Lecture #8

Basic Intent

This lecture is intended to overview basic techniques for sensing temperature, and study some product examples. Following that, some techniques for the measurement of flow will be briefly highlighted.

Thermometers

There are a number of well-known historical technologies for the measurement of temperature. Everyone is familiar with the mercury thermometer, in which a reservoir of mercury is sealed in a glass container under vacuum.

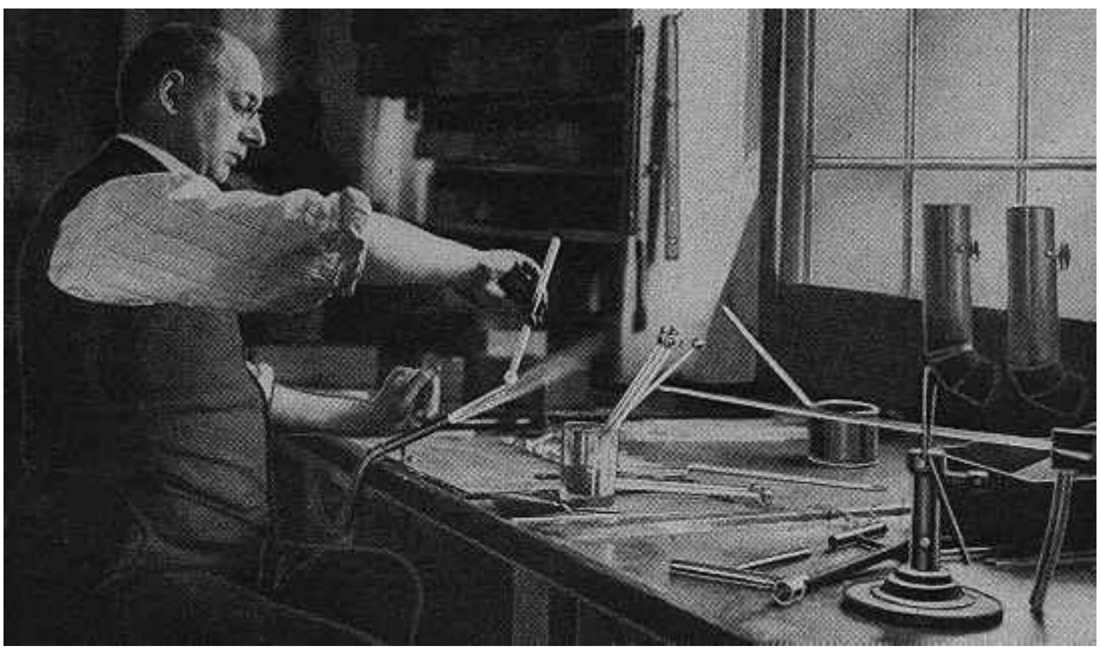

Fig. 1: Filling of a Mercury Thermometer

When the reservoir is heated, the mercury expands, rising through a long thin column, upon which a graded ruler has been etched. What sort of sensitivity can be expected for such a system?

Well, the thermal expansion coefficient of mercury is well known to be about 30 PPM/K. If we assume that dimensions of the container do not change appreciably, **(Is this a good assumption?)** then the mercury in the column expands linearly with temperature.

$$\frac{\Delta V}{V} = \frac{\Delta T}{T}$$

$$\Delta V = \pi R^2 \Delta L$$

Therefore,

$$\Delta L = \frac{V}{\pi R^2}\frac{\Delta T}{T}$$

If we want 1 mm/K at room temperature, and we have a reservoir volume of 0.1 cm^3, we need:

$$R = \sqrt{\frac{1 \times 10^{-7} m^3}{\pi \cdot 1 \times 10^{-3}\, {}^{m}\!/\!_{K}}\frac{1}{300K}} = 3.3 \times 10^{-4} m$$

Clearly, the sensitivity depends very strongly on the diameter of the column. Historically, makers of thermometer tubes worked very hard to control column diameter. Nevertheless, it was important to calibrate each thermometer with an ice point and a boiling water reference.

