Chapter 6: DEFLECTION OF BEAM
(Chương 6: CHUYỂN VỊ CỦA DẦM)

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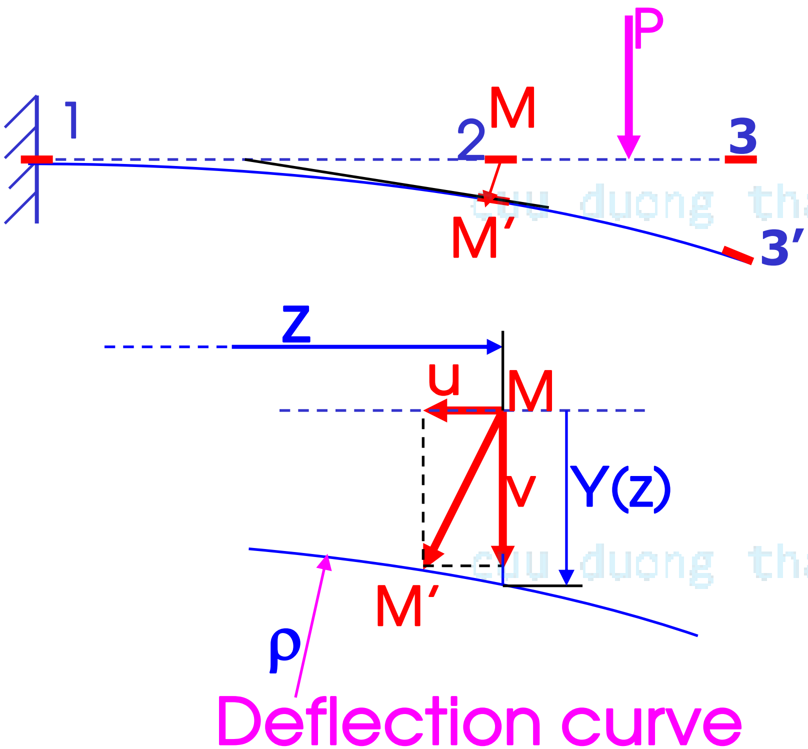
I. INTRODUCTION (GIỚI THIỆU)



$\overrightarrow{MM'}$: displacement vector

u: horizontal displace-ment

v: vertical displace-ment

ρ: curve radius of deflection curve

I. INTRODUCTION (GIỚI THIỆU) (tt)

- Why must we study the deflection of beams? (Tại sao chúng ta cần nghiên cứu chuyển vị của dầm?)

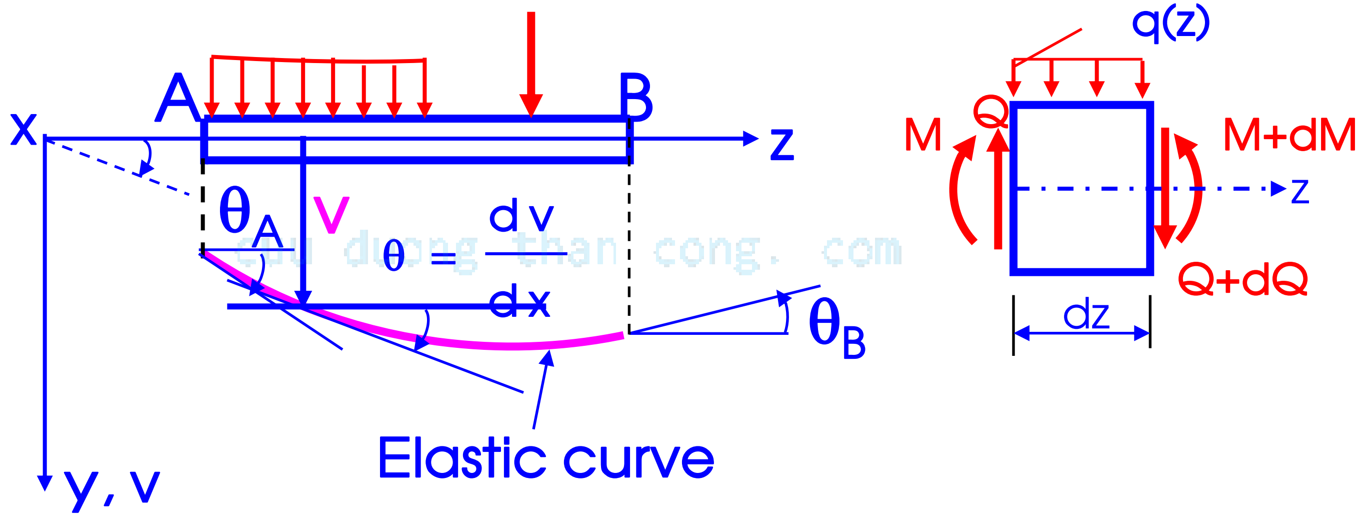
  ✓ The deflections help to verify the rigidity of structures (buildings, shafts……)

