

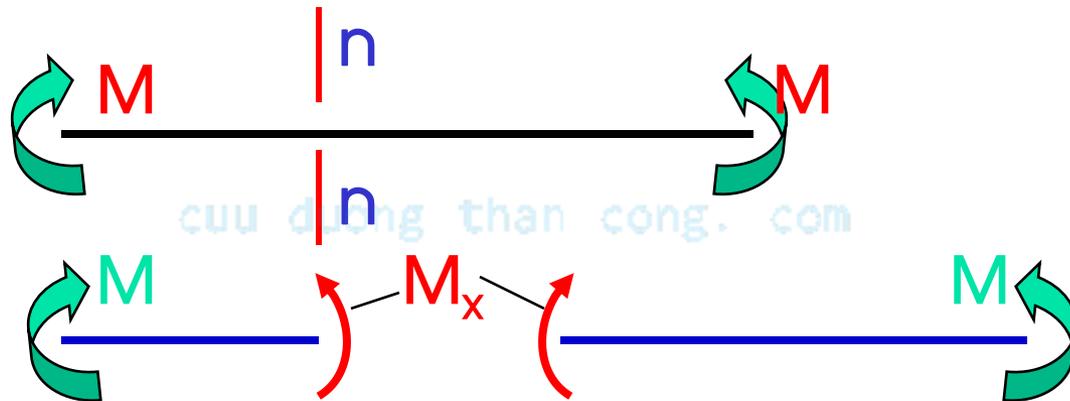
CHAPTER 5 (CHƯƠNG 5) PLANAR FLEXURE (UỐN PHẪNG)

PGS. TS. Bùi Công Thành
KHOA KỸ THUẬT XÂY DỰNG

I/ INTRODUCTION (GIỚI THIỆU)

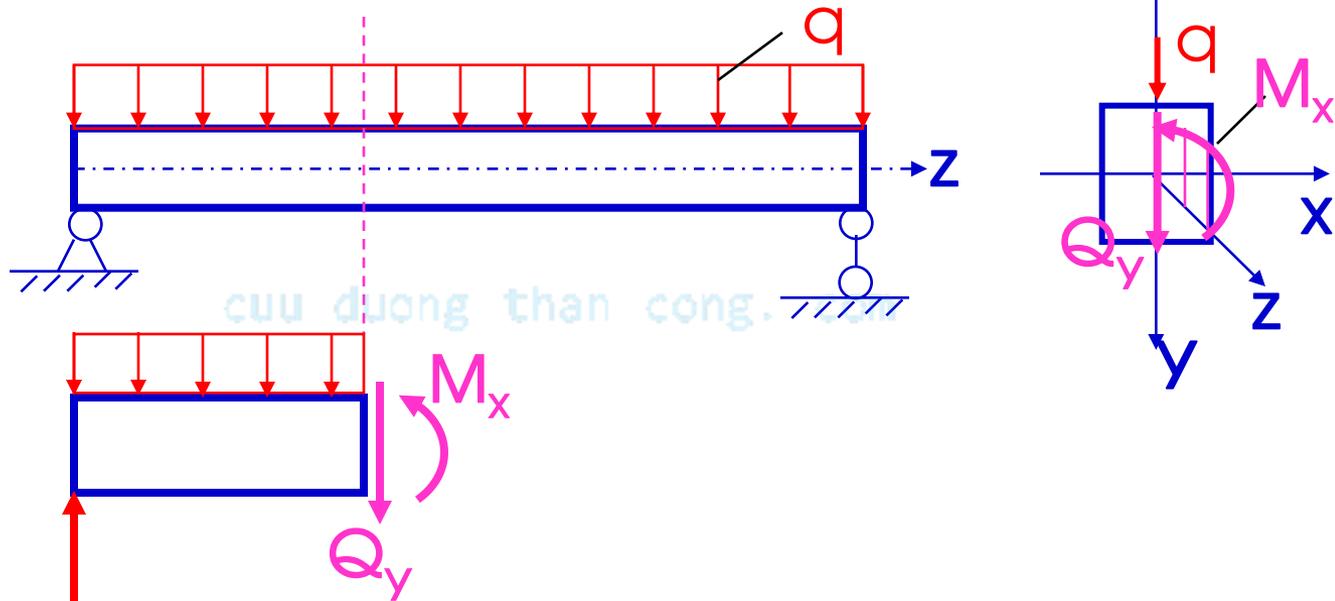
- **Notion (Khái niệm):**

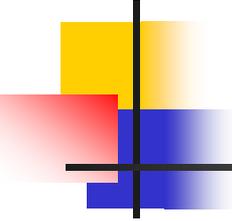
✓ **Pure bending** when only one internal bending moment (chỉ có 1 t/p momen uốn nội lực)



I/ INTRODUCTION (GIỚI THIỆU)

✓ Planar Flexure (Uốn phẳng): when all internal forces are in the same plane containing the beam axis



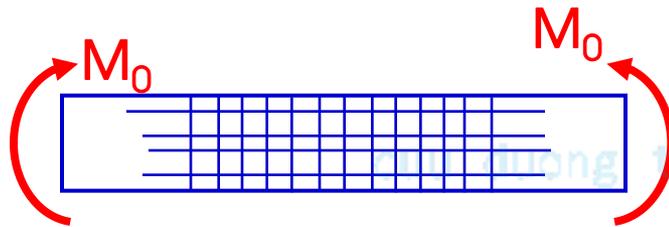


I/ INTRODUCTION (GIỚI THIỆU)

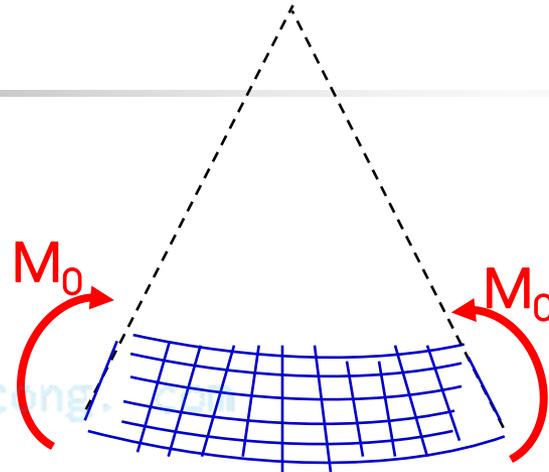
- **Hypotheses (Giả thiết)**
 - ✓ The section has at least one symmetric-al axis (Tiết diện có ít nhất 1 trục đối xứng)
 - ✓ The external moments are acting in the symmetrical plane containing the bar axis (Momen ngoại tác dụng trong mặt phẳng đối xứng chứa trục thanh)

II/ PURE BENDING (UỐN THUẦN TÚY) (cont.)

■ Test (Thí nghiệm)



Lưới ô vuông

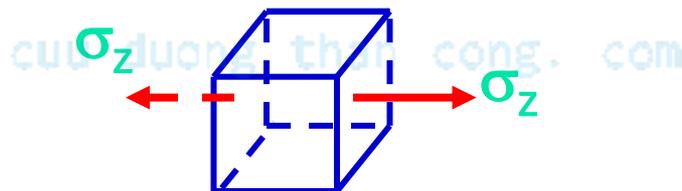


- ✓ After deformation, the bar axis is bent (Sau biến dạng, trục thanh bị uốn cong)
- ✓ Right angles are conserved (Góc vuông vẫn bảo toàn) → no angular strain (0 biến dạng góc)
- ✓ Từ thớ chịu kéo → nén: thớ trung hòa (neutral axis)

II/ PURE BENDING (UỐN THUẦN TÚY)

HYPOTHESES (GIẢ THIẾT TÍNH TOÁN):

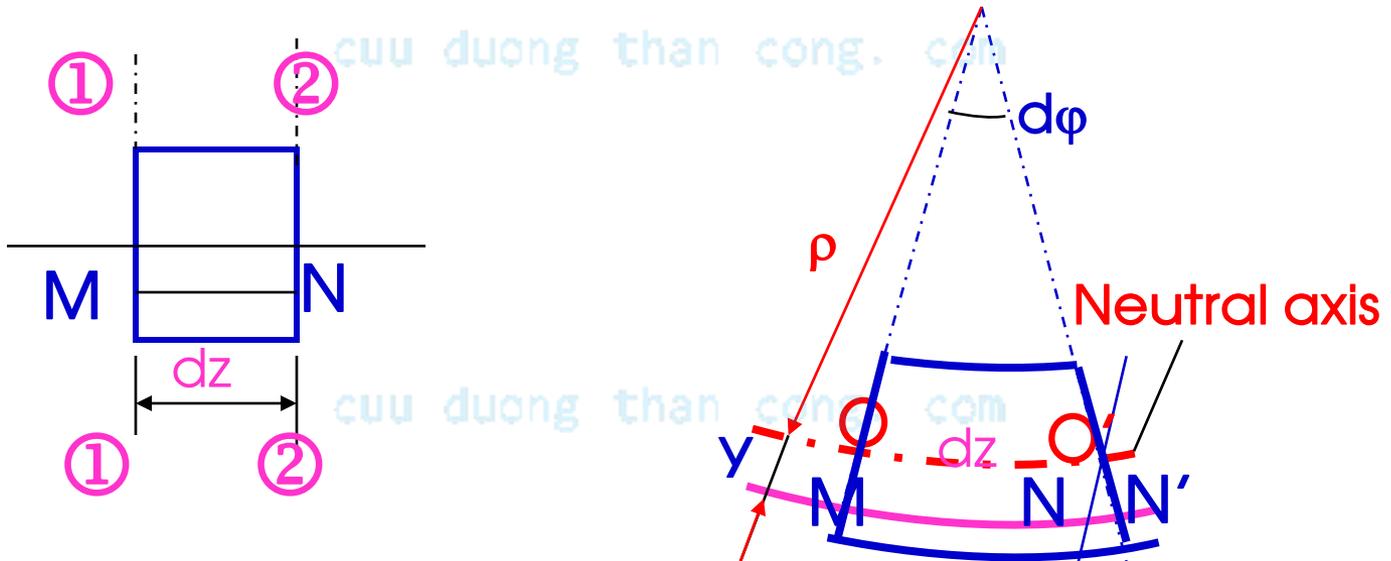
- ✓ Bernoulli hypothesis: cross-sections are plane and orthogonal to axis
 - ✓ Hypothesis on longitudinal fibers:
 - ✓ Hypothesis on materials: linearly elastic
- Only normal stress along its axis



II/ PURE BENDING (UỐN THUẦN TÚY) (cont.) – Normal Stress (σ/s pháp)

■ FORMULATION OF NORMAL STRESS

✓ Normal Strain (Biến dạng dọc trục)



$$\epsilon_z = \frac{MN' - MN}{MN} = \frac{y}{\rho}$$

II/ PURE BENDING (UỐN THUẦN TÚY) (cont.) – Normal Stress (ứ/s pháp) (cont.)

