Lecture #4

Basic Intent

This lecture will provide an overview of the use of capacitance measurements in sensors, and describe the fundamentals of accelerometers. At the end of the lecture, the student should be familiar with capacitance measuring systems, limiting factors of the measurement, and obtainable performance levels. Also the student should be familiar with the fundamentals of accelerometer operation, including the relationship between the mechanical characteristics of the sensor and its performance, and the limitations of the performance of most accelerometers.

Capacitive sensing

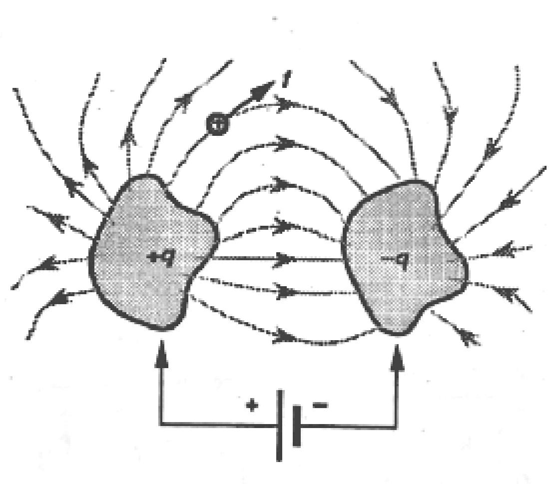

Fig. 1: Two Objects in Space

First, what is capacitance? Any two metallic objects, positioned in space, can have voltage applied between them (Fig. 1). Depending on their separation and orientation (sự định hướng) , the amount of charge that must be applied to the two elements to establish a certain voltage level varies. The capacitance is defined as the ratio of the charge to the voltage for a given physical situation. If the capacitance is large, more charge is needed to establish a given voltage difference.

In practice, capacitance between two objects can be measured experimentally. Predicting the capacitance between a pair of arbitrary (tùy ý) objects is very complicated, because it is necessary to know the electric field throughout the space between the objects. The field distribution is affected by the charge distribution, which is, in turn, affected by the field distribution. Iterative analytical techniques are generally required, and accurate calculations are very costly.

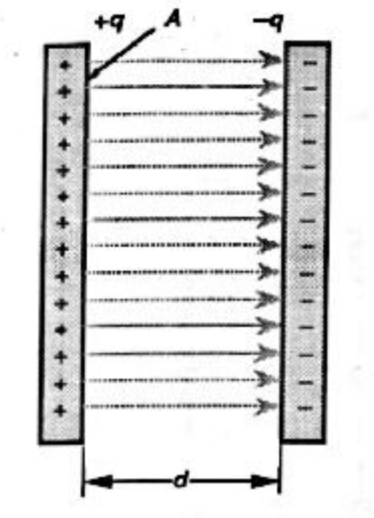

Fig. 2: Parallel Plate Capacitor

However, for simple geometry, the capacitance may be estimated very easily. For a pair of parallel plates (Fig. 2), separated by a distance which is much smaller than the lateral dimensions of the plates, the capacitance is given by:

$$C = \frac{\varepsilon \varepsilon_0 A}{d}$$

Clearly, the capacitance is increased by increasing the area of the electrodes or by reducing the separation between the plates. In addition, the capacitance can be increased by filling the gap with a medium with a large dielectric constant.

If you were to try to make a capacitor by placing electrodes close together, you could take electrodes with area 1cm x 1cm, and the best you could hope for would be a separation no smaller than 1um. This would amount to a capacitance of about 1000 pF, which isn't very big but still about the biggest you would ever expect to find in a real sensor. More generally, capacitive sensors have capacitance closer to 100 pF or less.

