Chapter 1: Introduction

- Pseudocode
- Abstract data type
- Algorithm efficiency

Pseudocode

- What is an algorithm?

Pseudocode

- What is an algorithm?
  - The logical steps to solve a problem.

Pseudocode

- What is a program?
  - Program = Data structures + Algorithms (Niklaus Wirth)

Pseudocode

- The most common tool to define algorithms.
- English-like representation of the code required for an algorithm.

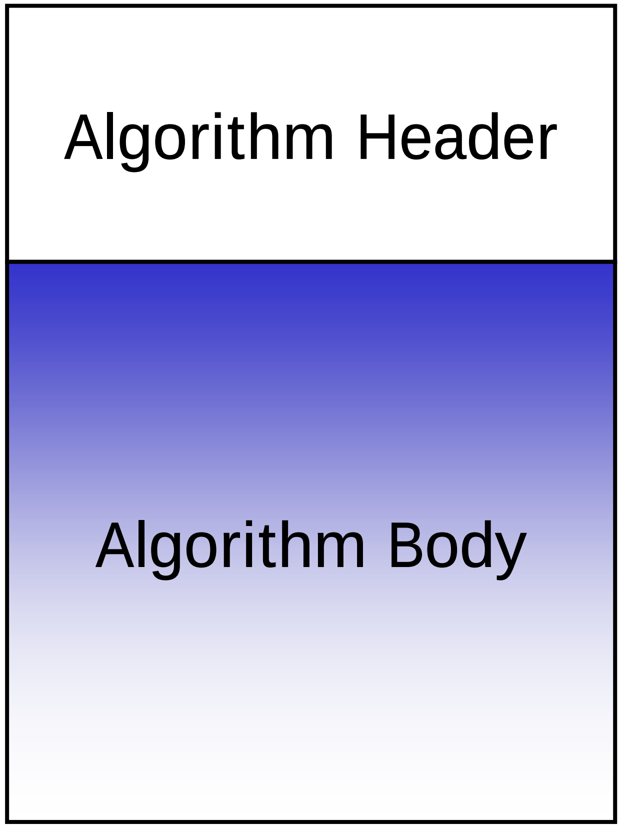
Pseudocode

- Pseudocode = English + Code

relaxed syntax being easy to read

instructions using basic control structures (sequential, conditional, iterative)

Pseudocode



Pseudocode

- Algorithm Header:
  - Name
  - Parameters and their types
  - Purpose
    - what the algorithm does
  - Precondition
    - precursor requirements for the parameters
  - Postcondition
    - taken action and status of the parameters
  - Return condition
    - returned value

Pseudocode

- Algorithm Body:
  - Statements
  - Statement numbers
    - decimal notation to express levels
  - Variables
    - important data
  - Algorithm analysis
    - comments to explain salient points
  - Statement constructs
    - sequence, selection, iteration

Example

Algorithm average
  Pre   nothing
  Post   numbers read and their average printed

```
1   i = 0
2   loop (all data not read)
    1  i = i + 1
    2  read number
    3  sum = sum + number
3   average = sum / i
4   print average
5   return
```

End   average

Algorithm Design

- Divide-and-conquer
- Top-down design
- Abstraction of instructions
- Step-wise refinement

