Lesson 4: Consumer Behavior

1. Uncertainty
2. Market Demand
3. Market equilibrium

1. Uncertainty
Uncertainty is Pervasive

- What is uncertain in economic systems?
  - tomorrow's prices
  - future wealth
  - future availability of commodities
  - present and future actions of other people.

Uncertainty is Pervasive

- What are rational responses to uncertainty?
  - buying insurance (health, life, auto)
  - a portfolio of contingent consumption goods.

States of Nature

- **Possible states of Nature:**
  - **"car accident" (a)**
  - **"no car accident" (na).**
- **Accident occurs with probability π_a, does not with probability π_{na} ;**

$$\pi_a + \pi_{na} = 1.$$

- **Accident causes a loss of \$L.**

Contingencies

- A contract implemented only when a particular state of Nature occurs is **state-contingent**.
- E.g. the insurer pays only if there is an accident.

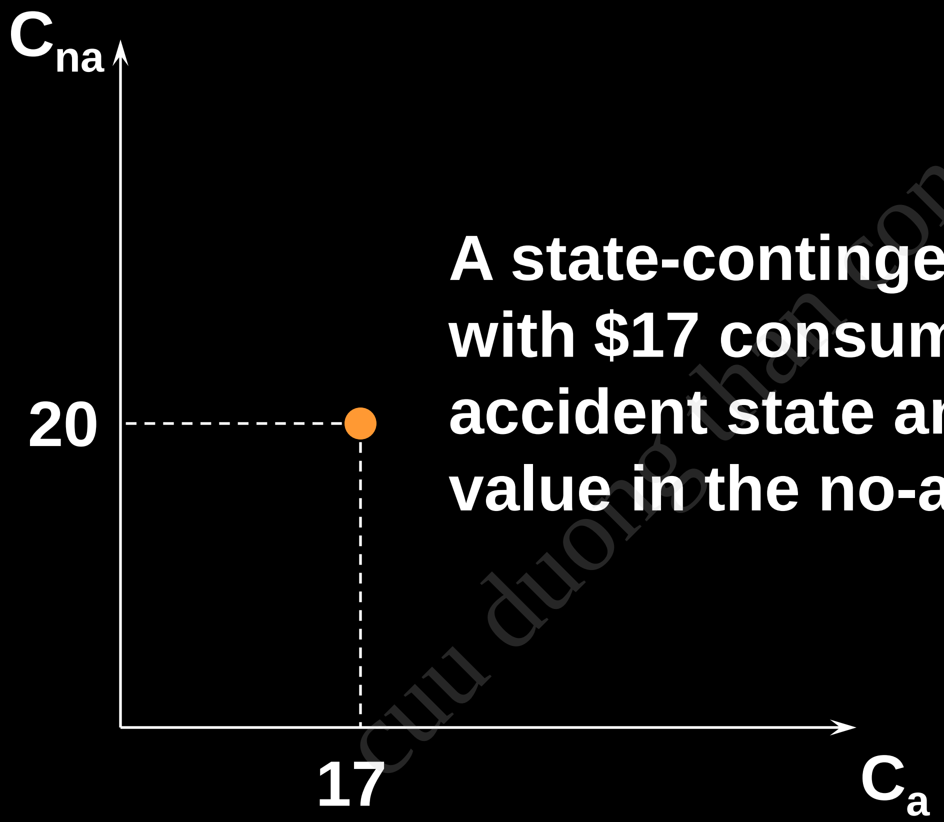
Contingencies

- A state-contingent consumption plan is implemented only when a particular state of Nature occurs.
- E.g. take a vacation only if there is no accident.

State-Contingent Budget Constraints

- Each \$1 of accident insurance costs γ.
- Consumer has \$m of wealth.
- C_{na} is consumption value in the no-accident state.
- C_a is consumption value in the accident state.

State-Contingent Budget Constraints

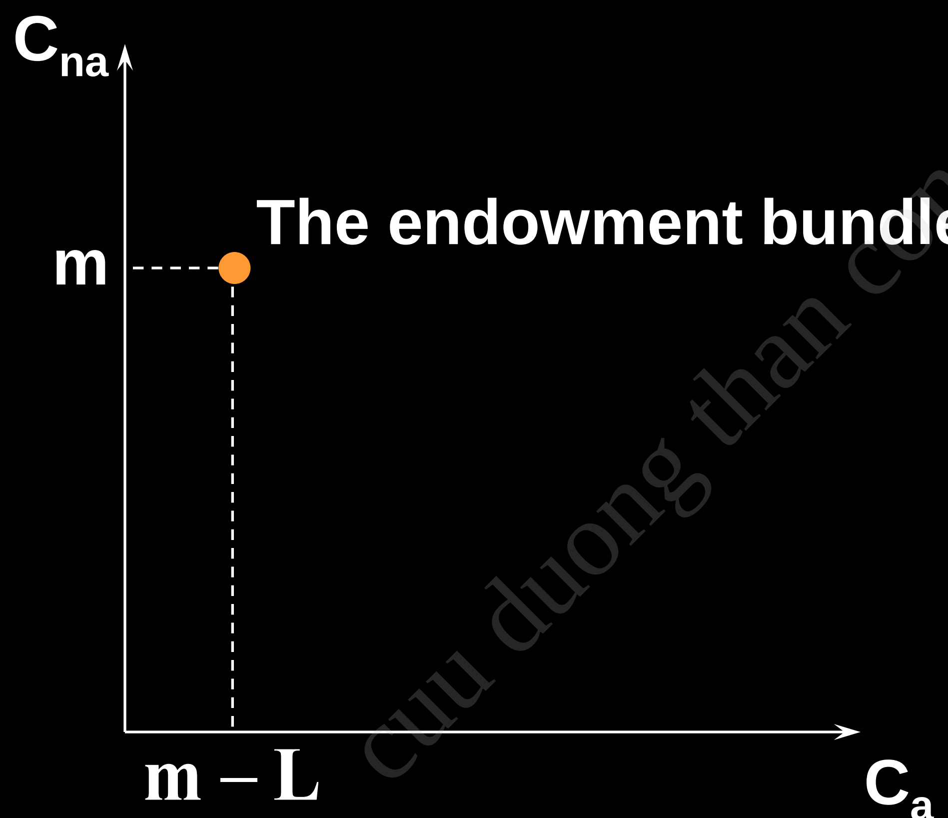


A state-contingent consumption with $17 consumption value in the accident state and $20 consumption value in the no-accident state.

State-Contingent Budget Constraints

- **Without insurance,**
- **$C_a = m - L$**
- **$C_{na} = m.$**

State-Contingent Budget Constraints



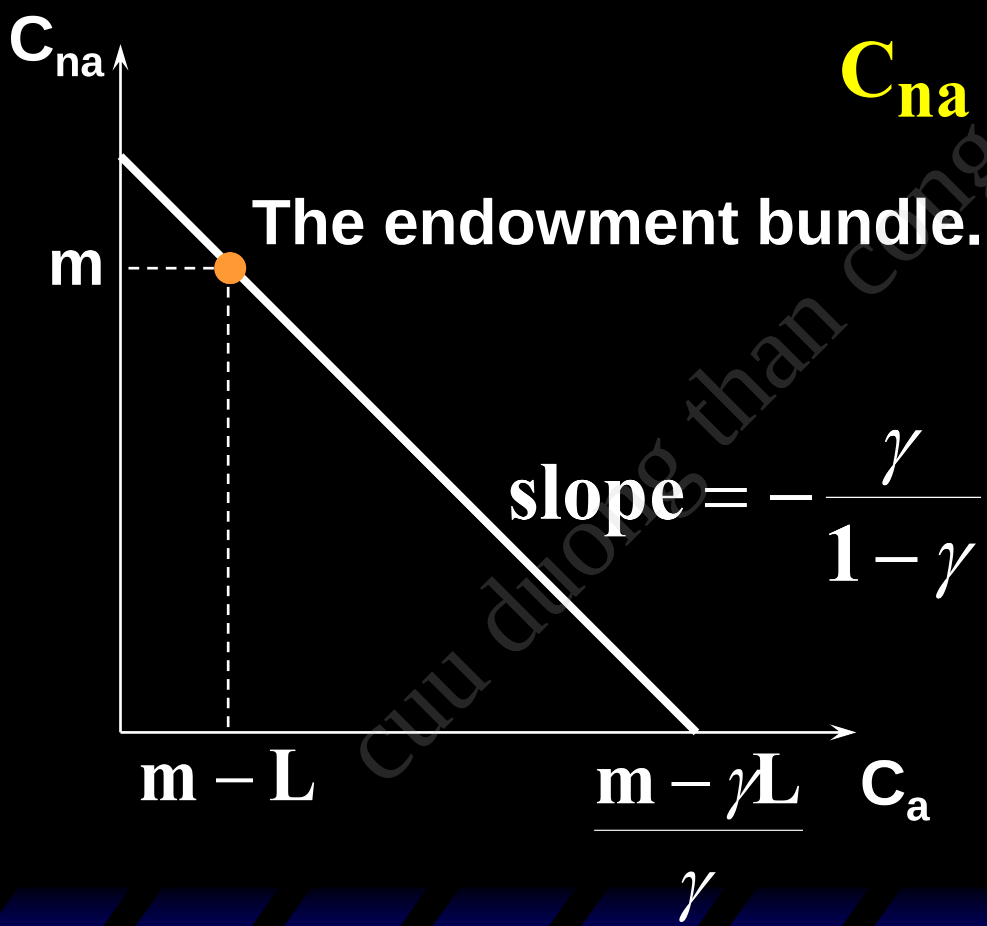
State-Contingent Budget Constraints

- Buy \$K of accident insurance.
- $C_{na} = m - \gamma K$.
- $C_a = m - L - \gamma K + K = m - L + (1- \gamma)K$.
- So $K = (C_a - m + L)/(1- \gamma)$
- And $C_{na} = m - \gamma (C_a - m + L)/(1- \gamma)$
- I.e.

$$C_{na} = \frac{m - \gamma L}{1 - \gamma} - \frac{\gamma}{1 - \gamma} C_a$$

State-Contingent Budget Constraints

$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$



Where is the most preferred state-contingent consumption plan?

Preferences Under Uncertainty

- Think of a lottery.
- Win \$90 with probability 1/2 and win \$0 with probability 1/2.
- U(\$90) = 12,   U(\$0) = 2.
- Expected utility is

$$EU = \frac{1}{2} \times U(\$90) + \frac{1}{2} \times U(\$0)$$

$$= \frac{1}{2} \times 12 + \frac{1}{2} \times 2 = 7.$$

Preferences Under Uncertainty

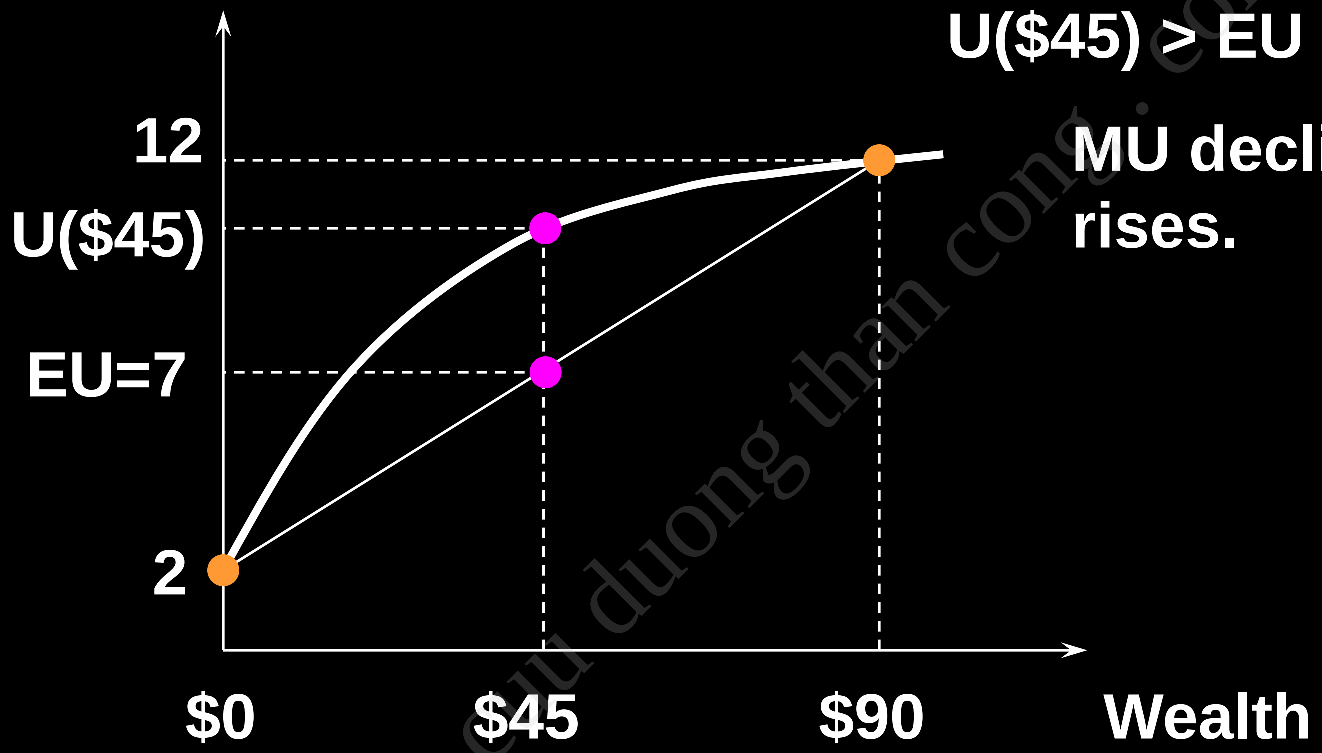
- Think of a lottery.
- Win \$90 with probability 1/2 and win \$0 with probability 1/2.
- Expected money value of the lottery is

$$EM = \frac{1}{2} \times \$90 + \frac{1}{2} \times \$0 = \$45.$$

Preferences Under Uncertainty

- **EU = 7 and EM = $45.**
- **U($45) > 7 ⇒ $45 for sure is preferred to the lottery ⇒ risk-aversion.**
- **U($45) < 7 ⇒ the lottery is preferred to $45 for sure ⇒ risk-loving.**
- **U($45) = 7 ⇒ the lottery is preferred equally to $45 for sure ⇒ risk-neutrality.**

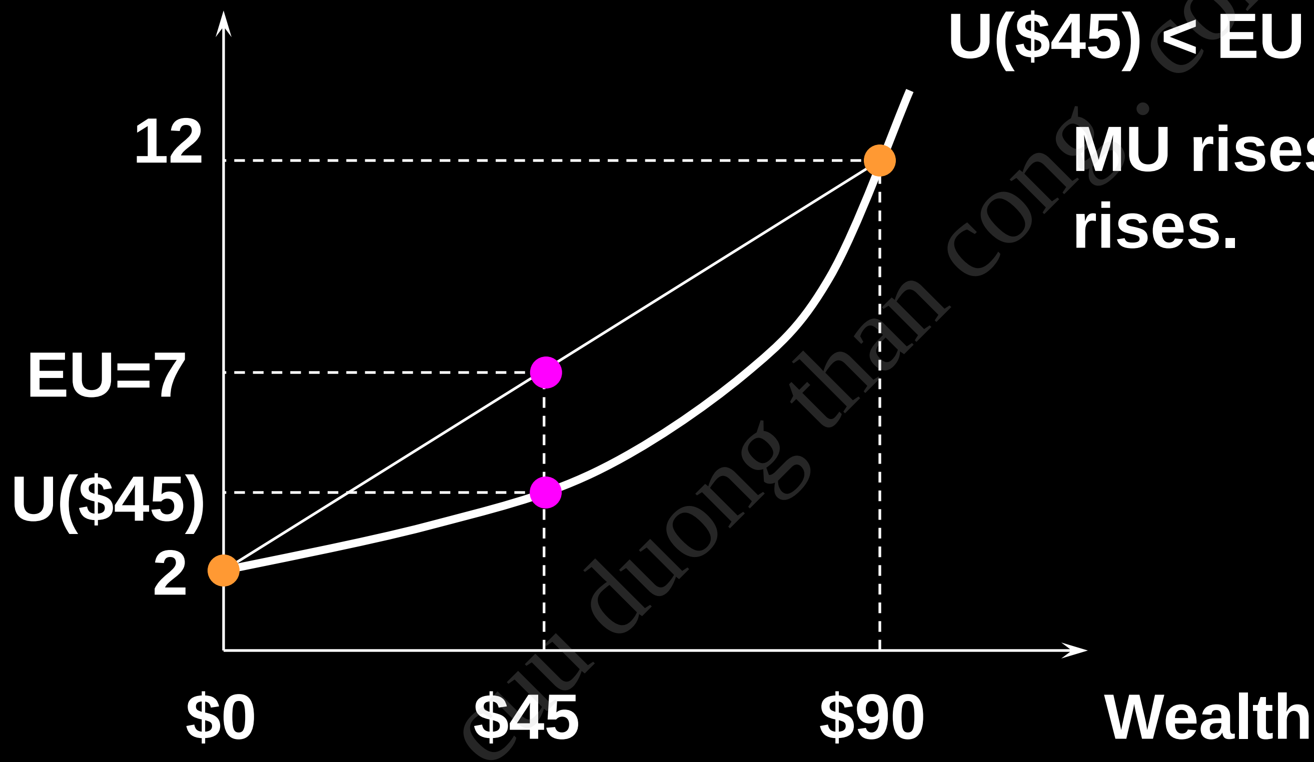
Preferences Under Uncertainty



U($45) > EU ⇒ risk-aversion.

MU declines as wealth rises.

Preferences Under Uncertainty

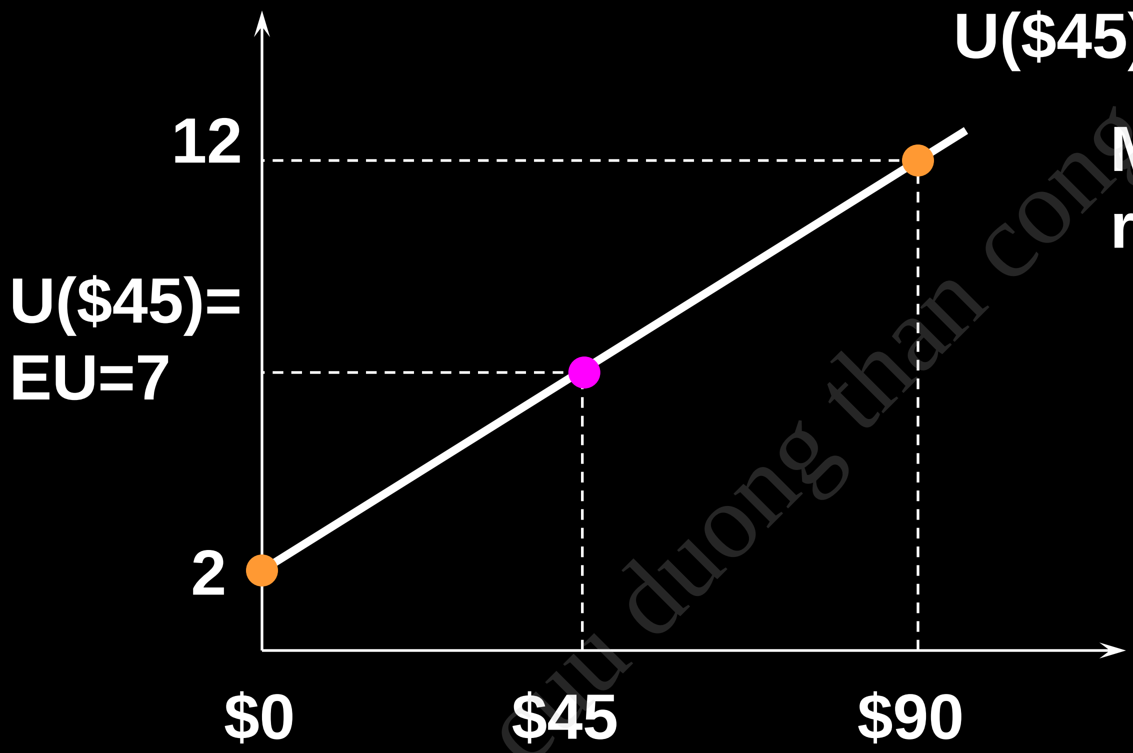


U($45) < EU ⇒ risk-loving.

MU rises as wealth rises.

Preferences Under Uncertainty

U($45) = EU ⇒ risk-neutrality.

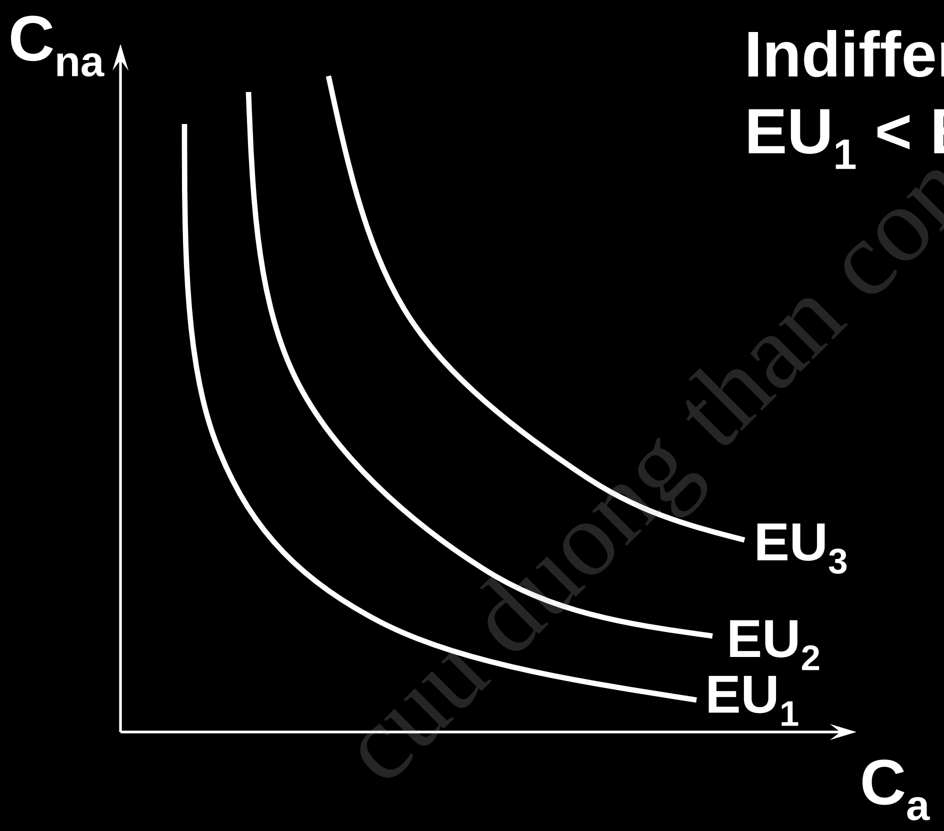


MU constant as wealth rises.

Preferences Under Uncertainty

- **State-contingent consumption plans that give equal expected utility are equally preferred.**

Preferences Under Uncertainty



Indifference curves

$EU_1 < EU_2 < EU_3$

Preferences Under Uncertainty

- **What is the MRS of an indifference curve?**
- **Get consumption c_1 with prob. π_1 and c_2 with prob. π_2 ($\pi_1 + \pi_2 = 1$).**
- **EU = $\pi_1 U(c_1) + \pi_2 U(c_2)$.**
- **For constant EU, dEU = 0.**

Preferences Under Uncertainty

$$EU = \pi_1 U(c_1) + \pi_2 U(c_2)$$

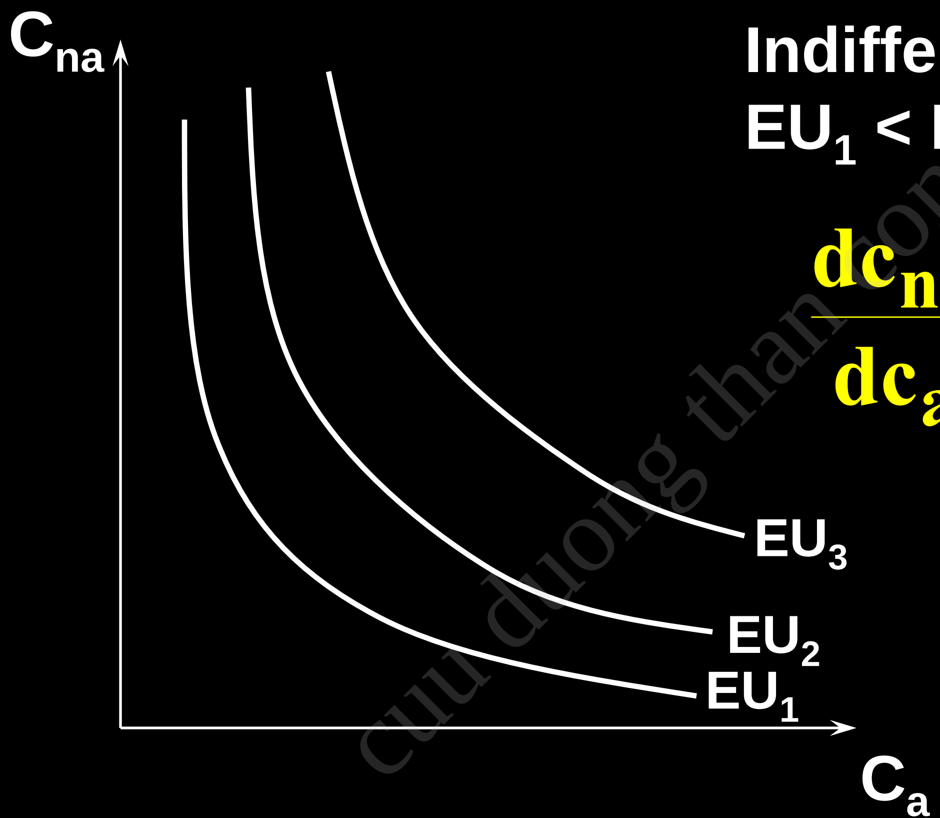
$$dEU = \pi_1 MU(c_1) dc_1 + \pi_2 MU(c_2) dc_2$$

$$dEU = 0 \Rightarrow \pi_1 MU(c_1) dc_1 + \pi_2 MU(c_2) dc_2 = 0$$

$$\Rightarrow \pi_1 MU(c_1) dc_1 = -\pi_2 MU(c_2) dc_2$$

$$\Rightarrow \frac{dc_2}{dc_1} = -\frac{\pi_1 MU(c_1)}{\pi_2 MU(c_2)}.$$

Preferences Under Uncertainty



Indifference curves

$EU_1 < EU_2 < EU_3$

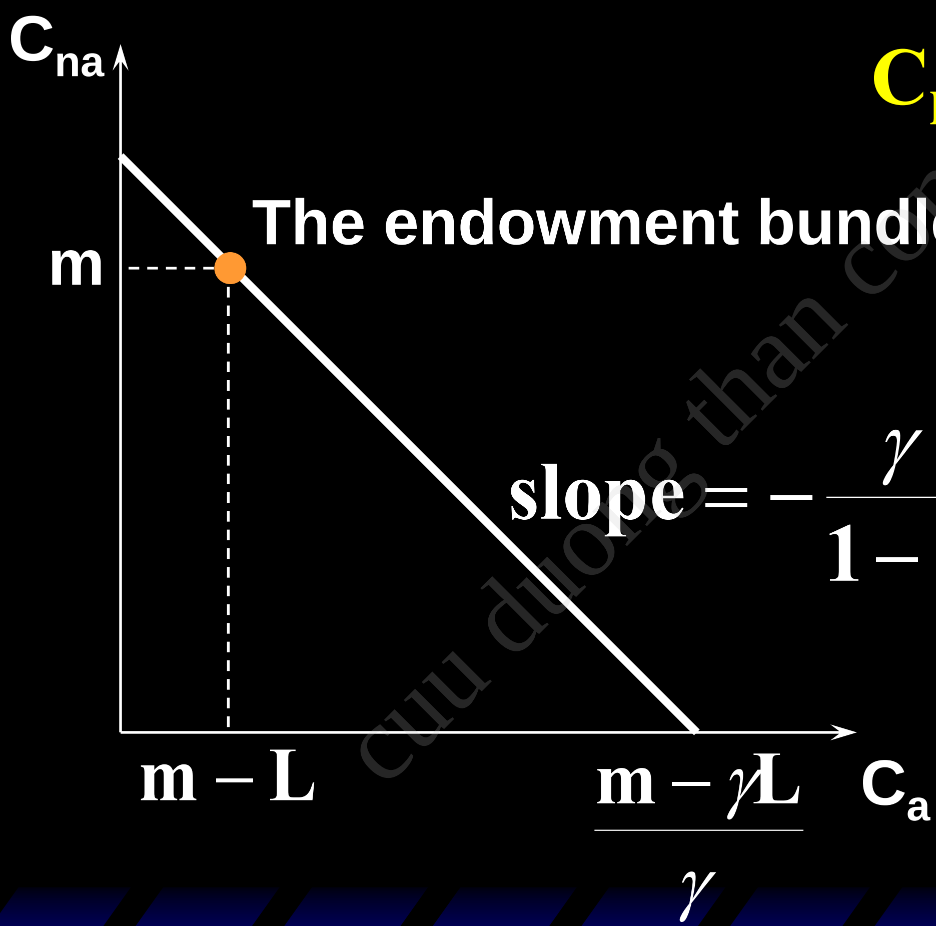
$$\frac{dc_{na}}{dc_a} = -\frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

