Digital Signal Processing

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Chapter 0

Introduction

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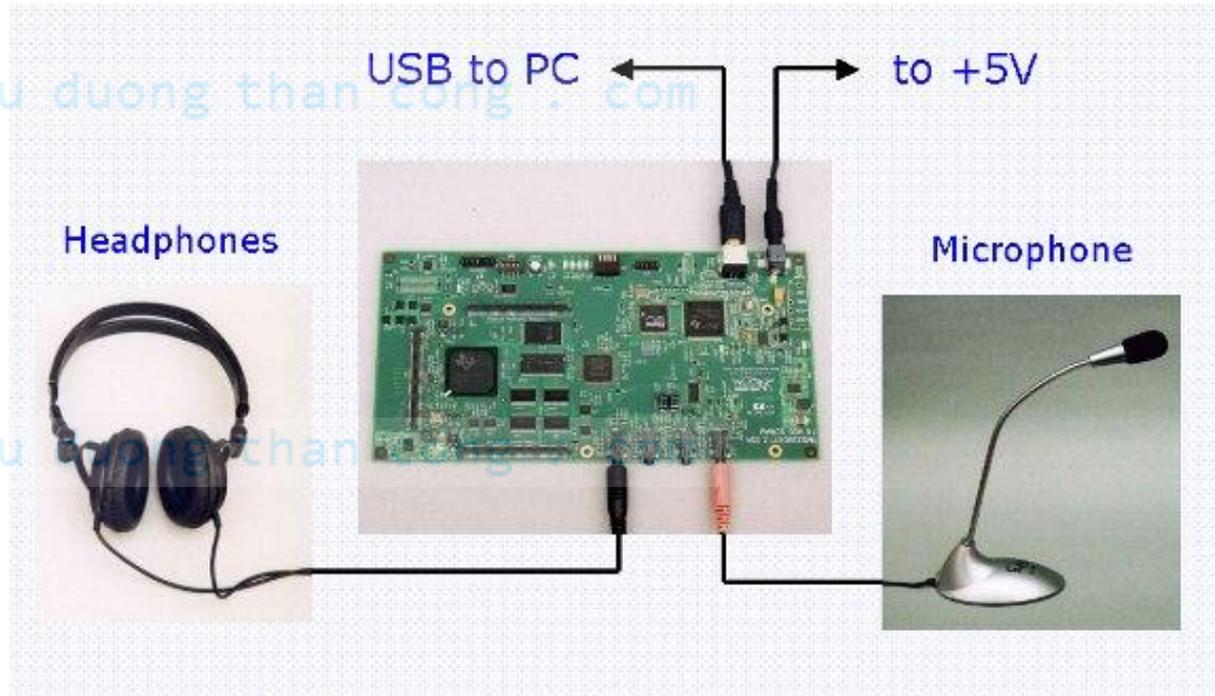
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1. Signal and System

❖ A **signal** is defined as any physical quantity that varies with time, space, or any other independent variable(s).

  ❑ Speech, image, video and electrocardiogram signals are information-bearing signals.

❖ Mathematically, we describe a signal as a function of one or more independent variables.

  ❑ Examples:
$$x(t) = 110\sin(2\pi \cdot 50t)$$
$$I(x, y) = 3x + 2xy + 10y^2$$

❖ A **system** is defined as a physical device that performs any operation on a signal.

  ❑ A filter is used to reduce noise and interference corrupting a desired information-bearing signal.

1. Signal and System

❖ **Signal processing** is to pass a signal through a system.
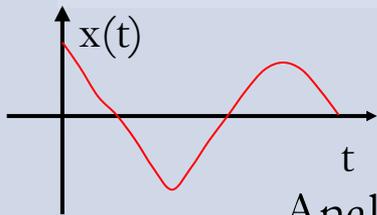
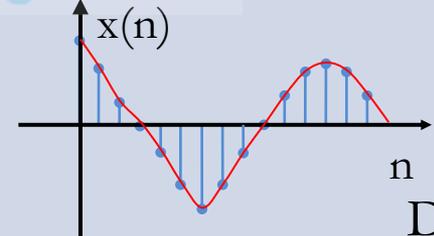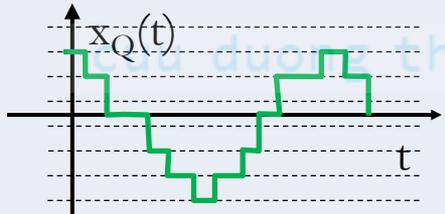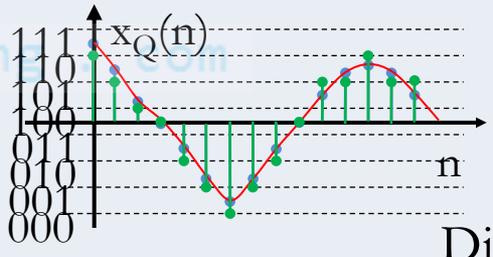❖ A **digital system** can be implemented as a combination of hardware and software (program, algorithm).

Multichannel and Multidimensional signals

❖ Signals which are generated by multiple sources or multiple sensors can be represented in a vector form. Such a vector of signals is referred to as a multichannel signals

❑ Ex: 3-lead and 12-lead electrocardiograms (ECG) are often used in practice, which results in 3-channel and 12-channel signals.

❖ A signal is called M-dimensional if its value is a function of M independent variable

❑ Picture: the intensity or brightness $I(x,y)$ at each point is a function of 2 independent variables

❑ TV picture is 3-dimensional signal $I(x,y,t)$

2. Classification of Signals

Continuous-time versus discrete-time signal

❖ Signals can be classified into four different categories depending on the characteristics of the time variable and the values they take.

Time Amplitude	Continuous	Discrete
Continuous	$x(t)$ t Analog signal	$x(n)$ n Discrete signal
Discrete	$x_Q(t)$ t Quantized signal	$x_Q(n)$ n Digital signal

3. Basic elements of a DSP system

❖ Most of the signals encountered in science and engineering are analog in nature. To perform the processing digitally, there is a need for an interface between the analog signal and the digital processor.
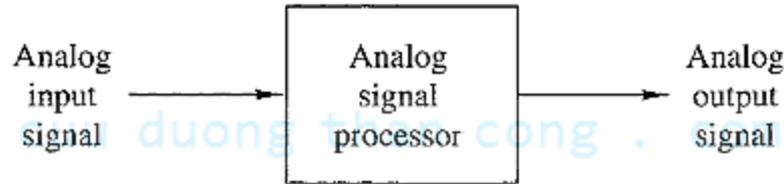

Fig 0.1: Analog signal processing

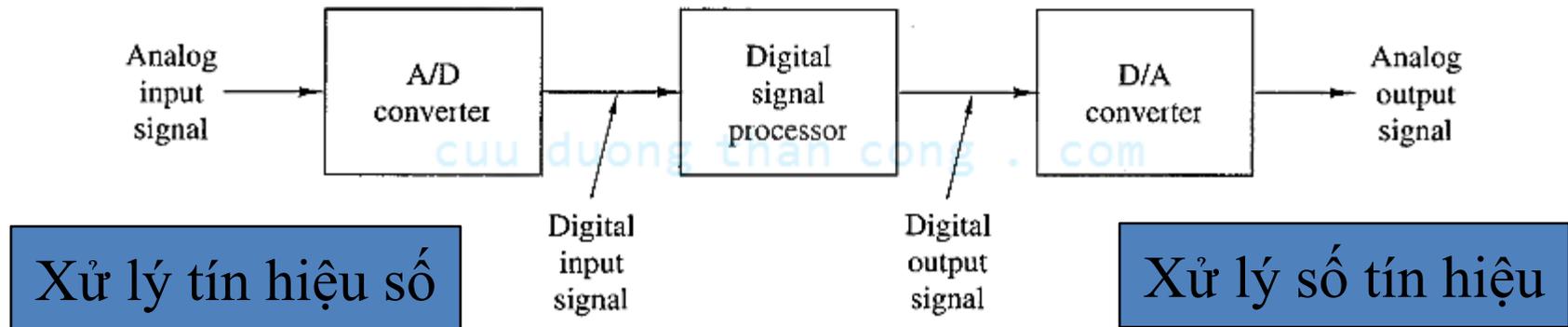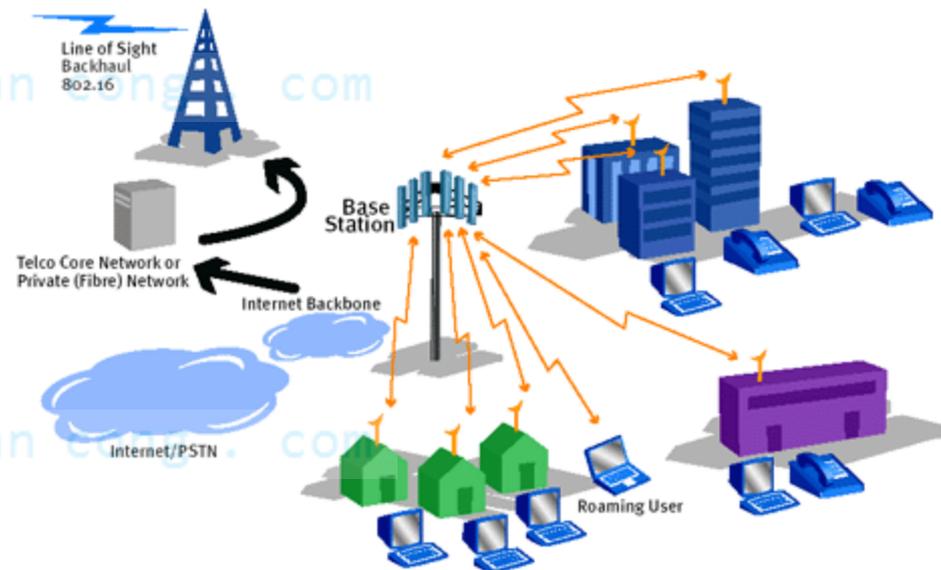Xử lý tín hiệu số