Other traditional techniques for temperature measurement based on thermal expansion are very popular even today. As we saw in the thermostat lab, a great many home thermostats still rely on differential expansion in a bimorph to close a switch. Also, toaster ovens that are a few years old still feature bimetal temperature switches to operate the timing feature of the oven. Bimetal switches are fairly inexpensive, can operate reliably for many cycles, and may still be the correct choice for temperature sensing applications.

In many cases, bimetal temperature switches are not accurate enough, or do not allow operation over a broad temperature range.

For high temperature applications, thermocouple thermometers are often used. The thermocouple is an interesting device from a physics standpoint, but its operation can be easily understood from a thermal model.

All metal wires may be considered as tubes filled with a fluid of electrons. The electrons are more or less free to move about in the tube, and certainly move in a preferential direction when a voltage is applied (voltage acts like a pressure). If the density of the electrons is non-uniform, the non-uniform charge distribution exerts a force on the electrons, tending to even them out. This charge effect is similar to a finite compressibility in a normal fluid.

Now, suppose a tube filled with electrons is heated at one end. The effect of heat is to increase the average thermal velocity (because 1/2 mv^2 is proportional to 1/2 k_BT) of the heated electrons. In this situation, the effect of heat is to increase the average velocity of the electrons on the heated end of the wire. Because of this velocity increase, `warm' electrons will leave the heated end of the wire faster than they can be replenished by `cold' electrons from the other end. This will lead to a non-uniform density distribution which gradually increases until the electrostatic pressure (because of the charges) is large enough to balance it.

If one attached the leads of a voltmeter to this wire, there would be an excess of electrons at the cold end, and a net voltage difference across the wire. Since the cold end has an excess of electrons, it is repulsive to additional electrons, and is therefore at a `low' voltage with respect to positive charges.

The amount of voltage difference is approximately proportional to temperature, and depends on materials properties (e.g. electron mobility and thermal conductivity). Tables of thermocouple properties are widely published.

Now, it is generally inconvenient to attach the leads of a voltmeter across the wire, especially since one end is at the point of temperature measurement. Instead, it is common to use a pair of wires made from 2 different materials. The wires are joined at the end which is to be the point of temperature measurement (the `junction'), and the voltage is measured across the other two ends. If the materials have different thermocouple effects, there will be a voltage difference across those wires. Tables of the thermocouple voltages for a set of standard pairs are also widely published. A good pair of materials for a thermocouple are any materials which can each survive the environment of the measurement, do not react with each other, and have suitable different thermocouple coefficients.

The voltages generated by such effects are fairly small. A good thermocouple exhibits a voltage signal of only 10 uV/Kelvin. Therefore, accurate measurements of small temperature changes require very well-designed electronics. For measurements which require accuracy of +/- 10 K, and need to be carried out at temperatures near 1000K, thermocouples are definitely the way to go.

There are also many examples of thermometers based on resistance changes. We have already seen several examples of resistance changes which are considered to be a problem in measurements of other quantities (piezoresistive strain gauges, for example).

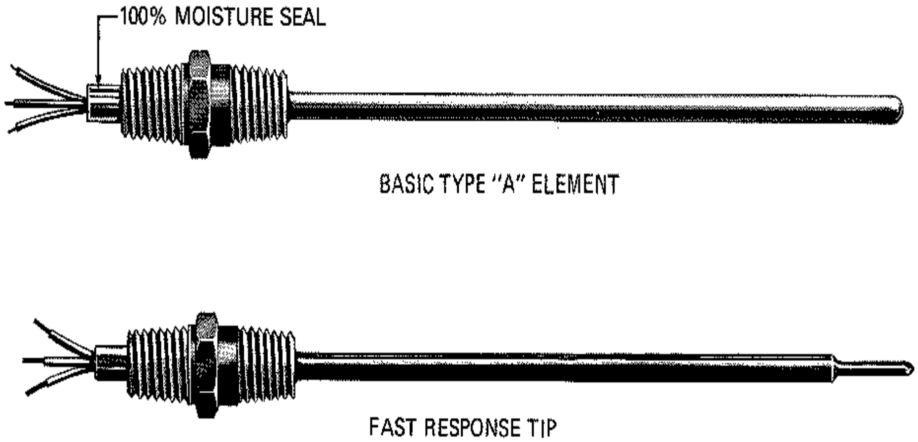

Fig. 2: RTD probes.

