Digital Signal Processing

405109

Chapter 6

Transfer function and Digital Filter Realization

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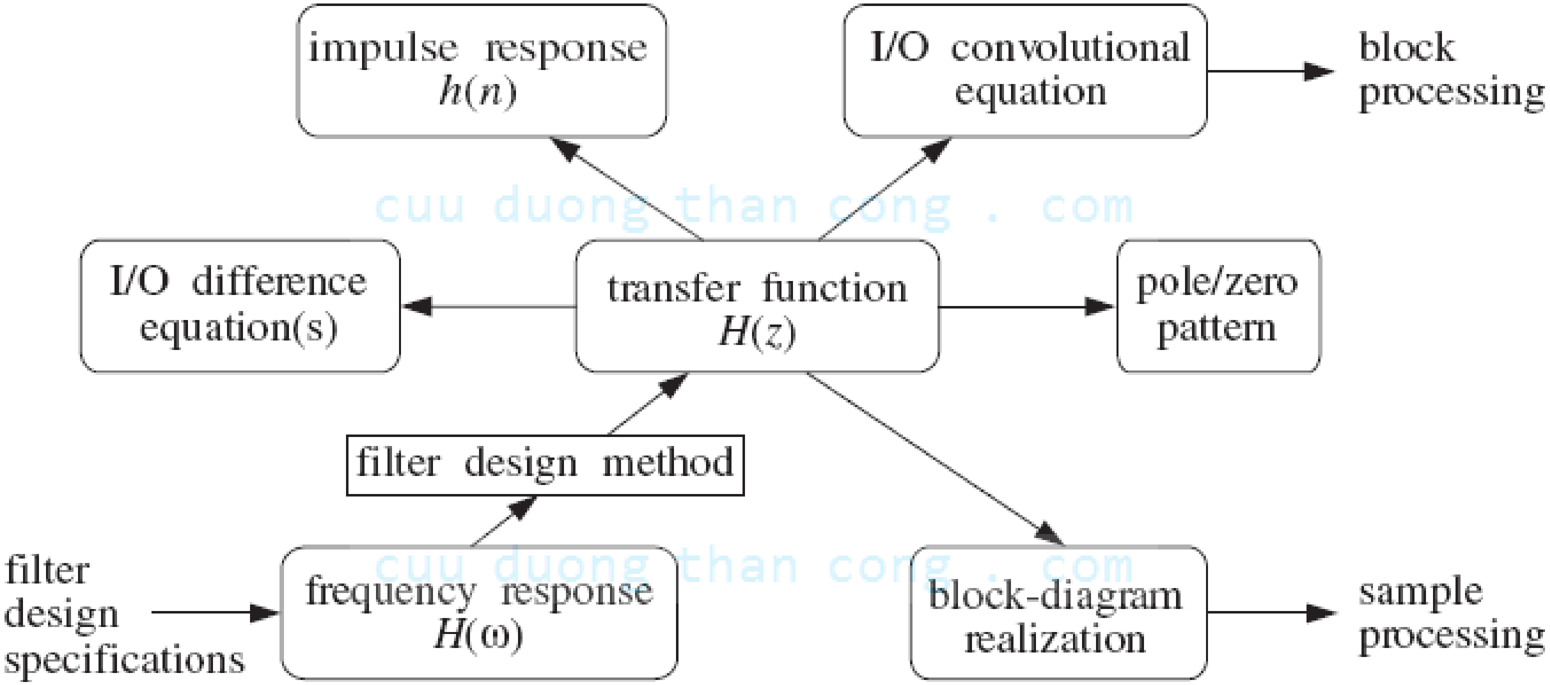
❖ With the aid of *z*-transforms, we can describe the FIR and IIR filters in several mathematically equivalent way



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1. Transfer functions

❖ Given a transfer functions H(z) one can obtain:

(a) the impulse response h(n)

(b) the difference equation satisfied the impulse response

(c) the I/O difference equation relating the output y(n) to the input x(n).

(d) the block diagram realization of the filter

(e) the sample-by-sample processing algorithm

(f) the pole/zero pattern

(g) the frequency response H(w)

Impulse response

❖ Taking the inverse z-transform of H(z) yields the impulse response h(n)

Example: consider the transfer function $H(z) = \dfrac{5 + 2z^{-1}}{1 - 0.8z^{-1}}$

To obtain the impulse response, we use partial fraction expansion to write

$$H(z) = \frac{5 + 2z^{-1}}{1 - 0.8z^{-1}} = A_0 + \frac{A_1}{1 - 0.8z^{-1}} = -2.5 + \frac{7.5}{1 - 0.8z^{-1}}$$

Assuming the filter is causal, we find

$$h(n) = -2.5\delta(n) + 7.5(0.8)^n u(n)$$

Difference equation for impulse response

❖ The standard approach is to eliminate the denominator polynomial of H(z) and then transfer back to the time domain.

Example: consider the transfer function $H(z) = \frac{5 + 2z^{-1}}{1 - 0.8z^{-1}}$

Multiplying both sides by denominator, we find

$$(1 - 0.8z^{-1})H(z) = 5 + 2z^{-1} \quad \Rightarrow \quad H(z) = 0.8z^{-1}H(z) + 5 + 2z^{-1}$$

Taking inverse z-transform of both sides and using the linearity and delay properties, we obtain the difference equation for h(n):

$$h(n) = 0.8h(n-1) + 5\delta(n) + 2\delta(n-1)$$

I/O difference equation

❖ Write $Y(z) = H(z)X(z)$ then eliminate the denominators and go back to the time domain.