Abstract Data Type

- What is a data type?
  - Class of data objects that have the same properties

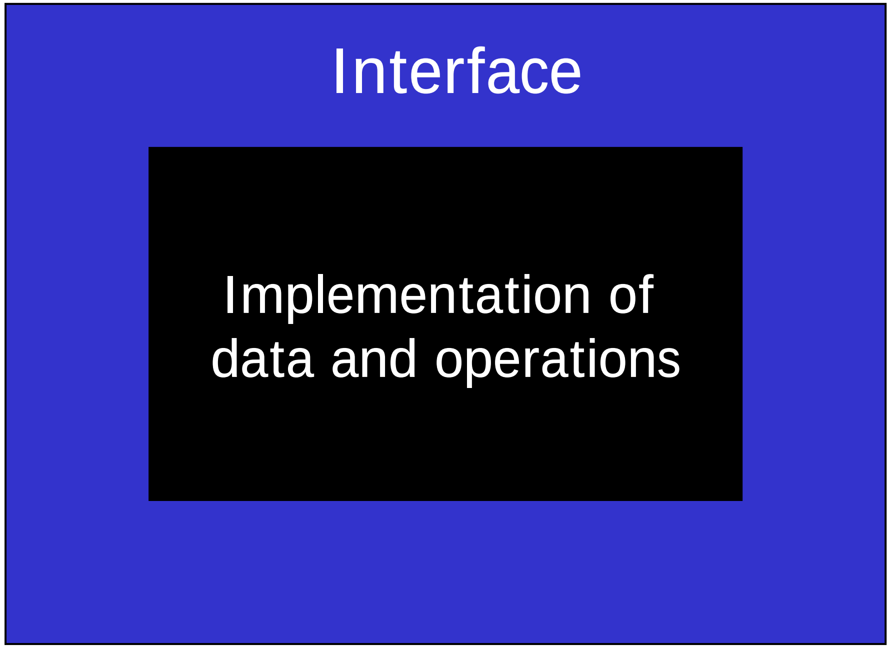
Abstract Data Type

- Development of programming concepts:
  - GOTO programming
    - control flow is like spaghetti on a plate
  - Modular programming
    - programs organized into subprograms
  - Structured programming
    - structured control statements (sequence, selection, iteration)
  - Object-oriented programming
    - encapsulation of data and operations

Abstract Data Type

- ADT = Data structures + Operations

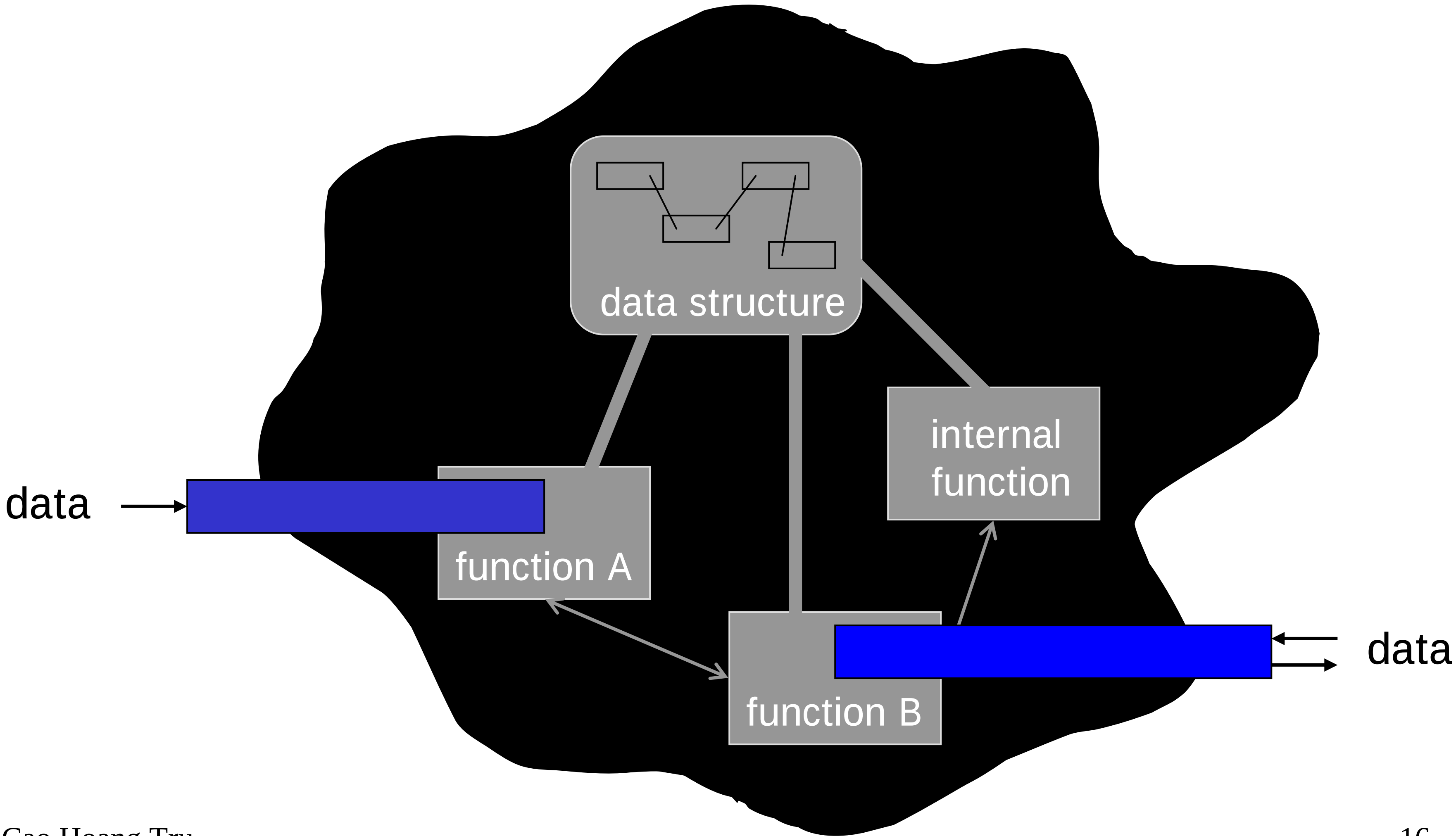
Abstract Data Type



User knows what a data type can do.