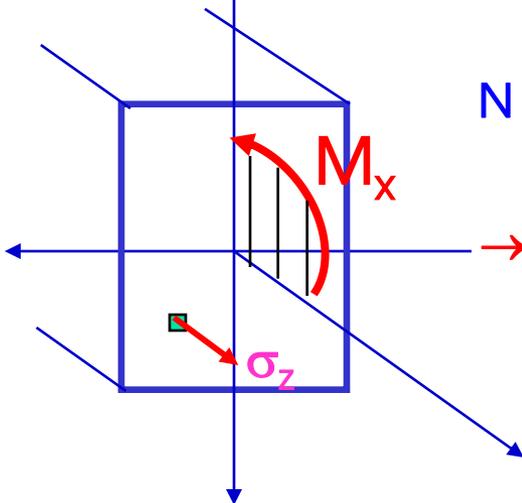
✓ Stress-Strain Relation (Quan hệ ứng suất- biến dạng):

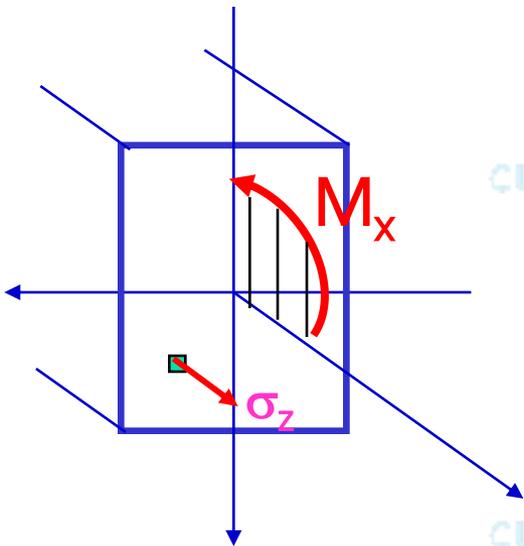
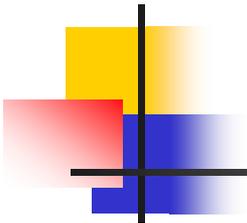
$$\sigma = E \varepsilon \quad (b)$$

✓ Stress - Internal Forces Relations:

$$N_z = \int_A \sigma_z dA = \int_A \frac{E}{\rho} y dA = 0 \rightarrow \int_A y dA = 0$$

→ trục trung hòa qua trọng tâm





$$M_y = \int_A \sigma_z x dA = \frac{E}{\rho} \int_A xy dA = 0$$

$$\rightarrow I_{G_{xy}} = \int_A xy dA = 0$$

$$M_x = \int_A \sigma_z y dA = \frac{E}{\rho} \int_A y^2 dA = \frac{EI_{G_x}}{\rho}$$

$$\rightarrow \frac{1}{\rho} = \frac{M_x}{EI_{G_x}} \quad (c)$$

ρ – neutral fiber radius (bk thớ trung hòa)

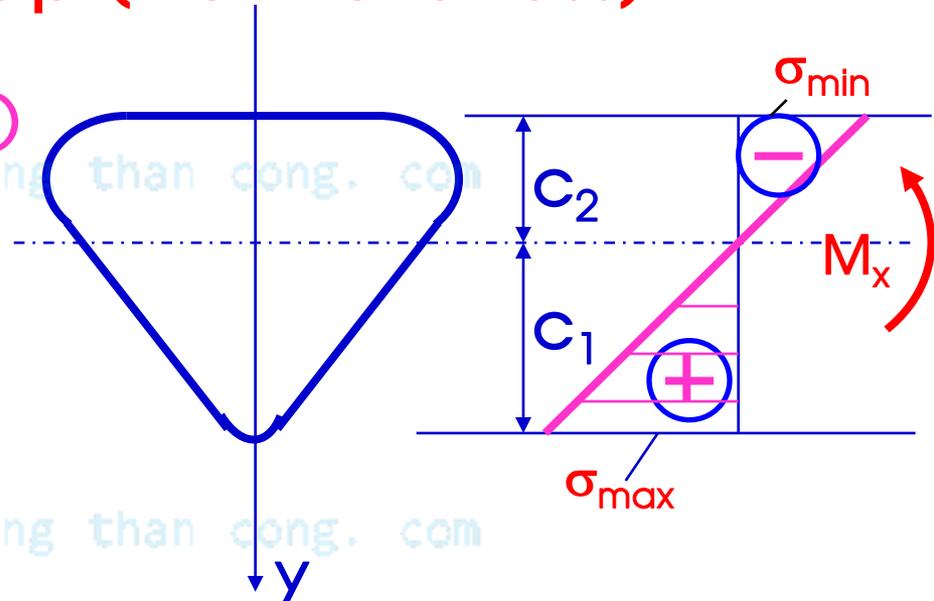
II/ PURE BENDING (UỐN THUẦN TÚY) (cont.) – Normal Stress (u/s pháp) (cont.)

■ Ứng suất pháp (Normal Stress)

$$\sigma_z = \frac{M_x}{I_{Gx}} y \quad (1)$$

$$\sigma_{\max} = + \frac{|M_x|}{W_x^k};$$

$$\sigma_{\min} = - \frac{|M_x|}{W_x^n}$$



■ With (với)

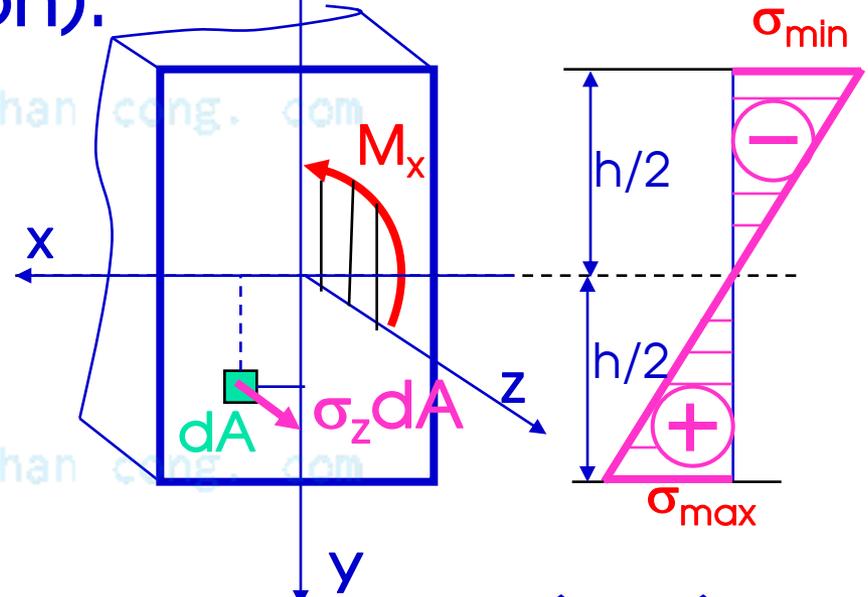
$$W_x^k = \frac{I_{Gx}}{C_1}; \quad W_x^n = \frac{I_{Gx}}{C_2}$$

II/ PURE BENDING (UỐN THUẦN TÚY) (cont.) – Normal Stress (u/s pháp) (cont.)

✓ Tiết diện có 2 trục đối xứng (Biaxial symmetry section):

$$W_x^k = W_x^n = W_x = \frac{I_{Gx}}{h/2}$$

$$\sigma_{\max} = -\sigma_{\min} = \frac{M_x}{W_x}$$



W_x – elastic section modulus (suất tiết diện đàn hồi hay momen chống uốn)

II/ PURE BENDING (UỐN THUẦN TÚY) (cont.) – Normal Stress (σ/s pháp) (cont.)

- Elastic section modulus of some sections
(Suất tiết diện của một số m/c ngang)

✓ Rectangular cross section (M/c chữ nhật)

$$I_{Gx} = \frac{bh^3}{12}; \quad W_x = \frac{bh^2}{6}$$

✓ Circular cross section (M/c ngang hình tròn)

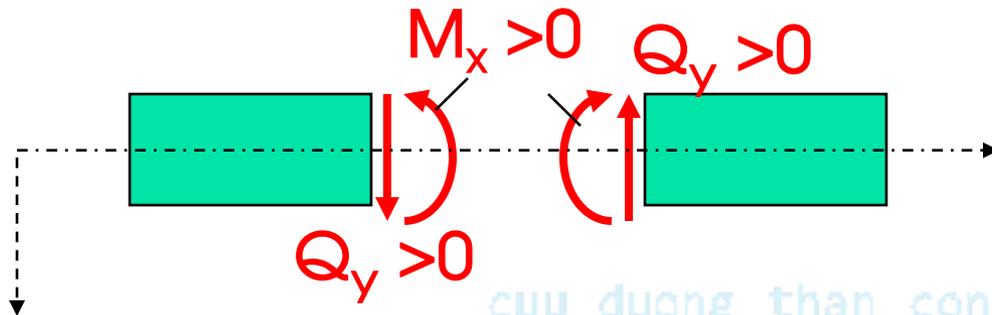
$$I_{Gx} = \frac{\pi d^4}{64}; \quad W_x = \frac{\pi d^3}{32}$$

III/ PLANAR FLEXURE OF BEAM (UỐN PHẪNG)

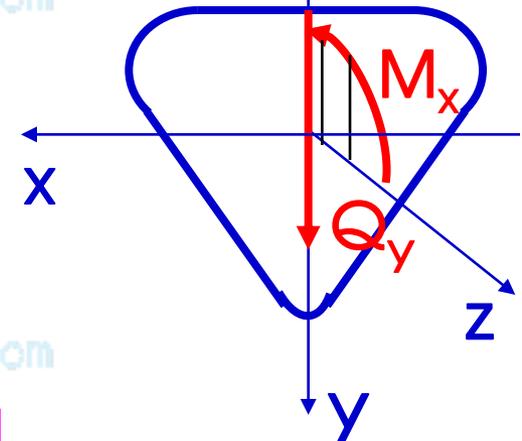
Notions (Khái niệm)

- ✓ Internal forces (Nội lực): M_x, Q_y
- ✓ Sign convention (Quy ước dấu):