Example: consider the transfer function $H(z) = \dfrac{5 + 2z^{-1}}{1 - 0.8z^{-1}}$

We have

$$Y(z) = H(z)X(z) = \frac{5 + 2z^{-1}}{1 - 0.8z^{-1}} X(z) \quad \Rightarrow \quad (1 - 0.8z^{-1})Y(z) = (5 + 2z^{-1})X(z)$$

which can write $Y(z) - 0.8z^{-1}Y(z) = 5X(z) + 2z^{-1}X(z)$

Taking the inverse z-transforms of both sides, we have

$$y(n) - 0.8y(n-1) = 5x(n) + 2x(n-1)$$

Thus, the I/O difference equation is $y(n) = 0.8y(n-1) + 5x(n) + 2x(n-1)$

Block diagram

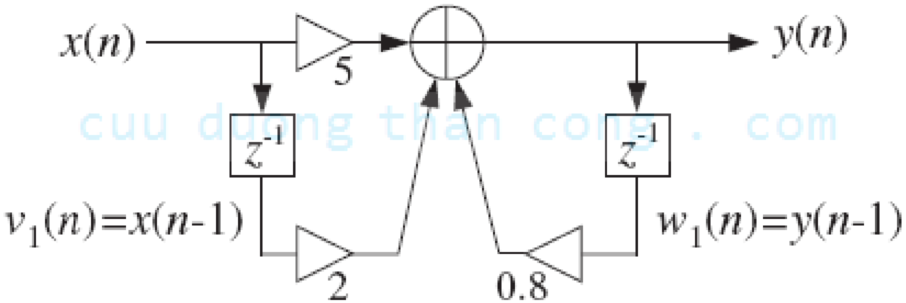
❖ One the I/O difference equation is determined, one can mechanize it by block diagram

Example: consider the transfer function $H(z) = \dfrac{5 + 2z^{-1}}{1 - 0.8z^{-1}}$

We have the I/O difference equation

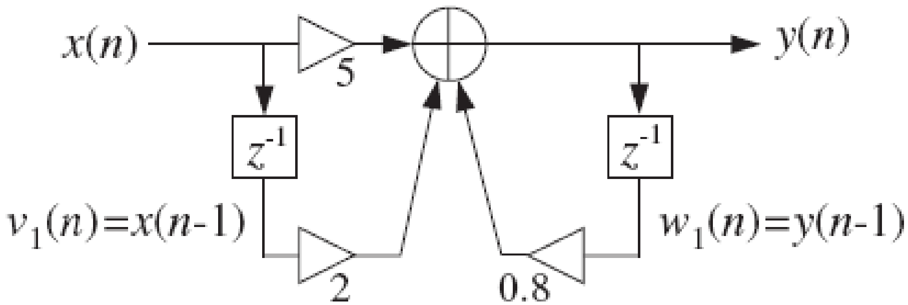
$$y(n) = 0.8y(n-1) + 5x(n) + 2x(n-1)$$

The direct form realization is given by



Sample processing algorithm

❖ From the block diagram, we assign internal state variables to all the delays:



We define $v_1(n)$ to be the content of the x-delay at time n:

$$v_1(n) = x(n-1) \quad \Rightarrow \quad v_1(n+1) = x(n)$$

Similarly, $w_1(n)$ is the content of the y-delay at time n:

$$w_1(n) = y(n-1) \quad \Rightarrow \quad w_1(n+1) = y(n)$$

(compute output) $y(n) = 0.8w_1(n) + 5x(n) + 2v_1(n)$

(update states) $v_1(n+1) = x(n)$

$w_1(n+1) = y(n)$

for each input sample x do:
$y = 0.8w_1 + 5x + 2v_1$
$v_1 = x$
$w_1 = y$

(direct form)

Frequency response and pole/zero pattern

- Given H(z) whose ROC contains unit circle, the frequency response H(w) can be obtained by replacing $z=e^{jw}$.

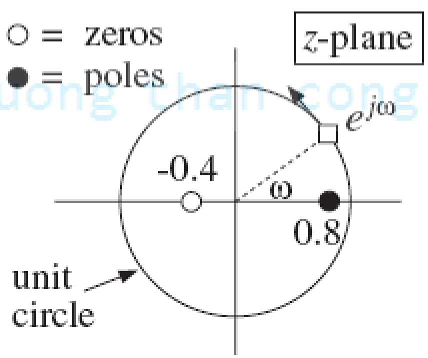
Example: $$H(z) = \frac{5(1+0.4z^{-1})}{1-0.8z^{-1}} \Rightarrow H(\omega) = \frac{5(1+0.4e^{-j\omega})}{1-0.8e^{-j\omega}}$$

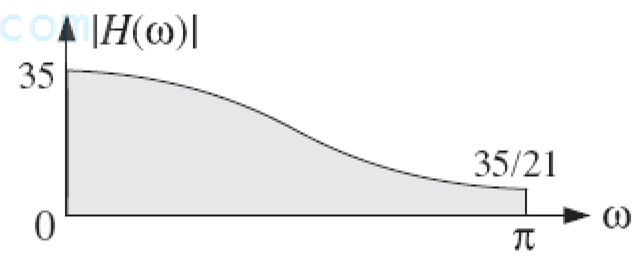
Using the identity $|1-ae^{-j\omega}| = \sqrt{1-2a\cos\omega + a^2}$

we obtain an expression for the magnitude response

$$|H(\omega)| = \frac{5\sqrt{1+0.8\cos\omega+0.16}}{\sqrt{1-1.6\cos\omega+0.64}}$$

