# Chapter 1: Introduction

***Signal***: is defined any physical quantity that varies with time, space, or any other independent variable or variables

The example: The functions of signal:

$$S_1(t) = 5t\ ; \quad S_2(t) = 20t^2 \qquad (1.1.1)$$



$$S(x,y) = 3x + 2xy + 10y^2 \qquad (1.1.2)$$

Speech signal:

$$\sum_{i=1}^{N} A_i(t)\sin[2\pi F_i(t)t + \theta_i(t)] \qquad (1.1.3)$$

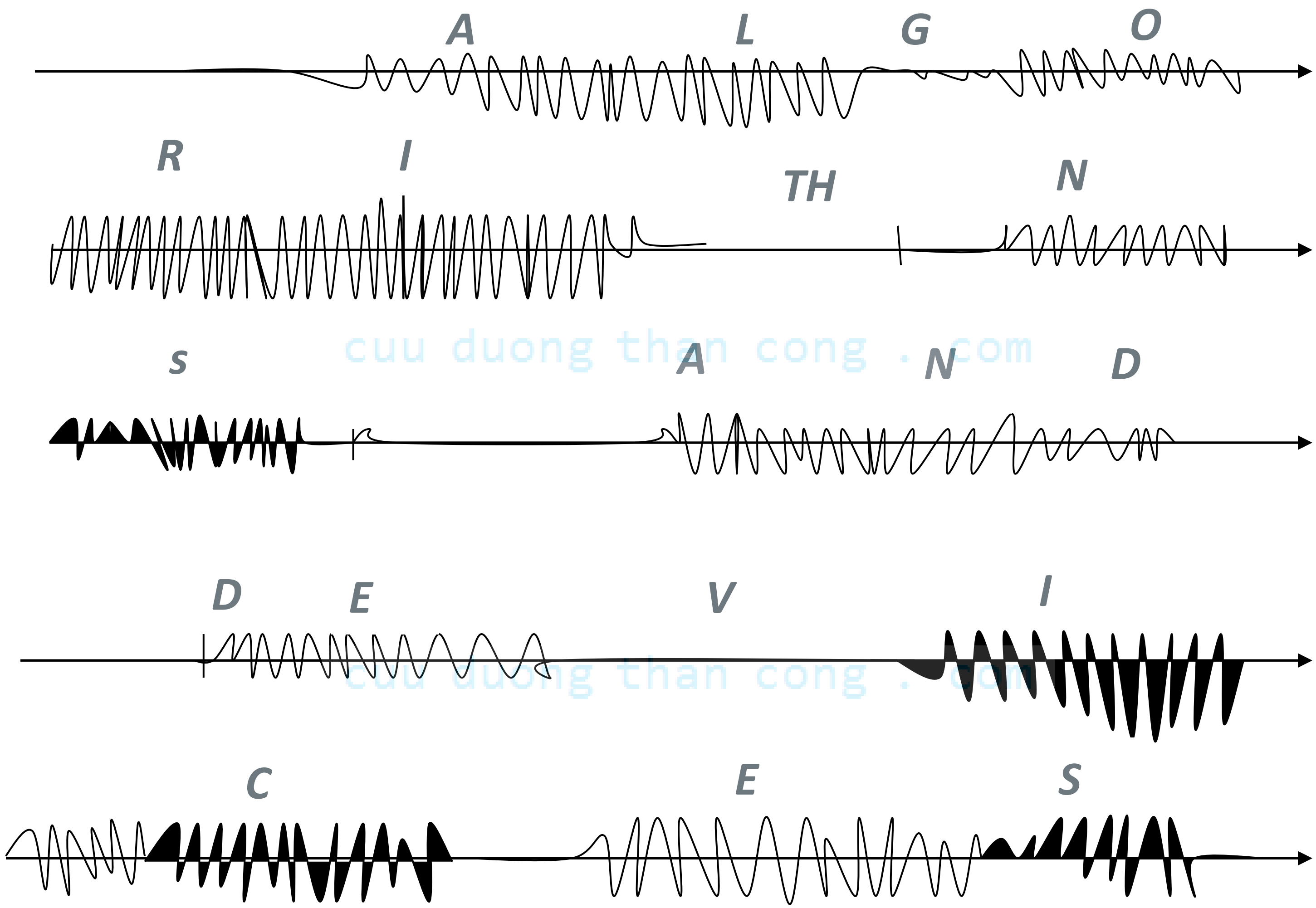


Where : { $A_i(t)$ } - amplitudes

{ $F_i(t)$ } - frequencies { $\theta_i(t)$ } - phases.

# 1.1: Signals, systems, and signal processing

Figure 1.1. Example of a speech signal



# 1.1: Signals, systems, and signal processing

- *Electrocardiogram* (ECG) signal provides a doctor with information about the condition of the patient' s heart.



- *Electroencephalogram* (ECG) signal provides information about the activity of the brain.



- ***System*** may be defined as a physical device that performs an operation on a signal.

# 1.1: Signals, systems, and signal processing

- A *filter* used to reduce the *noise* and *interference* corrupting a desired in formation-bearing signal in called a ***system***.
- When we pass a signal through a system, as in filtering, we say that we have processed the signal (***signal processing***).
- If the operation is linear, the system is called ***linear***. If the operation on the signal is nonlinear, the system is said to be ***nonlinear***…



# 1.1 : Basic Elements of a Digital Signal Processing System

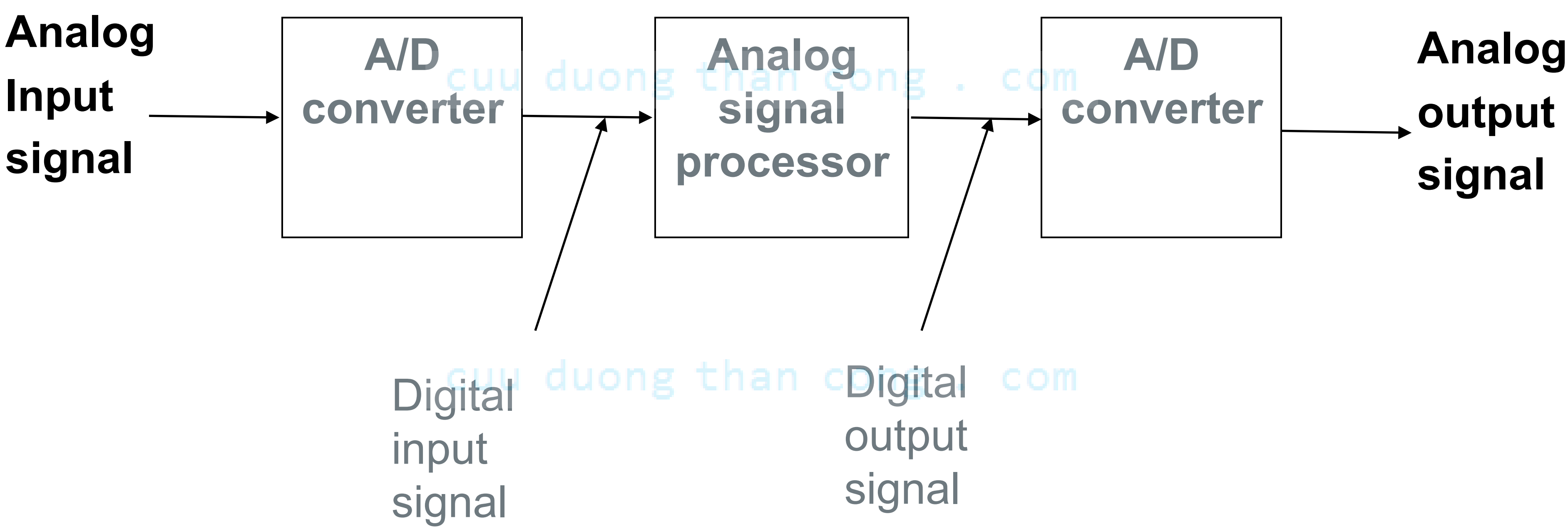
- To broaden the definition of a system to include not only physical devices (***hardware)***, but also ***software*** realizations of operations on a signal.



- The method or set of rule for implementing the system by a program that performs the corresponding mathematical operations, is called an ***algorithm.***

# 1.1 : Basic Elements of a Digital Signal Processing System

Figure 1.3 Block diagram of a digital processing system



# 1.1 : Basic Elements of a Digital Signal Processing System

- To perform the processing digitally, there is a need for an interface between the analog signal and the digital processor, in called an ***analog- to-digital (A/D) converter***.

- We must provide another interface from the digital domain to the analog domain, is called a ***digital-to-analog (D/A) converter***.

# 1.2 Classification of Signals

- For example

  If $S_k(t)$, $k = 1,2,3$, denotes the electrical signal from the *kth* sensor as a function of time

$$S_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

as ***a multichannel signal***.

- If the signal is a function of a single independent variable, the signal is called ***a one-dimensional signal.***
- A signal is called ***M-dimensional*** if its value is a function of *M* independent variables.

# 1.2.2 Continuous-Time Versus Discrete-Time Signal

- ***Continuous -time signal*** or ***analog signal*** are defined for every value of time they take on values in the continuous interval (*a*,*b*).



- ***Discrete-time signal*** are defined only at certain specific values of time .

- The signal $x(t_n) = e^{-|t_n|}$



  $n = 0, \pm 1, \pm 2, \ldots$ provides an example of a discrete-time signal.

