QUY TAÉC L'HOSPITALE

PHAÙT BIEÅU ÑÒNH LYÙ

1. Ñònh lyù 1: Cho f khaû vi trong (a, b) thoûa

i. $\lim_{x \to b^-} f(x) = 0, \lim_{x \to b^-} g(x) = 0$

ii. $g'(x) \neq 0, \forall x \in (a, b)$

iii. $\lim_{x \to b^-} \frac{f'(x)}{g'(x)} = A$

Khi ñoù: $\lim_{x \to b^-} \frac{f(x)}{g(x)} = A$

PHAÙT BIEÅU ÑÒNH LYÙ

2. Ñònh lyù : Cho f khaû vi trong (a, b) thoûa

i. $\lim_{x \to b^-} f(x) = \infty, \lim_{x \to b^-} g(x) = \infty$

ii. $g'(x) \neq 0, \forall x \in (a, b)$

iii. $\lim_{x \to b^-} \frac{f'(x)}{g'(x)} = A$

Khi ñoù: $\lim_{x \to b^-} \frac{f(x)}{g(x)} = A$

LÖU YÙ AÙP DUÏNG

1. **Quy taéc L'hopspitale chæ aùp duïng cho caùc daïng voâ ñònh** $\frac{0}{0}, \frac{\infty}{\infty}$
2. **Caùc keát quaû trong ñònh lyù vaãn ñuùng neáu thay** $x \to a^+$, $x \to x_0$, $x \to \infty$
3. **Neáu** $\frac{f'}{g'}$ **khoâng coù giôùi haïn, khoâng coù keát luaän gì cho** $\frac{f}{g}$
4. **Keát hôïp vôùi VCB vaø VCL ñeå coù keát quaû nhanh hôn**

Ví duï

$$1/\lim_{x\to 0}\frac{x-\tan x}{x^3+x^2\sin x}\qquad \left(\frac{0}{0}\right)$$

$$=\lim_{x\to 0}\frac{x-\tan x}{2x^3}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{1-(1+\tan^2 x)}{3x^2}$$

$$=\frac{1}{2}\lim_{x\to 0}\frac{-\tan^2 x}{3x^2}=-\frac{1}{6}$$

$$2 / \lim_{x \to 0} \left[\frac{1}{x(x+1)} - \frac{\ln(1+x)}{x^2} \right] \qquad (\infty - \infty)$$

$$= \lim_{x \to 0} \frac{x - (x+1)\ln(1+x)}{x^2(1+x)}$$

$$= \lim_{x \to 0} \frac{x - (x+1)\ln(1+x)}{x^2}$$

$$= \lim_{x \to 0} \frac{1 - \ln(1+x) - 1}{2x} = -\frac{1}{2}$$

$$3/\lim_{x\to 0}\left(\frac{1}{\sin^2 x}-\frac{1}{x^2}\right)=\lim_{x\to 0}\frac{x^2-\sin^2 x}{x^2\sin^2 x}\qquad(\infty-\infty)$$

$$=\lim_{x\to 0}\frac{(x-\sin x)(x+\sin x)}{x^4}$$

$$=\lim_{x\to 0}\frac{(x-\sin x)2x}{x^4}$$

$$=2\lim_{x\to 0}\frac{x-\sin x}{x^3}$$

$$=2\lim_{x\to 0}\frac{1-\cos x}{3x^2}=2\times\frac{1}{3}\times\frac{1}{2}=\frac{1}{3}$$

$$4/\ \lim_{x\to 0^+} x\ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} (-x) = 0$$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = \lim_{x\to 0}\left(1+\frac{\sin x}{x}-1\right)^{\frac{1}{x^2}} = \lim_{x\to 0}\left(1+\frac{\sin x-x}{x}\right)^{\frac{1}{x^2}}$$

$$= \lim_{x\to 0}\left[\left(1+\frac{\sin x-x}{x}\right)^{\frac{x}{\sin x-x}}\right]^{\frac{\sin x-x}{x^3}}$$

$$= e^{-\frac{1}{6}} \qquad \text{vì } \lim_{x\to 0}\frac{\sin x-x}{x^3} = \lim_{x\to 0}\frac{\cos x-1}{3x^2} = -\frac{1}{6}$$