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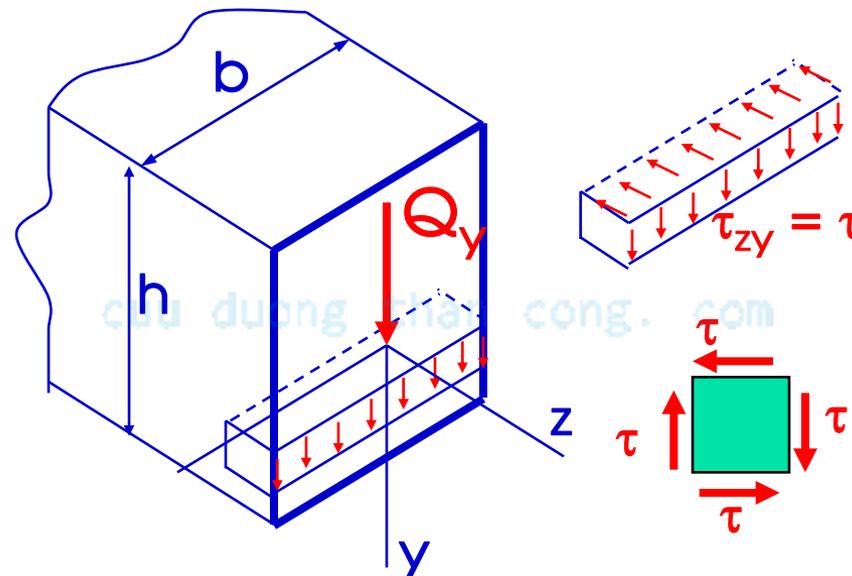


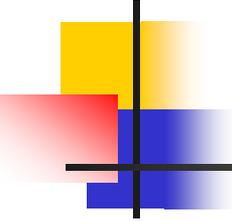
Normal stress:

$$\sigma_z = \frac{M_x}{I_{Gx}} y$$

III/ PLANAR FLEXURE OF BEAM (UỐN PHẪNG) – SHEAR STRESSES (Ứ/S TIẾP)

Shear Stresses associated with the shear force (Ứ/s tiếp trong dầm)





III/ PLANAR FLEXURE OF BEAM (UỐN PHẪNG) – SHEAR STRESSES (U'/'S TIẾP)

✓ Hypotheses (Giả thiết):

The shear stresses τ // shear force Q_y
(U'/'s tiếp τ // lực cắt Q)

The distribution of shear stresses is uniform across the width of the beam
(U'/'s tiếp phân bố đều trên bề rộng)

III/ PLANAR FLEXURE OF BEAM (UỐN PHẪNG) – SHEAR STRESSES (Ứ/S TIẾP)

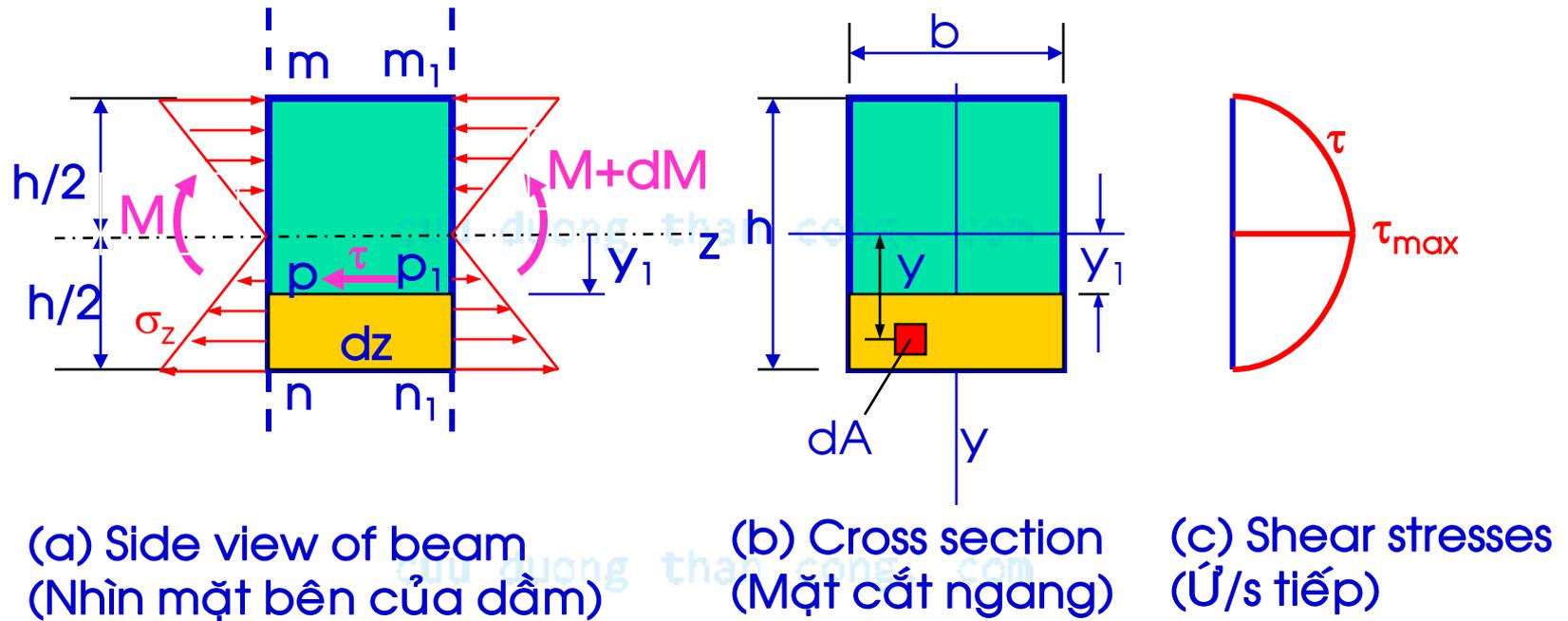


Fig. Shear stresses in a beam of rectangular cross section

III/ PLANAR FLEXURE OF BEAM (UỐN PHẪNG) – SHEAR STRESSES (U' / S TIẾP)

- Formulation (Thiết lập công thức)

Consider the equilibrium of an element pp_1n_1n (Xét cân bằng phần tử pp_1n_1n)

On face pn :
$$F_1 = \int_{A_c} \sigma_z dA = \int_{A_c} \frac{M_x}{I_{Gx}} y dA$$

On face p_1n_1 :
$$F_2 = \int_{A_c} \sigma_z^{(2)} dA = \int_{A_c} \frac{M_x + dM_x}{I_{Gx}} y dA$$

On the top face pp_1 :
$$F_3 = \tau b dz$$

III/ PLANAR FLEXURE OF BEAM (UỐN PHẪNG) – SHEAR STRESSES (U' / S TIẾP)

Equilibrium equation in z direction:
(P/t cân bằng theo phương z)

$$F_3 = F_2 - F_1$$

$$\tau b dz = \int_{A_c} \frac{M_x + dM_x}{I_{Gx}} y dA - \int_{A_c} \frac{M_x}{I_{Gx}} y dA$$

$$\tau = \frac{dM_x}{dz} \left(\frac{1}{I_{Gx} b} \right) \int_{A_c} y dA$$

$$S_x^c = \int_{A_c} y dA \quad \rightarrow \quad \tau = \frac{Q_y S_x^c}{I_{Gx} b_c}$$

III/ PLANAR FLEXURE OF BEAM – SHEAR STRESSES (U' S TIẾP) (†)

✓ Beams of rectangular cross section

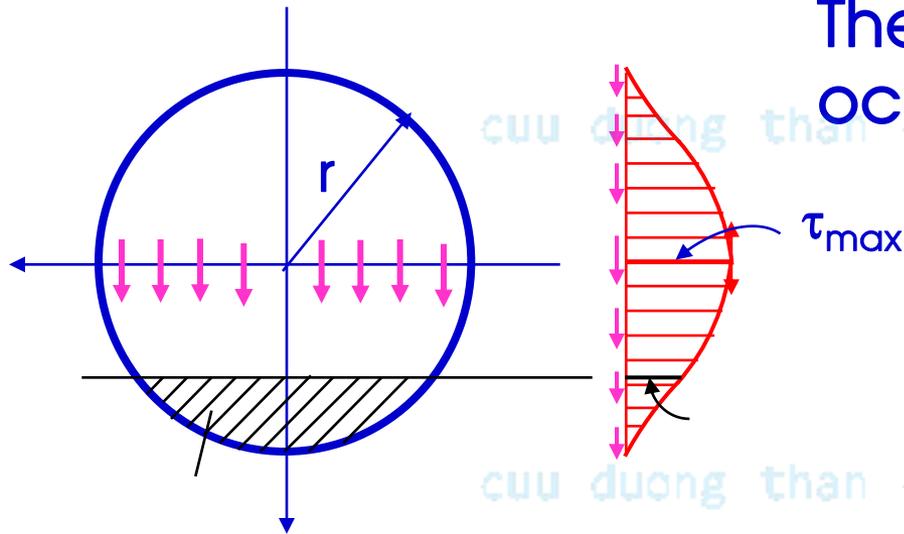
$$S_x^c = b \left(\frac{h}{2} - y_1 \right) \left(y_1 + \frac{\frac{h}{2} - y_1}{2} \right) = \frac{b}{2} \left(\frac{h^2}{4} - y_1^2 \right)$$

$$\rightarrow \tau = \frac{Q_y}{2I_{Gx}} \left(\frac{h^2}{4} - y_1^2 \right)$$

$$\tau_{\max} = \frac{Q_y h^2}{8I_{Gx}} = \frac{3}{2} \frac{Q_y}{A} \quad \text{with } A = bh$$

III/ PLANAR FLEXURE OF BEAM – SHEAR STRESSES (U/S TIẾP) (†)

✓ Beams of circular cross section



The largest shear stresses occur at the neutral axis

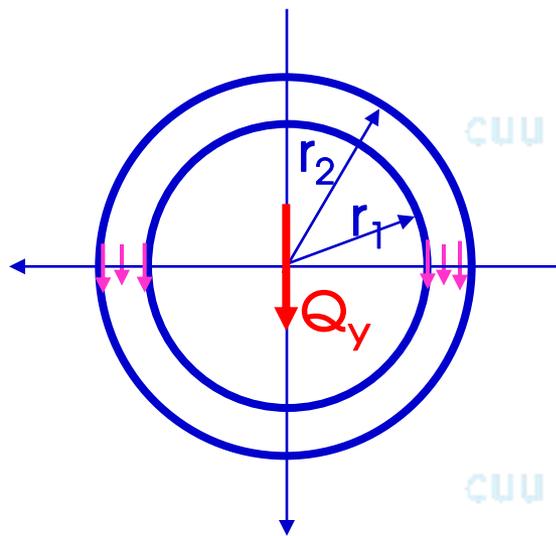
$$I_{Gx} = \frac{\pi r^4}{4}; \quad b = 2r$$

$$S_x^c = S_{max} = \frac{2r^3}{3}$$

$$\tau_{max} = \frac{4}{3} \frac{Q_y}{A} \quad \text{with} \quad A = \pi r^2$$

III/ PLANAR FLEXURE OF BEAM – SHEAR STRESSES (U/S TIẾP) (†)

