# *Chapter I:*
# Introduction to Signals and Systems





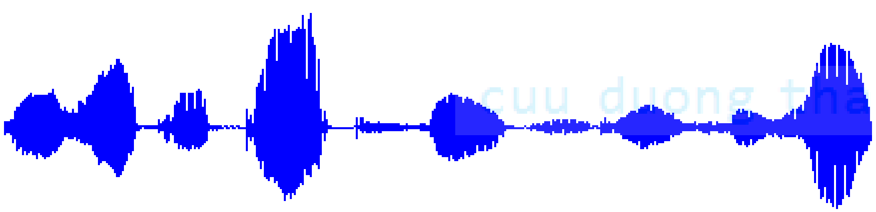


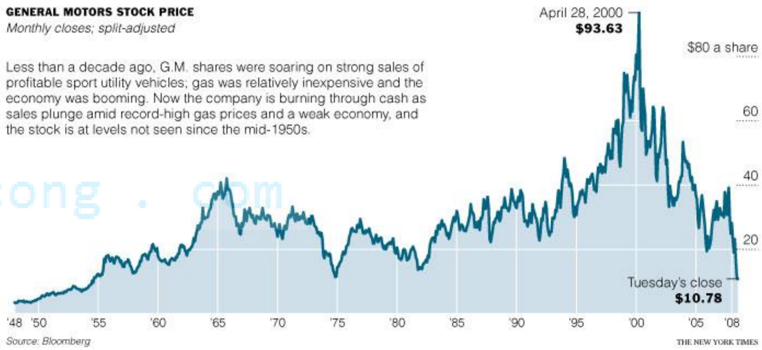
*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

# Signals

Signals are functions of independent variables that carry information. For example:

- Electrical signals: voltages and currents in a circuit.
- Acoustic signals: audion or speech signals.
- Video signals: intensity variations in an image
- Biological signals: sequence of bases in a gene
- Financial signals: stock price…

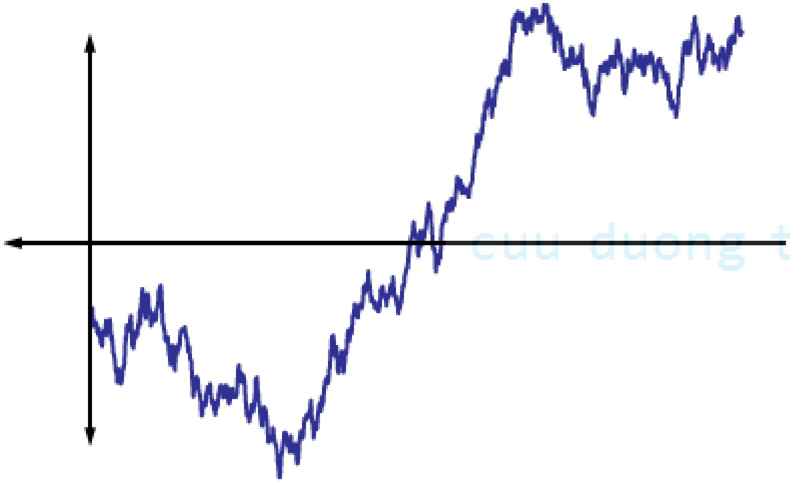


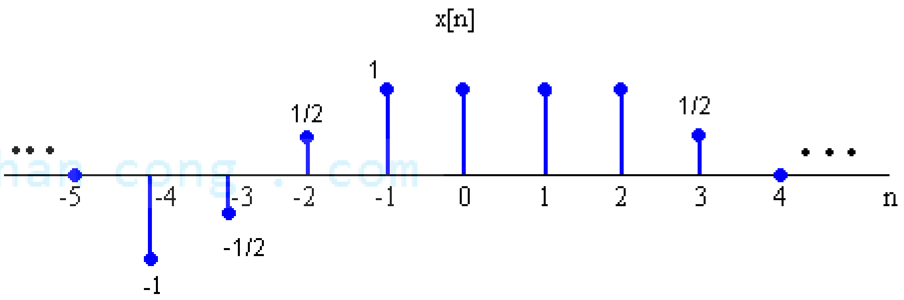



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## The independent variables

- Can be continuous or discrete
- Can be 1-D, 2-D, 3-D,…
- For this course: focus on a single independent variable which we call "time":
    - Continuous-Time (CT) signals: $f(t)$ – $t$: continuous values
    - Discrete-Time (DT) signals: $f[n] = f(nT)$ – $n$: integer values only



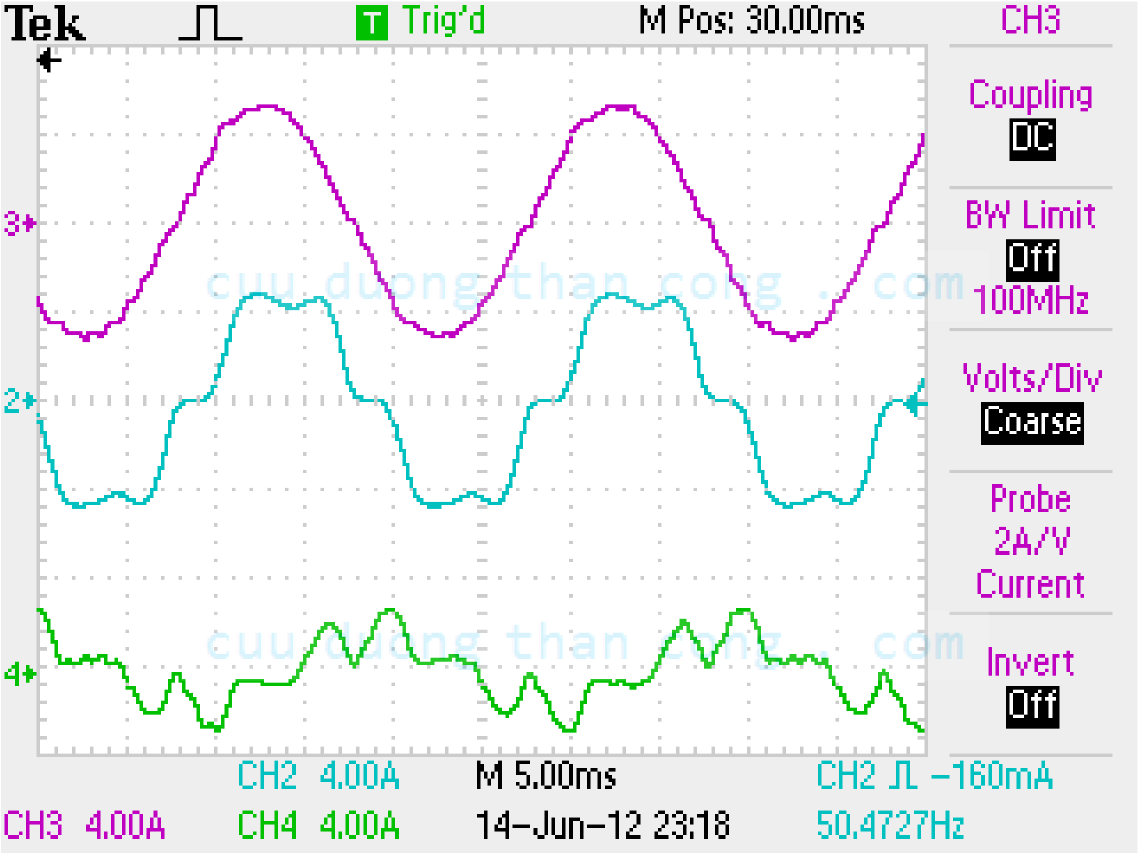


## Representation of signals

- In time domain $f(t)$
  - Signal is a sequence of values in time, very simple representation.
  - Useful for saying what is happening at a particular time.
  - Not so useful for capturing the overall characteristics of the signal.
  - Hard to sovle some problem directly.

# Representation of signals

- In time domain $f(t)$



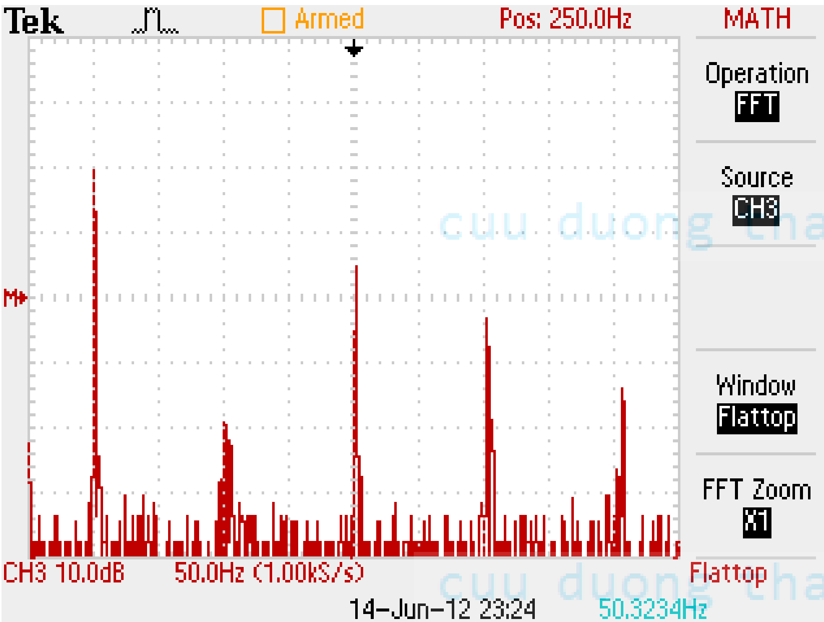

*Created and edited by:* **Nguyen Phuoc Bao Duy**
***HCMC University of Technology***

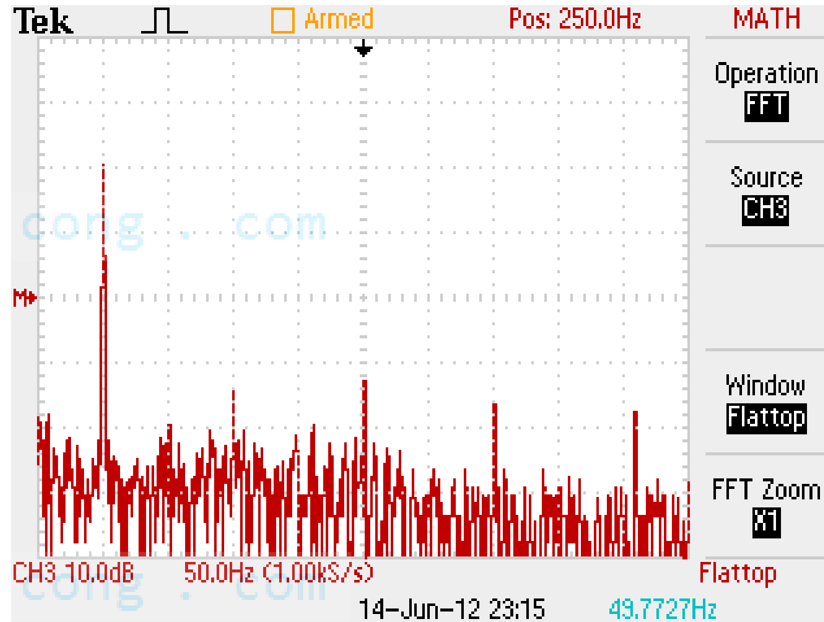
## Representation of signals

- In frequency domain $F(\omega)$
  - Simpler for many types of signals.
  - Reveals the fundamental characteristics of a signal or system.
  - Many systems are easier to analyze from this perspective (Linear Systems).