How it is done is hidden.

Abstract Data Type



Example: Variable Access

- Rectangle: r
  - length: x
  - width: y

- Rectangle: r
  - length: x (hidden)
  - width: y (hidden)
  - get_length()
  - get_width()

Example: List

- Interface:
  - Data:
    - sequence of components of a particular data type
  - Operations:
    - accessing
    - insertion
    - deletion
- Implementation:
  - Array, or
  - Linked list

Algorithm Efficiency

- How fast an algorithm is?
- How much memory does it cost?
- Computational complexity: measure of the difficulty degree (time or space) of an algorithm.

Algorithm Efficiency

- General format:

$$f(n)$$

n is the size of a problem (the key number that determines the size of input data)

Linear Loops

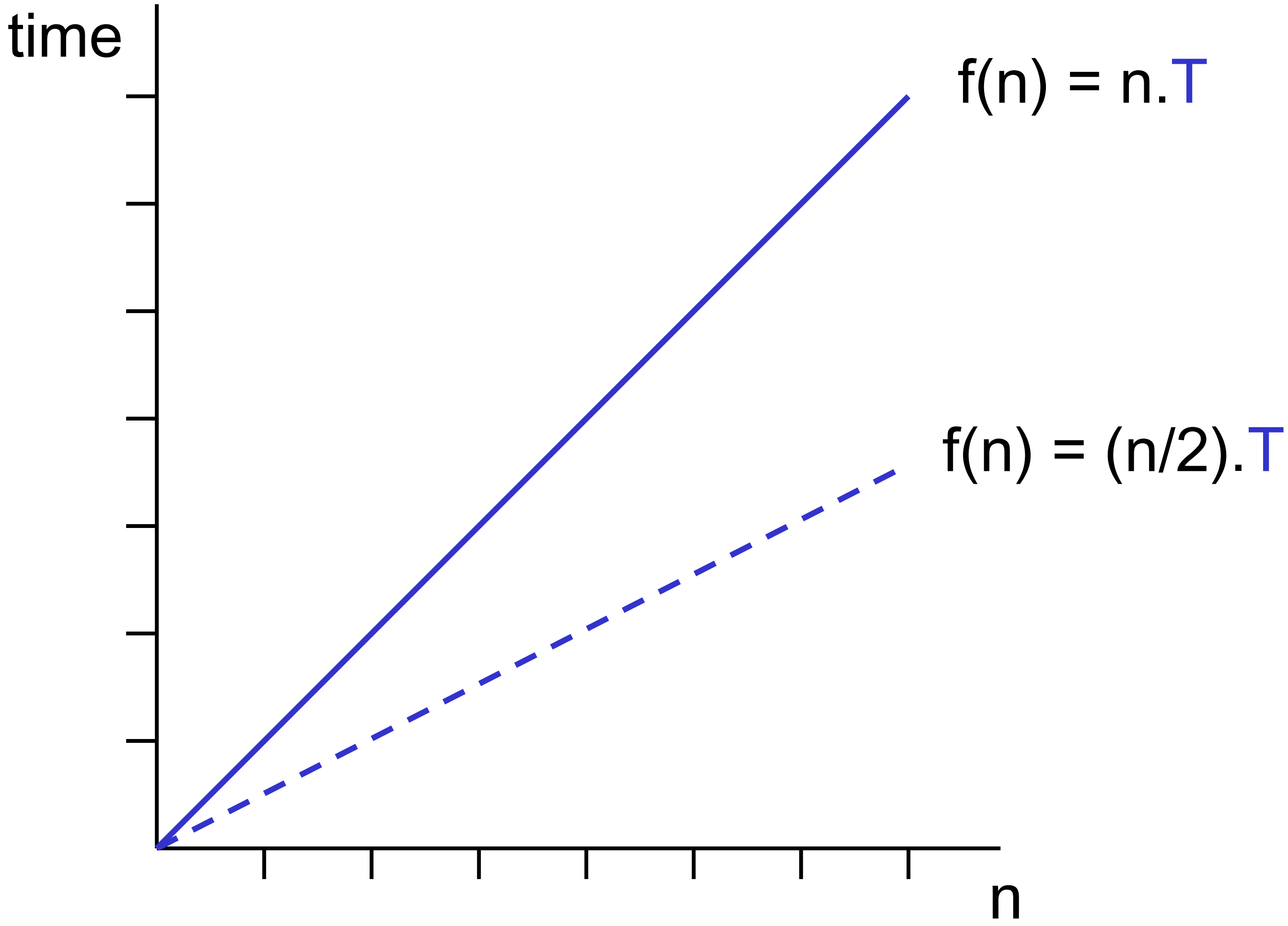
```
1  i = 1
2  loop (i <= 1000)
   1  application code
   2  i = i + 1
```