Xử lý số tín hiệu

Fig 0.2: Digital signal processing
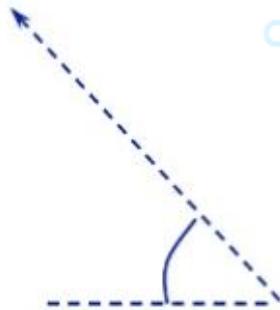
4. DSP applications-Communications

❖ Telephony: transmission of information in digital form via telephone lines, modem technology, mobile phone.

❖ Encoding and decoding of the information sent over physical channels (to optimize transmission, to detect or correct errors in transmission)

4. DSP applications-Radar and Sonar

❖ Target detection: position and velocity estimation
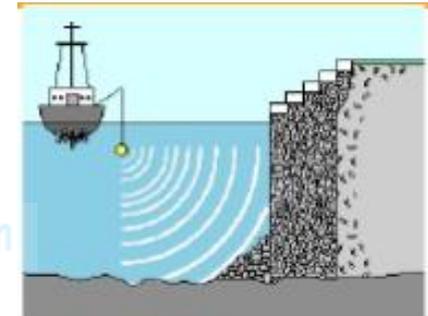
❖ Tracking

4. DSP applications-Biomedical

❖ Analysis of biomedical signals, diagnosis, patient monitoring, preventive health care, artificial organs.

❖ Examples:

➢ Electrocardiogram (ECG) signal provides information about the condition of the patient's heart.

➢ Electroencephalogram (EEG) signal provides information about the activity of the brain.

4. DSP applications-Speech

❖ **Noise reduction**: reducing background noise in the sequence produced by a sensing device (a microphone).

❖ **Speech recognition**: differentiating between various speech sounds.

❖ **Synthesis of artificial speech**: text to speech systems.
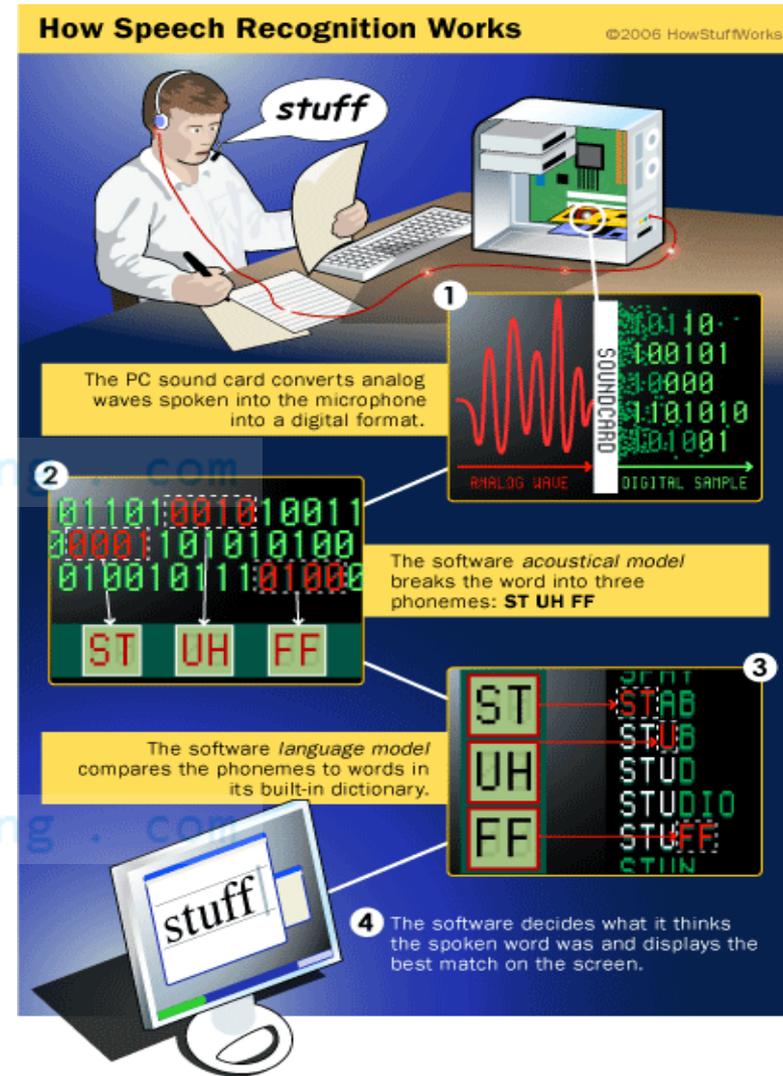
4. DSP applications-Image Processing

❖ **Content based image retrieval**: browsing, searching and retrieving images from database.

Before After
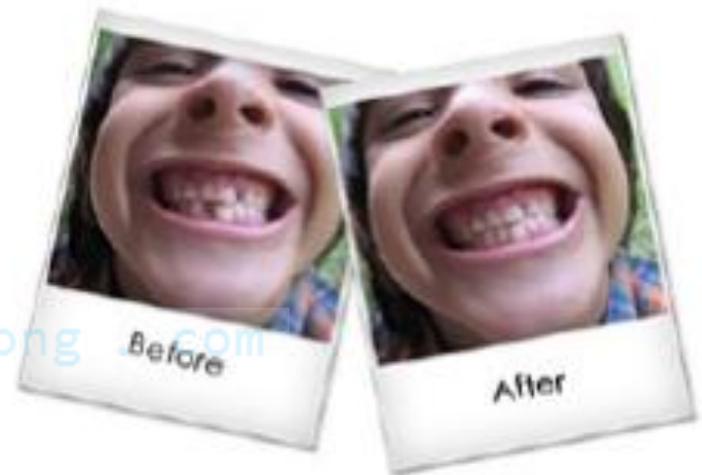
❖ **Image enhancement**

❖ **Compression**: reducing the redundancy in the image data to optimize transmission/storage

4. DSP applications-Multimedia

❖ Generation, storage and transmission of sound, still images, motion pictures.

❖ Digital TV

❖ Video conference

"Learning digital signal processing **is not something you accomplish**; it's a journey you take".

R.G. Lyons, Understanding Digital Signal Processing

5. Advantages of digital over analog signal processing

❖ A digital programmable system allows flexibility in reconfiguring the DSP operations simply by changing the program.

❖ A digital system provides much better control of accuracy requirements.

❖ Digital signals are easily stored.

❖ DSP methods allow for implementation of more sophisticated signal processing algorithms.

❖ Limitation: Practical limitations of DSP are the quantization errors and the speed of A/D converters and digital signal processors -> not suitable for analog signals with large bandwidths.

