CHAPTER 2:
STRESS AND STRAIN CONCEPTS

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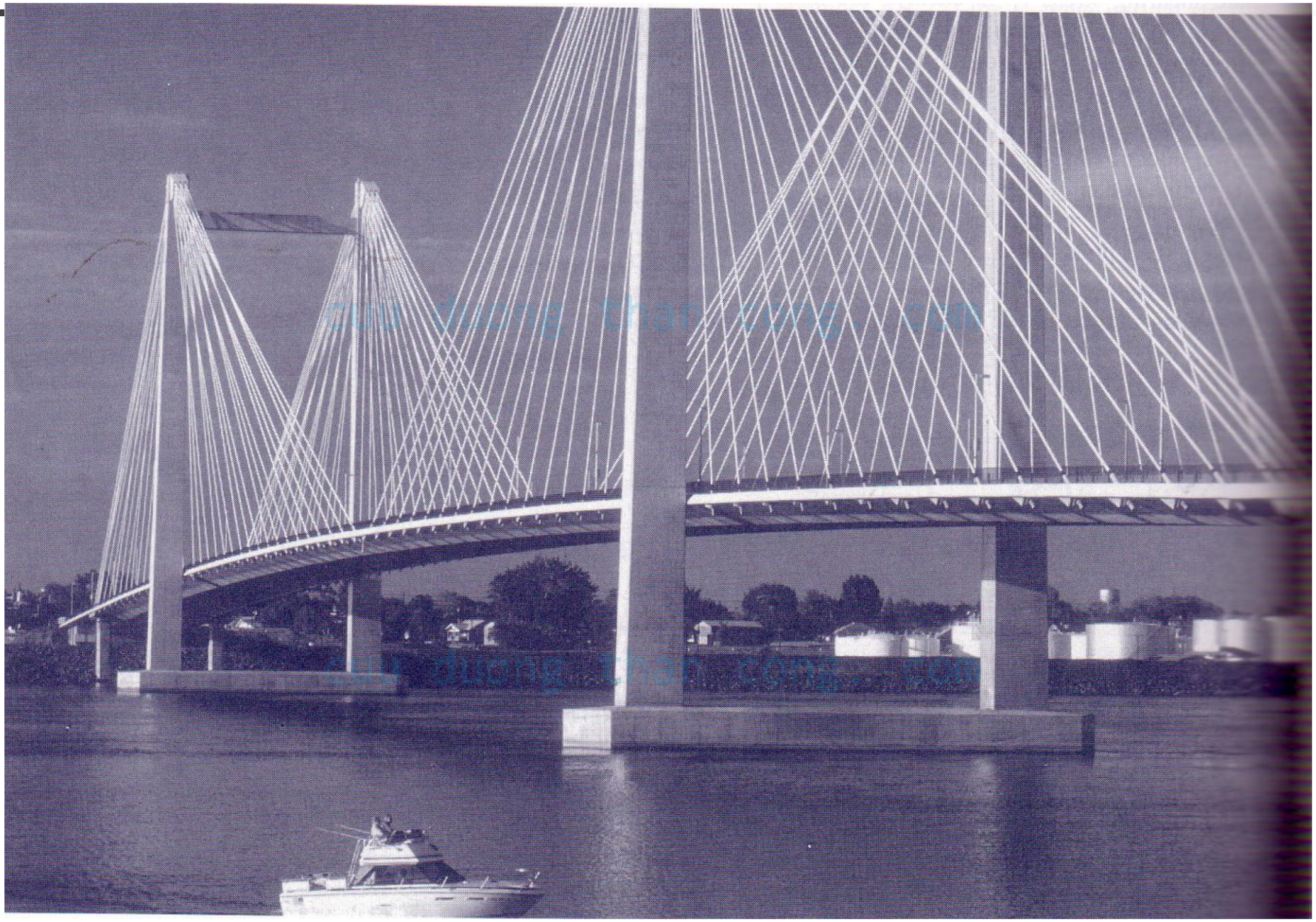

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CONTENTS (NỘI DUNG)

- Introduction (Giới thiệu chung)
- Concept of Stress (Khái niệm ứ/s)
- Uniaxial Problems (Bài toán 1 chiều)
- Shear Stress – Strain (Ư/s – b/d cắt)
- Stresses on Inclined Sections
- Stress – Strain Diagrams
- Safety Factor – Allowable Stresses

I/ INTRODUCTION (GIỚI THIỆU)

- This chapter → Mechanics of Materials (Strength of Materials or Mechanics of Solids)
- Mechanics of Materials is a branch of Mechanics that develops relationships between the external loads applied to a deformable body (vật thể biến dạng được) and the internal forces (nội lực) acting within the body

I/ INTRODUCTION (GIỚI THIỆU) (continued)

- Mechanics of materials → stresses, strains and deflections based on 3 fundamental principles:

✓ Statics: Equilibrium of forces (stresses)