The number of times the body of the loop is replicated is 1000

```
1  i = 1
2  loop (i <= 1000)
   1  application code
   2  i = i + 2
```

The number of times the body of the loop is replicated is 500

Linear Loops



Logarithmic Loops

Multiply loops

```
1  i = 1
2  loop (i <= 1000)
   1  application code
   2  i = i × 2
```

The number of times the body of the loop is replicated is $\log_2 n$

Logarithmic Loops

Multiply loops

```
1  i = 1
2  loop (i <= 1000)
   1  application code
   2  i = i × 2
```

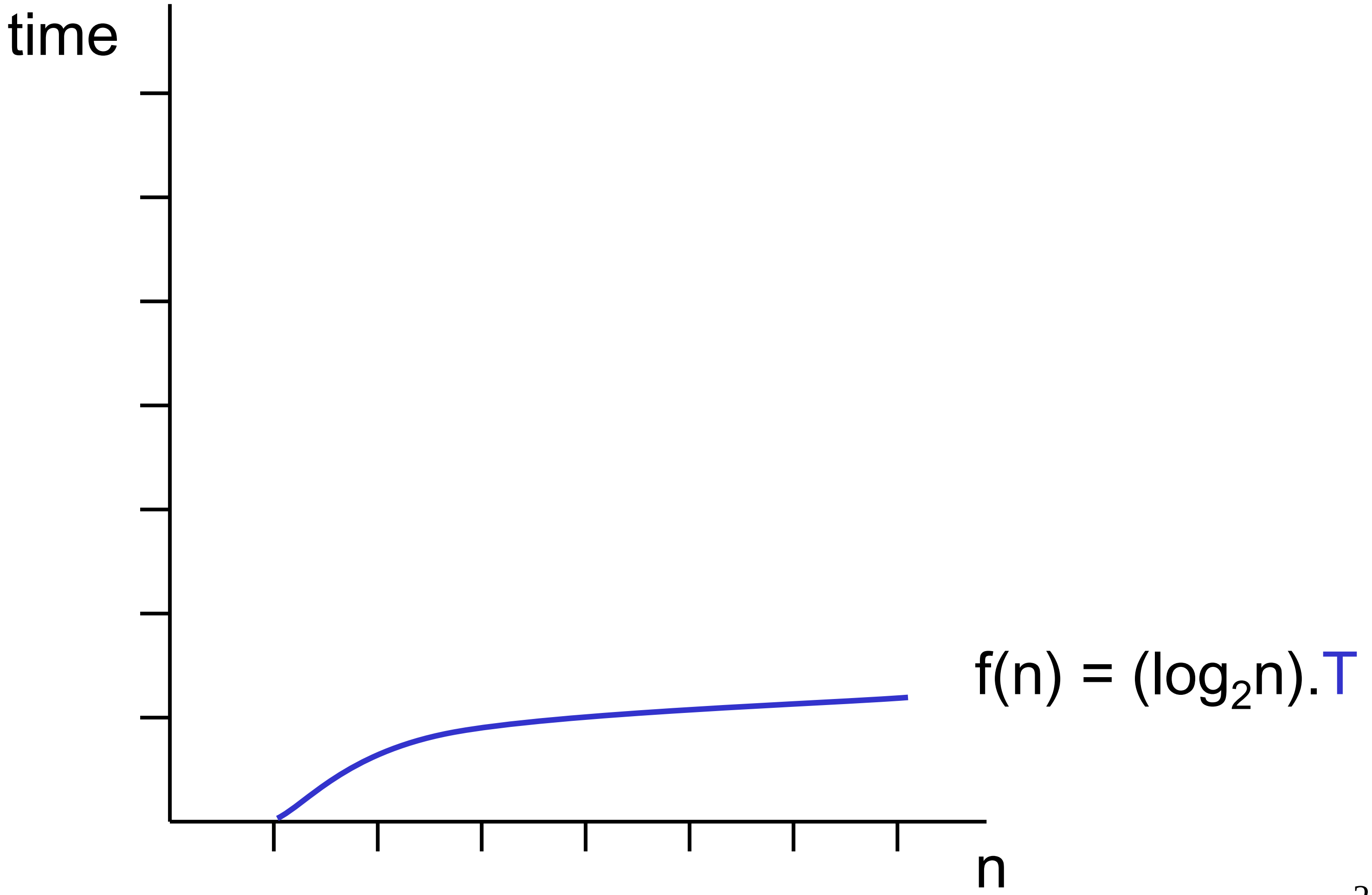
Divide loops

```
1  i = 1000
2  loop (i >= 1)
   1  application code
   2  i = i / 2
```

