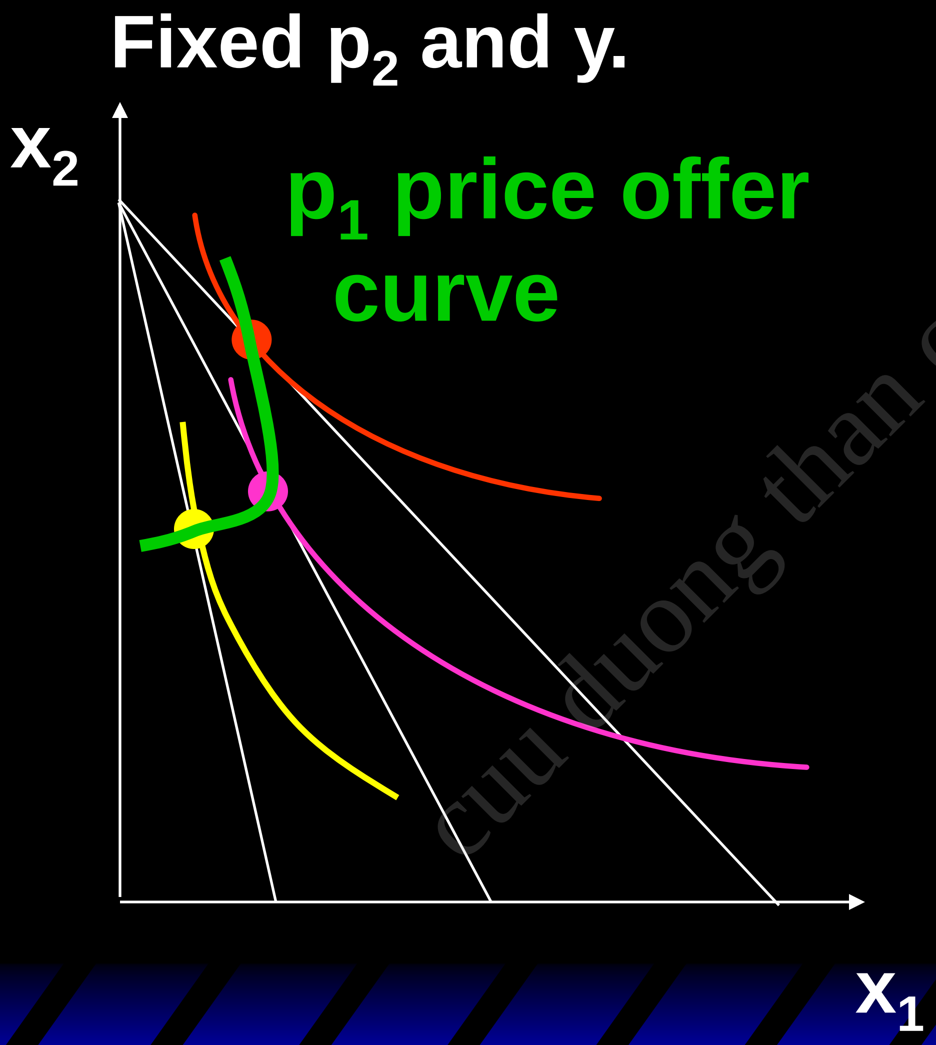
- If, for **some** values of its own price, the quantity demanded of a good rises as its own-price increases then the good is called **Giffen**.

Ordinary Goods

Fixed p_2 and y.

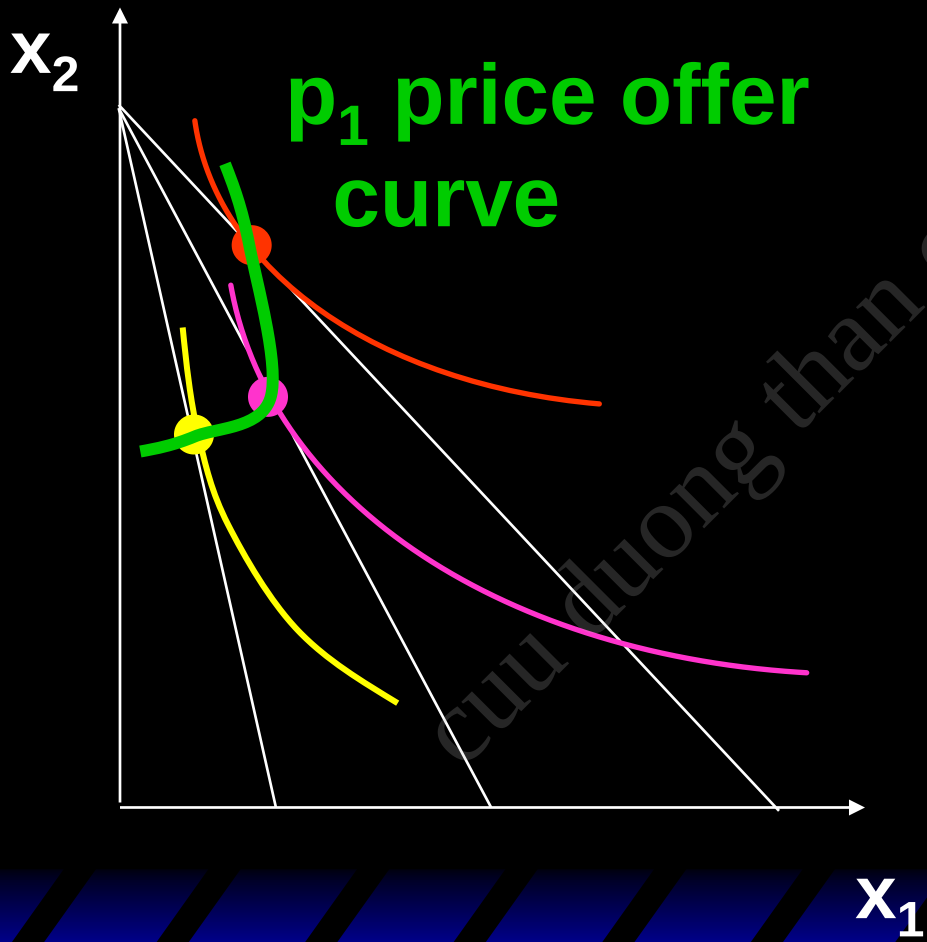


Ordinary Goods



Ordinary Goods

Fixed p_2 and y.



Demand curve has a positively sloped part

⇕

Good 1 is **Giffen**

Cross-Price Effects

- If an increase in p_2
  - **increases** demand for commodity 1 then commodity 1 is a **gross substitute** for commodity 2.
  - **reduces** demand for commodity 1 then commodity 1 is a **gross complement** for commodity 2.

Cross-Price Effects

A perfect-complements example:

$$x_1^* = \frac{y}{p_1 + p_2}$$

so

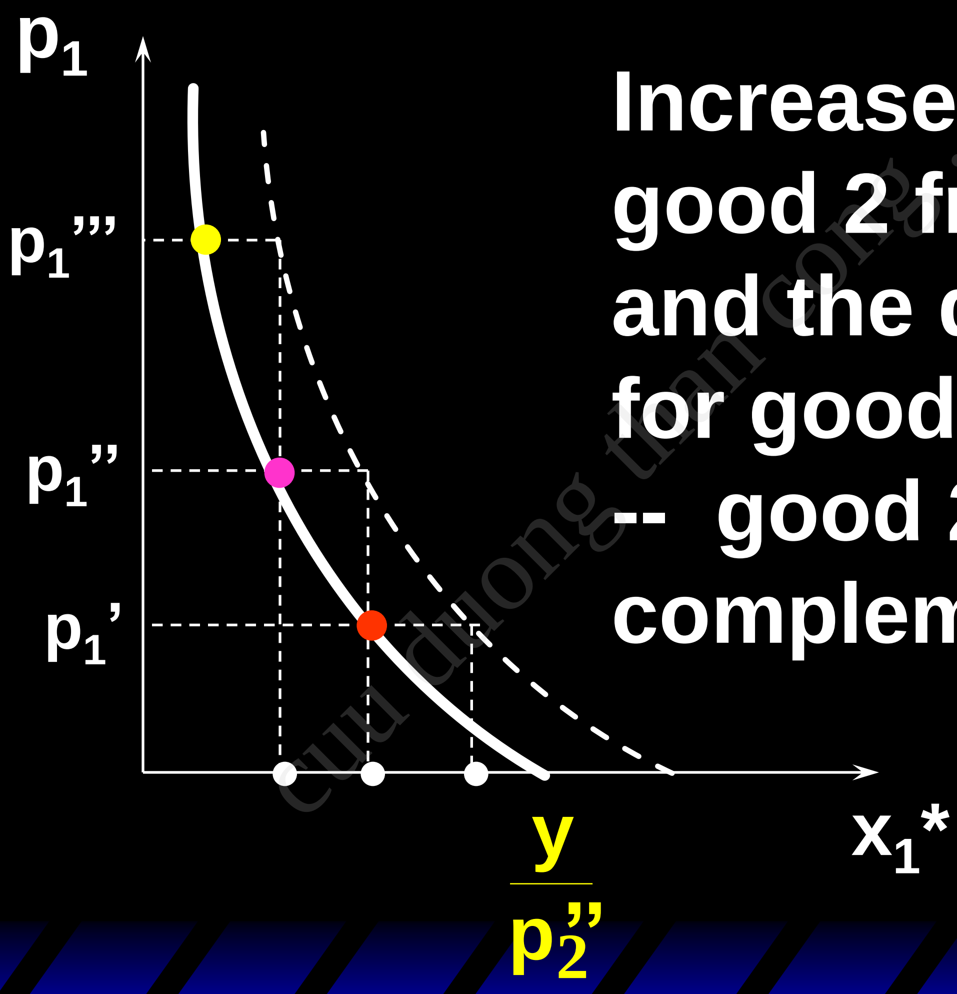
$$\frac{\partial x_1^*}{\partial p_2} = -\frac{y}{(p_1 + p_2)^2} < 0.$$

Therefore commodity 2 is a gross complement for commodity 1.

Cross-Price Effects

Increase the price of good 2 from p_2' to p_2'' and

Cross-Price Effects



Increase the price of good 2 from p_2' to p_2'' and the demand curve for good 1 shifts inwards -- good 2 is a complement for good 1.

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

Cross-Price Effects

A Cobb- Douglas example:

$$x_2^* = \frac{by}{(a+b)p_2}$$

so

$$\frac{\partial x_2^*}{\partial p_1} = 0.$$

Therefore commodity 1 is neither a gross complement nor a gross substitute for commodity 2.

3. Intertemporal Choice

- Preferences for consumption
- Inflation
- Present value

Intertemporal Choice

- **Persons often receive income in "lumps"; *e.g.* monthly salary.**
- **How is a lump of income spread over the following month (saving now for consumption later)?**
- **Or how is consumption financed by borrowing now against income to be received at the end of the month?**

Present and Future Values

- Begin with some simple financial arithmetic.
- Take just two periods; 1 and 2.
- Let r denote the interest rate per period.

Future Value

- E.g., if r = 0.1 then $100 saved at the start of period 1 becomes $110 at the start of period 2.
- The value next period of $1 saved now is the **future value** of that dollar.

Future Value

- **Given an interest rate r the future value one period from now of $1 is**

$$FV = 1 + r.$$

- **Given an interest rate r the future value one period from now of $m is**

$$FV = m(1 + r).$$

Present Value

- Suppose you can pay now to obtain \$1 at the start of next period.
- What is the most you should pay?
- \$1?
- No. If you kept your \$1 now and saved it then at the start of next period you would have \$(1+r) > \$1, so paying \$1 now for \$1 next period is a bad deal.

Present Value

- Q: How much money would have to be saved now, in the present, to obtain $1 at the start of the next period?
- A: $m saved now becomes $m(1+r) at the start of next period, so we want the value of m for which

$$m(1+r) = 1$$

That is, $m = 1/(1+r)$,

the **present-value** of $1 obtained at the start of next period.