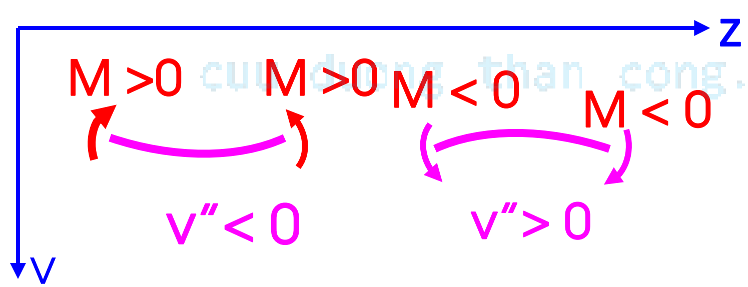
  ✓ The calculation of deflections is essential → the analysis of statically indeterminate beams

I. INTRODUCTION (GIỚI THIỆU) (tt)

- Basic assumptions (Giả thiết cơ bản):
  - ✓ Only deflections caused by Bending Moments are considered (B.d. do uốn)
  - ✓ The axis of bending is one of the Principal Axes of a beam cross-section
  - ✓ Plane sections remain plane during deformation (M/c ngang luôn phẳng)
  - ✓ Small deflections w/r the beam-span length (Chuyển vị bé)

II/ DIFFERENTIAL EQUATIONS OF THE DEFLECTION CURVE (P/T VI PHÂN CỦA ĐƯỜNG ĐÀN HỒI





II/ DIFFERENTIAL EQUATIONS OF THE DEFLECTION CURVE (P/T VI PHÂN CỦA ĐƯỜNG ĐÀN HỒI)(continued)

- From previous chapter , we have the moment-curvature relationship:

$$\frac{1}{\rho} = \kappa = \frac{M}{EI}$$

- In analytic geometry, the curvature of the deflection curve can be written:

$$\frac{1}{\rho} = \frac{|v''|}{\left[1 + (v')^2\right]^{3/2}}$$

II/ DIFFERENTIAL EQUATIONS OF THE DEFLECTION CURVE (P/T VI PHÂN CỦA ĐƯỜNG ĐÀN HỒI) (CONTINUED)

- With the sign convention of M and Q, it follows:

$$v'' = \frac{d^2v}{dx^2} = -\frac{M(x)}{EI} \rightarrow \qquad EIv'' = -M(x)$$

$$v''' = -\frac{1}{EI}\frac{dM}{dx} = -\frac{Q(x)}{EI} \rightarrow \qquad EIv''' = -Q(x)$$

$$v^{IV} = -\frac{1}{EI}\frac{dQ}{dx} = -\frac{q(x)}{EI} \rightarrow \qquad EIv^{IV} = -q(x)$$