# 1.2.2 Continuous-Time Versus Discrete-Time Signal

- For example :

$$X(n) = \begin{cases} 0.8^n, & \text{if } n \geq 0 \\ 0, & \text{otherwise} \end{cases} \qquad (1.2.1)$$

is a discrete-time signal which is represented graphically as

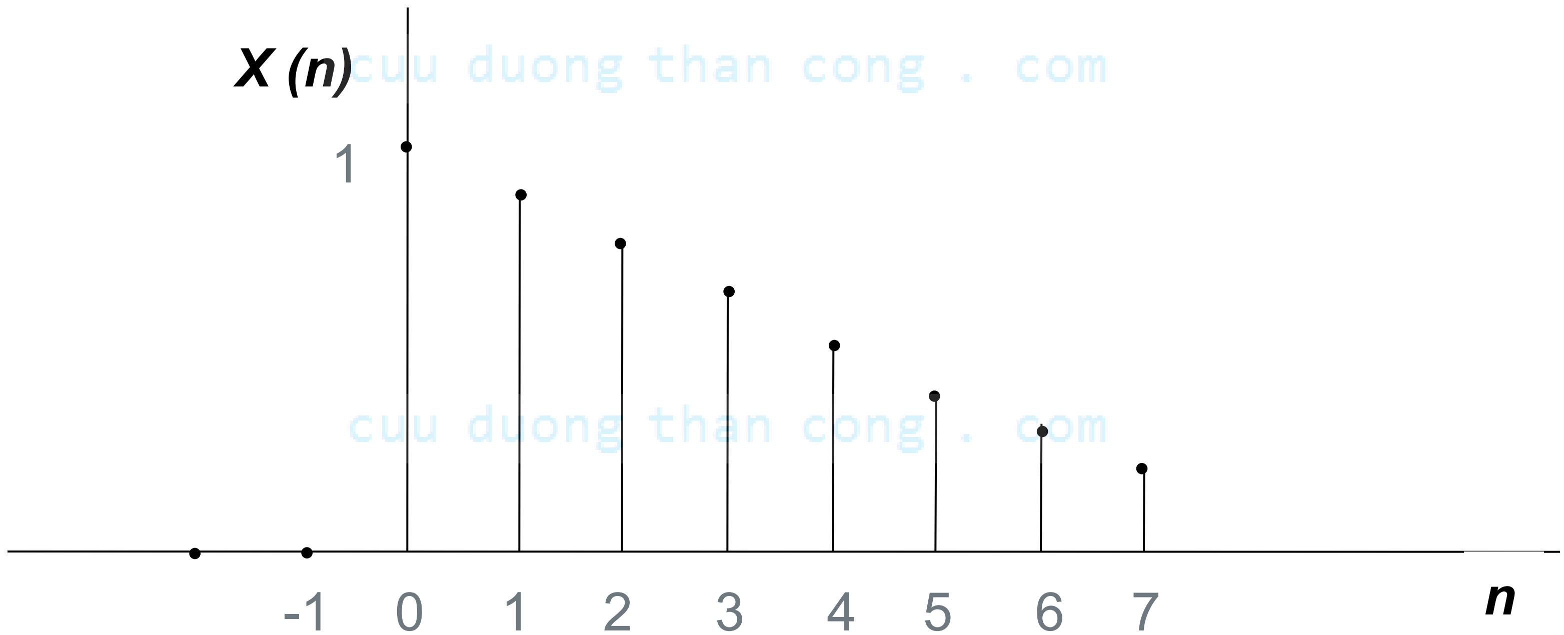



Figure 1.6

# 1.2.3 Continuous-Valued versus Discrete-Valued Signals

- By selecting values of as analog signal at discrete-time instants. This process is called ***sampling***.

- A discrete-time signal having a set of discrete values is called a ***digital signal*** that takes on one of four possible values. Fig.1.8



- In order for a signal to be processed digitally it must be discrete in time and *its values* ***must be discrete***.

# 1.2.3 Continuous-Valued versus Discrete-Valued Signals

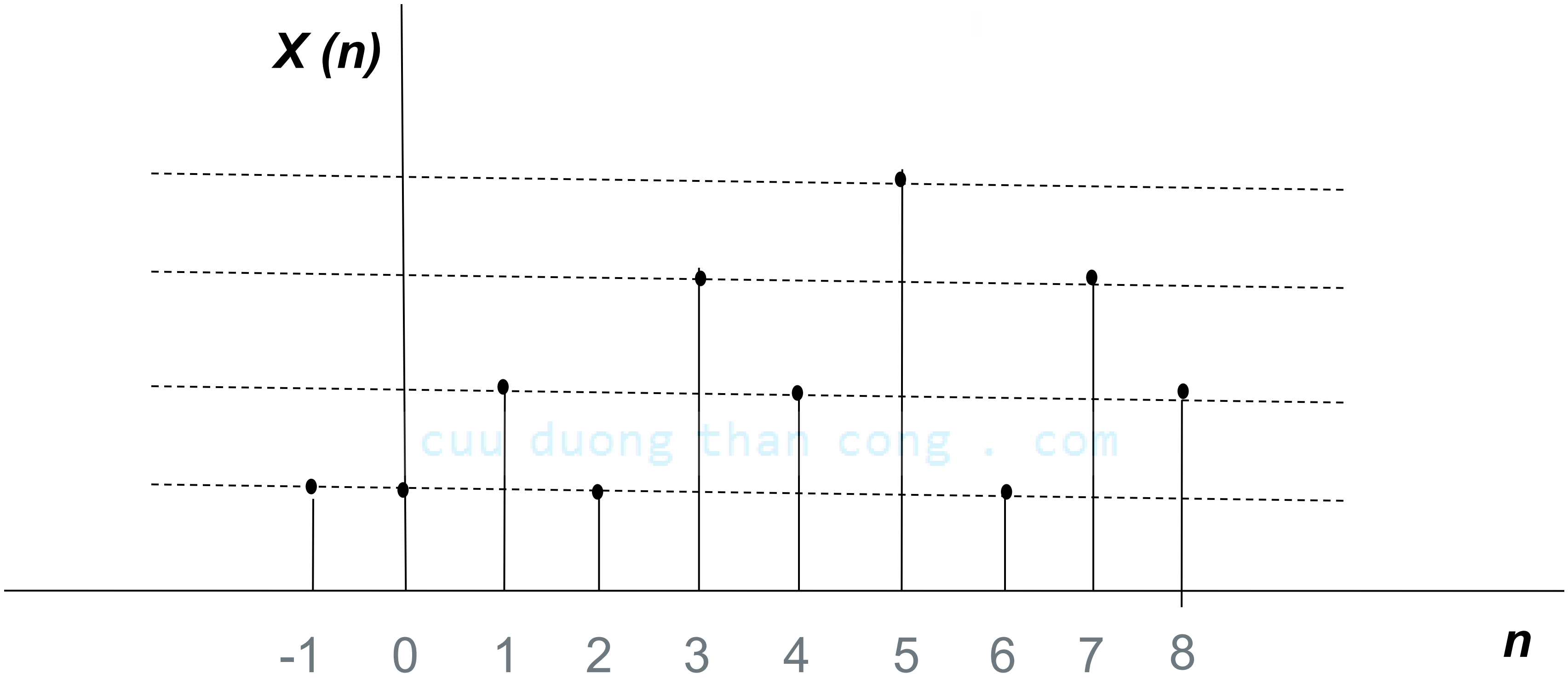



Figure 1.8: Digital signal with four different amplitude values

# 1.2.3 Continuous-Valued versus Discrete-Valued Signals

- The process of converting a continuous – valued signal in to a discrete valued signal, called ***quantization***, is basically an approximation process.
- Any signal that can be uniquely described by an explicit mathematical expression, a table of data, or a well defined rule is called ***deterministic***.



The signals evolve in time in an unpredictable manner, which is as ***random***.

# 1.3 The concept of frequency in continuous-time and discrete-time Signals

- ***Continuous-time sinusoidal signal***.

$$X_a(t) = A\cos(\Omega t + \theta), \qquad -\infty < t < \infty \qquad (1.3.1)$$

$$\Omega = 2\Pi F \text{ (rad/s)};$$

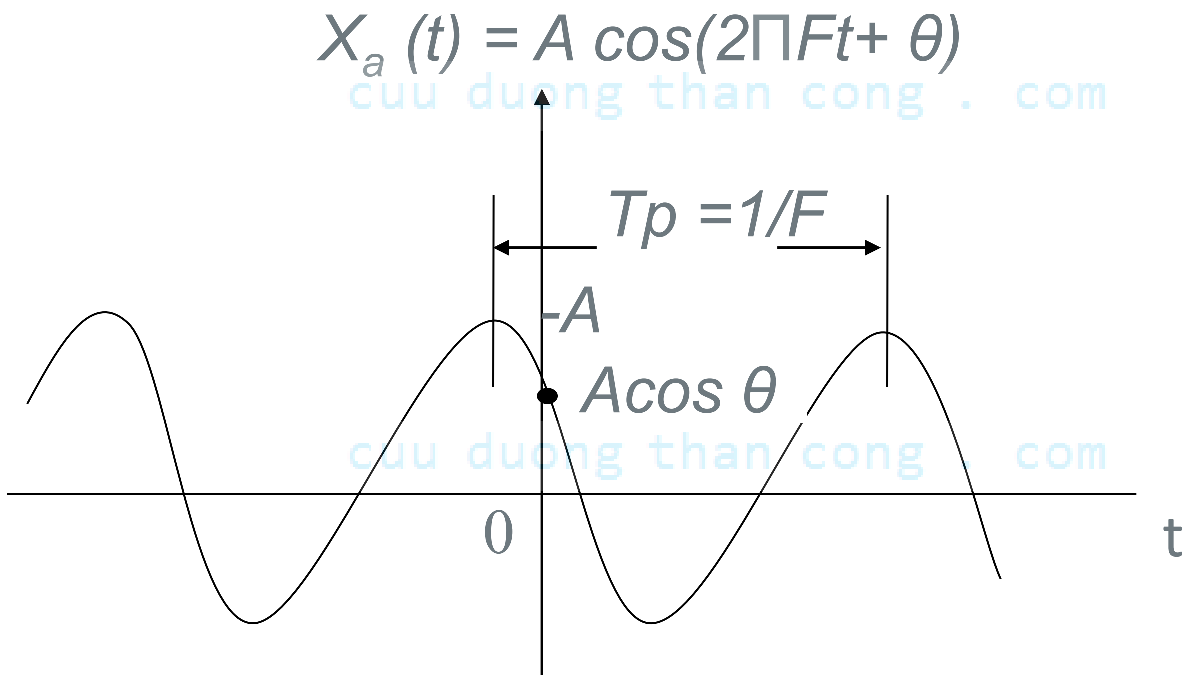




Figure 1.10 Example of an analog sinusoidal signal

## 1.3.1 The concept of frequency in continuous-time and discrete-time Signals

- The relationships we have described for sinusoidal signal carry over to the class of complex exponential signal.

$$x_a(t) = Ae^{j(\Omega t + \theta)} \qquad (1.3.4)$$



- Using the ***euler*** identity.