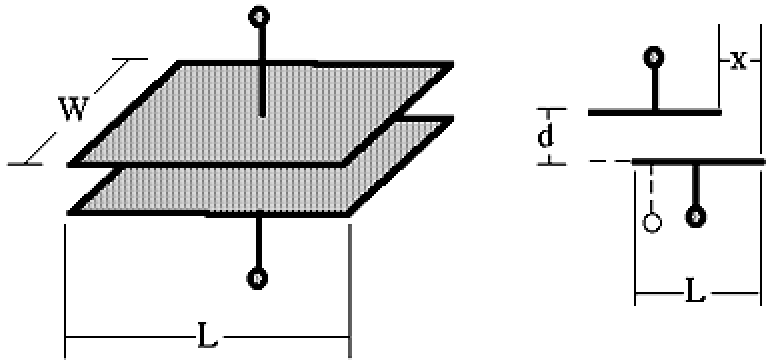

Fig. 3: Change in Capacitance due to the Lateral Movement of the Plates

Capacitance measurement is used to detect the motion of a sensor element. A simple example would involve the motion of one electrode in the plane parallel to the electrodes. Assume a pair of rectangular electrodes, as shown in figure 3, with dimensions Length (L) and Width (W). If one of the electrodes moves laterally a distance x, the capacitance changes

$$\text{from } \frac{\varepsilon\varepsilon_0 LW}{d} \text{ to } \frac{\varepsilon\varepsilon_0 W}{d}(L-x)$$

So, in this case, the capacitance signal changes linearly with displacement. To implement such a sensor, it is necessary to guarantee that the lateral motion does not also affect the separation between the electrodes, d. Also, this approach is difficult to use for measurement of very small lateral displacements, since a small lateral displacement would represent a very small fractional change in the capacitance of the sensor. For example, a 1um lateral displacement would cause only 10 PPM change in the capacitance of the capacitor geometry worked out earlier.

Lateral displacement capacitive transducers are useful for many applications, though. For example, rotary capacitive transducers were in wide use as tuning elements for AM/FM radios in recent years, and rotary variable capacitors are still available as adjustable circuit elements from electronic part suppliers.

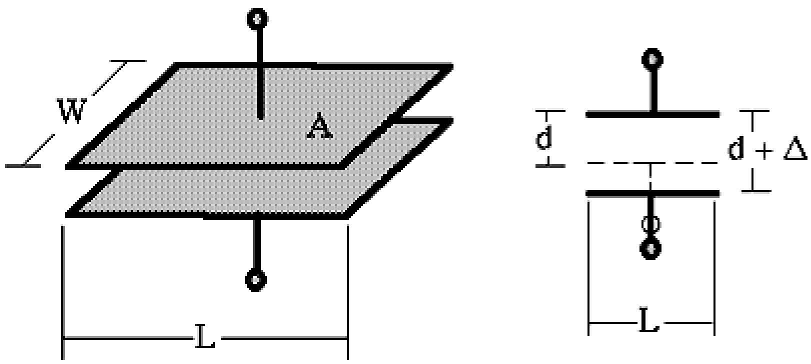

Fig. 4: Change in Capacitance due to the Change in Plate Separation

The most common use of capacitive detection for sensors is based on signals which are coupled to changes in the electrode separation, d. As shown in Fig. 4, consider a pair of electrodes with area A and separation d. A physical signal causes the separation to increase by a small quantity Delta. The capacitance changes

$$\text{from } \frac{\varepsilon\varepsilon_0 A}{d} \text{ to } \frac{\varepsilon\varepsilon_0 A}{d+\Delta}$$

Now, the relationship between the displacement and change in capacitance isn't obviously (rõ ràng) linear, but for small changes in separation, we can approximate the capacitance through use of a Taylor series expansion. In general, any function, *F*(*d*) can be approximated in the neighborhood of some nominal value d(0) as follows :

$$F(d_0+\Delta) = F(d_0) + \Delta\frac{\partial F(d_0)}{\partial d} + \frac{\Delta^2}{2}\frac{\partial^2 F(d_0)}{\partial d^2} + \ldots$$

For the expression above, this expansion takes the form :

$$C \approx \frac{\varepsilon\varepsilon_0 A}{d}\left(1 - \frac{\Delta}{d} + \frac{\Delta}{d^2}\right)$$
$$\text{where } \Delta = d - d_0$$

So, for $\Delta << d$, the capacitance change is linear with respect to displacement. The nonlinearity appears as a correction term of order $\frac{\Delta^2}{d^2}$. As long as we aren't concerned about errors of this order, the signal is very nearly linear.

If the initial separation between the capacitive electrodes is a few microns, a 1% change in the capacitance would indicate a displacement of a few tens of nanometers, which is a very small deflection. Such a measurement should be considered well within the capabilities of capacitive sensing.