since $\frac{d^2M}{dx^2} = \frac{dQ}{dx} = -q(x)$

III/ METHOD OF DIRECT INTEGRATION

$$v'' = \frac{d^2v}{dz^2} = -\frac{M(z)}{EI}$$

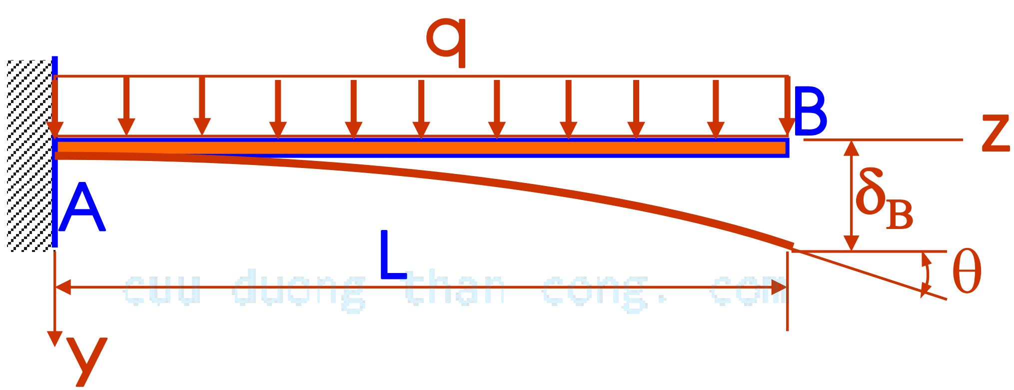
$$v' = \frac{dv}{dz} = \int\left(-\frac{M(z)}{EI}\right)dz + C$$

$$v = \int\left(\int\left(-\frac{M(z)}{EI}\right)dz\right)dz + Cz + D$$

C & D are determined by boundary conditions

III/ METHOD OF DIRECT INTEGRATION – Example 1: Cantiliver beam

- Example 1: Determine the equation of the deflection curve for a cantiliver beam with a uniform load



III/ METHOD OF DIRECT INTEGRATION – Example 1 (continued)

- Bending moment at distance z from A: (Momen uốn tại khỏang cách z từ A)

$$M_x = -\frac{q(L-z)^2}{2} \qquad \text{(a)}$$

- The differential equation of deflection curve (p/t vi phân đường đàn hồi)

$$EIv'' = \frac{q(L-z)^2}{2} \qquad \text{(b)}$$

III/ METHOD OF DIRECT INTEGRATION – Example 1 (continued)

- The consecutive integration of (b) gives:

$$EIv' = -\frac{q(L-z)^3}{6} + C_1 \qquad (c)$$

$$EIv = \frac{q(L-z)^4}{24} + C_1 z + C_2 \qquad (d)$$

- Boundary conditions: $v'(0) = v(0) = 0$

→

$$C_1 = qL^3/6 \; ; \qquad C_2 = 0$$

III/ METHOD OF DIRECT INTEGRATION – Example 1 (continued)

- The slope of the curve (Độ dốc của ĐĐH)

$$v' = \frac{qz}{6EI}\left(3L^2 - 3Lz + z^2\right)$$

- The deflection equation (P/t ĐĐH)

$$v = \frac{qz^2}{24EI}\left(6L^2 - 4Lz + z^2\right)$$

- The angle of rotation, θ_B and the deflect-ion, δ_B

$$\theta_B = v'(L) = \frac{qL^3}{6EI}$$

$$\delta_b = v(L) = \frac{qL^4}{8EI}$$

IV/ METHOD OF CONJUGATE BEAM (PHƯƠNG PHÁP DẦM GIẢ TẠO)

- Differential relations (Các liên hệ vi phân)