Present Value

- The **present value** of \$1 available at the start of the next period is

$$PV = \frac{1}{1+r}.$$

- And the present value of \$m available at the start of the next period is

$$PV = \frac{m}{1+r}.$$

Present Value

- E.g., if r = 0.1 then the most you should pay now for $1 available next period is

$$\text{PV} = \frac{1}{1+0\cdot 1} = \$0\cdot 91.$$

- And if r = 0.2 then the most you should pay now for $1 available next period is

$$\text{PV} = \frac{1}{1+0\cdot 2} = \$0\cdot 83.$$

The Intertemporal Choice Problem

- Let m_1 and m_2 be incomes received in periods 1 and 2.
- Let c_1 and c_2 be consumptions in periods 1 and 2.
- Let p_1 and p_2 be the prices of consumption in periods 1 and 2.

The Intertemporal Choice Problem

- **The intertemporal choice problem: Given incomes m_1 and m_2, and given consumption prices p_1 and p_2, what is the most preferred intertemporal consumption bundle (c_1, c_2)?**
- **For an answer we need to know:**
  - **the intertemporal budget constraint**
  - **intertemporal consumption preferences.**

The Intertemporal Budget Constraint

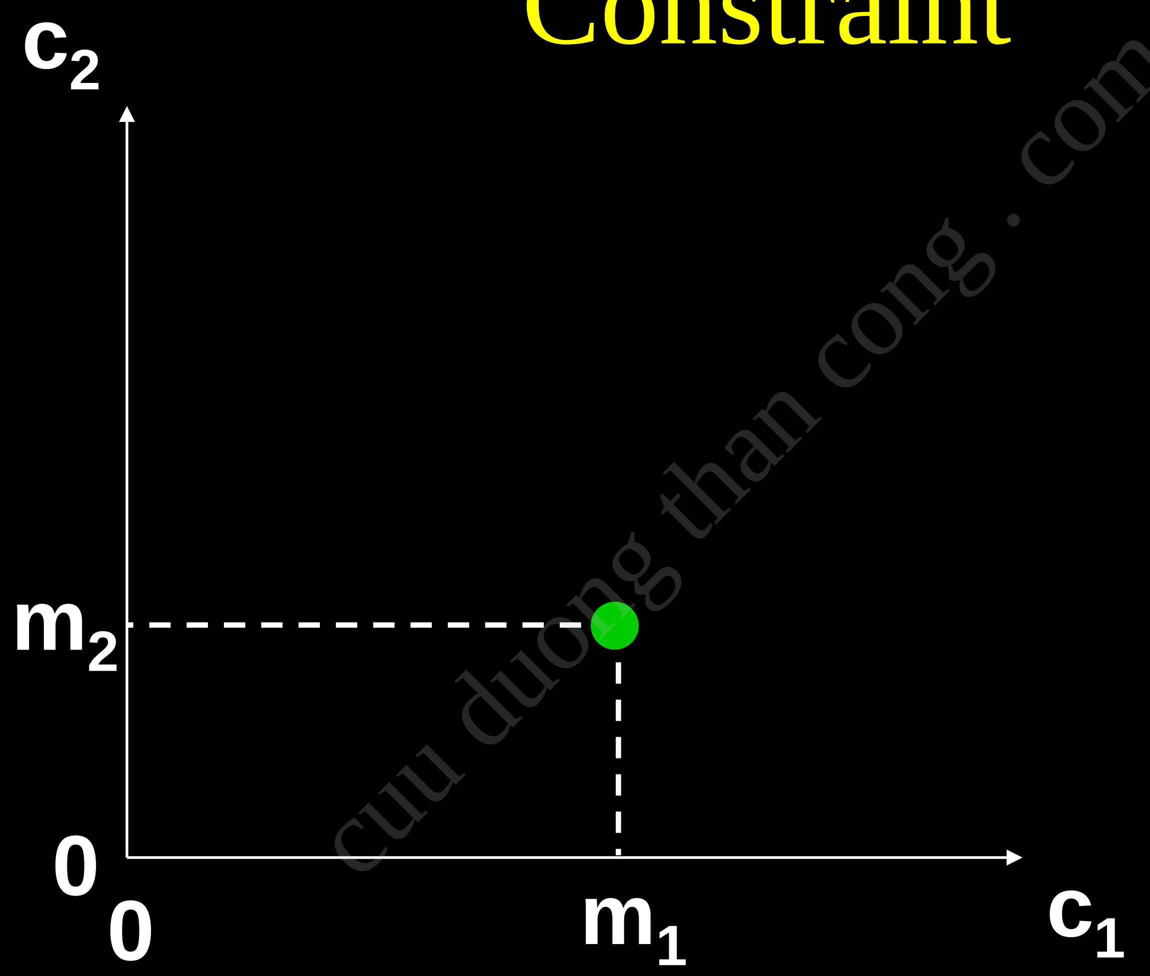
- **To start, let's ignore price effects by supposing that**

$$p_1 = p_2 = \$1.$$

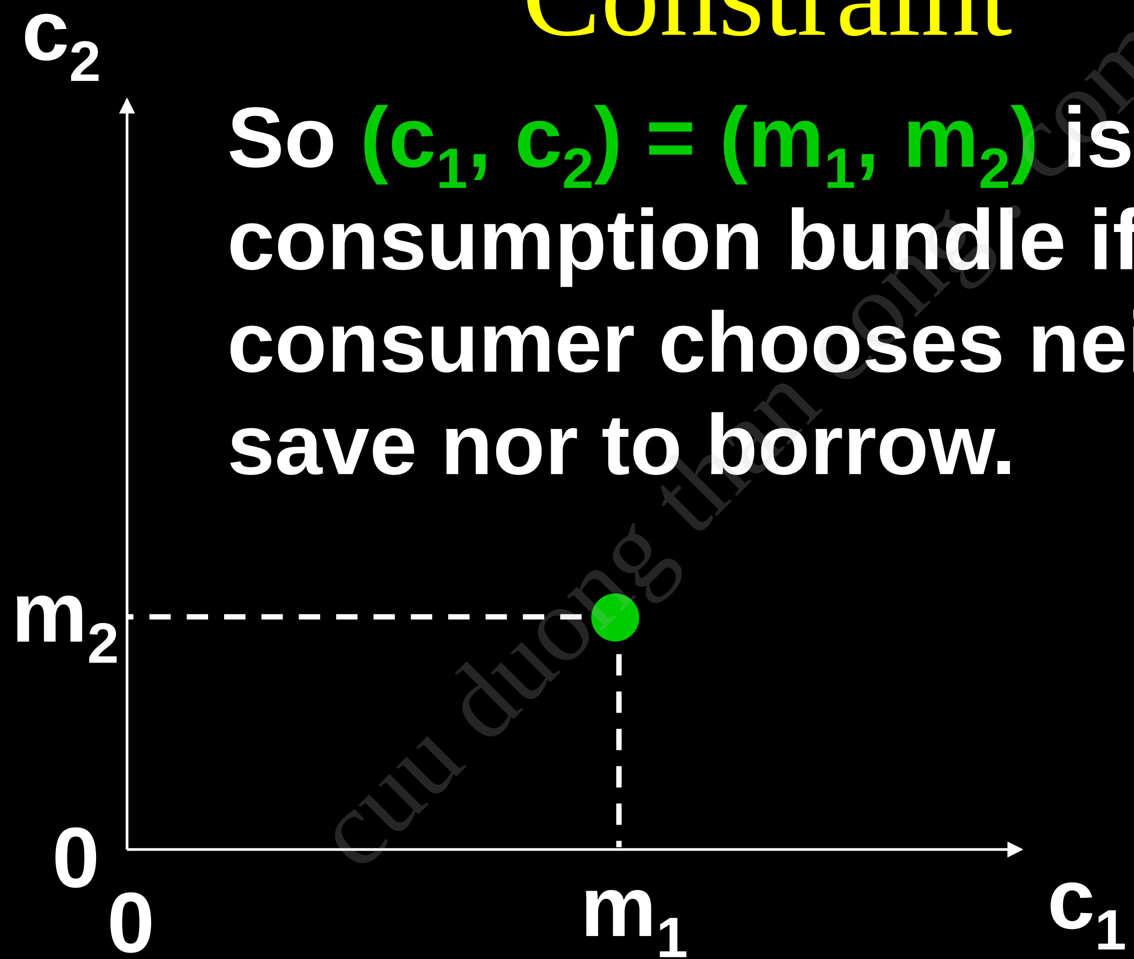
The Intertemporal Budget Constraint

- Suppose that the consumer chooses not to save or to borrow.
- Q: What will be consumed in period 1?
- A: $c_1 = m_1$.
- Q: What will be consumed in period 2?
- A: $c_2 = m_2$.

The Intertemporal Budget Constraint



The Intertemporal Budget Constraint



So $(c_1, c_2) = (m_1, m_2)$ is the consumption bundle if the consumer chooses neither to save nor to borrow.

The Intertemporal Budget Constraint

- Now suppose that the consumer spends nothing on consumption in period 1; that is, $c_1 = 0$ and the consumer saves

  $$s_1 = m_1.$$

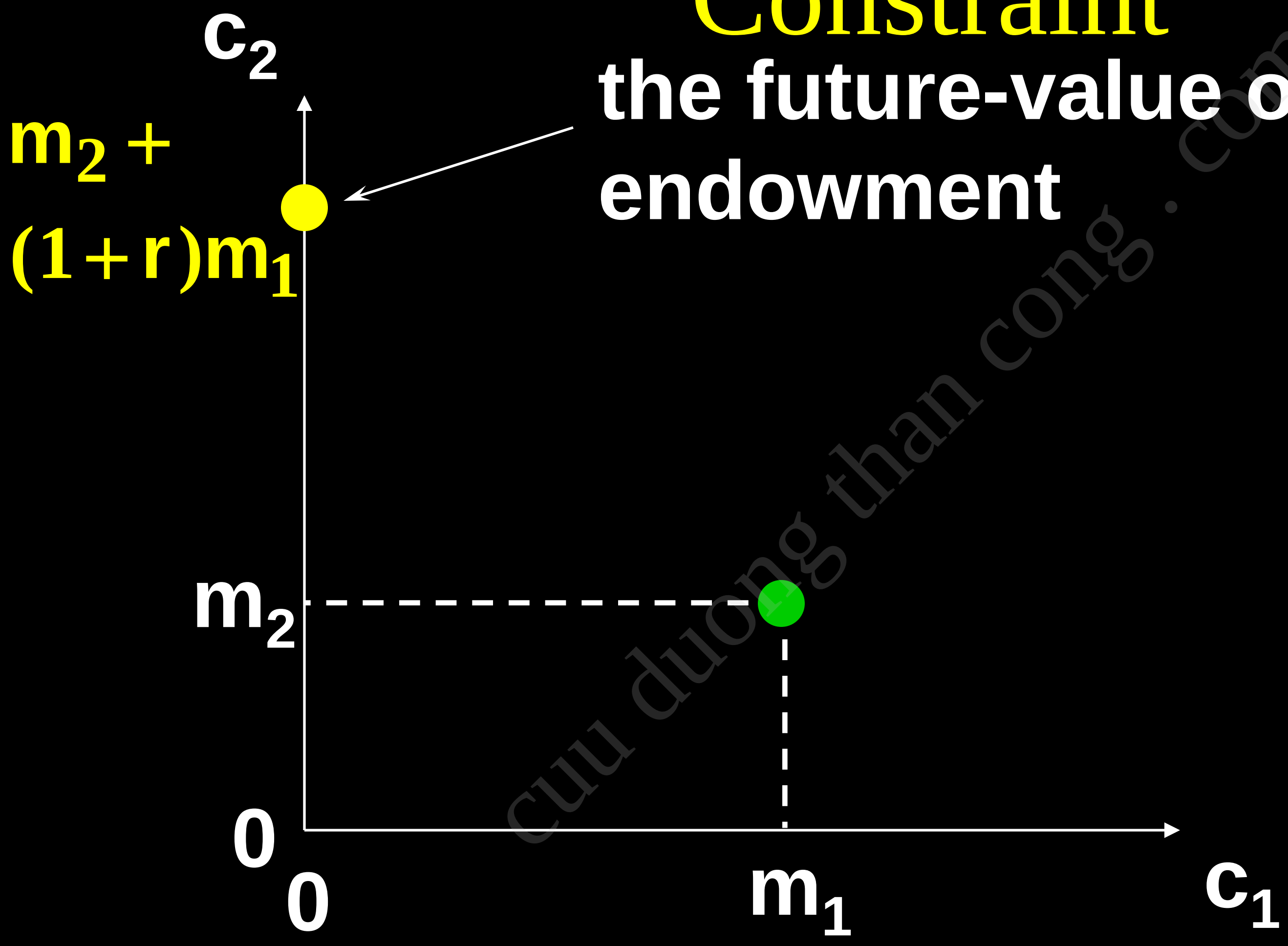
- The interest rate is r.
- What now will be period 2's consumption level?