Course overview

❖ Chapter 0: Introduction to Digital Signal Processing (3 periods)

❖ Chapter 1: Sampling and Reconstruction (6 periods)

❖ Chapter 2: Quantization (3 periods)

❖ Chapter 3: Analysis of linear time invariant systems (LTI) (6 periods)

❖ Chapter 4: Finite Impulse Response and convolution (3 periods)

❖ Chapter 5: Z-transform and its applications (6 periods)

❖ Chapter 6: Transfer function and filter realization (3 periods)

❖ Chapter 7: Fourier transform and FFT algorithm (6 periods)

❖ Chapter 8: FIR and IIR filter designs (6 periods)

❖ Review and mid-term exam: 3 periods

References

❖ **Text books:**

[1] S. J. Orfanidis, **Introduction to Signal Processing**, Prentice-Hall Publisher 2010.

[2] J. Proakis, D. Manolakis, **Digital Signal Processing**, Macmillan Publishing Company, 1989.

❖ **Reference books:**

[3] V. K. Ingle, J. Proakis, **Digital Signal Processing Using Matlab**, Cengage Learning, 3 Edt, 2011.

Learning outcomes

❖ Understand how to convert the analog to digital signal

❖ Have a thorough grasp of signal processing in linear time-invariant systems.

❖ Understand the z-transform and Fourier transforms in analyzing the signal and systems.

❖ Be able to design and implement FIR and IIR filters.

Assessment

❖ Mid-term test: 20%

❖ Homework: 20%

❖ Final exam: 60%

❖ Bonus: added to
    Test and Homework

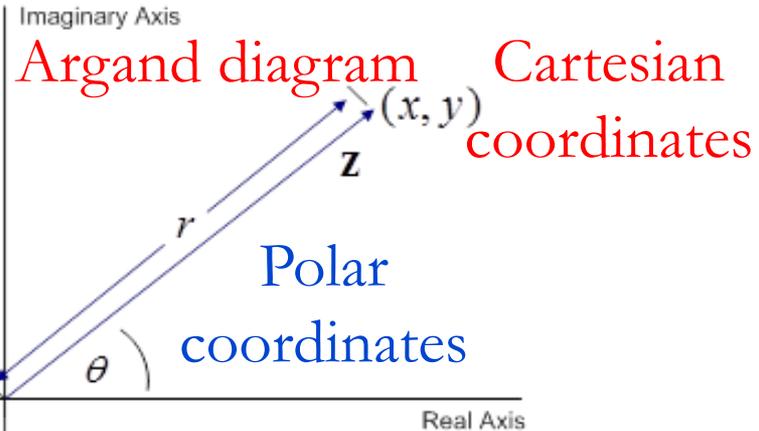| Test and Homework (40%)	Final exam (60%)		Final Mark (100%)
0.0	7.5	4.50	4.5
2.5	6.0	4.60	4.5
3.0	6.0	4.80	5.0
4.0	5.5	4.90	5.0
5.5	4.5	4.90	5.0
6.0	4.0	4.80	5.0
7.0	3.5	4.90	5.0
7.5	3.0	4.80	5.0
7.0	3.0	4.60	4.5
10.0	2.5	5.50	2.5
10.0		4.00	Absent

Điểm ghi trên *Bảng điểm kiểm tra, Bảng điểm thi* và *Bảng điểm tổng kết* được **làm tròn đến 0,5**. (từ 0 đến dưới 0,25 làm tròn thành 0; từ 0,25 đến dưới 0,75 làm tròn thành 0,5; từ 0,75 đến dưới 1,0 làm tròn thành 1,0)

Nếu điểm thi nhỏ hơn 3 và nhỏ hơn điểm tổng kết tính từ các điểm thành phần (kể cả điểm thi) thì lấy điểm thi làm điểm tổng kết.

Timetable

Time	Class
Monday (T1-3)	DD13BK01-A02 **314B1**
Tuesday (T7-9)	DD13KSTD **206B1**
Wednesday (T10-12)	DD13LT04-A04 **303B1**

Review of complex number

❖ Rectangular form: $\mathbf{z} = x + iy$ Argand diagram   Cartesian coordinates

  ➢ Real part: $x = r\cos\theta$

  Polar coordinates

  ➢ Imaginary part: $y = r\sin\theta$

❖ Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$

$$\arg(zw) = \arg(z) + \arg(w)$$
$$|zw| = |z||w|$$

❖ Polar form: $\mathbf{z} = re^{i\theta} = r\angle\theta$

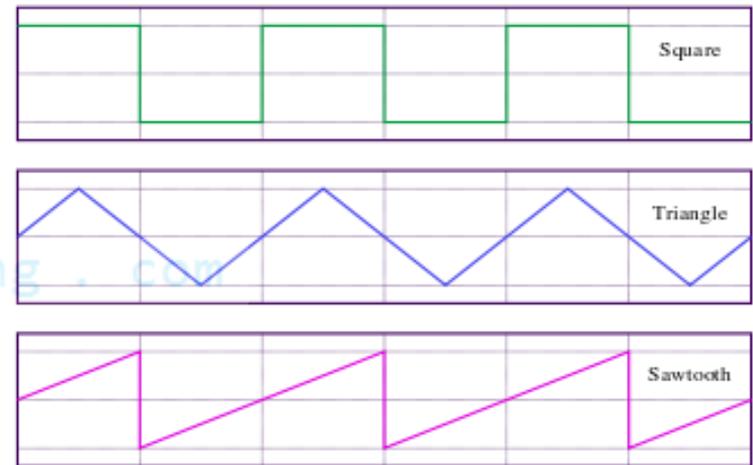 ➢ Absolute value (modulus, magnitude): $r = |\mathbf{z}| = \sqrt{x^2 + y^2}$

  $(-\pi \quad , \quad \pi]$

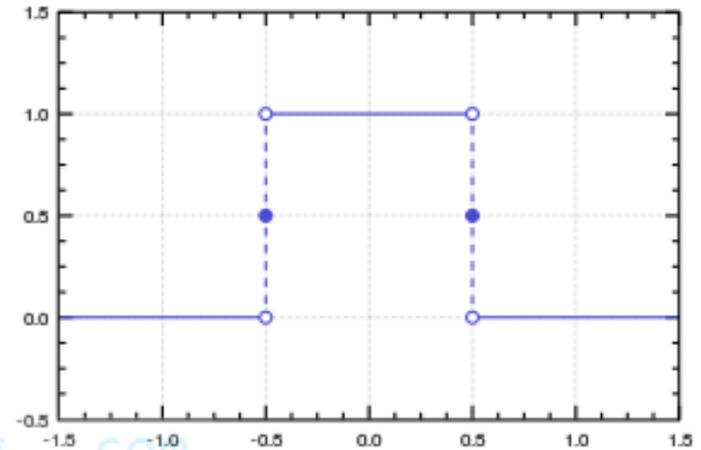 ➢ Argument (angle): $-180° < \theta \le 180°$ $\theta = \arg(\mathbf{z}) = \tan^{-1}\dfrac{y}{x}$

Review of periodic signals

❖ Definition: x(t) = x(t + T) ∀t

❖ Fundamental period (cycle duration): smallest T

❖ Ordinary frequency: f = 1/T (cps or Hz) --> F
❖ Radial (angular) frequency: $\omega = 2\pi f$ (rad/s) --> Ω

Review of special functions
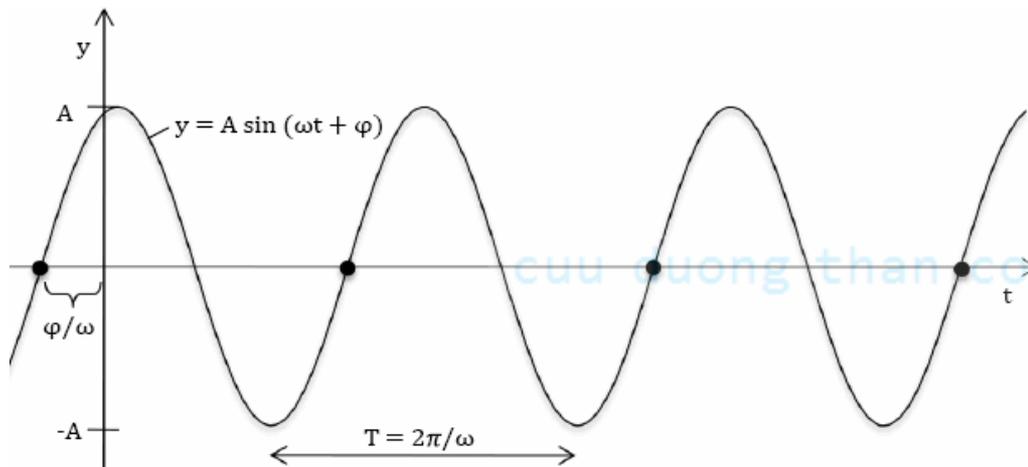
❖ **Rectangular (rect)**

$$\text{rect}(t) = \Pi(t) = \begin{cases} 0 & \text{if } |t| > \frac{1}{2} \\ \frac{1}{2} & \text{if } |t| = \frac{1}{2} \\ 1 & \text{if } |t| < \frac{1}{2}. \end{cases}$$

❖ **Cardinal sine (sinc)**

➤ Unnormalized: $\text{sinc}(x) = \dfrac{\sin(x)}{x}$.