✓ Beams of hollow circular cross section



It's assumed that the shear stresses along the neutral axis → // y-axis and uniformly distributed

$$S_x^c = S_{\max} = \frac{2}{3}(r_2^3 - r_1^3)$$

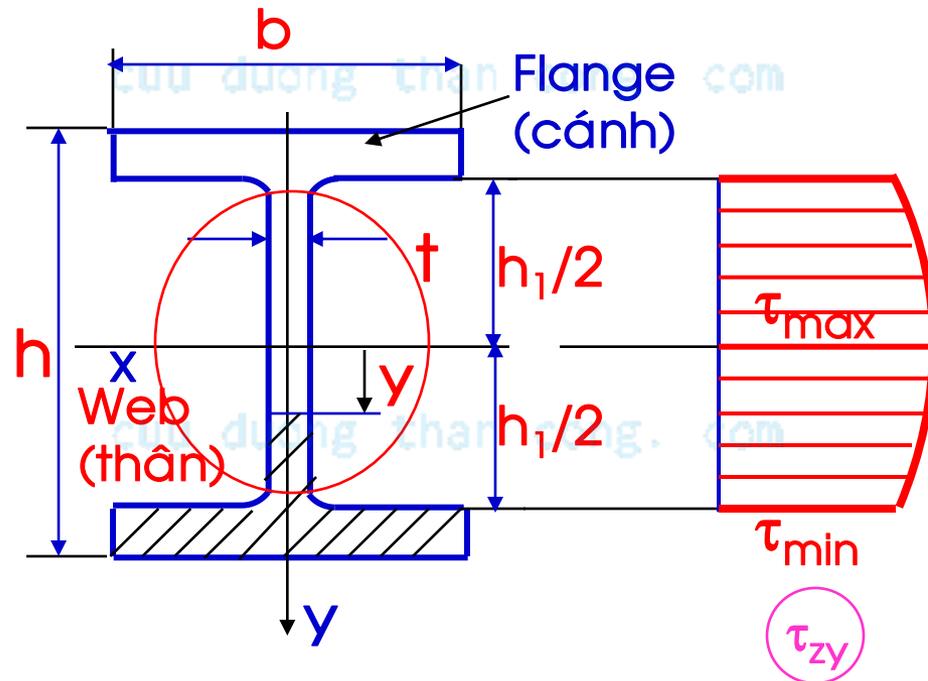
$$b^c = 2(r_2 - r_1) \quad I_{Gx} = \frac{\pi}{4}(r_2^4 - r_1^4)$$

$$\tau_{\max} = \frac{Q_y S_x^c}{I_{Gx} b^c} = \frac{4 Q_y}{3 A} \frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}$$

with $A = \pi (r_2^2 - r_1^2)$

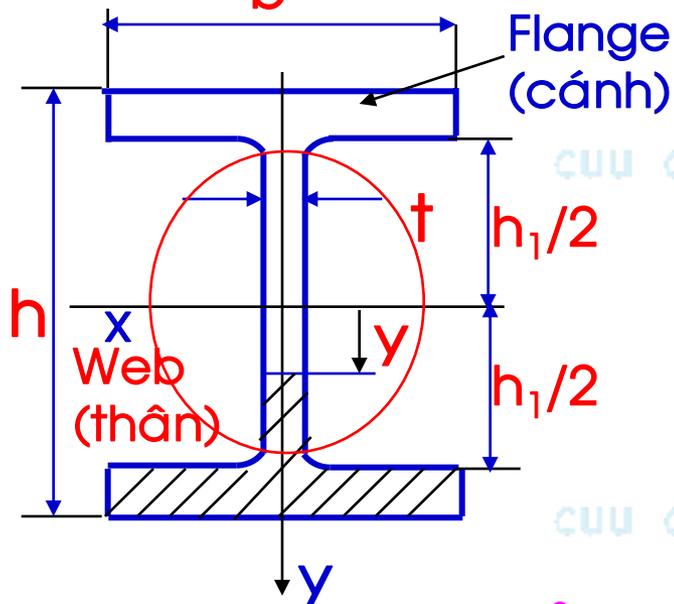
III/ PLANAR FLEXURE OF BEAM – SHEAR STRESSES (Ứ/S TIẾP) (†)

✓ Shear stresses in the webs of beams with flanges (Ứ/s tiếp ở thân các dầm có cánh)



III/ PLANAR FLEXURE OF BEAM – SHEAR STRESSES (U' S TIẾP) (†)

✓ Shear stress at a point at a distance y



$$A_f = b \left(\frac{h}{2} - \frac{h_1}{2} \right); \quad A_w = t \left(\frac{h_1}{2} - y \right)$$

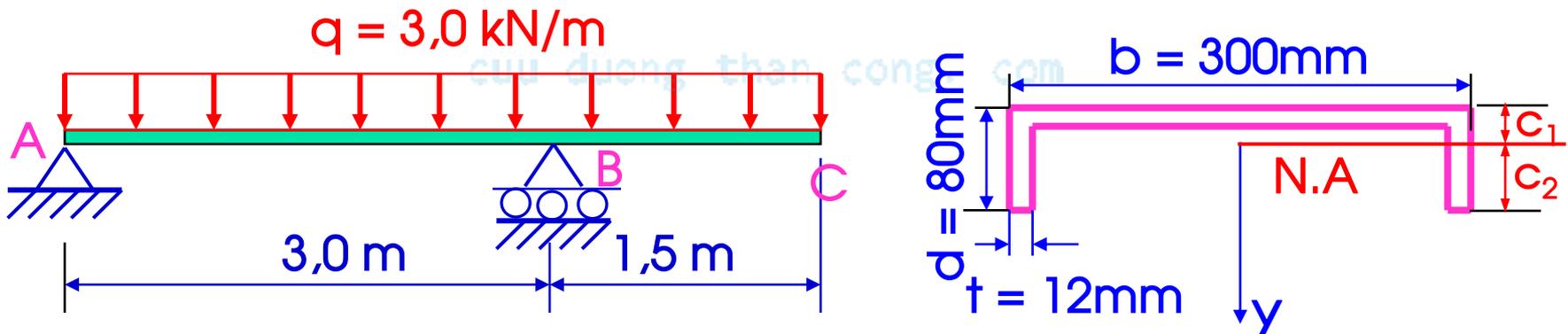
$$S_x^c = \frac{b}{8} (h^2 - h_1^2) + \frac{t}{8} (h_1^2 - 4y^2)$$

$$I_{Gx} = \frac{1}{12} (bh^3 - bh_1^3 + th_1^3)$$

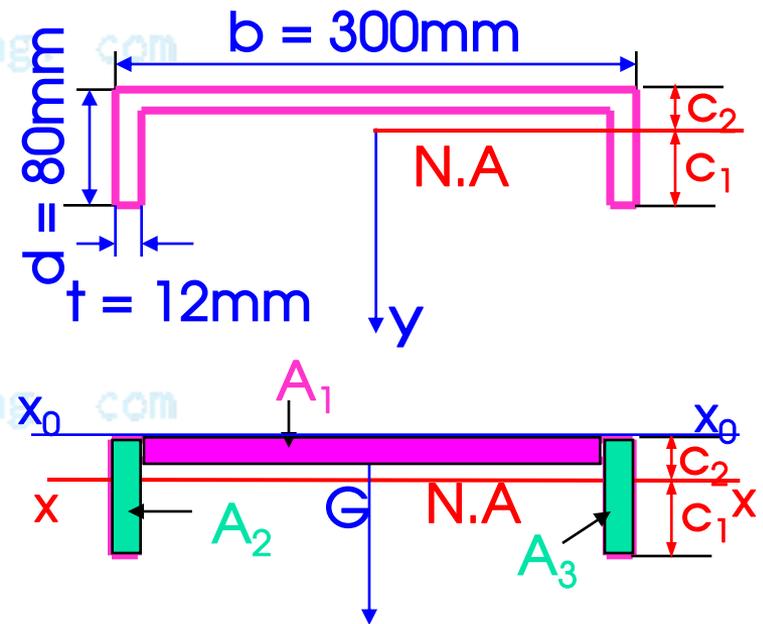
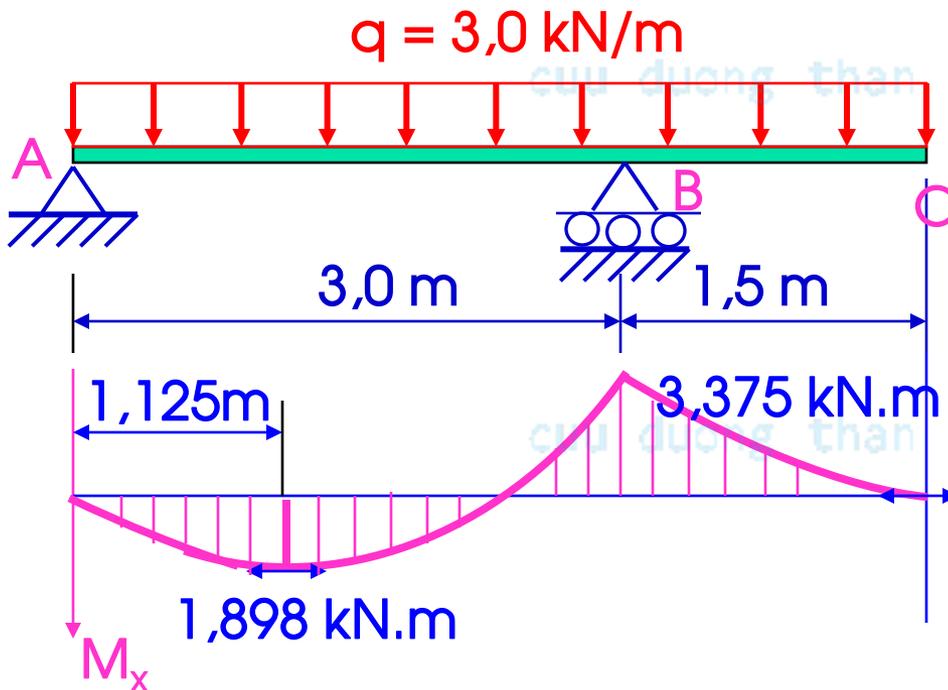
$$\tau_{zy} = \frac{Q_y S_x^c}{I_{Gx} t} = \frac{3Q_y (bh^2 - bh_1^2 + th_1^2 - 4ty^2)}{2t(bh^3 - bh_1^3 + th_1^3)}$$

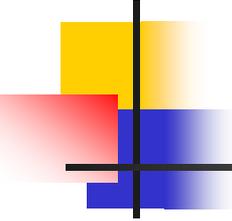
EXAMPLE 1: Calculate stresses

Calculate the maximum tensile and compressive stresses (Tính ú/s kéo và nén lớn nhất)?



EXAMPLE 1: Calculate stresses (cont.)





EXAMPLE 1: Calculate stresses (cont.)