The Intertemporal Budget Constraint

- Period 2 income is m_2.
- Savings plus interest from period 1 sum to $(1 + r)m_1$.
- So total income available in period 2 is $m_2 + (1 + r)m_1$.
- So period 2 consumption expenditure is

$$c_2 = m_2 + (1 + r)m_1$$

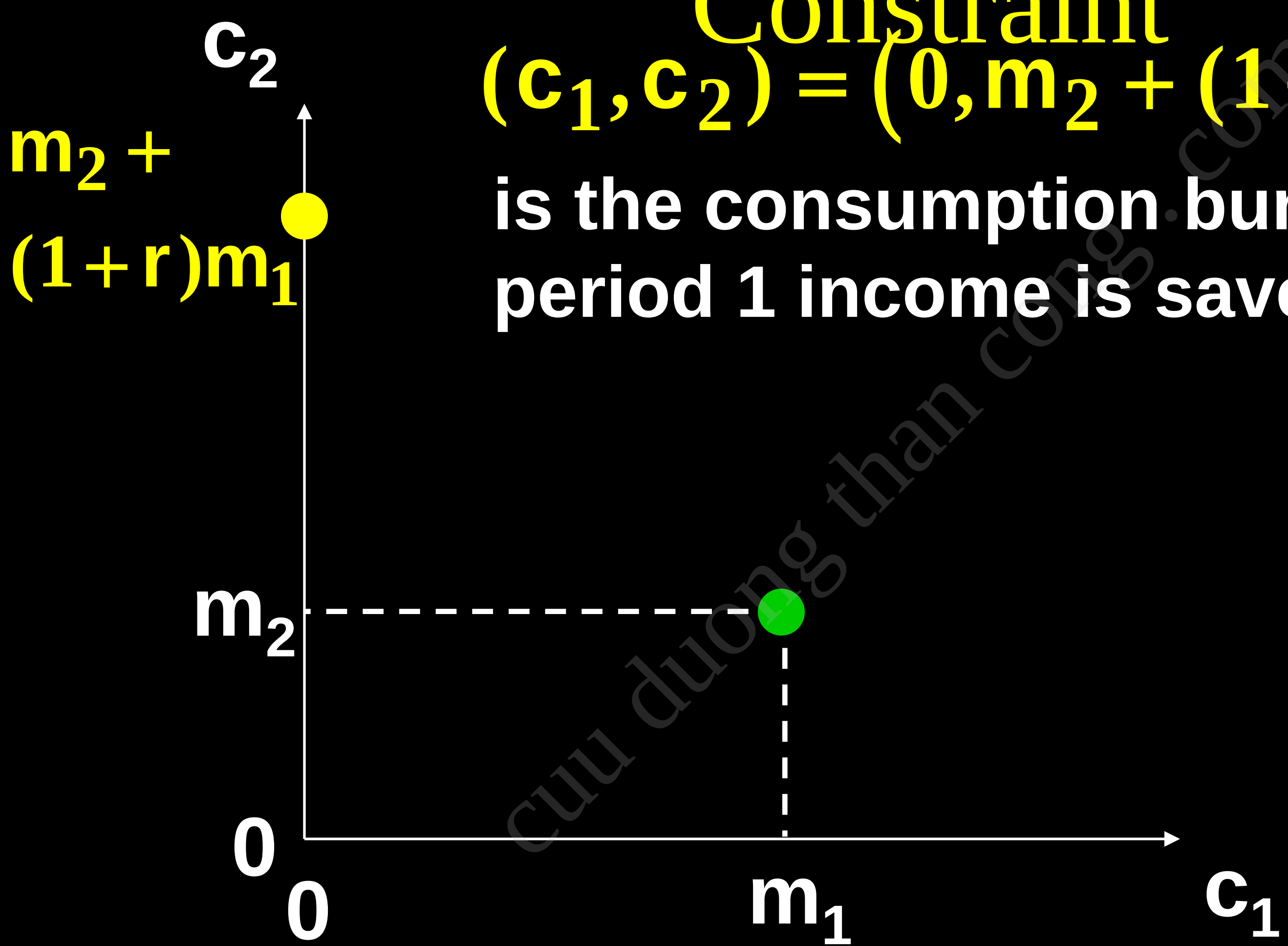
The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when all period 1 income is saved.



The Intertemporal Budget Constraint

- Now suppose that the consumer spends everything possible on consumption in period 1, so $c_2 = 0$.
- What is the most that the consumer can borrow in period 1 against her period 2 income of $\$m_2$?
- Let b_1 denote the amount borrowed in period 1.

The Intertemporal Budget Constraint

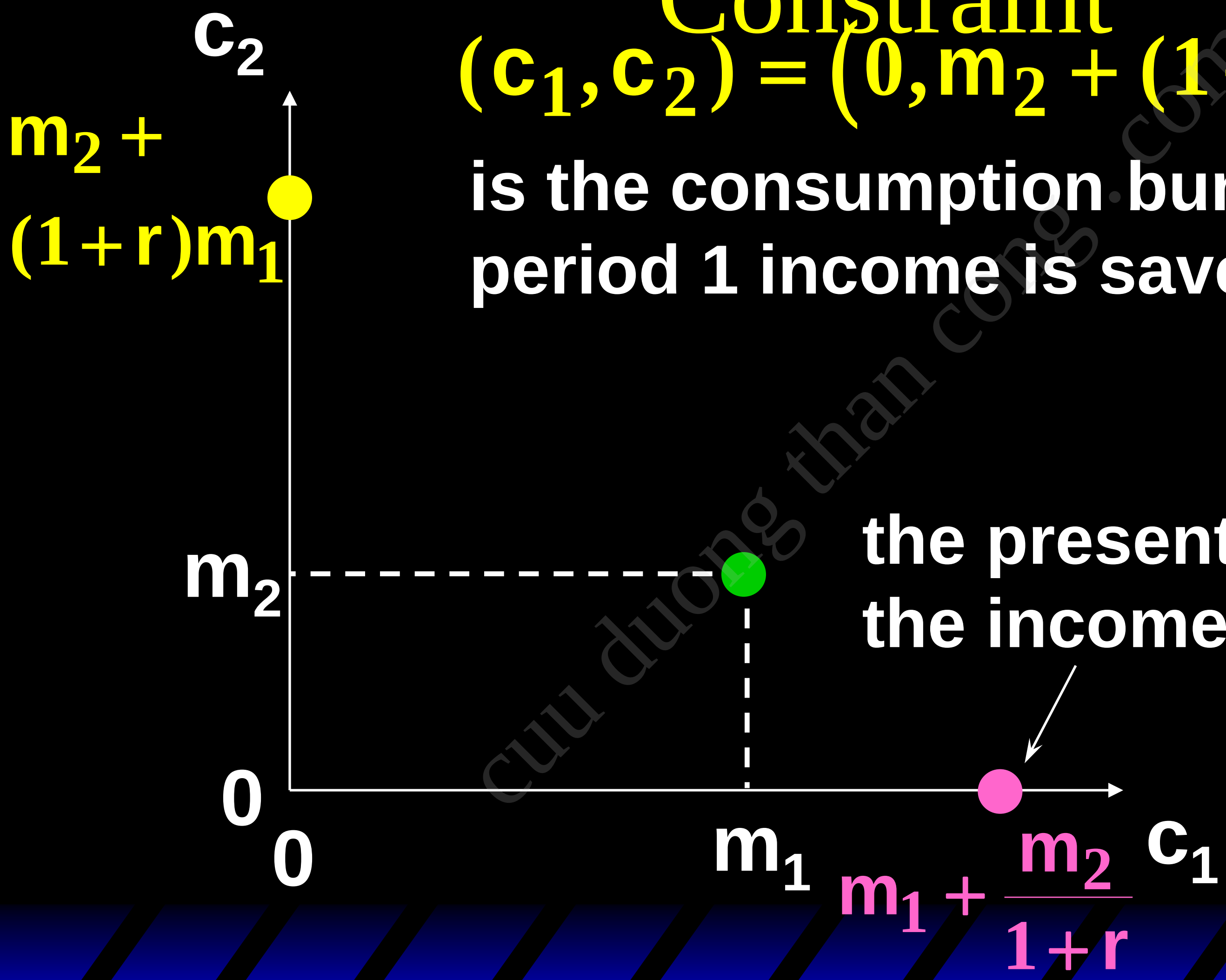
- **Only $\$m_2$ will be available in period 2 to pay back $\$b_1$ borrowed in period 1.**
- **So $b_1(1 + r) = m_2$.**
- **That is, $b_1 = m_2 / (1 + r)$.**
- **So the largest possible period 1 consumption level is**

$$c_1 = m_1 + \frac{m_2}{1+r}$$

The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when all period 1 income is saved.



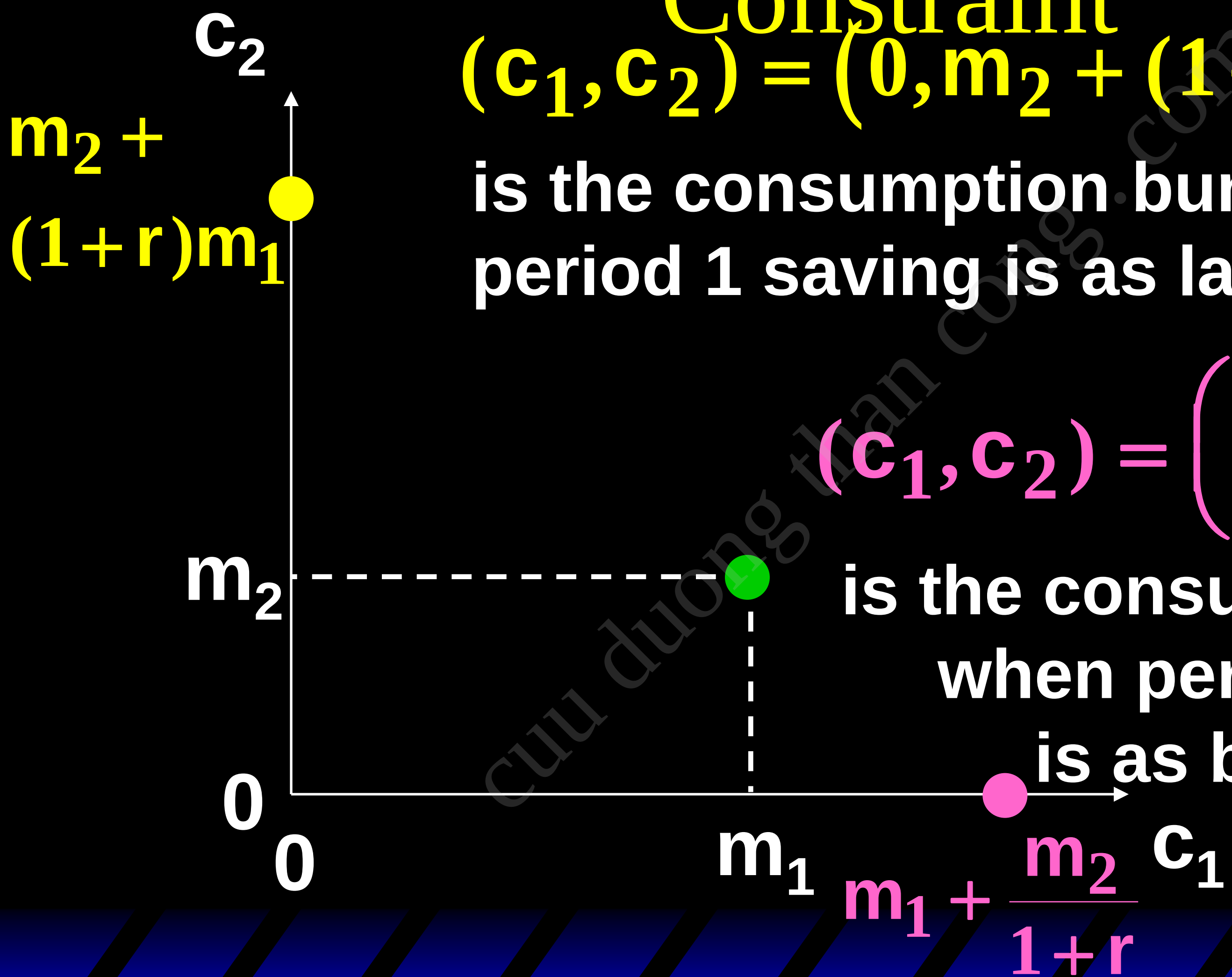
The Intertemporal Budget Constraint

$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

is the consumption bundle when period 1 saving is as large as possible.