➤ **Normalized:** $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$.
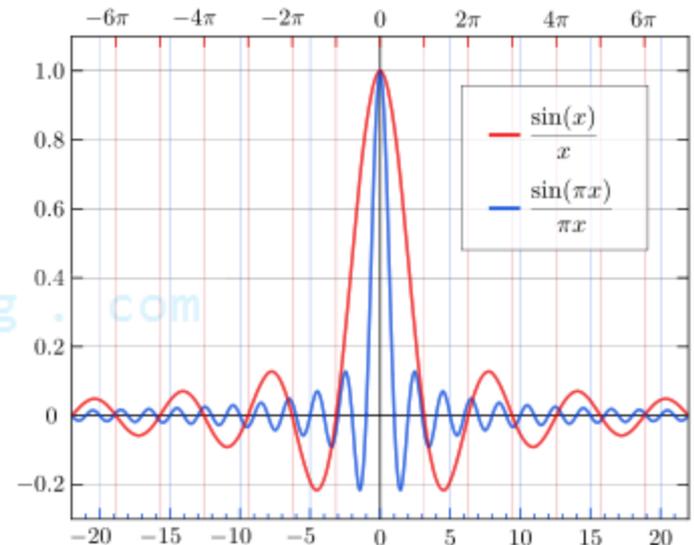
Dirac delta:
$$\delta(x) = \begin{cases} +\infty, & x = 0 \\ 0, & x \neq 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1.$$

$\frac{1}{\epsilon}\Pi\left(\frac{t}{\epsilon}\right)$

➤ **Properties:**

$$\int_{-\infty}^{\infty} f(x)\delta(x)\, dx = f(0)$$

$$\delta(\alpha x) = \frac{\delta(x)}{|\alpha|}$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - T)\, dt = f(T).$$

$A\delta(t-t_d)$

Review of special functions
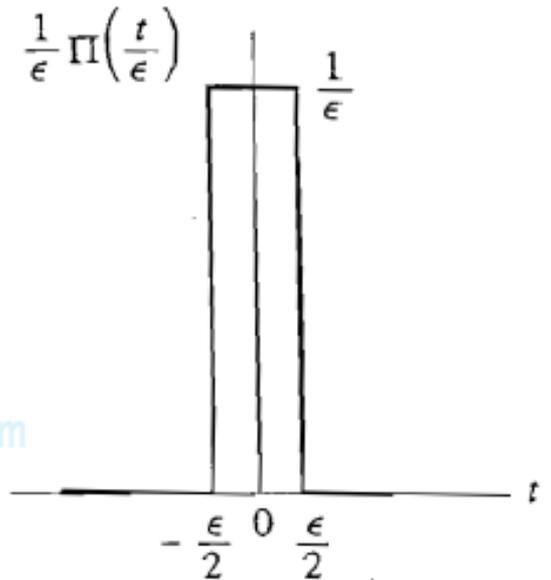
❖ **Dirac comb (impulse train, sampling function):**

$$\mathrm{III}_T(t) \overset{\mathrm{def}}{=} \sum_{k=-\infty}^{\infty} \delta(t - kT) = \frac{1}{T} \mathrm{III}\left(\frac{t}{T}\right)$$
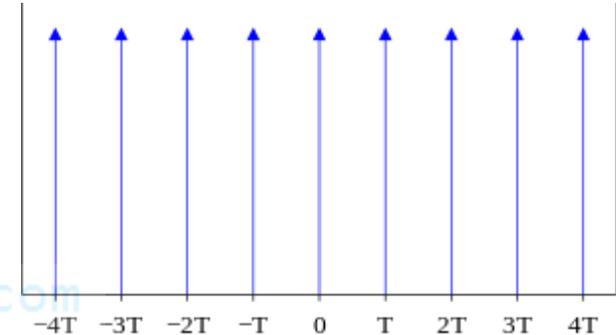
➤ **Properties:**

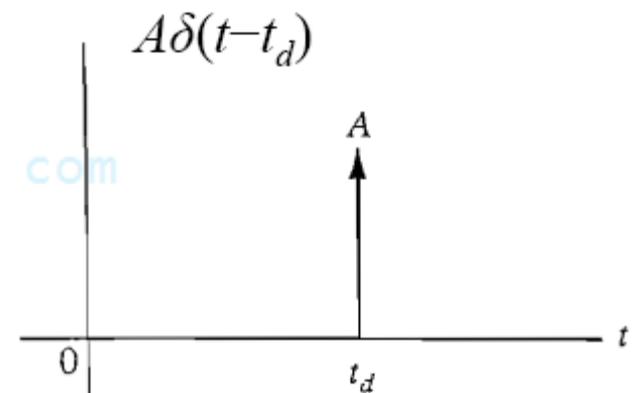$$\mathrm{III}_T(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{T}\mathrm{III}_{\frac{1}{T}}(f) = \sum_{n=-\infty}^{\infty} e^{-i2\pi fnT}. \qquad \mathrm{III}_{aT}(t) = \frac{1}{a}\mathrm{III}_T\left(\frac{t}{a}\right)$$

$$(\mathrm{III}_T\, x)(t) = \sum_{k=-\infty}^{\infty} x(t)\delta(t - kT) = \sum_{k=-\infty}^{\infty} x(kT)\delta(t - kT).$$

$$(\mathrm{III}_{\frac{1}{T}} * X)(f) = \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T}\right)$$

Review of spectral analysis

❖ **Periodic signal:** Fourier series (line spectrum) $\qquad f_0 = \dfrac{1}{T_0}$

$$V(f) = \sum_{n=-\infty}^{\infty} c_n \delta(f - nf_0) \qquad c_n = \frac{1}{T_0} \int_{T_0} v(t) e^{-j2\pi n f_0 t} \, dt$$

$$v(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi n f_0 t} \qquad c_n = |c_n| e^{j\arg c_n}$$

❖ **Aperiodic signal:** Fourier transform

$$V(f) = \mathrm{F}[v(t)] = \int_{-\infty}^{\infty} v(t) e^{-j2\pi f t} \, dt$$

$$v(t) = \mathrm{F}^{-1}[V(f)] = \int_{-\infty}^{\infty} V(f) e^{j2\pi f t} \, df$$

$$\cos(2\pi F_0 t) \xleftrightarrow{\;FT\;} \frac{1}{2}[\delta(F + F_0) + \delta(F - F_0)]$$

$$\sin(2\pi F_0 t) \xleftrightarrow{\;FT\;} \frac{1}{2} j[\delta(F + F_0) - \delta(F - F_0)]$$

$$A \leftrightarrow A\delta(f) \qquad A\delta(t) \leftrightarrow A$$

$$\int_{-\infty}^{\infty} \text{sinc}(t)\, e^{i2\pi f t}\, dt = \text{rect}(f),$$

$$\int_{-\infty}^{\infty} \text{rect}(t) \cdot e^{-i2\pi f t}\, dt = \frac{\sin(\pi f)}{\pi f} = \text{sinc}(\pi f),$$

Review of Fourier transform properties

❖ **Linear (superposition):**

$$a_1 x_1(t) + a_2 x_2(t) \qquad\qquad a_1 X_1(f) + a_2 X_2(f)$$

❖ **Delay:**

$$x(t - t_0) \qquad\qquad X(f)e^{-j2\pi f t_0}$$

❖ **Convolution:**

$$\mathscr{F}\{g(t) * h(t)\} = G(f)H(f)$$

Review of trigonometric formulas

$$\cos(a)\cos(b) = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin(a)\sin(b) = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$

$$\sin(a)\cos(b) = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$A * \sin(\omega t) + B * \cos(\omega t) = C * \sin(\omega t + \Phi)$$

$$C = \sqrt{(A^2 + B^2)}$$

$$\Phi = \arctan(A/B)$$

Review of Poisson summation formula

❖ **Statement:** $\displaystyle\sum_{n=-\infty}^{\infty} f(n) = \sum_{k=-\infty}^{\infty} \hat{f}(k), \quad \hat{f}(\nu) = \mathcal{F}\{f(x)\}.$