- Drawing peaks when passing near poles
- Drawing dips when passing near zeros





Example

❖ Consider the system which has the I/O equation:

$$y(n) = 0.25y(n-2) + x(n)$$

a) Determine the transfer function

b) Determine the casual impulse response

c) Determine the frequency response and plot the magnitude response of the filter.

d) Plot the block diagram of the system and write the sample processing algorithm

2. Digital filter realizations

❖ Construction of block diagram of the filter is called a realization of the filter.

❖ Realization of a filter at a block diagram level is essentially a flow graph of the signals in the filter.

❖ It includes operations: delays, additions and multiplications of signals by a constant coefficients.

❖ The block diagram realization of a transfer function is not unique.

❖ Note that for implementation of filter we must concerns the accuracy of signal values, accuracy of coefficients and accuracy of arithmetic operations. We must analyze the effect of such imperfections on the performance of the filter.

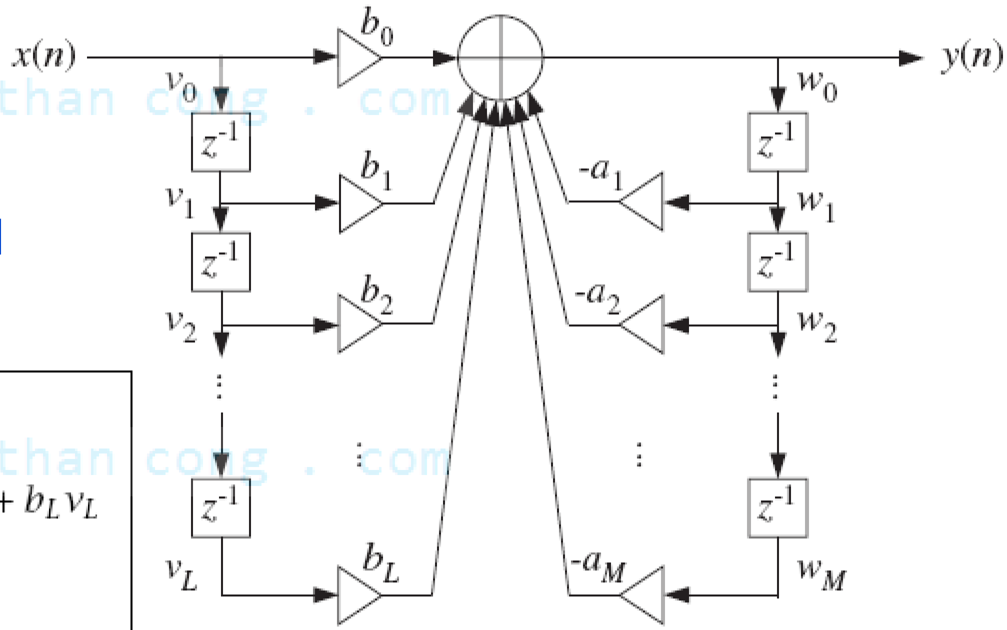
Direct form realization

❖ Use the I/O difference equation

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_L z^{-L}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} - \cdots - a_M y_{n-M} + b_0 x_n + b_1 x_{n-1} + \cdots + b_L x_{n-L}$$

- The b-multipliers are feeding forward
- The a-multipliers are feeding backward



for each input sample x do:

$v_0 = x$

$w_0 = -a_1 w_1 - \cdots - a_M w_M + b_0 v_0 + b_1 v_1 + \cdots + b_L v_L$

$y = w_0$

$v_i = v_{i-1}, \quad i = L, L-1, \ldots, 1$

$w_i = w_{i-1}, \quad i = M, M-1, \ldots, 1$

Example

❖ Consider IIR filter with $h(n)=0.5^n u(n)$

a) Draw the direct form realization of this digital filter ?

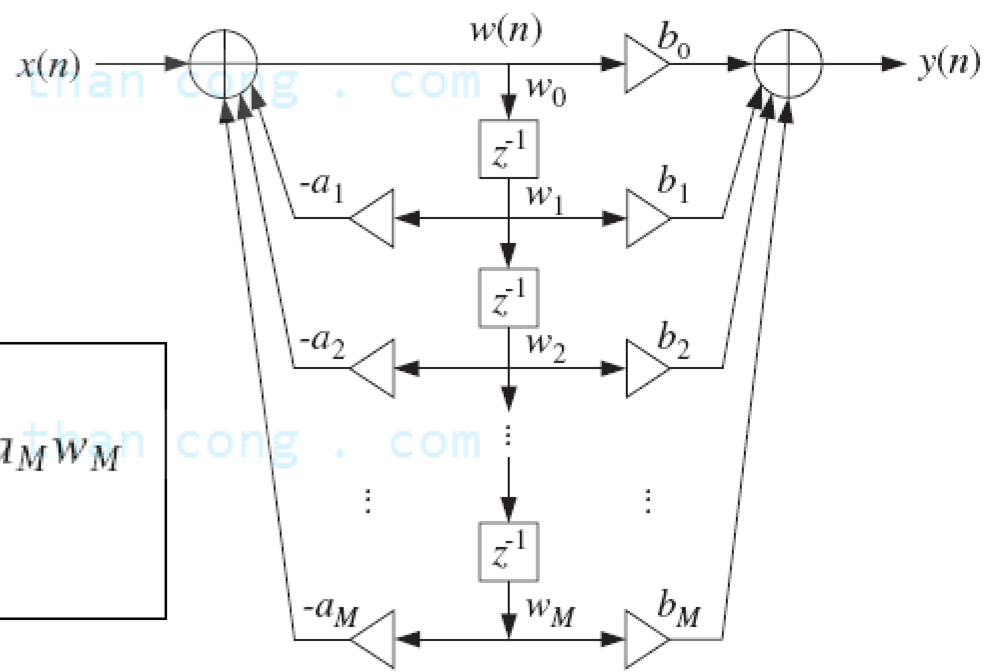
b) Given x=[2, 8, 4], find the first 6 samples of the output by using the sample processing algorithm ?