- The bending moment diagram gives (Biểu đồ momen uốn cho):

$$M_{\max} = 1,898 \text{ kN.m} ; M_{\min} = -3,375 \text{ kN.m}$$

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- Position of neutral axis (Vị trí trục trung hòa)

$$c_2 = 18,48 \text{ mm} ; c_1 = 61,52 \text{ mm}$$

- Moment of inertia of the entire cross sectional area (Momen quán tính của toàn tiết diện)

$$I_{Gx} = I_{Gx(1)} + I_{Gx(2)} + I_{Gx(3)} = 2,469 \times 10^6 \text{ mm}^4$$

EXAMPLE 1: Calculate stresses (cont.)

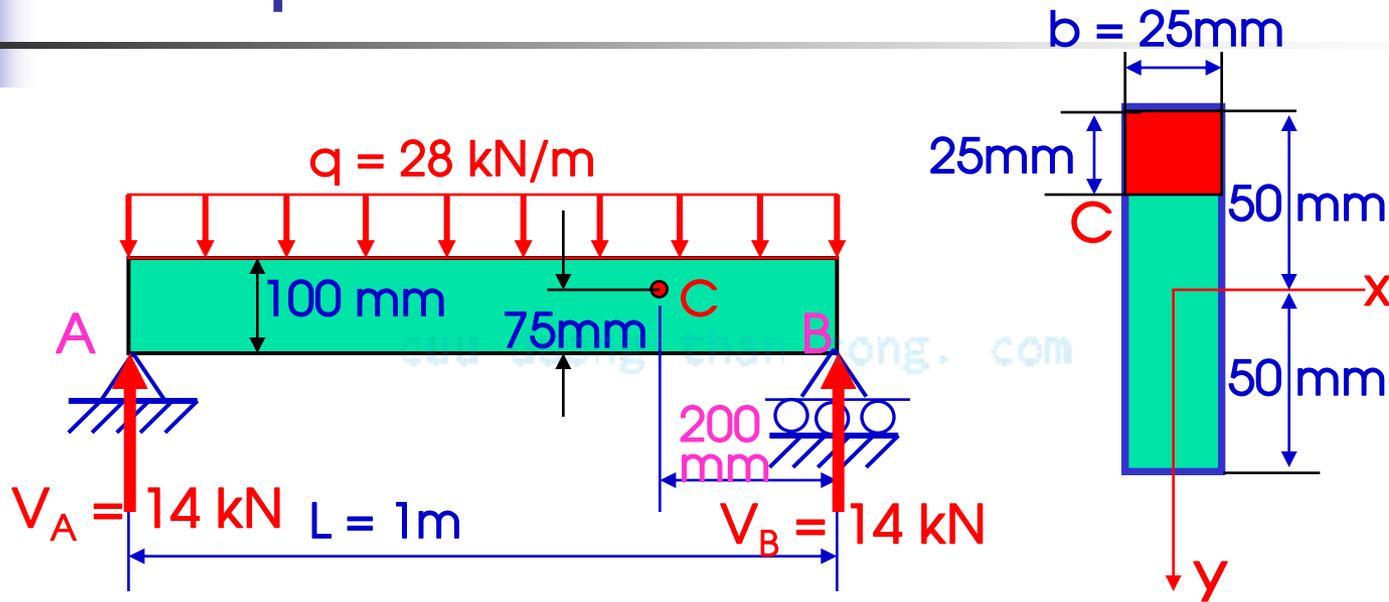
- The section moduli for the bottom and top of the beam (Suất tiết diện đ/v đáy và đỉnh của dầm)

$$W_1 = \frac{I_{Gx}}{C_1} = 40100 \text{ mm}^4; \quad W_2 = \frac{I_{Gx}}{C_2} = 133600 \text{ mm}^4$$

- At the cross section of M_{\max} (Tại đ/c có M_{\max})
 $\sigma_{\max} = \frac{M_{\max}}{W_1} = 47,3 \text{ MPa}; \quad \sigma_{\min} = -\frac{M_{\max}}{W_2} = -14,2 \text{ MPa}$

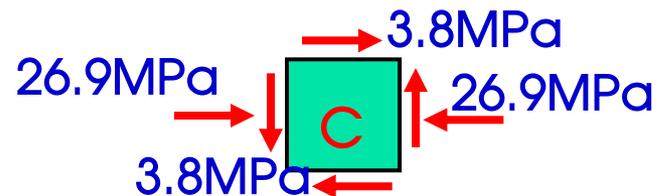
- At the cross section of $M_{\min} = 3,375 \text{ kN.m}$
 $\sigma'_{\max} = \frac{|M|}{W_2} = 25,3 \text{ MPa}; \quad \sigma'_{\min} = -\frac{|M|}{W_1} = -84,2 \text{ MPa}$

Example 2:

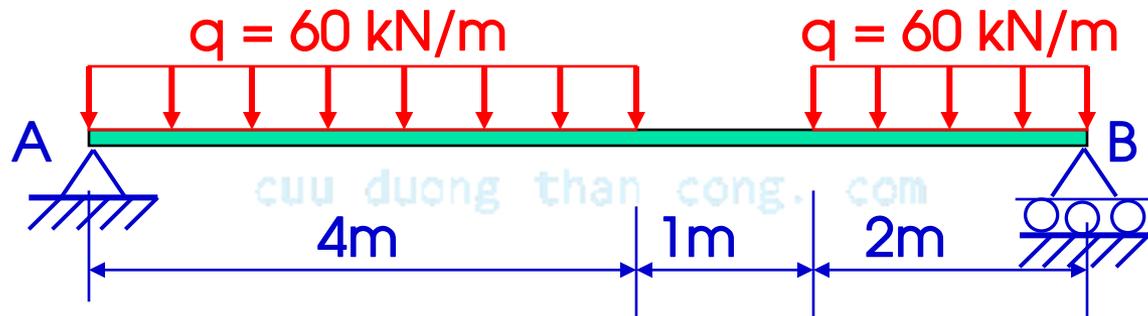


Calculate the normal and shear stresses at point C in the steel beam AB.

Response (ĐS):



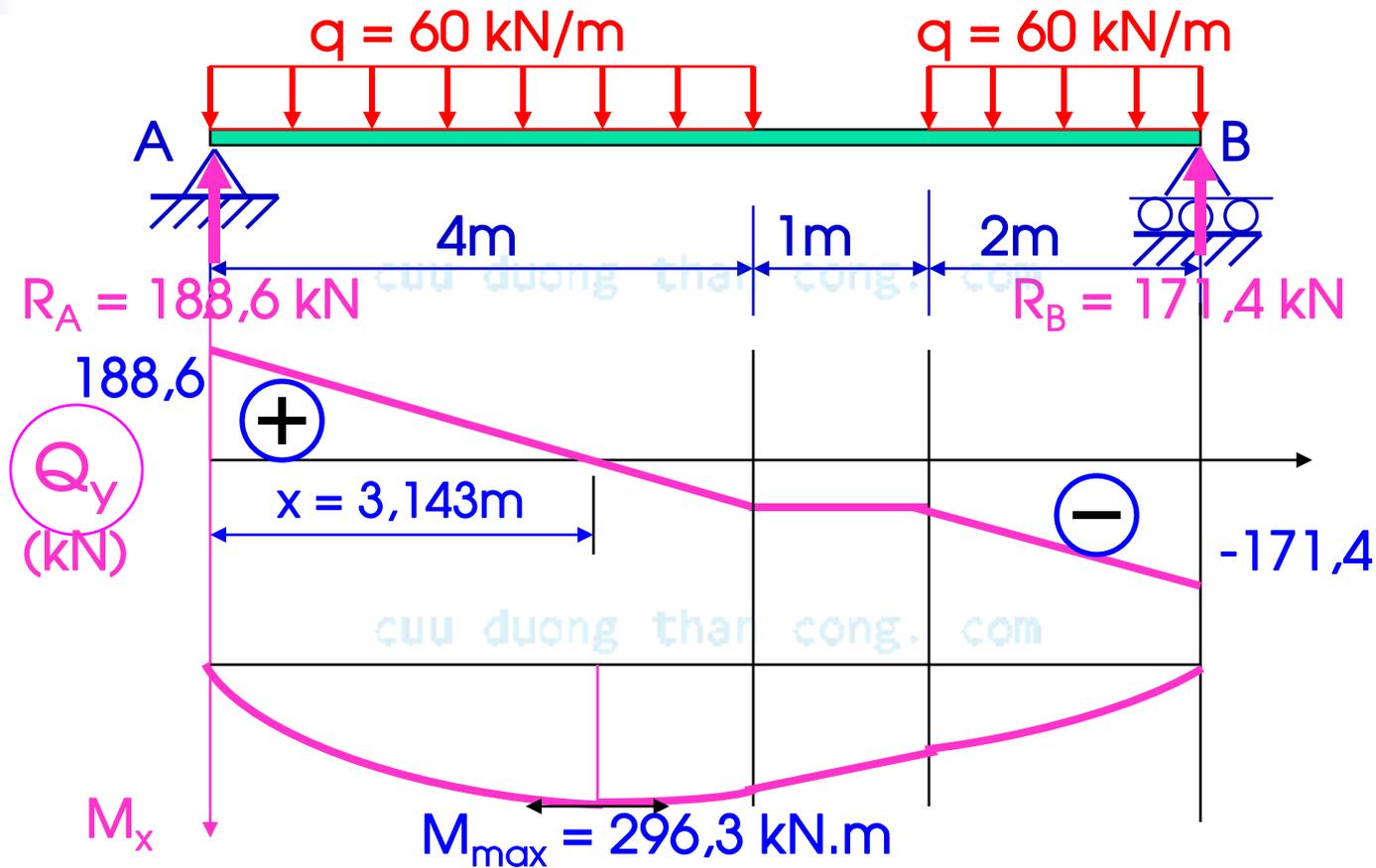
Example 3: Design of beams (Thiết kế dầm) – Choice of section (chọn m/c)



Determine the require section modulus, W_x , and then select the I-section in the table (Xác định suất tiết diện hay momen chống uốn yêu cầu và chọn tiết diện chữ I trong bảng). $[\sigma] = 110 \text{ MPa}$

Ex 3: Design of beams (Thiết kế dầm)

- Choice of section (chọn m/c) (tt)



Ex 3: Design of beams (Thiết kế dầm)

– Choice of section(chọn m/c) (tt)

- The max. moment → at $x = 3.143\text{m}$

$$M_{\max} = 296,3 \text{ kN.m}$$

- The required section modulus, W_x ,
(disregarding the weight of the beam):