Platinum wires are commonly used for resistance thermometry. Even thought platinum is quite expensive, it is favored for these applications for several very good reasons.

Do you know what they are?

For narrower temperature ranges, there are a large assortment of resistance thermometers. ordinary carbon resistors can be used, but companies such as **Thermometrics** offer a very broad collection of resistance thermometers in different shapes, sizes and characteristics.

An important parameter for a resistance thermometer is the temperature coefficient, generally denoted as **Alpha**. The temperature coefficient is defined as the fractional change in resistance per unit change in temperature. This definition is convenient because of the way it emerges from expressions which are associated with voltage divider-based resistance measurement.

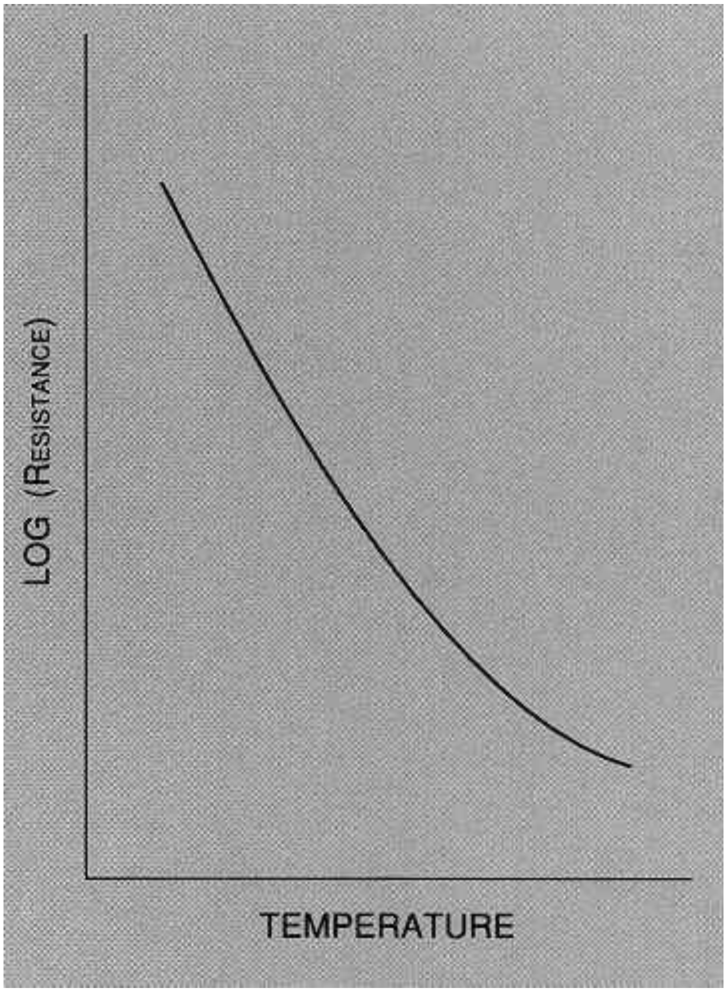

Fig. 3: NTC thermistor.