❖ **Condition:** $\displaystyle f(x) \ll \frac{1}{1+|x|^2} \text{ and } f'(x) \ll \frac{1}{1+|x|^2}.$

$$\sum_{k=-\infty}^{\infty} \hat{f}(k) = \sum_{k=-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(x)\, e^{-i2\pi kx} dx \right) = \int_{-\infty}^{\infty} f(x) \underbrace{\left(\sum_{k=-\infty}^{\infty} e^{-i2\pi kx} \right)}_{\sum_{n=-\infty}^{\infty} \delta(x-n)} dx$$

$$= \sum_{n=-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(x)\, \delta(x-n)\, dx \right) = \sum_{n=-\infty}^{\infty} f(n).$$

Review of convolution and correlation

❖ **Convolution:**

$$g(t) * h(t) = \int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau$$

$$v * w = w * v$$

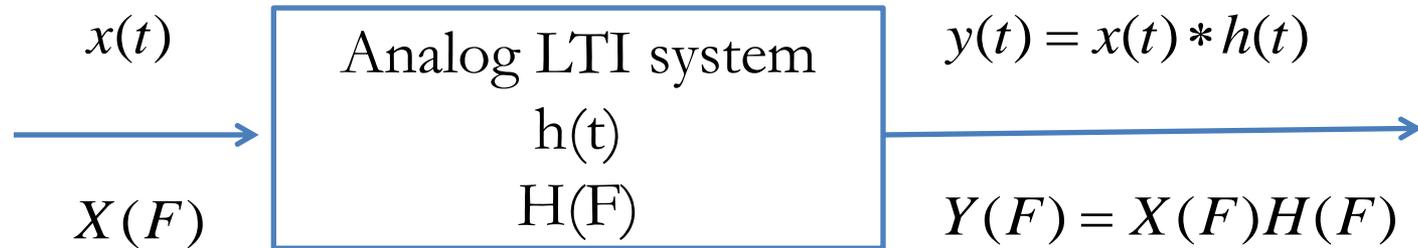$$v * (w * z) = (v * w) * z$$

$$v * (w + z) = (v * w) + (v * z)$$

❖ **Correlation:**

$$\phi_{sh}(\tau) = \int_{-\infty}^{+\infty} s(t)\, h(t+\tau)\, dt$$

➢ **Auto-correlation:**

$$\phi_{ss}(\tau) = \int_{-\infty}^{+\infty} s(t)\, s(t+\tau)\, dt$$

$$\phi_{ss}(\tau) \longleftrightarrow \left| S(f) \right|^2,$$

Review of analog linear time-invariant system

$$x(t)$$

$$\boxed{\text{Analog LTI system} \\ h(t) \\ H(F)}$$

$$y(t) = x(t) * h(t)$$

$$X(F)$$

$$Y(F) = X(F)H(F)$$

$$x(t) = A\cos(2\pi F_0 t + \theta) \qquad y(t) = A\,|\,H(F_0)\,|\cos(2\pi F_0 t + \theta + \arg\{H(F_0)\})$$

❖ Linear: $\qquad$ if $\qquad x(t) = \sum_k a_k x_k(t) \qquad$ then $\qquad y(t) = \sum_k a_k F[x_k(t)]$

❖ Time-invariant: $\qquad F[x(t - t_d)] = y(t - t_d)$

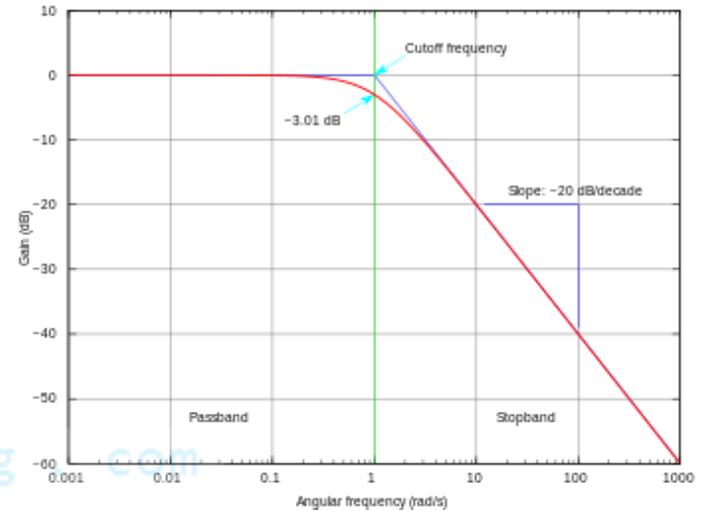❖ Impulse response: $\qquad h(t) = F[\delta(t)]$

❖ Frequency response: $\qquad H(f) = F[h(t)] = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt$
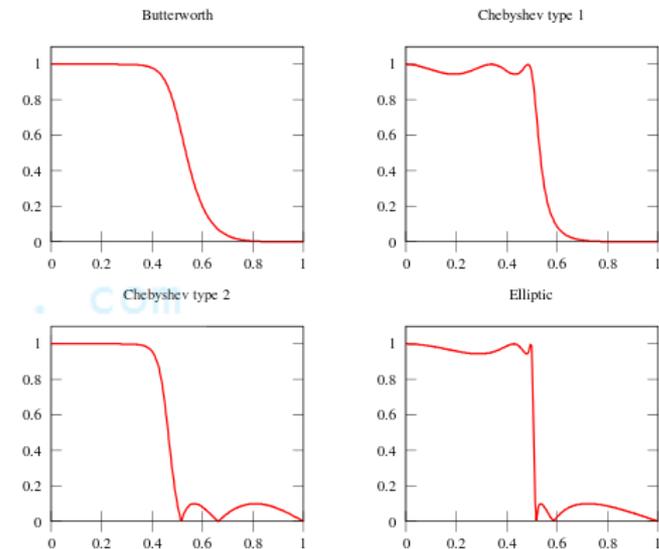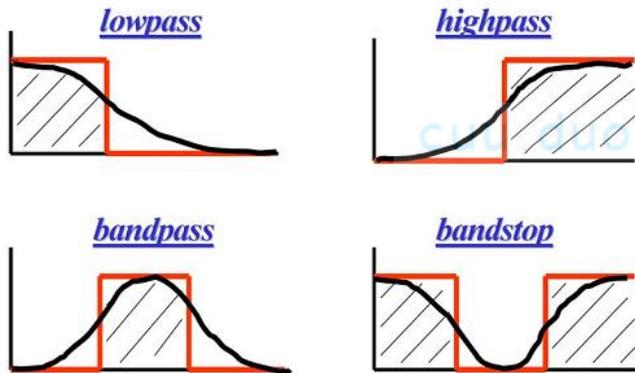
➢ Amplitude (magnitude): $|H(F)|$
➢ Phase: $\arg\{H(F)\}$

Review of analog filters

❖ Decibel: $|A|_{dB} = 20\log_{10}|A|$

❖ Logarithmic scales:
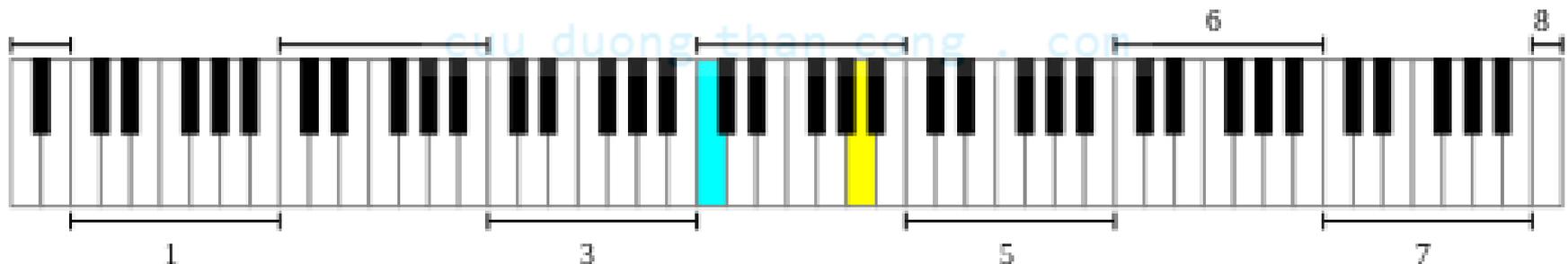 ➢ Decade: decades $= \log_{10}(F2/F1)$
  ➢ Octave: octaves $= \log_2(F2/F1)$