$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0\right)$$

is the consumption bundle when period 1 borrowing is as big as possible.



The Intertemporal Budget Constraint

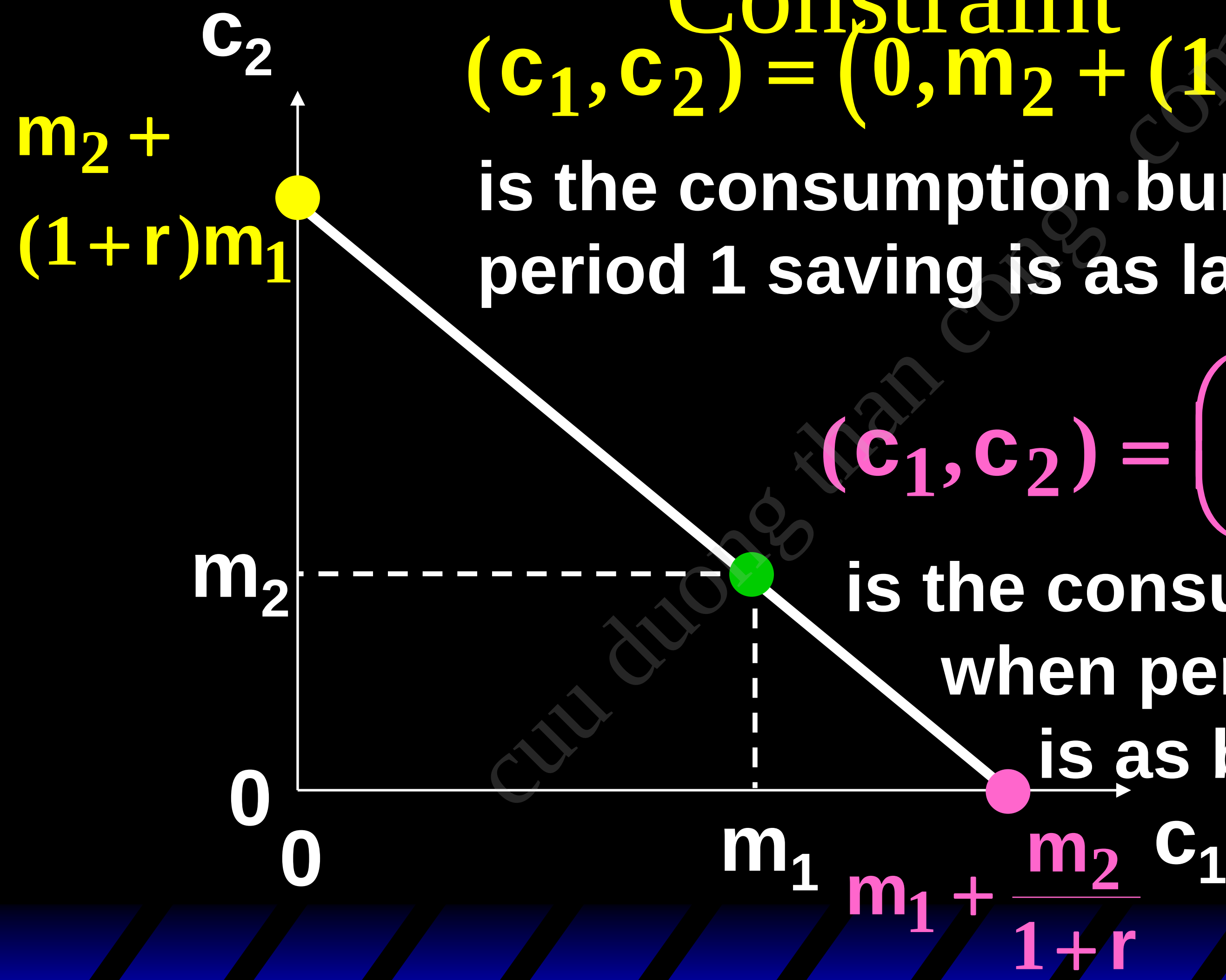
- Suppose that c_1 units are consumed in period 1. This costs $\$c_1$ and leaves $m_1 - c_1$ saved. Period 2 consumption will then be

$$c_2 = m_2 + (1 + r)(m_1 - c_1)$$

which is

$$c_2 = \underbrace{-(1 + r)}_{\text{slope}} c_1 + \underbrace{m_2 + (1 + r)m_1}_{\text{intercept}}.$$

The Intertemporal Budget Constraint



$$(c_1, c_2) = (0, m_2 + (1+r)m_1)$$

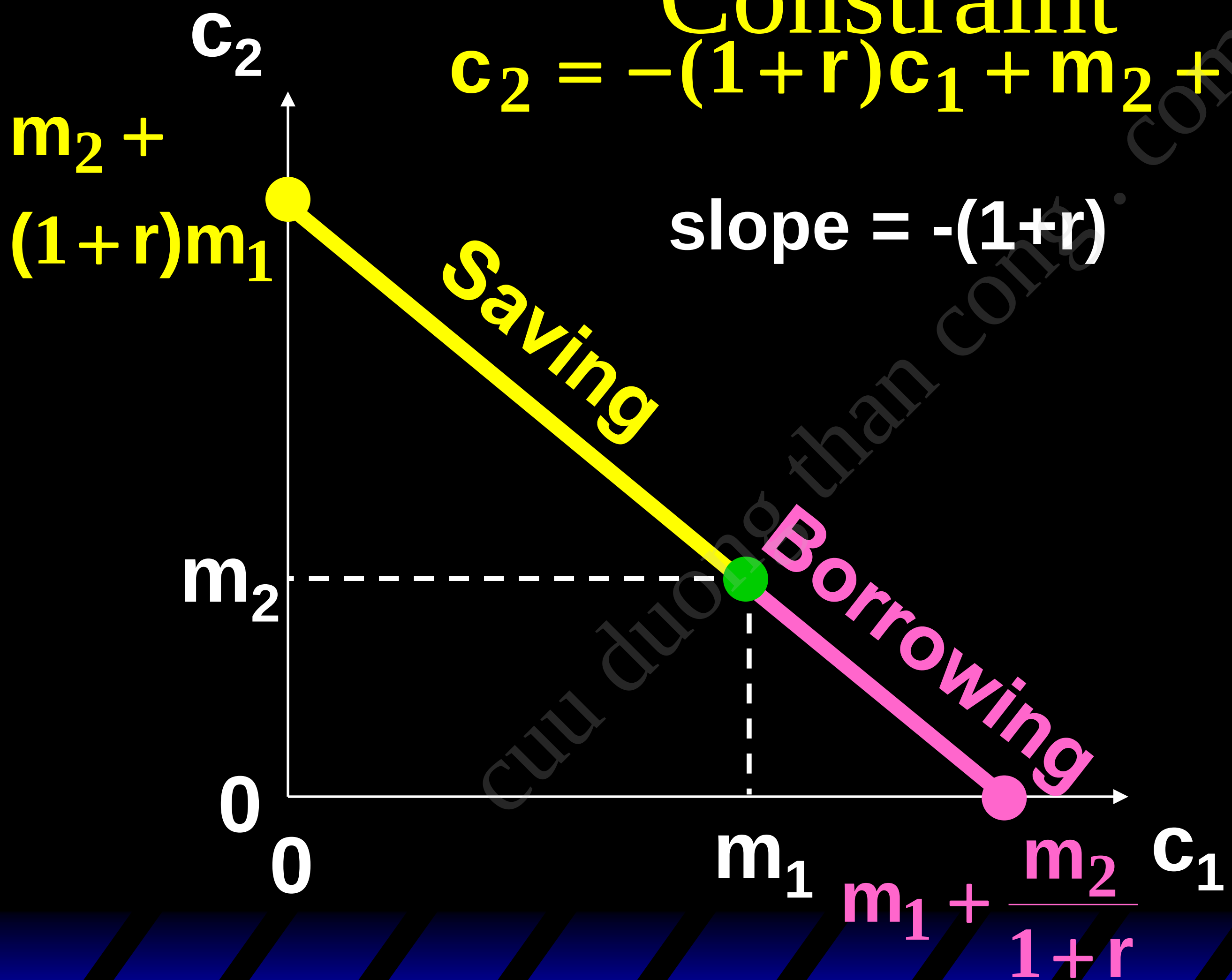
is the consumption bundle when period 1 saving is as large as possible.

$$(c_1, c_2) = \left(m_1 + \frac{m_2}{1+r}, 0\right)$$

is the consumption bundle when period 1 borrowing is as big as possible.

The Intertemporal Budget Constraint

$$c_2 = -(1+r)c_1 + m_2 + (1+r)m_1.$$



The Intertemporal Budget Constraint

$$(1+r)c_1 + c_2 = (1+r)m_1 + m_2$$

is the "future-valued" form of the budget constraint since all terms are in period 2 values. This is equivalent to

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

which is the "present-valued" form of the constraint since all terms are in period 1 values.

The Intertemporal Budget Constraint

- Now let's add prices p_1 and p_2 for consumption in periods 1 and 2.
- How does this affect the budget constraint?