$$\frac{dM}{dz} = Q \qquad \frac{dQ}{dz} = q$$

and

$$\frac{dv}{dz} = \theta = v' \qquad \frac{d\theta}{dz} = v'' = -\frac{M}{EI}$$

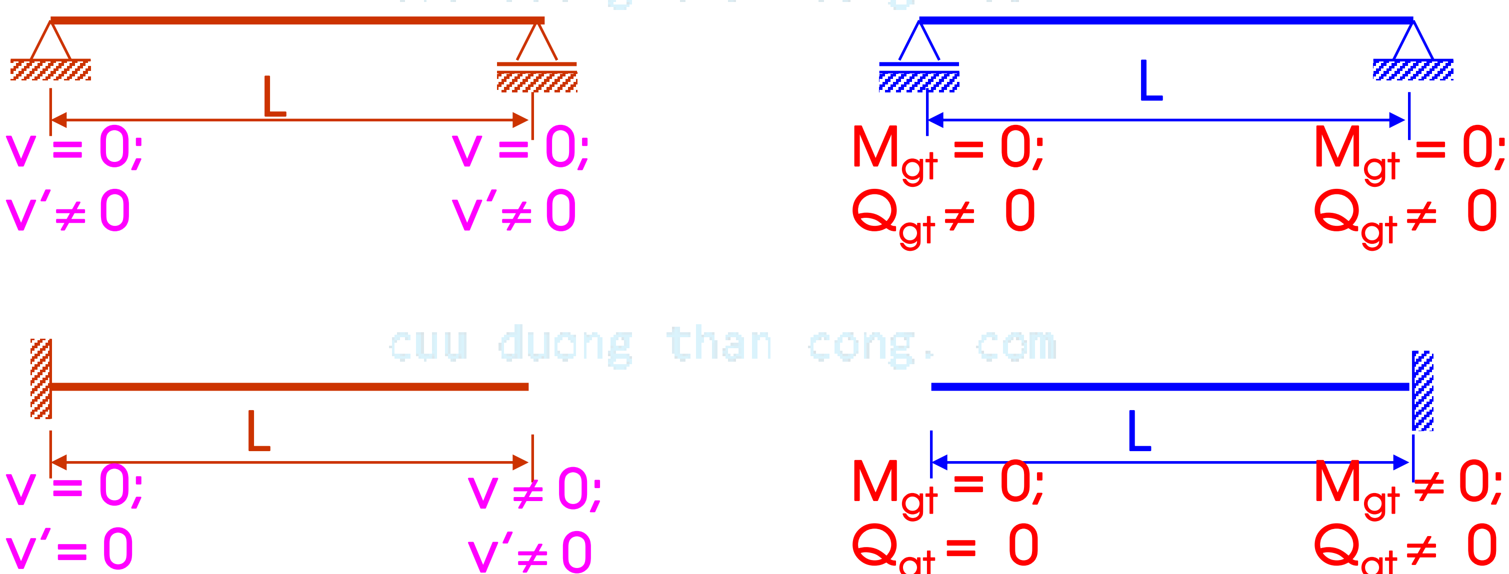
- If we imagine a fictitious beam subjected to:

$$q_{gt} = -\frac{M}{EI} \rightarrow \begin{cases} Q_{gt} = v'_{thực} \\ M_{gt} = v_{thực} \end{cases}$$

IV/ METHOD OF CONJUGATE BEAM (PHƯƠNG PHÁP DẦM GIẢ TẠO) (tt)

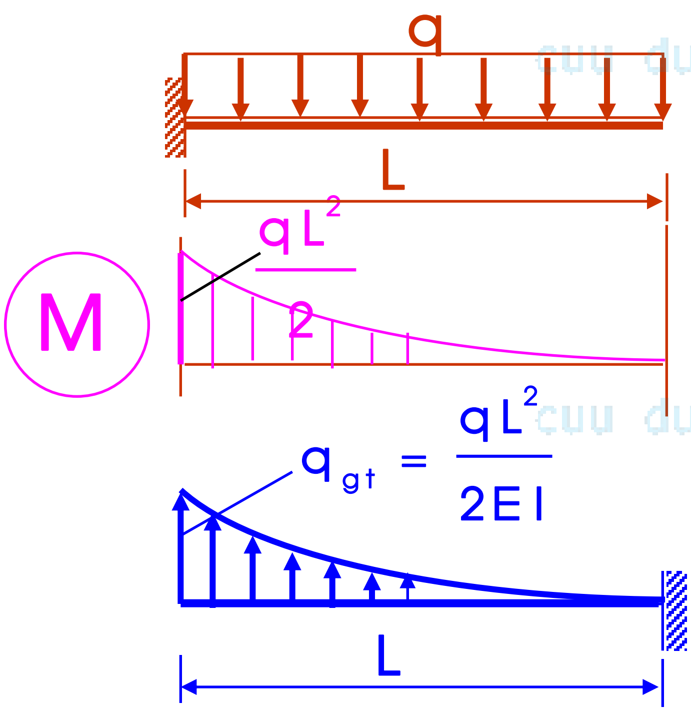
- Conjugate beams (Dầm giả tạo):