# Representation of signals

- In frequency domain $F(\omega)$

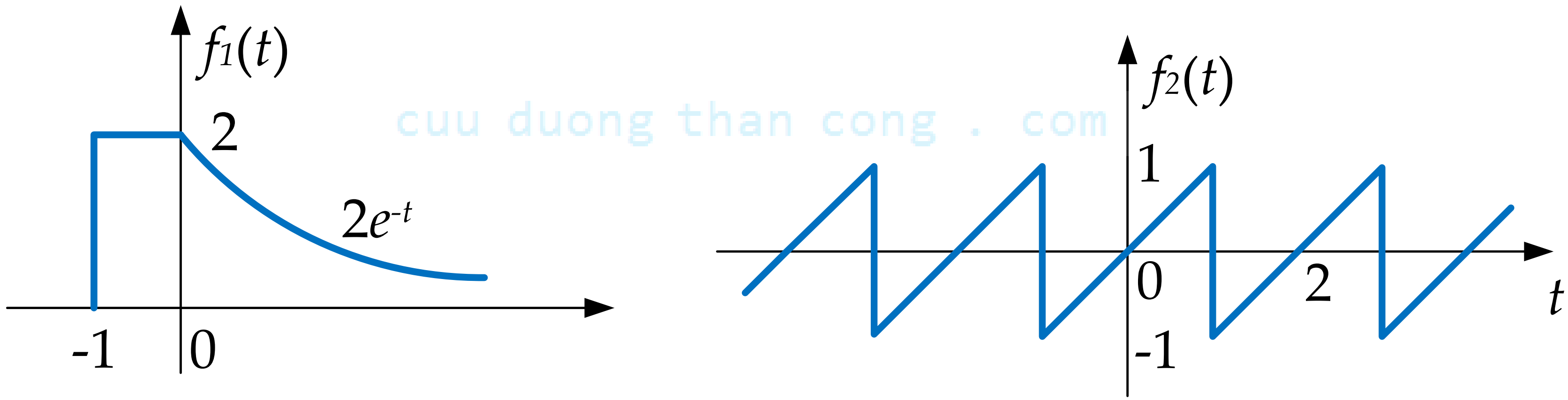


## Energy of a signal $f(t)$:

$$E_f = \int_{-\infty}^{\infty} \left| f(t) \right|^2 dt$$

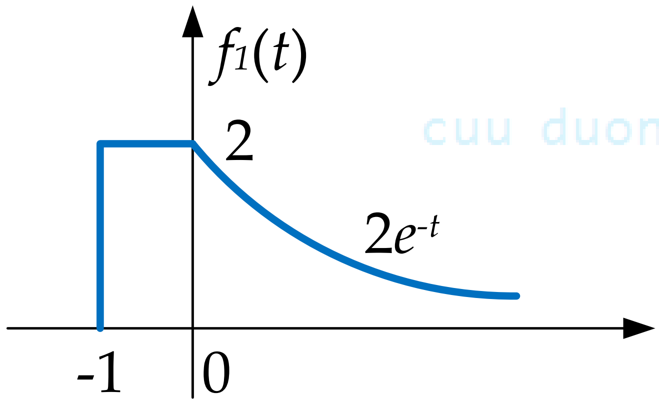
- *Example 1.01:* Find the energy of following signals

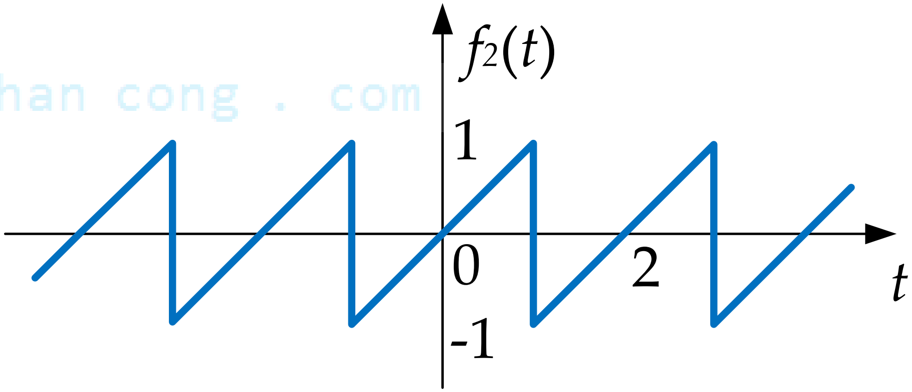


## Power of a signal $f(t)$:

$$P_f = \lim_{T \to +\infty} \left[ \frac{1}{T} \int_{-T/2}^{T/2} \left| f(t) \right|^2 dt \right] = \frac{1}{T} \int_{-T/2}^{T/2} \left| f(t) \right|^2 dt \text{ (for periodic signal)}$$

- *Example 1.02:* Find the power of following signals



# *Chapter I:*
# Introduction to Signals and Systems



1. *Introduction to Signals*
2. ***Classification of Signals***
3. *Some Signal Operations*
4. *Some Signal Models*
5. *Systems and Classification of System*
6. *System Model*

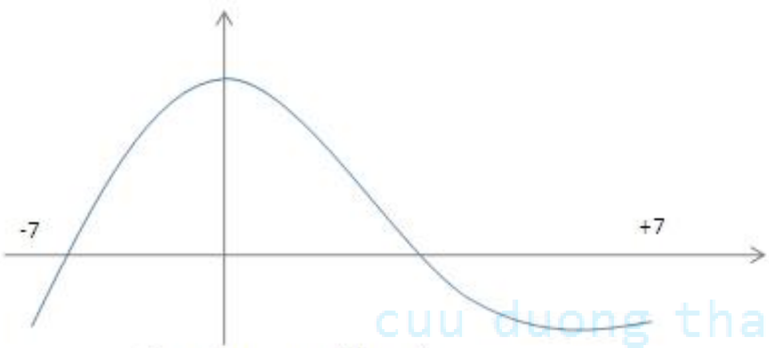


*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

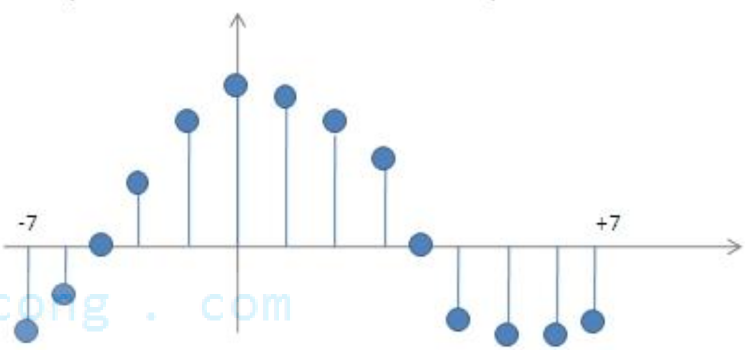
# Continuous-Time and Discrete-Time signals:

- **Continuous-Time** (CT) Signals:
  - Specified for **every value** of **time** *t*
  - Most of the signals in the physical world are CT signals: voltage & current, pressure, temperature, velocity, etc.






Continuous Signal

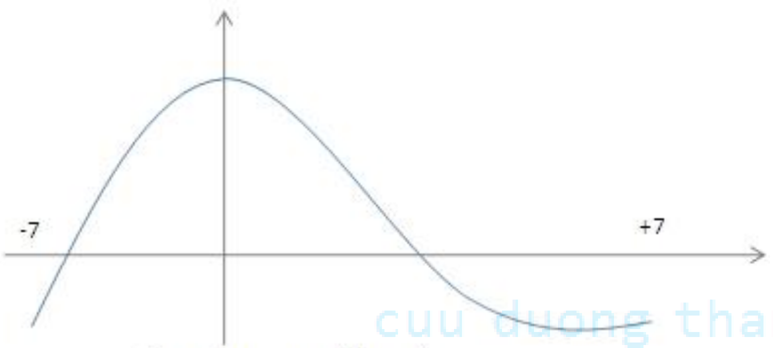



Discrete Signal

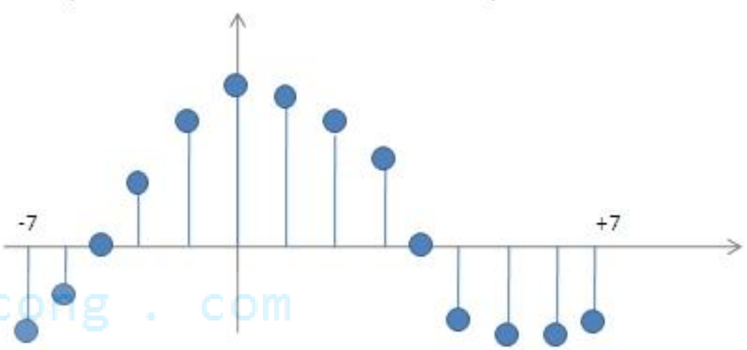
## Continuous-Time and Discrete-Time signals:

- **Discrete-Time** (DT) Signals:
  - Specified only at **discrete values** of **time** $t$
  - Can be processed by modern digital computers and digital signal processors (DSPs).