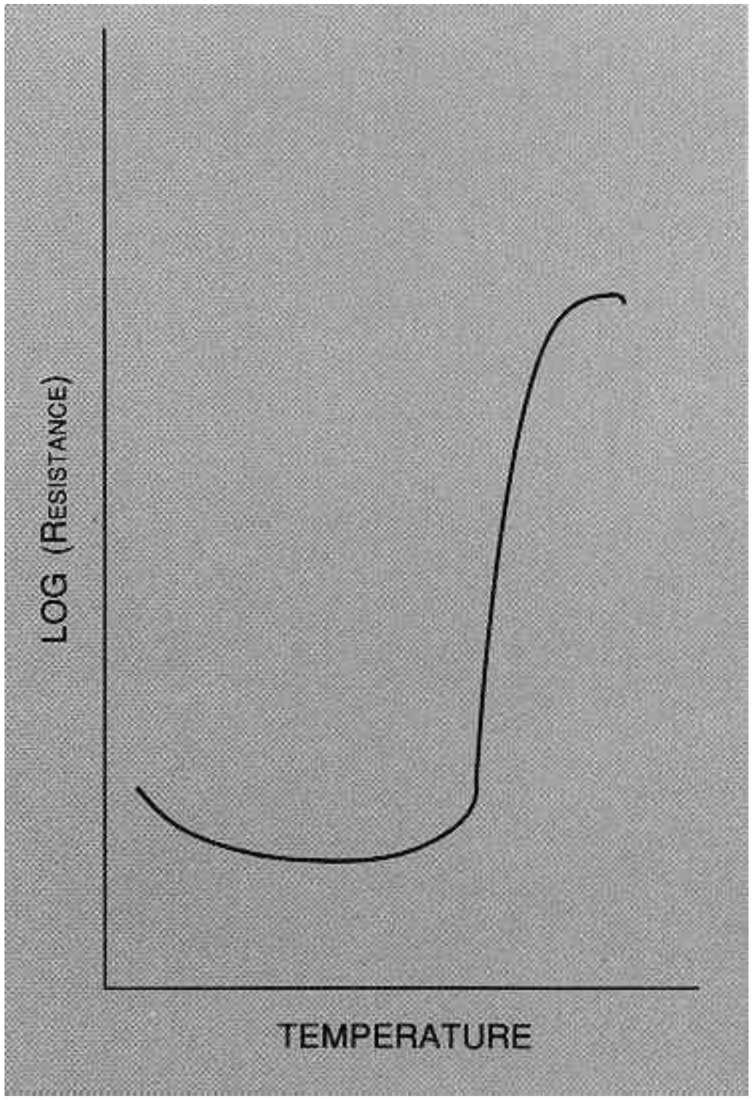

Fig. 4: PTC thermistor.

For a voltage divider with a thermistor of average resistance R_1 and a load resistor with resistance R_L, the voltage at the output is given by

$$V_{out} = V_{in}\frac{R_1}{R_1 + R_L}$$

Now, if $R_L >> R_1$, we have

$$V_{out} = V_{in}\frac{R_1}{R_L}$$

If the temperature of R_1 changes by 1 K, the resistance changes by (*Alpha* R_1), so the voltage changes by *Alpha*(V_{in} R_1/R_L). The definition of the temperature coefficient as a fractional change in resistance per unit change in temperature produces a result in which the fractional change in voltage per unit change in temperature is given by alpha as well.

When using a thermistor to measure small temperature changes, noise can impose a limitation. For example, all resistive elements exhibit voltage noise known as Johnson noise with density given by

$$V_{noise} = \sqrt{4k_B RT\Delta F} \text{ (in volts)}$$

The resolution of a thermistor limited by Johnson noise would be given by:

$$\text{Resolution} = \frac{\text{Noise}}{\text{Sensitivity}}$$

$$\frac{V_{noise}}{\Delta V/\Delta T} = \frac{\sqrt{4k_B RT\Delta F}}{V_{in}\alpha \frac{R_1}{R_L}}$$

We can see from this that the resolution is improved by reducing the temperature, the bandwidth, and the load resistor, and by increasing the temperature coefficient and the bias voltage, $\boldsymbol{V_{in}}$. Of all of these parameters, it may be easiest to increase the bias voltage.

However, there is a problem to watch out for. The bias in this system also causes power to be dissipated in the sense resistor. We may assume that the sense resistor is attached to the object of interest with a finite thermal conductance **G**. There will be a temperature difference between the sense resistor and the object of interest given by:

$$\Delta T = \frac{P}{G} = \frac{V_s^2}{R_1 G} = \frac{V_{in}^2 R_1}{RL^2 G}$$

So, we can see that increasing the bias voltage can easily lead to problems. Since thermal conductance between resistance thermometers and the surface are generally of order 10^{-2} to 1 W/K, we see that power dissipation in the mW range can cause substantial errors.

Because of this, manufacturers of resistance thermometers generally provide extensive information on electrical measurement conditions to be used with their products. Thermometrics, for instance, provides a lengthy (35 page) tutorial on their products in their standard catalog.