Dầm thực　　　　　　　　Dầm giả tạo



IV/ METHOD OF CONJUGATE BEAM (PHƯƠNG PHÁP DẦM GIẢ TẠO) (tt)

- Example : Cantiliver beam subjected to uniform load q. Find θ_B , v_B . EI = constant



Conjugate-beam

$$\theta_B = Q_{gt}^B = \frac{1}{3} \times \frac{qL^2}{2EI} \times L = \frac{qL^3}{6EI}$$

$$v_B = M_{gt}^B = \left(\frac{1}{3} \times \frac{qL^2}{2EI} \times L \right) \times \frac{3L}{4} = \frac{qL^4}{8EI}$$

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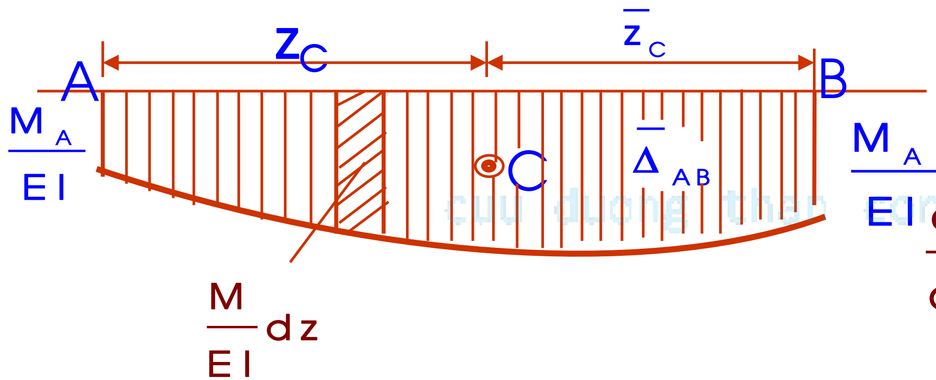
V/ MOMENT AREA METHOD
(PP DIỆN TÍCH MOMEN)

First moment-area theorem



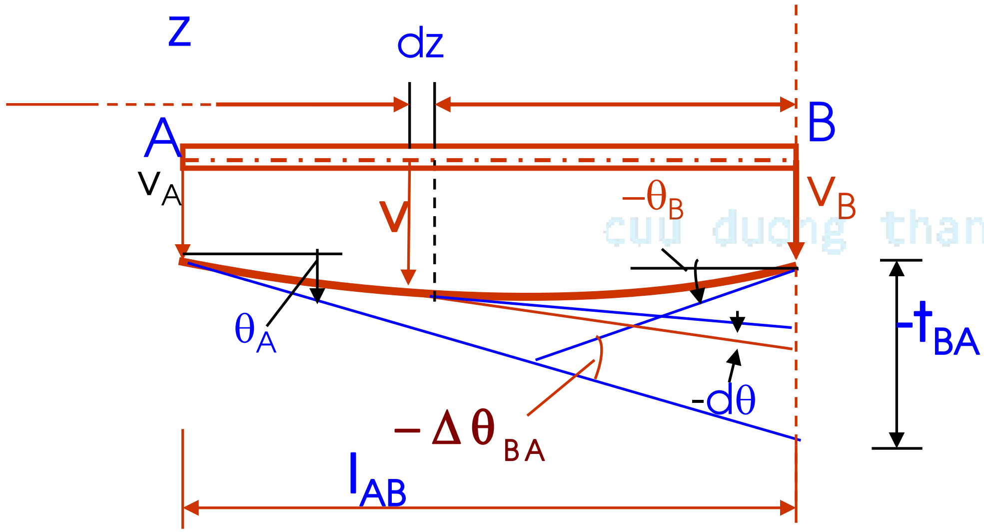
$$\theta \approx \tan\theta = \frac{dv}{dx} = v'$$