Intertemporal Choice

- Given her endowment (m_1,m_2) and prices p_1, p_2 what intertemporal consumption bundle (c_1^*,c_2^*) will be chosen by the consumer?
- Maximum possible expenditure in period 2 is $m_2 + (1+r)m_1$ so maximum possible consumption in period 2 is $c_2 = \frac{m_2 + (1+r)m_1}{p_2}.$

Intertemporal Choice

- **Similarly, maximum possible expenditure in period 1 is**

$$m_1 + \frac{m_2}{1+r}$$

so maximum possible consumption in period 1 is

$$c_1 = \frac{m_1 + m_2/(1+r)}{p_1}.$$

Intertemporal Choice

- Finally, if c_1 units are consumed in period 1 then the consumer spends p_1c_1 in period 1, leaving $m_1 - p_1c_1$ saved for period 1. Available income in period 2 will then be

$$m_2 + (1+r)(m_1 - p_1c_1)$$

so

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1).$$

Intertemporal Choice

$$p_2c_2 = m_2 + (1+r)(m_1 - p_1c_1)$$

rearranged is

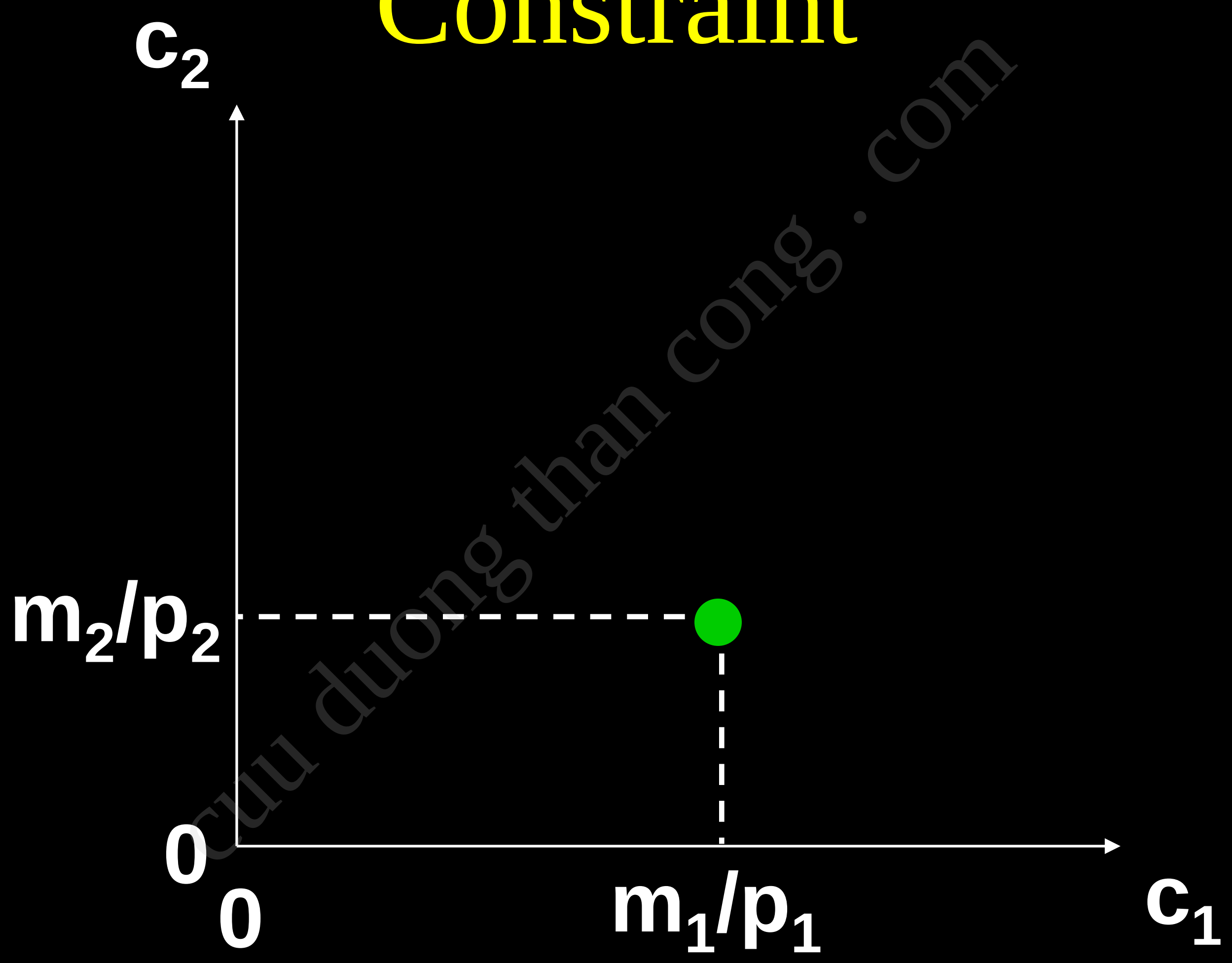
$$(1+r)p_1c_1 + p_2c_2 = (1+r)m_1 + m_2.$$

This is the "future-valued" form of the budget constraint since all terms are expressed in period 2 values. Equivalent to it is the "present-valued" form

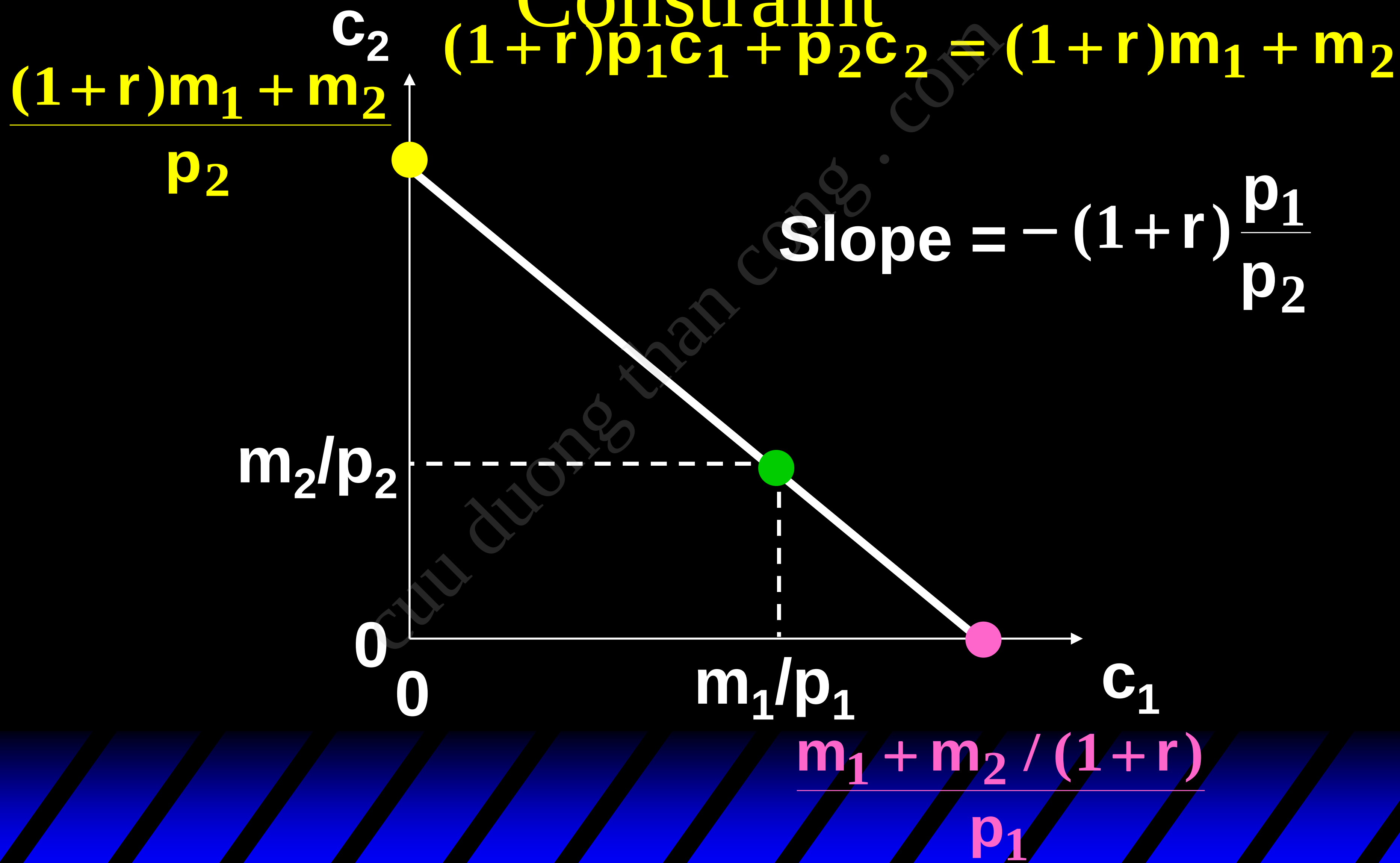
$$p_1c_1 + \frac{p_2}{1+r}c_2 = m_1 + \frac{m_2}{1+r}$$

where all terms are expressed in period 1 values.

The Intertemporal Budget Constraint

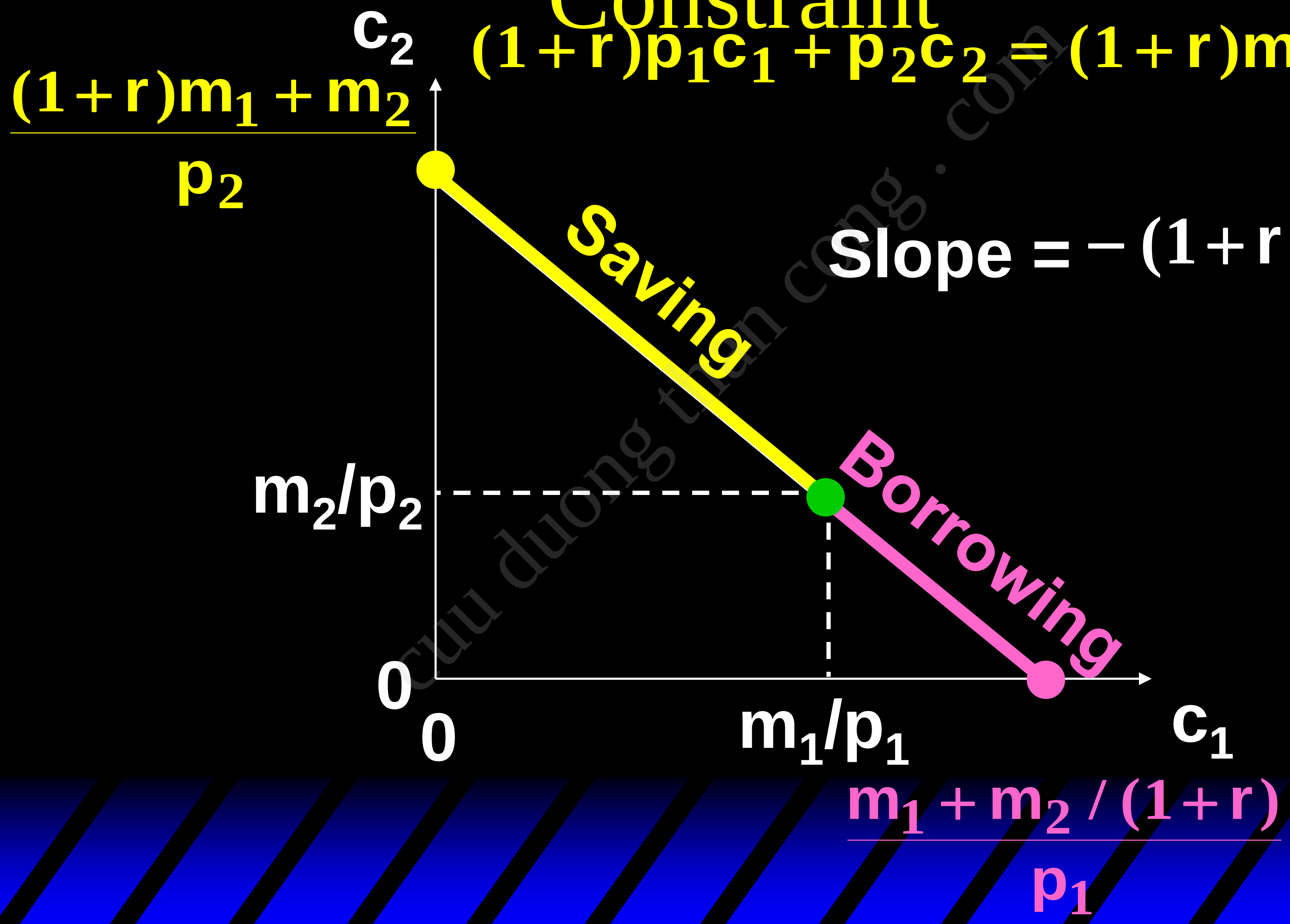


The Intertemporal Budget Constraint



The Intertemporal Budget Constraint

$$(1+r)p_1c_1 + p_2c_2 = (1+r)m_1 + m_2$$



Price Inflation

- Define the inflation rate by π where

$$p_1(1+\pi) = p_2.$$

- For example,
$\pi = 0.2$ means 20% inflation, and
$\pi = 1.0$ means 100% inflation.

Price Inflation

- We lose nothing by setting $p_1=1$ so that $p_2 = 1+\pi$.
- Then we can rewrite the budget constraint

$$p_1 c_1 + \frac{p_2}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

as

$$c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

Price Inflation

$$c_1 + \frac{1+\pi}{1+r} c_2 = m_1 + \frac{m_2}{1+r}$$

rearranges to

$$c_2 = -\frac{1+r}{1+\pi} c_1 + (1+\pi)\left(\frac{m_1}{1+r} + m_2\right)$$

so the slope of the intertemporal budget constraint is

$$-\frac{1+r}{1+\pi}.$$

Price Inflation

- When there was no price inflation ($p_1=p_2=1$) the slope of the budget constraint was -(1+r).
- Now, with price inflation, the slope of the budget constraint is $-(1+r)/(1+\pi)$. This can be written as

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

ρ is known as the **real interest rate**.