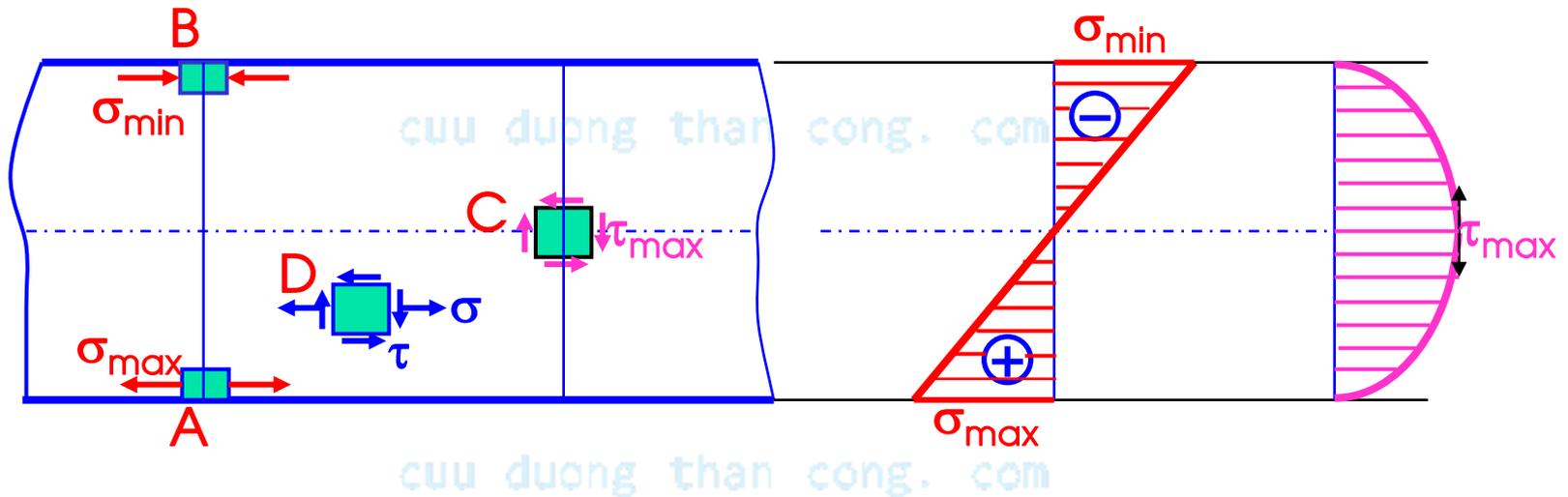
$$W_x^{\text{re}} = \frac{M_{\max}}{[\sigma]} = \frac{296,3 \times 10^6 \text{ (N.m m)}}{110 \text{ (MPa)}} = 2694 \times 10^3 \text{ m m}^3$$

- In the table, we can select a Universal Beam having a section modulus greater than 2694cm^3

→ UB 762 x 267 x 147 kg: $A = 187,8 \text{ cm}^2$, $b = 265,3 \text{ mm}$, $h = 753,9\text{mm}$, $I_{Gx} = 168535\text{cm}^4$, $W_x = 4471 \text{ cm}^4$

IV/VERIFICATION OF STRENGTH CONDITION (KIỂM TRA ĐIỀU KIỆN BỀN)

- Elements to be verified (Các loại phân tử cần kiểm tra)



- ✓ Elements at Uniaxial Stress State : A và B
- ✓ Elements at Pure Shear Stress State : C
- ✓ Elements at Plane Stress State : D

IV/ VERIFICATION OF STRENGTH (KIỂM TRA ĐIỀU KIỆN BỀN) (†)

- Kiểm tra phân tử ở TTƯS đơn:

- ✓ For ductile materials (Đ/v vật liệu dẻo)

$$\max |\sigma| \leq [\sigma]$$

- ✓ For brittle materials (Đ/v vật liệu giòn)

$$\sigma_{\max} \leq [\sigma_k]; \quad |\sigma_{\min}| \leq [\sigma_n]$$

IV/ VERIFICATION OF STRENGTH (KIỂM TRA ĐIỀU KIỆN BỀN) (†)

- Element of pure shear stress (Phân tố ở TTUS trượt thuần túy)
- ✓ For ductile materials (Đ/v vật liệu dẻo)

$$\tau_{\max} \leq [\tau] = \frac{[\sigma]}{2} \rightarrow \text{strength theory of maximum shear stress}$$

$$\tau_{\max} \leq [\tau] = \frac{[\sigma]}{\sqrt{3}} \rightarrow \text{strength theory of maximum strain energy}$$

IV/ VERIFICATION OF STRENGTH (KIỂM TRA ĐIỀU KIỆN BỀN) (tt)

- Element of plane stress state (Phân tố ở TTUS phẳng):
 - ✓ For ductile materials (Đ/v vật liệu dẻo)

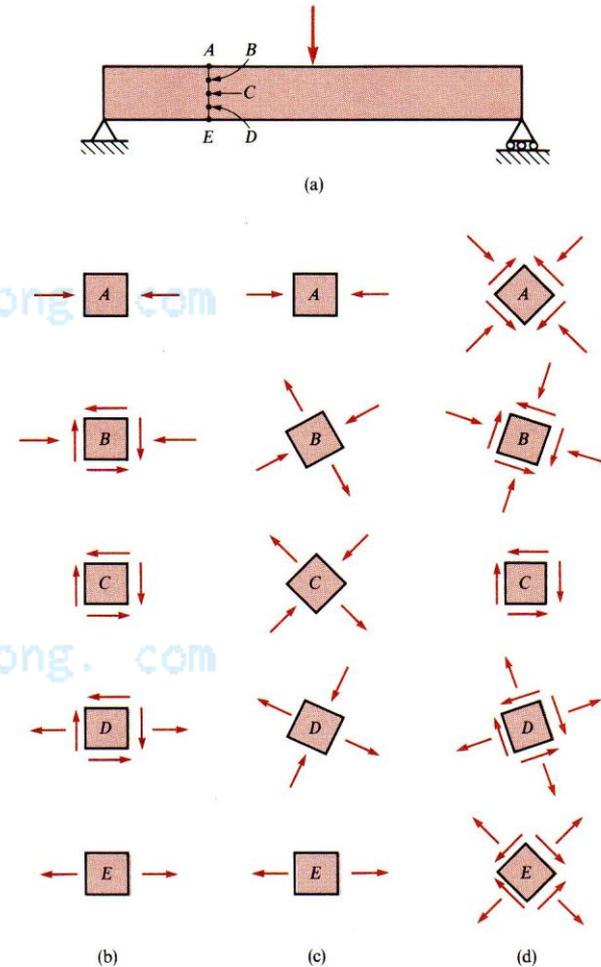
$$\sigma_{t3} = \sqrt{\sigma_z^2 + 4\tau_{zy}^2} \leq [\sigma] \rightarrow \text{Strength Theory 3}$$

$$\sigma_{t4} = \sqrt{\sigma_z^2 + 3\tau_{zy}^2} \leq [\sigma] \rightarrow \text{Strength Theory 4}$$

V/ PRINCIPAL STRESSES IN BEAM (ỨNG SUẤT CHÍNH TRONG DẦM)

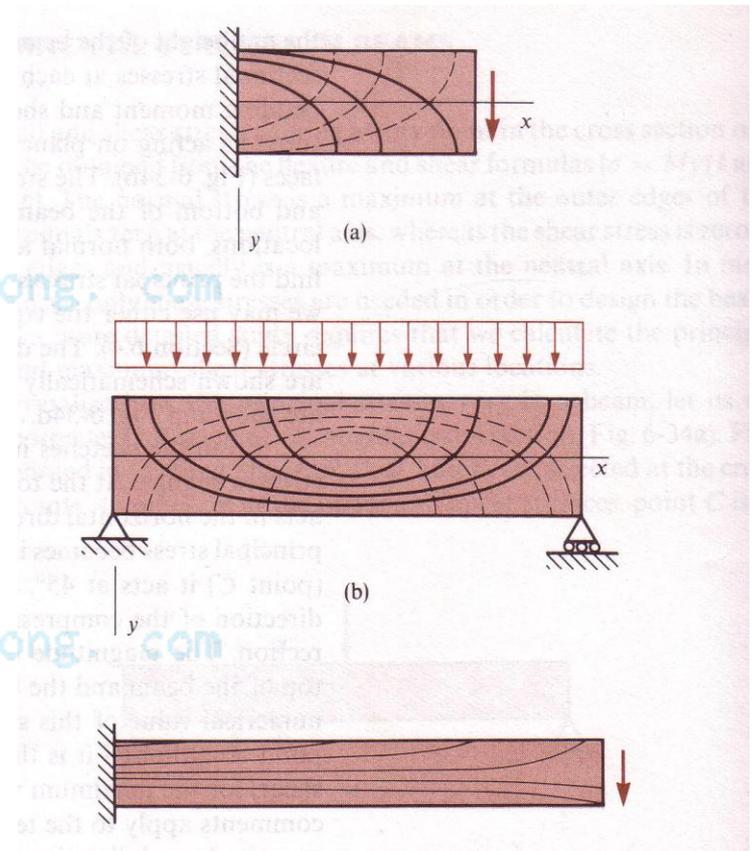
- Stresses in beam of rectangular cross section: (ứng suất trong dầm t/d chữ nhật):

(a) points A, B, C, D and E;
 (b) normal and shear stresses acting on horizontal and vertical planes;
 (c) principal stresses, and (d) maximum shear stresses

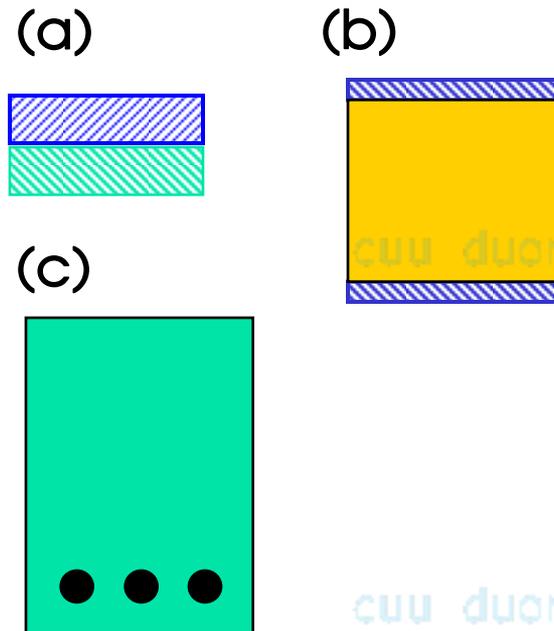


PRINCIPAL STRESS TRAJECTORIES (QUỸ ĐẠO ỨNG SUẤT CHÍNH)

- Fig. Principal Stress trajectories of beams of rectangular cross section: (a) cantiliver beam; (b) simple beam
- Fig. Typical stress contours (tensile principal stresses only) for a cantiliver beam