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta \qquad (1.3.5)$$

- The sinusoidal signal (1.3.1) may be expressed as.



$$x_a(t) = A\cos(\Omega t + \theta) = A/2\, e^{j(\Omega t + \theta)} + A/2 e^{-j(\Omega t + \theta)} \qquad (1.3.6)$$

# 1.3.1 The concept of frequency in continuous-time and discrete-time Signals

- A discrete-time sinusoidal signal may be expressed as.

  $x(n) = A\cos(\omega n + \theta), \quad -\infty < n < \infty \qquad (1.3.7)$

- Where ***n*** is an integer variable, called the ***sample number***, ***A*** is the ***amplitude*** of the sinusoid, ***ω*** is the ***frequency*** in radians per sample, and ***θ*** is the ***phase*** in radians.



$$\omega \equiv 2\Pi f \qquad (1.3.8)$$

$$x(n) = A\cos(2\Pi f\, n + \theta), \quad -\infty < n < \infty \quad (1.3.9)$$

# 1.3.2 Discrete-time Sinusoidal Signals

$$x(n) = A\cos(\omega n + \theta)$$

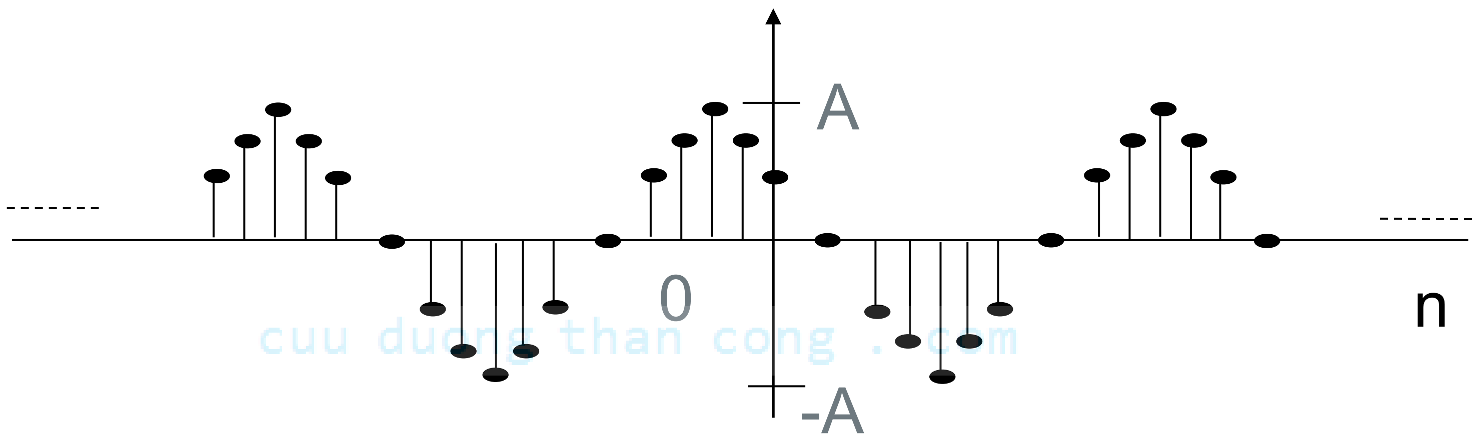





Figure 1.12: Example of a discrete-time sinusoidal signal ($\omega = \Pi/6$ and $\theta = \Pi/3$).

# 1.3.2 Discrete-time Sinusoidal Signals

- The discrete-time sinusoid are characterized by the following properties:

**B1.** *A discrete-time sinusoid is periodic only if its frequency f is a* ***rational number.***

$x(n + N) = x(n)$ for all $n$ (1.3.10)



the smallest value of $N$ is a periodic, is called the ***fundamental period*** .

$\cos[2\Pi f_0(N + n) + \theta] = \cos(2\Pi f_0 n + \theta)$



This relation is true if and only If there exists an integer $k$ such that $2\Pi f_0 N = 2k\Pi$

or, equivalently $\boldsymbol{f_0 = k/N}$ ( 1.3.11)

# 1.3.2 Discrete-time Sinusoidal Signals

- **B2.** *Discrete-time sinusoid whose frequencies are separated by an integer multiple of **2Π** are **identical**.*

$$Cos[(\omega_0 + 2\Pi)n + \theta] = cos(\omega_0 n + 2\Pi n + \theta)$$

$$= cos\,(\omega_0 n + \theta) \qquad (1.3.12)$$

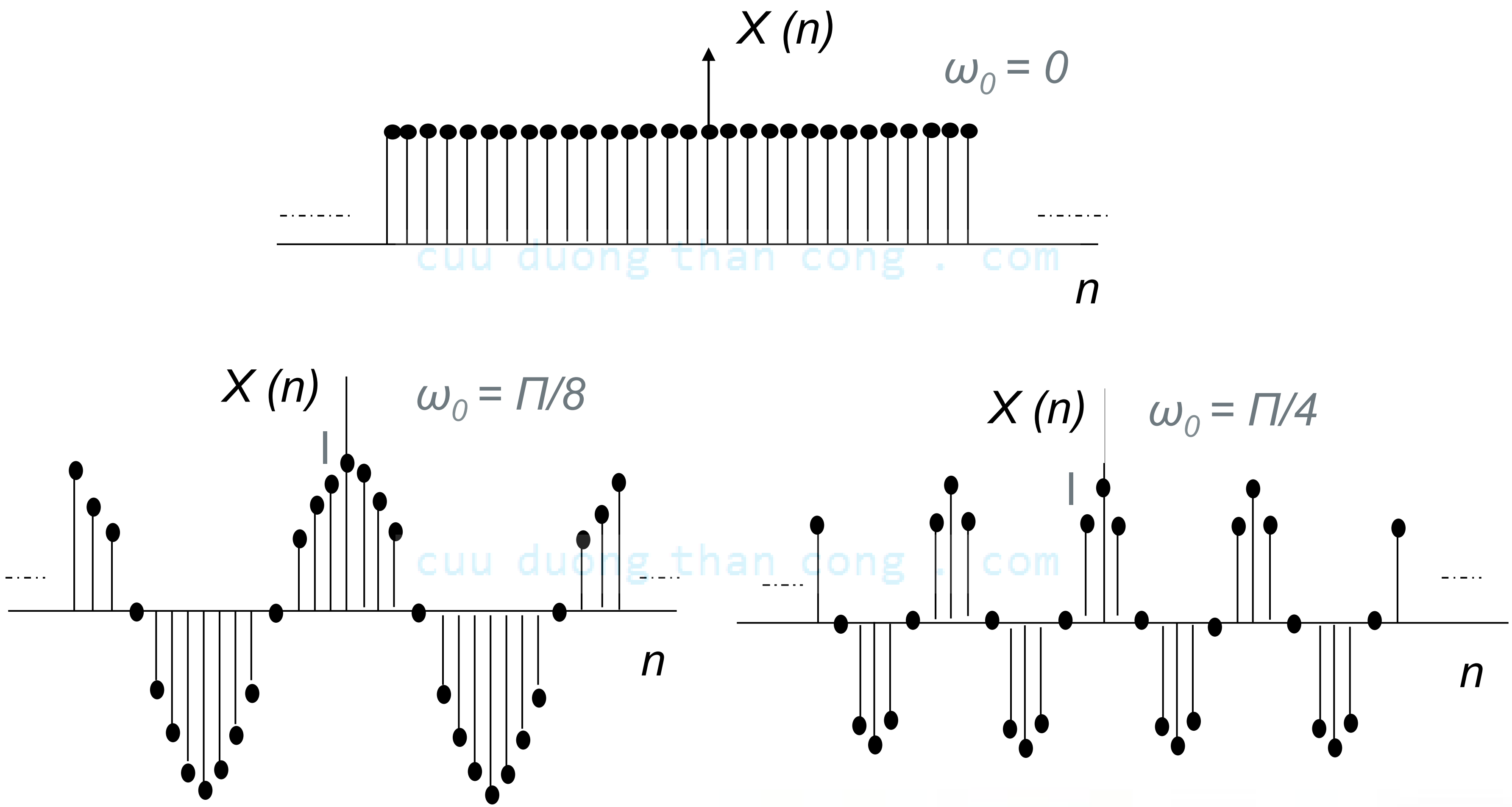
$$X_k(n) = Acos(\omega_k n + \theta), \quad k = 0,1,2,\ldots \qquad (1.3.13)$$