Canonical form realization

❖ Note that $Y(z) = H(z)X(z) = N(z)\dfrac{1}{D(z)}X(z) = \dfrac{1}{D(z)}N(z)X(z)$

$Y(z) = N(z)W(z)$ and $W(z) = \dfrac{1}{D(z)}X(z)$

$$w(n) = x(n) - a_1 w(n-1) - \cdots - a_M w(n-M)$$
$$y(n) = b_0 w(n) + b_1 w(n-1) + \cdots + b_L w(n-L)$$



❑ The maximum number of common delays: K=max(L,M)

for each input sample x do:

$$w_0 = x - a_1 w_1 - a_2 w_2 - \cdots - a_M w_M$$
$$y = b_0 w_0 + b_1 w_1 + \cdots + b_L w_L$$
$$w_i = w_{i-1}, \quad i = K, K-1, \ldots, 1$$

Cascade form

❖ The cascade realization form of a general functions assumes that the transfer functions is the product of such second-order sections (SOS):

$$H(z) = \prod_{i=0}^{K-1} H_i(z) = \prod_{i=0}^{K-1} \frac{b_{i0} + b_{i1}z^{-1} + b_{i2}z^{-2}}{1 + a_{i1}z^{-1} + a_{i2}z^{-2}}$$

❖ Each of SOS may be realized in direct or canonical form.

for each input sample x do:

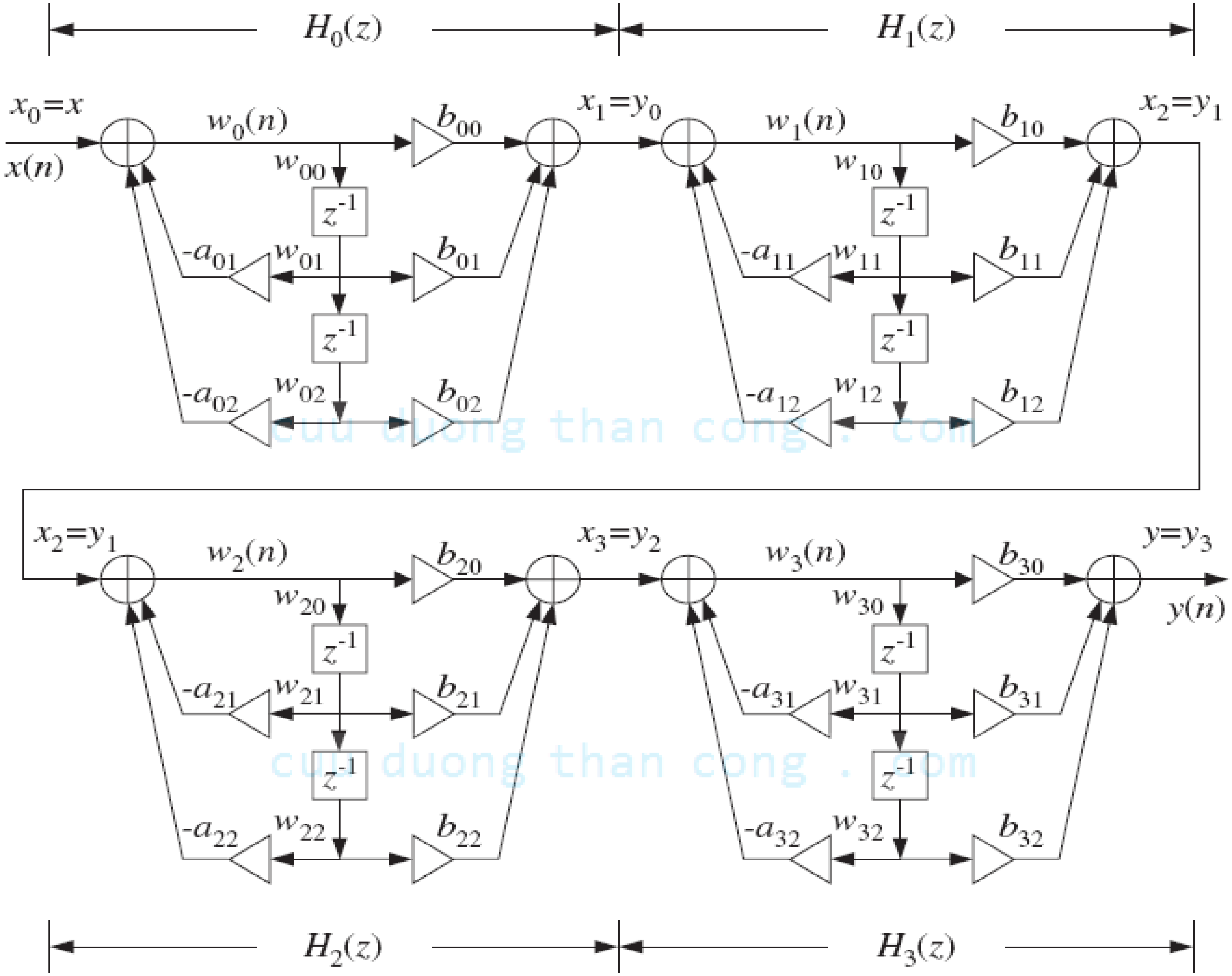
$$x_0 = x$$

for $i = 0, 1, \ldots, K-1$ *do:*

$$w_{i0} = x_i - a_{i1}w_{i1} - a_{i2}w_{i2}$$
$$y_i = b_{i0}w_{i0} + b_{i1}w_{i1} + b_{i2}w_{i2}$$
$$w_{i2} = w_{i1}$$
$$w_{i1} = w_{i0}$$
$$x_{i+1} = y_i$$

$$y = y_{K-1}$$

Cascade form



Review

Homework 1

Determine the transfer function $H(z)$ and the corresponding I/O difference equation relating $x(n)$ and $y(n)$ of the linear filters having the following impulse responses:

a. $h(n) = \delta(n-5)$

b. $h(n) = u(n-5)$

c. $h(n) = (0.8)^n u(n)$

d. $h(n) = (-0.8)^n u(n)$

e. $h(n) = (-0.8)^n [u(n) - u(n-8)]$

f. $h(n) = (0.8)^n u(n) + (-0.8)^n u(n)$

g. $h(n) = 2(0.8)^n \cos(\pi n/2) u(n)$

h. $h(n) = (0.8j)^n u(n) + (-0.8j)^n u(n)$

In each case, determine also the *frequency response* $H(\omega)$, the *pole/zero* pattern of the transfer function on the z-plane, draw a rough sketch of the *magnitude response* $|H(\omega)|$ over the right half of the Nyquist interval $0 \le \omega \le \pi$, and finally, draw the direct and canonical realizations implementing the I/O difference equation and state the corresponding *sample-by-sample* processing algorithms.

Homework 2

A digital reverberation processor has frequency response:

$$H(\omega) = \frac{-0.5 + e^{-j\omega 8}}{1 - 0.5e^{-j\omega 8}}$$

where ω is the digital frequency in [radians/sample]. Determine the *causal* impulse response $h(n)$, for all $n \geq 0$, and sketch it versus n. [*Hint:* Do not use partial fractions.]

Homework 3

The first few Fibonacci numbers are:

$$\mathbf{h} = [0, 1, 1, 2, 3, 5, 8, 13, 21, \dots]$$

where each is obtained by summing the previous two.

a. Determine the linear system $H(z)$ whose causal impulse response is $\mathbf{h}$, and express it as a rational function in z^{-1}.

b. Using partial fractions, derive an expression for the nth Fibonacci number in terms of the poles of the above filter.

c. Show that the ratio of two successive Fibonacci numbers converges to the *Golden Section*, that is, the positive solution of the quadratic equation $\phi^2 = \phi + 1$, namely, $\phi = (1 + \sqrt{5})/2$.

d. Show that the filter's poles are the two numbers $\{\phi, -\phi^{-1}\}$. Show that the geometric sequence:

$$\mathbf{y} = [0, 1, \phi, \phi^2, \phi^3, \dots]$$

satisfies the same recursion as the Fibonacci sequence (for $n \geq 3$). Show that $\mathbf{y}$ may be considered to be the output of the filter $\mathbf{h}$ for a particular input. What is that input?

Homework 4

For a particular causal filter, it is observed that the input signal $(0.5)^n u(n)$ produces the output signal $(0.5)^n u(n) + (0.4)^n u(n)$. What *input* signal produces the output signal $(0.4)^n u(n)$?

For a particular filter, it is observed that the input signal $a^n u(n)$ causes the output signal $a^n u(n) + b^n u(n)$ to be produced. What output signal is produced by the input $c^n u(n)$, where $c = (a + b)/2$?

The signal $(0.7)^n u(n)$ is applied to the input of an unknown causal LTI filter, and the signal $(0.7)^n u(n) + (0.5)^n u(n)$ is observed at the output. What is the causal input signal that will cause the output $(0.5)^n u(n)$? What is the transfer function $H(z)$ of the system? Determine its causal impulse response $h(n)$, for all $n \geq 0$.

Homework 5

Design a resonator filter of the form $H(z) = \dfrac{1}{1 + a_1 z^{-1} + a_2 z^{-2}}$, which has a peak at $f_0 = 250$ Hz and a 3-dB width of $\Delta f = 20$ Hz and is operating at a rate of $f_s = 5$ kHz. What are the values of a_1 and a_2? Show that the *time constant* of the resonator is given approximately by

$$n_{\text{eff}} = -\frac{2\ln\epsilon}{\Delta\omega}$$

which is valid for small $\Delta\omega$. For the designed filter, calculate the 40-dB value of n_{eff}, that is, corresponding to $\epsilon = 10^{-2}$. Compare the approximate and exact values of n_{eff}.