The number of times the body of the loop is replicated is

$\log_2 n$

Logarithmic Loops



Nested Loops

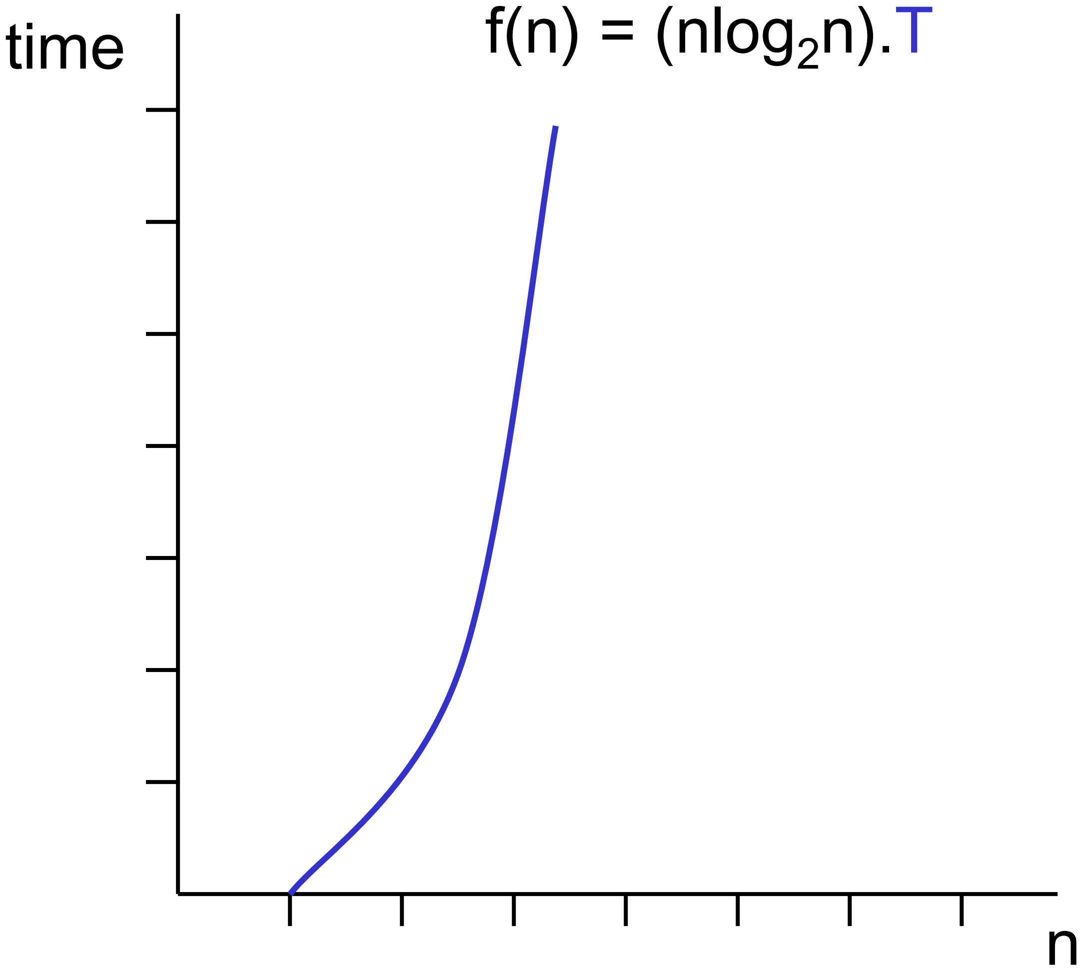
Iterations = Outer loop iterations × Inner loop iterations

Linear Logarithmic Loops

```
1   i = 1
2   loop (i <= 10)
    1   j = 1
    2   loop (j <= 10)
        1   application code
        2   j = j × 2
    3   i = i + 1
```

The number of times the body of the loop is replicated is $n\log_2 n$

Linear Logarithmic Loops



Quadratic Loops

```
1  i = 1
2  loop (i <= 10)
   1  j = 1
   2  loop (j <= 10)
      1  application code
      2  j = j + 1
   3  i = i + 1
```