Choice Under Uncertainty

- **Q: How is a rational choice made under uncertainty?**
- **A: Choose the most preferred affordable state-contingent consumption plan.**

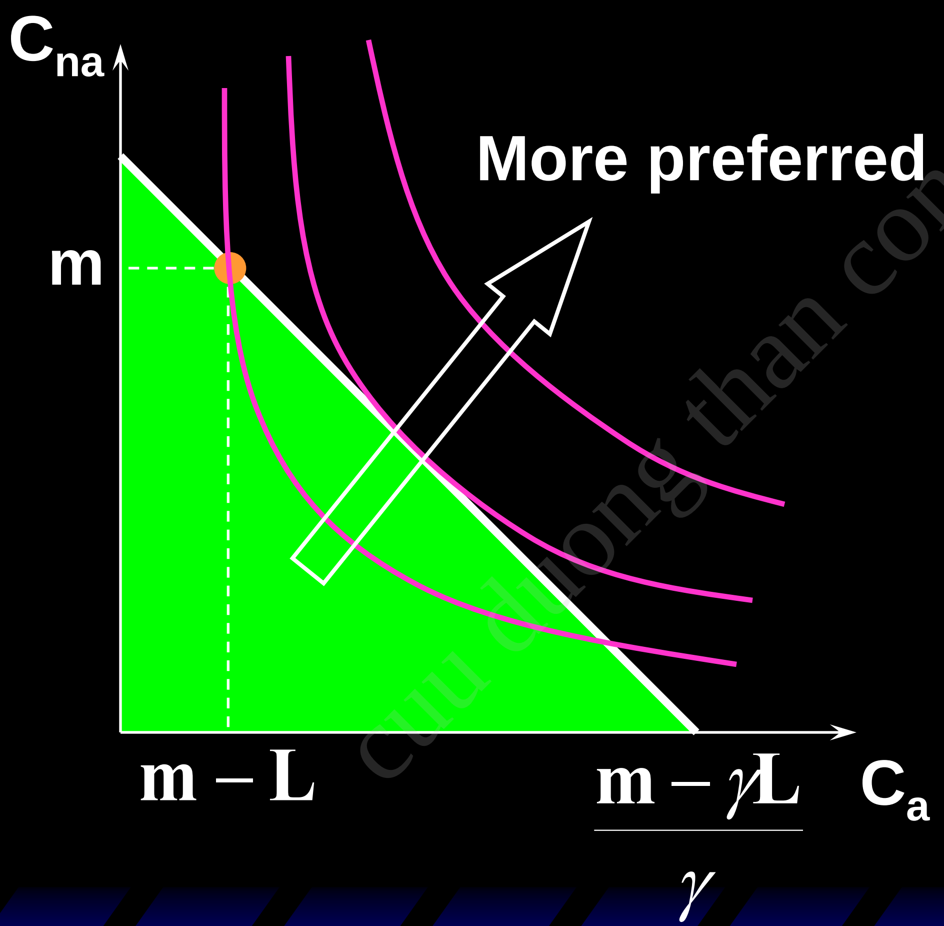
State-Contingent Budget Constraints

$$C_{na} = \frac{m - \gamma L}{1-\gamma} - \frac{\gamma}{1-\gamma} C_a$$



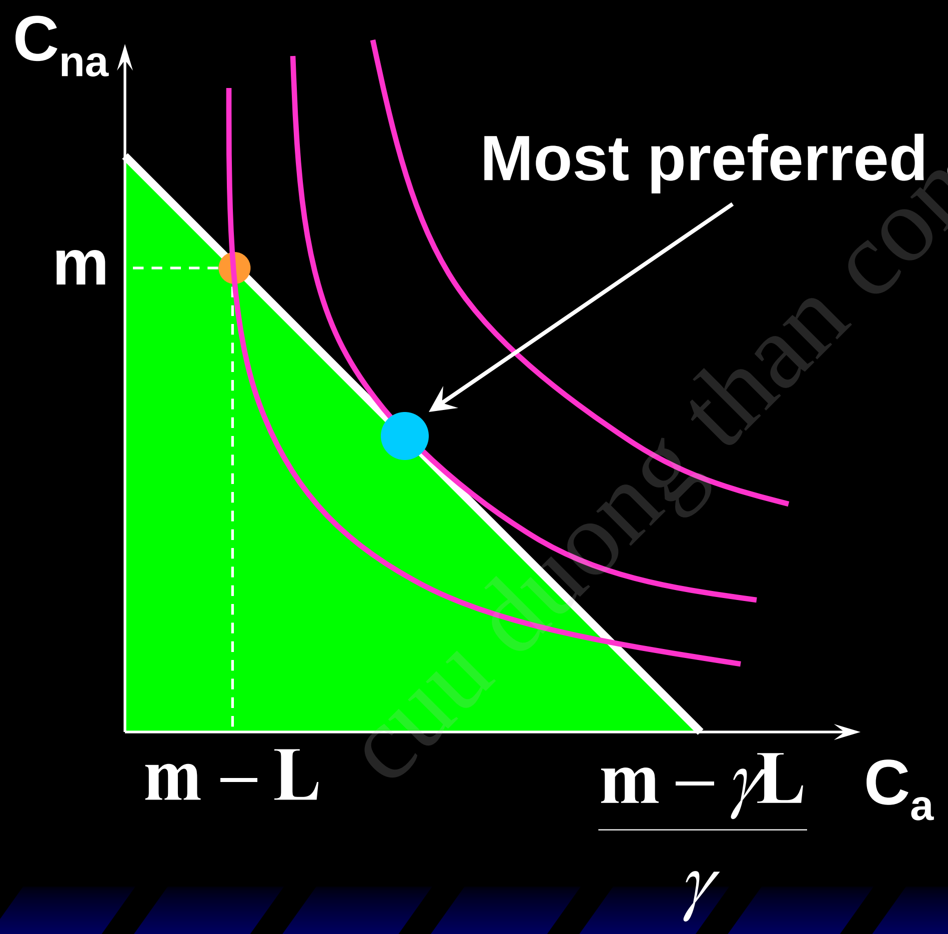
Where is the most preferred state-contingent consumption plan?

State-Contingent Budget Constraints

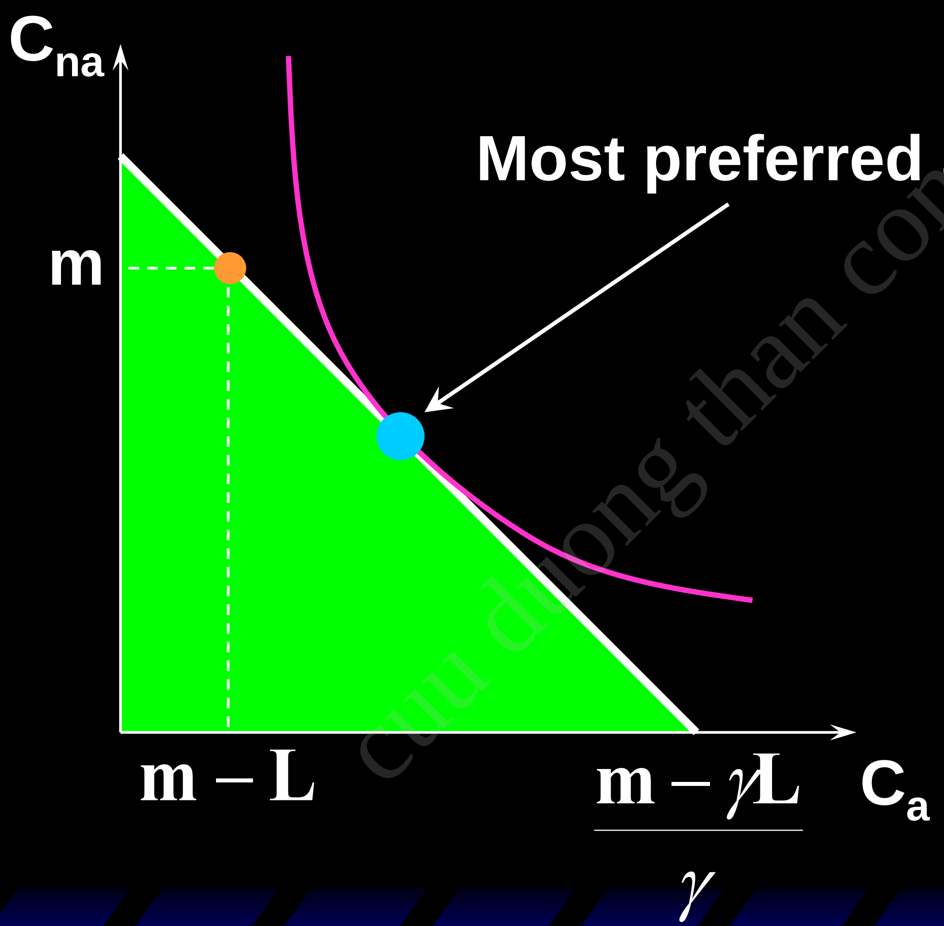


Where is the most preferred state-contingent consumption plan?

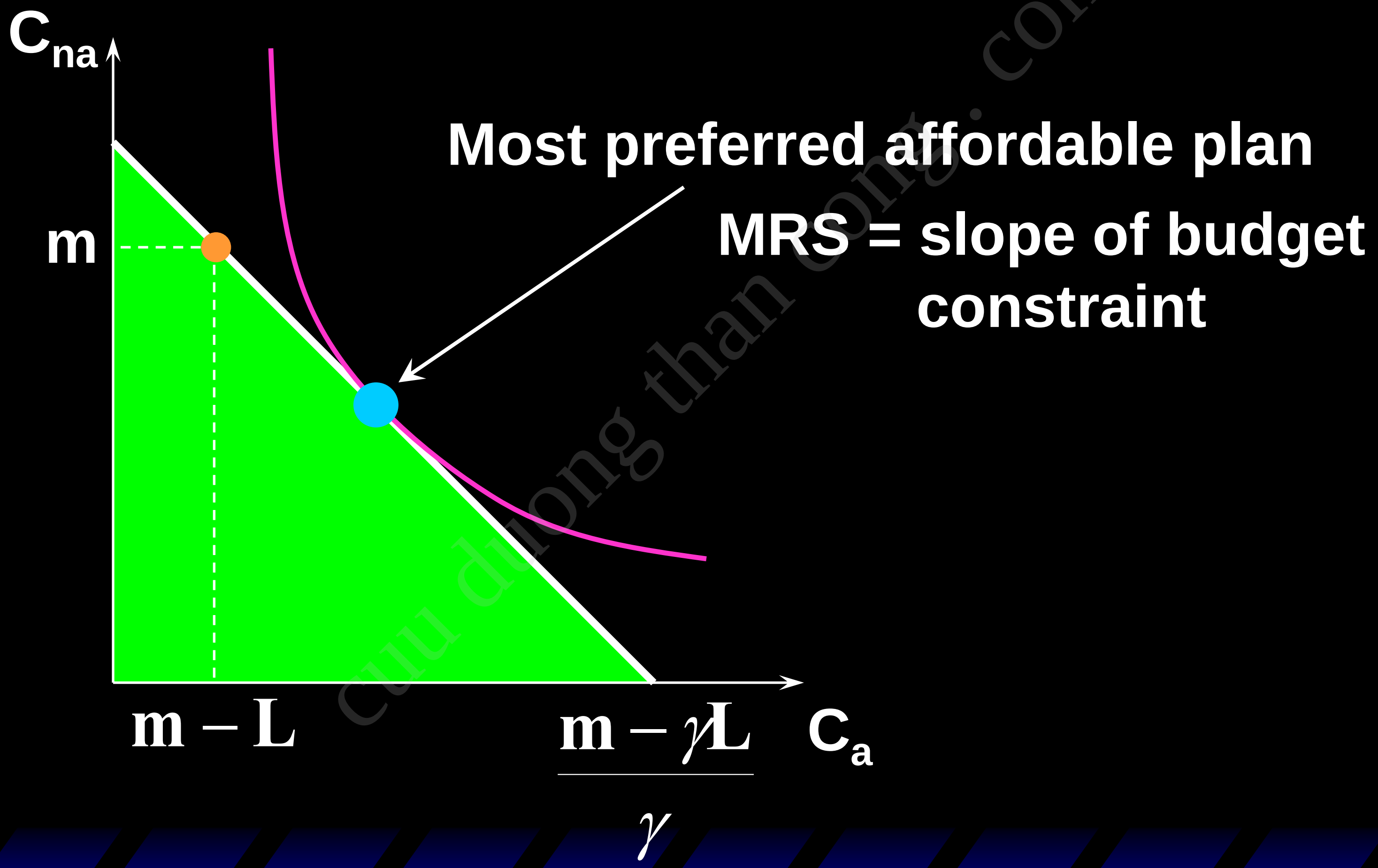
State-Contingent Budget Constraints



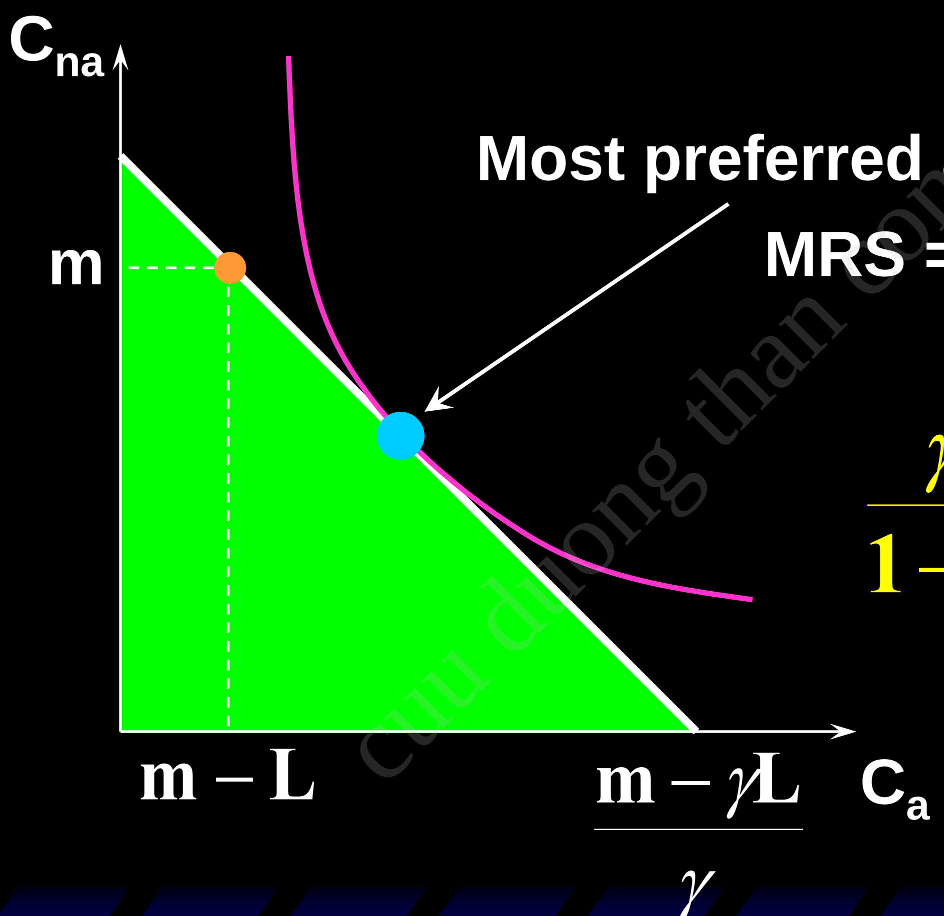
State-Contingent Budget Constraints



State-Contingent Budget Constraints



State-Contingent Budget Constraints



Most preferred affordable plan

MRS = slope of budget constraint; i.e.

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a MU(c_a)}{\pi_{na} MU(c_{na})}$$

Competitive Insurance

- **Suppose entry to the insurance industry is free.**
- **Expected economic profit = 0.**
- **I.e.** $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K = 0.$
- **I.e. free entry** $\Rightarrow \gamma = \pi_a$.
- **If price of $1 insurance = accident probability, then insurance is fair.**

Competitive Insurance

- When insurance is fair, rational insurance choices satisfy

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a}{1-\pi_a} = \frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

- I.e. $\mathbf{MU(c_a) = MU(c_{na})}$
- Marginal utility of income must be the same in both states.

Competitive Insurance

- How much fair insurance does a risk-averse consumer buy?

$$MU(c_a) = MU(c_{na})$$

- Risk-aversion $\Rightarrow$ MU(c) $\downarrow$ as c $\uparrow$.
- Hence $c_a = c_{na}.$
- I.e. full-insurance.

"Unfair" Insurance

- **Suppose insurers make positive expected economic profit.**
- **I.e.** $\gamma K - \pi_a K - (1 - \pi_a)0 = (\gamma - \pi_a)K > 0$.
- **Then** $\Rightarrow \gamma > \pi_a \Rightarrow \dfrac{\gamma}{1-\gamma} > \dfrac{\pi_a}{1-\pi_a}$.

“Unfair” Insurance

- Rational choice requires

$$\frac{\gamma}{1-\gamma} = \frac{\pi_a \mathbf{MU(c_a)}}{\pi_{na} \mathbf{MU(c_{na})}}$$

- Since $\frac{\gamma}{1-\gamma} > \frac{\pi_a}{1-\pi_a}$, $\mathbf{MU(c_a) > MU(c_{na})}$
- Hence $\mathbf{c_a < c_{na}}$ for a risk-averter.
- I.e. a risk-averter buys less than full “unfair” insurance.

Uncertainty is Pervasive

- What are rational responses to uncertainty?

✓ – buying insurance (health, life, auto)

? – a portfolio of contingent consumption goods.

Diversification

- Two firms, A and B. Shares cost $10.
- With prob. 1/2 A's profit is $100 and B's profit is $20.
- With prob. 1/2 A's profit is $20 and B's profit is $100.
- You have $100 to invest. How?

Diversification

- Buy only firm A's stock?
- $100/10 = 10 shares.
- You earn $1000 with prob. 1/2 and $200 with prob. 1/2.
- Expected earning: $500 + $100 = $600

Diversification

- Buy only firm B's stock?
- \$100/10 = 10 shares.
- You earn \$1000 with prob. 1/2 and \$200 with prob. 1/2.
- Expected earning: \$500 + \$100 = \$600

Diversification

- Buy 5 shares in each firm?
- You earn $600 **for sure**.
- Diversification has maintained expected earning and lowered risk.
- Typically, diversification lowers expected earnings in exchange for lowered risk.

Risk Spreading/Mutual Insurance

- **100 risk-neutral persons each independently risk a $10,000 loss.**
- **Loss probability = 0.01.**
- **Initial wealth is $40,000.**
- **No insurance: expected wealth is**

$$0 \cdot 99 \times \$40,000 + 0 \cdot 01(\$40,000 - \$10,000)$$
$$= \$39,900.$$

Risk Spreading/Mutual Insurance

- **Mutual insurance: Expected loss is**

$$0 \cdot 01 \times \$10{,}000 = \$100.$$

- **Each of the 100 persons pays $1 into a mutual insurance fund.**
- **Mutual insurance: expected wealth is**

$$\$40{,}000 - \$1 = \$39{,}999 > \$39{,}900.$$

- **Risk-spreading benefits everyone.**

2. Market Demand

From Individual to Market Demand Functions

- Think of an economy containing n consumers, denoted by i = 1, … ,n.
- Consumer i's ordinary demand function for commodity j is

$$x_j^{*i}(p_1, p_2, m^i)$$

From Individual to Market Demand Functions

- When all consumers are price-takers, the market demand function for commodity j is

$$X_j(p_1,p_2,m^1,\Lambda,m^n) = \sum_{i=1}^{n} x_j^{*i}(p_1,p_2,m^i).$$

- If all consumers are identical then

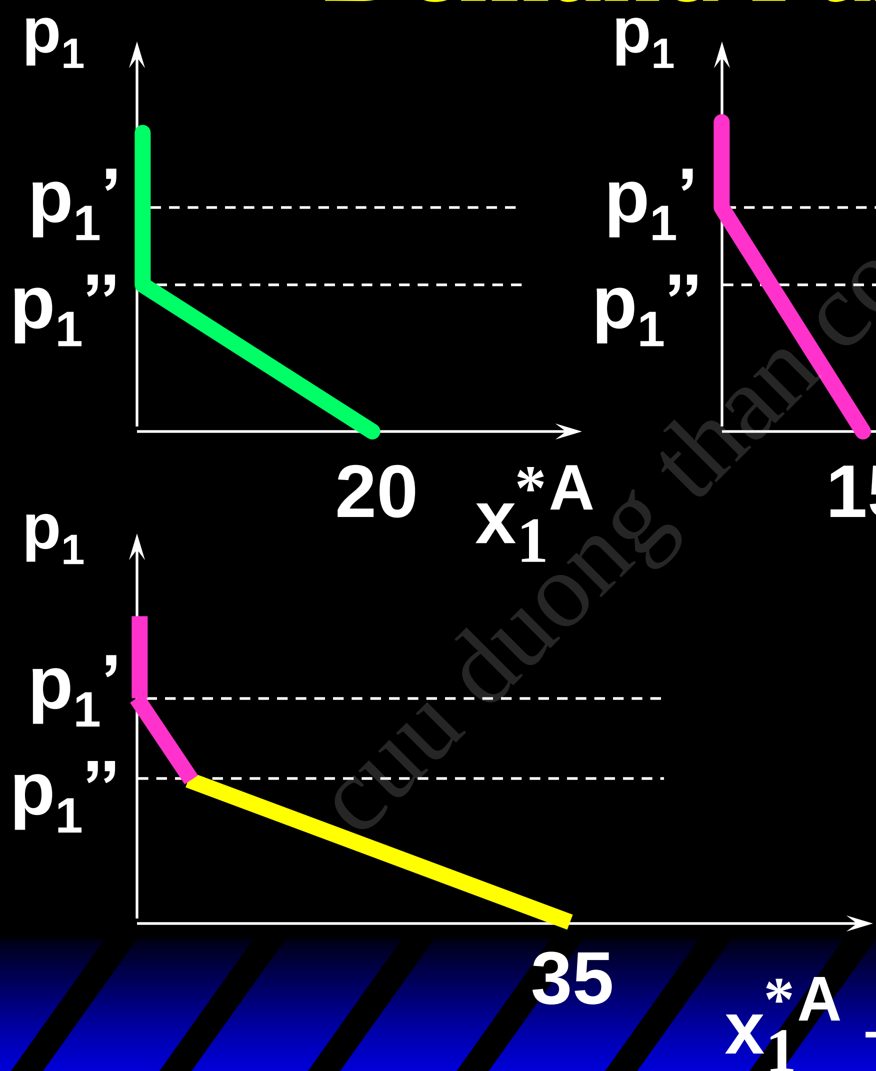
$$X_j(p_1,p_2,M) = n \times x_j^*(p_1,p_2,m)$$

where M = nm.

From Individual to Market Demand Functions

- The market demand curve is the "horizontal sum" of the individual consumers' demand curves.
- E.g. suppose there are only two consumers; i = A,B.

From Individual to Market Demand Functions



The "horizontal sum" of the demand curves of individuals A and B.

Elasticities

- Elasticity measures the "sensitivity" of one variable with respect to another.
- The elasticity of variable X with respect to variable Y is

$$\varepsilon_{x,y} = \frac{\%\Delta x}{\%\Delta y}.$$

Economic Applications of Elasticity

- Economists use elasticities to measure the sensitivity of
  - quantity demanded of commodity i with respect to the price of commodity i (own-price elasticity of demand)
  - demand for commodity i with respect to the price of commodity j (cross-price elasticity of demand).