In addition, there are a number of solid-state thermometer technologies which have recently become commercially feasible. National Semiconductor, among other companies, is marketing a family of thermometers based on diodes. The leakage current in a forward-biased diode is generated by thermal excitation of electrons over an energy barrier. At temperatures near room temperature, the voltage across a current-biased diode is small and linearly dependent on temperature. **National Semiconductor** makes a series of such devices which produce very nice voltage outputs over a decent range, and at low cost. These devices look like 3-terminal transistors, require 5V and ground, and produce an easily measured voltage output.

Finally, a number of commercially-available `non-contact' temperature sensors are available which are based on measurement of infrared radiation. This approach can be expensive, and can suffer from some calibration errors. The infrared radiation from any object is given by:

$$\text{Radiated Power} = (\text{Emissivity})\left(5.67 \times 10^{-12}\, W/cm^2K^4\right)\left(T^4\right)$$

For a room temperature object, this comes to 45 mW/cm^2. This isn't much power, but is easily detectable with a good sensor.

The difficulty lies in that the emissivity may be very difficult to know with any precision, since it depends critically on surface finish, thin coatings of residue, and may depend on temperature. Also, the radiation emitted by the object of interest may be absorbed by the atmosphere. The atmosphere is reasonably transparent in some regions of the IR (3-5 microns, 8-12 microns), and basically opaque in others (6-7 microns, 14-20 microns). Therefore, all of the radiation emitted by the surface cannot be expected to arrive at the detector.

Finally, the radiation emitted by the surface is distributed more or less uniformly over the entire field of view. Therefore, the amount available for detection is substantially less, unless the instrument being used has a very large primary optical element.

All things considered, this is a difficult approach to use, and can be expensive. Good IR detectors must be cooled, required an expensive package and operating system. Nevertheless, these products are becoming popular for manufacturing applications, and device and packaging costs can be expected to continue falling as the market grows...

For any thermometer, there are issues associated with measurement of changing temperature that need to be considered. Consider a general situation in which a thermometer is attached to an object with a thermal conductance of G (W/K). The thermometer is a physical object, and has a heat capacity C (J/K). Assume that some power, P_{in}, is being applied to the thermometer (bias currents?) Finally, assume that the temperature of the object is oscillating in time according to:

$$T_o = T_{o1} + T_{o2}e^{i\omega t}$$

where w is the frequency of oscillation. We assume that the temperature of the thermometer is also oscillating, and that the frequencies are the same, so the temperature of the thermometer may be expressed :

$$T_s = T_{s1} + T_{s2}e^{i\omega t}$$

From energy balance, we know that the energy into the thermometer equals the change in energy of the thermometer:

$$P + G(T_o - T_s) = C\frac{dT_s}{dt}$$

We can plug in our expressions for T_o and T_s, and we have:

$$P + G\left(T_{o1} + T_{o2}e^{i\omega t} - T_{s1} - T_{s2}e^{i\omega t}\right) = i\omega C T_{s2}e^{i\omega t}$$

We may separate the static and oscillating parts to give equations:

$$P + G(T_{o1} - T_{s1}) = 0$$
$$\Rightarrow T_{s1} = \frac{P}{G} + T_{o1}$$

$$G(T_{o2} - T_{s2}) = i\omega C T_{s2}$$
$$\Rightarrow T_{s2} = \frac{T_{o2}}{1 + i\omega\frac{C}{G}}$$

So, we should expect a finite temperature offset due to the bias power (P/G), and an oscillation amplitude which varies with frequency. At low frequency, T_{s2} is nearly equal to T_{o2}, as we would hope for. At higher frequencies, the thermal time constant associated with the heat capacity of the thermometer can cause a reduce oscillation and a phase lag. These issues are important to keep in mind for measurements of time-varying temperatures.