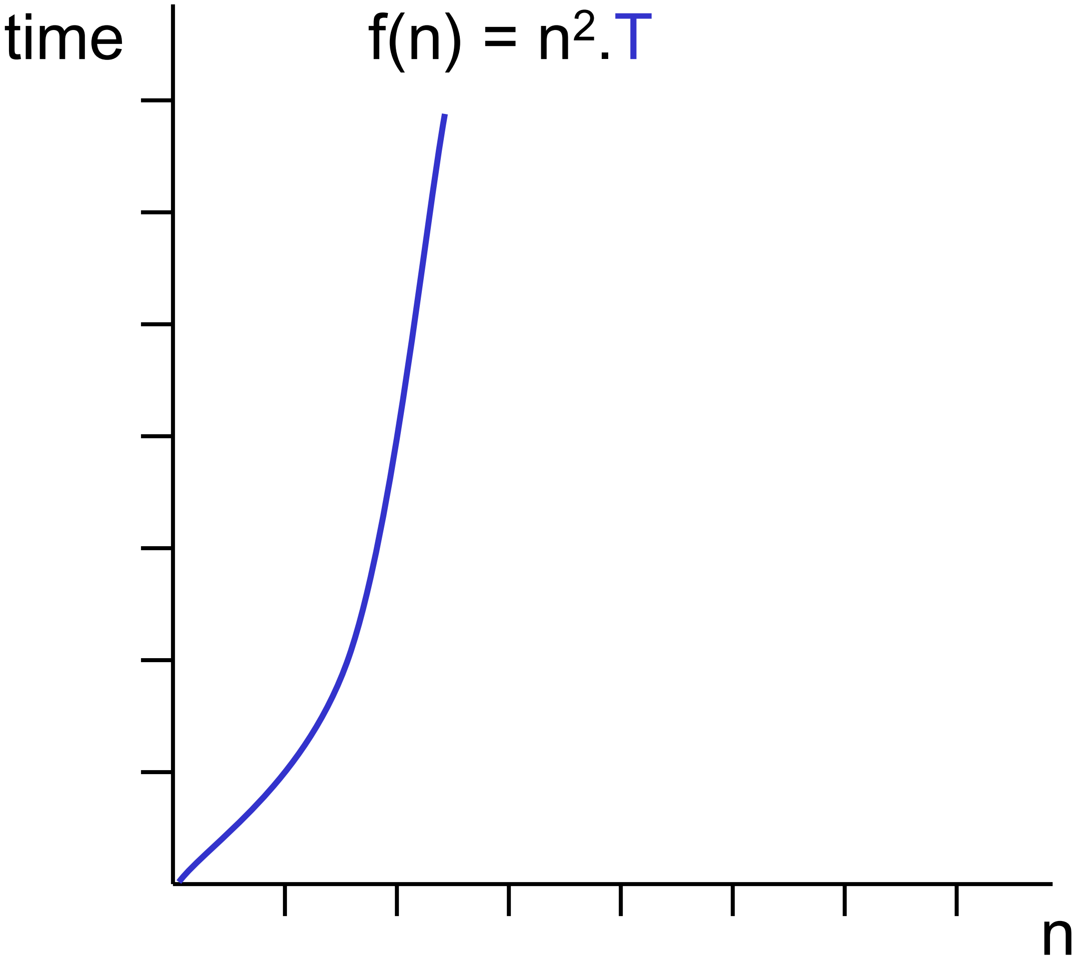
The number of times the body of the loop is replicated is n^2

Dependent Quadratic Loops

```
1  i = 1
2  loop (i <= 10)
   1  j = 1
   2  loop (j <= i)
      1  application code
      2  j = j + 1
   3  i = i + 1
```

The number of times the body of the loop is replicated is
$1 + 2 + \ldots + n = n(n + 1)/2$

Quadratic Loops



Asymptotic Complexity

- Algorithm efficiency is considered with only big problem sizes.
- We are not concerned with an exact measurement of an algorithm's efficiency.
- Terms that do not substantially change the function's magnitude are eliminated.

Big-O Notation

- $f(n) = c.n \Rightarrow f(n) = O(n)$.
- $f(n) = n(n + 1)/2 = n^2/2 + n/2 \Rightarrow f(n) = O(n^2)$.

Big-O Notation

- Set the coefficient of the term to one.
- Keep the largest term and discard the others.