Real Interest Rate

$$-(1+\rho) = -\frac{1+r}{1+\pi}$$

gives

$$\rho = \frac{r-\pi}{1+\pi}.$$

For low inflation rates ($\pi \approx 0$), $\rho \approx r - \pi$. For higher inflation rates this approximation becomes poor.

Real Interest Rate

r	**0.30**	**0.30**	**0.30**	**0.30**	**0.30**
π	**0.0**	**0.05**	**0.10**	**0.20**	**1.00**
r - π	**0.30**	**0.25**	**0.20**	**0.10**	**-0.70**
ρ	**0.30**	**0.24**	**0.18**	**0.08**	**-0.35**

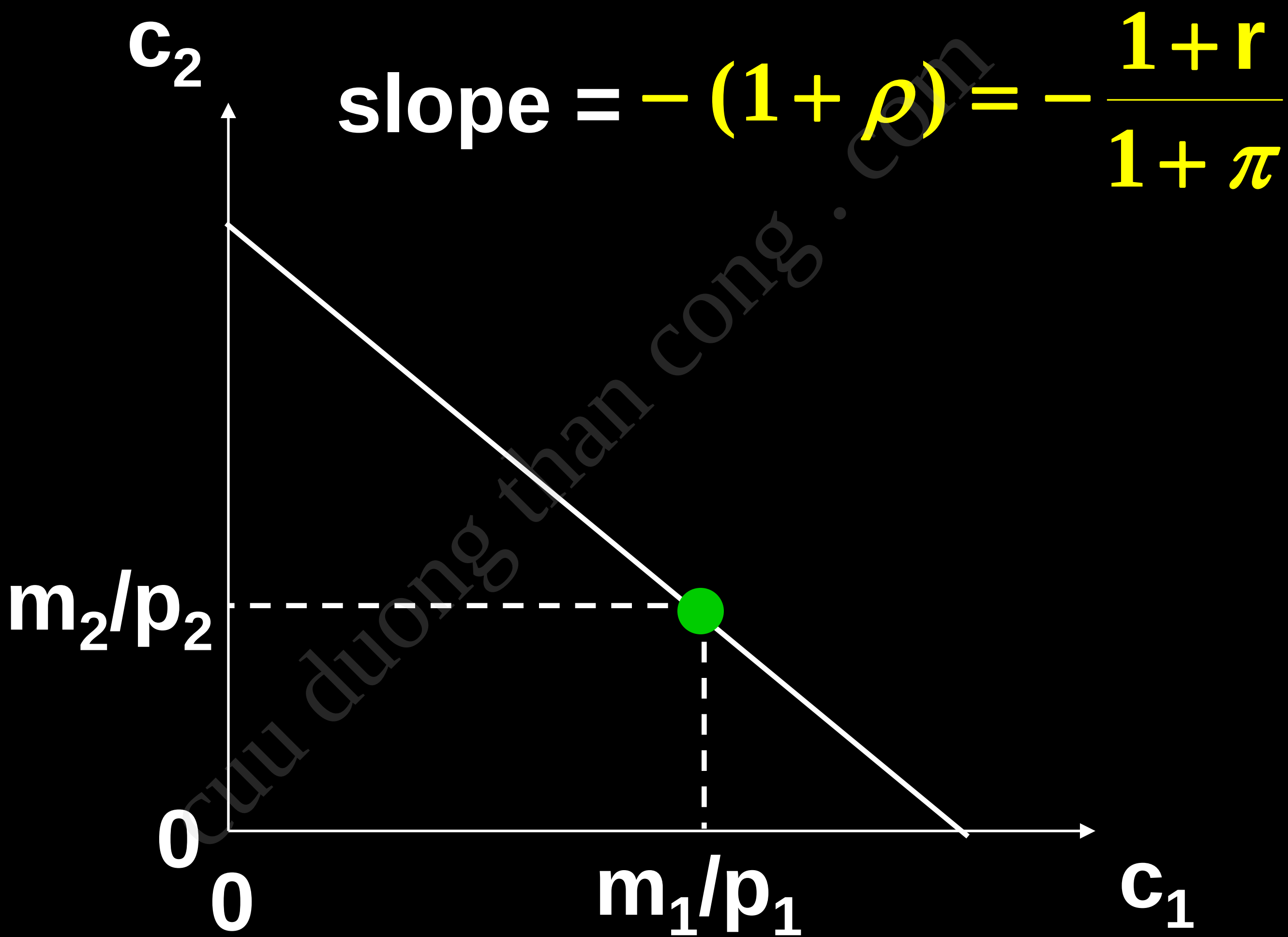
Comparative Statics

- The slope of the budget constraint is

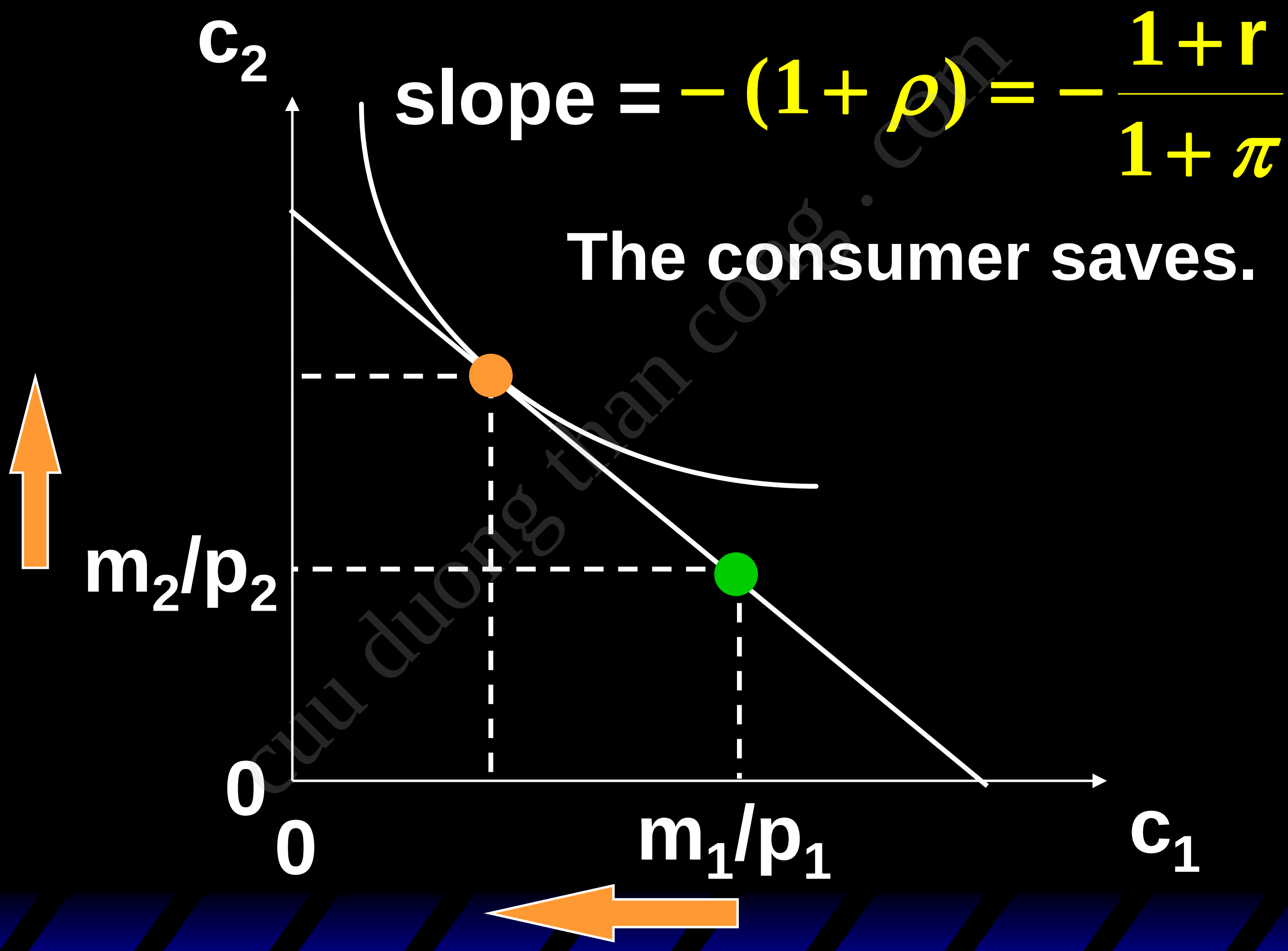
$$-(1+\rho) = -\frac{1+r}{1+\pi}.$$

- The constraint becomes flatter if the interest rate r falls or the inflation rate π rises (both decrease the real rate of interest).