Economic Applications of Elasticity

- demand for commodity i with respect to income (income elasticity of demand)
- quantity supplied of commodity i with respect to the price of commodity i (own-price elasticity of supply)

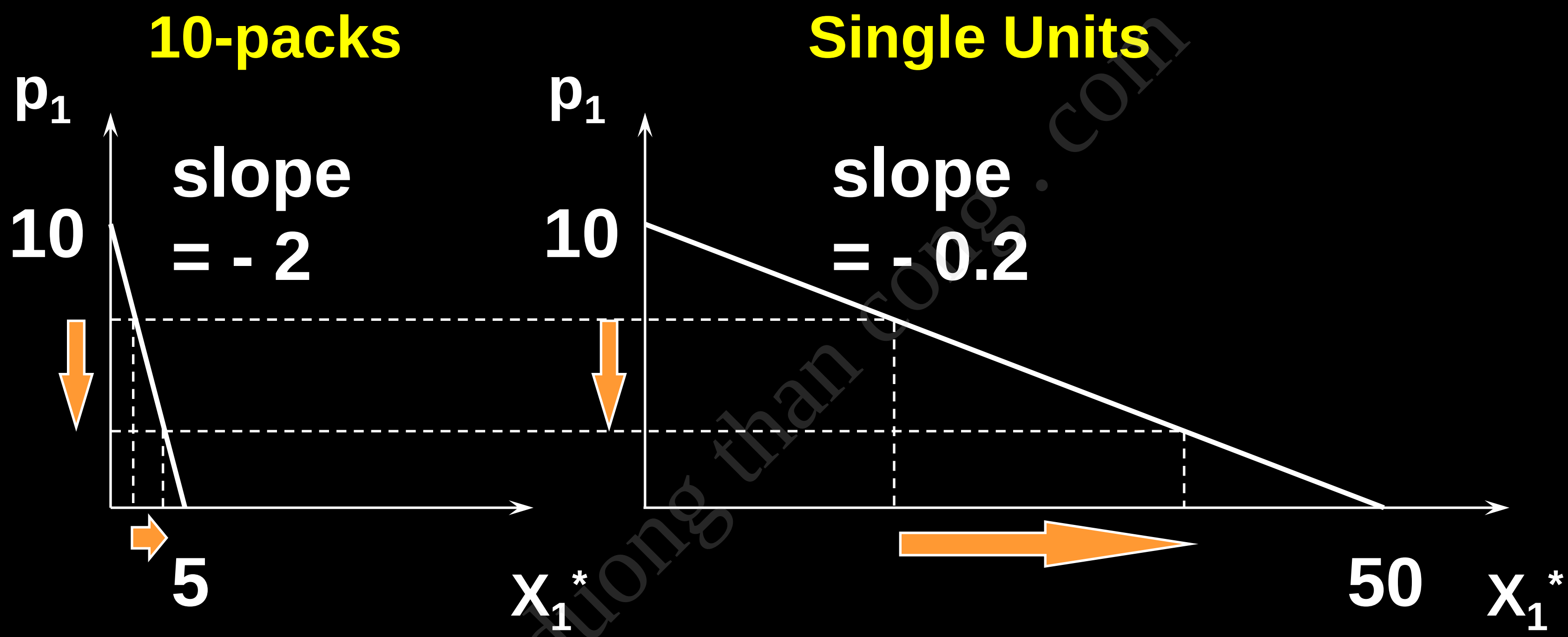
Economic Applications of Elasticity

- quantity supplied of commodity i with respect to the wage rate (elasticity of supply with respect to the price of labor)
- and many, many others.

Own-Price Elasticity of Demand

- **Q: Why not use a demand curve's slope to measure the sensitivity of quantity demanded to a change in a commodity's own price?**

Own-Price Elasticity of Demand



In which case is the quantity demanded X_1* more sensitive to changes to p_1?
It is the same in both cases.

Own-Price Elasticity of Demand

- Q: Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity's own price?
- A: Because the value of sensitivity then depends upon the (arbitrary) units of measurement used for quantity demanded.

Own-Price Elasticity of Demand

$$\varepsilon_{x_1^*,p_1} = \frac{\%\Delta x_1^*}{\%\Delta p_1}$$

is a ratio of percentages and so has no units of measurement.

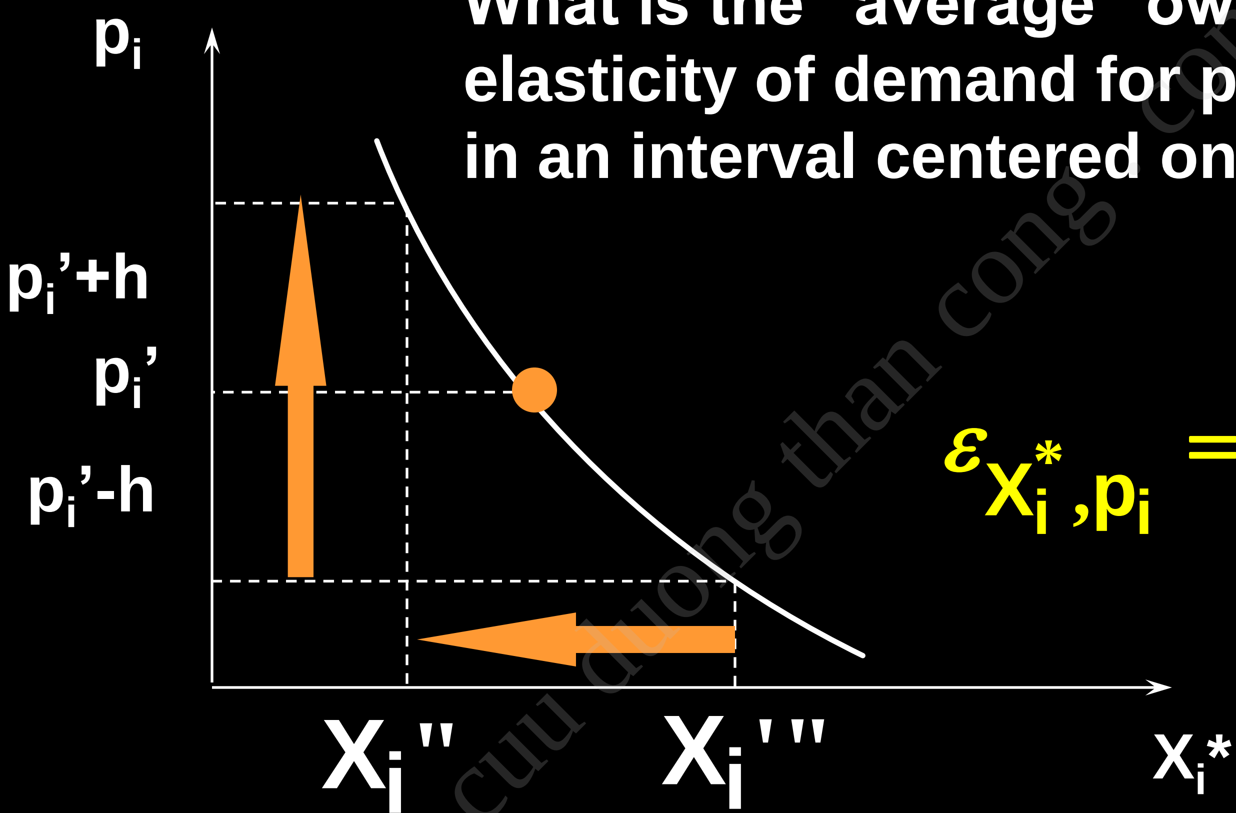
Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.

Arc and Point Elasticities

- An “average” own-price elasticity of demand for commodity i over an **interval of values for p_i** is an **arc-elasticity**, usually computed by a mid-point formula.
- Elasticity computed for a **single value of p_i** is a **point elasticity**.

Arc Own-Price Elasticity

What is the "average" own-price elasticity of demand for prices in an interval centered on p_i'?



$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'} \qquad \%\Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

Arc Own-Price Elasticity

$$\%\Delta p_i = 100 \times \frac{2h}{p_i'}$$

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i}$$

$$\%\Delta X_i^* = 100 \times \frac{(X_i'' - X_i''')}{(X_i'' + X_i''')/2}$$

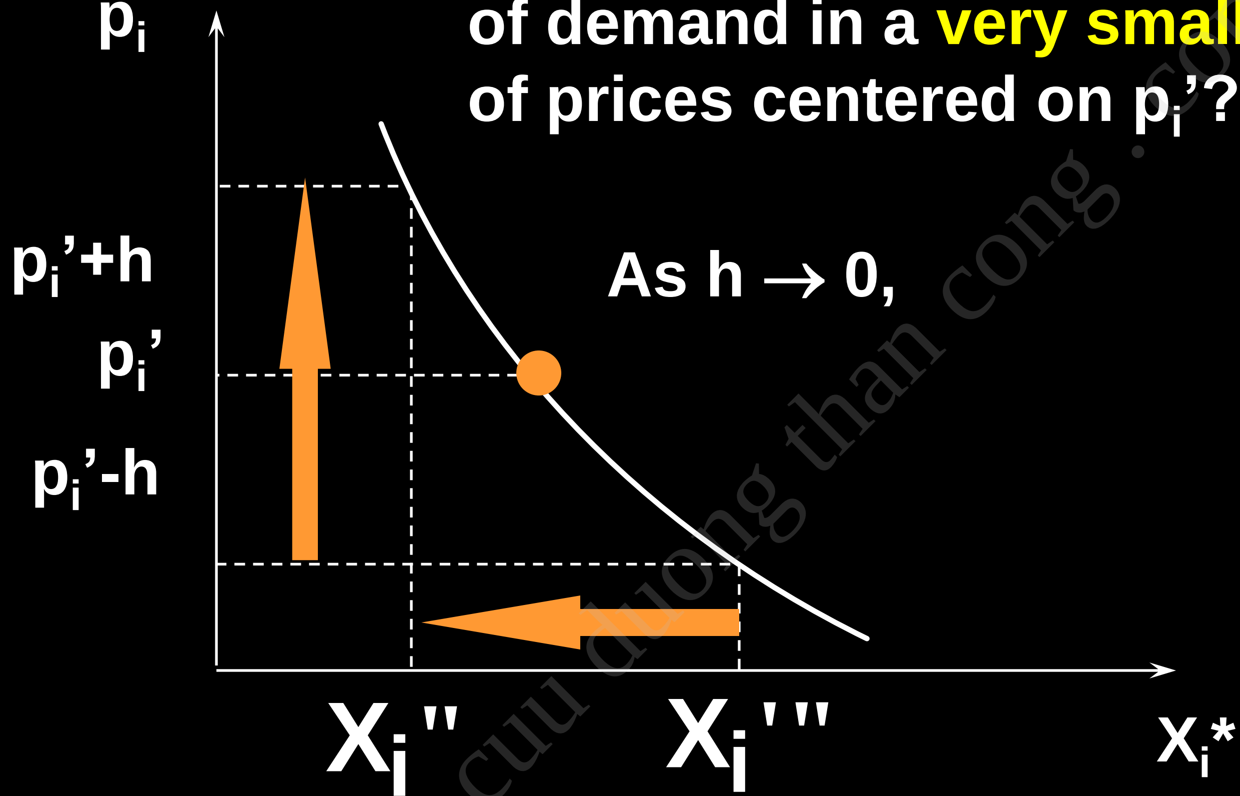
So

$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

is the arc own-price elasticity of demand.

Point Own-Price Elasticity

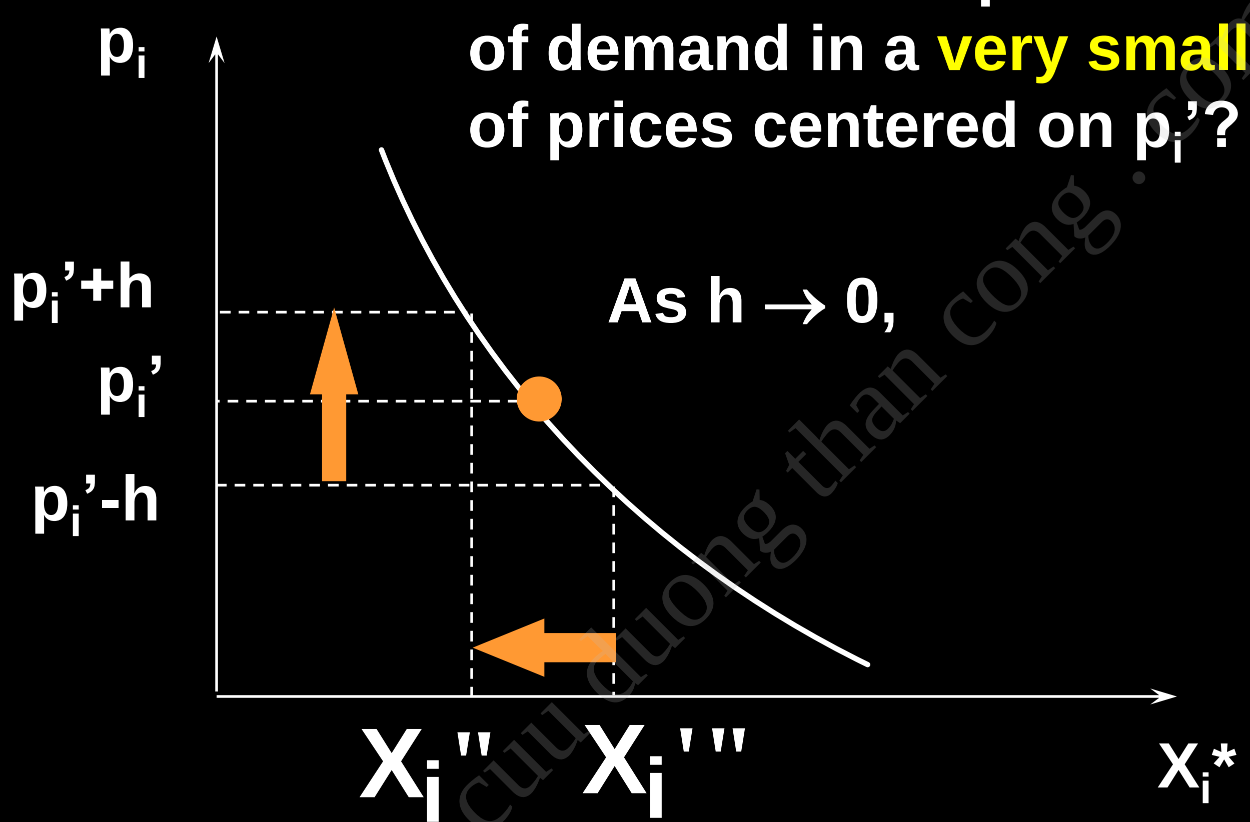
What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i'?



$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

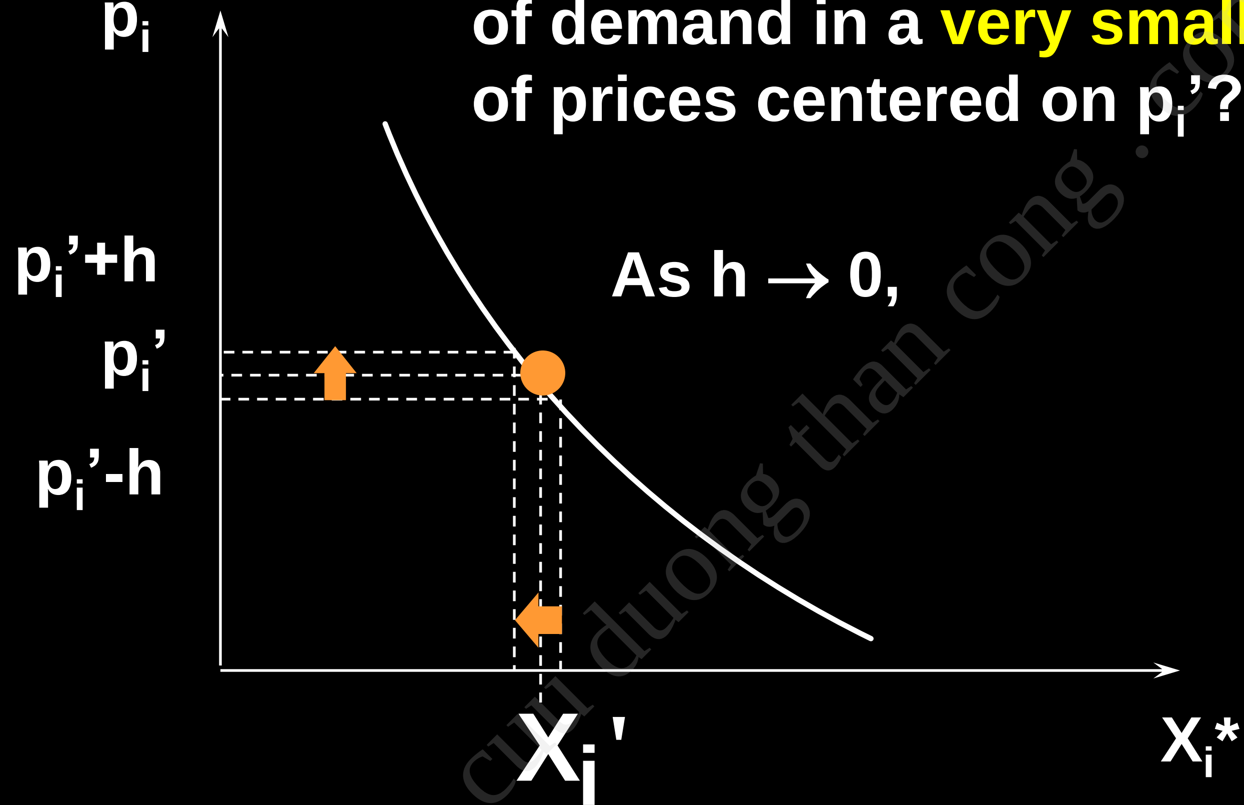
What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i'?



$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i" + X_i''')/2} \times \frac{(X_i" - X_i''')}{2h}.$$

Point Own-Price Elasticity

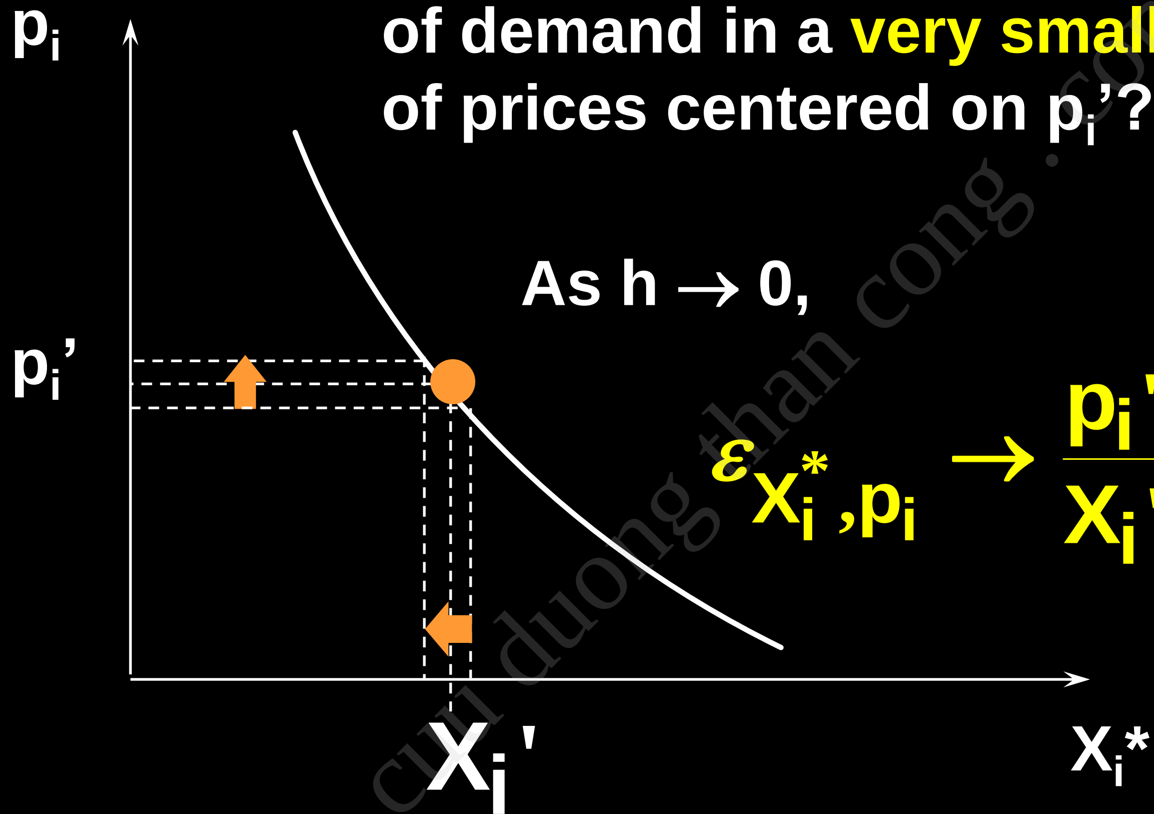
What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i'?



$$\varepsilon_{X_i^*,p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i'?



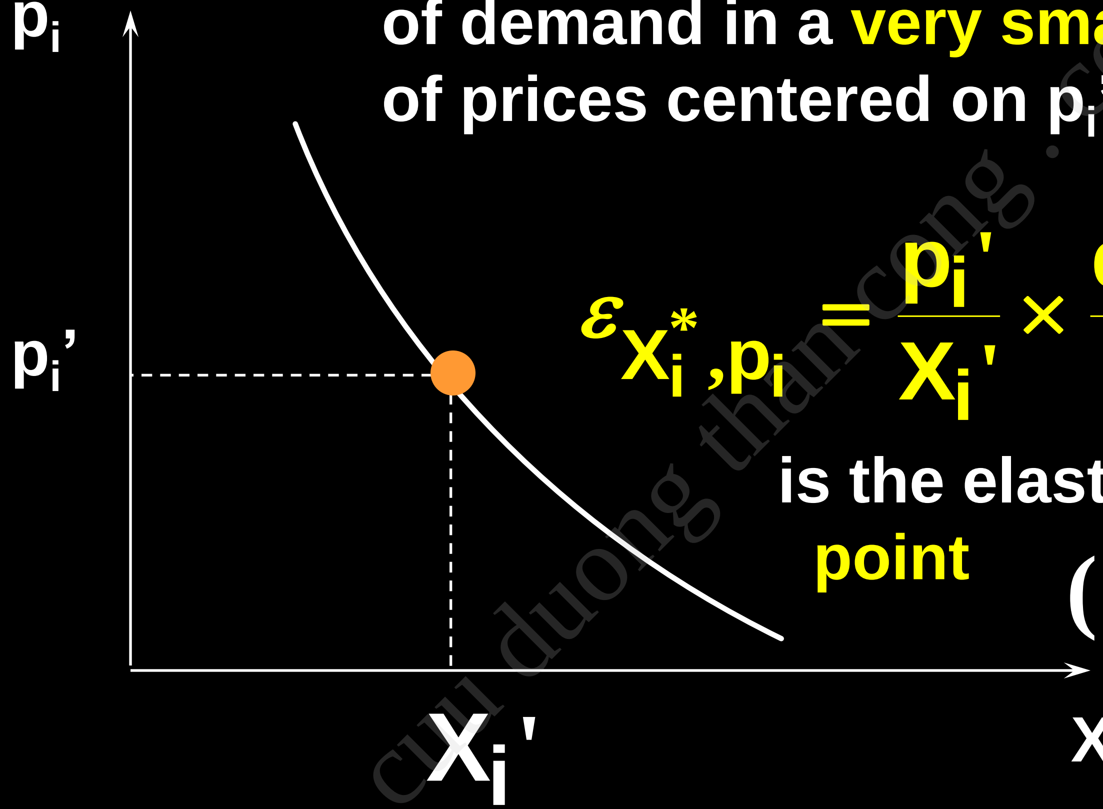
As $h \to 0$,

$$\varepsilon_{X_i^*, p_i} \to \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

$$\varepsilon_{X_i^*, p_i} = \frac{\%\Delta X_i^*}{\%\Delta p_i} = \frac{p_i'}{(X_i'' + X_i''')/2} \times \frac{(X_i'' - X_i''')}{2h}.$$

Point Own-Price Elasticity

What is the own-price elasticity of demand in a **very small interval** of prices centered on p_i'?



$$\varepsilon_{X_i^*, p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i^*}{dp_i}$$

is the elasticity **at the point** (X_i', p_i').

Point Own-Price Elasticity

$$\varepsilon_{X_i^*,p_i} = \frac{p_i}{X_i^*} \times \frac{dX_i^*}{dp_i}$$

E.g. Suppose $p_i = a - bX_i$.
Then $X_i = (a-p_i)/b$ and

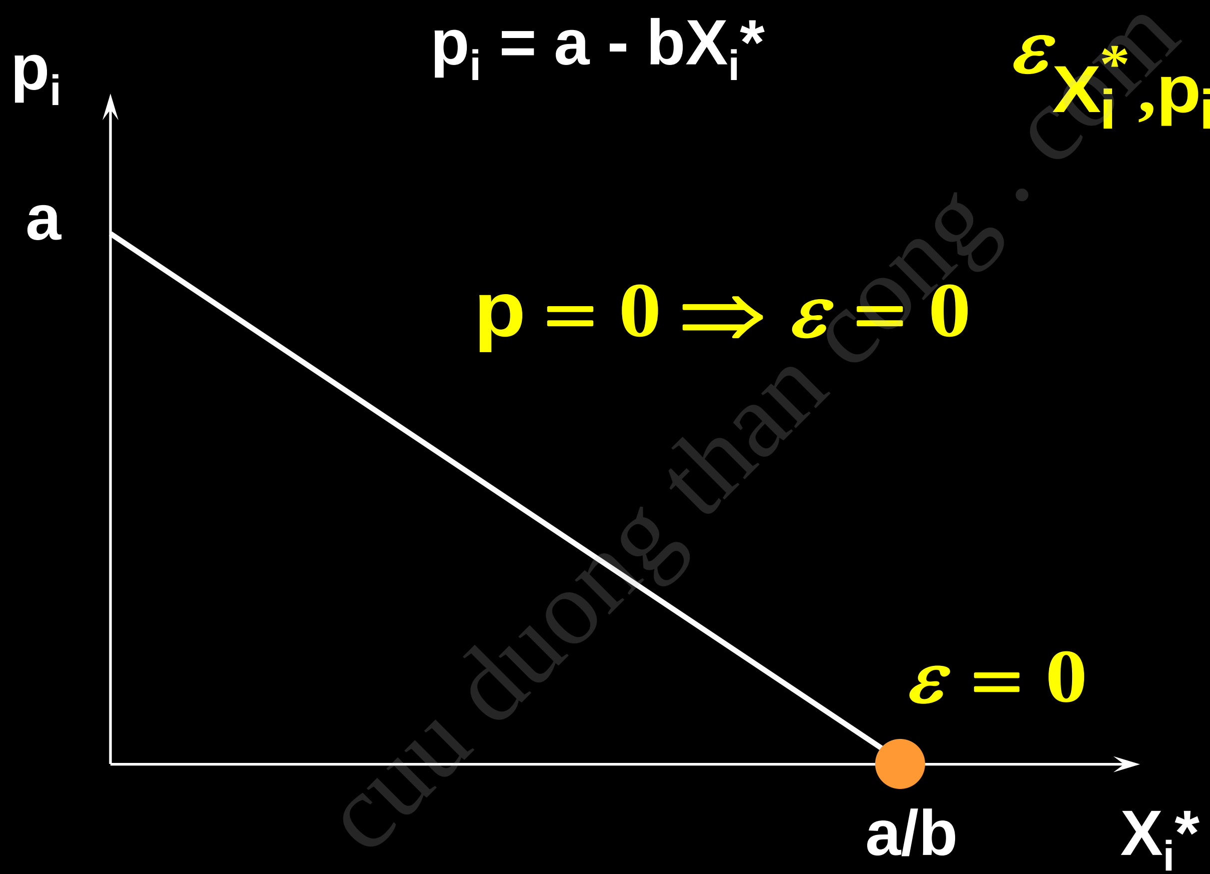
$$\frac{dX_i^*}{dp_i} = -\frac{1}{b}.$$ Therefore,

$$\varepsilon_{X_i^*,p_i} = \frac{p_i}{(a-p_i)/b} \times \left(-\frac{1}{b}\right) = -\frac{p_i}{a-p_i}.$$