$\log_2 n \quad n \quad n\log_2 n \quad n^2 \quad n^3 \quad \ldots \quad n^k \quad \ldots \quad 2^n \quad n!$

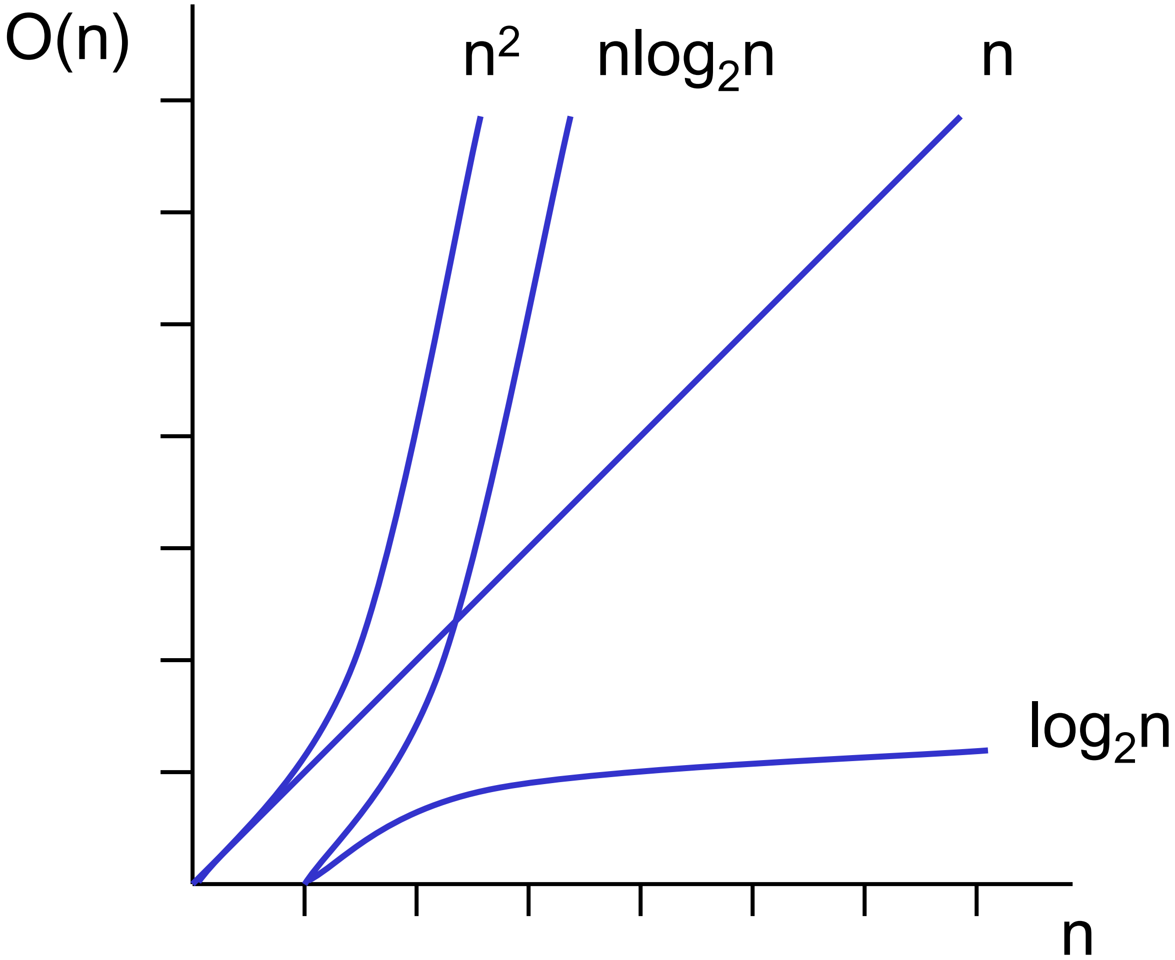
Standard Measures of Efficiency

Efficiency	Big-O	Iterations	Est. Time
logarithmic	$O(\log_2 n)$	14	microseconds
linear	$O(n)$	10,000	.1 seconds
linear logarithmic	$O(n\log_2 n)$	140,000	2 seconds
quadratic	$O(n^2)$	$10{,}000^2$	15-20 min.
polynomial	$O(n^k)$	$10{,}000^k$	hours
exponential	$O(2^n)$	$2^{10{,}000}$	intractable
factorial	$O(n!)$	10,000!	intractable

Assume instruction speed of 1 microsecond and 10 instructions in loop.

$n = 10{,}000$

Standard Measures of Efficiency



Big-O Analysis Examples

```
Algorithm  addMatrix (val matrix1 <matrix>, val matrix2 <matrix>,
                      val size <integer>, ref matrix3 <matrix>)
Add matrix1 to matrix2 and place results in matrix3
Pre     matrix1 and matrix2 have data
        size is number of columns and rows in matrix
Post    matrices added - result in matrix3
1   r = 1
2   loop (r <= size)
    1   c = 1
    2   loop (c <= size)
        1   matrix3[r, c] = matrix1[r, c] + matrix2[r, c]
        2   c = c + 1
    3   r = r + 1
3   return
End   addMatrix
```

Big-O Analysis Examples

```
Algorithm  addMatrix (val matrix1 <matrix>, val matrix2 <matrix>,
                      val size <integer>, ref matrix3 <matrix>)
Add matrix1 to matrix2 and place results in matrix3
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1   r = 1
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    2   loop (c <= size)
        1   matrix3[r, c] = matrix1[r, c] + matrix2[r, c]
        2   c = c + 1
    3   r = r + 1
3   return
End     addMatrix
```

Nested linear loop: $f(size) = O(size^2)$

Time Costing Operations

- The most time consuming: data movement to/from memory/storage.
- Operations under consideration:
  - Comparisons
  - Arithmetic operations
  - Assignments

Recurrence Equation

- An equation or inequality that describes a function in terms of its value on smaller input.