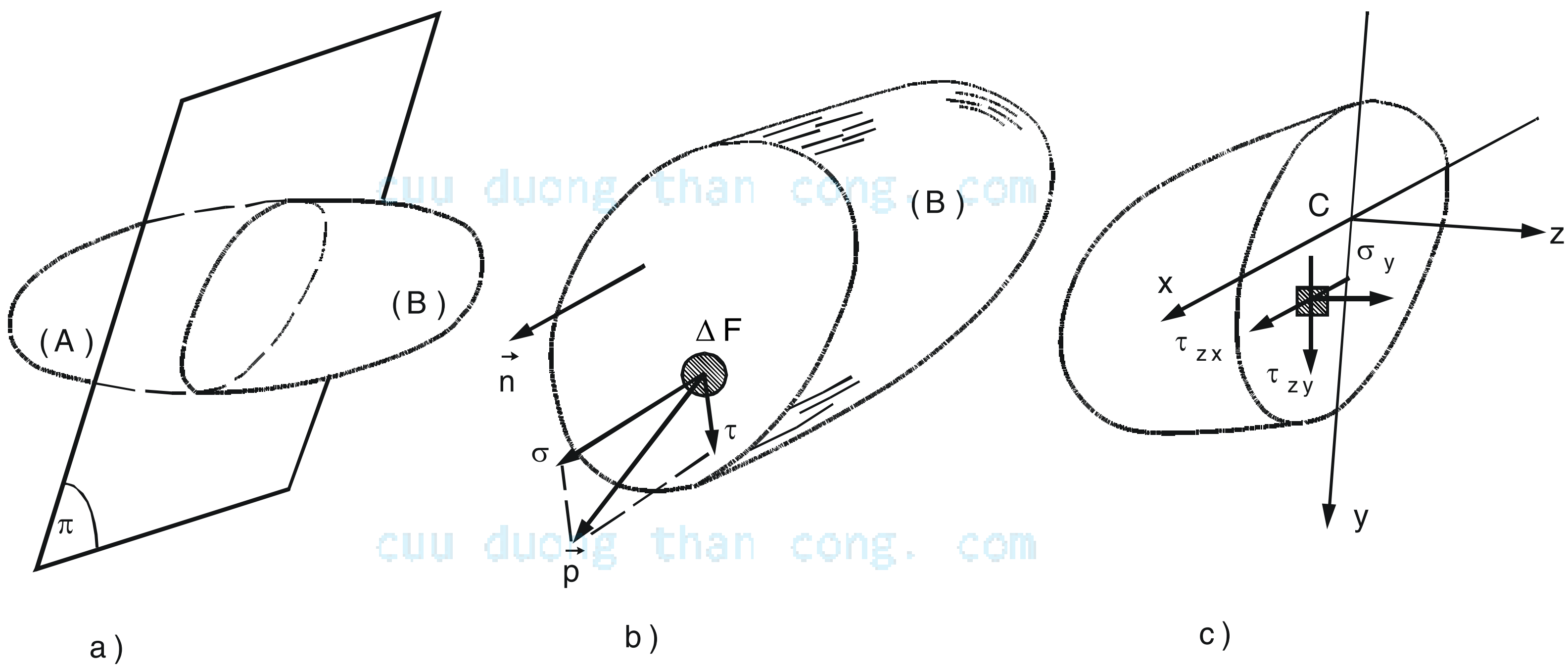
✓ Geometry: Compatibility deformations:

✓ Constitutive Laws: stress-strain relations

I/ INTRODUCTION (GIỚI THIỆU) (continued)

- Boundary conditions: (điều kiện biên)
  - ✓ Static ones (điều kiện biên tĩnh học)
  - ✓ Kinematical ones (đ/k biên động học)
- Material Hypotheses (Giả thiết v/l):
  - ✓ Perfect Elasticity
  - ✓ Continuity, Homogeneity, Isotropy

II/ CONCEPT OF STRESS (KHÁI NIỆM VỀ ỨNG SUẤT)



A body in equilibrium (Vật thể cân bằng)

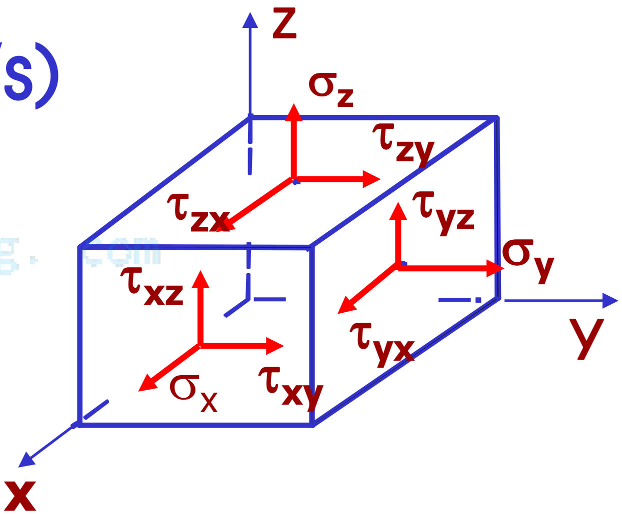
II/ CONCEPT OF STRESS (KHÁI NIỆM VỀ ỨNG SUẤT)

- Stresses at a point (Ư/suất tại 1 điểm)

$$\vec{p} = \lim_{\Delta F \to 0} \frac{\overrightarrow{\Delta P}}{\Delta F} \quad \begin{cases} \sigma : \text{ normal stress} \\ \tau : \text{ shearing stress} \end{cases}$$

- Stress tensor (tenxơ ư/s)

$$\sigma = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$



II/ CONCEPT OF STRESS (KHÁI NIỆM VỀ ỨNG SUẤT)

- Sign convention of normal stresses:
  - ✓ $\sigma_x > 0$: tensile stress
  - ✓ $\sigma_x < 0$: compression stress

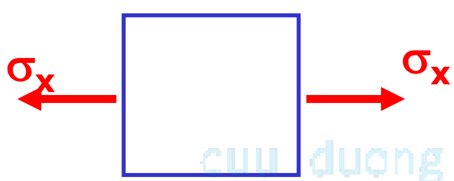
- From the equil. eqs. of moments about the x-, y-, & z-axes, one has:

$$\tau_{yz} = \tau_{zy};\ \tau_{xz} = \tau_{zx};\ \tau_{xy} = \tau_{yx}$$

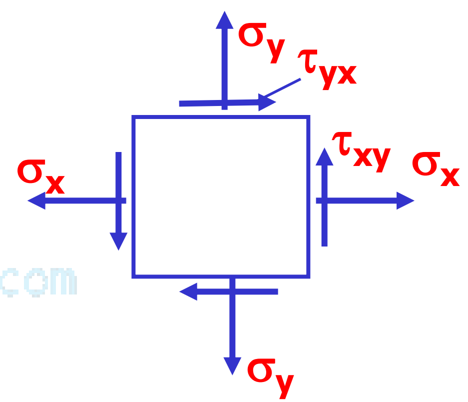
→ **6 independent stress components**

II/ CONCEPT OF STRESS (KHÁI NIỆM VỀ ỨNG SUẤT)