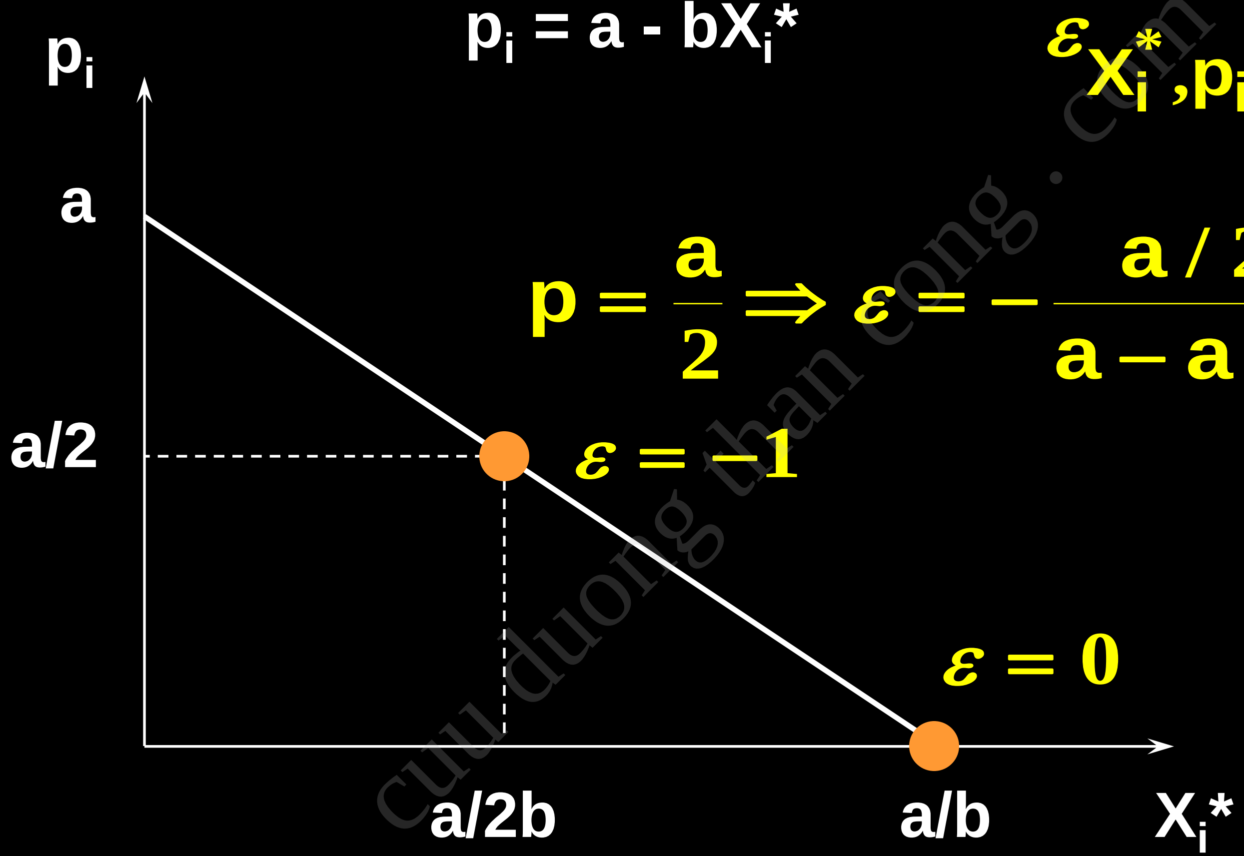
Point Own-Price Elasticity

$$\varepsilon_{X_i^*,p_i} = -\frac{p_i}{a - p_i}$$

$$p = 0 \Rightarrow \varepsilon = 0$$



Point Own-Price Elasticity



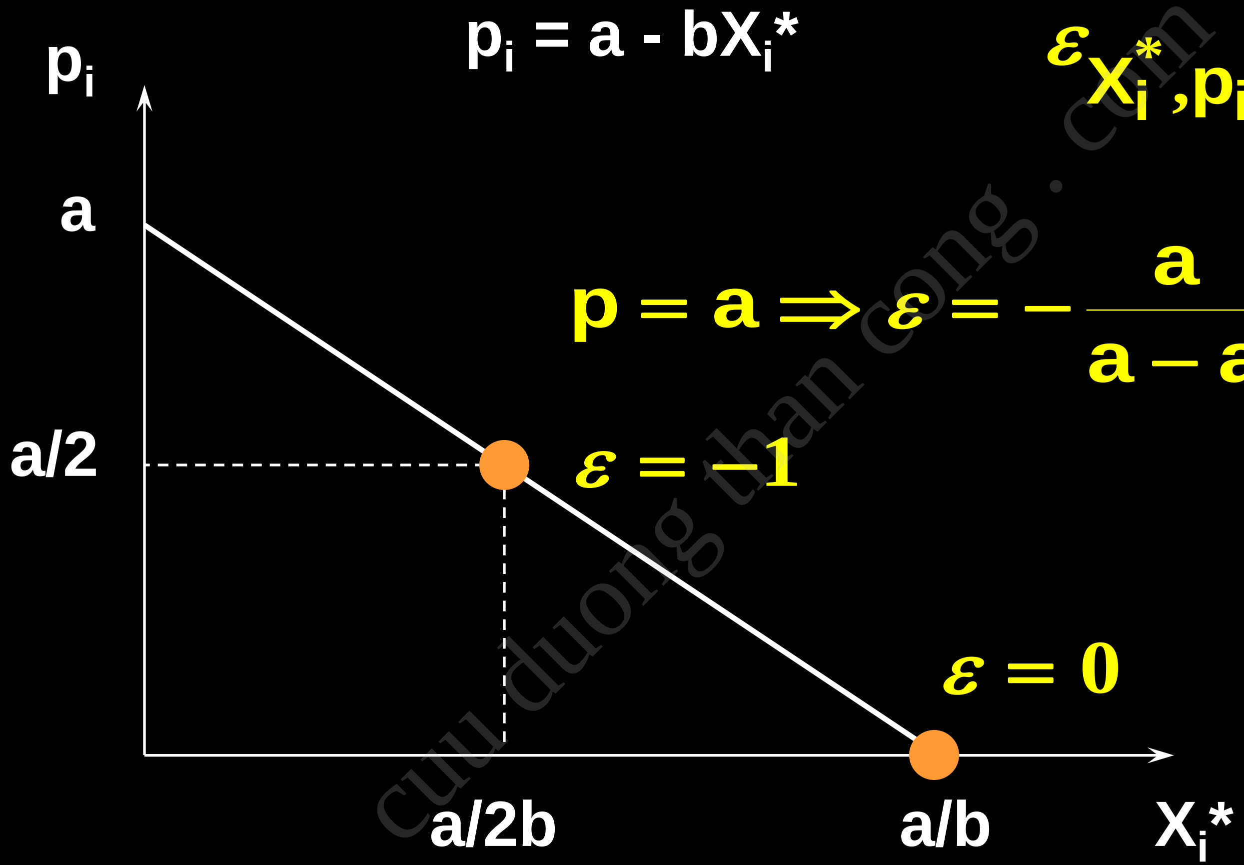
$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = \frac{a}{2} \Rightarrow \varepsilon = -\frac{a/2}{a - a/2} = -1$$

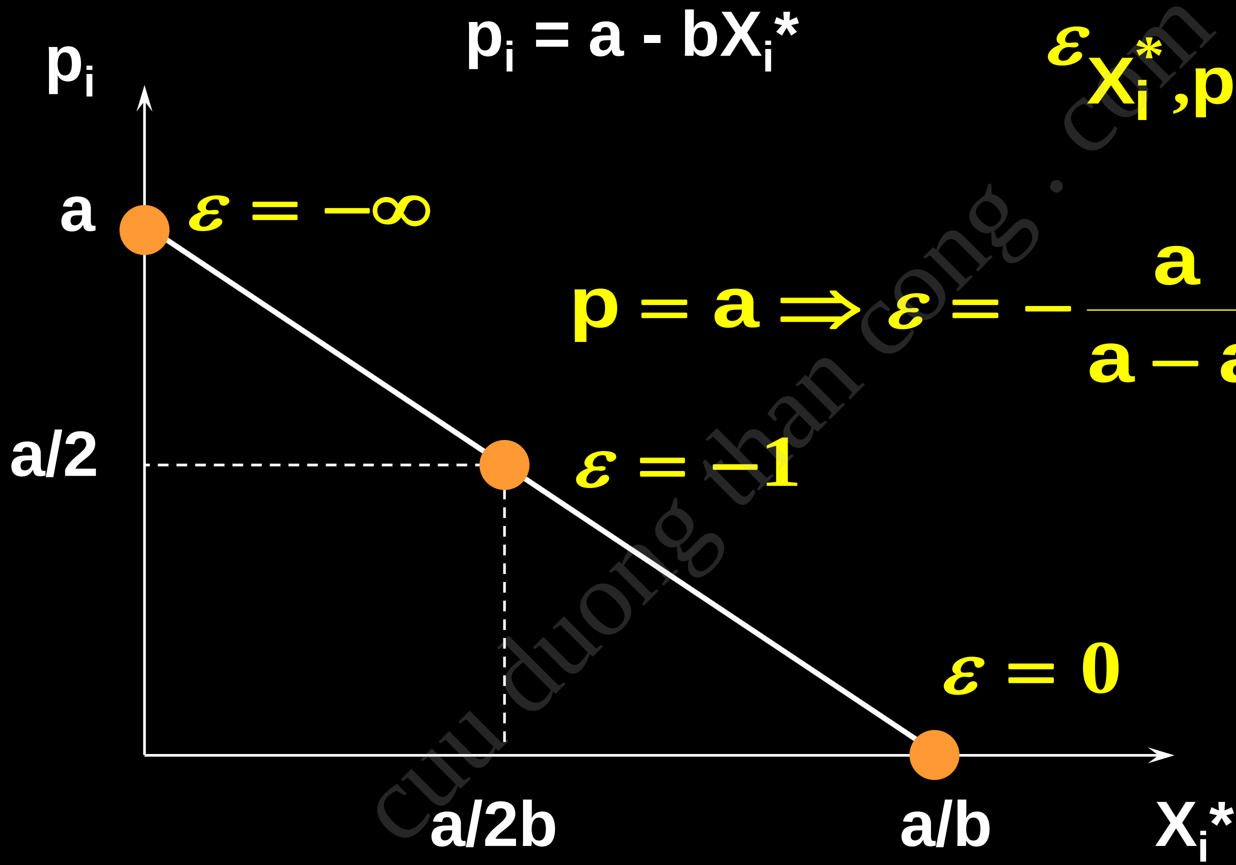
Point Own-Price Elasticity

$$\varepsilon_{X_i^*,p_i} = -\frac{p_i}{a - p_i}$$

$$p = a \Rightarrow \varepsilon = -\frac{a}{a-a} = -\infty$$



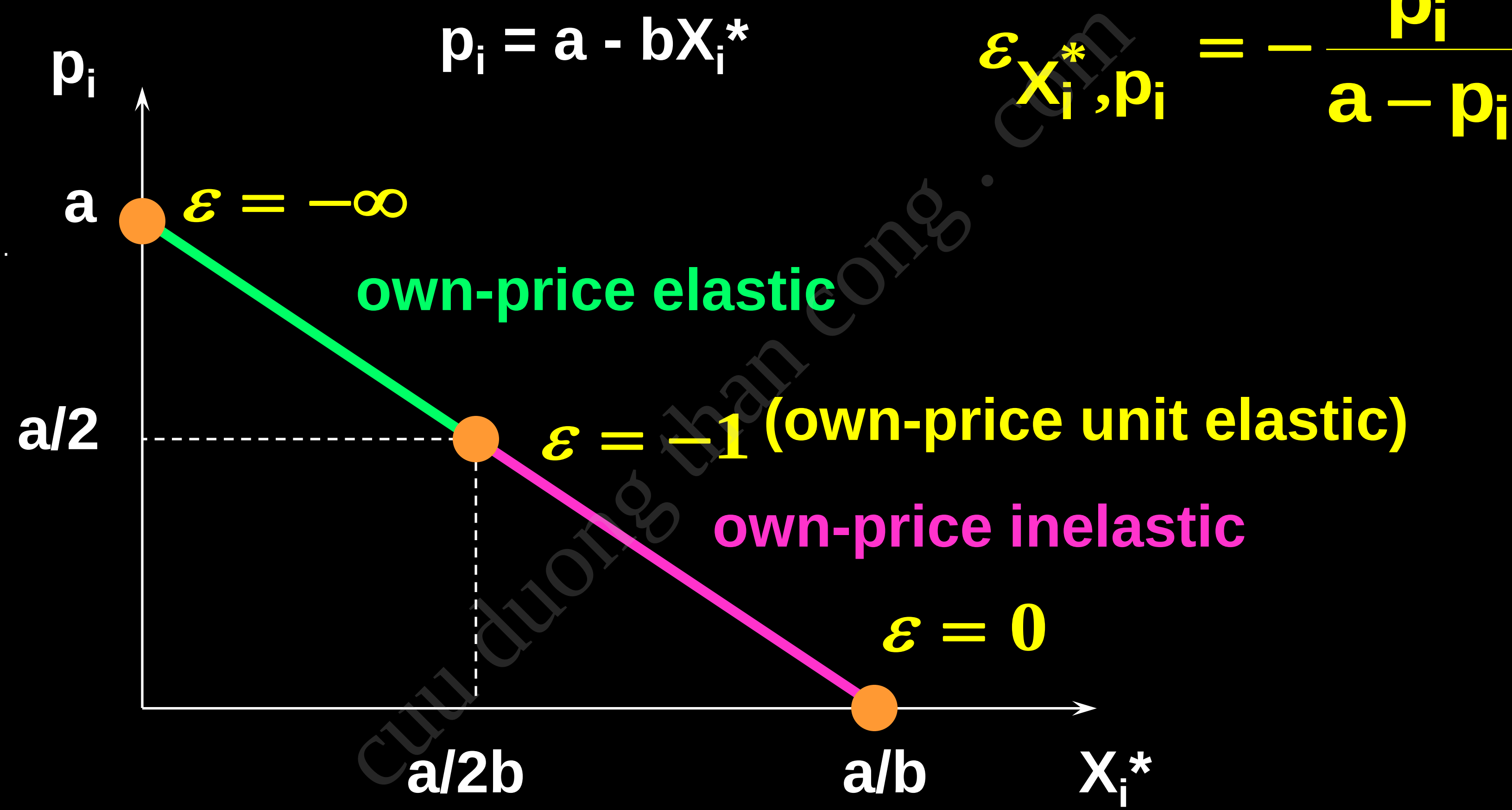
Point Own-Price Elasticity



$$\varepsilon_{X_i^*, p_i} = -\frac{p_i}{a - p_i}$$

$$p = a \Rightarrow \varepsilon = -\frac{a}{a - a} = -\infty$$

Point Own-Price Elasticity



Point Own-Price Elasticity

$$\varepsilon_{x_i^*,p_i} = \frac{p_i}{x_i^*} \times \frac{dx_i^*}{dp_i}$$

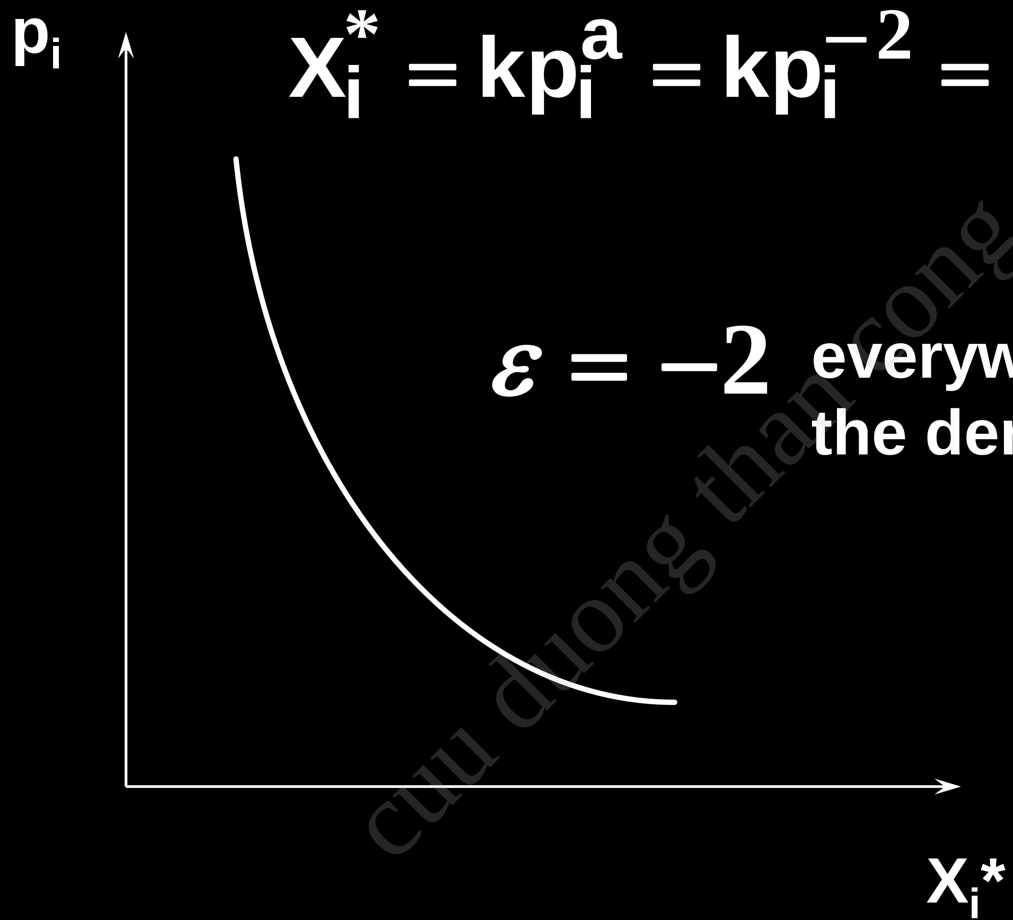
E.g. $x_i^* = kp_i^a$. Then $\frac{dx_i^*}{dp_i} = ap_i^{a-1}$

so

$$\varepsilon_{x_i^*,p_i} = \frac{p_i}{kp_i^a} \times kap_i^{a-1} = a\frac{p_i^a}{p_i^a} = a.$$

Point Own-Price Elasticity

$$X_i^* = kp_i^a = kp_i^{-2} = \frac{k}{p_i^2}$$



$\varepsilon = -2$ everywhere along the demand curve.

Revenue and Own-Price Elasticity of Demand

- If raising a commodity’s price causes little decrease in quantity demanded, then sellers’ revenues rise.
- Hence own-price **inelastic** demand causes sellers’ revenues to rise as price rises.

Revenue and Own-Price Elasticity of Demand

- If raising a commodity's price causes a large decrease in quantity demanded, then sellers' revenues fall.
- Hence own-price **elastic** demand causes sellers' revenues to fall as price rises.

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

Revenue and Own-Price Elasticity of Demand

Sellers' revenue is $R(p) = p \times X^*(p).$

So $$\frac{dR}{dp} = X^*(p) + p\frac{dX^*}{dp}$$

$$= X^*(p)\left[1 + \frac{p}{X^*(p)}\frac{dX^*}{dp}\right]$$

$$= X^*(p)[1 + \varepsilon].$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)\left[1 + \varepsilon\right]$$

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1 + \varepsilon]$$

so if $\varepsilon = -1$ then $\frac{dR}{dp} = 0$

and a change to price does not alter sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1+\varepsilon]$$

but if $-1 < \varepsilon \leq 0$ then $\frac{dR}{dp} > 0$

and a price increase raises sellers' revenue.

Revenue and Own-Price Elasticity of Demand

$$\frac{dR}{dp} = X^*(p)[1 + \varepsilon]$$

And if $\varepsilon < -1$ then $\frac{dR}{dp} < 0$

and a price increase reduces sellers' revenue.

Revenue and Own-Price Elasticity of Demand

In summary:

Own-price inelastic demand; price rise causes rise in sellers' revenue. $-1 < \varepsilon \le 0$

Own-price unit elastic demand; price rise causes no change in sellers' revenue. $\varepsilon = -1$

Own-price elastic demand; price rise causes fall in sellers' revenue. $\varepsilon < -1$

Marginal Revenue and Own-Price Elasticity of Demand

- A seller's **marginal revenue** is the rate at which revenue changes with the number of units sold by the seller.

$$MR(q) = \frac{dR(q)}{dq}.$$

Marginal Revenue and Own-Price Elasticity of Demand

p(q) denotes the seller's inverse demand function; i.e. the price at which the seller can sell q units. Then

$$R(q) = p(q) \times q$$

so

$$MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} q + p(q)$$

$$= p(q)\left[1 + \frac{q}{p(q)} \frac{dp(q)}{dq}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{q}{p(q)}\frac{dp(q)}{dq}\right].$$

and

$$\varepsilon = \frac{dq}{dp} \times \frac{p}{q}$$

so

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right].$$

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$ says that the rate

at which a seller's revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; *i.e.*, upon the of the own-price elasticity of demand.

Marginal Revenue and Own-Price Elasticity of Demand

$$MR(q) = p(q)\left[1 + \frac{1}{\varepsilon}\right]$$

If $\varepsilon = -1$ then $MR(q) = 0.$

If $-1 < \varepsilon \le 0$ then $MR(q) < 0.$

If $\varepsilon < -1$ then $MR(q) > 0.$

Marginal Revenue and Own-Price Elasticity of Demand

If $\varepsilon = -1$ then **MR(q) = 0.**

Selling one more unit does not change the seller's revenue.

If $-1 < \varepsilon \leq 0$ then **MR(q) < 0.**

Selling one more unit reduces the seller's revenue.

If $\varepsilon < -1$ then **MR(q) > 0.**

Selling one more unit raises the seller's revenue.

Marginal Revenue and Own-Price Elasticity of Demand

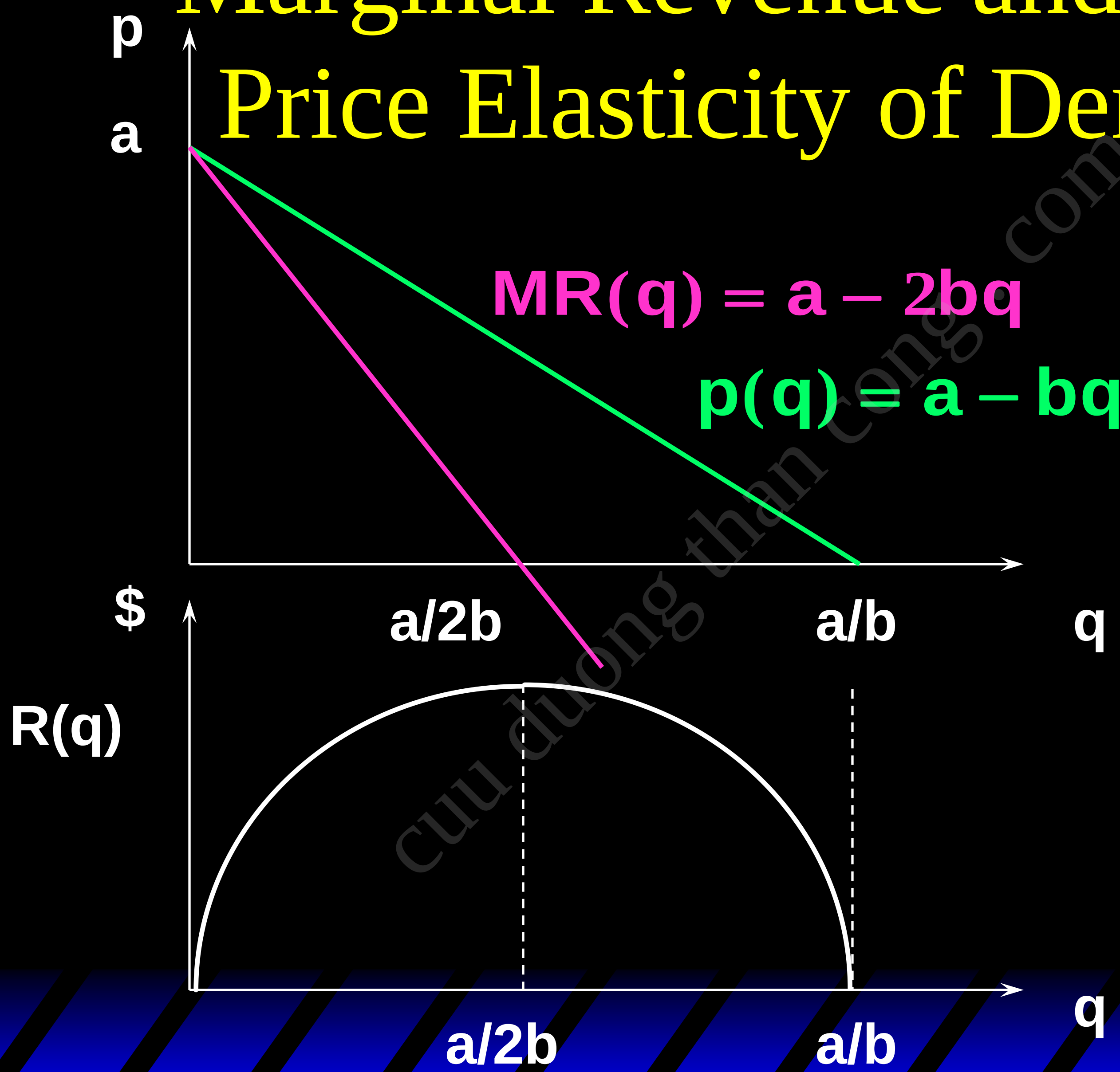
An example with linear inverse demand.