Recurrence Equation

- Example: binary search.

a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]	a[10]	a[11]	a[12]
4	7	8	10	14	21	22	36	62	77	81	91

Recurrence Equation

- Example: binary search.

a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]	a[10]	a[11]	a[12]
4	7	8	10	14	21	22	36	62	77	81	91

$f(n) = 1 + f(n/2) \Rightarrow f(n) = O(\log_2 n)$

Best, Average, Worst Cases

- Best case: when the number of steps is smallest.
- Worst case: when the number of steps is largest.
- Average case: in between.

Best, Average, Worst Cases

- Example: sequential search.

a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]	a[10]	a[11]	a[12]
4	8	7	10	21	14	22	36	62	91	77	81

Best case: f(n) = O(1)

Worst case: f(n) = O(n)

Best, Average, Worst Cases

- Example: sequential search.

a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]	a[10]	a[11]	a[12]
4	8	7	10	21	14	22	36	62	91	77	81

Average case: $f(n) = \sum i.p_i$

p_i: probability for the target being at a[i]

$p_i = 1/n \Rightarrow f(n) = (\sum i)/n = O(n)$

P and NP Problems

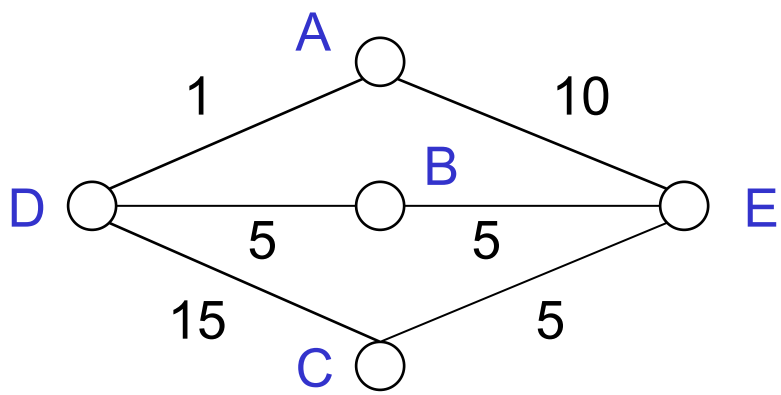
- P: Polynomial (can be solved in polynomial time on a deterministic machine).
- NP: Nondeterministic Polynomial (can be solved in polynomial time on a non-deterministic machine).

P and NP Problems

Travelling Salesman Problem:

A salesman has a list of cities, each of which he must visit exactly once. There are direct roads between each pair of cities on the list.

Find the route the salesman should follow for the shortest possible round trip that both starts and finishes at any one of the cities.

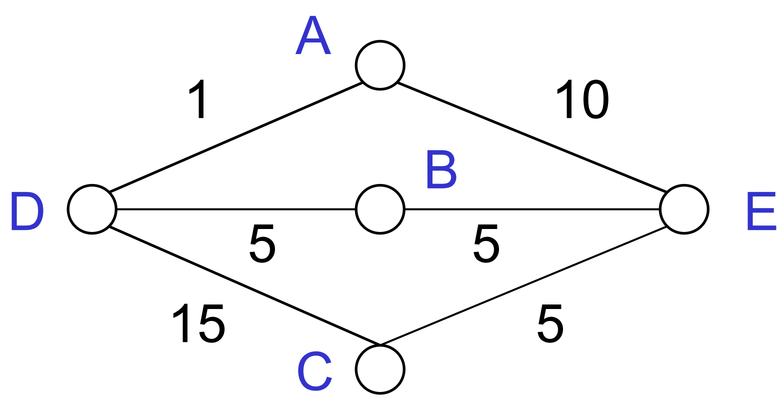


P and NP Problems

Travelling Salesman Problem:

Deterministic machine: f(n) = n(n-1)(n-2) … 1 = O(n!)

⇒ NP problem



P and NP Problems

- NP-complete: NP and every other problem in NP is polynomially reducible to it.
- Open question: P = NP?