EXAMPLE 6.1.1

To illustrate this computational procedure, let us consider the computation of an $N = 15$ point DFT.

Since $N = 5 \times 3 = 15$, we select $L = 5$ and $M = 3$.

In other words, we store the 15-point sequence $x(n)$ column-wise as follows:

Row 1:	$x(0,0) = x(0)$	$x(0,1) = x(5)$	$x(0,2) = x(10)$
Row 2:	$x(1,0) = x(1)$	$x(1,1) = x(6)$	$x(1,2) = x(11)$
Row 3:	$x(2,0) = x(2)$	$x(2,1) = x(7)$	$x(2,2) = x(12)$
Row 4:	$x(3,0) = x(3)$	$x(3,1) = x(8)$	$x(3,2) = x(13)$
Row 5:	$x(4,0) = x(4)$	$x(3,1) = x(9)$	$x(4,2) = x(14)$

EXAMPLE 6.1.1

Now, we compute the three-point DFTs for each of the five rows. This leads to the following *5 x 3* array:

F (0, 0)	*F (0, 1)*	*F (0, 2)*
F (1, 0)	*F (1, 1)*	*F (1, 2)*
F (2, 0)	*F (2, 1)*	*F(2, 2)*
F (3, 0)	*F (3, 1)*	*F (3, 2)*
F (4, 0)	*F (4, 1)*	*F (4, 2)*

EXAMPLE 6.1.1

The next step is to multiply each of the terms *F(l, q)* by the phase factors $W_N^{lq} = W_{15}^{lq}$, $0 \le l \le 4$ and $0 \le q \le 2$. This compute results in the *5 x 3* array:

Column 1	Column 2	Column 3
G (0, 0)	*G (0, 1)*	*G (0, 2)*
G (1, 0)	*G (1, 1)*	*G (1, 2)*
G (2, 0)	*G (2, 1)*	*G (2, 2)*
G (3, 0)	*G (3, 1)*	*G (3, 2)*
G (4, 0)	*G (4, 1)*	*G (4, 2)*

EXAMPLE 6.1.1

The final step is to compute the five-point DFTs for each of the three columns. This computation yields the desired values of the DFT in the form :

$$
\begin{array}{lll}
X(0,0) = X(0) & X(0,1) = X(1) & X(0,2) = X(2) \\
X(1,0) = X(3) & X(1,1) = X(4) & X(1,2) = X(5) \\
X(2,0) = X(6) & X(2,1) = X(7) & X(2,2) = X(8) \\
X(3,0) = X(9) & X(3,1) = X(10) & X(3,2) = X(11) \\
X(4,0) = X(12) & X(4,1) = X(13) & X(4,2) = X(14)
\end{array}
$$

EXAMPLE 6.1.1

It is interesting to view the segmented data sequence and the resulting DFT in terms of one-dimensional arrays. When the input sequence *x(n)* and the output DFT *X(k)* in the two-dimensional arrays are read across from row *1* through row *5*, we obtain the following sequences:

INPUT ARRAY

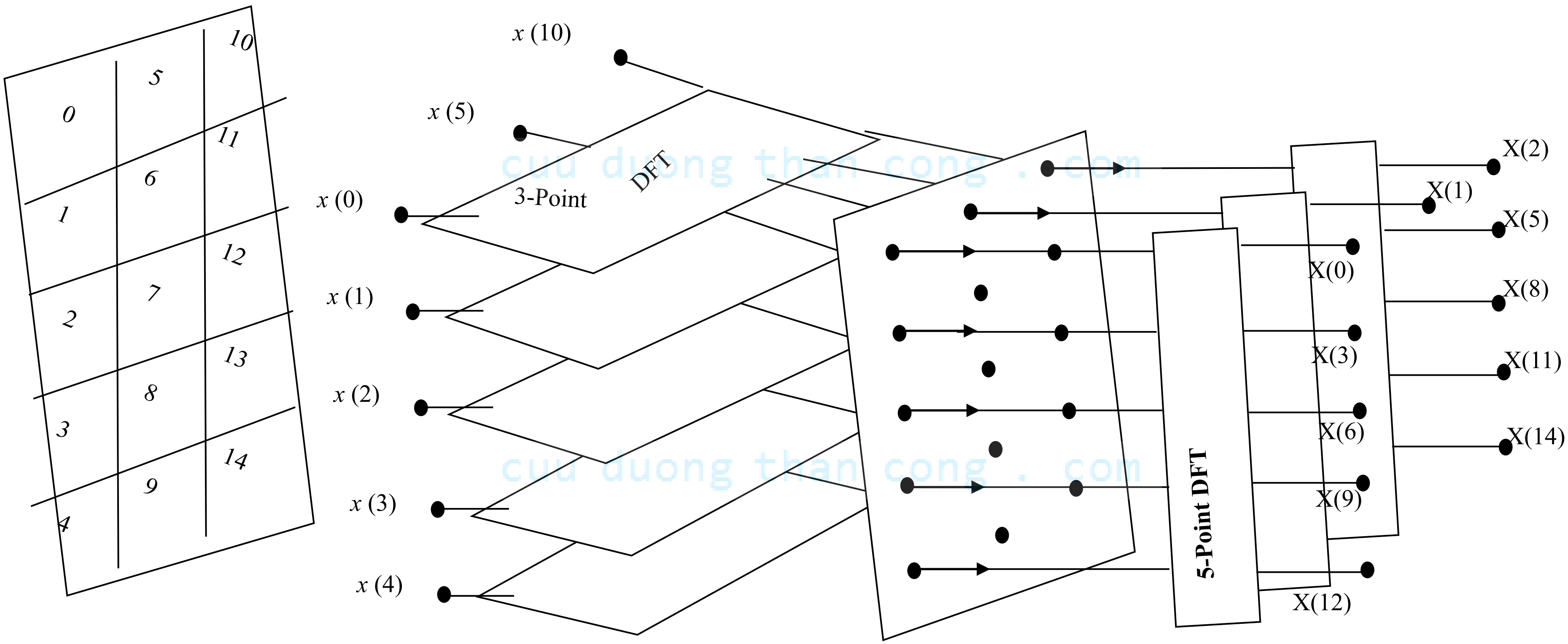
x(0) x(5) x(10) x(1) x(6) x(11) x(2) x(7) x(12) x(3) x(8) x(13) x(4) x(9) x(14)

OUTPUT ARRAY

X(0) X(1) X(2) X(3) X(4) X(5) X(6) X(7) X(8) X(9) X(10) X(11) X(12) X(13) X(14)

EXAMPLE 6.1.1

Figure 6.3 Computation of $N = 15$-point DFT by means of 3-point and 5-point DFTs



EXAMPLE 6.1.1

We observe that the input data sequence is shuffled from the normal order in the computation of the DFT. On the other hand, the output sequence occurs in normal order. In this case the rearrangement of the input data array is due to the segmentation of the one-dimensional array into a rectangular array and the order in which the DFTs are computed. This shuffling of either the input data sequence or the output DFT sequence is a characteristic of most FFT algorithms.