Parts after this may be repeated: We build a resistance measuring circuit in the form of a voltage divider with a load resistor (RL) in series with the thermistor (Rs). As always, the voltage at the output is given by

$$V_{out} = V_{in}\frac{R_s}{R_s + R_L}$$

The resistance of the thermistor is

$$R_s = R_o + \alpha R_o \Delta T$$

So,

$$V_{out} = V_{in}\frac{R_o + \alpha R_o \Delta T}{R_L + R_o + \alpha R_o \Delta T}$$

Now, this is clearly a mess, and it is hard to see what the actual response function will look like. To make some sense of it, we will do a Taylor series expansion of $V_{out}(R_s)$ =

V_{in} $R_s/(R_s + R_L)$ about $\Delta T = 0$, and extract the offset, the slope and the nonlinearity of this response.

As mentioned in an earlier lecture, the Taylor series expansion is:

$$F(x) = F(x_0) + (x - x_0)\frac{\partial F(x)}{\partial x} + \frac{(x - x_0)^2}{2}\frac{\partial F^2(x)}{\partial x^2} + \ldots$$

For our expression for $V_{out}(R_s)$, we have:

$$\frac{V_{out}}{V_{in}} = \frac{R_o}{R_L + R_o} + (R_s - R_o)\frac{R_L}{(R_L + R_o)^2} - \frac{(R_s - R_o)^2}{2}\frac{2R_L}{(R_L + R_o)^3}$$

Since Rs - Ro = Alpha Ro ΔT, we have

$$\frac{V_{out}}{V_{in}} = \frac{R_o}{R_L + R_o} + (\alpha R_o \Delta T)\frac{R_L}{(R_L + R_o)^2} - (\alpha R_o \Delta T)^2\frac{R_L}{(R_L + R_o)^3}$$

The sensitivity is defined as the derivative of the voltage with respect to the temperature evaluated at $\Delta T = 0$, so we have

$$\frac{\partial V_{out}}{\partial T} = V_{in}\frac{\alpha R_o R_L}{(R_L + R_o)^2}$$

The linearity of this system is essentially the maximum fractional error between the true response and the linear response. A good approximation to this quantity may be calculated by taking the ratio of the quadratic term in the expansion to the linear term.

$$\text{Linearity} = \frac{\text{Quadratic Term}}{\text{Linear Term}}$$
$$\text{Linearity} = \frac{(\alpha R_o \Delta T)^2 \frac{R_L}{(R_L + R_o)^3}}{(\alpha R_o \Delta T)\frac{R_L}{(R_L + R_o)^2}}$$

which simplifies to:

$$\text{Linearity} = \frac{\alpha R_o \Delta T}{R_L + R_o}$$

We see that the linearity is improved (by reducing it) by taking $R_L \gg R_o$, and by keeping either ΔT or Alpha small. Simply put, if we need a certain ΔT and a certain linearity, we can select the thermometer Alpha and the load resistance to meet our needs.

When using a thermistor to measure small temperature changes, noise can impose a limitation. For example, all resistive elements exhibit voltage noise known as Johnson noise with density given by

$$V_{noise} = \sqrt{4k_B RT\Delta F} \text{ (in volts)}$$

The resolution of a thermistor limited by Johnson noise would be given by:

$$\text{Resolution } = \frac{V_{noise}}{dV/dT} = \frac{\sqrt{4k_B RT\Delta F}}{V_{in}\frac{\alpha R_o R_L}{(R_L + R_o)^2}}$$

If we have made the simplifying assumption that $R_L >> R_o$, we have

$$\text{Resolution } = \frac{\sqrt{4k_B RT\Delta F}}{V_{in}\frac{\alpha R_o}{R_L}}$$

We can see from this that the resolution is improved by reducing the temperature, the bandwidth, and the load resistor, and by increasing the temperature coefficient and the bias voltage, V_{in}. Of all of these parameters, it may be easiest to increase the bias voltage.

However, increasing the bias voltage can cause heating problems.