- Uniaxial Stress State (Trạng thái ư/s đơn)
- State of plane stresses/ two dimensional stress state. (Trạng thái ứng suất phẳng)

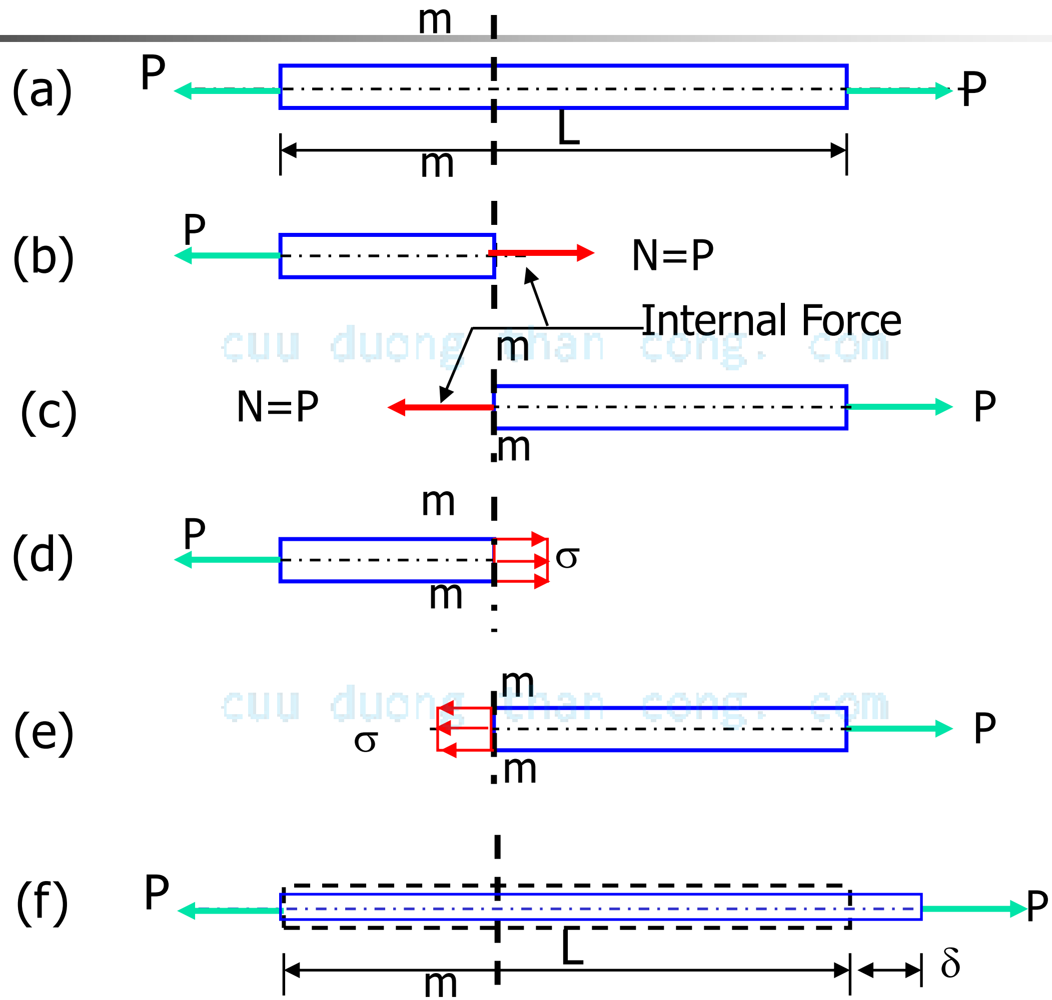


Uniaxial Stress State(TTƯS đơn)



Plane Stress State (TTƯS phẳng)

III/UNIAXIAL PROBLEMS (BÀI TOÁN ĐƠN TRỤC)



III/UNIAXIAL PROBLEMS (BÀI TOÁN ĐƠN TRỤC)

- Internal force: From a cut section → N = P represents also the resultant of normal, tensile stress
- Normal stress: *at a section* far from the bar ends, the normal stress can be regarded *uniform*, or considered as average value

$$\sigma = \frac{N}{A} = \frac{P}{A}$$

III/UNIAXIAL PROBLEMS (BÀI TOÁN ĐƠN TRỤC)

- Deformation (Biến dạng):
  - ✓ Longitudinal strain (Biến dạng dài):

$$\varepsilon = \frac{\delta}{L}$$

  - ✓ Lateral strain (Biến dạng ngang):

$$\varepsilon_{lateral} = -\nu\varepsilon$$

ν- Poisson's ratio, determined by experiment

III/UNIAXIAL PROBLEMS (BÀI TOÁN ĐƠN TRỤC)

- Dilatation = unit volume change (Độ dãn thể tích):

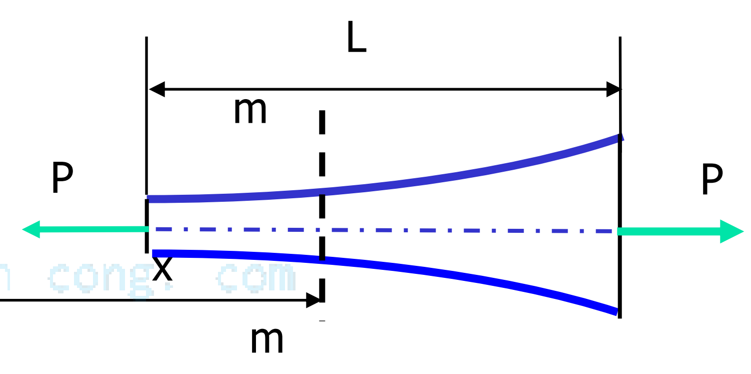
$$\theta = \frac{\Delta V}{V} = \varepsilon(1 - 2\nu)$$

(ν cannot pass the value 0,5, since any larger value would mean that the volume of a bar decreases when it is being stretched!)