Homework 6

It is desired to generate the following *periodic* waveform:

$$h(n) = [1, 2, 3, 4, 0, 0, 0, 0, 1, 2, 3, 4, 0, 0, 0, 0, \cdots]$$

where the dots indicate the periodic repetition of the 8 samples $[1, 2, 3, 4, 0, 0, 0, 0]$.

a. Determine the filter $H(z)$ whose impulse response is the above periodic sequence. Express $H(z)$ as a ratio of two polynomials of degree less than 8.

b. Draw the canonical and direct realization forms of $H(z)$. Write the corresponding sample processing algorithms.

Homework 7

A digital sawtooth generator filter has a periodic impulse response:

$$\mathbf{h} = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3, \cdots]$$

where the dots indicate the periodic repetition of the length-4 sequence $\{0, 1, 2, 3\}$.

a. Determine the transfer function $H(z)$.

b. Draw the *direct* and *canonical* realization forms. Factor $H(z)$ into second-order sections with real coefficients. Draw the corresponding *cascade* realization.

c. For each of the above three realizations, write the corresponding I/O time-domain difference equations and sample-by-sample processing algorithms.

d. Using partial fractions, do an inverse z-transform of $H(z)$ and determine a closed form expression for the above impulse response $h(n)$ in the form

$$h(n) = A + B(-1)^n + 2C\cos\left(\frac{\pi n}{2}\right) + 2D\sin\left(\frac{\pi n}{2}\right), \qquad n \geq 0$$

What are the values of A, B, C, D ?

Homework 8

Consider the system: $H(z) = \dfrac{1 + z^{-1} + z^{-2} + z^{-3}}{1 - z^{-7}}$.

a. Without using partial fractions, determine the causal impulse response of the system. Explain your reasoning.

b. Draw the *canonical* realization form of the system. Write the I/O *difference equations* and the *sample processing algorithm* describing this realization.

c. The length-3 input signal $\mathbf{x} = [3, 2, 1]$ is applied as input to the system. Using any method, determine the output signal $y(n)$ for all $n \geq 0$. Explain your method.

Homework 9

A causal filter has transfer function: $H(z) = \dfrac{1 + z^{-1} + z^{-2} + z^{-3}}{1 - z^{-2}}$.

a. Determine the numerical values of the causal impulse response $h(n)$, for all $n \geq 0$.

b. Draw the canonical realization form of this filter and write the sample processing algorithm describing it.

Homework 10

A digital filter has transfer function, where $0 < a < 1$:

$$H(z) = \frac{1 - z^{-16}}{1 - az^{-16}}$$

a. What are the poles and zeros of this filter? Show them on the z-plane.

b. Draw a rough sketch of its magnitude response $|H(\omega)|$ over the frequency interval $0 \le \omega \le 2\pi$.

c. Determine the causal/stable impulse response $h(n)$ for all $n \ge 0$. Sketch it as a function of n. [*Hint*: Do not use PF expansions.]

d. Draw the *canonical* realization form and write the corresponding sample processing algorithm. (You may make use of the `delay` routine to simplify the algorithm.)

Homework 11

Let $H(z) = \dfrac{1 - a}{1 - az^{-1}}$ be a first-order lowpass filter (also called a first-order smoother), where $0 < a < 1$. Draw the canonical realization. Draw another realization that uses only one multiplier, (that is, a), one delay, and one adder and one subtractor. For both realizations, write the sample-by-sample processing algorithms. What would you say is the purpose of the chosen gain factor $1 - a$?

Homework 12

A discrete system is described by the difference equation

$$y(n) = 2.5y(n-1) - y(n-2) + 3x(n) + 3x(n-2)$$

Using z-transforms, find *all* possible impulse responses $h(n)$ and indicate their causality and stability properties.

For the causal filter, determine the output $y(n)$ if the input is $x(n) = g(n) - 2g(n-1)$, where $g(n) = \cos(\pi n/2)u(n)$.

Homework 13

A system has transfer function:

$$H(z) = \frac{z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4}}{1 - z^{-5}}$$

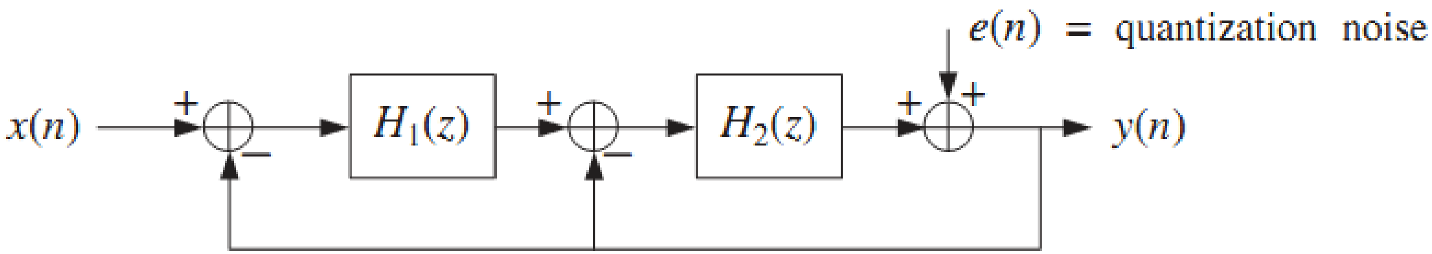
a. *Without* using partial fractions, determine the *causal* impulse response $h(n)$ of this system, for *all* $n \geq 0$, and sketch it versus n.

b. Draw the *direct* and *canonical* realization forms. Write the *difference equations* describing these realizations. Then, write the corresponding *sample processing* algorithms.

c. Factor this transfer function in the form $H(z) = H_1(z)H_2(z)$, where $H_1(z)$ is the ratio of two first-order polynomials, and $H_2(z)$ has numerator of degree 3 and denominator of degree 4. Draw the corresponding cascade realization, with each factor realized in its *canonical* form. Write the *difference equations* describing this realization, and the corresponding *sample processing* algorithm.