❖ Cut-off (-3dB) frequency
❖ Bandwidth

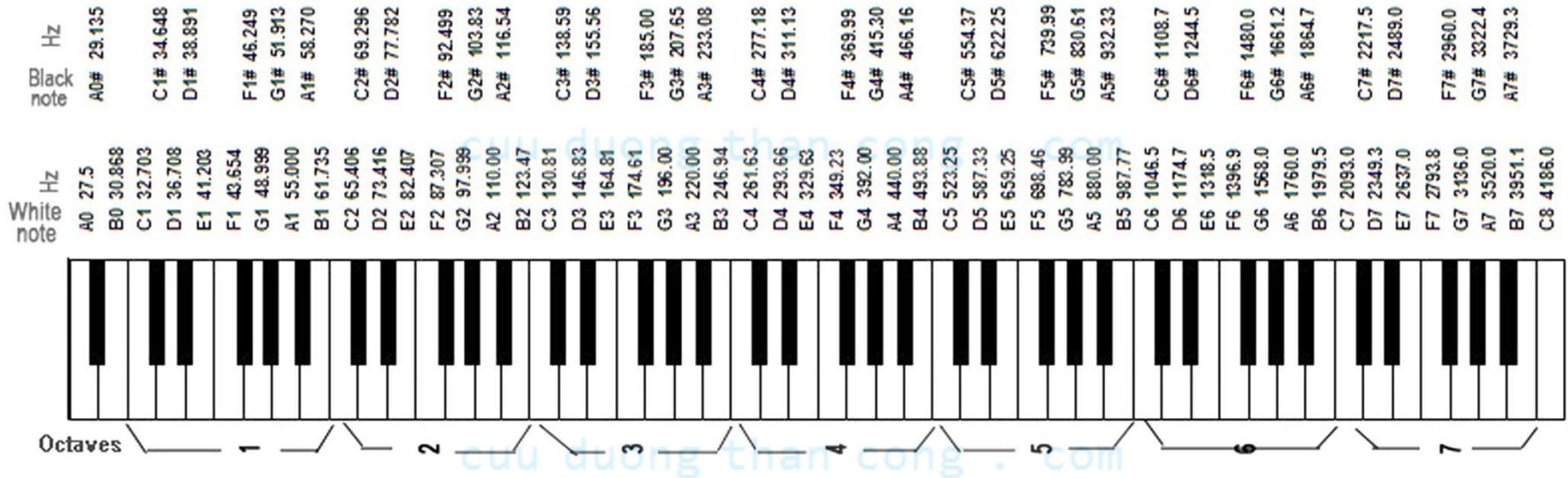# Example of octave scale

❖ An 88-key piano in twelve-tone equal temperament, with the octaves numbered and Middle C (cyan) and A440 (yellow) highlighted.

$$n = 12 \log_2 \left(\frac{f}{440\,\mathrm{Hz}} \right) + 49 \qquad f(n) = \left(\sqrt[12]{2} \right)^{n-49} \times 440\,\mathrm{Hz}$$
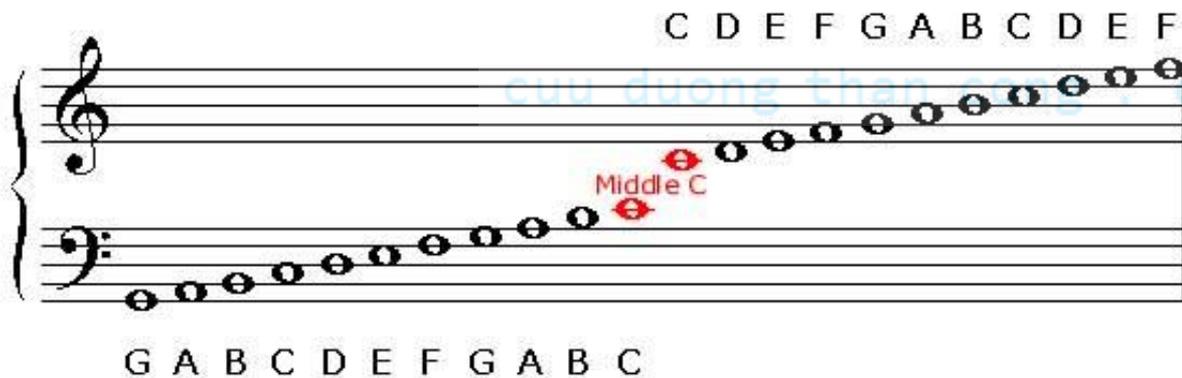
❖ Write a program generating tones of an 88-key piano in twelve-tone equal temperament with A440 standard.

Bonus 2

❖ Write a program generating tones of a guitar with standard below.

	OPEN	1F	2F	3F	4F	5F	6F	7F	8F	9F	10F	11F	12F	13F	14F	15F	16F	17F	18F	19F	20F
1st	E 329	F 349	F# 370	G 392	G# 415	A 440	A# 466	B 494	C 523	C# 554	D 587	D# 622	E 659	F 698	F# 740	G 784	G# 831	A 880	A# 932	B 988	C 1047
2nd	B 247	C 262	C# 277	D 294	D# 311	E 329	F 349	F# 370	G 392	G# 415	A 440	A# 466	B 494	C 523	C# 554	D 587	D# 622	E 659	F 698	F# 740	G 784
3rd	G 196	G# 208	A 220	A# 233	B 247	C 262	C# 277	D 294	D# 311	E 329	F 349	F# 370	G 392	G# 415	A 440	A# 466	B 494	C 523	C# 554	D 587	D# 622
4th	D 147	D# 156	E 165	F 175	F# 185	G 196	G# 208	A 220	A# 233	B 247	C 262	C# 277	D 294	D# 311	E 329	F 349	F# 370	G 392	G# 415	A 440	A# 466
5th	A 110	A# 117	B 123	C 131	C# 139	D 147	D# 156	E 165	F 175	F# 185	G 196	G# 208	A 220	A# 233	B 247	C 262	C# 277	D 294	D# 311	E 329	F 349
6th	E 82	F 87	F# 92	G 98	G# 104	A 110	A# 117	B 123	C 131	C# 139	D 147	D# 156	E 165	F 175	F# 185	G 196	G# 208	A 220	A# 233	B 247	C 262

❖ Write a program plotting the waveform of signal below.

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(2\pi * n\omega t) + \sum_{n=1}^{\infty} B_n \cos(2\pi * n\omega t)$$

❖ Write a program plotting the spectrum of signal below.

$$f(t) = A_0 + \sum_{n=1}^{\infty} A_n \sin(2\pi * n\omega t) + \sum_{n=1}^{\infty} B_n \cos(2\pi * n\omega t)$$

Greek alphabet

A	alpha	α	N	nu	ν
B	beta	β	Ξ	xi	ξ
Γ	gamma	γ	O	omicron	o
Δ	delta	δ	Π	pi	π
E	epsilon	ϵ	P	rho	ρ
Z	zeta	ζ	Σ	sigma	σ
H	eta	η	T	tau	τ
Θ	theta	θ	Υ	upsilon	υ
I	iota	ι	Φ	phi	ϕ
K	kappa	κ	X	chi	χ
Λ	lambda	λ	Ψ	psi	ψ
M	mu	μ	Ω	omega	ω

| Greek Letter		Name	Equivalent	Sound When Spoken
A	α	Alpha	A	al-fah
B	β	Beta	B	bay-tah
Γ	γ	Gamma	G	gam-ah
Δ	δ	Delta	D	del-tah
E	ϵ	Epsilon	E	ep-si-lon
Z	ζ	Zeta	Z	zay-tah
H	η	Eta	E	ay-tay
Θ	θ	Theta	Th	thay-tah
I	ι	Iota	I	eye-o-tah
K	κ	Kappa	K	cap-ah
Λ	λ	Lambda	L	lamb-dah
M	μ	Mu	M	mew
N	ν	Nu	N	new
Ξ	ξ	Xi	X	zzEye
O	o	Omicron	O	om-ah-cron
Π	π	Pi	P	pie
P	ρ	Rho	R	row
Σ	σ	Sigma	S	sig-ma
T	τ	Tau	T	tawh
Υ	υ	Upsilon	U	oop-si-lon
Φ	ϕ	Phi	Ph	figh or fie
X	χ	Chi	Ch	kigh
Ψ	ψ	Psi	Ps	sigh
Ω	ω	Omega	O	o-may-gah

Portraits of Scientists and Inventors

❖ **René Descartes** (1596-1650): French philosopher, mathematician and scientist. "Cogito, ergo sum" ("Tôi tư duy, vậy tôi tồn tại").