Nice features associated with such a measurement include good sensitivity to very small deflections and no natural sensitivity to temperature. Precision fabrication (chế tạo 1 cách chính xác) is required, since it is necessary to produce electrodes which are very close to one another and highly parallel. So, capacitive sensing is generally used for situations in which a precision measurement is required, and the expense associated with the sensor fabrication is acceptable.

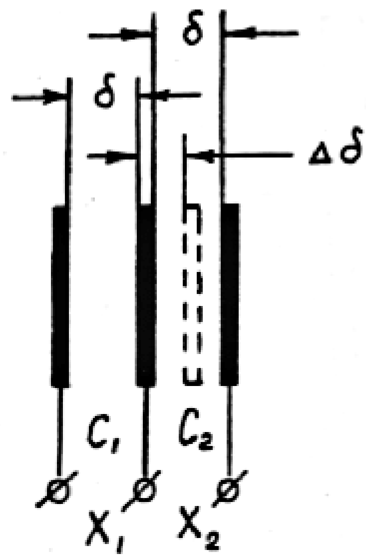

Fig. 5: Differential Capacitor

One technique for reducing the effect of the nonlinearity relies on the use of a differential capacitor, as is shown in Fig. 5. In this case, the capacitance measuring circuit is set up to measure the difference between the two capacitances, which is expressed as :

$$\Delta C = C_2 - C_1 \approx \frac{\varepsilon\varepsilon_0 A}{d-\Delta} - \frac{\varepsilon\varepsilon_0 A}{d+\Delta}$$

$$\Delta C \approx \frac{\varepsilon\varepsilon_0 A}{d}\left(1+\frac{\Delta}{d}+\frac{\Delta^2}{d^2}\right) - \frac{\varepsilon\varepsilon_0 A}{d}\left(1-\frac{\Delta}{d}+\frac{\Delta^2}{d^2}\right)$$

$$\Delta C \approx \frac{\varepsilon\varepsilon_0 A}{d}\left(\frac{2\Delta}{d}\right)$$

In this case, the nonlinearity associated with the $\frac{\Delta^2}{d^2}$ term is subtracted away, and the first nonlinearity appears as a cubic $\frac{\Delta^3}{d^3}$ term, which should be substantially smaller than the squared term.

Why do we care so much about linearity in capacitive sensors. Generally, capacitive measuring techniques are only applied in cases where precision (chính xác) measurement is necessary - otherwise, a strain gauge based measurement would suffice. One example of such a measurement is the measurement of acceleration for inertial navigation applications. A common problem in navigation situations is due to vibrations of the vehicle.

In inertial navigation, one is generally taking the output of an accelerometer, and integrating twice with respect to time to obtain displacement. Because of the nature of this integration, offset errors in the output of the accelerometer accumulate as errors in position as t^2. Therefore, inertial navigation applications are especially concerned about offset errors.

If an accelerometer with a small nonlinearity in the form of a Δ^2 term is used in a situation which includes a vibration, there will be a displacement of the form $\Delta \sin(\omega t)$. There will be a term in the output of the sensor of the form

$$\frac{\Delta^2}{d^2} \approx \frac{\Delta^2 \sin^2(\omega t)}{d^2} \approx \frac{\Delta^2 (1 - \cos(2\omega t))}{2d^2}.$$

Note that this expression includes an oscillating (dao động) term and a static (tĩnh) term. Generally, this phenomenon is referred to as vibration rectification - the process of generating a dc offset signal from a vibration signal. As described above, inertial navigation is one application which is particularly concerned about such phenomena, and so cancellation of nonlinearities in capacitive sensing is very important for such applications.

The textbook gives several good examples of capacitive sensing circuits and applications. The switched capacitor sensing circuit shown on page 353 (fig. 7-5) is a particularly good example of the use of a square-wave oscillation and FET switches to sample and rectify a waveform in order to convert a capacitance to a voltage. Such circuits are becoming very common because it is a simple matter to design an entire circuit of this type as an integrated circuit on a single silicon chip.

Among the kinds of sensors which can use capacitance measurement to detect physical signals are pressure sensors, accelerometers, position detectors, level sensors, and many others, all of which are shown as examples in the book. Several of these examples will be studied in more detail later in the course.