$$p(q) = a - bq.$$

Then $R(q) = p(q)q = (a - bq)q$

and $MR(q) = a - 2bq.$

Marginal Revenue and Own-Price Elasticity of Demand

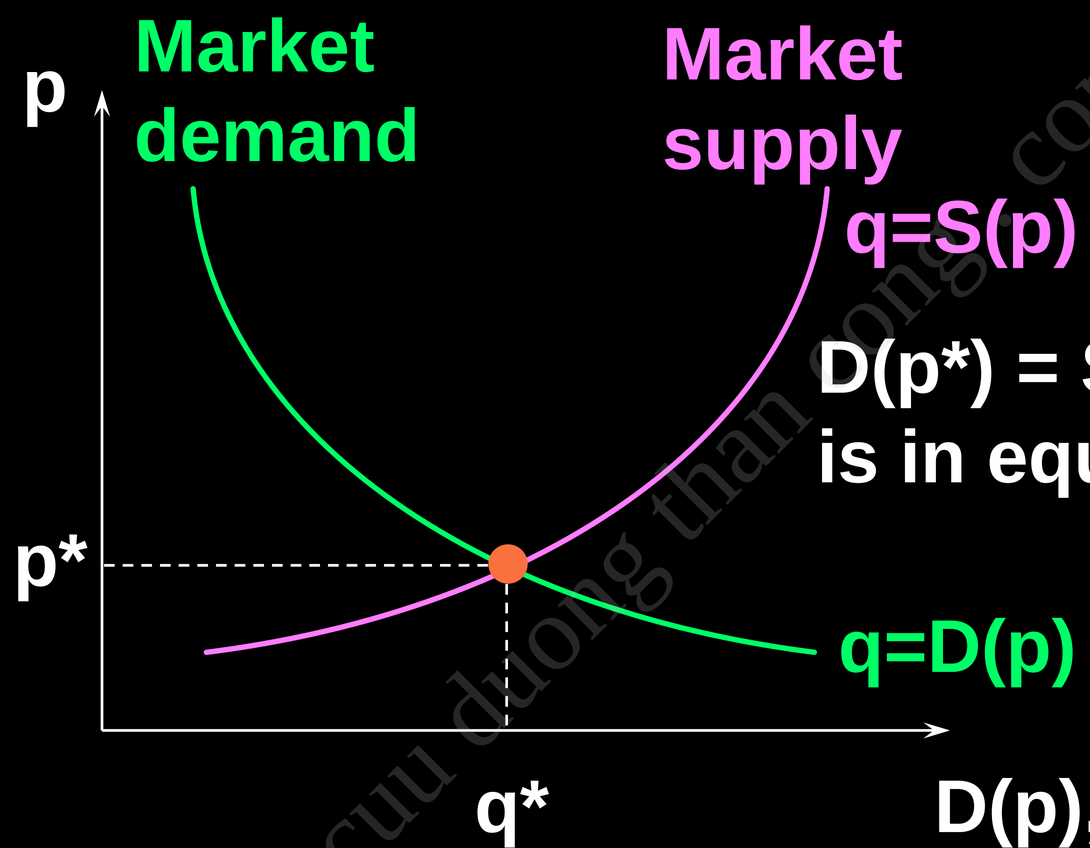


3. Equilibrium

Market Equilibrium

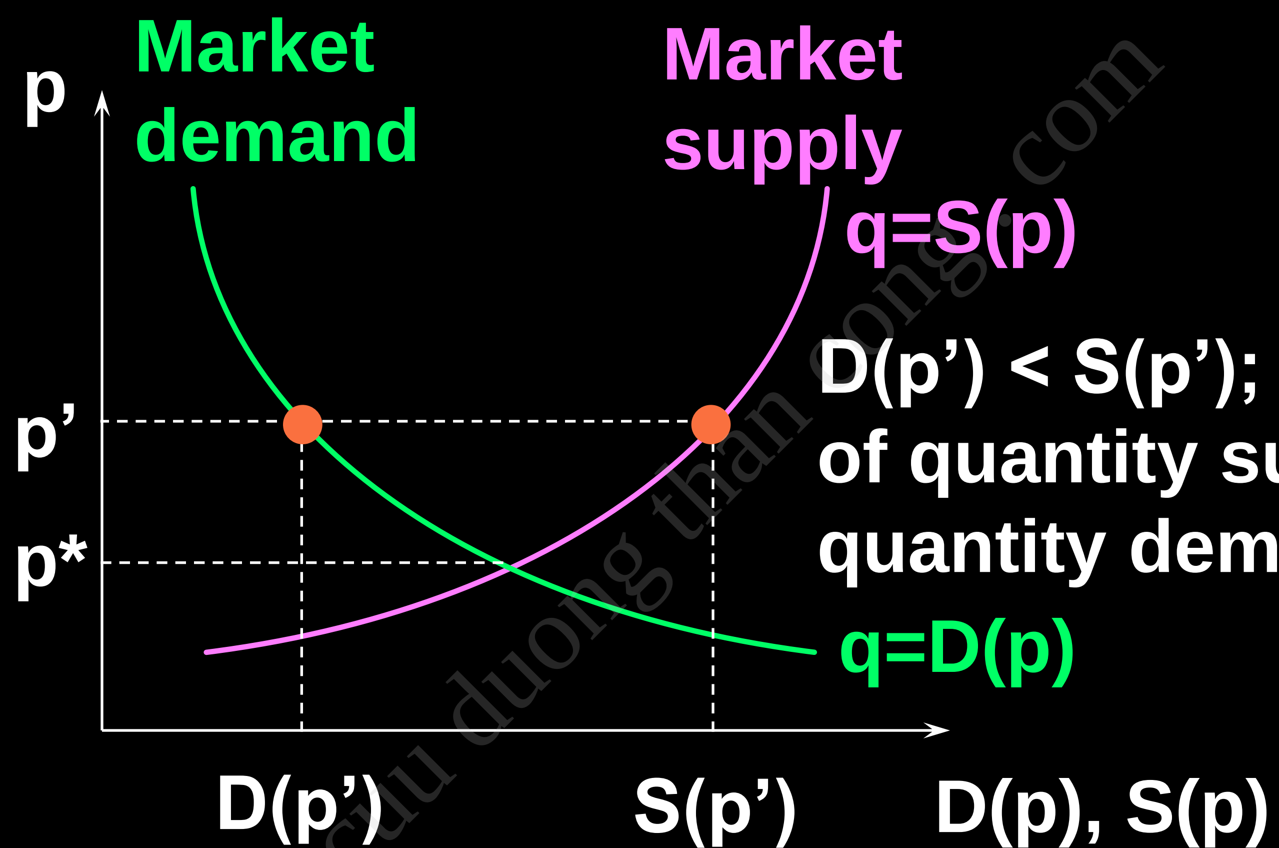
- A market is in **equilibrium** when total quantity demanded by buyers equals total quantity supplied by sellers.

Market Equilibrium



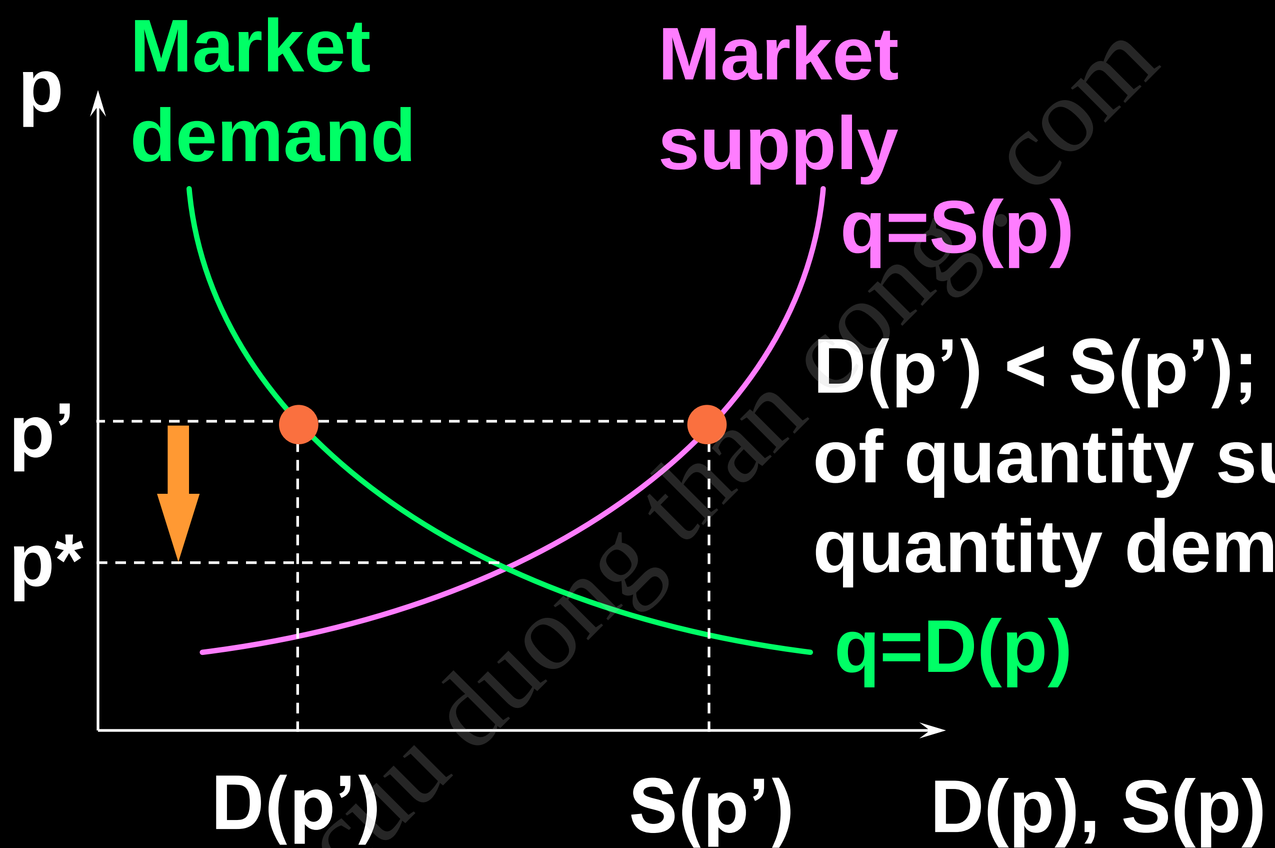
D(p*) = S(p*); the market is in equilibrium.

Market Equilibrium



D(p') < S(p'); an excess of quantity supplied over quantity demanded.

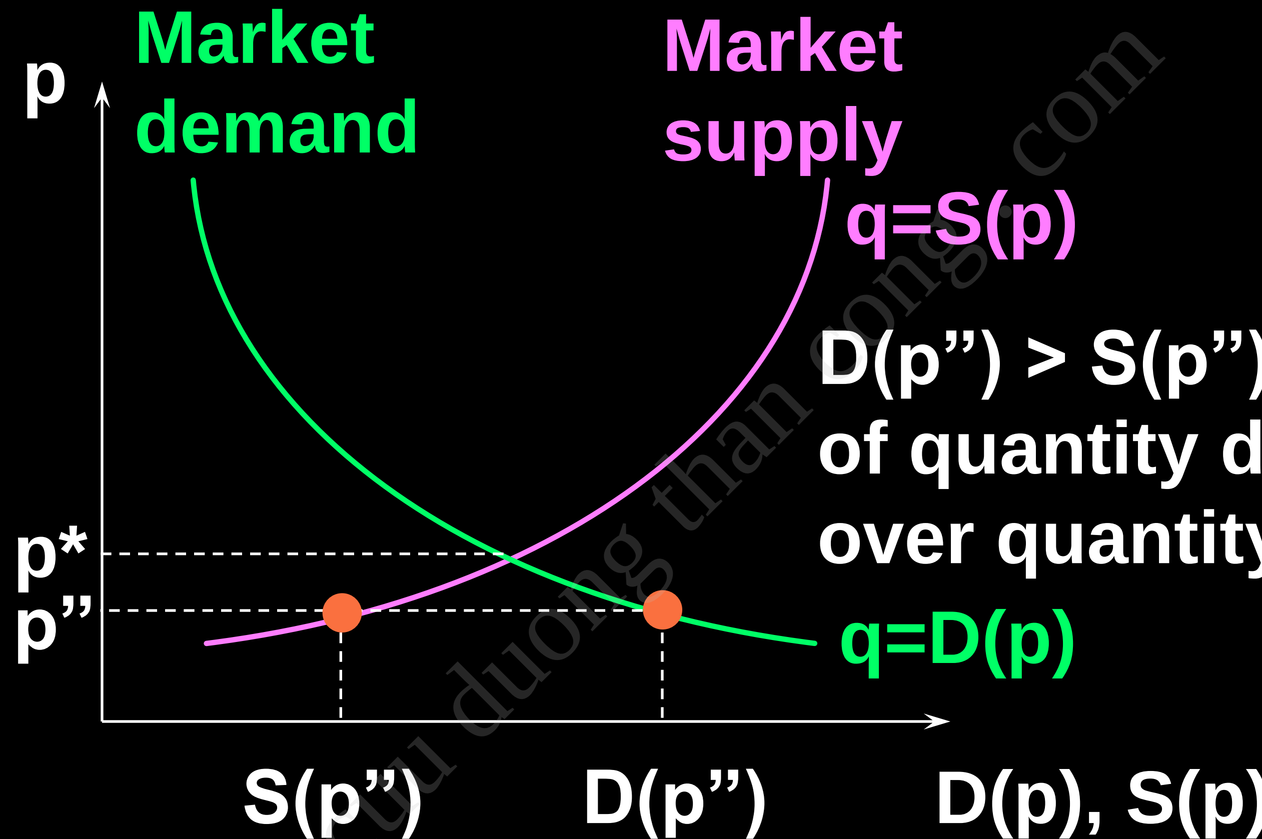
Market Equilibrium



D(p’) < S(p’); an excess of quantity supplied over quantity demanded.

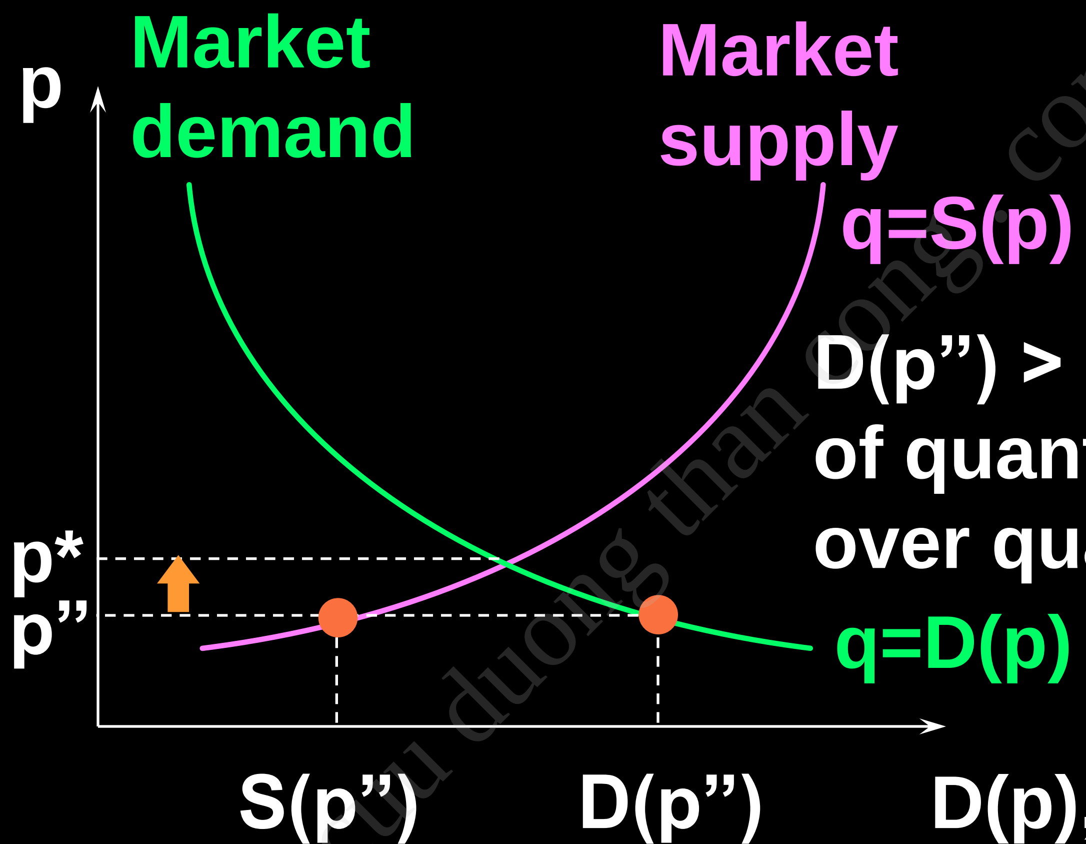
Market price must fall towards p*.

Market Equilibrium



D(p”) > S(p”); an excess of quantity demanded over quantity supplied.

Market Equilibrium



D(p") > S(p"); an excess of quantity demanded over quantity supplied.

Market price must rise towards p*.

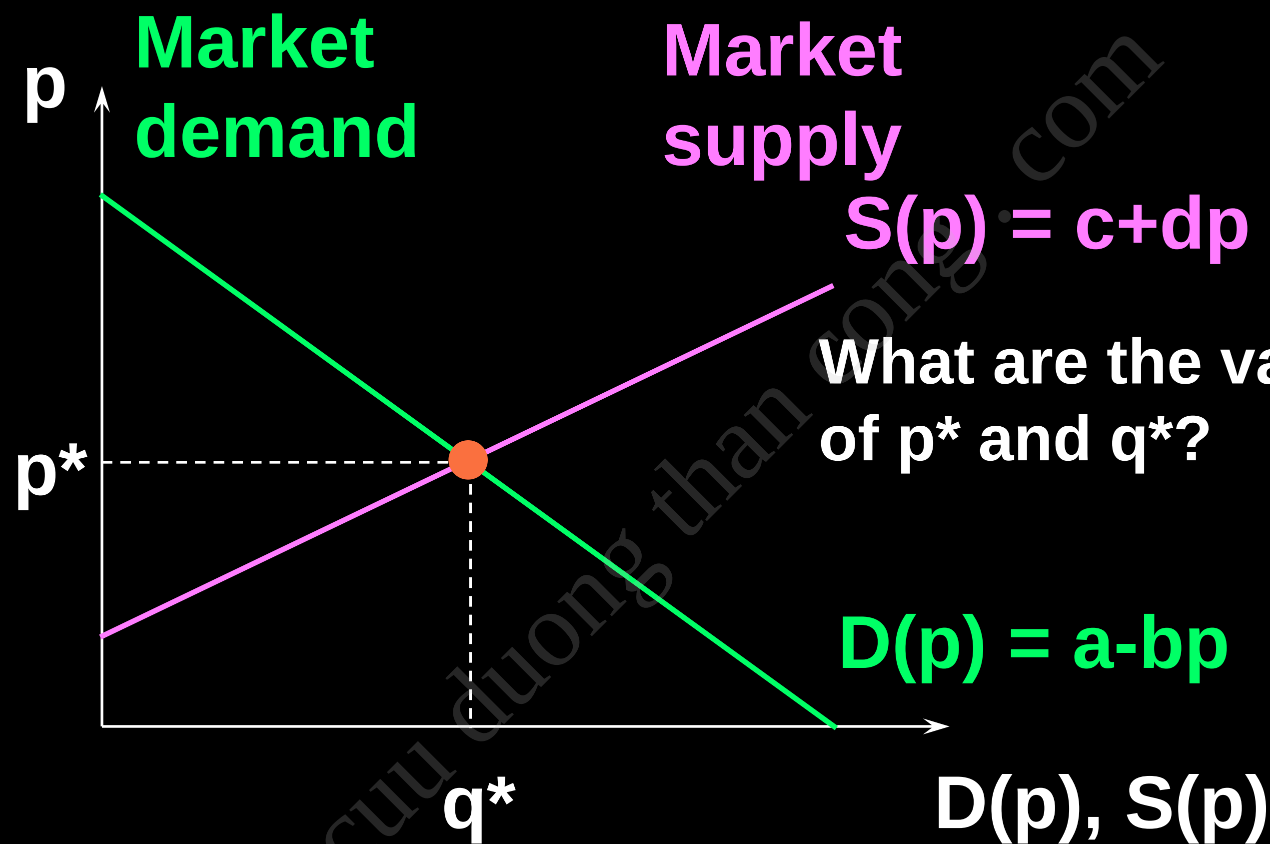
Market Equilibrium

- **An example of calculating a market equilibrium when the market demand and supply curves are linear.**

$$D(p) = a - bp$$

$$S(p) = c + dp$$

Market Equilibrium



What are the values of p* and q*?

Market Equilibrium

$$D(p) = a - bp$$

$$S(p) = c + dp$$

At the equilibrium price p*, D(p*) = S(p*).
That is,

$$a - bp^* = c + dp^*$$

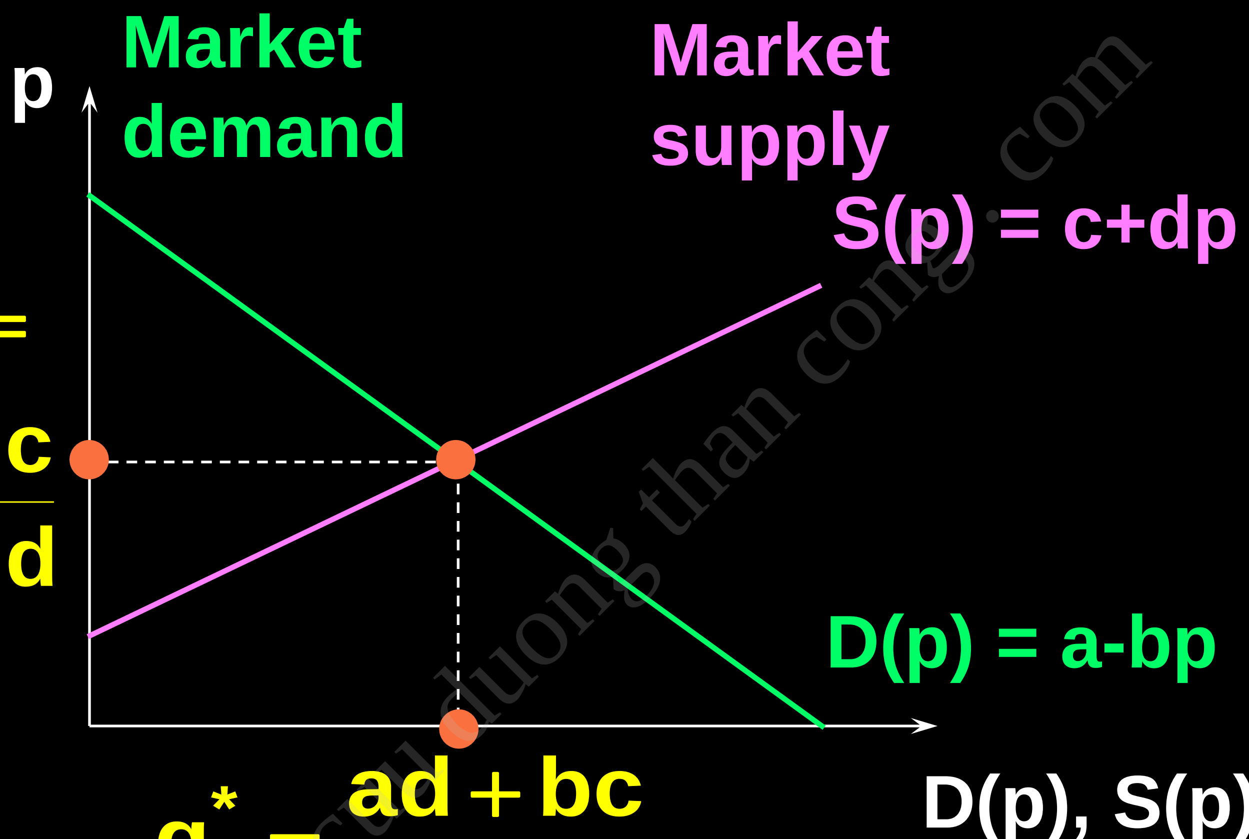
which gives

$$p^* = \frac{a - c}{b + d}$$

and

$$q^* = D(p^*) = S(p^*) = \frac{ad + bc}{b + d}.$$

Market Equilibrium



$$p^* = \frac{a - c}{b + d}$$

$$q^* = \frac{ad + bc}{b + d}$$

Market Equilibrium

- Can we calculate the market equilibrium using the inverse market demand and supply curves?
- Yes, it is the same calculation.

Market Equilibrium

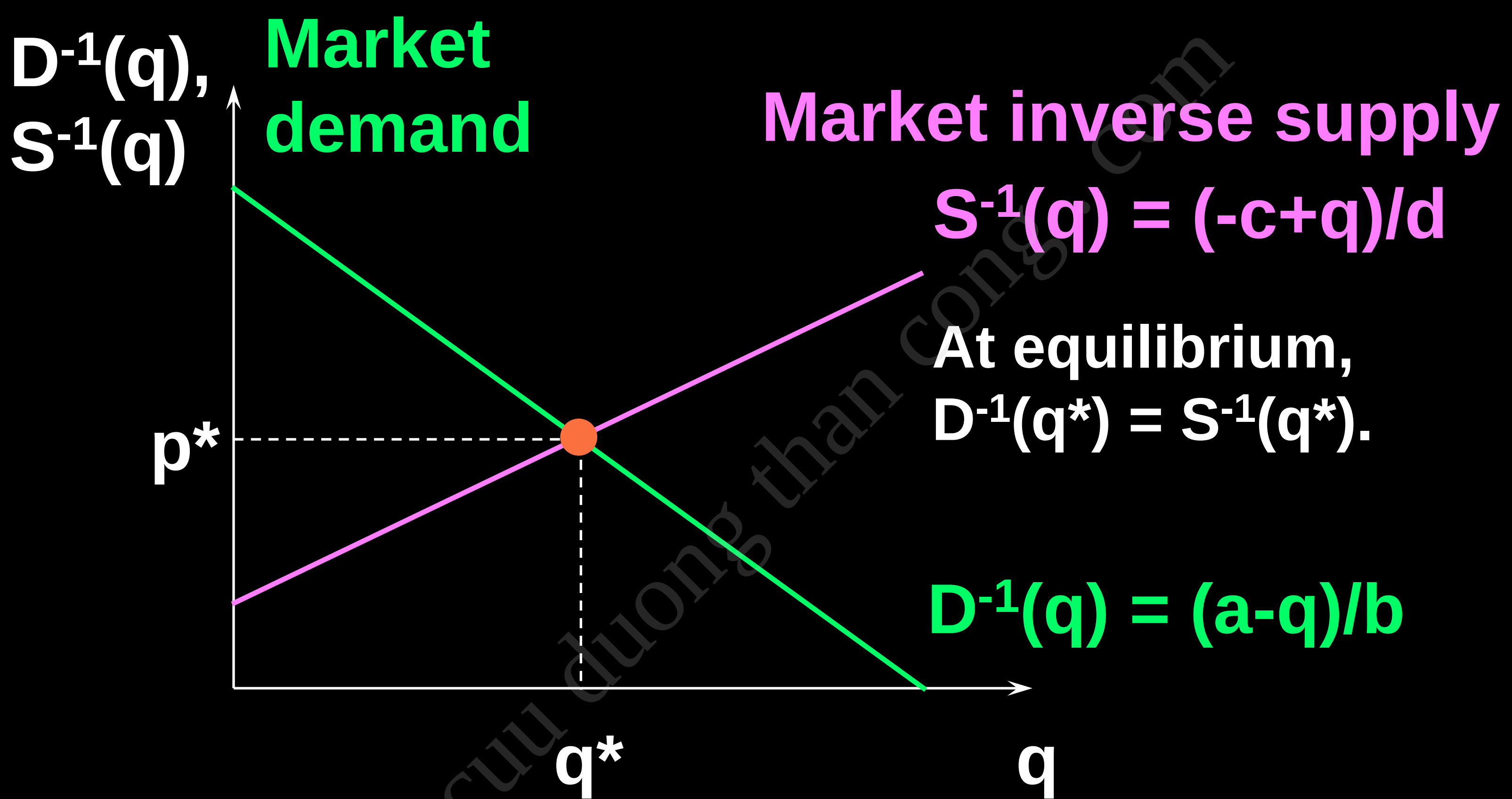
$$q = D(p) = a - bp \Leftrightarrow p = \frac{a - q}{b} = D^{-1}(q),$$

the equation of the inverse market demand curve. And

$$q = S(p) = c + dp \Leftrightarrow p = \frac{-c + q}{d} = S^{-1}(q),$$

the equation of the inverse market supply curve.