III/UNIAXIAL PROBLEMS (BÀI TOÁN ĐƠN TRỤC)

- Axially Loaded Straight Bar of Varying Cross Section (Thanh có tiết diện thay đổi chịu tải dọc trục)

$$\delta = \int_0^L \varepsilon(x)\,dx$$



IV/ SHEARING STRESS & STRAIN (ỨNG SUẤT – BIẾN DẠNG CẮT)

- Shear stress = stress component acting in the plane of considered section

  In many engineering prolems, shear stress t is assumed as uniformly distributed over certain sections for simplicity

$$\tau_{aver} = \frac{V}{A}$$

IV/ SHEARING STRESS & STRAIN (ỨNG SUẤT – BIẾN DẠNG CẮT)

- State of pure shear (TTƯS trượt thuần túy)

An element being acted upon by shear stress only is in *state of pure shear*

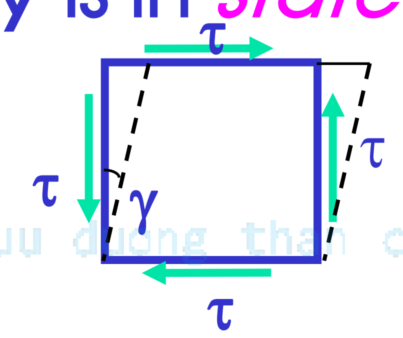
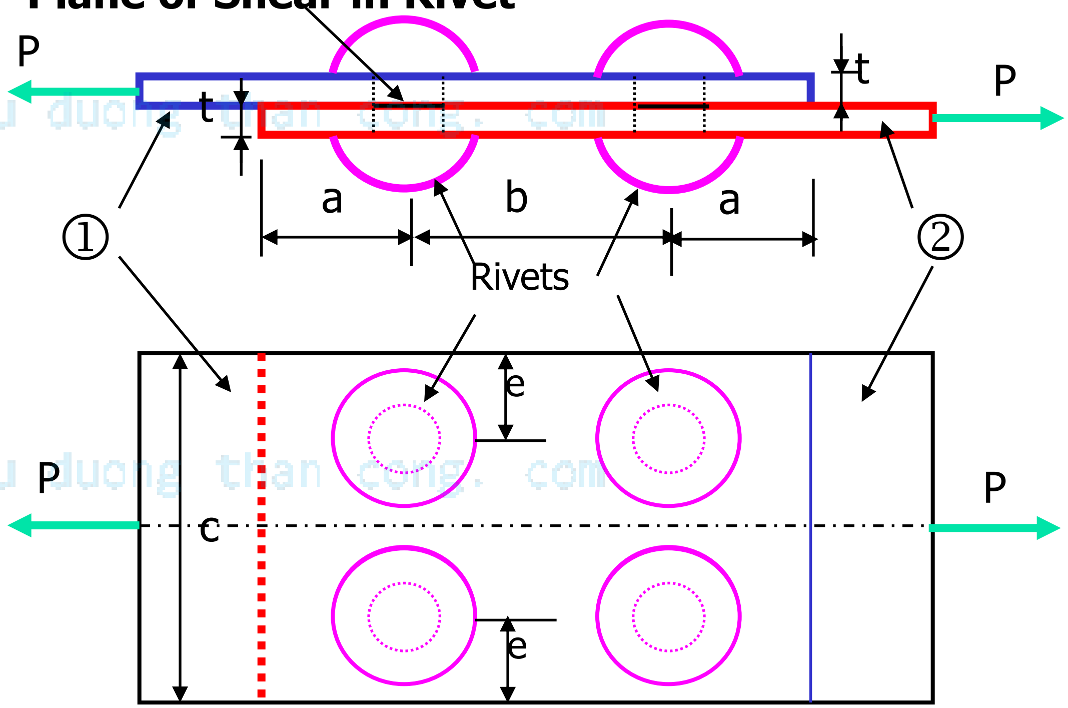


Fig.2.7 State of pure shear

IV/ SHEARING STRESS & STRAIN (ỨNG SUẤT – BIẾN DẠNG CẮT)

- Shear Strain, γ: the change in right angle due to τ.