In general, capacitance detection is a good way to measure displacement. If implemented carefully, very small displacements may be measured. Measurement of such small displacements requires fabrication of precise mechanical structures with small gaps between the electrodes of the transducer. In addition, well designed and very well-packaged circuitry is necessary to carry out precision measurements. As a result, capacitance detection is best suited to applications which require better performance than can be obtained from a strain gauge, and where the added cost of the capacitance detection is allowed.

Capacitive detection has the advantage that it is not directly sensitive to temperature. However, the output of a capacitive transducer is not immediately linear. If linearity is important, differential capacitance schemes are advisable.

Accelerometer overview

Accelerometers are devices that produce voltage signals in proportion to the acceleration experienced. There are several techniques for converting an acceleration to an electrical signal. We will overview the most general such technique, and then look briefly at a few others.

The most general approach to acceleration measurement is to take advantage of Newton's law, which states that any mass that undergoes an acceleration is responding to a force given by F = ma.

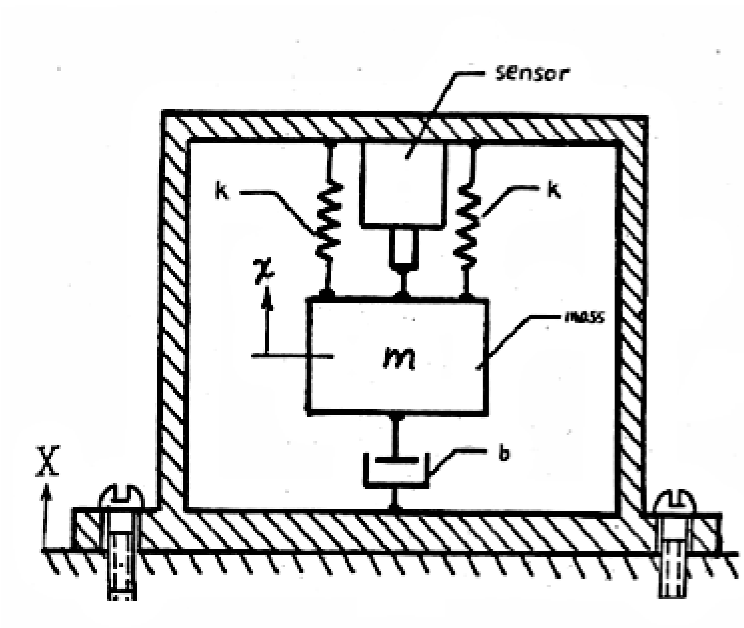

Fig. 6: A General Accelerometer

The most general way to take advantage of this force is to suspend a mass on a linear spring from a frame which surrounds the mass, as shown in Fig. 6. When the frame is shaken, it begins to move, pulling the mass along with it. If the mass is to undergo the same acceleration as the frame, there needs to be a force exerted on the mass, which will lead to an elongation of the spring. We can use any of a number of displacement transducers (such as a capacitive transducer) to measure this deflection.

For the case shown in Fig. 6, the sum of the forces on the mass is equal to the acceleration of the mass:

$$k(X-x)+b\frac{d(X-x)}{dt}=m\frac{d^2x}{dt^2}$$

We make assignments

$$Z=X-x$$
$$x=X-Z$$

and we have :

$$m\frac{d^2X}{dt^2}=m\frac{d^2Z}{dt^2}+kZ+b\frac{dZ}{dt}$$

Now, since X is the position of the frame, we can impose an acceleration on this problem by forcing X to take the form $X=X_0e^{i\omega t}$. If we assume all the time varying quantities also oscillate, we need $Z=Z_0e^{i\omega t}$. Substituting these into the above equation, we have:

$$-m\omega^2X_0e^{i\omega t}=-m\omega^2Z_0e^{i\omega t}+kZ_0e^{i\omega t}+i\omega bZ_0e^{i\omega t}$$

Canceling, assigning $\omega_0=\sqrt{\frac{k}{m}}$, and rearranging gives:

$$Z_0=\frac{m\omega^2X_0}{m\omega^2-k-i\omega b}=\frac{X_0}{\sqrt{\left(1-\frac{\omega_0^2}{\omega^2}\right)-\frac{b^2}{m^2\omega^2}}}$$