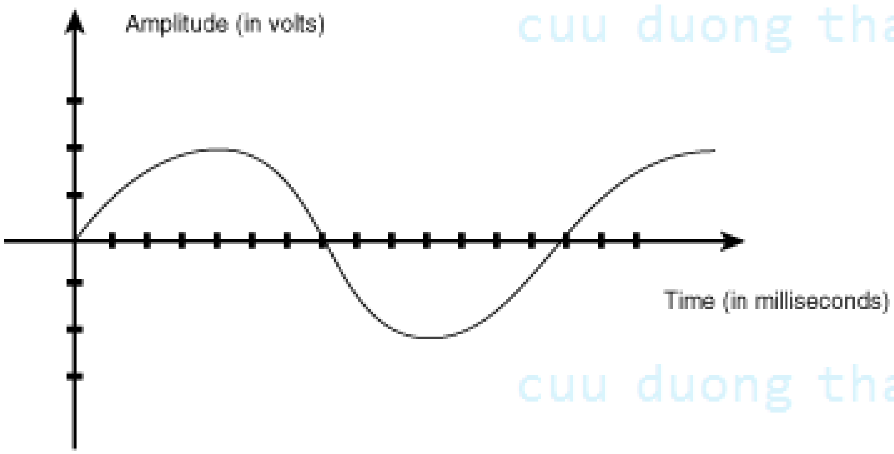

Continuous Signal

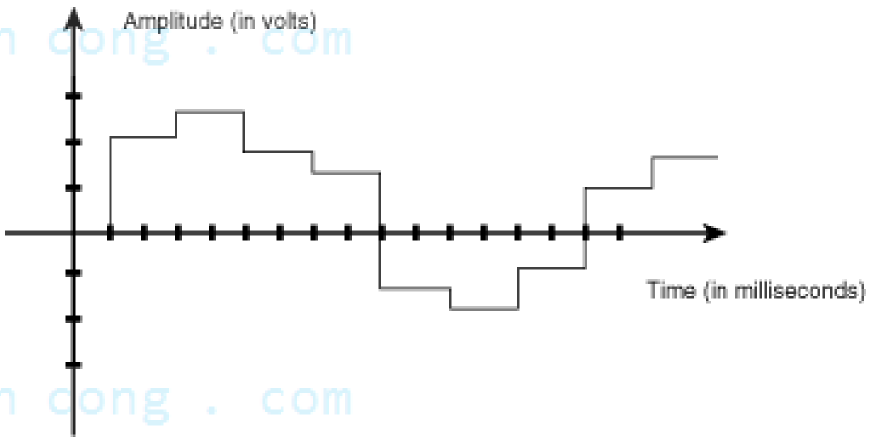



Discrete Signal

# Analog and Digital signals:

- **Analog** signals: the **amplitude** can take on **any value** in a continuous range.
- **Digital** signals: the amplitude can take on **only a finite number of value**.



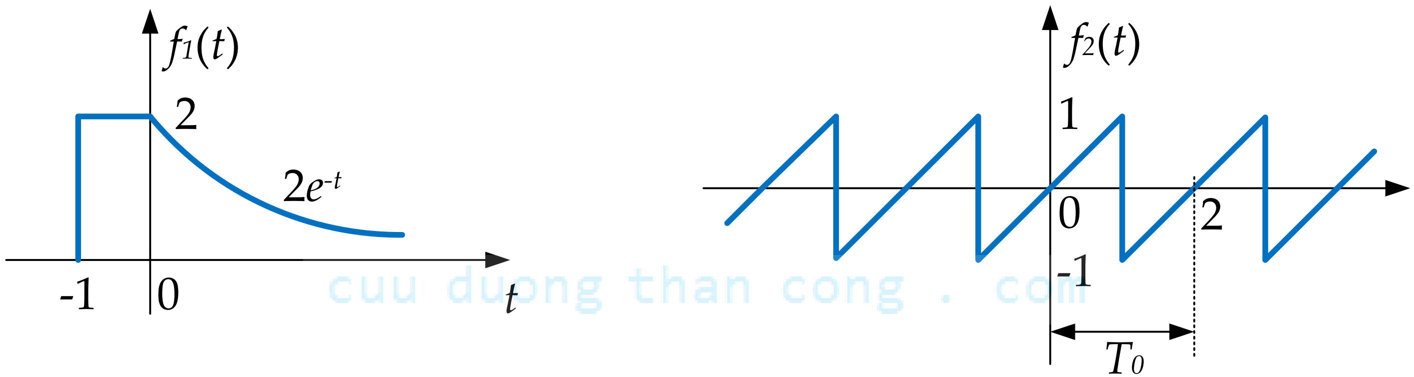



# Periodic and Aperiodic Signals:

- A signal $f(t)$ is said to be **periodic** if:

$$f(t) = f(t + T_0), \text{ for all } t$$

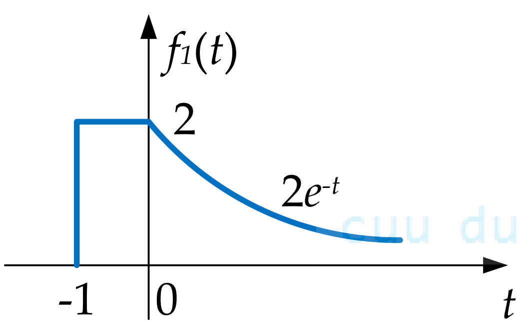
- The smallest value of $T_0$ is the period of $f(t)$.
- A periodic signal must start at $t = -\infty$ and continuing forever.
- A signal is **aperiodic** if it is **not periodic**.



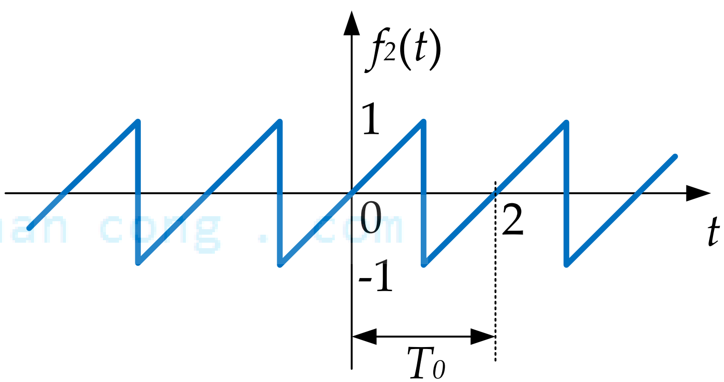

*aperiodic* *periodic*

# Energy and Power Signals:

- A signal with **finite energy** is an **energy signal**.
- A signal with **finite and nonzero power** is a **power signal**.
- A signal cannot both be an energy and a power signal.
- Periodic signal is one kind of power signal.
- There are signals that are neither energy nor power signals.



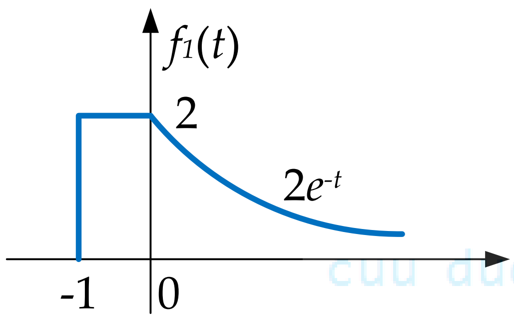

*energy signal*




*power signal*

# Deterministic and Random signals:

- A signal whose physical **description is know completely** is a **deterministic signal**.
- A signal whose values **cannot be predicted** precisely but are known only in terms of **probabilistic description** is a **random signal**.
- We shall exclusively deal with deterministic signals.

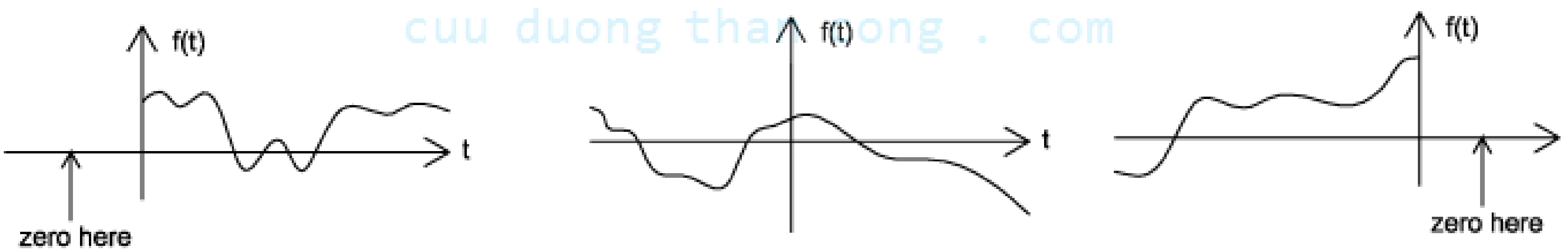



*deterministic signal*


*random signal*

# <u>Causal and Noncausal signals:</u>

- A signal that does not start before $t = 0$ is a **causal** signal.
- A signal that starts before $t = 0$ is a **non-causal** signal.
- A signal that is zero for all $t \geq 0$ is an **anti-causal** signal.




*causal* *noncausal* *anticausal*

# *Chapter I:*
# Introduction to Signals and Systems



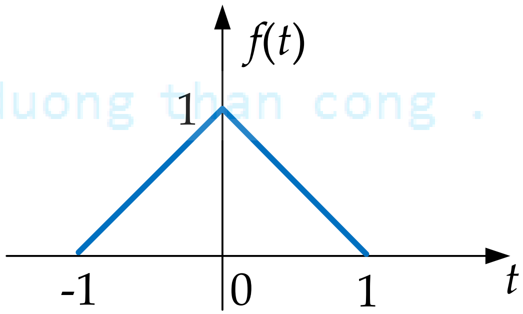
1. *Introduction to Signals*
2. *Classification of Signals*
3. ***Some Signal Operations***
4. *Some Signal Models*
5. *Systems and Classification of System*
6. *System Model*

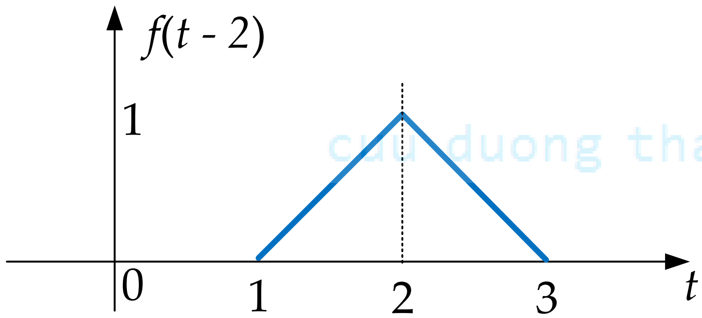


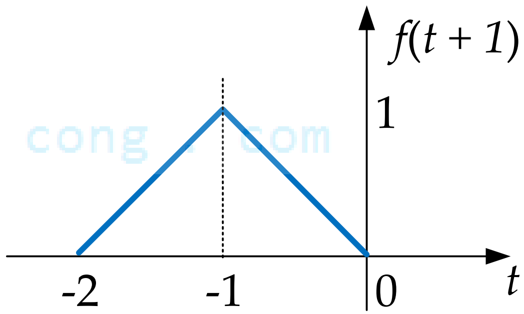
*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

# Time shifting:

- Let $f(t)$ be a CT signal, $T$ is a constant, then:
  - If $T > 0$: $f(t + T)$ is a left-shift signal.
  - If $T < 0$: $f(t + T)$ is a righ-shift signal (delay).