Homework 14

A discrete-time model for a second-order delta-sigma A/D converter is shown below:



a. Show that the output z-transform $Y(z)$ is related to $X(z)$ and $E(z)$ by a transfer function relationship of the form:

$$Y(z) = H_x(z)X(z) + H_e(z)E(z)$$

Express the transfer functions $H_x(z)$ and $H_e(z)$ *in terms of* the loop filters $H_1(z)$ and $H_2(z)$.

b. Determine $H_1(z)$ and $H_2(z)$ in order for $H_x(z)$ to act as a single delay and $H_e(z)$ as a second-order noise shaper, that is,

$$H_x(z) = z^{-1} \qquad \text{and} \qquad H_e(z) = (1 - z^{-1})^2$$

Homework 15

A digital filter has transfer function:

$$H(z) = \frac{z^{-1}(1 + 2z^{-2})(1 + 3z^{-2})}{1 - z^{-6}}$$

a. Draw the *direct* form realization (direct form I). Write the I/O difference equation and corresponding sample processing algorithm for this realization.

b. Draw the *canonical* form realization (direct form II). Write the I/O difference equations and corresponding sample processing algorithm for this realization.

c. Factor $H(z)$ into *second-order sections* with real-valued coefficients, and draw the corresponding *cascade* realization. Write the I/O difference equations and corresponding sample processing algorithm for this realization.

d. *Without* using partial fractions, determine the causal impulse response $h(n)$ of this filter for all n. Explain your reasoning.

Homework 16

A linear system is described by the system of difference equations:

$$v(n) = x(n) + v(n-1)$$

$$y(n) = v(n) + v(n-2) + v(n-4)$$

Determine the transfer function from $x(n)$ to $y(n)$. Draw the direct, the canonical, and the cascade of SOS realizations (with real coefficients). In each case, state the sample-by-sample processing algorithm.

Homework 17

Draw the three realizations: (1) direct, (2) canonical, and (3) cascade of second-order sections for the following filter:

$$H(z) = \frac{(2 - 3z^{-1})(1 + z^{-2})}{1 - 0.25z^{-4}}$$

For each realization write the corresponding: (a) I/O difference equations and (b) sample processing algorithm.

Homework 18

A filter has transfer function:

$$H(z) = \frac{5}{1 + 0.25z^{-2}} - \frac{4}{1 - 0.25z^{-2}} = \frac{1 - 2.25z^{-2}}{(1 + 0.25z^{-2})(1 - 0.25z^{-2})}$$

a. Determine *all possible* impulse responses $h(n)$ and their ROCs.

b. Draw the *direct realization form* of $H(z)$.

c. Draw the *canonical realization form.*

d. Draw the *cascade form.*

In all cases, write all the *I/O difference equations* describing the realization in the time domain, and the *sample processing algorithm* implementing it.

Homework 19

An allpass digital reverberator with delay of 10 time units and having input $x(n)$ and overall output $y(n)$, is described by the system of difference equations:

$$w(n) = 0.75w(n-10) + x(n)$$

$$y(n) = -0.75w(n) + w(n-10)$$

a. Draw a block diagram realization of this filter. The realization must use only one 10-fold delay.

b. Write the sample processing algorithm for this filter. Then, convert this algorithm into a C routine that implements it.

c. Show that the magnitude response of the filter is identically equal to one, that is, $|H(\omega)| = 1$ for all ω.

Homework 20

For the following three filters,

$$H(z) = (1 + z^{-2})^3, \qquad H(z) = \frac{1}{1 + 0.81z^{-2}}, \qquad H(z) = \frac{1 - z^{-4}}{1 - 0.9z^{-1}}$$

a. Determine *all possible* impulse responses $h(n)$, corresponding ROCs, stability, and causality properties.

b. Draw the *direct*, *canonical*, and *cascade of SOS* realization forms. Write the I/O difference equations for each realization. State the sample-by-sample processing algorithm for each realization.

c. Determine the corresponding pole/zero plots and then make a rough sketch of the magnitude responses $|H(\omega)|$ versus ω.

Homework 21

Consider a stable system with transfer function $H(z) = \dfrac{\frac{1}{16} + z^{-4}}{1 + \frac{1}{16} z^{-4}}$.

a. Determine the poles and zeros of $H(z)$ and place them on the complex z-plane. Pair them in conjugate pairs to write $H(z)$ as a cascade of second-order sections with real coefficients.

b. Draw the direct, canonical, and cascade realization forms. In each case, write the corresponding sample processing algorithm.

c. Determine the impulse response $h(n)$ for all n. And, finally show that $|H(\omega)| = 1$ for all ω, that is, it is an allpass filter.