Where $\omega_k = \omega_0 + 2k\Pi, \quad -\Pi \le \omega_0 \le \Pi$

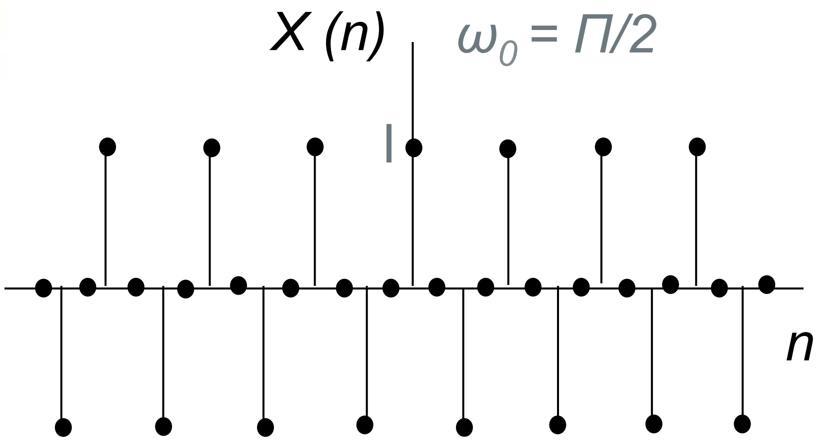
- **B3.** *The **highest rate** of oscillation in a discrete-time sinusoid is attained when **ω = Π** (or **ω = -Π**) or, equivalently, **f = 1/2** (or **f = -1/2**)*

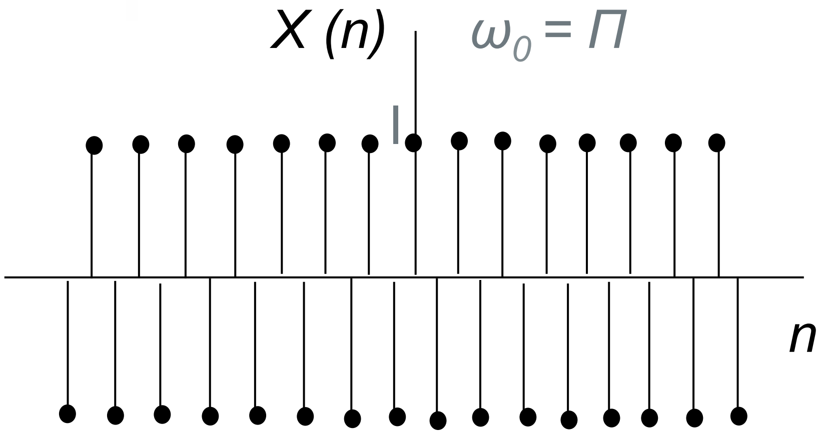
# 1.3.2 Discrete-time Sinusoidal Signals

Figure 1.13: Signal $X(n) = \cos\omega_0 n$ for various values of the frequency $\omega_0$



# 1.3.2 Discrete-time Sinusoidal Signals






Continuous-time exponentials

$$S_k(t) = e^{jk\Omega_0 t} = e^{j\,2\Pi\,k\,F_0 t}, \qquad k = 0, \pm 1, \pm 2, \ldots \qquad (1.3.16)$$



$$X_a(t) = \sum_{k=-\infty}^{\infty} C_k S_k(t) = \sum_{k=-\infty}^{\infty} C_k\, e^{-jk\Omega_0 t} \qquad (1.3.17)$$

# 1.3.2 Discrete-time Sinusoidal Signals

- Discrete-time exponentials

$$S_k(n) = e^{j2\pi k f_0 n} \quad , \quad k = 0, \pm 1, \pm 2 \ldots \qquad (1.3.18)$$

$$X(n) = \sum_{k=o}^{n-1} C_k S_k(n) = \sum_{k=0}^{n-1} C_k e^{j2\pi k/N} \qquad (1.3.20)$$



Where $N = \dfrac{1}{f_0}$

- To process analog signal by digital means, it is convert them to sequence of numbers having finite precision. This procedure is called ***analog-to-digital (A/D) conversion***

# 1.3.2 Discrete-time Sinusoidal Signals

- A/D conversion has ***three-step*** process.

***1. Sampling***: conversion of a continuous-time signal into a discrete-time signal.

$X_a(nT) \equiv X(n)$ $\qquad$ *T*- sampling interval.



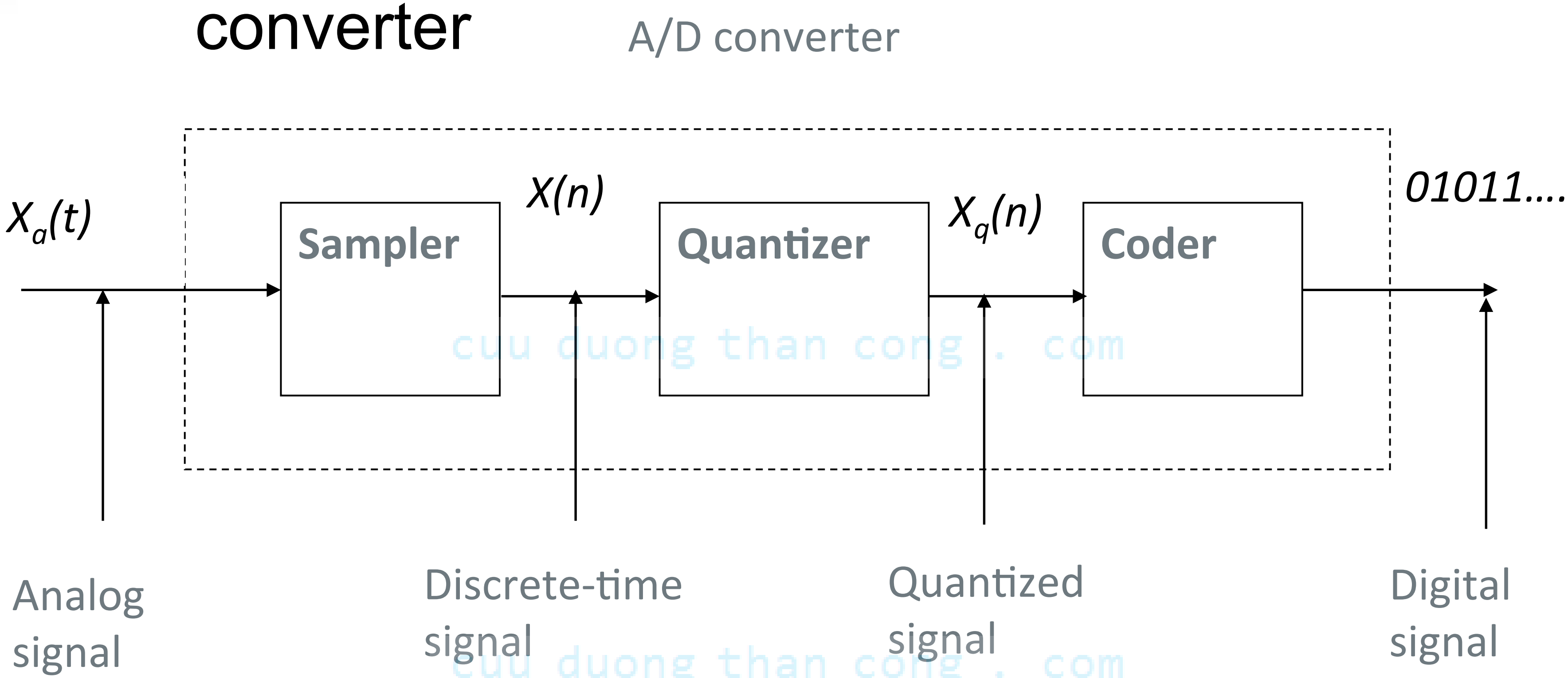
***2. Quantization***: conversion of discrete-time continuous-valued signal into a discrete-time, discrete-value (digital) signal.



***3. coding*** : In the coding process, each discrete-value, *Xq(n)* is represented by *b*-bit binary sequence .

# 1.4 Analog-to-Digital and Digital-to-Analog Conversion

Figure 1.14: Basic parts of an analog-to-digital (A/D) converter






- The process of converting a digital signal into an analog signal is known as ***digital-to-analog (A/D) conversion***.

# 1.4.1 Sampling of Analog Signals

$$X(n) = X_a(nT), \qquad -\infty < n < \infty \qquad (1.4.1)$$

$X(n)$ is discrete-time signal

$X_a(nT)$ - analog signal every $T$ seconds



- The time interval $T$ between successive sampling is called the ***sampling period*** or ***sample interval*** and its reciprocal $1/T = F_s$ is called the ***sampling rate*** or the ***sampling frequency***.