Comparative Statics

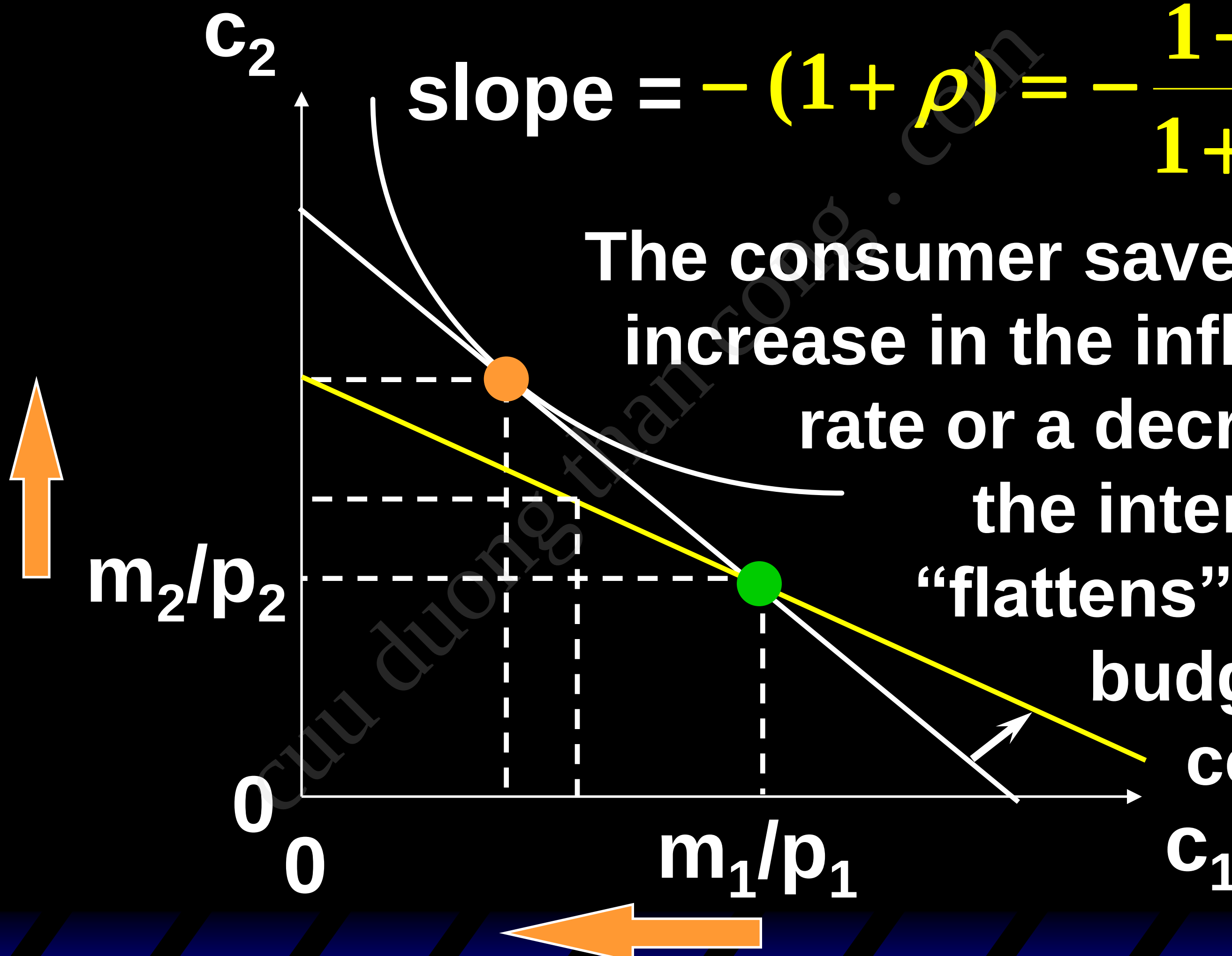


Comparative Statics



Comparative Statics

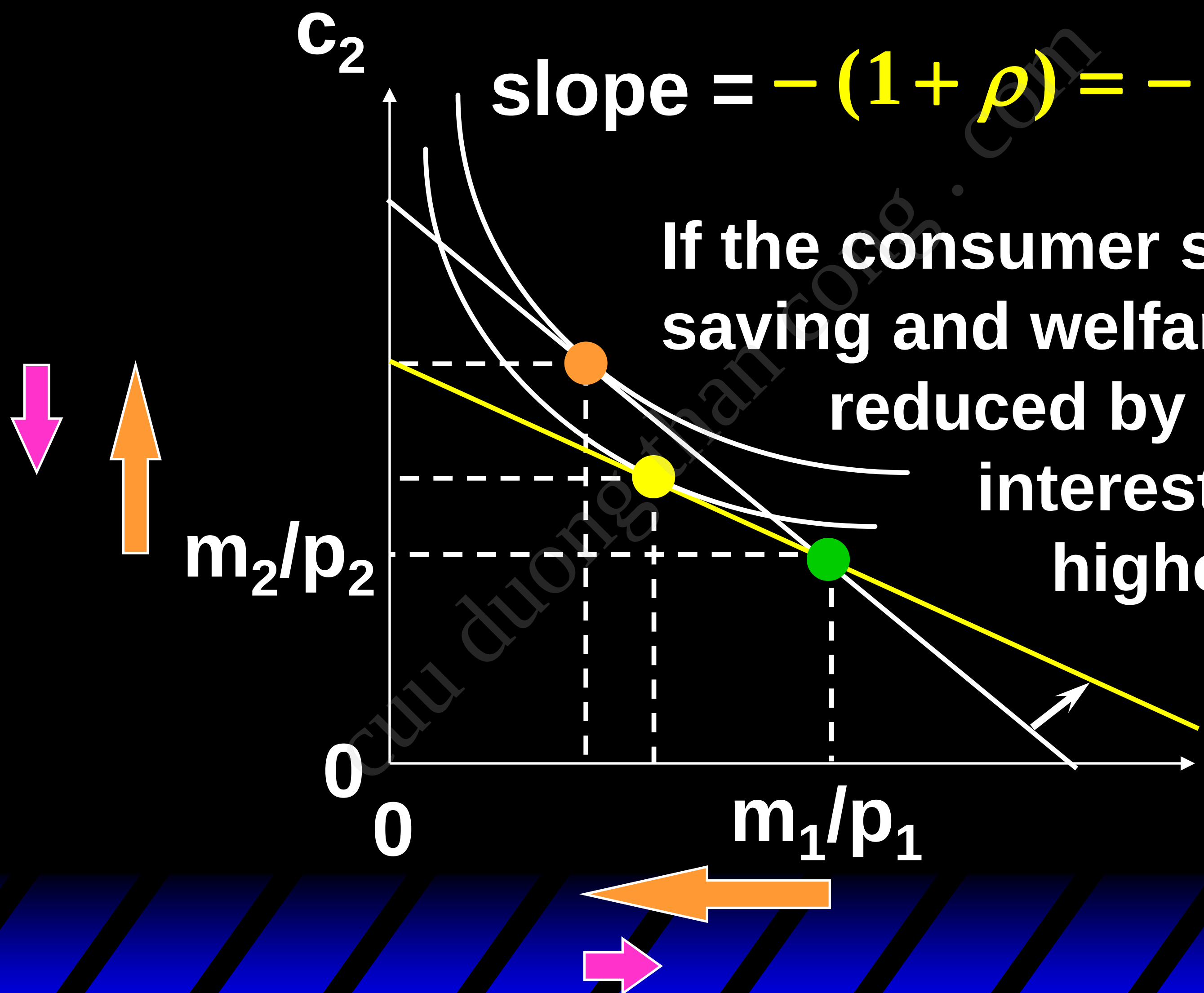
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



The consumer saves. An increase in the inflation rate or a decrease in the interest rate "flattens" the budget constraint.

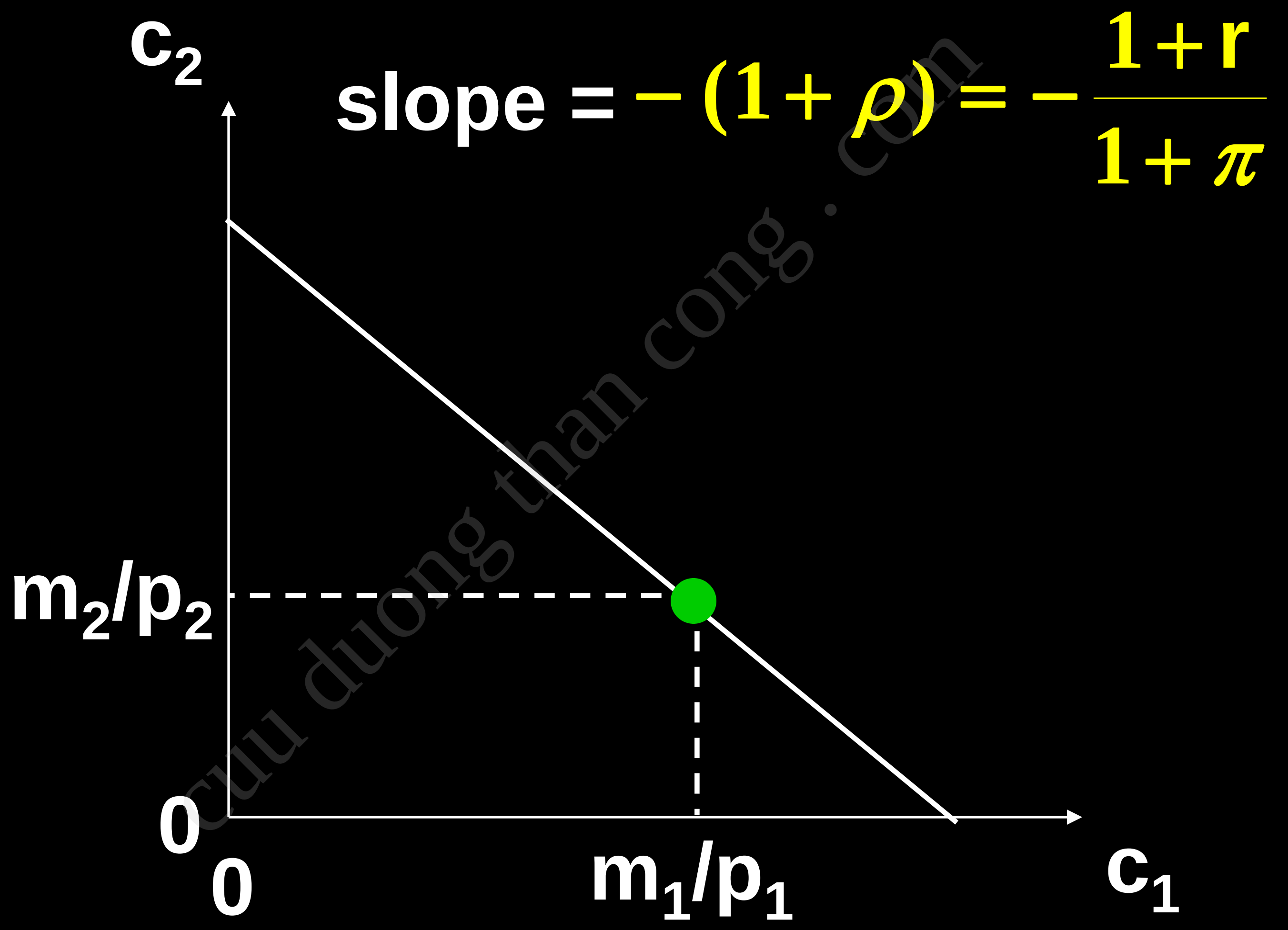
Comparative Statics

$$\text{slope} = -(1+\rho) = -\frac{1+r}{1+\pi}$$



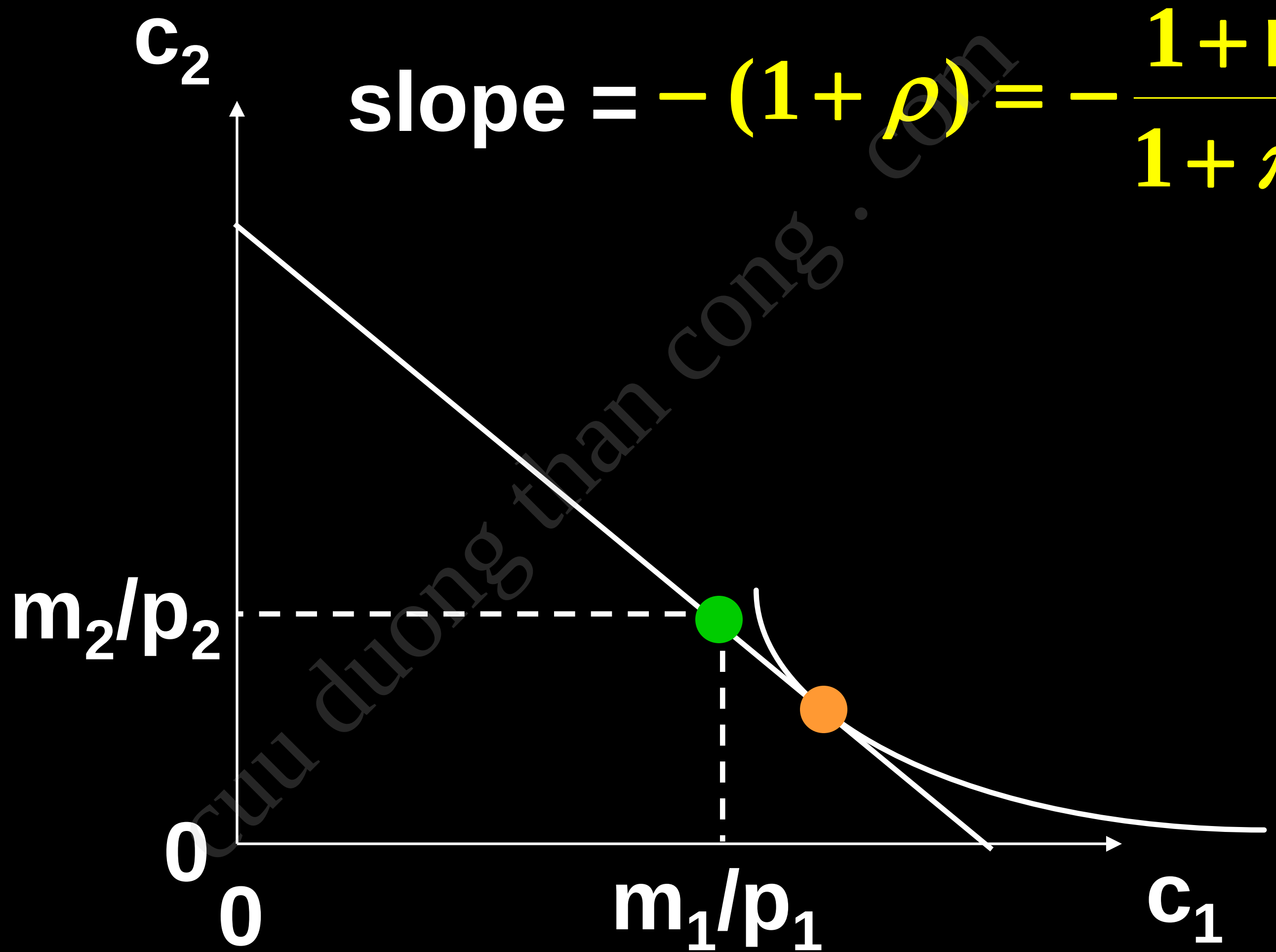
If the consumer saves then saving and welfare are reduced by a lower interest rate or a higher inflation rate.