$$\frac{d\theta}{dx} = v'' = -\frac{M}{EI} \rightarrow d\theta = -\frac{M}{EI}dx$$

$$\Delta\theta_{BA} = \theta_B - \theta_A = \int_{x_A}^{x_B}\left(-\frac{M}{EI}\right)dx = -\bar{\Delta}_{AB}$$

"The angle θ_{ba} between the tangents to the deflection curve at 2 points A and B is equal to the negative of the area of the M/EI diagram between theses two points"

V/ MOMENT AREA METHOD
(PP DIỆN TÍCH MOMEN) (tt)



$$dt = \bar{x}\, d\theta = -\bar{x}\frac{M}{EI}dx$$

$$t_{BA} = -\int_{x_A}^{x_B} \bar{x}\frac{M}{EI}dx = -\bar{x}_C \bar{A}_{AB}$$

The offset t_{BA} of point B from the tangent at A is equal to the negative of the first moment of the area of M/EI diagram between A and B, taken with respect to B

V/ MOMENT AREA METHOD
(PP DIỆN TÍCH MOMEN) (tt)

- The deflection of B (Chuyển vị của B)

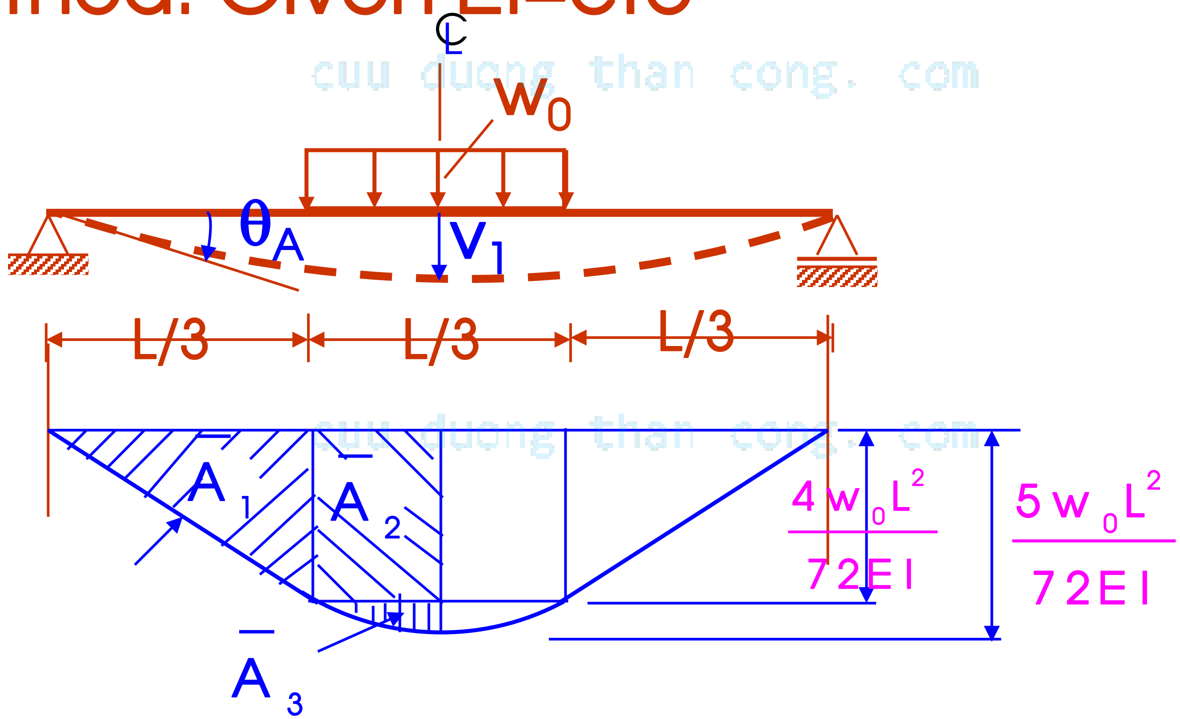
$$v_B = v_A + \theta_A l_{AB} + t_{BA} = v_A + \theta_A l_{AB} - \bar{x}_C \bar{A}_{AB}$$