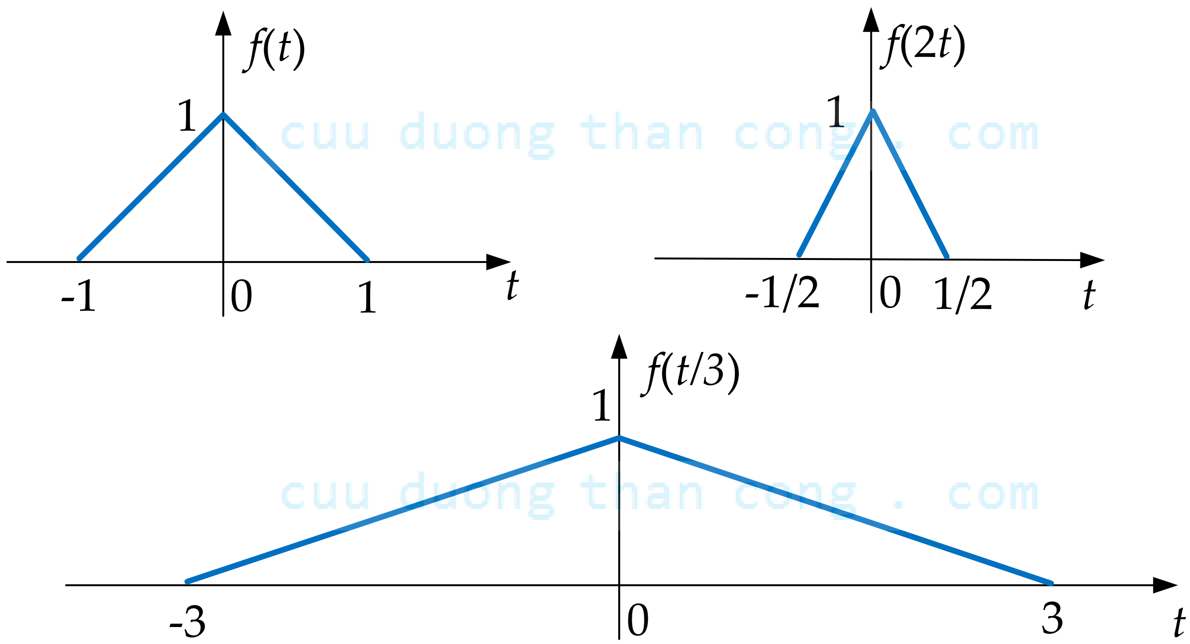


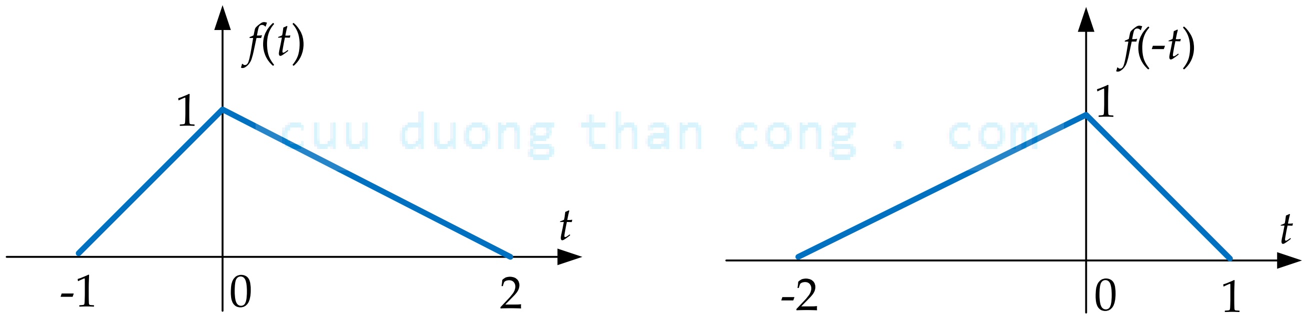
# Time scaling:

- Let $f(t)$ be a CT-signal, $b$ is a positve constant.
  - $0 < b < 1$: $f(bt)$ is an expanded signal
  - $b > 1$: $f(bt)$ is a compressed signal



# Time inversion:

- CT-signal: replace $t$ with $-t$ (time scaling with $b = -1$)



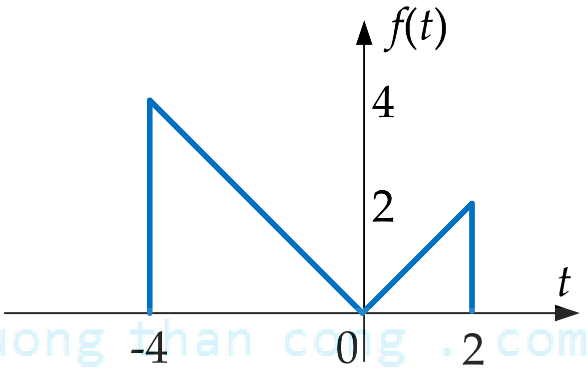



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

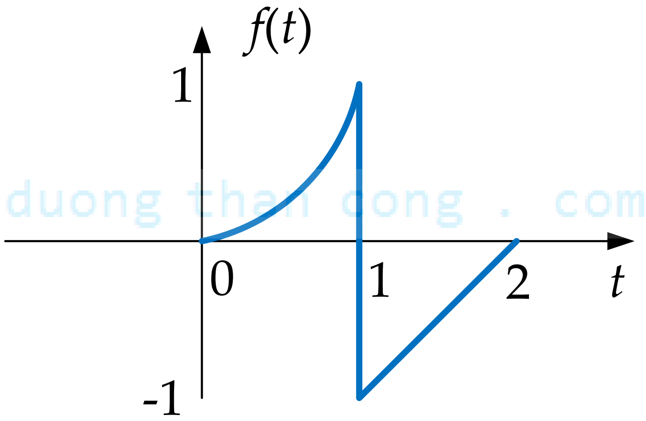
## Combined operation:

*Example 1.03:* For the signal $f(t)$ depicted below, sketch the signals:

a. $f(t-4)$
b. $f(t/2)$
c. $f(2t-4)$
d. $f(2-t)$

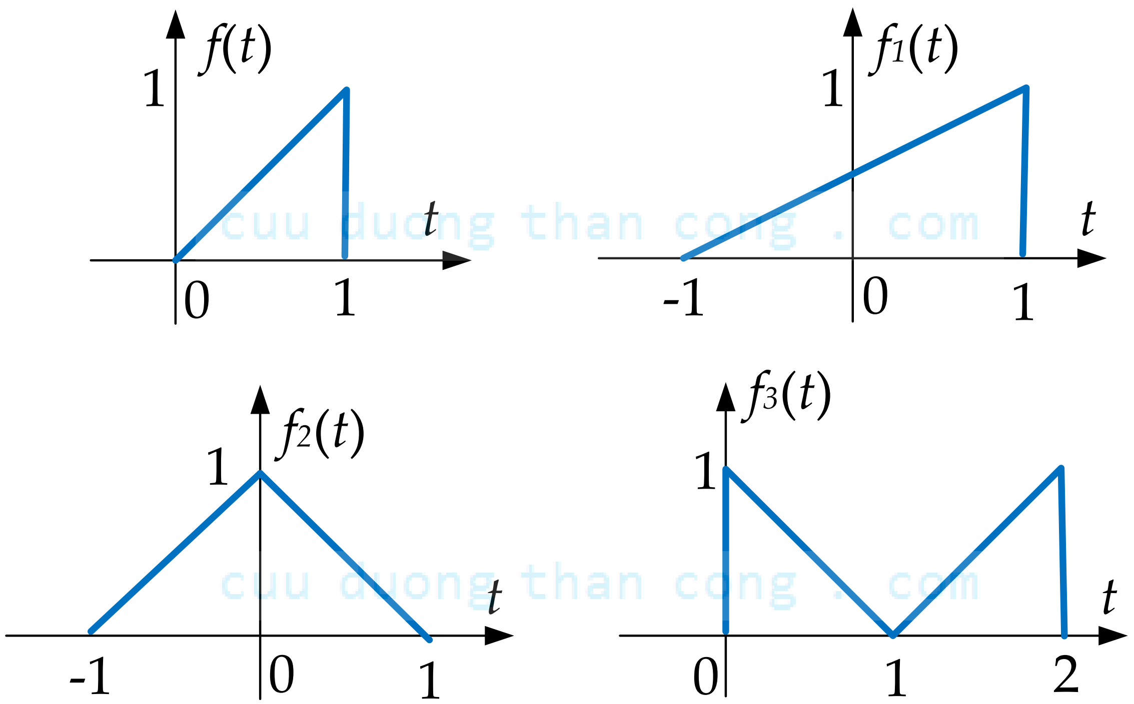



*Example 1.04:* Repeat example 1.03 for the signal depicted below:



# <u>Combined operation:</u>

<u>*Example 1.05:*</u> Express signals $f_1(t)$, $f_2(t)$, and $f_3(t)$ in terms of signals $f(t)$ and $f(-t)$.






*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

# *Chapter I:*
# Introduction to Signals and Systems



1. *Introduction to Signals*
2. *Classification of Signals*
3. *Some Signal Operations*
4. ***Some Signal Models***
5. *Systems and Classification of System*
6. *System Model*



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## Unit Step Function $u(t)$:

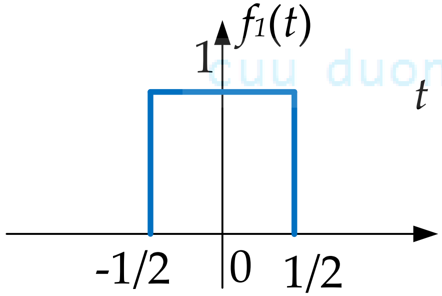
- Definition:

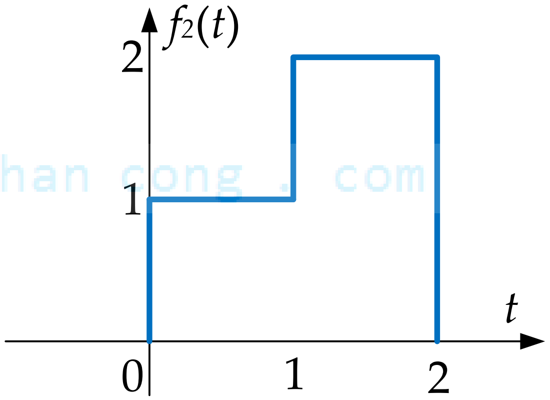
$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

- Combinations of unit steps to create other signals.
- Use $u(t)$ to extract part of another signal.



*Example 1.06:* Express $f_1(t)$ and $f_2(t)$ as weighted sum of step functions.