Next some pictures considered in the development of these pages:

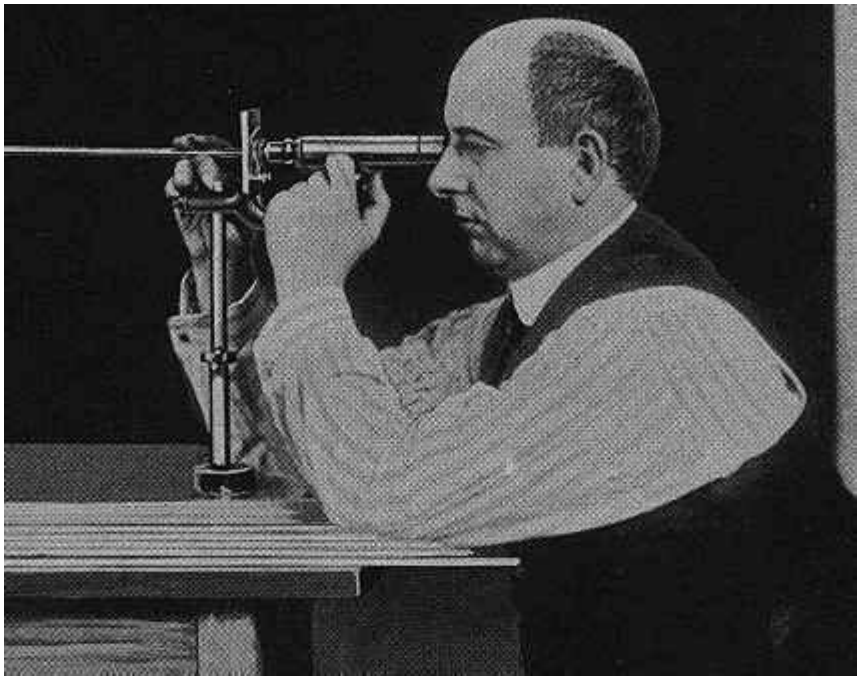

Fig. 5: Measuring the bore of a tube.

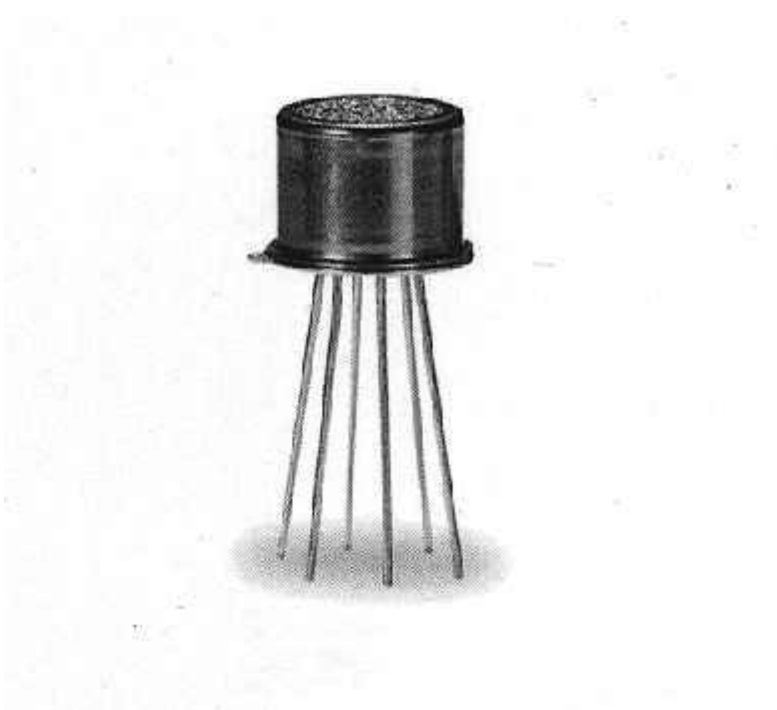

Fig. 6: Package of a thermometer.

Fig. 7: Thermometer probes.