Market Equilibrium



Market Equilibrium

$$p = D^{-1}(q) = \frac{a - q}{b} \quad \text{and} \quad p = S^{-1}(q) = \frac{-c + q}{d}.$$

At the equilibrium quantity q*, $D^{-1}(p^*) = S^{-1}(p^*)$.
That is,

$$\frac{a - q^*}{b} = \frac{-c + q^*}{d}$$

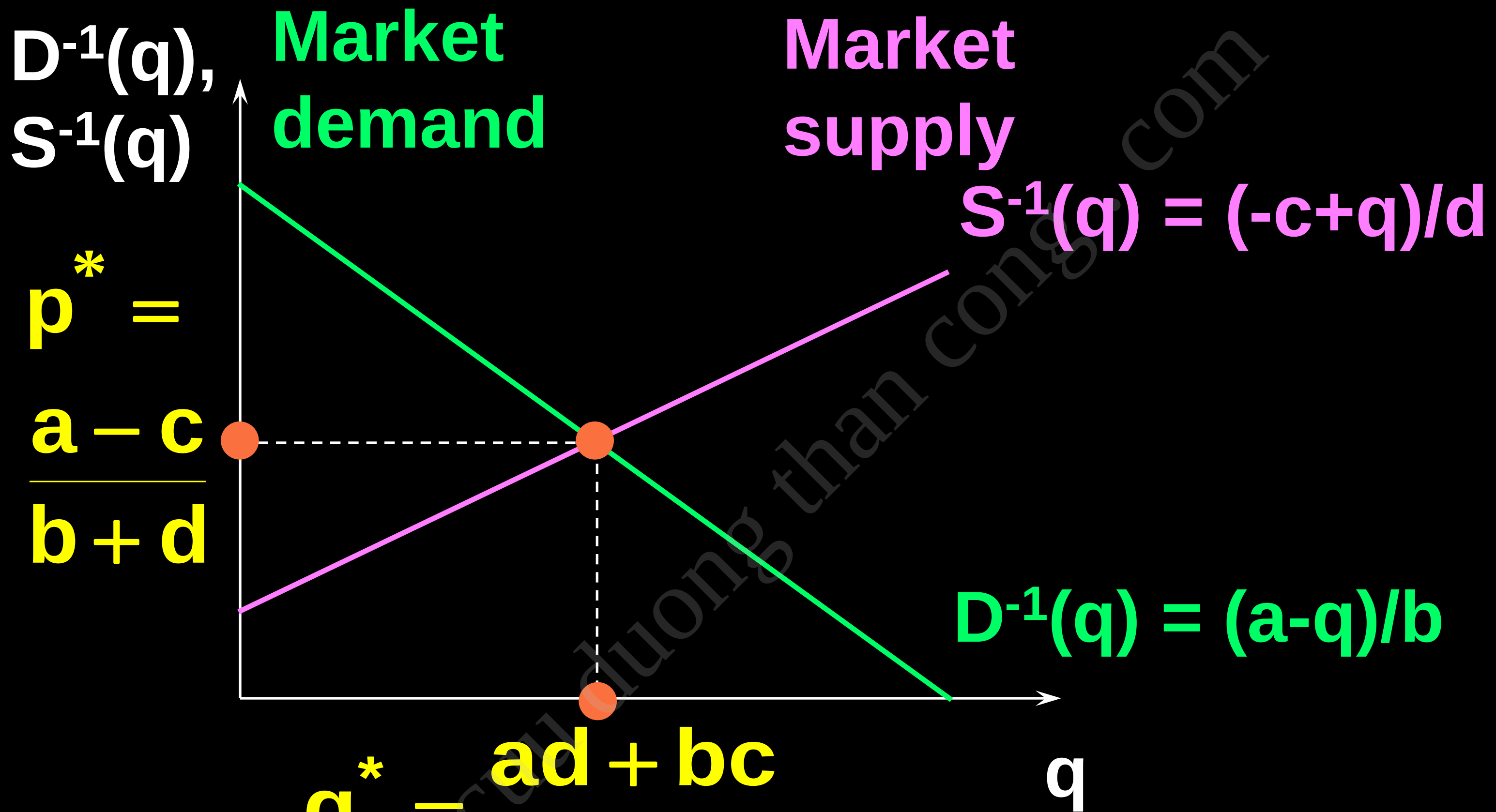
which gives

$$q^* = \frac{ad + bc}{b + d}$$

and

$$p^* = D^{-1}(q^*) = S^{-1}(q^*) = \frac{a - c}{b + d}.$$

Market Equilibrium



$$p^* = \frac{a-c}{b+d}$$

$$q^* = \frac{ad+bc}{b+d}$$

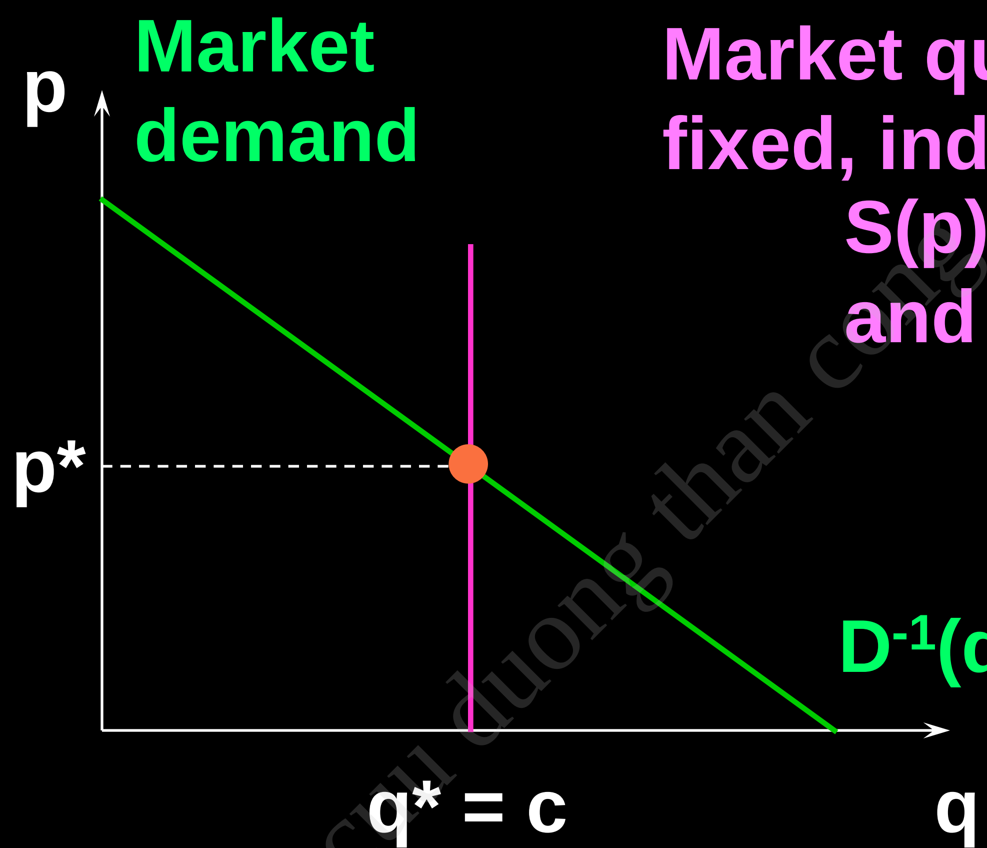
Market Equilibrium

- Two special cases:
  - quantity supplied is fixed, independent of the market price, and
  - quantity supplied is extremely sensitive to the market price.

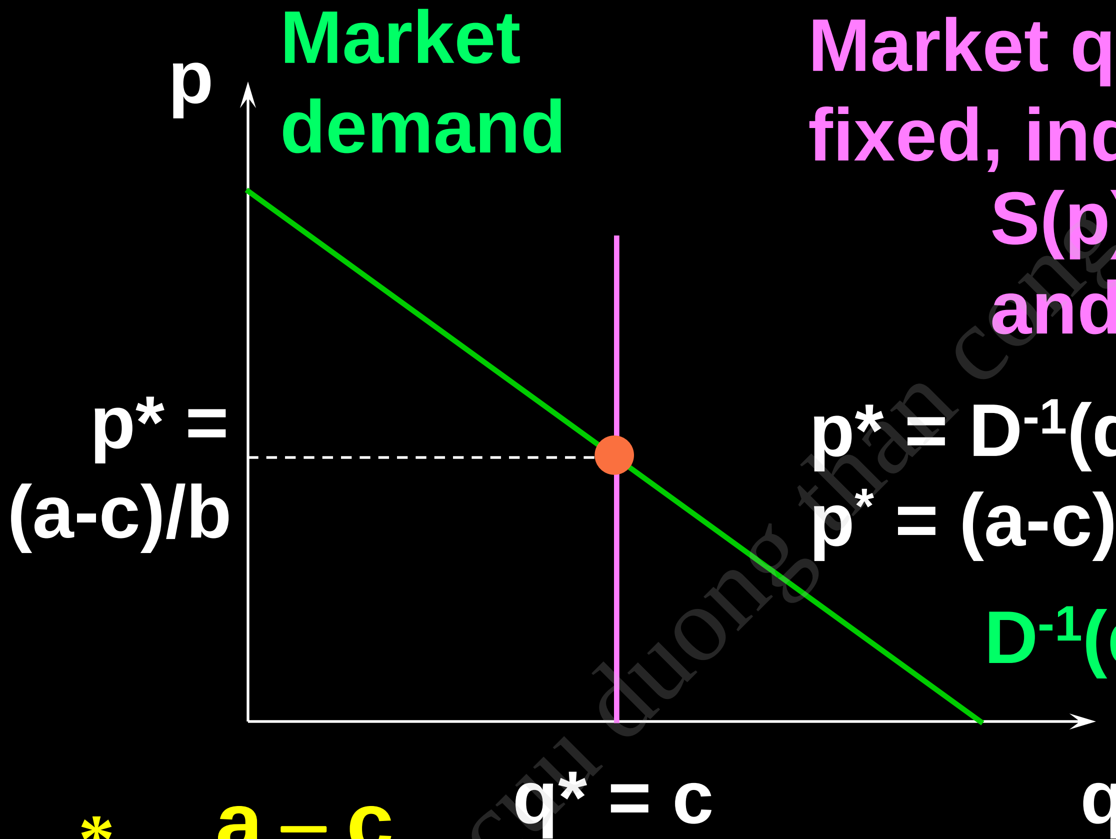
Market Equilibrium

Market quantity supplied is fixed, independent of price.

$S(p) = c+dp$, so $d=0$ and $S(p) \equiv c$.



Market Equilibrium



Market quantity supplied is fixed, independent of price.

S(p) = c+dp, so d=0 and S(p) ≡ c.

p* = D^{-1}(q*); that is,
p* = (a-c)/b.

D^{-1}(q) = (a-q)/b

$$p^* = \frac{a - c}{b + d}$$

$$q^* = \frac{ad + bc}{b + d}$$

with d = 0 give

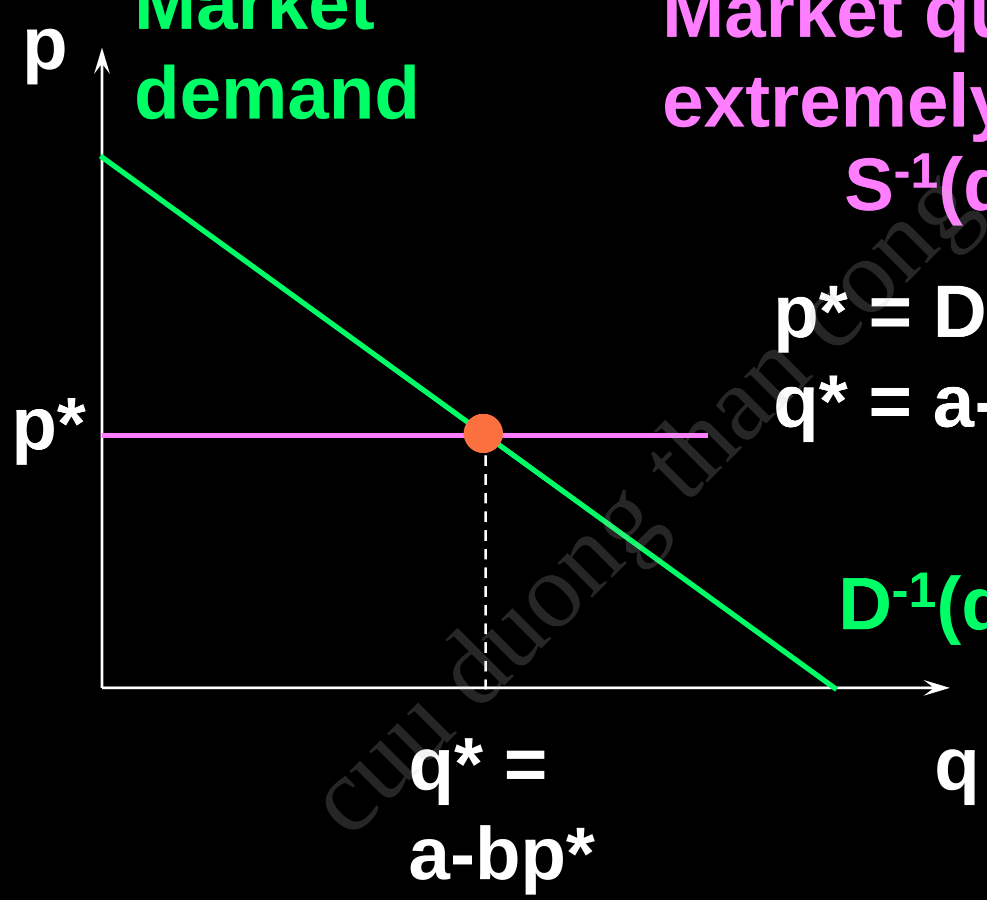
$$p^* = \frac{a - c}{b}$$

$$q^* = c.$$

Market Equilibrium

- Two special cases are
  - when quantity supplied is fixed, independent of the market price, and
  - when quantity supplied is extremely sensitive to the market price.

Market Equilibrium



Market quantity supplied is extremely sensitive to price.

$S^{-1}(q) = p^*$.

$p^* = D^{-1}(q^*) = (a-q^*)/b$ so

$q^* = a-bp^*$

$D^{-1}(q) = (a-q)/b$

Quantity Taxes

- A quantity tax levied at a rate of $t is a tax of $t paid on each unit traded.
- If the tax is levied on sellers then it is an **excise tax**.
- If the tax is levied on buyers then it is a **sales tax**.

Quantity Taxes

- What is the effect of a quantity tax on a market's equilibrium?
- How are prices affected?
- How is the quantity traded affected?
- Who pays the tax?
- How are gains-to-trade altered?

Quantity Taxes

- A tax rate t makes the price paid by buyers, p_b, higher by t from the price received by sellers, p_s.

$$p_b - p_s = t$$

Quantity Taxes

- **Even with a tax the market must clear.**
- **I.e. quantity demanded by buyers at price p_b must equal quantity supplied by sellers at price p_s.**

$$D(p_b) = S(p_s)$$

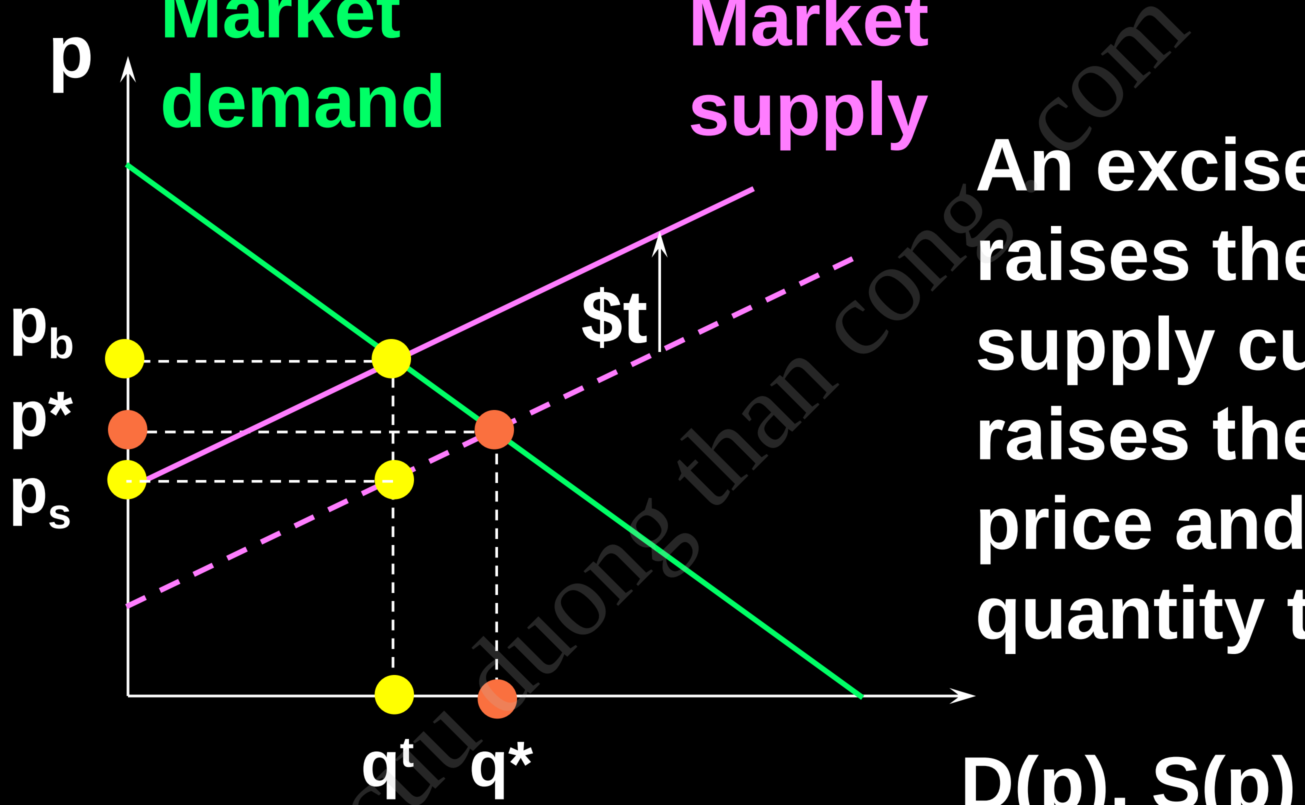
Quantity Taxes

$p_b - p_s = t$ and $D(p_b) = S(p_s)$

describe the market's equilibrium.
Notice that these two conditions apply no matter if the tax is levied on sellers or on buyers.

Hence, a sales tax rate $t has the same effect as an excise tax rate $t.

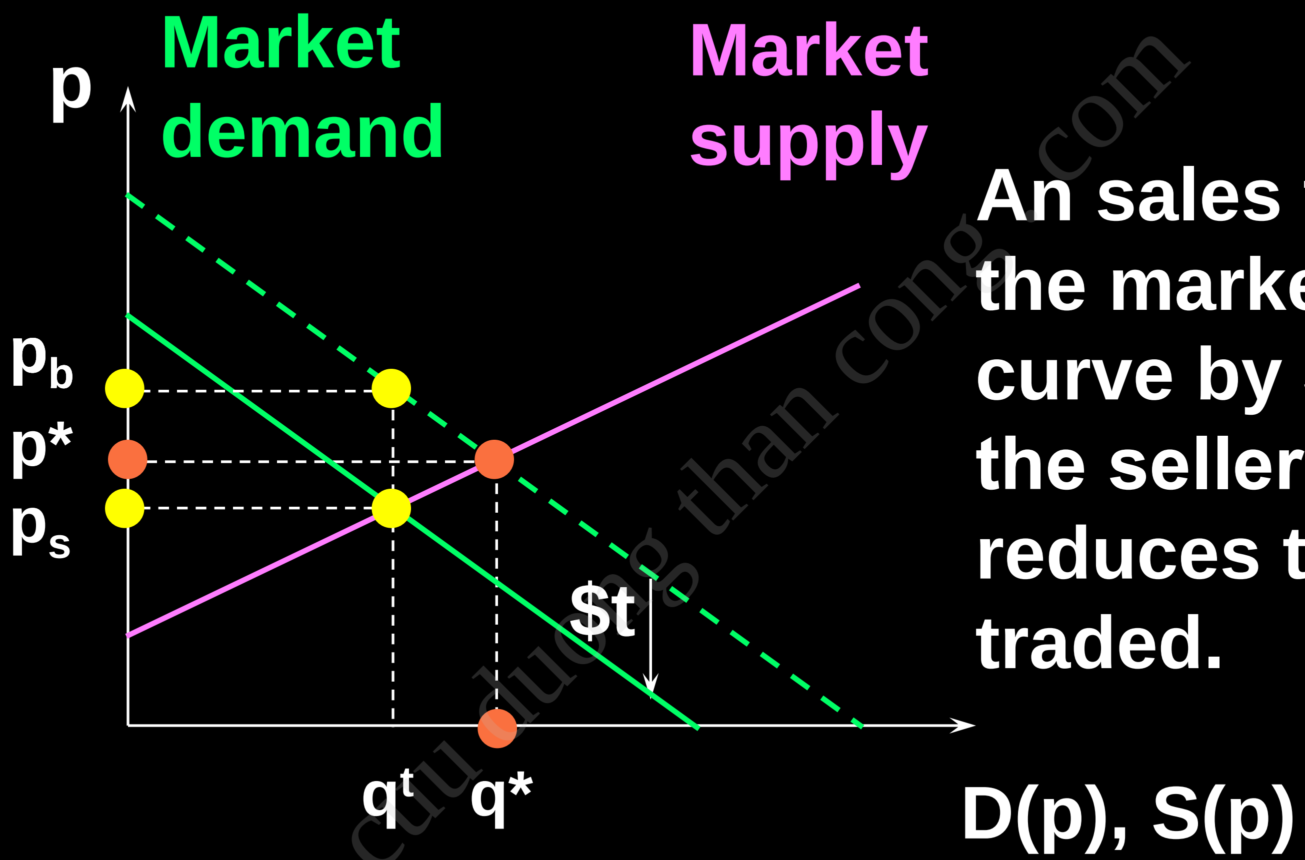
Quantity Taxes & Market Equilibrium



An excise tax raises the market supply curve by $t, raises the buyers' price and lowers the quantity traded.

And sellers receive only $p_s = p_b - t$.

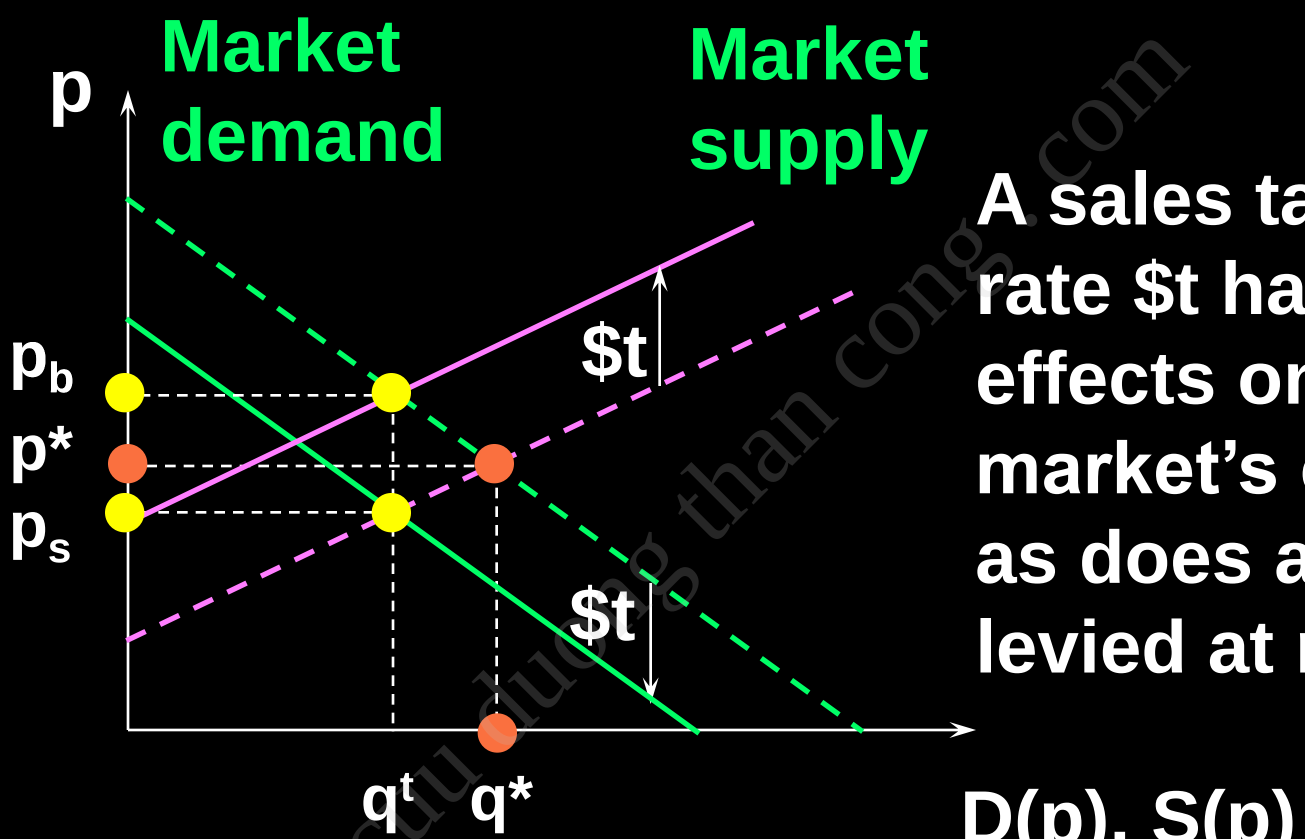
Quantity Taxes & Market Equilibrium



An sales tax lowers the market demand curve by $t, lowers the sellers' price and reduces the quantity traded.

And buyers pay $p_b = p_s + t$.

Quantity Taxes & Market Equilibrium

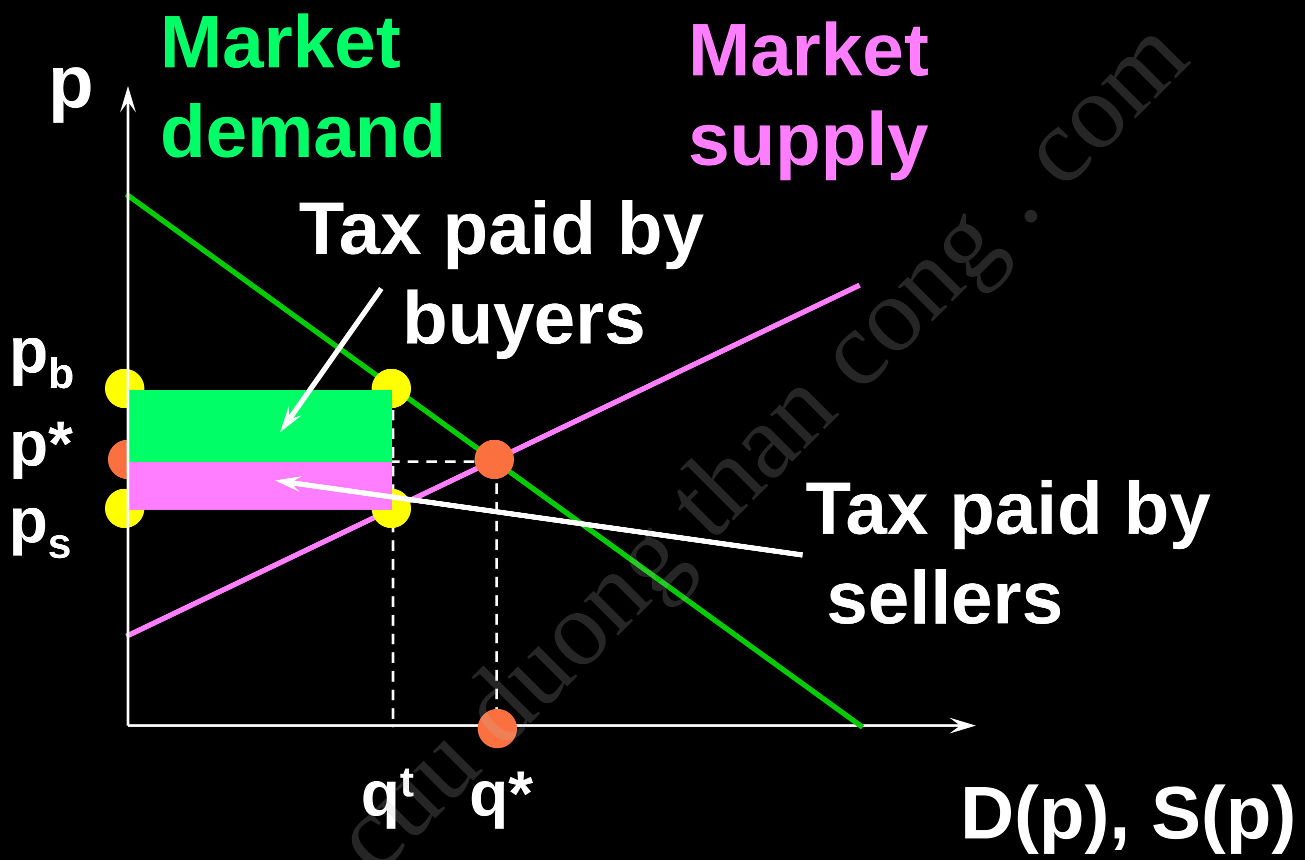


A sales tax levied at rate $t has the same effects on the market's equilibrium as does an excise tax levied at rate $t.