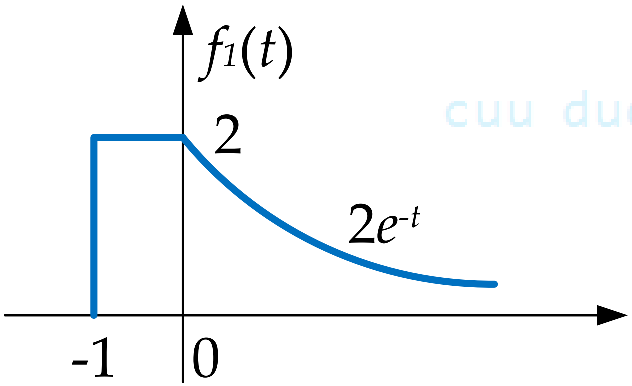



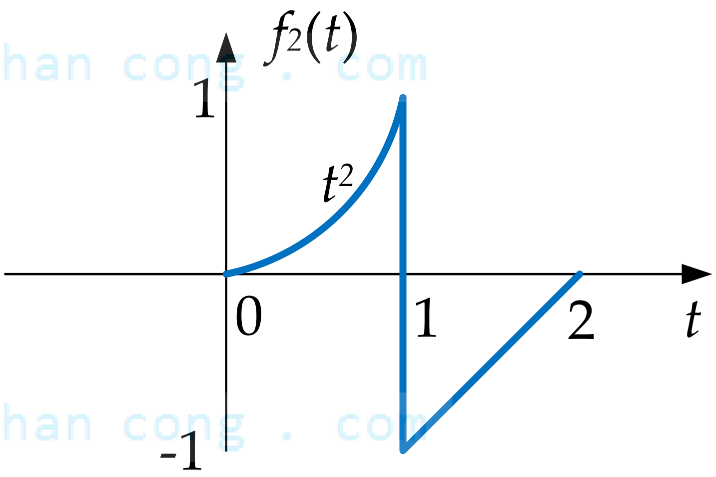



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

# Unit Step Function $u(t)$:

*Example 1.07:* Express each of the signals $f_1(t)$ and $f_2(t)$ by a single expression of variable $t$.





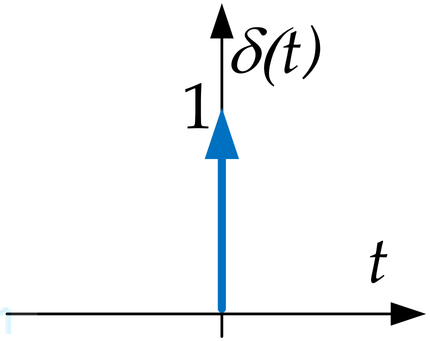



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

# The Unit Impluse Function $\delta(t)$:

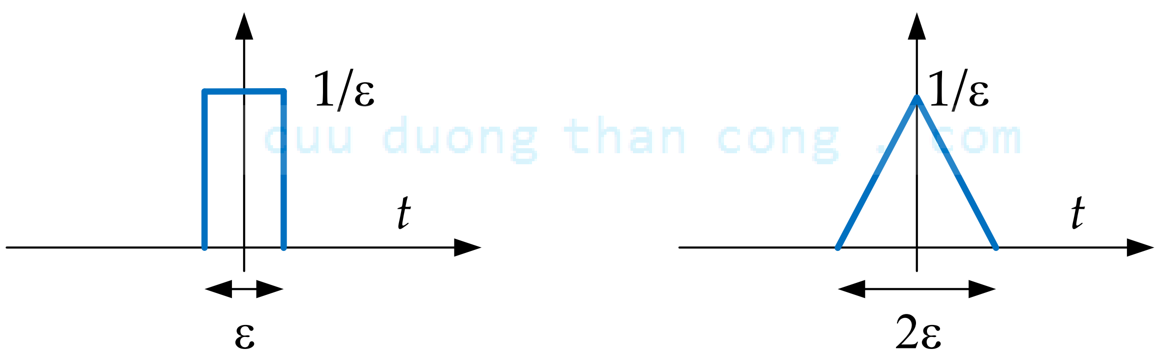
- The Unit Impulse Funtion (or Dirac Function) is defined as:

$$\delta(t)=\begin{cases}\infty, & t=0\\ 0, & t\neq 0\end{cases} \quad \text{and} \quad \int_{-\infty}^{\infty}\delta(t)dt=1$$






- No ordinary function behaves this way!
- Some real impulse approximations:



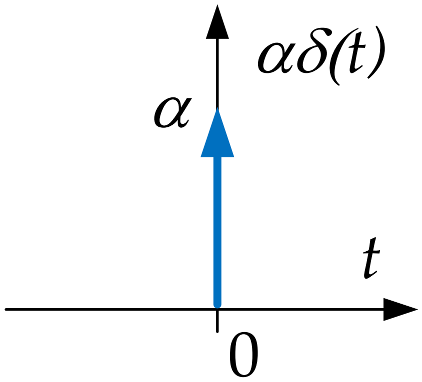



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

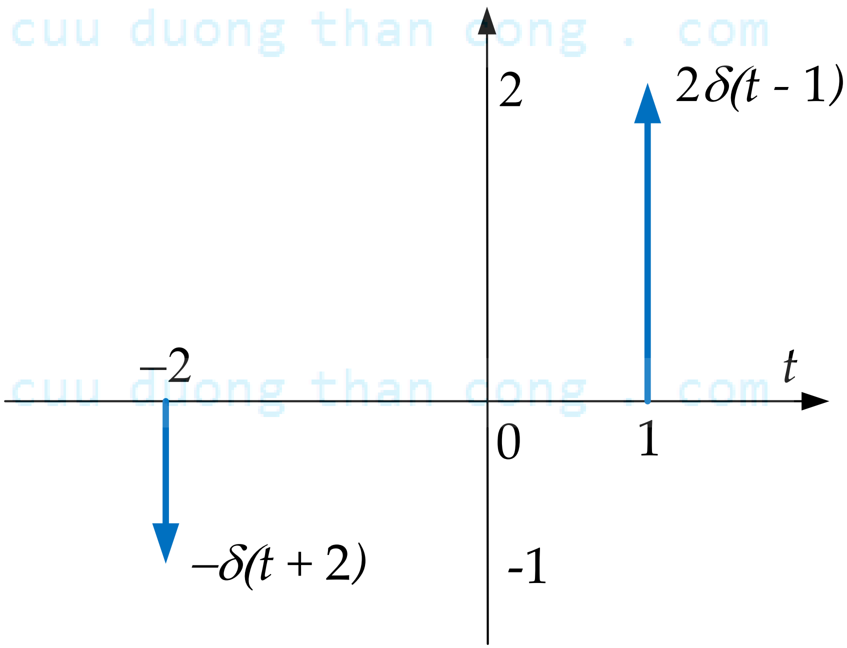
# Properties of $\delta(t)$:

- Scaled Impulses

$$\alpha\delta(t) = \begin{cases} \infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \text{ and } \int_{-\infty}^{\infty} \alpha\delta(t)dt = \alpha$$




- Shifting: $\delta(t - T)$ is an impulse at $t = T$.




*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## Properties of $\delta(t)$:

- Multiplication of a Function by an Impulse:
  - $f(t)\delta(t) = f(0)\delta(t)$ - $f(t)$ is continuous at 0
  - $f(t)\delta(t - T) = f(T)\delta(t - T)$ - $f(t)$ is continuous at $T$

- Sampling:



$$\int_a^b f(t)\delta(t)dt = \begin{cases} f(0) & \text{if } a < 0 < b \\ 0, & \text{otherwise} \end{cases}$$

$$\int_a^b f(t)\delta(t-T)dt = \begin{cases} f(T) & \text{if } a < T < b \\ 0, & \text{otherwise} \end{cases}$$

## Properties of $\delta(t)$:

- Symmetry
  - $\delta(t) = \delta(-t)$
  - $\delta(t - T) = \delta(T - t)$
- Define integral:

$$\int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$$

$$\int_{-\infty}^{t} \delta(\tau - T)d\tau = u(t - T)$$





- Unit step derivative:

$$\frac{du(t)}{dt} = \delta(t); \qquad \frac{du(t-T)}{dt} = \delta(t-T)$$

*Created and edited by:* **Nguyen Phuoc Bao Duy**
***HCMC University of Technology***
 

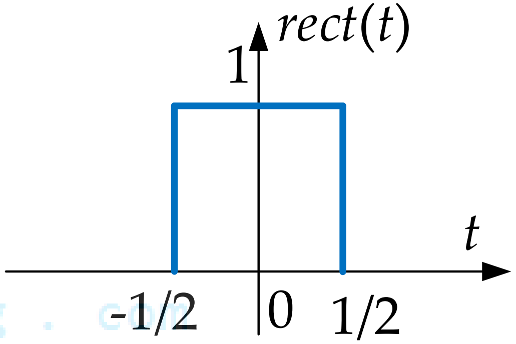
# Properties of $\delta(t)$:

*Example 1.08:* Find the derivative of following functions

a. $f(t) = t^2u(t)$

b. $g(t) = e^{-2t}u(t)$

c. $y(t) = \text{rect}(t)$






*Example 1.09:* Simplify the following expression:

$$a.\, f(t) = (1+t^2)\left[\delta(t) - 2\delta(t-4)\right]$$

$$b.\, z(t) = \int_{-\infty}^{t} \delta(\tau+1)d\tau$$

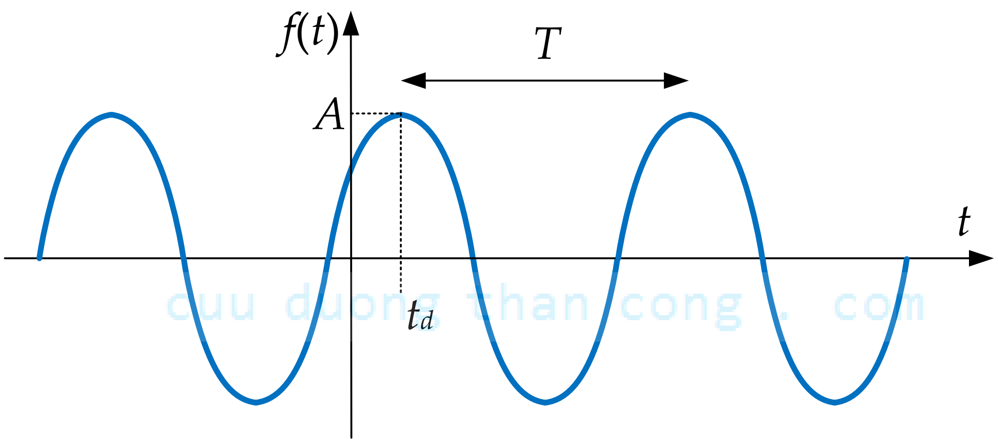


$$c.\, y(t) = \int_{-\infty}^{\infty} \delta(\tau)f(t-\tau)d\tau$$

# Sinusoidal signal $f(t) = A\cos(\omega_0 t + \varphi)$:

- $A$ = amplitude
- $\omega_0$ = frequency in rads/second
- $\varphi$ = phase in radians
- Delay time $t_d = -\varphi/\omega_0$.