- The deflection of A (Chuyển vị của A)

$$v_A = v_B - \theta_B l_{AB} + t_{BA} = v_A - \theta_B l_{AB} - \bar{x}_C \bar{A}_{AB}$$

APPLICATION (ÁP DỤNG)

Determine θ_A and v_1 by the moment-area method. Given EI=cte



APPLICATION (ÁP DỤNG) (tt)

- Using symmetry and Moment-Area Theorem I between point A (x=0) and point 1 (x=L/2):

$$\theta_1 - \theta_A = -\overline{A}_{A1} = -\overline{A}_1 - \overline{A}_2 - \overline{A}_3$$

in which $\theta_1 = 0$

$$\theta_A = \overline{A}_1 + \overline{A}_2 + \overline{A}_3$$

$$= \frac{w_0L^2}{18EI}\cdot\frac{L}{2x3} + \frac{w_0L^2}{18EI}\cdot\frac{L}{6} + \frac{w_0L^2}{72EI}\cdot\frac{2}{3}\cdot\frac{L}{6} = \frac{13w_0L^3}{648EI}$$

APPLICATION (ÁP DỤNG) (tt)

- Applying the Moment-Area Theorem II:

$$v_1 = v_A + \theta_A \frac{L}{2} - \bar{x}_C \bar{A}_{A1}$$

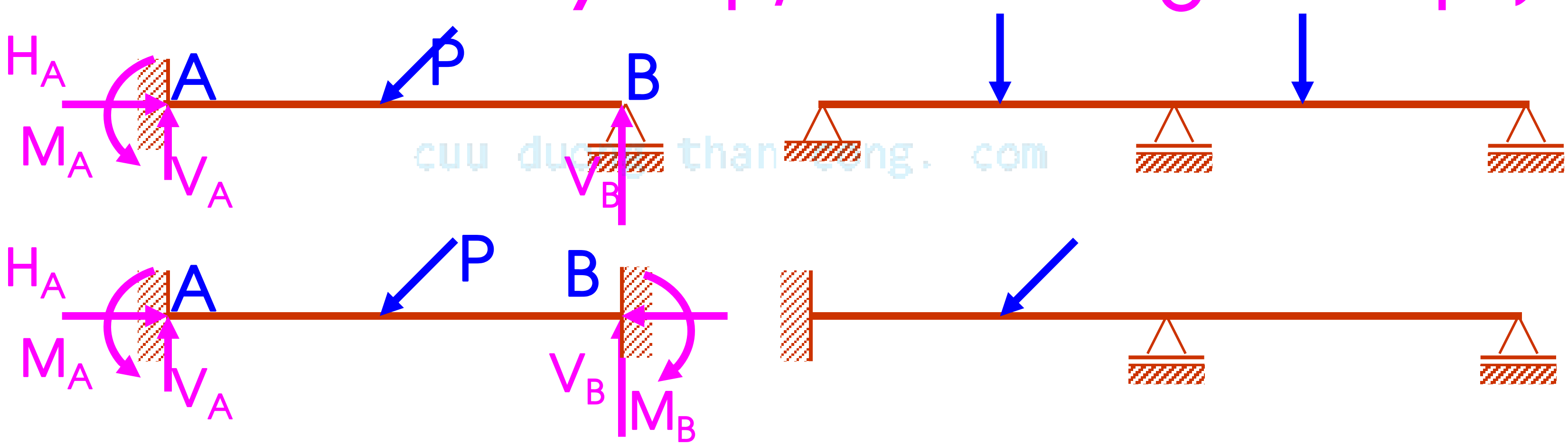
$$= 0 + \frac{13 w_0 L^4}{1296EI} - \bar{x}_C^{(1)} \bar{A}_1 - \bar{x}_C^{(2)} \bar{A}_2 - \bar{x}_C^{(3)} \bar{A}_3$$