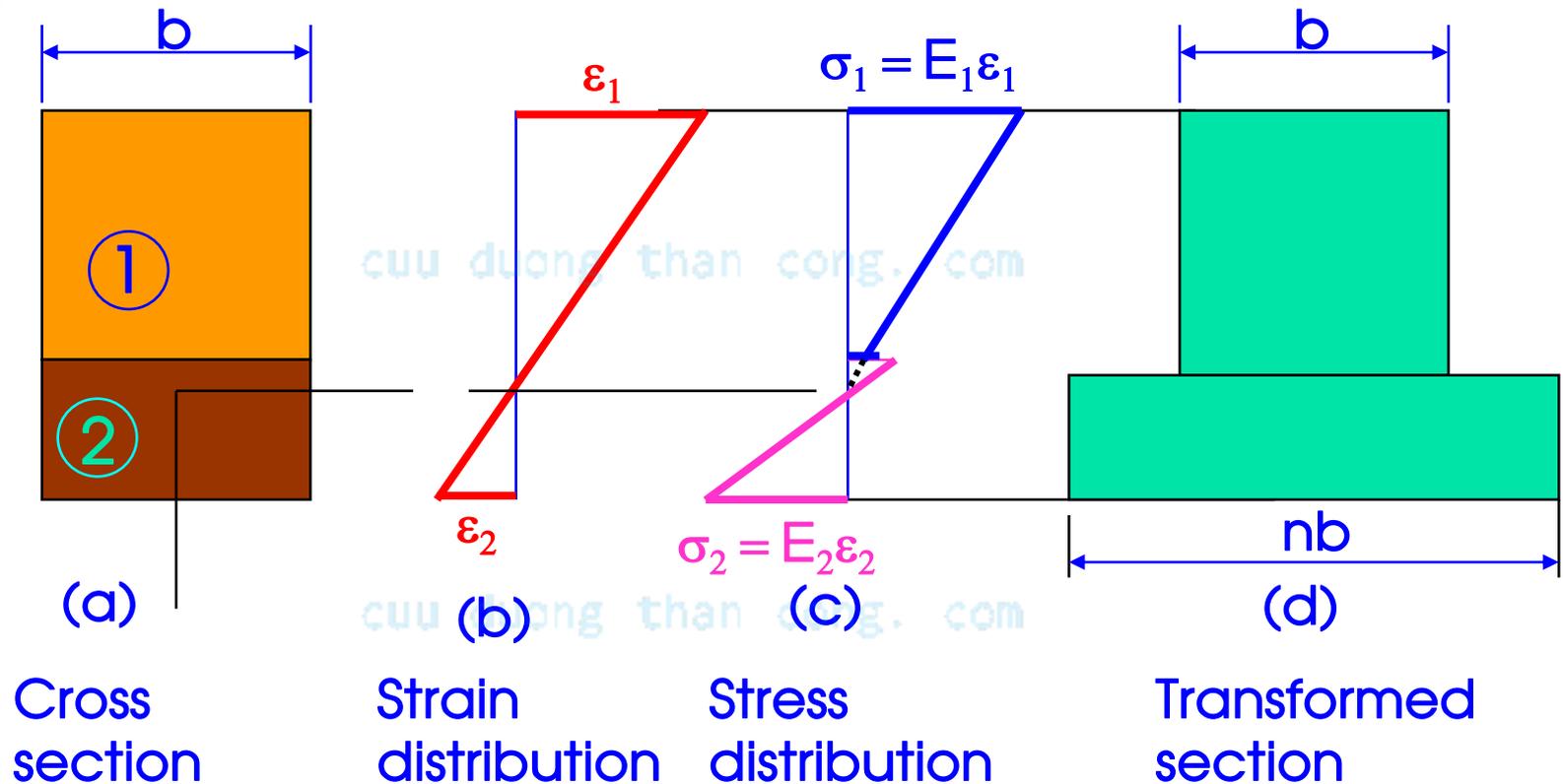
VI/ COMPOSITE BEAM* (DẦM HỖN HỢP) – DEFINITION & HYPOTHESES



- Composite beams are fabricated of more than 1 material
- The same hypotheses of plane cross-sections
→ ϵ_z vary linearly

Fig. Cross sections of composite beams: (a) bimetallic beam, (b) sandwich beam, and (c) reinforced concrete beam

VI/ COMPOSITE BEAM* (DẦM HỖN HỢP) – STRESS FORMULATION



Composite of two materials

VI/ COMPOSITE BEAM* (DẦM HỖN HỢP) – STRESS FORMULATION (cont.)

- Strain distribution for a beam made of two materials → Fig. (b) (Sự phân bố biến dạng cho dầm bằng 2 VL → H.b)
- The normal stresses σ_z are obtained from Hooke's law (Ú/s pháp σ_z có từ đ/l Húc) → Fig. (c) : $\sigma_{z1} = -E_1\kappa y$ $\sigma_{z2} = -E_2\kappa y$
- The position of the neutral axis (Vị trí trục TH):

$$N_z = 0 \rightarrow \int_1 \sigma_{z1} dA + \int_2 \sigma_{z2} dA = 0$$

$$\rightarrow E_1 \int_1 y dA + E_2 \int_2 y dA = 0$$

VI/ COMPOSITE BEAM* (DẦM HỖN HỢP) – STRESS FORMULATION (cont.)

- The relationships between M – stresses σ (Hệ thức giữa M – σ)

$$\begin{aligned} M &= \int \sigma_z y dA = \int_1 \sigma_{z1} y dA + \int_2 \sigma_{z2} y dA \\ &= -\kappa E_1 \int_1 y^2 dA - \kappa E_2 \int_2 y^2 dA \\ &= -\kappa (E_1 I_1 + E_2 I_2) \end{aligned}$$

I_1, I_2 – inertia moments about the neutral axis of cross sectional areas 1 and 2 (momen quán tính của các diện tích 1 và 2 đ/v trục trung hòa)

VI/ COMPOSITE BEAM* (DẦM HỖN HỢP) – STRESS FORMULATION (cont.)

- The curvature of the beam (Độ cong của dầm):

$$\kappa = \frac{1}{\rho} = - \frac{M}{E_1 I_1 + E_2 I_2}$$

- Normal Stresses (Ứng suất pháp):

$$\sigma_{z1} = \frac{M y E_1}{E_1 I_1 + E_2 I_2}; \quad \sigma_{z2} = \frac{M y E_2}{E_1 I_1 + E_2 I_2}$$

VII/ STRAIN ENERGY OF BENDING (THỂ NĂNG BIẾN DẠNG ĐÀN HỒI KHI UỐN)

■ Cách 1:

✓ Density of Strain Energy (Thế năng riêng)

$$u = \frac{U}{V} = \frac{1}{2E} \left[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \right]$$

✓ In case of plane stress as in beam bending (TƯỠI phẳng như dầm chịu uốn)

$$u = \frac{1}{2E} \left[\sigma_1^2 + \sigma_3^2 - 2\nu \sigma_1\sigma_3 \right] \text{ with } \sigma_{1,3} = \frac{\sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_z}{2} \right)^2 + \tau_{zy}^2}$$

VII/ STRAIN ENERGY OF BENDING (†) (THỂ NĂNG BIẾN DẠNG ĐÀN HỒI KHI UỐN)

- After reducing, one has (Rút gọn ta được)

$$u = \frac{\sigma_z^2}{2E} + \frac{\tau_{zy}^2}{2G}$$

- Replacing σ_z and τ_{zy} (Thay thế σ_z và τ_{zy})

$$u = \frac{M^2}{2EI^2} y^2 + \frac{Q^2 (S_x^c)^2}{2GI^2 (b^c)^2}$$

VII/ STRAIN ENERGY OF BENDING (†) (THẾ NĂNG BIẾN DẠNG ĐÀN HỒI KHI UỐN)

- Strain energy on a segment of length dz
(Thế năng biến dạng trên đoạn dz)

$$dU = \frac{M^2}{2EI} dz + \eta \frac{Q^2}{2GA} dz$$

- Strain energy on a beam of length L
(Thế năng biến dạng trên toàn dầm)

$$U = \int_0^L \frac{M^2}{2EI} dz + \int_0^L \eta \frac{Q^2}{2GA} dz$$

VII/ STRAIN ENERGY OF BENDING (†) (THỂ NĂNG BIẾN DẠNG ĐÀN HỒI KHI UỐN)

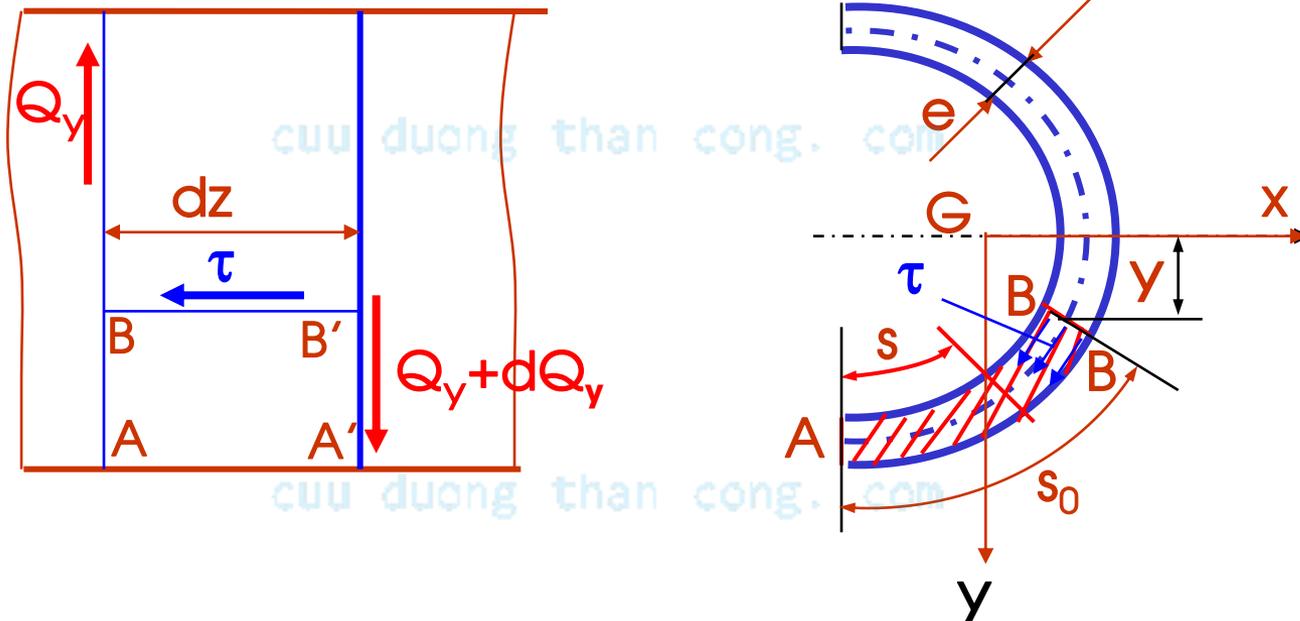
- With beams having varying rigidity or internal forces (M & Q) (Với dầm có độ cứng hay nội lực thay đổi):

$$U = \sum_{i=1}^n \int_0^{L_i} \frac{M_i^2}{2EI_i} dz + \sum_{i=1}^n \int_0^{L_i} \eta \frac{Q_i^2}{2GA_i} dz$$