Quantity Taxes & Market Equilibrium

- **Who pays the tax of $t per unit traded?**
- **The division of the $t between buyers and sellers is the incidence of the tax.**

Quantity Taxes & Market Equilibrium



Quantity Taxes & Market Equilibrium

- E.g. suppose the market demand and supply curves are linear.

$$D(p_b) = a - bp_b$$

$$S(p_s) = c + dp_s$$

Quantity Taxes & Market Equilibrium

$$D(p_b) = a - bp_b \quad \text{and} \quad S(p_s) = c + dp_s.$$

With the tax, the market equilibrium satisfies

$$p_b = p_s + t \quad \text{and} \quad D(p_b) = S(p_s) \quad \text{so}$$

$$p_b = p_s + t \quad \text{and} \quad a - bp_b = c + dp_s.$$

Substituting for p_b gives

$$a - b(p_s + t) = c + dp_s \Rightarrow p_s = \frac{a - c - bt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$ and $$p_b = p_s + t$$ give

$$p_b = \frac{a - c + dt}{b + d}$$

The quantity traded at equilibrium is

$$q^t = D(p_b) = S(p_s)$$

$$= a + bp_b = \frac{ad + bc - bdt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

As $t \rightarrow 0$, p_s and $p_b \rightarrow \dfrac{a - c}{b + d} = p^*$, the equilibrium price if there is no tax ($t = 0$) and $q^t \rightarrow \dfrac{ad + bc}{b + d}$, the quantity traded at equilibrium when there is no tax.

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

As t increases, p_s falls,

p_b rises,

and q^t falls.

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The tax paid per unit by the buyer is

$$p_b - p^* = \frac{a - c + dt}{b + d} - \frac{a - c}{b + d} = \frac{dt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

The tax paid per unit by the buyer is

$$p_b - p^* = \frac{a - c + dt}{b + d} - \frac{a - c}{b + d} = \frac{dt}{b + d}.$$

The tax paid per unit by the seller is

$$p^* - p_s = \frac{a - c}{b + d} - \frac{a - c - bt}{b + d} = \frac{bt}{b + d}.$$

Quantity Taxes & Market Equilibrium

$$p_s = \frac{a - c - bt}{b + d}$$

$$p_b = \frac{a - c + dt}{b + d}$$

$$q^t = \frac{ad + bc - bdt}{b + d}$$

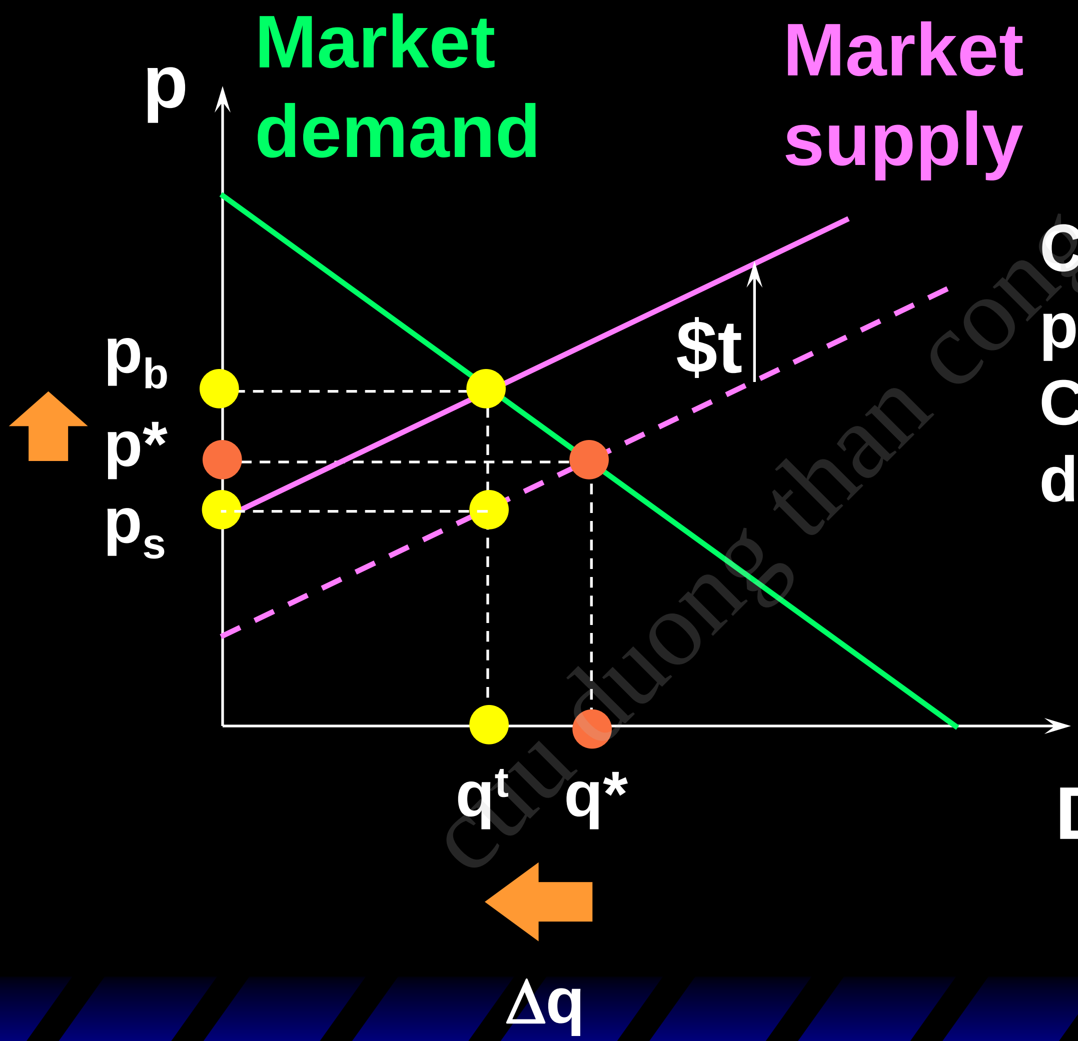
The total tax paid (by buyers and sellers combined) is

$$T = tq^t = t\frac{ad + bc - bdt}{b + d}.$$

Tax Incidence and Own-Price Elasticities

- **The incidence of a quantity tax depends upon the own-price elasticities of demand and supply.**

Tax Incidence and Own-Price Elasticities



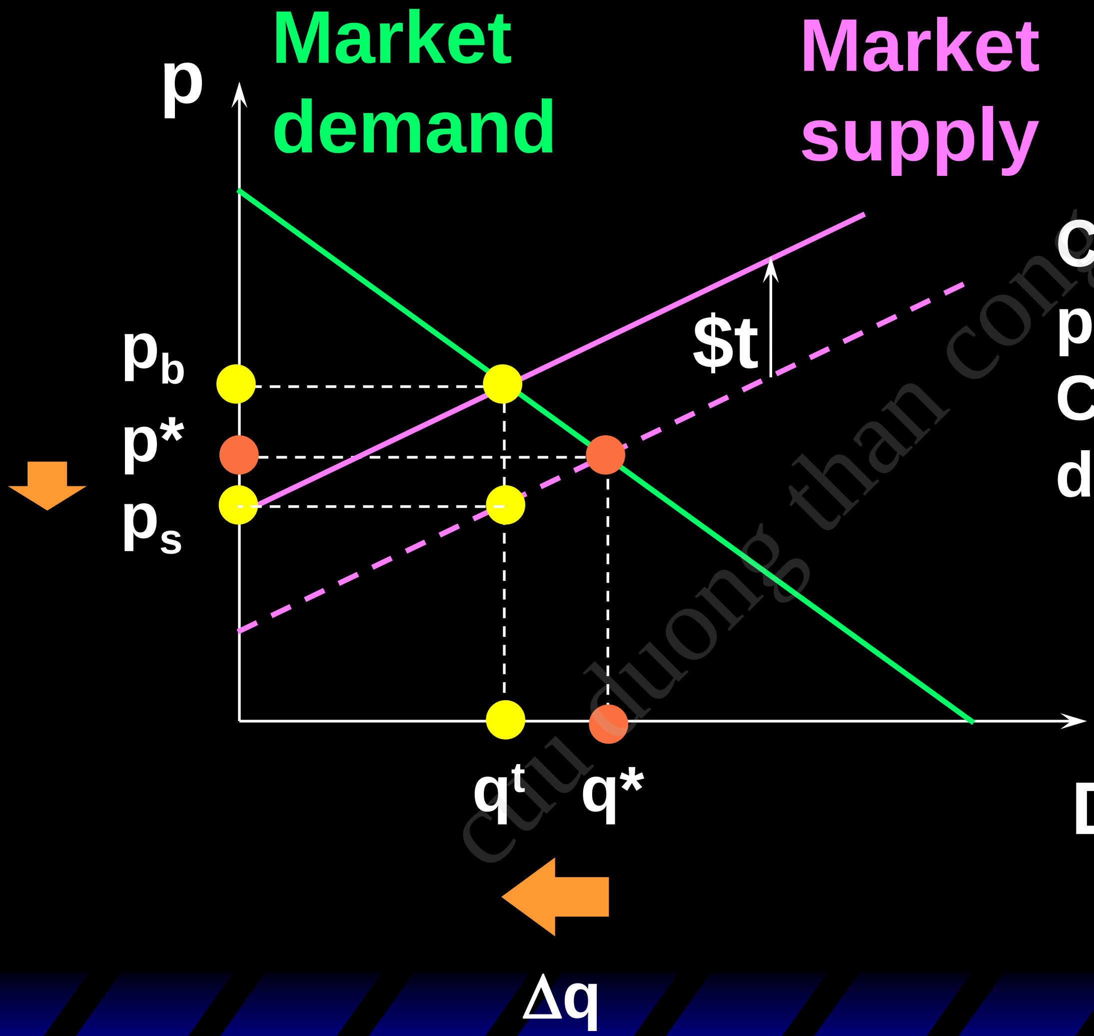
Change to buyers' price is p_b - p*.
Change to quantity demanded is Δq.

Tax Incidence and Own-Price Elasticities

Around p = p* the own-price elasticity of demand is approximately

$$\varepsilon_D \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_b - p^*}{p^*}} \quad \Rightarrow \quad p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}.$$

Tax Incidence and Own-Price Elasticities



Change to sellers' price is p_s - p*.
Change to quantity demanded is Δq.

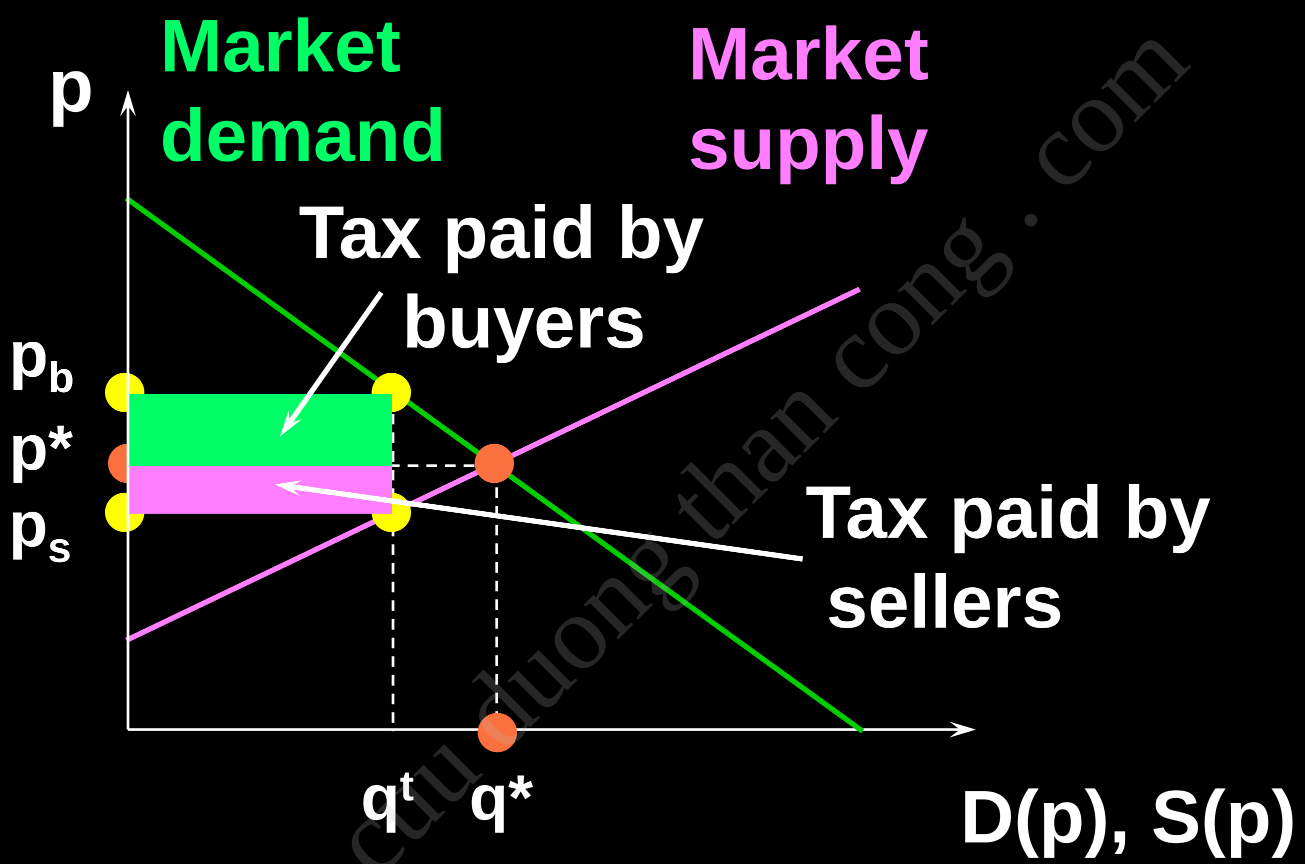
D(p), S(p)

Tax Incidence and Own-Price Elasticities

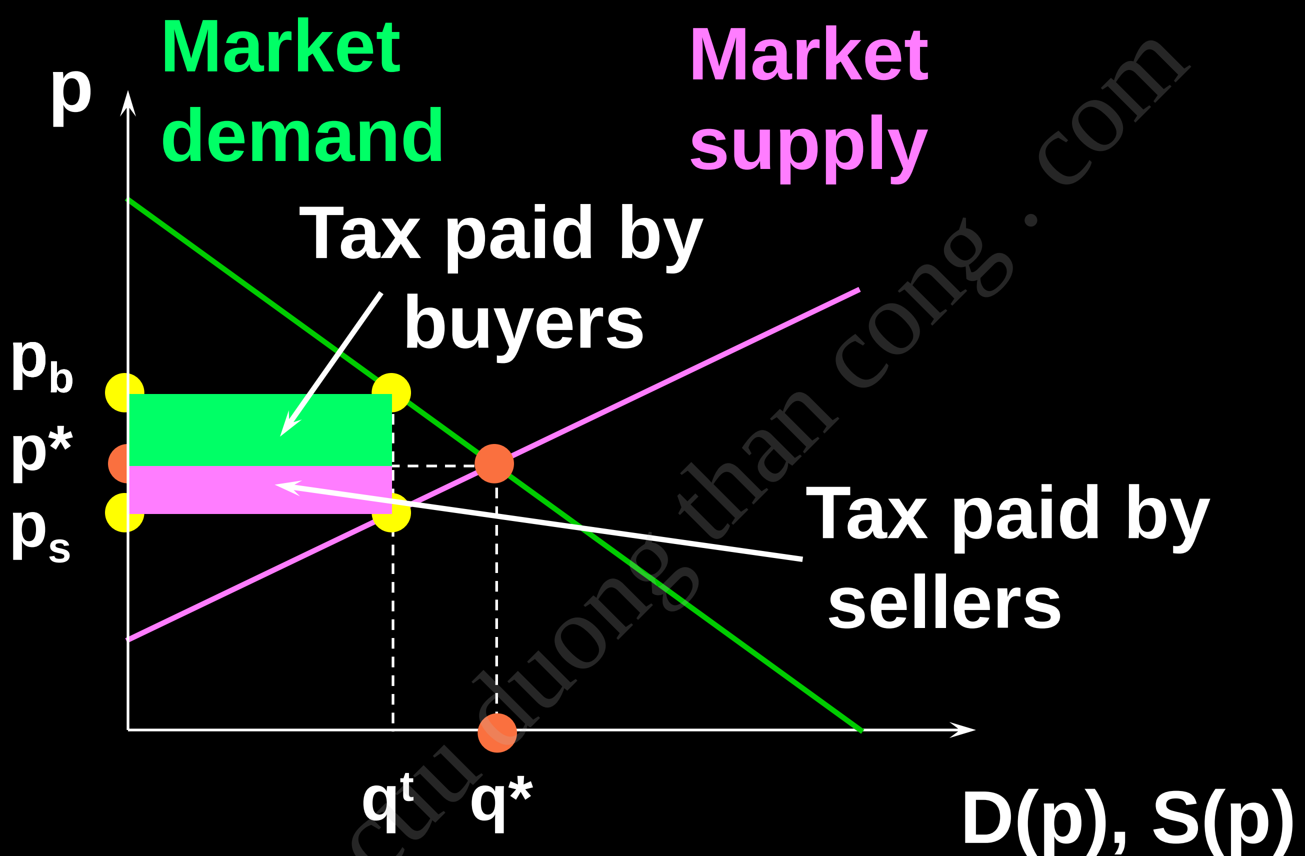
Around p = p* the own-price elasticity of supply is approximately

$$\varepsilon_S \approx \frac{\dfrac{\Delta q}{q^*}}{\dfrac{p_s - p^*}{p^*}} \quad \Rightarrow \quad p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

Tax Incidence and Own-Price Elasticities



Tax Incidence and Own-Price Elasticities



$$\text{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

Tax Incidence and Own-Price Elasticities

$$\text{Tax incidence} = \frac{p_b - p^*}{p^* - p_s}.$$

$$p_b - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_D \times q^*}. \qquad p_s - p^* \approx \frac{\Delta q \times p^*}{\varepsilon_S \times q^*}.$$

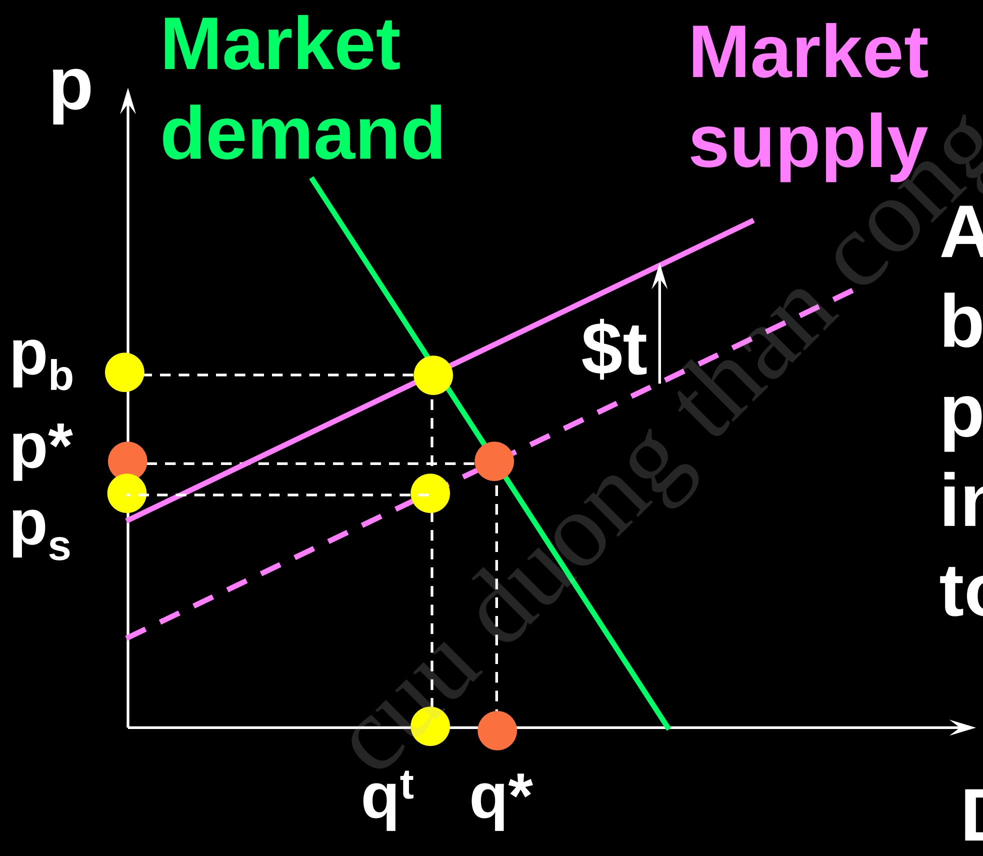
So $$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

Tax Incidence and Own-Price Elasticities

Tax incidence is $$\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$$

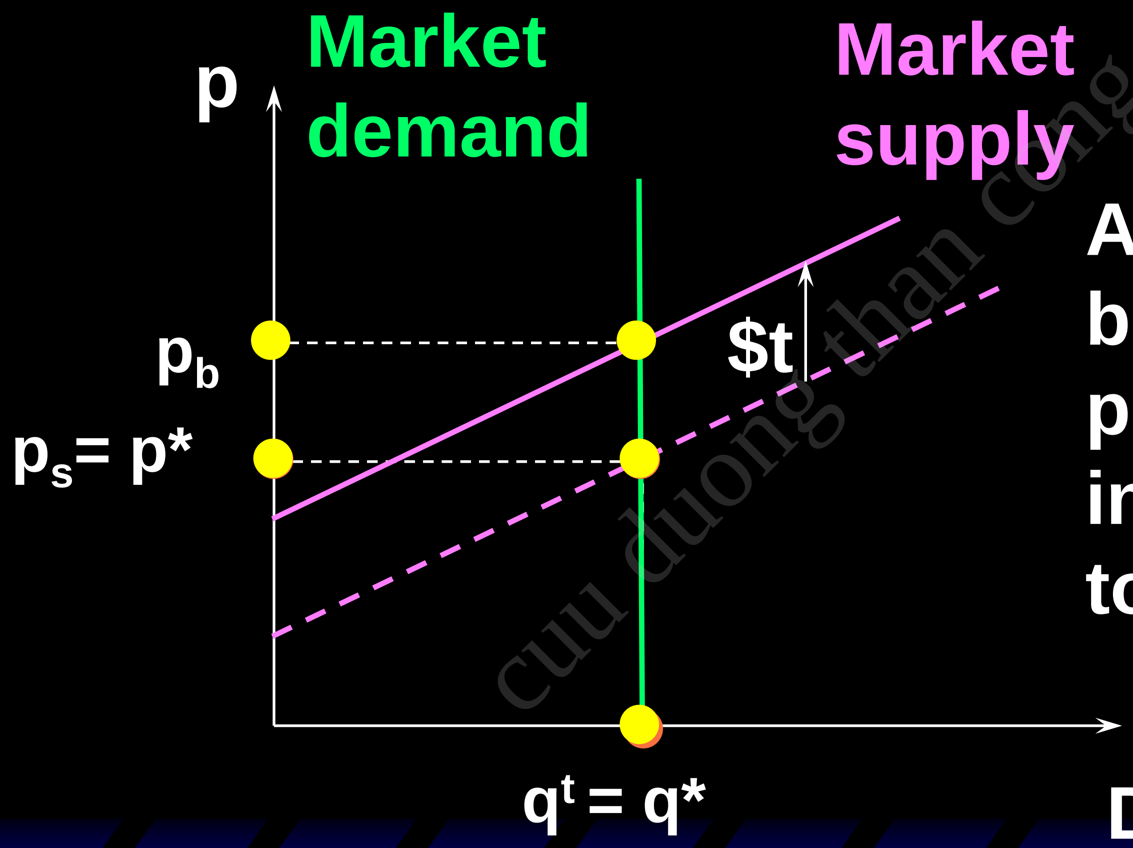
The fraction of a $t quantity tax paid by buyers rises as supply becomes more own-price elastic or as demand becomes less own-price elastic.

Tax Incidence and Own-Price Elasticities



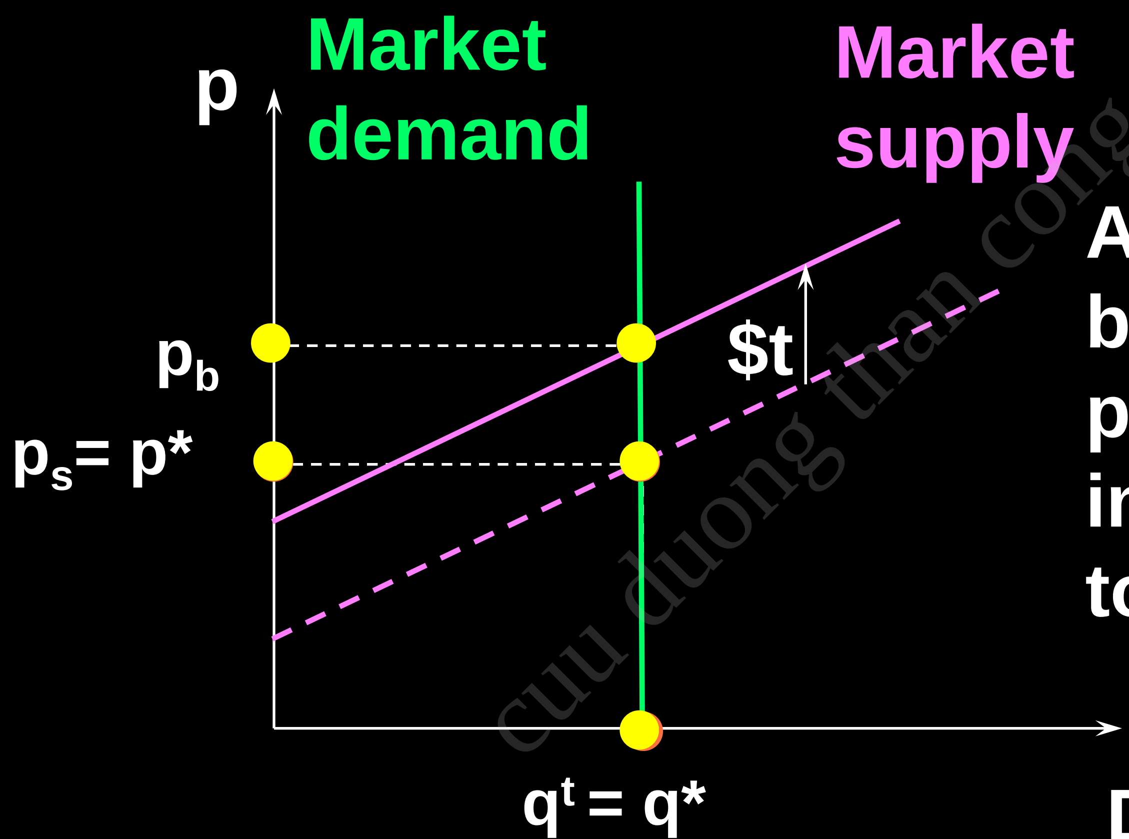
As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

Tax Incidence and Own-Price Elasticities



As market demand becomes less own-price elastic, tax incidence shifts more to the buyers.