❖ **Jean-Robert Argand** (1768-1822): French amateur mathematician.

❖ **Jean-Baptiste Joseph Fourier** (1768-1830): French mathematician and physicist.

❖ **Siméon Denis Poisson** (1781-1840): French mathematician, geometer, and physicist.

Portraits of Scientists and Inventors

❖ **Heinrich Rudolf Hertz** (1857-1894) was a German physicist who first conclusively proved the existence of electromagnetic waves.

❖ **Alexander Graham Bell** (1847-1922) was an eminent Scottish-born scientist, inventor, engineer and innovator who is credited with inventing the first practical telephone.
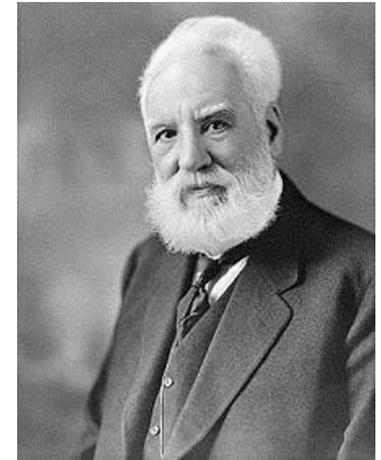
Homework 1

❖ For each case below, find the modulus and argument (both in radian and degree):

1) –2
2) –3i
3) –2 – 3i
4) –2 + 3i
5) 2 – 3i
6) 1/(2 – 3i)
7) (2 – 3i)/i
8) (2 – 3i)^2
9) (2 – 3i) + 1/(2 – 3i)
10) (2 – 3i).(–2 – 3i)
11) (2 – 3i)/(–2 – 3i)
12) (2 – 3i)/(2 + 3i)

Homework 2

❖ For each case below, find the modulus and argument (both in radian and degree):

1) $e^{\wedge}(i\pi)$
2) $e^{\wedge}(i\pi/2)$
3) $e^{\wedge}(-i\pi/2)$
4) $e^{\wedge}(i\pi/4)$
5) $e^{\wedge}(i\pi/2) + e^{\wedge}(i\pi/4)$
6) $1/e^{\wedge}(i\pi/4)$
7) $e^{\wedge}(i\pi/4) \, / \, e^{\wedge}(-i\pi/4)$
8) $e^{\wedge}(i\pi/4) + e^{\wedge}(-i\pi/4)$
9) $e^{\wedge}(i\pi/4) - e^{\wedge}(-i\pi/4)$
10) $1 + e^{\wedge}(i\pi/2)$
11) $1 - e^{\wedge}(i\pi/2)$
12) $(2 - 3i). \, e^{\wedge}(i\pi/4)$

❖ For each case below, sketch the locus of z on the complex plane:

1) $|z| = 1$

2) $|z - 2| = 1$

3) $|z - 1| = 2$

4) $|z - 1 - 2i| = 3$

5) $|z| < 3$

6) $|z| > 2$

7) $2 < |z| < 3$

8) $|z^{-1}| < 4$

9) $|z^{-1}| > 2$

10) $2 < |z^{-1}| < 4$

11) $z + z^{-1} \neq \infty$

12) $1 + z^{-2} \neq \infty$

❖ For each case below, sketch the waveform of the signal:

1) x(t) = 4sin(2t) (t:s)

2) x(t) = 4sin(2πt) (t:s)

3) x(t) = 4cos(2πt) (t:s)

4) x(t) = 4cos(10πt) (t:s)

5) x(t) = 4cos(10πt) (t:ms)

6) x(t) = 1 + 4cos(10πt) (t:s)

7) x(t) = 4cos(2πt) + 4cos(10πt) (t:s)

8) x(t) = 4sin^2(2πt) (t:s)

9) x(t) = 4sinc(2t) (t:s)

10) x(t) = 4Π{(t − 3)/2}

11) x(t) = Σ$_k${4Π{(t − k5 − 3)/2}}

12) x(t) = 4δ(t − 3) − 3δ(t + 4)

Homework 5

❖ For each case below, plot the magnitude spectrum of the signal:
1) A
2) A.cos(2πFt+ϕ)
3) A.cos(2πFt+ϕ) + B
4) A.cos(2πF$_1$t+ϕ$_1$) + B.cos(2πF$_2$t+ϕ$_2$)
5) A.cos(2πFt+ϕ$_1$) + B.cos(2πFt+ϕ$_2$)
6) A.cos(2πFt+ϕ$_1$) + A.cos(2πFt+ϕ$_2$)
7) A.cos(2πFt+ϕ) + A.sin(2πFt+ϕ)
8) x(t) = 10 – 4cos6πt (t: ms)
9) x(t) = 1 – 2cos6πt + 3sin14πt (t: ms)
10) x(t) = 3cos10^{3}πt – 4sin10^{4}πt (t: s)
11) x(t) = 14sin^{2}3πt + 3sin14πt (t: ms)
12) x(t) = 4cos^{2}2πt – 10sin10πt (t: ms)

❖ Suppose a filter has magnitude response as shown in figure below. Determine the expression (ignoring the phase) of the output signal and plot it's magnitude response for each case of the input signal:

1) $x(t) = 2$
2) $x(t) = 2\cos(2\pi t)$ (t:ms)
3) $x(t) = 2\cos(20\pi t)$ (t:ms)
4) $x(t) = 2\cos(200\pi t)$ (t:ms)
5) $x(t) = 2\cos(400\pi t)$ (t:ms)
6) $x(t) = 2\cos^2(400\pi t)$ (t:ms)
7) $x(t) = 2\cos(200\pi t).\sin(400\pi t)$ (t:ms)
8) $x(t) = 2\cos(200\pi t) - 2\cos(400\pi t)$ (t:ms)
9) $x(t) = 2\cos(200\pi t) + 2\sin(400\pi t)$ (t:ms)
10) $x(t) = 2\cos(200\pi t) + 2\sin(200\pi t)$ (t:ms)
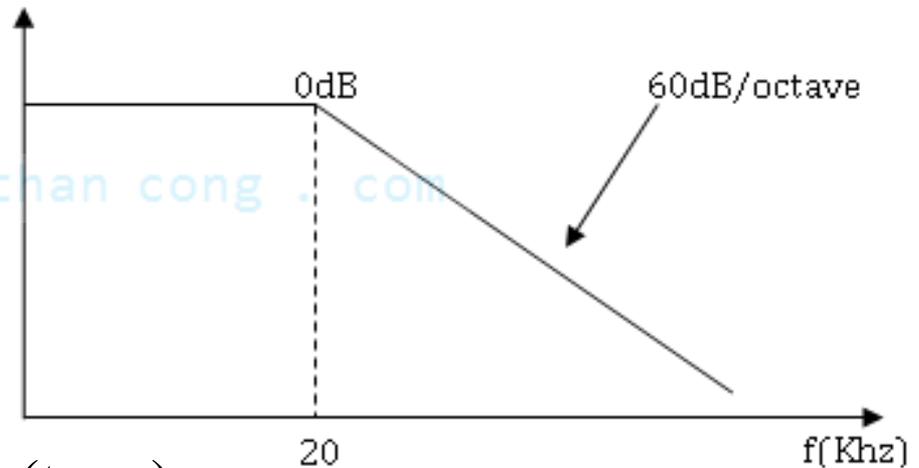
❖ Cho hệ thống tuyến tính bất biến có hàm truyền H(f) như hình:
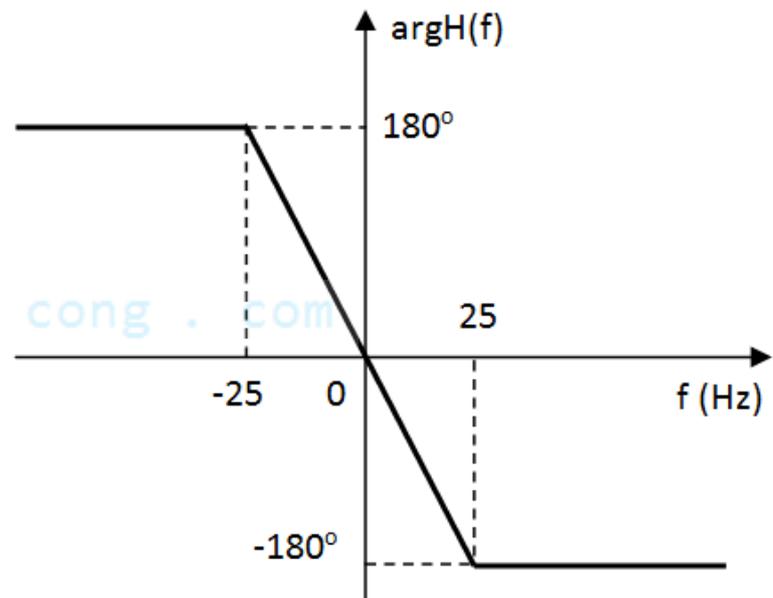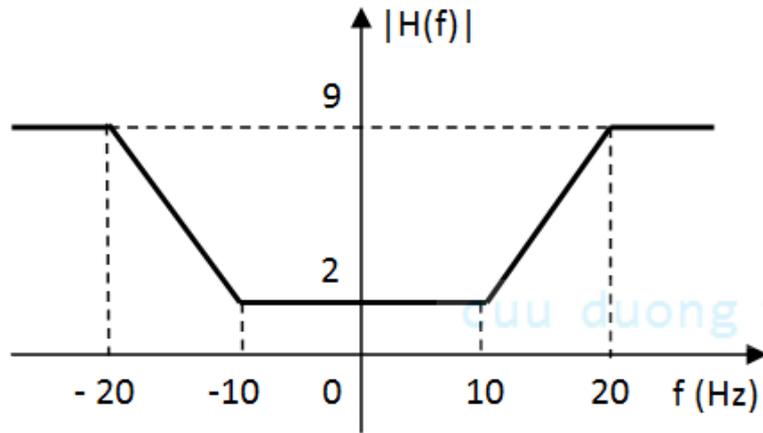
a) Xác định biểu thức đầy đủ của tín hiệu ngõ ra y(t) khi tín hiệu ngõ vào x(t) = 10cos2@πt – 30sin40πt (t:s).