- η – hệ số điều chỉnh sự phân bố không đều của ư/s tiếp:
 - ✓ Rectangular section $\rightarrow \eta = 1,2$
 - ✓ Circular section $\rightarrow \eta = 10/9$

VIII/ SHEAR STRESS IN BEAMS OF THIN-WALLED OPEN CROSS SECTION (Ư/s tiếp của dầm thành mỏng tiết diện hở)

- Introduction

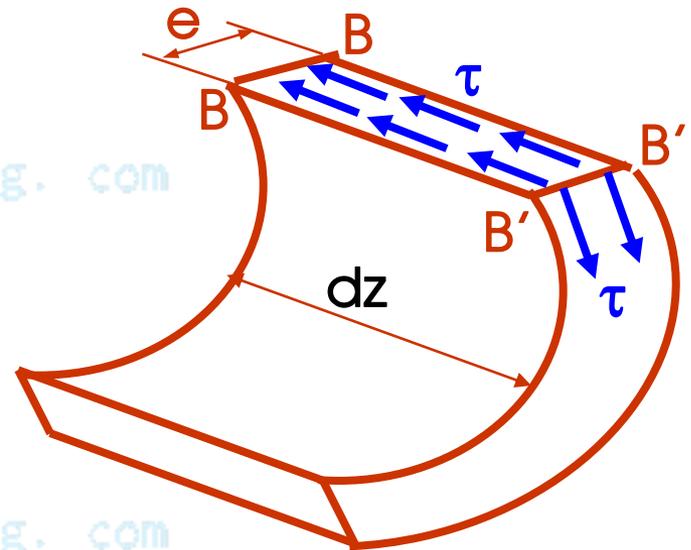


VIII/ SHEAR STRESS IN BEAMS OF THIN-WALLED OPEN CROSS SECTION (Ư/s tiếp của dầm thành mỏng tiết diện hở) (TT)

- ✓ Assumption:
 - τ is assumed to be uniformly distributed along e
- ✓ Formula

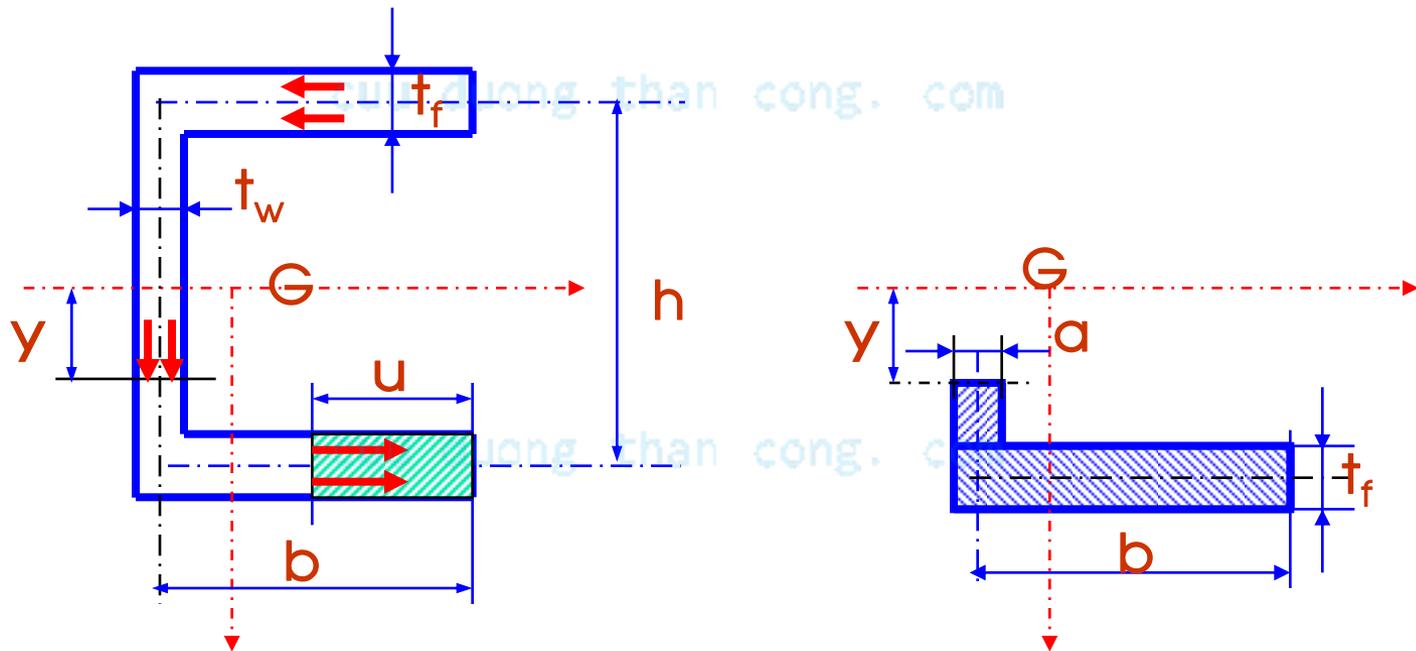
$$\tau = \frac{Q_y S_x^c}{I_x e}$$

with $S_x^c = \int_0^{s_0} y \cdot e \cdot ds$



VIII/ SHEAR STRESS IN BEAMS OF THIN-WALLED OPEN CROSS SECTION (U/s tiếp của dầm thành mỏng tiết diện hở) (TT)

■ U section beam



VIII/ SHEAR STRESS IN BEAMS OF THIN-WALLED OPEN CROSS SECTION (Ư/s tiếp của dầm thành mỏng tiết diện hở) (TT)

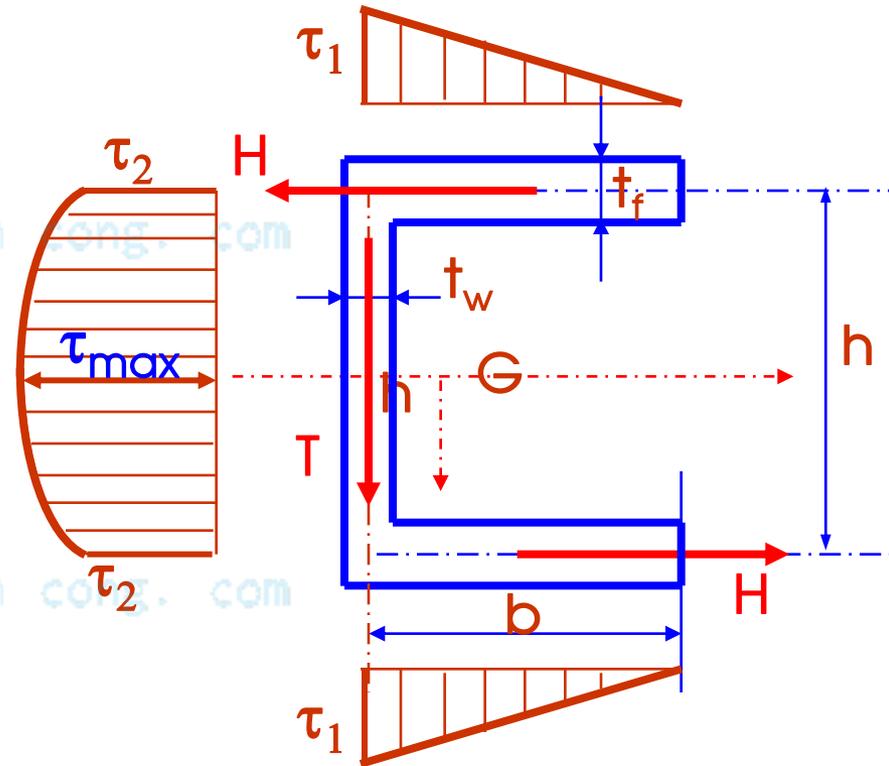
✓ Shear in the flange

$$\tau_1 = \frac{b h Q_y}{2 I_{Gx}}$$

✓ Shear in the web

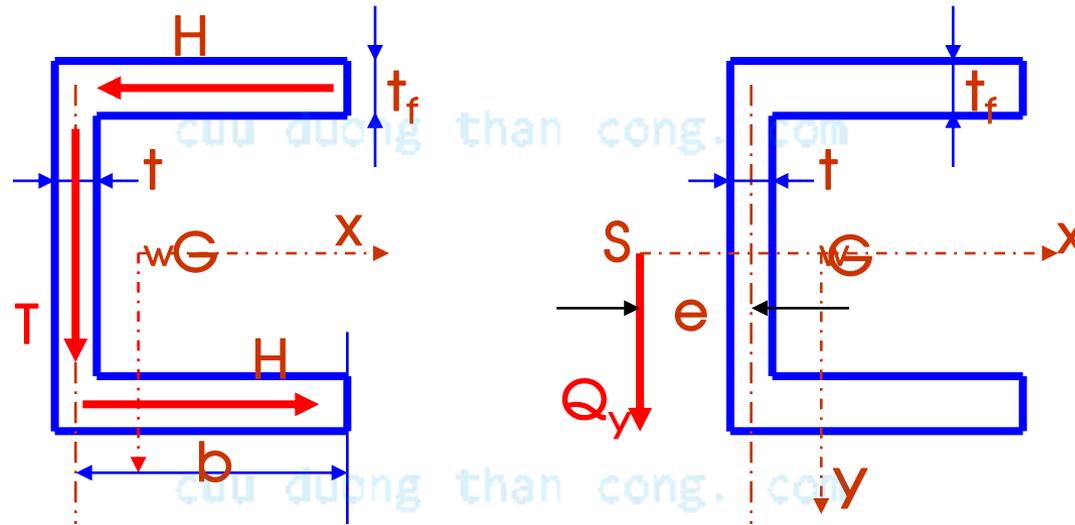
$$\tau_2 = \frac{b t_f h Q_y}{2 t_w I_{Gx}}$$

$$\tau_{max} = \left(\frac{b t_f}{t_w} + \frac{h}{4} \right) \frac{h Q_y}{2 I_{Gx}}$$



VIII/ SHEAR STRESS IN BEAMS OF THIN-WALLED OPEN CROSS SECTION (U/s tiếp của dầm thành mỏng tiết diện hở) (TT)

✓ Shear center (Tâm cắt)



$$Hh - Q_y e = 0 \rightarrow e = \frac{b^2 h^2 t_f}{4I_{Gx}}$$