Homework 22

Cho hệ thống rời rạc LTI có đáp ứng xung $h(n) = 0.5^n u(n-1)$.

1) Viết phương trình sai phân vào-ra và vẽ 1 sơ đồ khối thực hiện hệ thống.
2) Tìm giá trị của mẫu tín hiệu ngõ ra $y(n=2)$ khi tín hiệu ngõ vào $x(n) = \delta(n-1)$.
3) Tìm giá trị của mẫu tín hiệu ngõ ra $y(n=2)$ khi tín hiệu ngõ vào $x(n) = u(-n-1)$.
4) Tìm giá trị của mẫu tín hiệu ngõ ra $y(n=2)$ khi tín hiệu ngõ vào $x(n) = 1$.

Homework 23

Cho hệ thống LTI nhân quả có hàm truyền $H(z)=\dfrac{z^{-1}}{1-0.5z^{-1}}+\dfrac{2}{1+0.5z^{-1}}$.

1) Kiểm tra tính ổn định của hệ thống.
2) Tìm đáp ứng xung của hệ thống.
3) Viết phương trình sai phân vào-ra và vẽ 1 sơ đồ khối thực hiện hệ thống.
4) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào $x(n) = 4\delta(n) - \delta(n-2)$.

Homework 24

Cho hệ thống rời rạc LTI nhân quả có phương trình vào-ra $y(n) = x(n-1) - 0.5y(n-1)$.

1) Vẽ 1 sơ đồ khối thực hiện hệ thống.
2) Tìm đáp ứng xung của hệ thống.
3) Vẽ phác thảo biên độ đáp ứng tần số và xác định đặc tính tần số (thông thấp, thông cao, thông dải hay chắn dải) của hệ thống.
4) Tìm tín hiệu ngõ vào $x(n)$ để tín hiệu ngõ ra $y(n) = \delta(n-1)$.

Homework 25

Cho hệ thống rời rạc LTI nhân quả có phương trình vào-ra $y(n) = x(n-1) + 0.5y(n-1)$

1) Vẽ sơ đồ khối thực hiện hệ thống trên với số bộ trễ ít nhất có thể.
2) Tìm đáp ứng xung của hệ thống trên.
3) Tìm giá trị của mẫu tín hiệu ngõ ra $y(n=2)$ khi tín hiệu ngõ vào $x(n) = 2\delta(n-2)$.
4) Tìm giá trị của mẫu tín hiệu ngõ ra $y(n=2)$ khi tín hiệu ngõ vào $x(n) = u(-n-1)$.
5) Tìm giá trị của mẫu tín hiệu ngõ ra $y(n=2)$ khi tín hiệu ngõ vào $x(n) = 2$.

Homework 26

Cho hệ thống rời rạc LTI nhân quả có hàm truyền H(z) = $\dfrac{1+z^{-2}}{4-z^{-2}}$.

1) Vẽ sơ đồ cực-zero và kiểm tra tính ổn định của hệ thống trên.
2) Viết phương trình sai phân vào-ra và vẽ sơ đồ khối thực hiện hệ thống trên với số bộ trễ là ít nhất.
3) Vẽ phác họa đáp ứng biên độ và chỉ ra đặc tính tần số (thông thấp, thông cao, thông dải hay chắn dải) của hệ thống trên.
4) Xác định biểu thức và chỉ ra đặc tính đáp ứng xung (FIR hay IIR) của hệ thống trên.
5) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào x(n) = 2cos(πn/2)u(n).
6) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào x(n) = u(-n) – 2δ(n).
7) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào x(n) = 2.
8) Tìm tín hiệu ngõ vào x(n) để tín hiệu ngõ ra y(n) = {2, 0, 2}.

Homework 27

Cho hệ thống LTI nhân quả có hàm truyền $H(z) = \dfrac{z^{-1}}{1-0.5z^{-1}} + \dfrac{2}{1+0.5z^{-1}}$.

1) Vẽ sơ đồ cực-zero và kiểm tra tính ổn định của hệ thống trên.
2) Viết phương trình sai phân vào-ra và vẽ sơ đồ khối thực hiện hệ thống trên với số bộ trễ là ít nhất.
3) Vẽ phác họa đáp ứng biên độ và chỉ ra đặc tính tần số (thông thấp, thông cao, thông dải hay chắn dải) của hệ thống trên.
4) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào $x(n) = 2\delta(n-2)$.
5) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào $x(n) = \{2, 1\}$.
6) Tìm giá trị của mẫu tín hiệu ngõ ra y(n=2) khi tín hiệu ngõ vào $x(n) = 2$.

Homework 28

Cho hệ thống tuyến tính bất biến có tín hiệu ngõ ra y(n) = {1, 0, 0, 0, -1} khi tín hiệu ngõ vào x(n) = {1, 0, 1}.

1) Viết phương trình sai phân vào-ra của hệ thống trên.
2) Xác định tín hiệu ngõ ra y(n) khi tín hiệu ngõ vào x(n) = {1, 0, 0, 0, -1}.
3) Xác định tín hiệu ngõ vào x(n) để tín hiệu ngõ ra y(n) = {1, 1}.
4) Tìm đáp ứng xung nhân quả của hệ thống khôi phục ghép nối tiếp ngay sau hệ thống trên để ngõ ra hệ thống khôi phục đúng bằng tín hiệu ngõ vào của hệ thống ban đầu.