# 1.4.1 Sampling of Analog Signals

$$t = nT = \frac{n}{F_s} \qquad (1.4.2)$$

$$X_a(t) = A\cos(2\pi Ft + \theta) \qquad (1.4.3)$$

$$X_a(nT) \equiv x(n) = A\cos(2\pi FnT + \theta) \qquad (1.4.4)$$

$$= A\cos\left(\frac{2\pi nF}{F_s} + \theta\right)$$

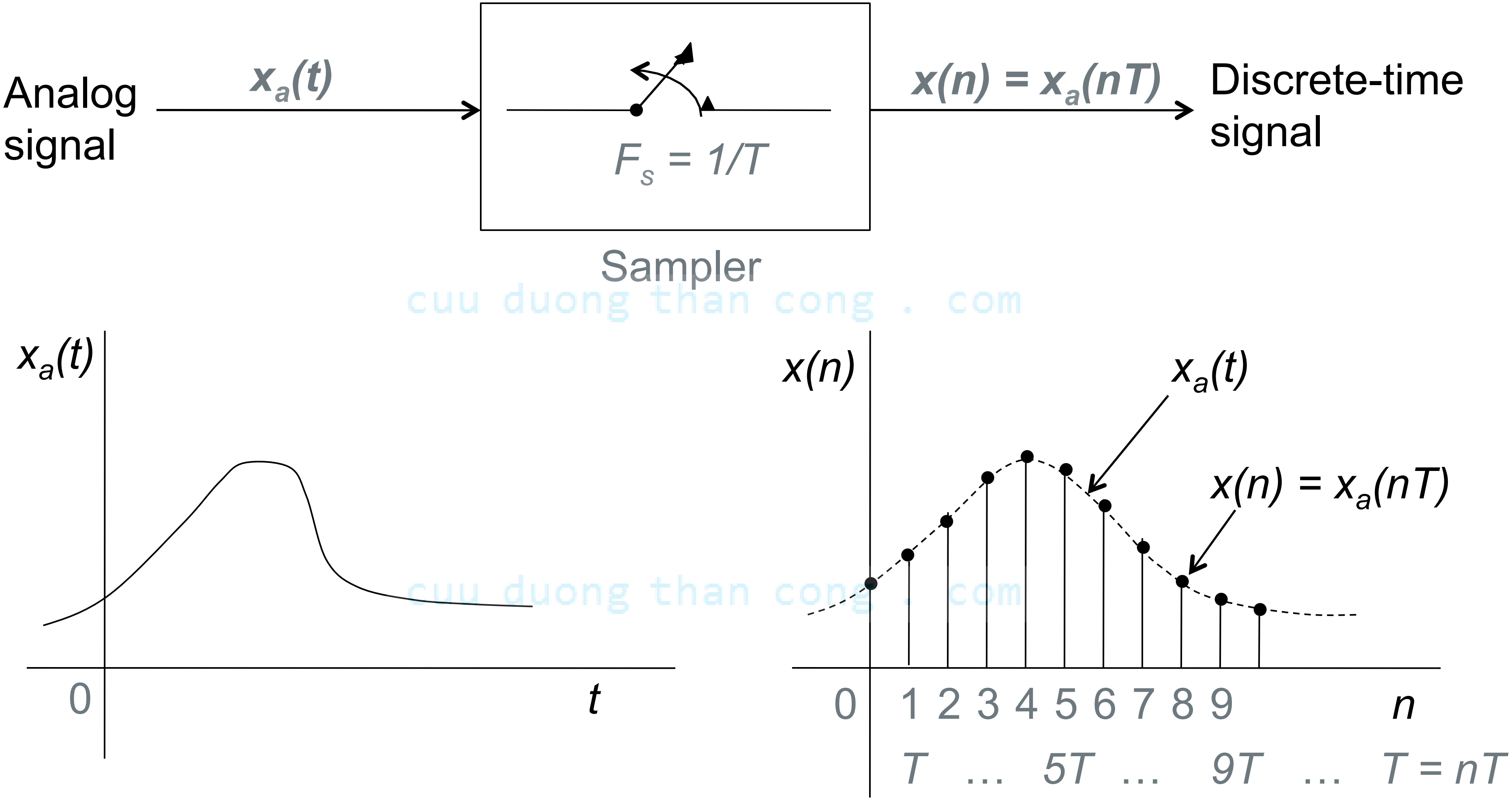
$$f = \frac{F}{F_s} \quad (1.4.5) \qquad \text{or equivalently, as} \qquad \omega = \Omega T \quad (1.4.6)$$

$$-\frac{1}{2} < f < \frac{1}{2} \qquad (1.4.8)$$

$$-\pi < \omega < \pi$$

# 1.4.1 Sampling of Analog Signals

Figure 1.16: Periodic sampling of an analog signal



# 1.4.1 Sampling of Analog Signals

- The frequency of the continuous-time sinusoid when sampled at a rate $F_s = 1/T$ ***must fall in the range***.

$$-\frac{1}{2T} = -\frac{F_s}{2} \le F \le \frac{F_s}{2} = \frac{1}{2T} \qquad (1.4.9)$$



$$-\frac{\pi}{T} = -\pi Fs \le \Omega \le \pi F_s = \frac{\pi}{2} \qquad (1.4.10)$$

- The ***highest frequency*** in a discrete-time signal



$$F_{max} = \frac{F_S}{2} = \frac{1}{2T}$$

$$\Omega_{max} = \pi F_S = \frac{\pi}{T} \qquad (1.4.11)$$

# 1.4.1 Sampling of Analog Signals

***In general***, the sampling of a continuous-time sinusoidal signal

$$X_a(t) = A\cos(2\pi F_0 t + \theta) \qquad (1.4.14)$$

With a sampling rate $F_s = 1/T$ results in a discrete-time signal



$$X(n) = A\cos(2\pi f_0 n + \theta) \qquad (1.4.15)$$

Where $f_0 = F_0 / F_s$ is the ***relative frequency*** of sinusoid.



If we assume that $-F_s/2 \le F_0 \le F_s/2$, then the frequency $f_0$ of $x(n)$ is in the range $-\frac{1}{2} \le f_0 \le \frac{1}{2}$.

# 1.4.1 Sampling of Analog Signals

On the other hand, if the sinusoids

$$X_a(t) = A\cos(2\pi F_k t + \theta) \qquad (1.4.16)$$

Where $\boldsymbol{F_k = F_0 + kF_s\,,\ k = \pm 1, \pm 2, \ldots}$ (1.4.17)

are sampled at a rate $F_s$.



The frequency $F_k$ is outside the fundamental frequency range $-F_s/2 \le F \le F_s/2$

Consequently, the sampled signal is



$$x(n) \equiv x_a(nT) = A\cos\left(2\pi \frac{F_0 + kF_s}{F_s} n + \theta\right)$$

$$= A\cos(2\pi f_0 n + \theta)$$

# 1.4.1 Sampling of Analog Signals

Which is ***identical*** to the discrete-time signal in (1.4.15) obtained by sampling (1.4.14).

Thus an infinite number of continuous-time sinusoids is represented by sampling the ***same*** discrete-time signal.

We can say that the frequencies

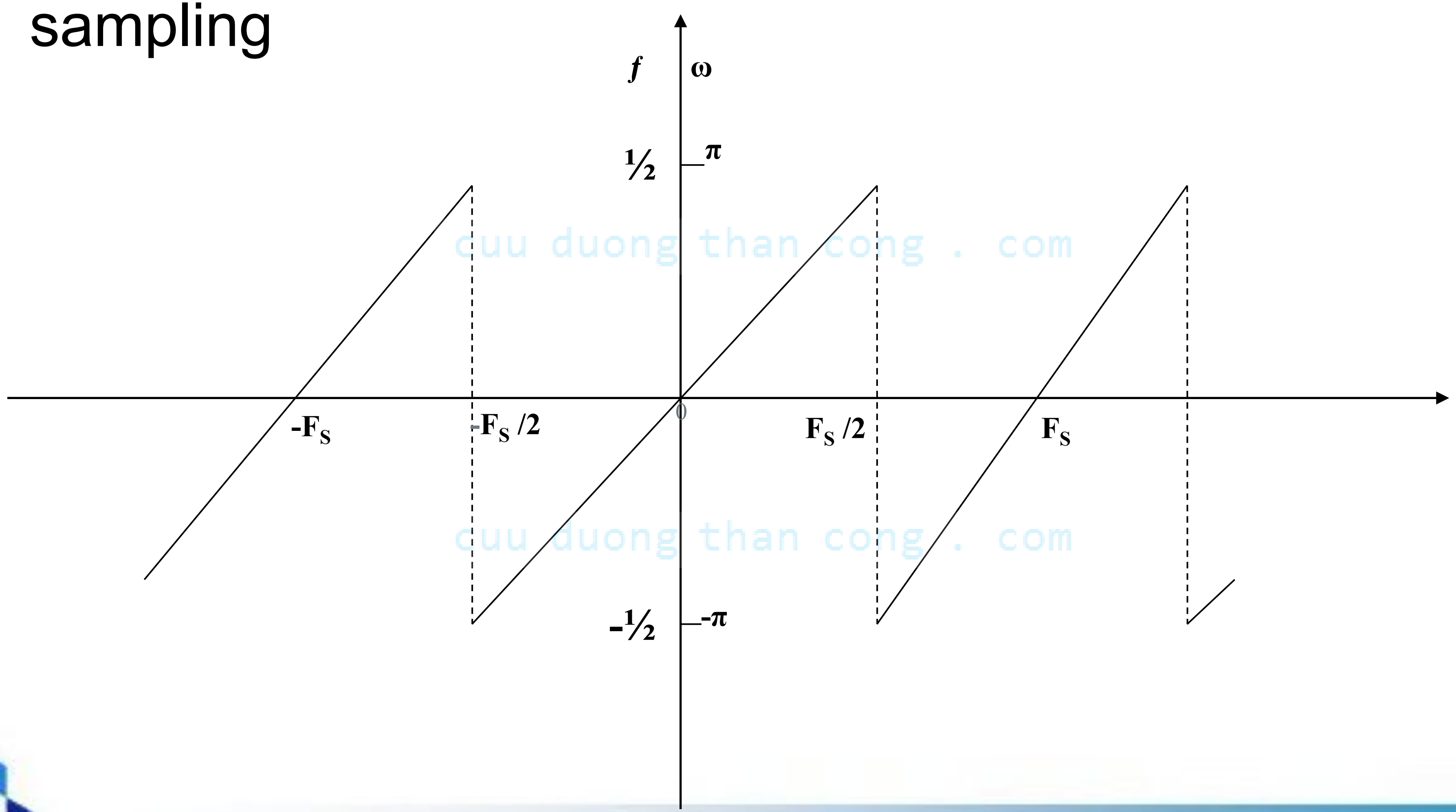
$$F_k = F_0 + k F_s, \; -\infty < k < \infty \quad (k \text{ integer})$$

are ***indistinguishable*** from the frequency $F_0$ after sampling and hence they are ***aliases*** of $F_0$.

To be illustrated in Fig.1.17.