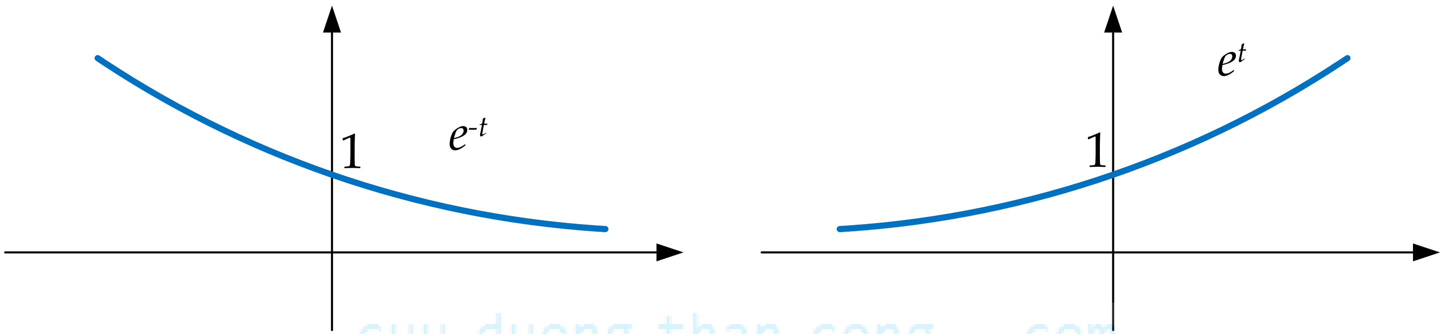





*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

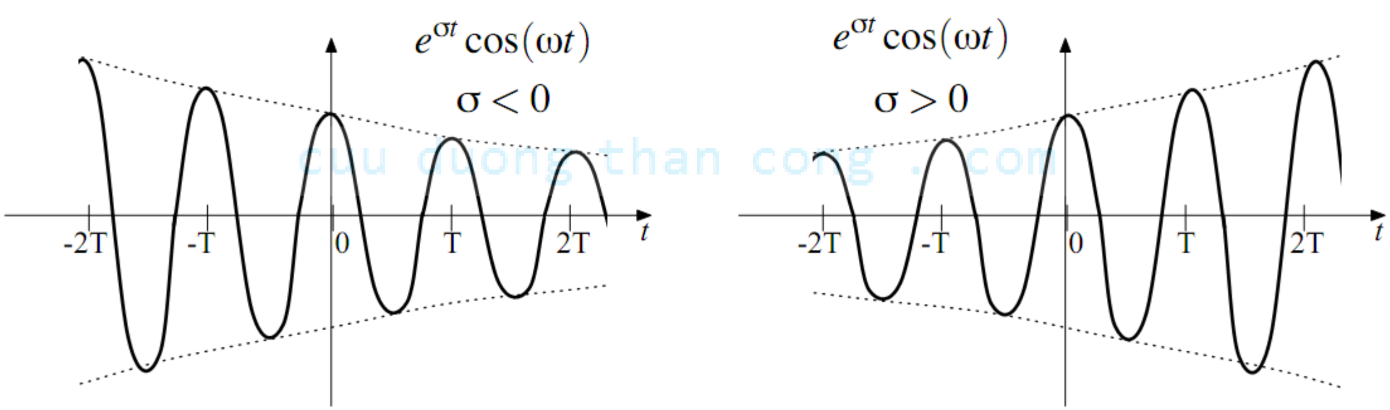
## Exponential signal $f(t) = e^{\sigma t}$:

- $\sigma < 0$: this is exponential decay.
- $\sigma > 0$: this is exponential growth.






## Damped or Growing Sinusoid signal $f(t) = e^{\sigma t}\cos(\omega_0 t)$:

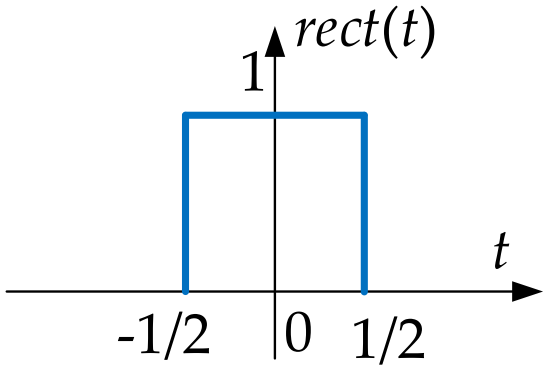





*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## Unit Rectangle:

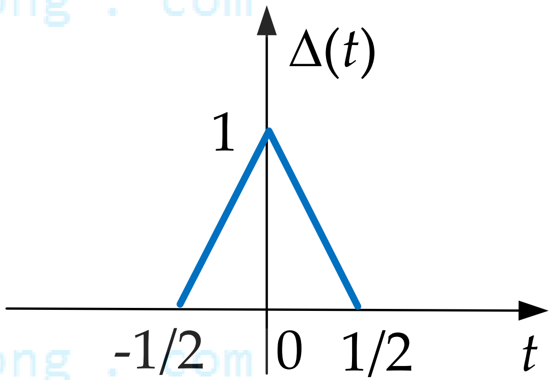
$$\text{rect}(t) = \begin{cases} 1 & \text{if } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$






## Unit Triangle

$$\Delta(t) = \begin{cases} 1 - 2|t| & \text{if } |t| \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$






*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

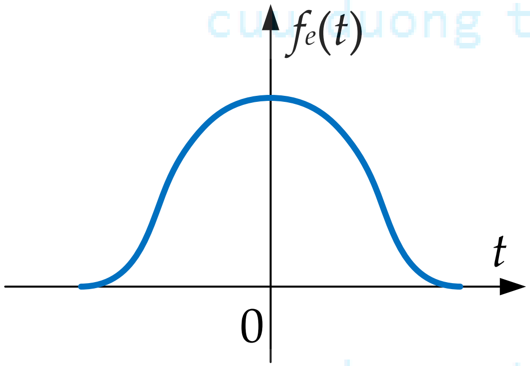
## Even and Odd signals:

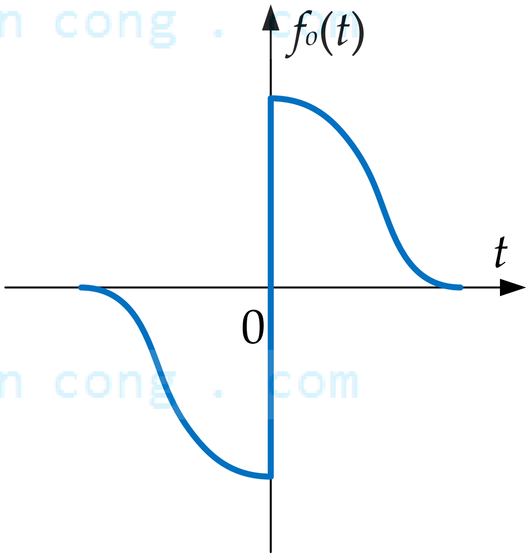
- An even signal is symmetric about the vertical axis:

$$f(t) = f(-t)$$

- An odd signal is symmetric about the origin:

$$f(t) = -f(-t)$$



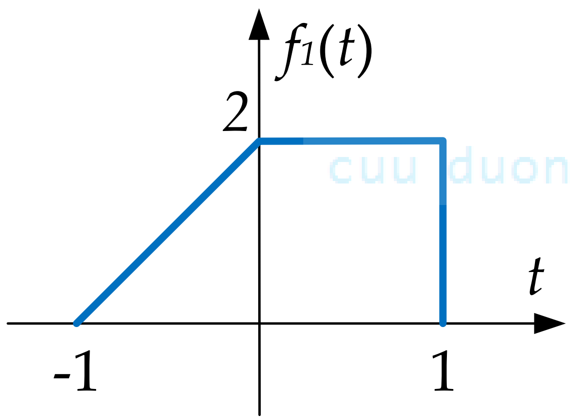
## Even and Odd signals:

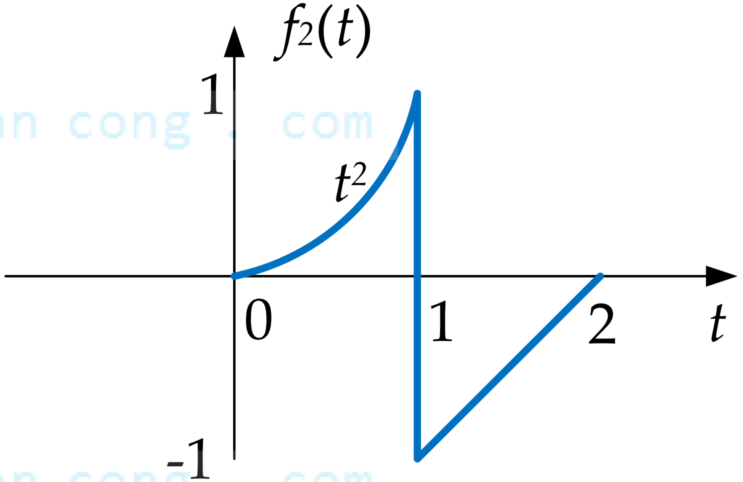
- Some properties of even and odd signals:
- Even signal x Even signal = Even signal
- Odd signal x Odd signal = Even signal
- Even signal x Odd signal = Odd signal
- Integral: $\int_{-a}^{a} f_e(t)dt = 2\int_{0}^{a} f_e(t)dt; \quad \int_{-a}^{a} f_o(t)dt = 0$
- Any signal can be decomposed into even and odd components.

$$f(t) = f_e(t) + f_o(t) \quad \text{with} \quad \begin{cases} f_e(t) = \frac{1}{2}\left[f(t) + f(-t)\right] \\ f_o(t) = \frac{1}{2}\left[f(t) - f(-t)\right] \end{cases}$$

## **Even and Odd signals:**

*Example 1.10:* Find even and odd components of following signals:








*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

*Chapter I:*

# Introduction to Signals and Systems



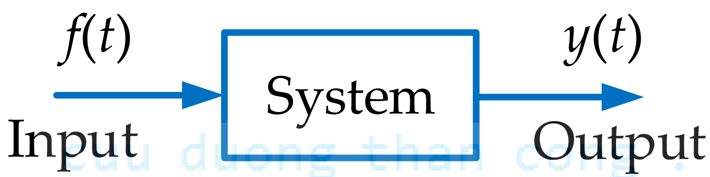
1. *Introduction to Signals*
2. *Classification of Signals*
3. *Some Signal Operations*
4. *Some Signal Models*
5. ***Systems and Classification of System*** ◀
6. *System Model*



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## Systems:

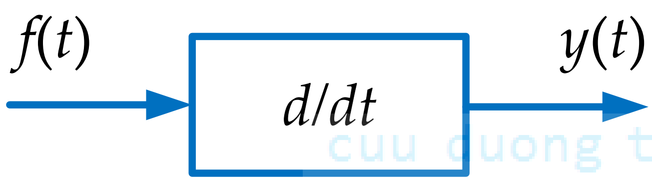
- A system transforms input signals into output signals.
- We will concentrate on systems with one input and one output (Single-Input, Single-Output – SISO).
- Systems often denoted by block diagram:

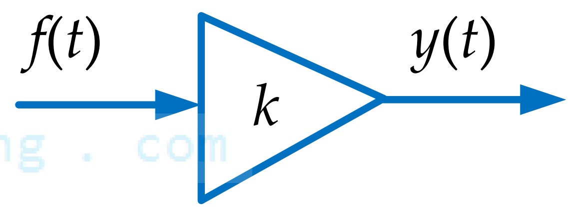



$$y(t) = S\left[ f(t) \right]$$



- Examples:



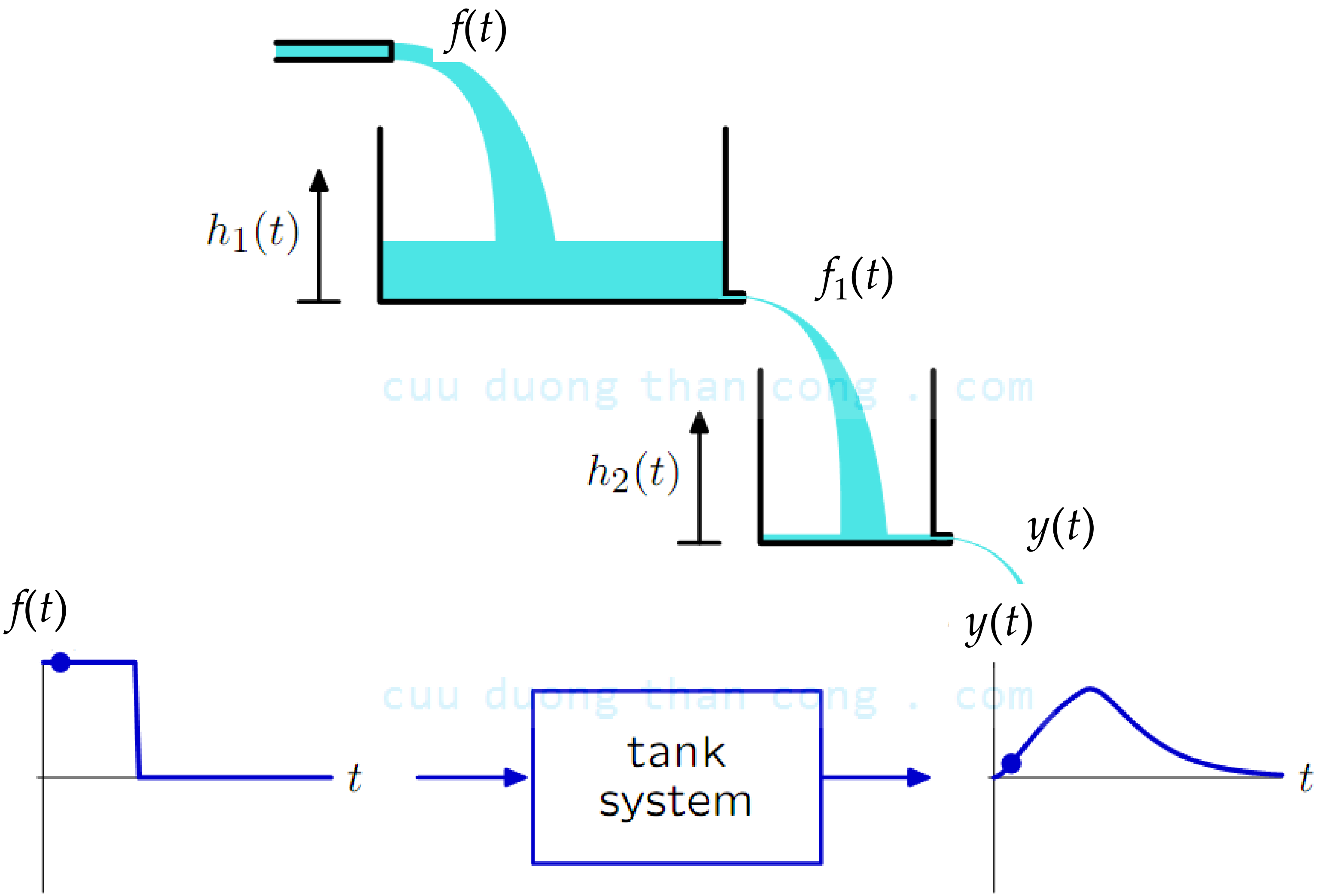





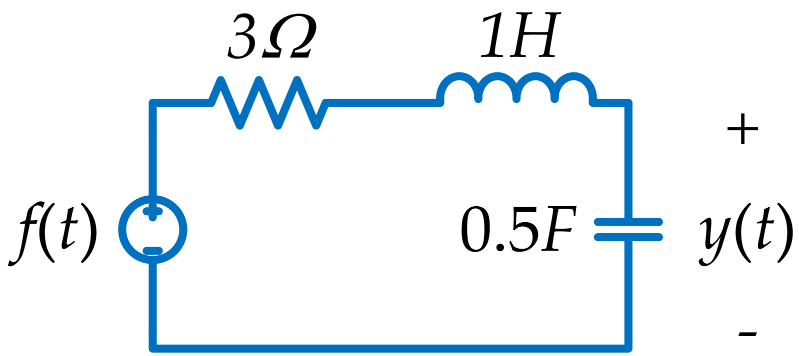

$$y(t) = \frac{df(t)}{dt} = f'(t)$$

$$y(t) = k.f(t)$$

# *Example 1.11:* Tank system



*Example 1.12:* Find the input-output relationship of electrical system discribed below:

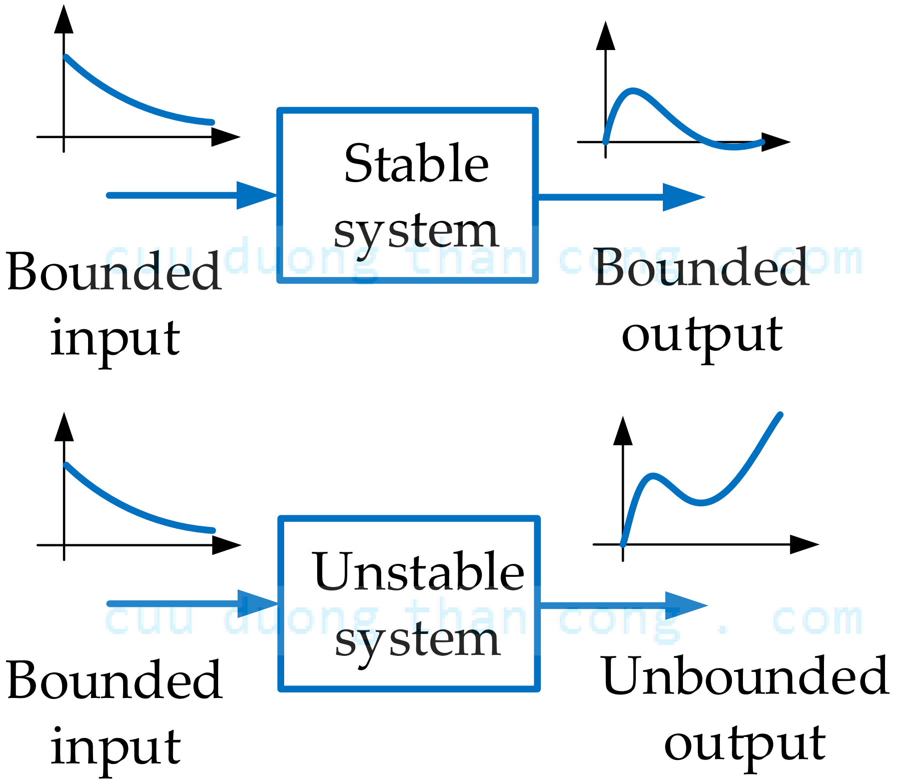





With $f(t) = e^{-3t}u(t)$, determine $y(t)$ when $t \geq 0$.

## System stability:

*The concept of Stability:* A **stable system** is a dynamic system with a bounded response to a bounded input (BIBO)






*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

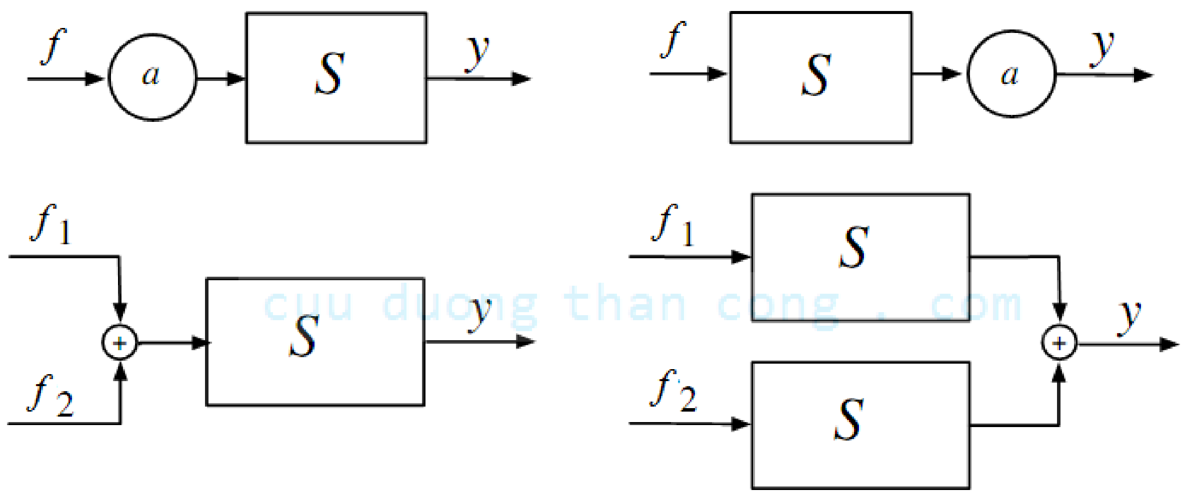
# Linear and Nonlinear systems:

A system S is **linear** if the following two properties hold:

- Homogeneity: $S(af) = aS(f)$; $a$ is any scalar.
- Superposition: $S(f_1 + f_2) = S(f_1) + S(f_2)$

Or the following pairs of block diagrams are equivalent:








*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## Linear and Nonlinear systems:

- Equivalent definition of **linear systems**: S is linear if:

$$S(a_1 f_1 + a_2 f_2) = a_1 S(f_1) + a_2 S(f_2)$$

*Examples:*

  - Scaling system, differentiator, integrator….
  - Systems that are described by a differential equation:

$$a_n y^{(n)}(t)+\ldots+a_1 y'(t)+a_0 y(t) = b_m f^{(m)}(t)+\ldots+b_1 f'(t)+b_0 f(t)$$



- **Nonlinear** system is one that does not satisfy the homogeneity or superposition principle.