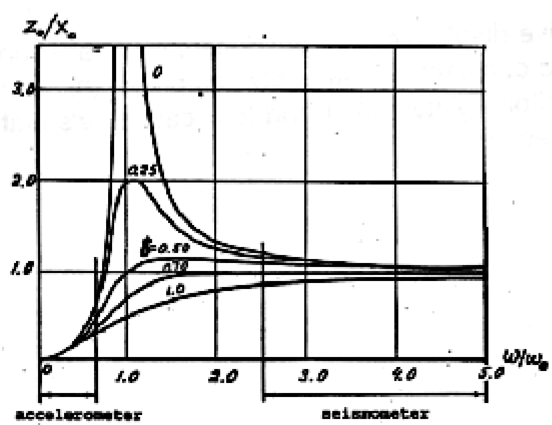

Fig. 7: Amplitude Response of Vibration-measuring Instruments

Things to note about this expression:

1. If b = 0 (no damping), this expression blows up at $\omega = \omega_0$. This means that signal which occurs at the resonance of an undamped accelerometer can lead to infinitely large. This is one of the reasons that accelerometer designers generally impose finite damping on the system.
2. If $\omega < \omega_0$, this expression simplifies to:

$$Z_0 = \frac{\omega^2 X_0}{\omega_0^2} = \frac{A}{\omega_0^2}$$

   In this case, the displacement of the mass is proportional to the acceleration of the frame. This is the response we would hope for from an accelerometer.

3. If $\omega > \omega_0$, the expression simplifies to:

$$Z_0 = X_0.$$

   This is the case for high frequency signals, during which the mass remains stationary, and the accelerometer frame shakes around it. In this case, the displacement between the mass and the frame is the same size as the motion of the frame. This mode of operation is generally referred to as `seismometer mode'. Seismometers are instruments which attempt to measure ground motion, rather than ground acceleration.

So, the general accelerometer consists of a mass, a spring, and a displacement transducer. The overall performance of accelerometers is generally limited by: the mechanical characteristics of the spring (linearity, dynamic range, cross-axis sensitivity), and the sensitivity of the displacement transducer.

Many different displacement transducers can be used in accelerometers. It is generally easy to design a mechanical system which is well enough behaved that the performance of the accelerometer is limited by the displacement transducer. Examples of displacement transducers which may be used include:

- capacitive transducers
- strain gage transducers
- optical transducers (laser interference measurement)
- resistive transducers
- electromagnetic transducers

In each case, the transducer is configured to measure the displacement of the mass relative to the frame.

How large are the displacements being measured?

Suppose we want a device to measure 1 milli-g accelerations with a bandwidth of 20 kHz, $\omega_0 = 2\pi \cdot 2 \times 10^4$. Then, we need a displacement transducer capable of resolving displacements given by:

$$Z_0 = \frac{\left(1 \times 10^{-3} g\right)\left(9.8\, m/s^2\right)}{\left(2\pi \cdot 2 \times 10^4\right)} = 6.2 \times 10^{-13} m$$

.

This is a very small displacement! Such a device is not easily made, and measurements with this level of accuracy are difficult to carry out. We can see that the difficulty comes in large part from the very large bandwidth (20 kHz) requested from this device. If we were to reduce this request to 1 kHz, the required resolution would increase by a factor of 400, which would make the displacement detection problem much easier.

Conclusions (Kết luận)

We have looked at the use of capacitance detection for use in physical sensors. For applications which require accurate measurement of small signals, capacitance detection is a good selection. The performance of a capacitive detection system is obtained at a cost, and this cost is generally more than 10x higher than a device made with piezoelectric or piezoresistive technology.

We then looked at the design and operation of accelerometers. Depending on the capabilities of the detection system, it is possible to have miniature devices for measurement of signals from the milli-gs down into the micro-gs. The best of these sensors is based on capacitance detection of a very small displacement and is on the market for a few thousand dollars. On the other end of the market, there are devices available for a few tens of dollars based on piezo technologies.

In the next lecture, we will look at a particular accelerometer which has recently entered the market, and examine some of the tradeoffs in its design, performance, and cost.

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