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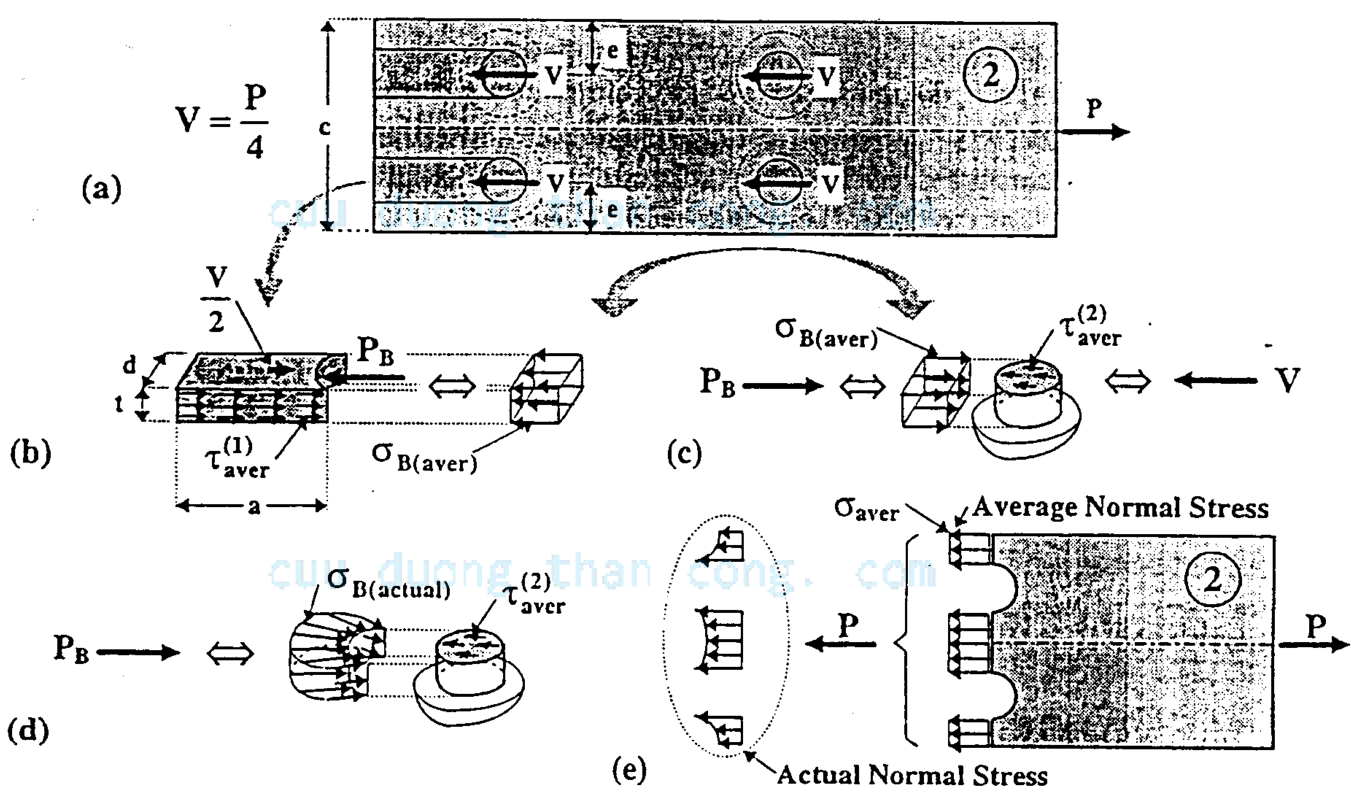
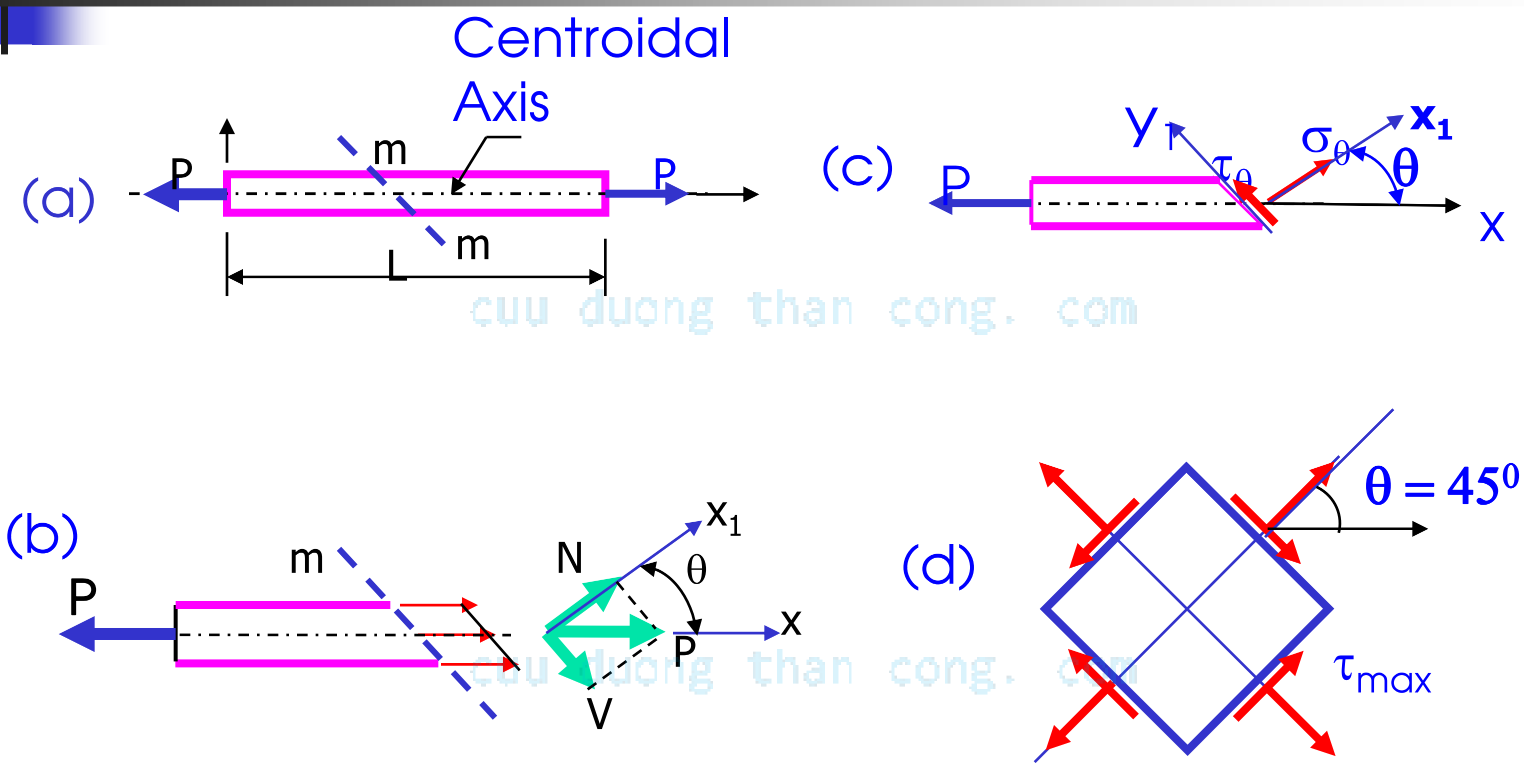