# 1.4.1 Sampling of Analog Signals

Figure 1.17 Relationship between the continuous and discrete-time frequency variables in the case of periodic sampling



## 1.4.2 The Sampling Theorem

- Given any analog signal, can be represented as a ***sum of sinusoids*** of different amplitudes, frequencies, and phases, that is,

$$X_a(t) = \sum_{i=1}^{N} A_i \cos(2\pi F_i t + \theta_i) \qquad (1.4.18)$$



  Where *N* denotes the number of frequency components.
- Suppose that to signal in the class do not exceed some known frequency *Fmax.*



- Thus to avoid the problem of ***aliasing***, $F_s$ is selected so that. $\boldsymbol{F_s > 2Fmax}$ (1.4.19)

# 1.4.2 The Sampling Theorem

If the ***highest frequency*** contained in an analog signal $X_a(t)$ *is Fmax = 2B.*

The signal is sampled at rate ***Fs > 2 Fmax*** then



$X_a(t)$ can be ***exactly recovered*** its sample values using the ***interpolation function***



.

$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt} \qquad (1.4.22)$$

# 1.4.2 The Sampling Theorem

$$X_a(t) = \sum_{n=-\infty}^{\infty} x_a\left(\frac{n}{F_S}\right) g\left(t - \frac{n}{F_S}\right) \qquad (1.4.23)$$



Where $X_a\left(\frac{n}{F_S}\right) = X_a(nT) \equiv x(n)$ are the samples of $X_a(t)$.

When the sampling of $X_a(t)$ is performed at the minimum sampling rate $Fs = 2B$,

# 1.4.2 The Sampling Theorem

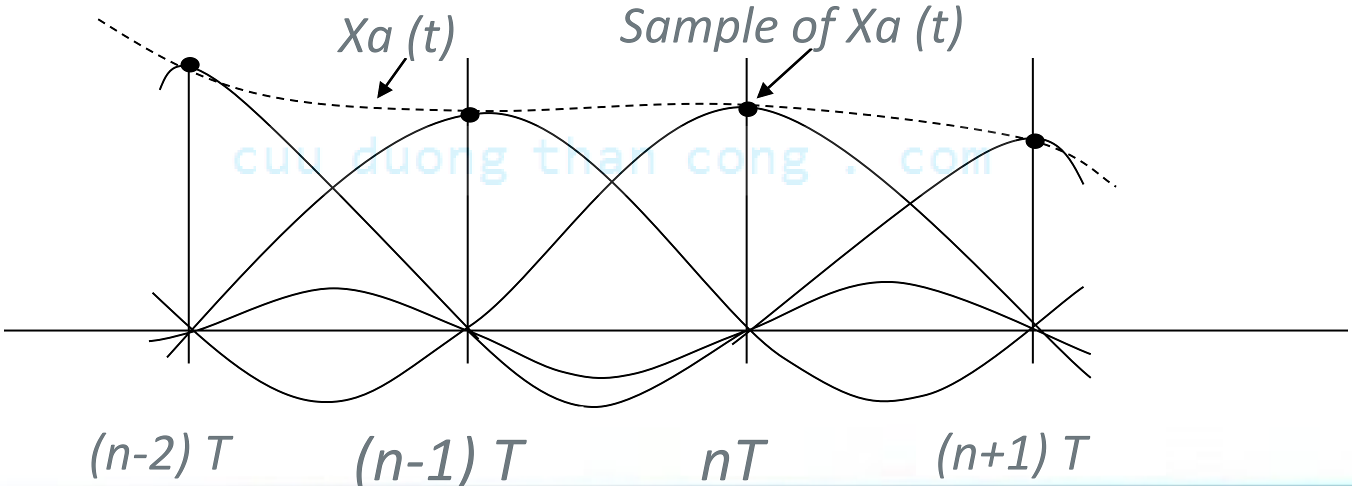
it becomes .

$$X_a(t) = \sum_{n=-\infty}^{\infty} X_a\left(\frac{n}{2B}\right) \frac{\sin 2\pi B\left(t - \frac{n}{2B}\right)}{2\pi B\left(t - \frac{n}{2B}\right)} \qquad (1.4.24)$$

The sampling rate $F_N = 2B = 2F_{max}$ is called the ***Nyquist rate***.



Figure 1.19 Ideal D/A conversion (interpolation).



# 1.4.3 Quantization of Continuous-Amplitude Signals

The process of converting a discrete-time continuous-amplitude signal into a digital signal by expressing each sample value as a ***finite number*** of ***digits*** is called ***quantization***.

- The error introduced in representing the continuous-value signal by a finites set of discrete values is called ***quantization error*** or ***quantization noise***.

# 1.4.3 Quantization of Continuous-Amplitude Signals

$$X_q(n) = Q[x(n)]$$

$X_q(n)$- sequence of quantized samples at the output of quantizer.

$$e_q(n) = X_q(n) - X(n) \quad (1.4.25)$$



$e_q(n)$- quantization error.

For example:

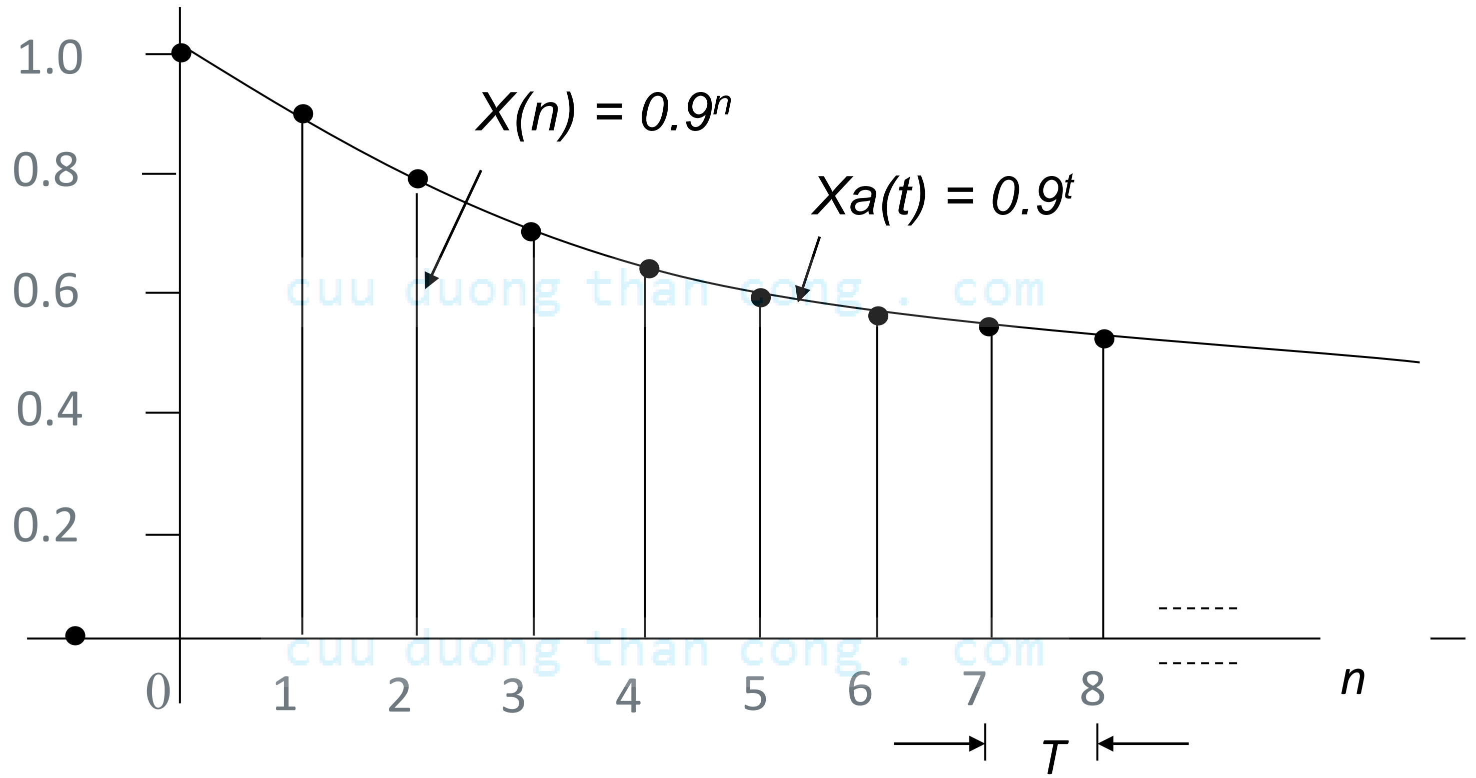
$$X(n) = \begin{cases} 0.9^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