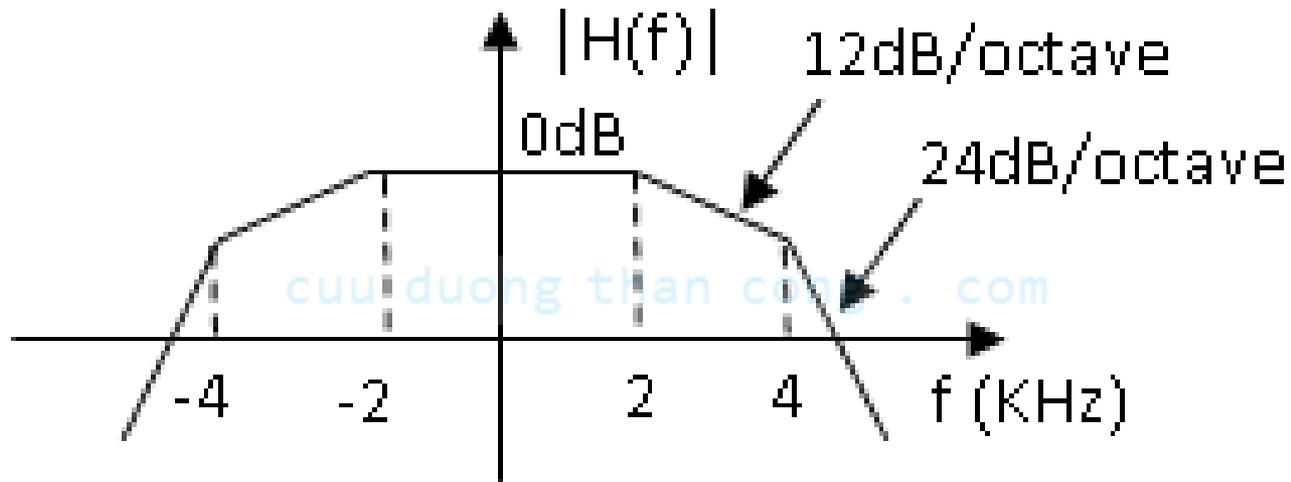b) Xác định biểu thức đầy đủ của tín hiệu ngõ vào x(t) để tín hiệu ngõ ra y(t) = 10cos2@πt (t:s).

❖ Cho các tín hiệu tương tự $x_1(t) = 2\cos^2 2\pi t$ (t: s) và $x_2(t) = 6\sin 6\pi t + 7\cos 7\pi t + 8\sin 8\pi t$ (t:s) lần lượt đi qua hệ thống tuyến tính bất biến có hàm truyền H(f) như hình:
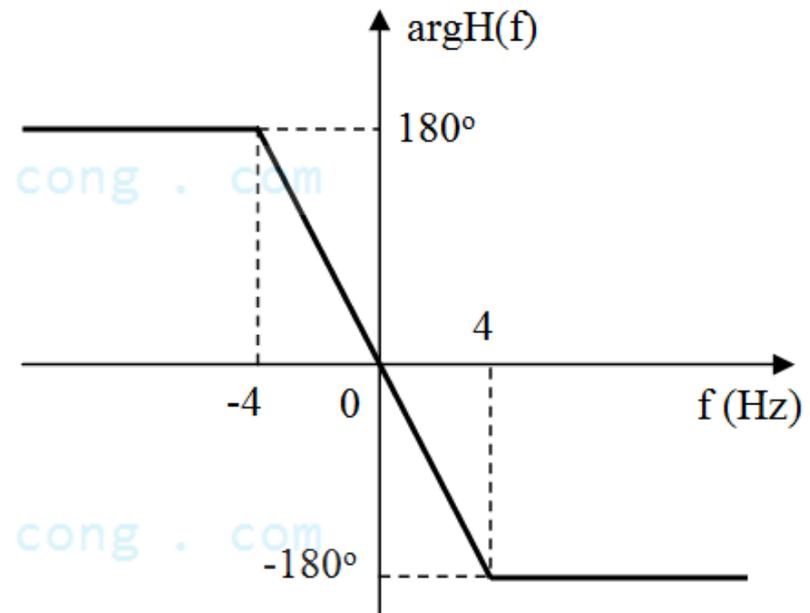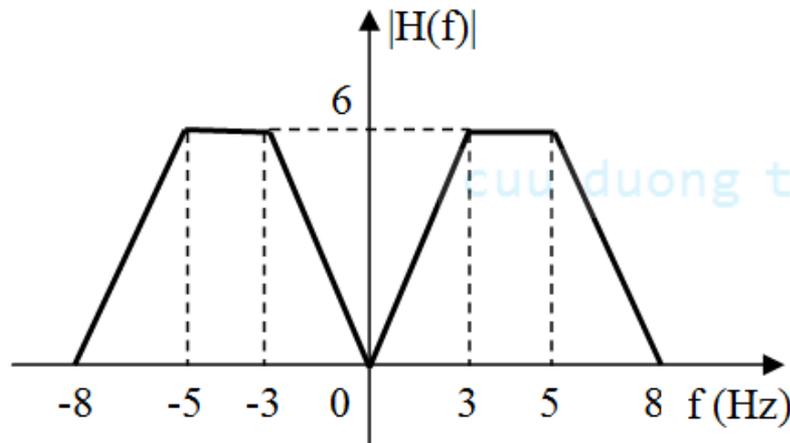
a) Xác định biểu thức (theo thời gian) của tín hiệu ngõ ra $y_1(t)$.
b) Tính giá trị của tín hiệu ngõ ra $y_2(t = 0.125s)$.

❖ Tìm giá trị đáp ứng biên độ |H(f)| tại các tần số sau:

a) 1KHz.
b) 3KHz.
c) 4KHz.
d) 5KHz.
e) 8KHz.

❖ Cho bộ lọc thông thấp có đáp ứng biên độ phẳng 0dB trong khoảng [0 ÷ 4]KHz, suy giảm với độ dốc 12dB/octave trong khoảng [4 ÷ 8]KHz và suy giảm với độ dốc 20dB/decade ngoài 8KHz. Tìm giá trị đáp ứng biên độ của bộ lọc tại các tần số sau:

a) 2KHz.

b) 3KHz.

c) 5KHz.

d) 6KHz.

e) 7KHz.

f) 8KHz.

g) 10KHz.

h) 12KHz.

i) 16KHz.

j) 20KHz.