V/ STRESSES ON INCLINED SECTIONS (Ư/s trên m/c nghiêng)



V/ STRESSES ON INCLINED SECTIONS (Ư/s trên m/c nghiêng)

- An axially loaded bar is cut by a section m-m inclined with respect to the longitudinal axis, and whose normal makes un angle of θ with the x-axis (Fig.a).
- We try to find σ_θ ? τ_θ ?

V/ STRESSES ON INCLINED SECTIONS (Ư/s trên m/c nghiêng)

- Hypotheses :

  ✓ The section m-m is relative far from the bar ends

  ✓ On m-m the stress is uniformly distribut-ed in x-direction, whose resultant is equal to P

V/ STRESSES ON INCLINED SECTIONS (Ư/s trên m/c nghiêng)

- The normal & shear stresses:

$$\sigma_\theta = \frac{N}{A_\theta} = \sigma_x \cos^2\theta \qquad \tau_\theta = -\frac{V}{A_\theta} = -\frac{\sigma_x}{2}\sin 2\theta$$

- Maximal shear stress & corres. norm. str.

$$\tau_{max} = \left| \tau_{\theta|\theta=45^0} \right| = \frac{\sigma_x}{2} \qquad \sigma_{\theta|\theta=45^0} = \frac{\sigma_x}{2}$$

VI/ STRESS-STRAIN DIAGRAMS

- Elastic and Linear-Elastic Material Behavior

  ✓ If removing the applied loads restores the original shape and size → elastic material

  ✓ A linear stress-strain relationship → linearly elastic material

VI/ STRESS-STRAIN DIAGRAMS (continued)

- Hooke's Law
  - ✓ The linear stress-strain relationships

$$\sigma = E\varepsilon \qquad \tau = G\gamma$$

E - Modulus of Elasticity or Young's Modulus

G – Shear Modulus of Elasticity

Unit of E, G: Pa, GPa = 109 Pa

VI/ STRESS-STRAIN DIAGRAMS (continued)

- Relation among E, G and n in Isotropic Materials (determined from theory and experiments)

$$G = \frac{E}{2(1+\nu)}$$

⇨ "The materials elastic properties E, G & ν are not independent"

VI/ STRESS-STRAIN DIAGRAMS (continued)

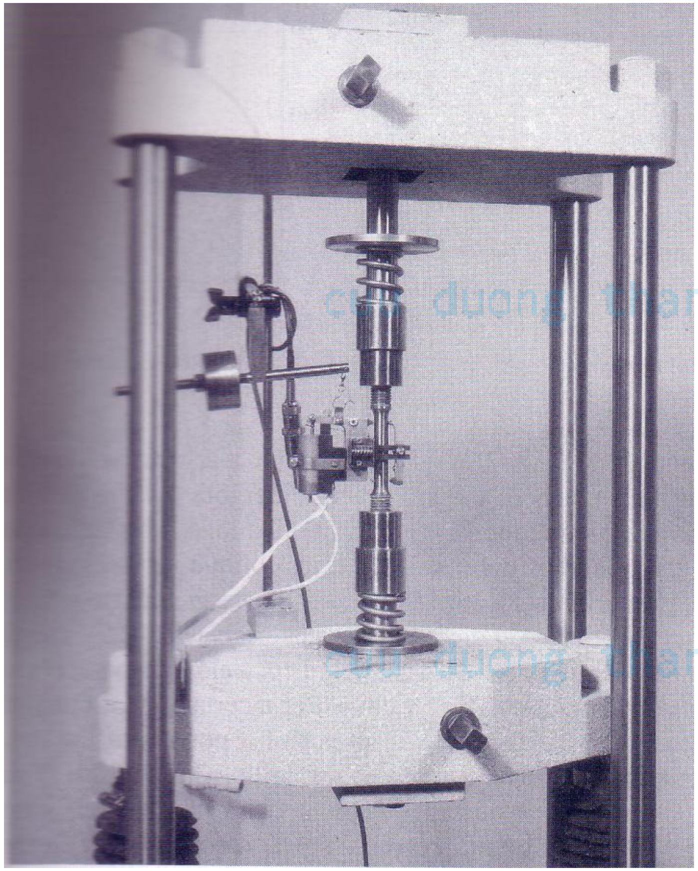
- Stress-Strain Diagrams. Tensile Test of Mild Steel

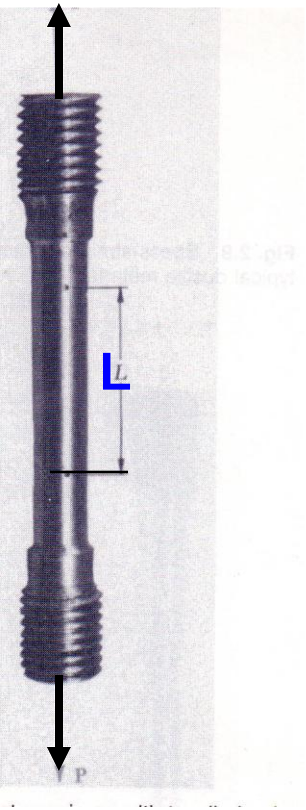
✓Generalities:

- ($\sigma-\varepsilon$) diagrams →basic mechanical properties of structural materials.
- From tensile & compression tests