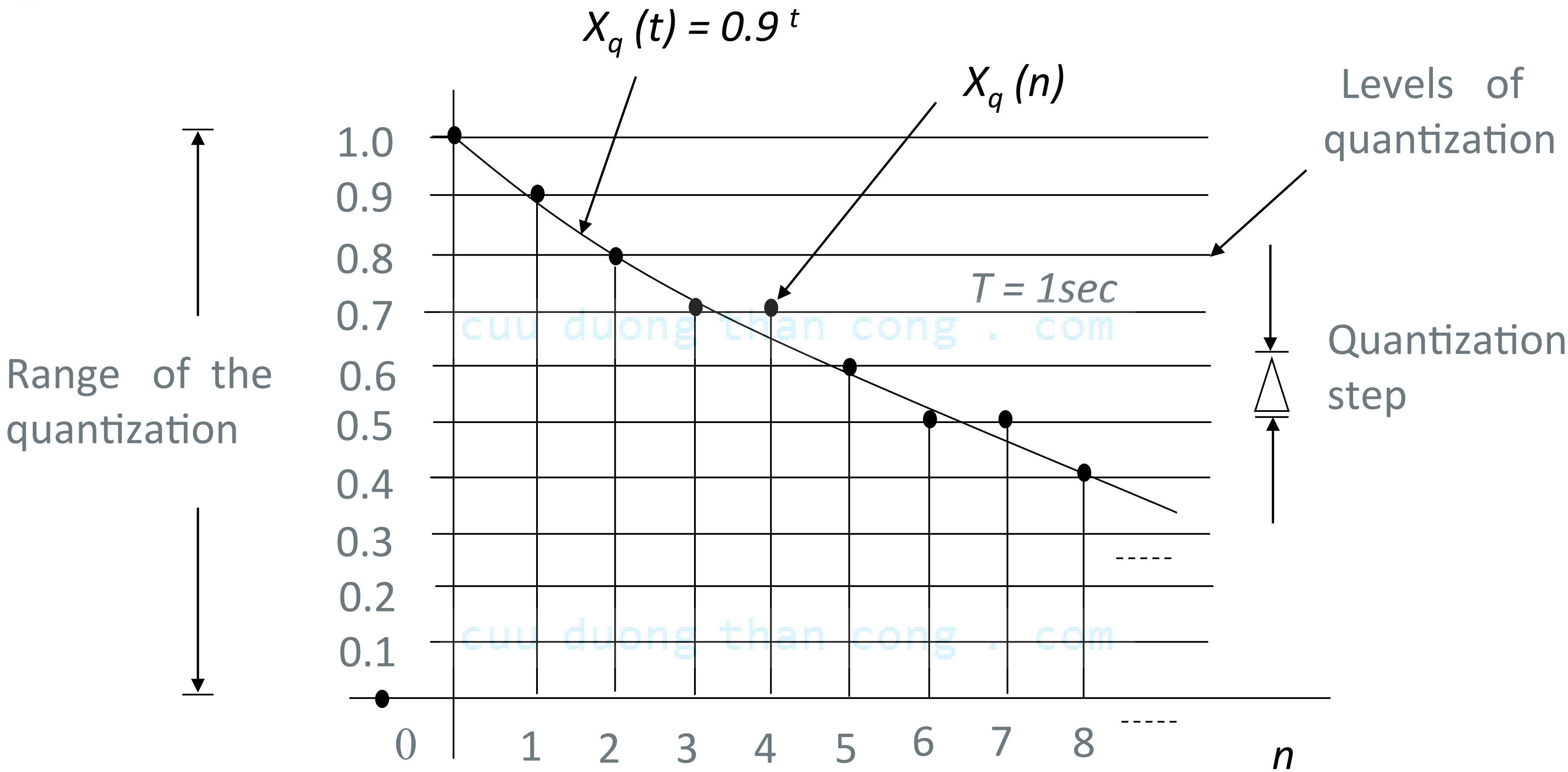
with $F_s = 1Hz$

# 1.4.3 Quantization of Continuous-Amplitude Signals

Figure 1.20 Illustration of quantization



# 1.4.3 Quantization of Continuous-Amplitude Signals



# 1.4.3 Quantization of Continuous-Amplitude Signals

To eliminate the excess digits, we can either simple ***discard*** them(***truncation***) or discard them by rounding the resulting number (***rounding***)

The value allowed in the digital signal are called the ***quantization levels***.

The distance $\Delta$ between two successive quantization levels is called the ***quantization step size*** or ***resolution***.

$$-\frac{\Delta}{2} \leq e_q(n) \leq \frac{\Delta}{2}$$

# 1.4.3 Quantization of Continuous-Amplitude Signals

If $X_{min}$ and $X_{max}$ represent the minimum and maximum value of *x(n)* and *L* is the number of quantization levels, then

$$\Delta = \frac{Xmax - Xmin}{L-1}$$



Dynamic range of the signal as $X_{max} - X_{min}$



In our example we have

$x_{max} = 1$, $x_{min} = 0$, and $L = 11$

## 1.4.4 Quantization of Sinusoidal Signals

- We have an analog sinusoidal signal

$$x_a(t) = A\cos\Omega_0 t$$

thus, from the original analog signal, we obtain



$$x(n) \equiv x_a(nT) - \text{discrete-time signal}$$

$x_q(nT)$- discrete-sampled signal after quantization

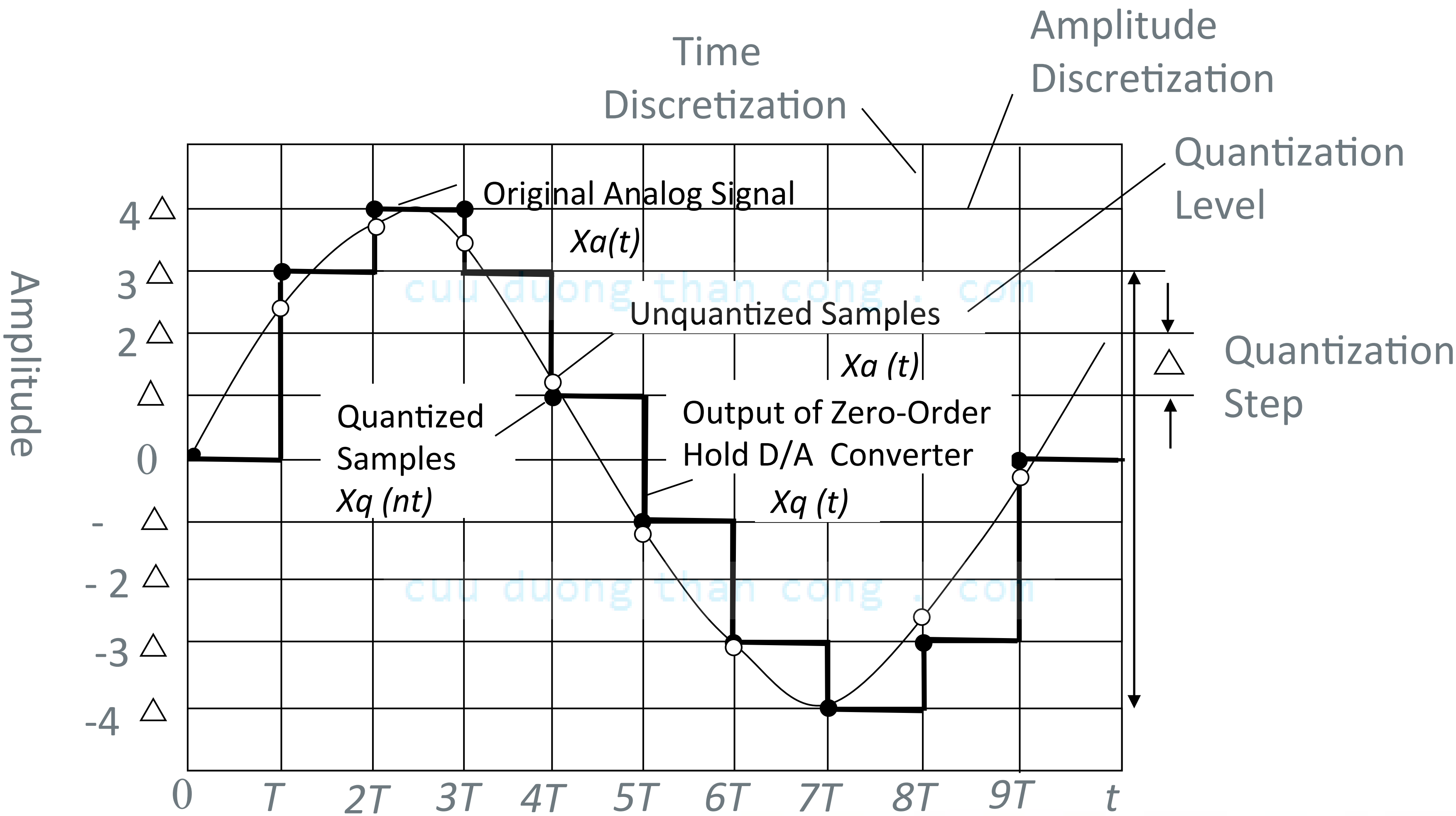
$f_s$ - sampling rate, satisfies the sampling theorem



$x_q(t)$ - staircase-signal

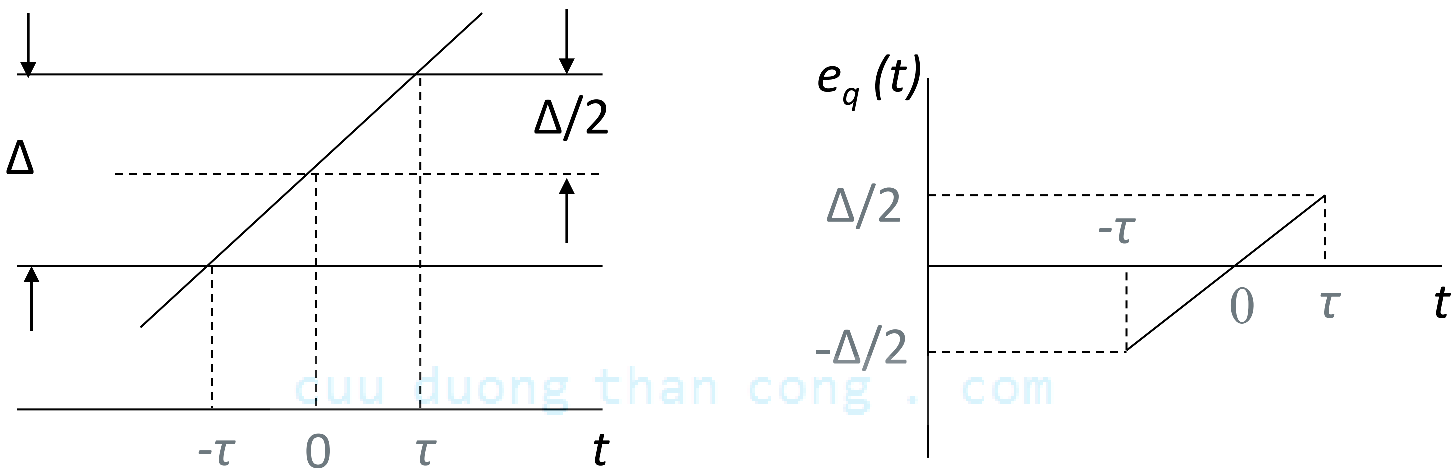
# 1.4.4 Quantization of Sinusoidal Signals

**Figure 1.21:** Sampling and quantization of a sinusoidal signal



# 1.4.4 Quantization of Sinusoidal Signals

**Figure 1.22** The quantization error $e_q(t) = x_a(t) - x_q(t)$






- The mean-square ***error power $P_q$***

$$P_q = \frac{1}{2\tau}\int_{-\tau}^{\tau} e_q^2(t)dt = \frac{1}{\tau}\int_{0}^{\tau} e_q^2(t)dt \qquad (1.4.28)$$



Since $e_q(t) = (\Delta/2\tau)t, \qquad -\tau \le t \le \tau$

# 1.4.4 Quantization of Sinusoidal Signals

$$P_q = \frac{\Delta^2}{12} = \frac{\Delta^2/3}{2^{2b}} \qquad (1.4.29)$$

*2A* – entire range of signal *b* – quantization of bit of accuracy.