Comparative Statics



Comparative Statics

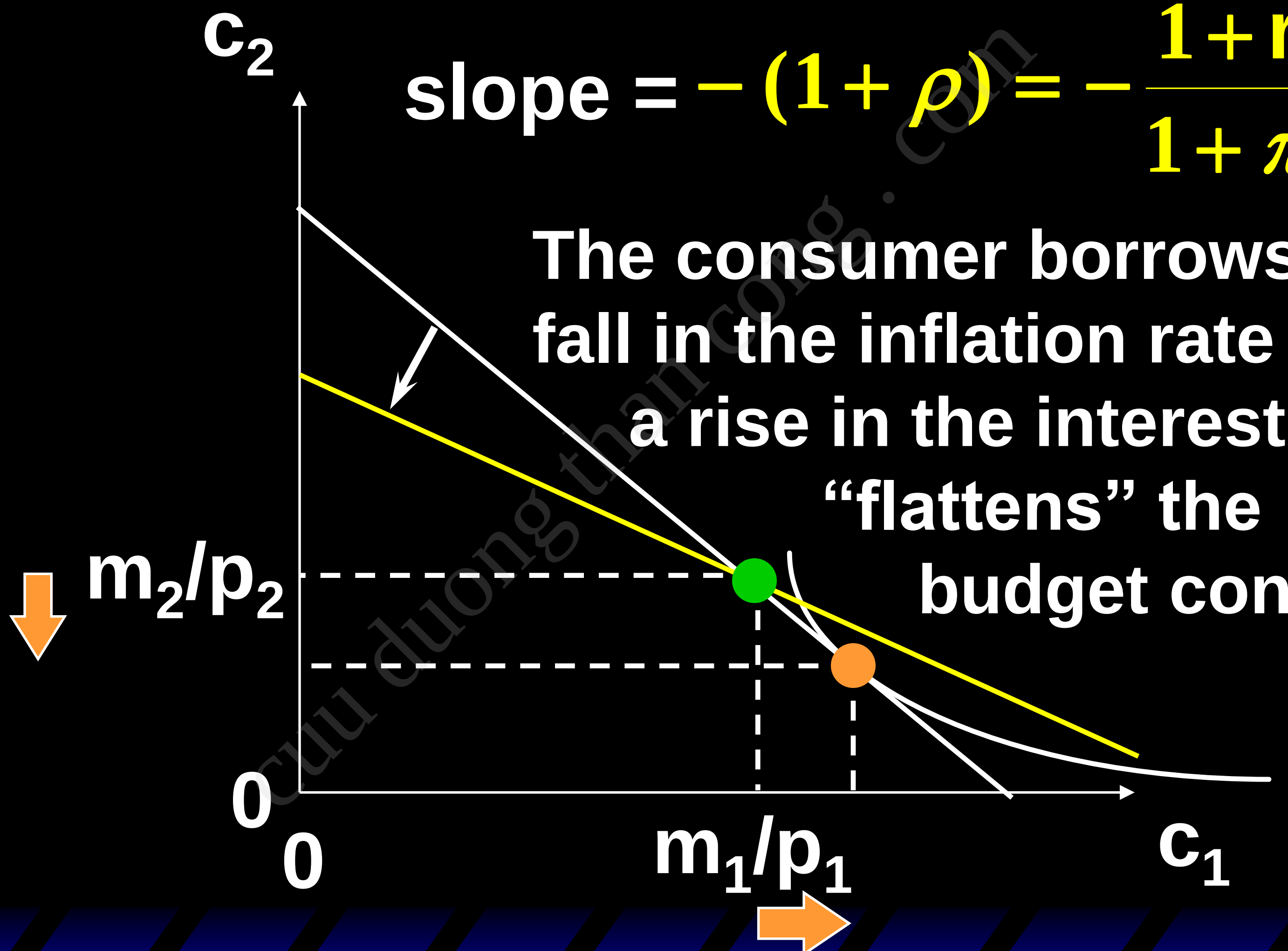
$$\text{slope} = -(1+\rho) = -\frac{1+r}{1+\pi}$$



Comparative Statics

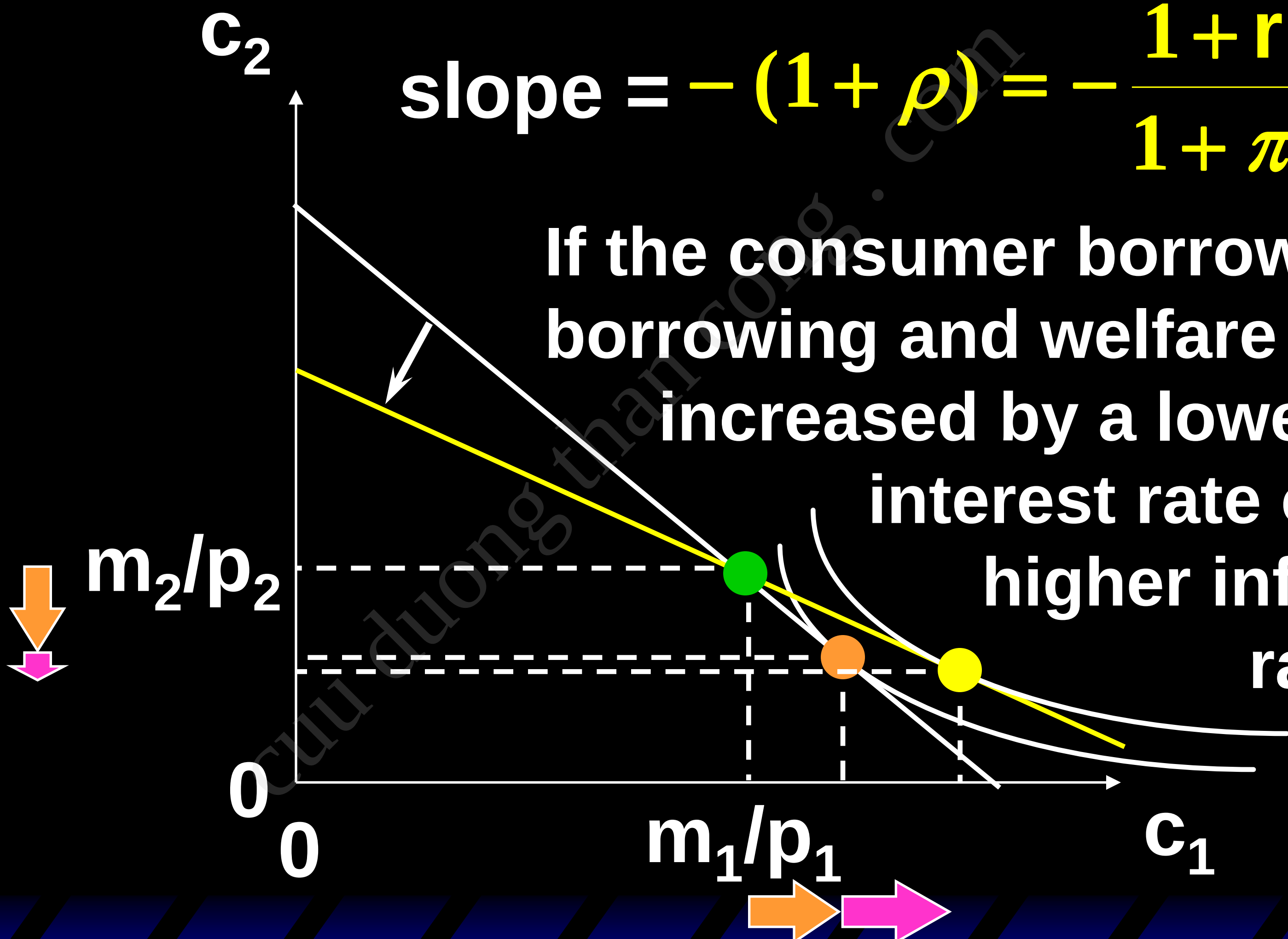
slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$

The consumer borrows. A fall in the inflation rate or a rise in the interest rate "flattens" the budget constraint.



Comparative Statics

slope $= -(1+\rho) = -\dfrac{1+r}{1+\pi}$



If the consumer borrows then borrowing and welfare are increased by a lower interest rate or a higher inflation rate.

Valuing Securities

- A **financial security** is a financial instrument that promises to deliver an income stream.
- E.g.; a security that pays
  - $\$m_1$ at the end of year 1,
  - $\$m_2$ at the end of year 2, and
  - $\$m_3$ at the end of year 3.
- What is the most that should be paid now for this security?

Valuing Securities

- The security is equivalent to the sum of three securities;
  - the first pays only $\$m_1$ at the end of year 1,
  - the second pays only $\$m_2$ at the end of year 2, and
  - the third pays only $\$m_3$ at the end of year 3.

Valuing Securities

- The PV of $\$m_1$ paid 1 year from now is
  $$m_1 / (1+r)$$
- The PV of $\$m_2$ paid 2 years from now is
  $$m_2 / (1+r)^2$$
- The PV of $\$m_3$ paid 3 years from now is
  $$m_3 / (1+r)^3$$
- The PV of the security is therefore

$$m_1 / (1+r) + m_2 / (1+r)^2 + m_3 / (1+r)^3.$$

Valuing Bonds

- A **bond** is a special type of security that pays a fixed amount $x for T years (its **maturity date**) and then pays its **face value** $F.
- What is the most that should now be paid for such a bond?

Valuing Bonds

End of Year	1	2	3	…	T-1	T
Income Paid	$x	$x	$x	$x	$x	$F
Present -Value	$\frac{\$x}{1+r}$	$\frac{\$x}{(1+r)^2}$	$\frac{\$x}{(1+r)^3}$	…	$\frac{\$x}{(1+r)^{T-1}}$	$\frac{\$F}{(1+r)^T}$

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + K + \frac{x}{(1+r)^{T-1}} + \frac{F}{(1+r)^T}.$$

Valuing Bonds

- **Suppose you win a State lottery. The prize is $1,000,000 but it is paid over 10 years in equal installments of $100,000 each. What is the prize actually worth?**

Valuing Bonds

$$PV = \frac{\$100{,}000}{1+0\cdot1} + \frac{\$100{,}000}{(1+0\cdot1)^2} + \ldots + \frac{\$100{,}000}{(1+0\cdot1)^{10}}$$

$$= \$614{,}457$$

is the actual (present) value of the prize.

Valuing Consols

- A **consol** is a bond which never terminates, paying $x per period forever.
- What is a consol's present-value?

Valuing Consols

End of Year	1	2	3	…	t	…
Income Paid	\$x	\$x	\$x	\$x	\$x	\$x
Present -Value	$\frac{\$x}{1+r}$	$\frac{\$x}{(1+r)^2}$	$\frac{\$x}{(1+r)^3}$	…	$\frac{\$x}{(1+r)^t}$	…

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + K + \frac{x}{(1+r)^t} + K.$$

Valuing Consols

$$PV = \frac{x}{1+r} + \frac{x}{(1+r)^2} + \frac{x}{(1+r)^3} + \cdots$$

$$= \frac{1}{1+r}\left[x + \frac{x}{1+r} + \frac{x}{(1+r)^2} + \cdots\right]$$

$$= \frac{1}{1+r}[x + PV].$$

Solving for PV gives

$$PV = \frac{x}{r}.$$

Valuing Consols

E.g. if r = 0.1 now and forever then the most that should be paid now for a console that provides $1000 per year is

$$PV = \frac{x}{r} = \frac{\$1000}{0 \cdot 1} = \$10,000.$$