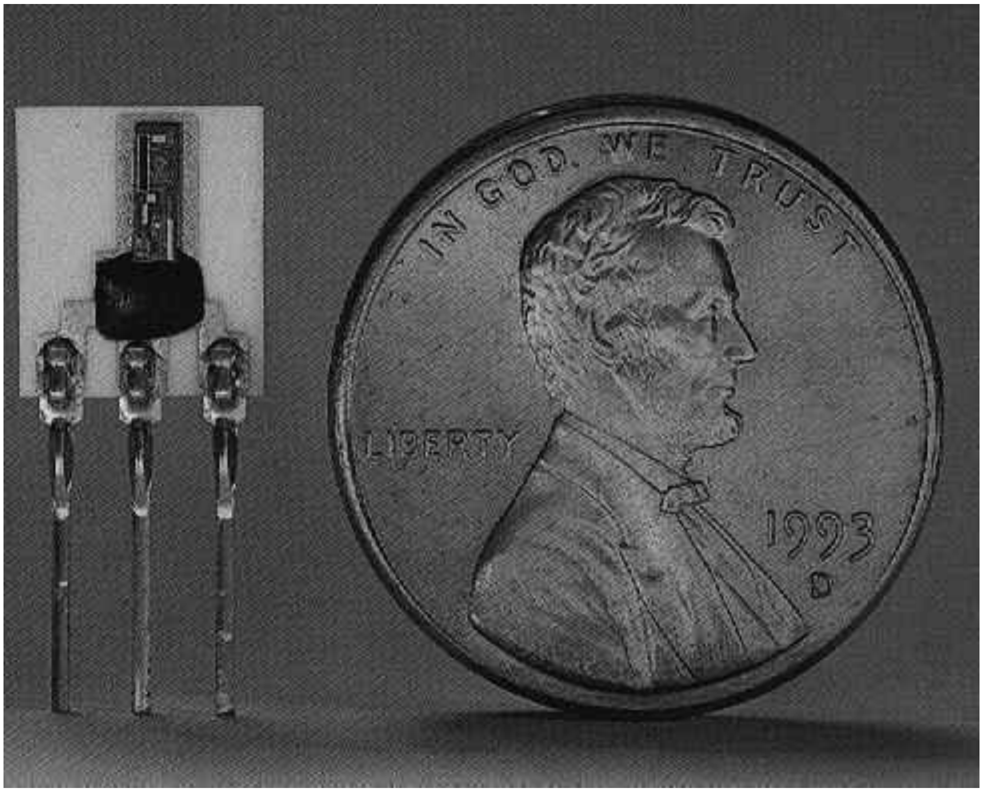

Fig. 8: Size of a thermometer.

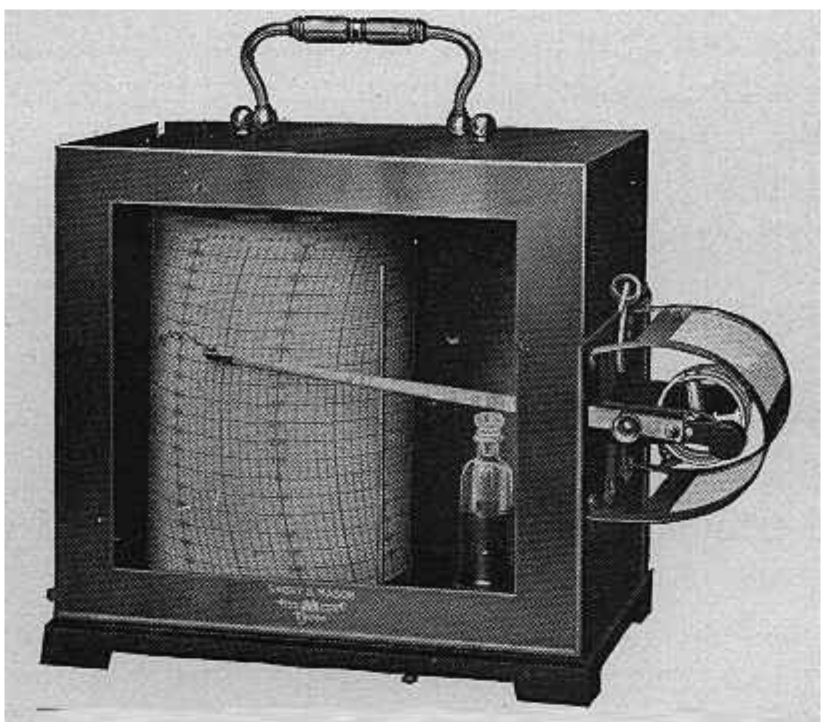

Fig. 9: Thermograph.