$\Delta = 2A/2^b$

The ***average power*** of the signal $x_a(t)$ is



$$Px = \frac{1}{Tp}\int_0^{Tp} (A\cos\Omega o t)^2\, dt = \frac{\Delta^2}{2} \qquad (1.4.30)$$

The ***signal-to-quantization noise ratio*** (***SQNR***)



$$SQNR = \frac{Px}{Pq} = \frac{3}{2}.2^{2b}$$

$$SQNR\,(dB) = 10log_{10}SQNR = 1.76 + 6.02b \qquad (1.4.31)$$

# 1.4.5 Coding of quantized samples

If we have *L* quantization levels, then

$2^b \geq L$ or $b \geq log_2 L$ ,

with a word length of ***b*** bit.

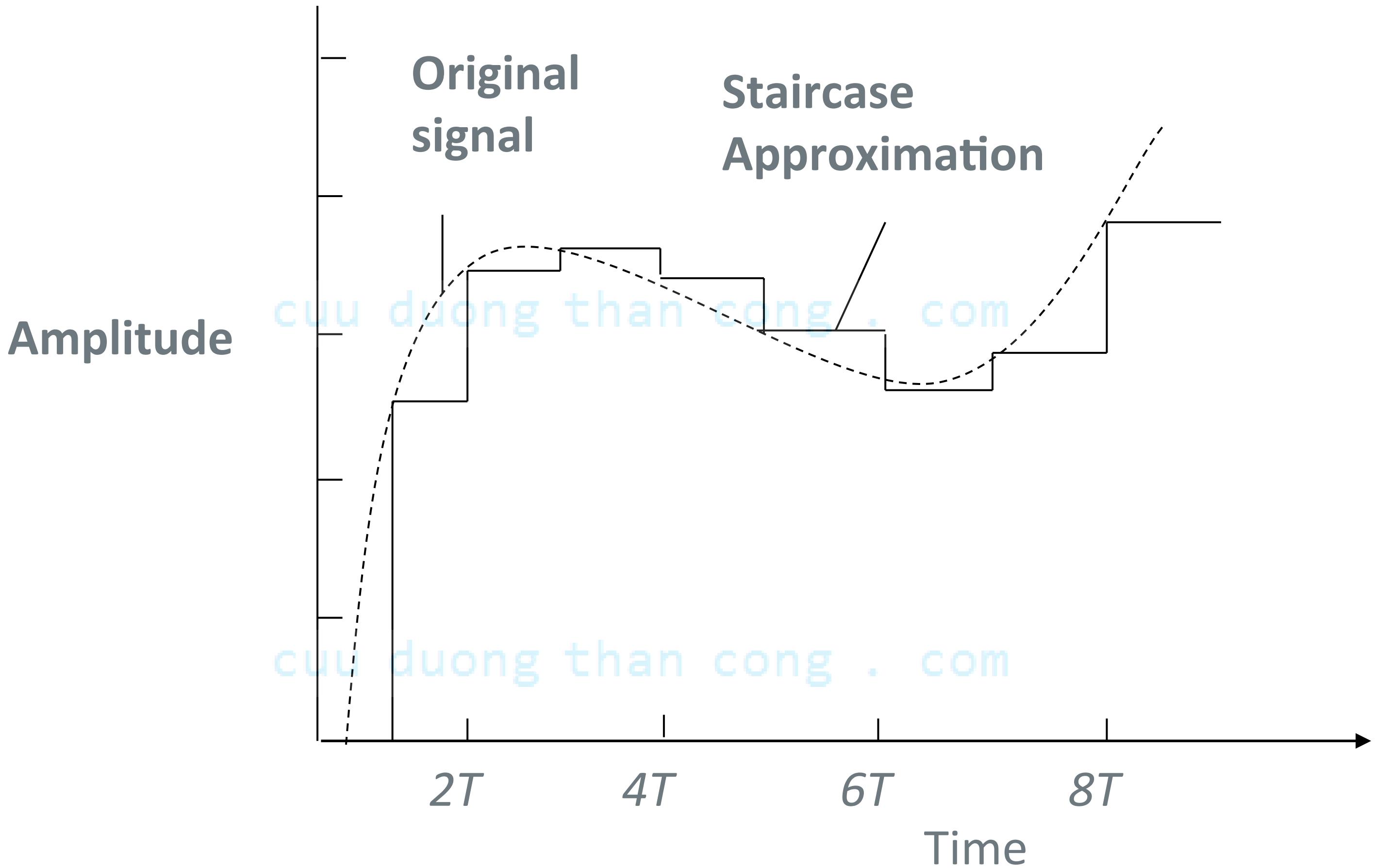
The task of a D/A converter is to interpolate between samples.

+ The simplest D/A converter is the ***zero-order hold***, which simply holds constant the value of one sample until the next one is received. (see Fig.1.15)

+ ***Interpolation*** to connect successive samples with *straight-line segments*. (see Fig.1.23)

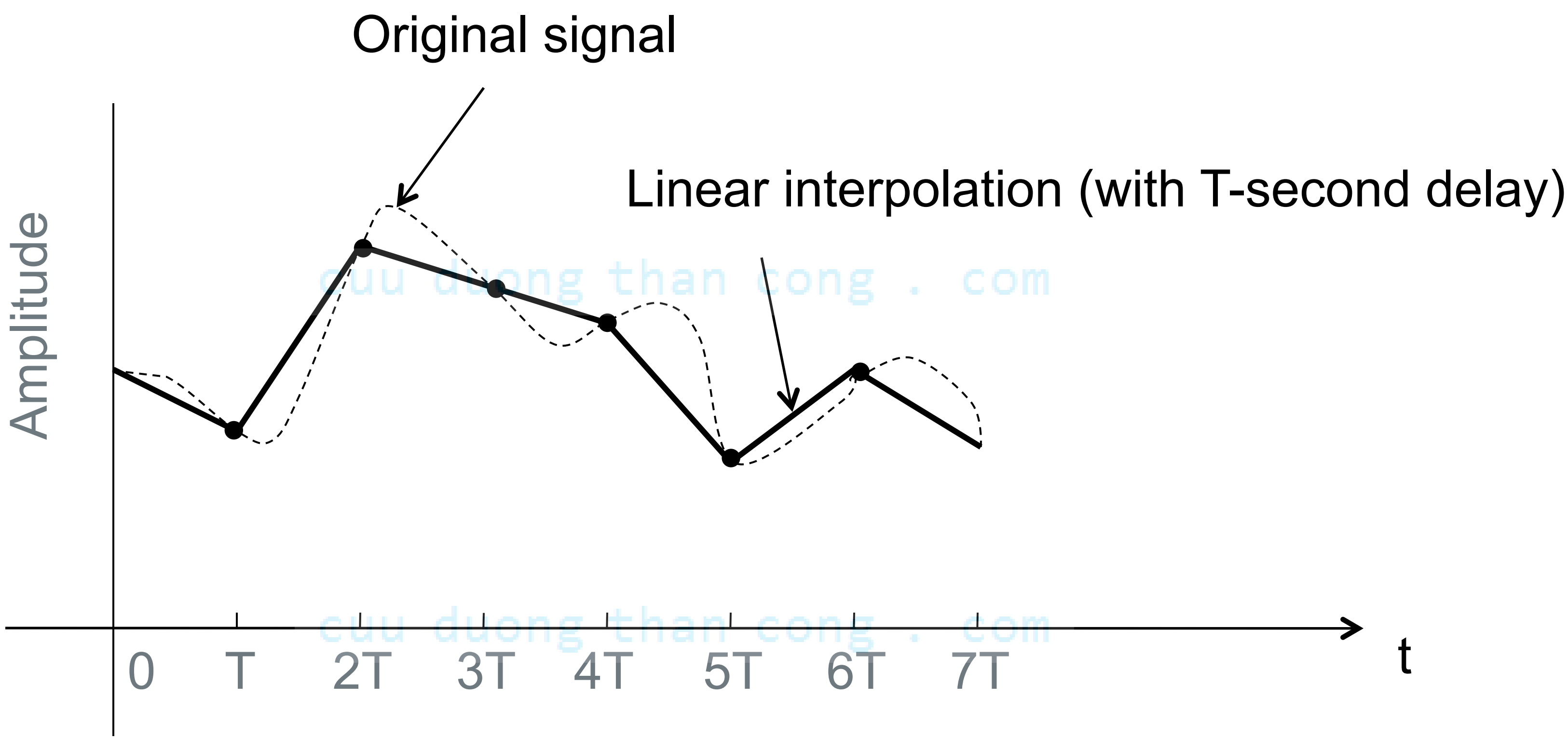
# 1.4 Analog-to-Digital and Digital-to-Analog Conversion

**Figure 1.15** Zero-order hold digital-to-analog (D/A) conversion



# 1.4.6 Digital-to Analog conversion

Figure 1.23: Linear point connector (with T-second delay).




Problems: 1.1, 1.2, 1.8, 1.9, 1.10, 1.11, 1.12, 1.13,1.14