*Example:* $y(t) = f^2(t)$

*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## **Linear and Nonlinear systems:**

*Example 1.13:* Linear or Nonlinear?

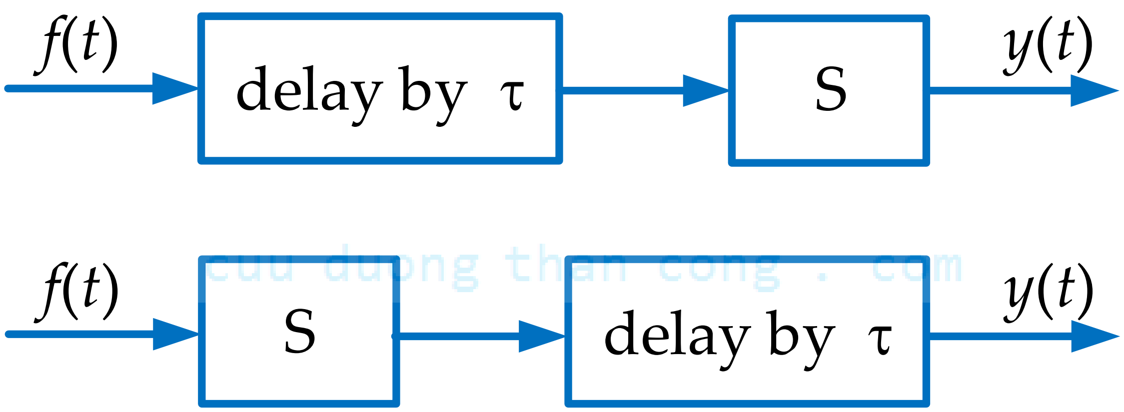
$$a)\ \frac{dy}{dt} + t^2 y(t) = (2t+3)f(t)$$

$$b)\ y(t)\frac{dy}{dt} + 3y(t) = f(t)$$

# Time-invariant and time-varying-parameter systems:

- A system is **time-invariant** (constant-parameter) if a time shift in the input only produces the same time shift in the output: $y(t) = S[f(t)] \leftrightarrow y(t-\tau) = S[f(t-\tau)]$
- If system S is **time-invariant**, these block diagrams are equivalent:





## Time-invariant and time-varying-parameter systems:

- Systems that are described by a differential equation:

$$a_n y^{(n)}(t) + \ldots + a_1 y'(t) + a_0 y(t) = b_m f^{(m)}(t) + \ldots + b_1 f'(t) + b_0 f(t)$$

  - are Linear Time-Invariant (LTI) systems if $a_i$ and $b_i$ are time-constant.
  - are Linear Time-varying system if $a_i$, $b_i$ are functions of time.

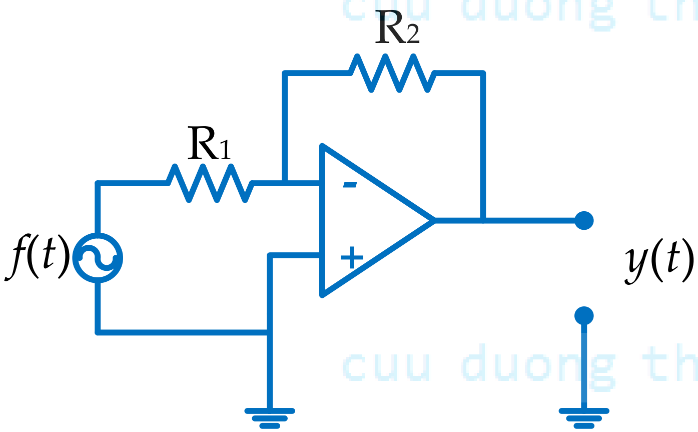
- Some **time-varying** systems:

$$y(t) = f(at),\ y(t) = f(T - t),\ y(t) = (\sin t) f(t - 2)\ldots$$

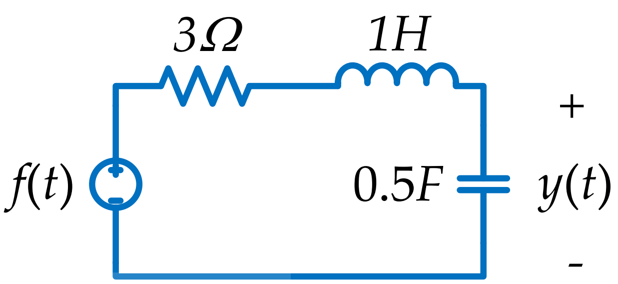
## Instantaneous (memoryless) systems and dynamic systems (systems with memory):

- A system is instantaneous (memoryless) if the output depends only on the present input.
- A system in dynamic (with memory) has an output signal that depends on inputs in the past.






*instantaneous system*




*dynamic system*

cuu duong than cong . com

# Causal and noncausal systems:

- A **causal** system has an output that depends only on the past or present inputs.
  *E.g:* Any real physical circuit, or mechanical system.

- A system that can respond to future inputs is a **noncausal** system.



*Examples:* causal or noncausal?

- $y(t) = f^2(t - 1)$
- $y(t) = f(t + 1)$

## **Lumped-parameter and distributed-parameter systems:**

- A system where all signals can be assumed to be functions of time alone is called **lumped-parameter** system.
  *E.g:* Electrical systems with physical dimensions that are small compared to the wavelength of the signal propagated. 

- A **distributed-parameter** system is a system where the signals are functions of space as well as of time.

## Continuous-time (CT) and discrete-time (DT) systems:

- Systems whose inputs and outputs are CT signals are **CT systems**.
- **DT systems** have DT input and DT output signals.



## Analog and Digital systems:

- Systems whose inputs and outputs are analog signals are **analog systems.**
- **Digital systems** have digital input and digital output signals.



*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

*Chapter I:*

# Introduction to Signals and Systems



1. *Introduction to Signals*
2. *Classification of Signals*
3. *Some Signal Operations*
4. *Some Signal Models*
5. *Systems and Classification of System*
6. ***System Model***

cuu duong than cong . com

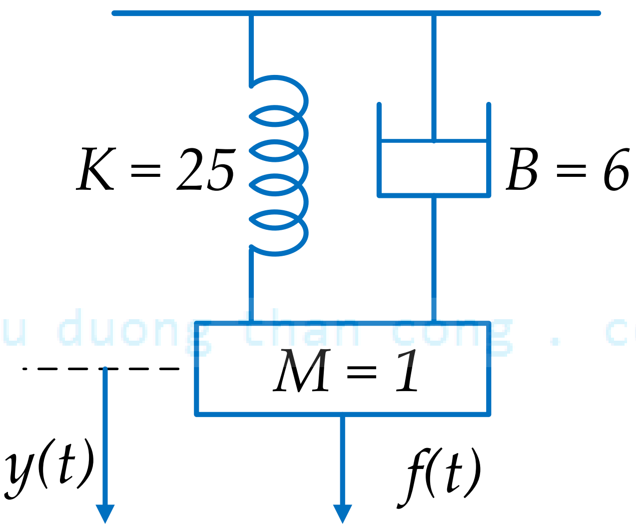
*Created and edited by:* **Nguyen Phuoc Bao Duy**
*HCMC University of Technology*

## System Model:

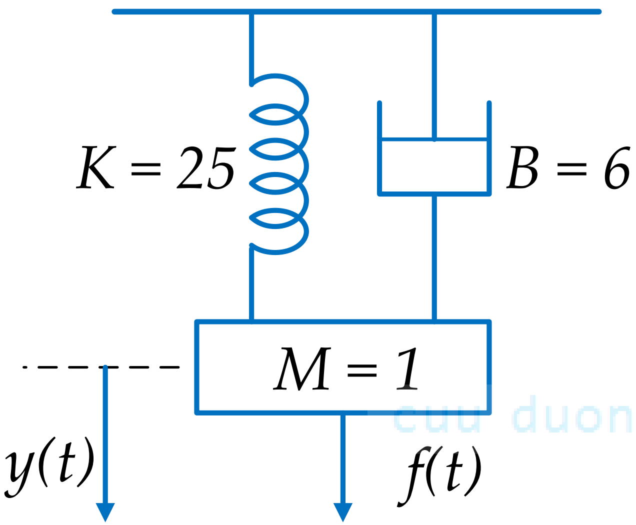
- To construct a system model, we must study the relationships between different variables in the system.
- Usually, a system is described by differential equations.

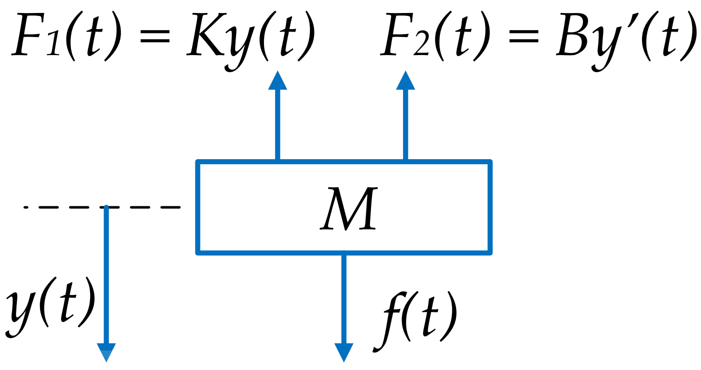
*Example 1.14:*





## Example 1.13:



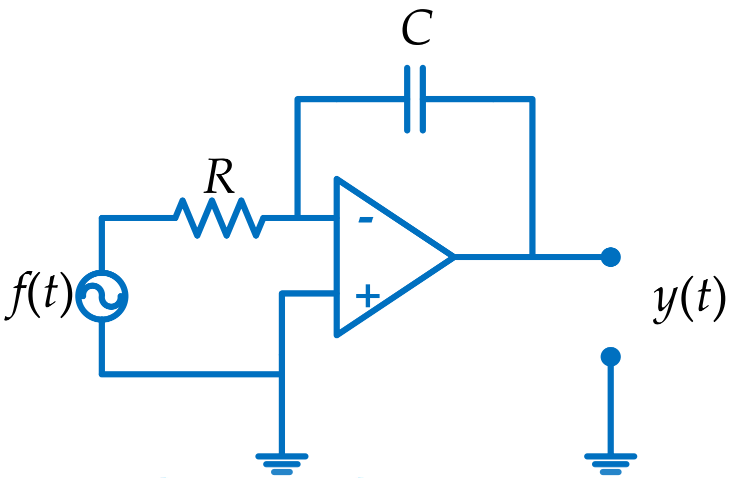

*Solution:* by Newton's law:

$$My''(t) = f(t) - F_1(t) - F_2(t)$$

$$\Leftrightarrow My''(t) = f(t) - Ky(t) - By'(t)$$

$$\Leftrightarrow My''(t) + By'(t) + Ky(t) = f(t)$$

*Example 1.15:*






*Solution:*

$$y'(t) = -\frac{1}{RC} f(t)$$