When $\varepsilon_D = 0$, buyers pay the entire tax, even though it is levied on the sellers.

Tax Incidence and Own-Price Elasticities

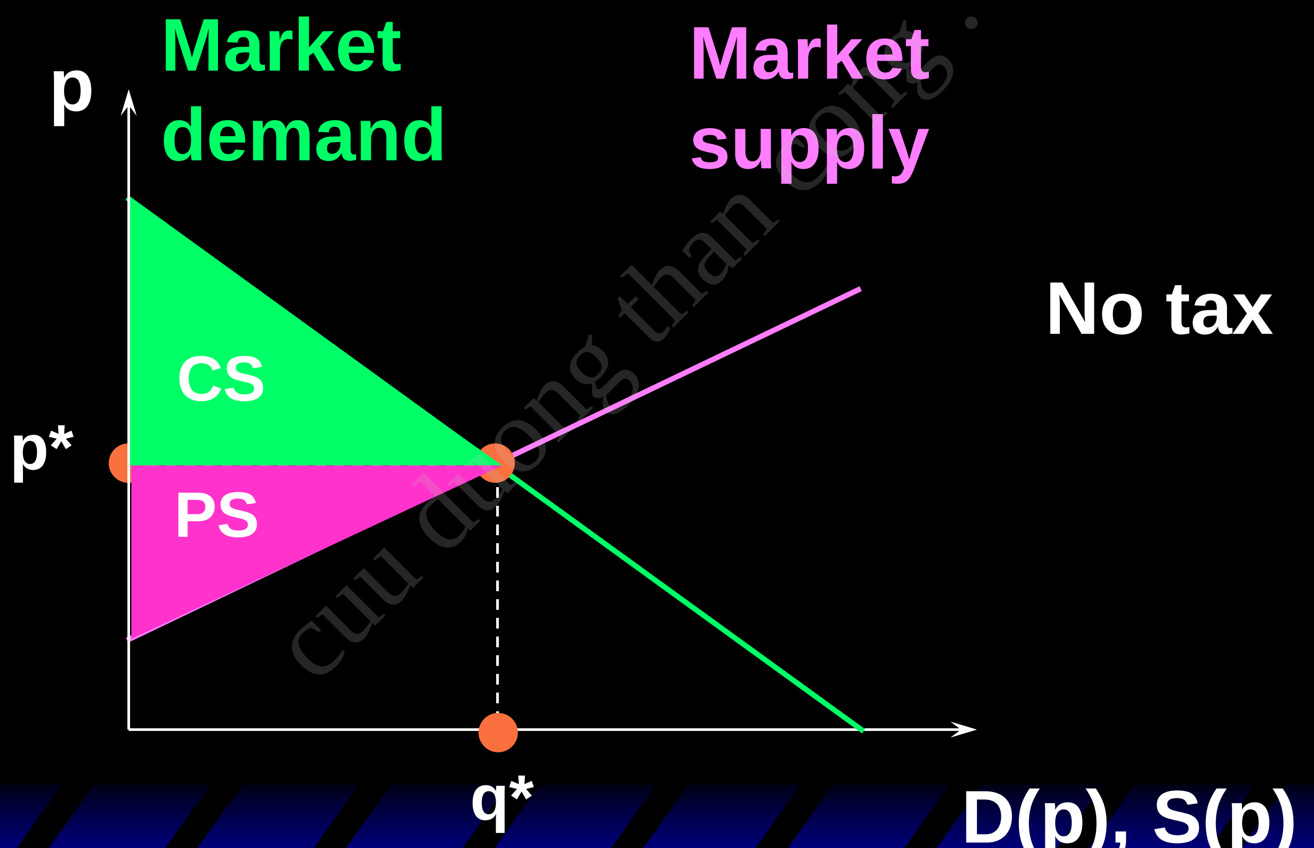
Tax incidence is $\frac{p_b - p^*}{p^* - p_s} \approx -\frac{\varepsilon_S}{\varepsilon_D}.$

Similarly, the fraction of a $t quantity tax paid by sellers rises as supply becomes less own-price elastic or as demand becomes more own-price elastic.

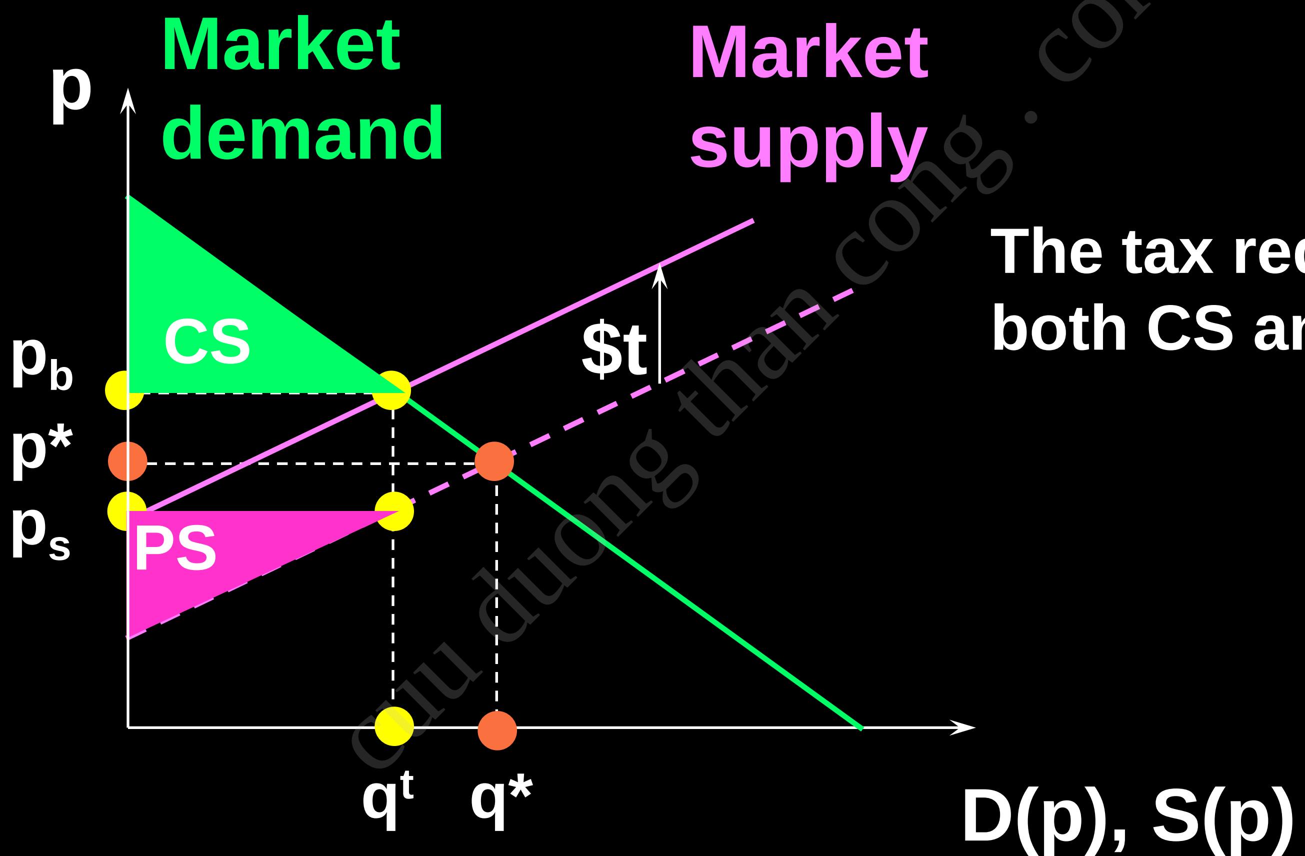
Deadweight Loss and Own-Price Elasticities

- **A quantity tax imposed on a competitive market reduces the quantity traded and so reduces gains-to-trade (*i.e.* the sum of Consumers' and Producers' Surpluses).**
- **The lost total surplus is the tax's deadweight loss, or excess burden.**

Deadweight Loss and Own-Price Elasticities

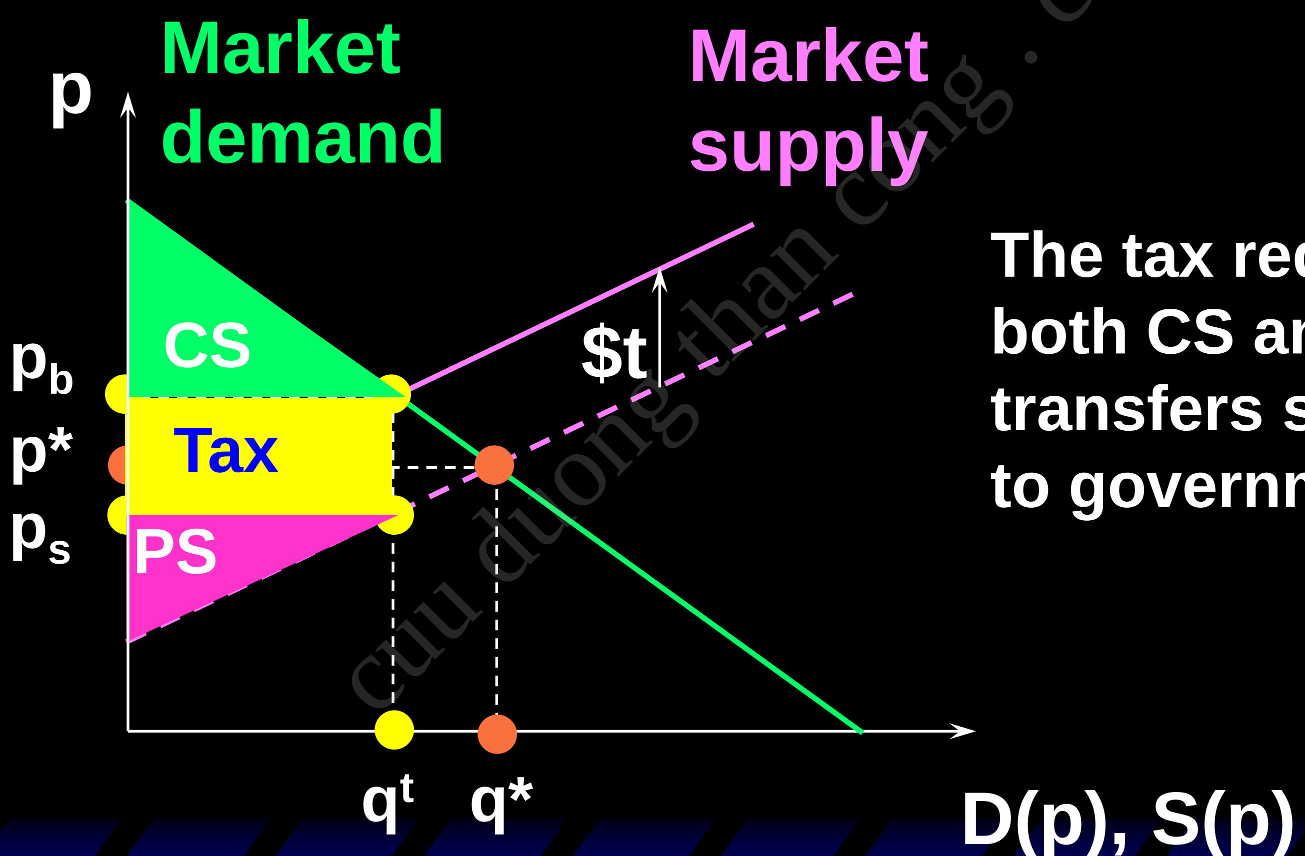


Deadweight Loss and Own-Price Elasticities



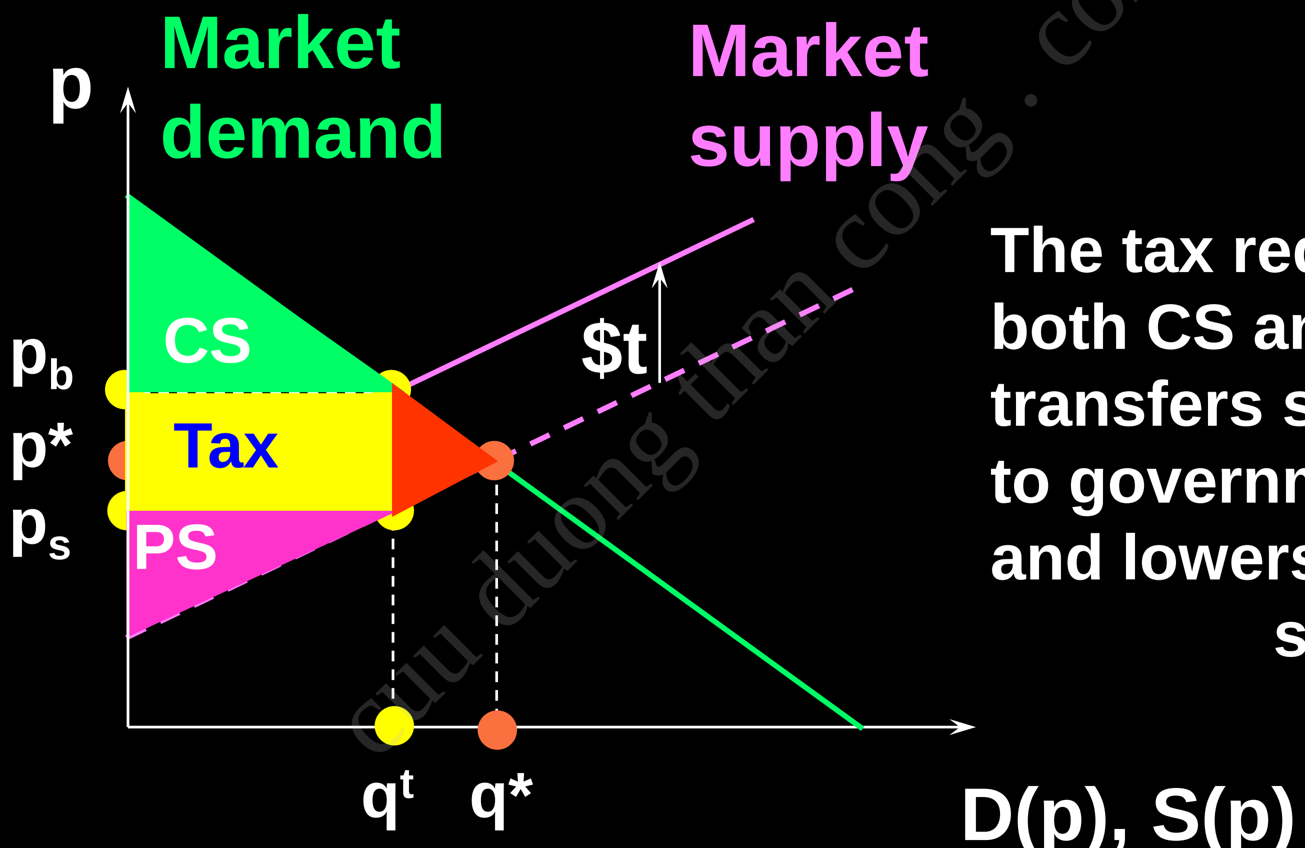
The tax reduces both CS and PS

Deadweight Loss and Own-Price Elasticities



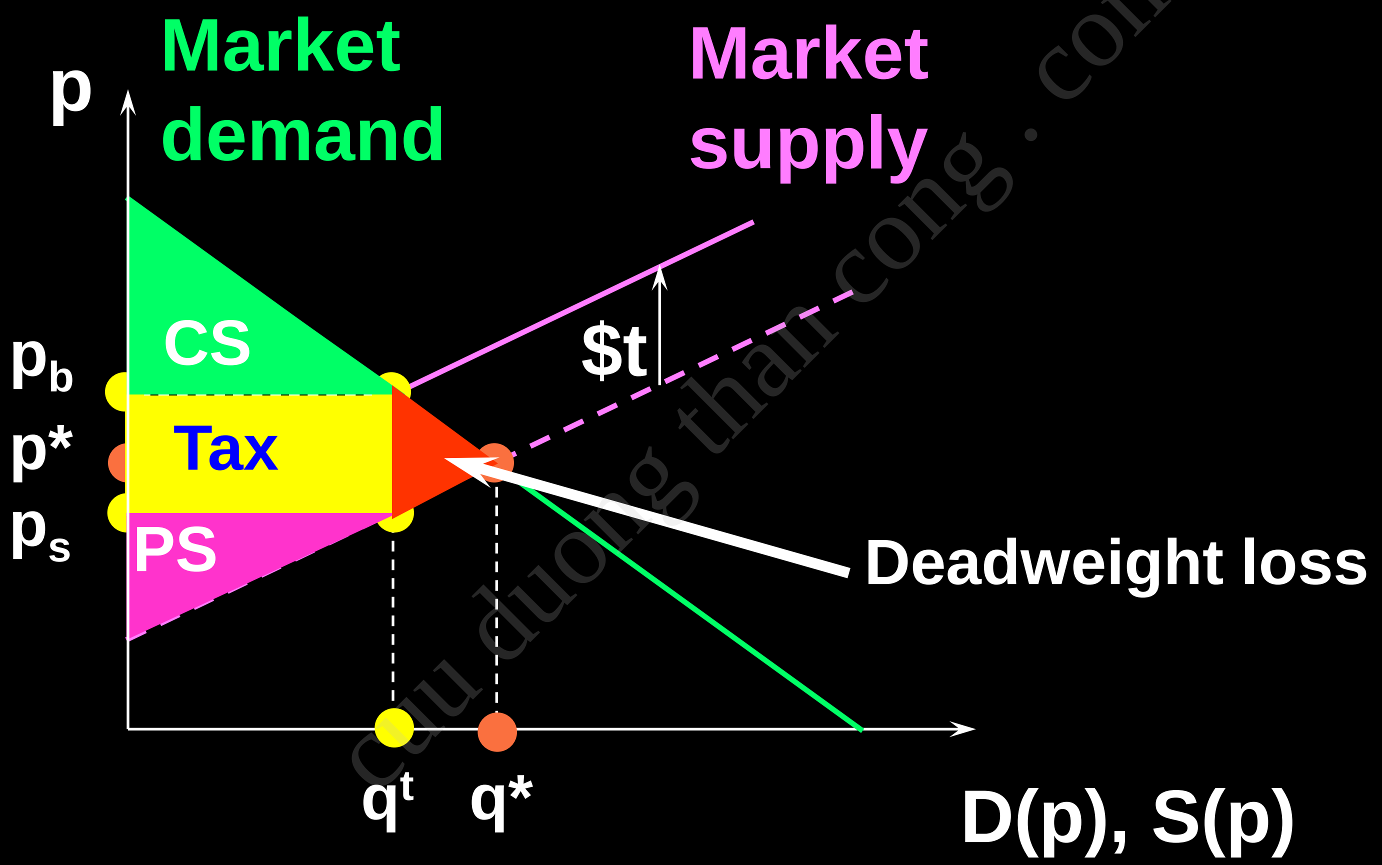
The tax reduces both CS and PS, transfers surplus to government

Deadweight Loss and Own-Price Elasticities

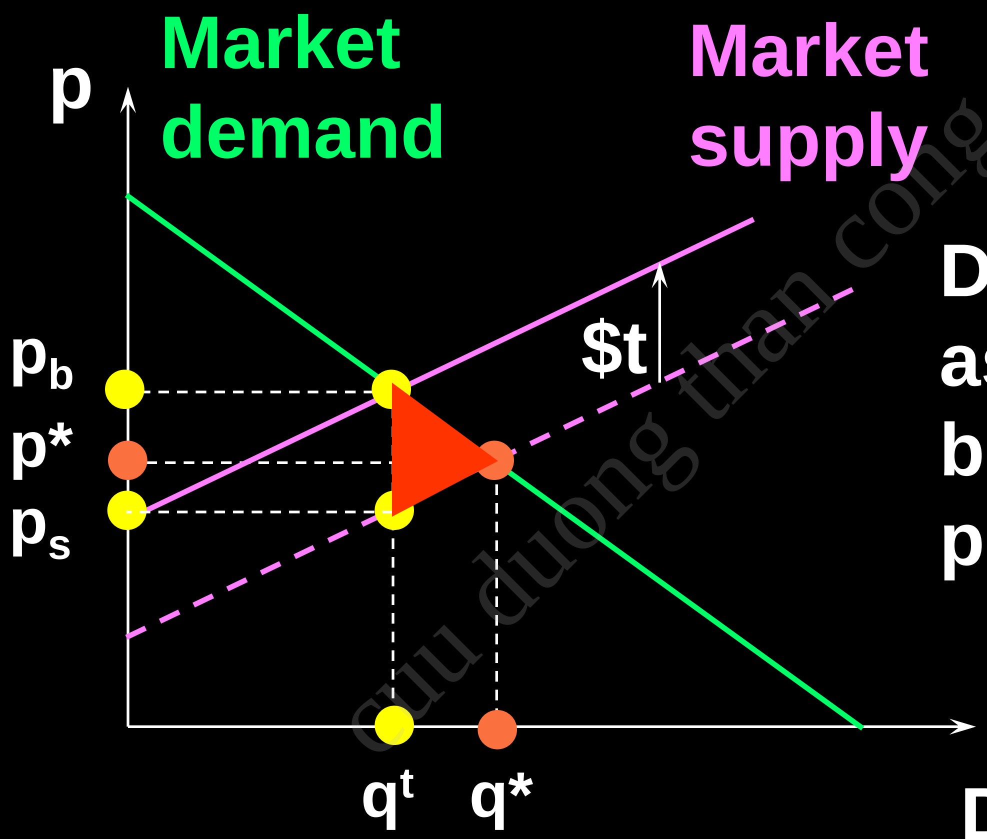


The tax reduces both CS and PS, transfers surplus to government, and lowers total surplus.

Deadweight Loss and Own-Price Elasticities

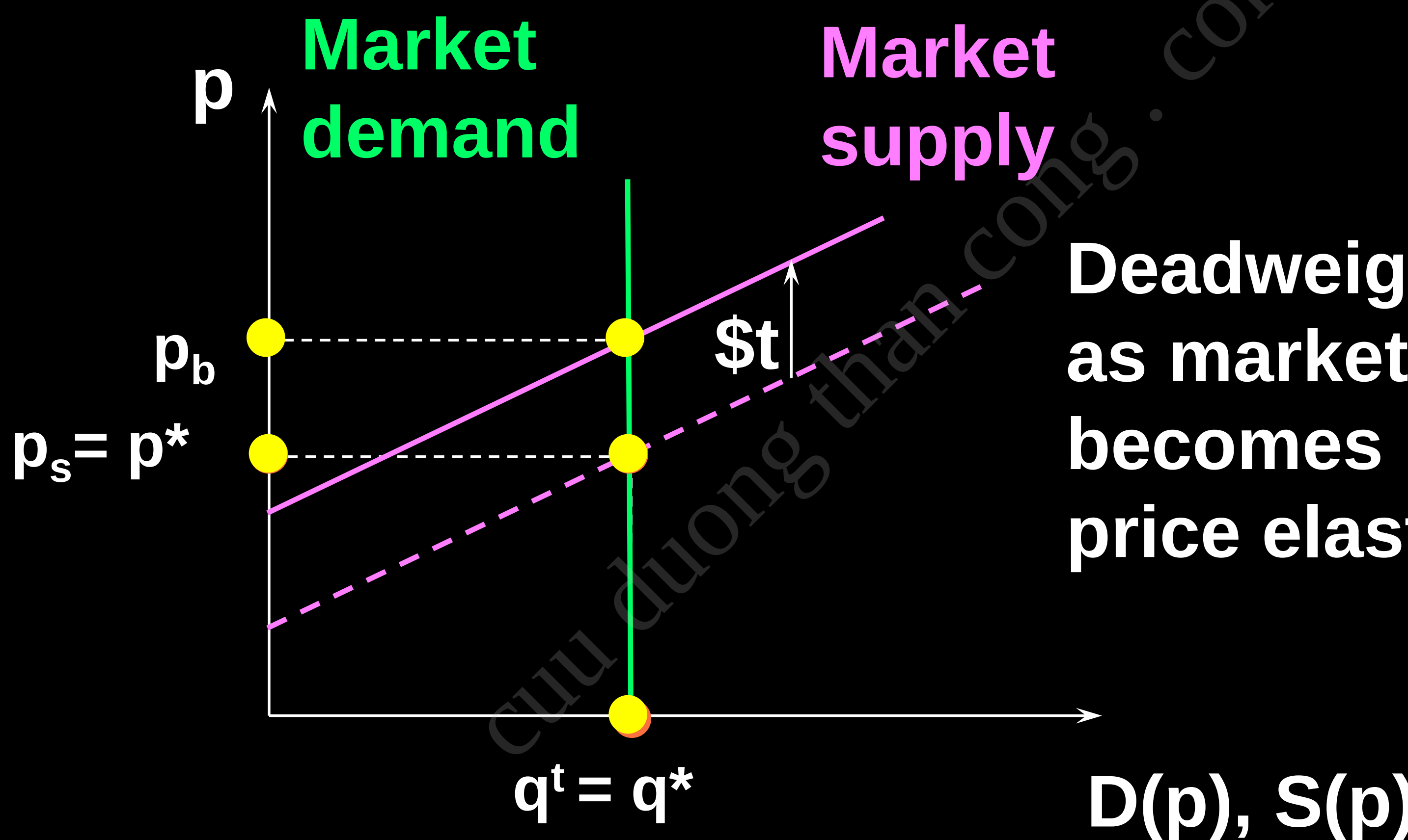


Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

Deadweight Loss and Own-Price Elasticities



Deadweight loss falls as market demand becomes less own-price elastic.

When $\varepsilon_D = 0$, the tax causes no deadweight loss.

Deadweight Loss and Own-Price Elasticities

- Deadweight loss due to a quantity tax rises as either market demand or market supply becomes more own-price elastic.
- If either $\varepsilon_D = 0$ or $\varepsilon_S = 0$ then the deadweight loss is zero.