- Category of Materials: ductile and brittle (fragile) materials

VI/ STRESS-STRAIN DIAGRAMS (continued)



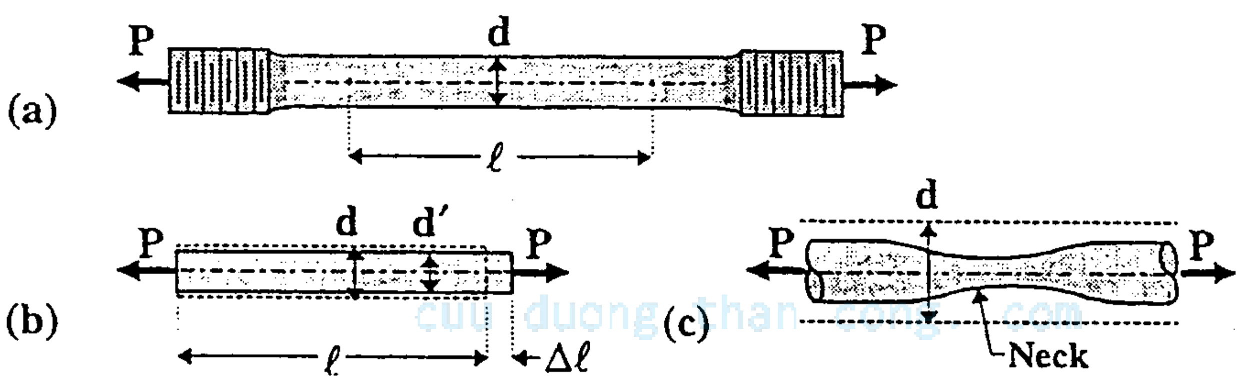


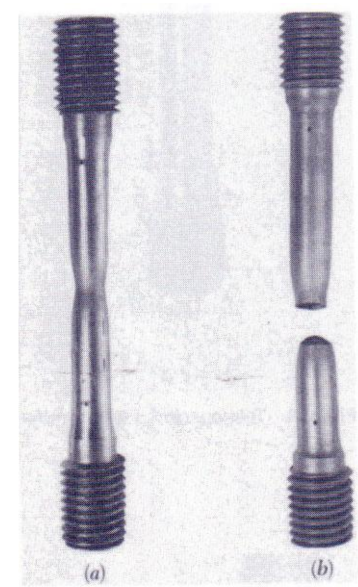
Máy vạn năng

Mẫu thử kéo

VI/HOOKE'S LAW. STRESS-STRAIN DIAGRAMS (continued)

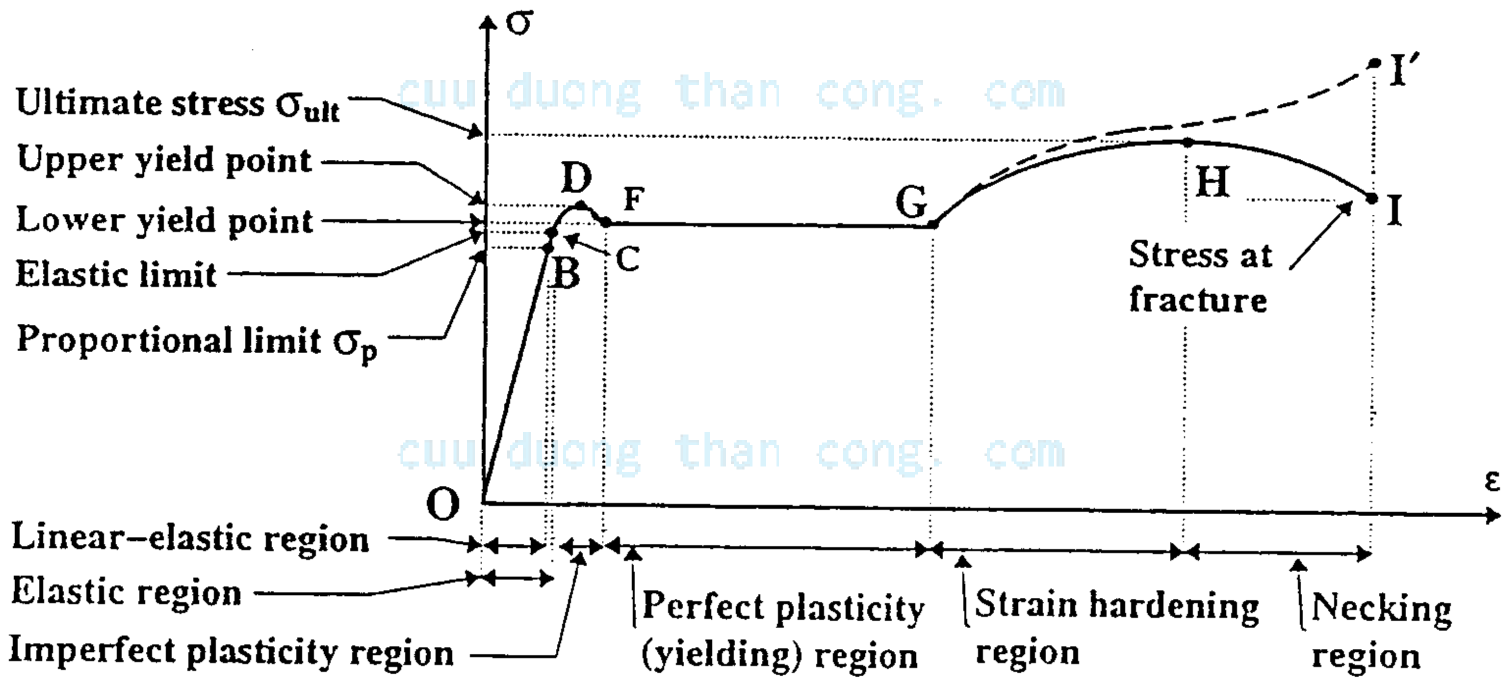
- ✓ Tensile Test Specimen of Mild Steel: a slender circular cross section bar:





VI/ STRESS-STRAIN DIAGRAMS (continued)

✓ Real stress-strain diagram



VI/ STRESS-STRAIN DIAGRAMS (continued)

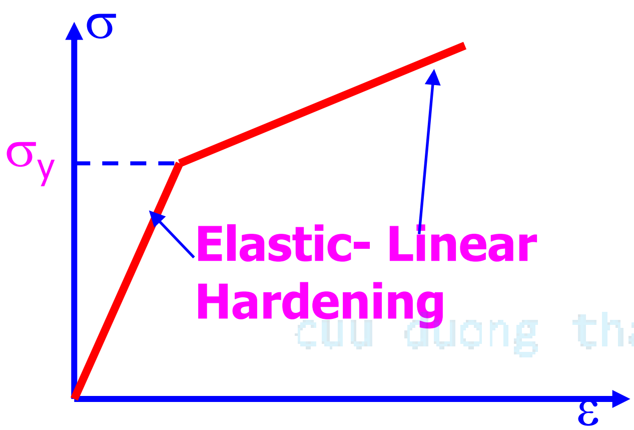
- ✓ Elastic domain: OB
  - linear stress–strain relationship
  - point B: proportional limit, σ_p
  - Const slope: *Modulus of Elasticity, E*
- ✓ Transition domain: BCDF
  - point C: Elastic Limit (B ≡ C)
  - Point D: the Upper Yield Point, $\sigma_y^{(u)}$
  - Point F: the Lower Yield Point, $\sigma_y^{(l)}$

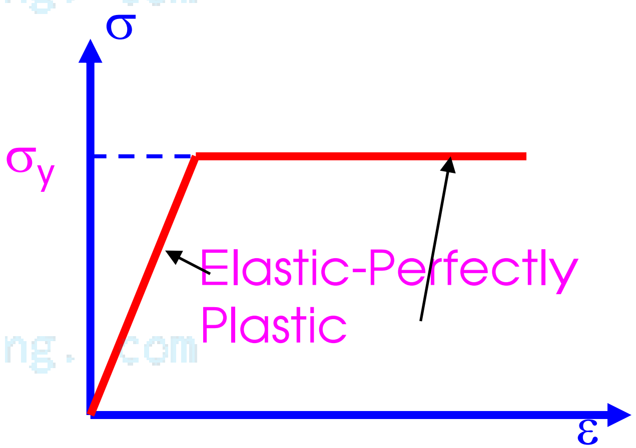
VI/ STRESS-STRAIN DIAGRAMS (continued)

- ✓ Region of Plasticity: FG
  - Deformation develops under constant yield stress, $\sigma_y^{(u)} = \sigma_y^{(l)} = \sigma_y$
  - G → end of the region of plasticity
- ✓ Strain-hardening & necking regions:
  - Nonlinear Stress-Strain Relationship
  - Point H: → the ultimate stress, σ_{ult}
  - Point I: → Fracture of the specimen

VI/ STRESS-STRAIN DIAGRAMS (continued)

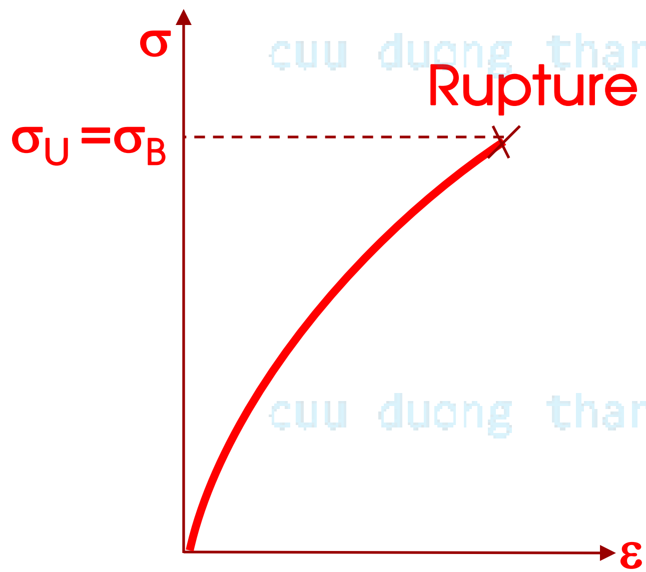
✓ Idealised mild steel stress-strain diagrams



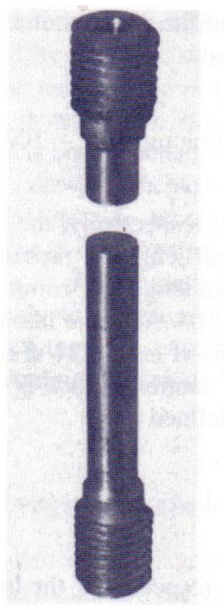


VI/ STRESS-STRAIN DIAGRAMS (continued)

✓ Tensile Test Specimen of brittle materials



Stress-strain diagram for a typical brittle material



Tested specimen of a brittle material

VII/ FACTOR OF SAFETY. ALLOWABLE STRESSES. SERVICE LOADS

- Factor of Safety:

$$\text{Factor of safety } n = \frac{\text{actual strength}}{\text{required strength}} > 1$$

- Allowable Stresses, σ_{allow} (or $[\sigma]$)

✓Ductile Materials: $\sigma_{allow} = \frac{\sigma_y}{n}$, $n \approx 1.67$

For mild steel, σ_y =250MPa ⇨ (σ) =150MPa

✓ Brittle Materials: $\sigma_{allow} = \frac{\sigma_{ult}}{n}$