$$= \frac{13 w_0 L^4}{1296EI} - \left(\frac{L}{6} + \frac{L}{9}\right)\frac{w_0 L^3}{108EI} - \frac{L}{12}\frac{w_0 L^3}{108EI} - \frac{3}{8}\frac{L}{6}\frac{w_0 L^3}{648EI}$$

$$= \frac{77 w_0 L^4}{11664EI}$$

VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH)

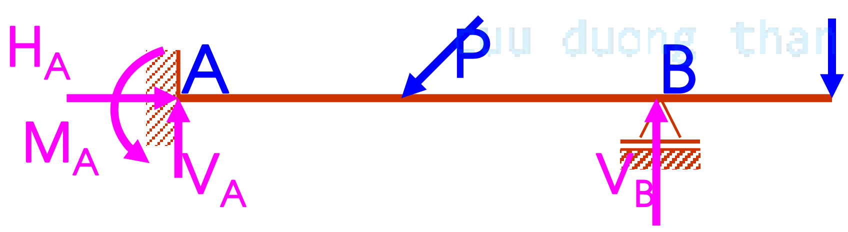
- Statically Indeterminate Beams: have a larger number of reactions than can be found from equilibrium equations (Dầm siêu tĩnh có nhiều hơn số phản lực có thể tìm thấy từ p/t cân bằng tĩnh học)



VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH) (continued)

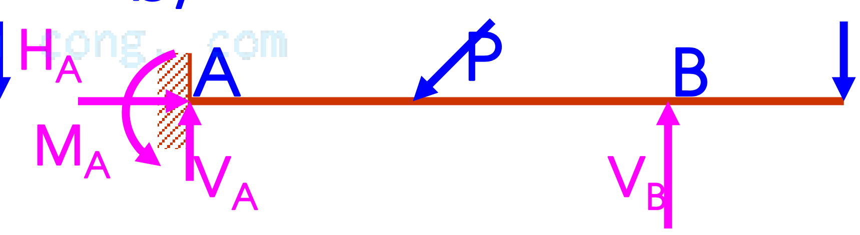
- Released structure or Primary Structure (Hệ cơ bản): can be deduced from the real structure by releasing all redundant supports and replacing their reactions
- Example:

a/



Real structure (K/c thật)

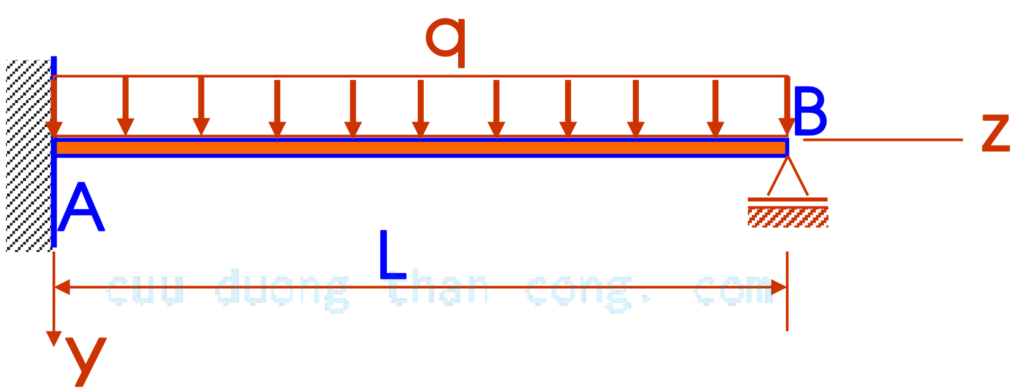
b/



Primary structure(Hệ CB)

VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH) (continued)

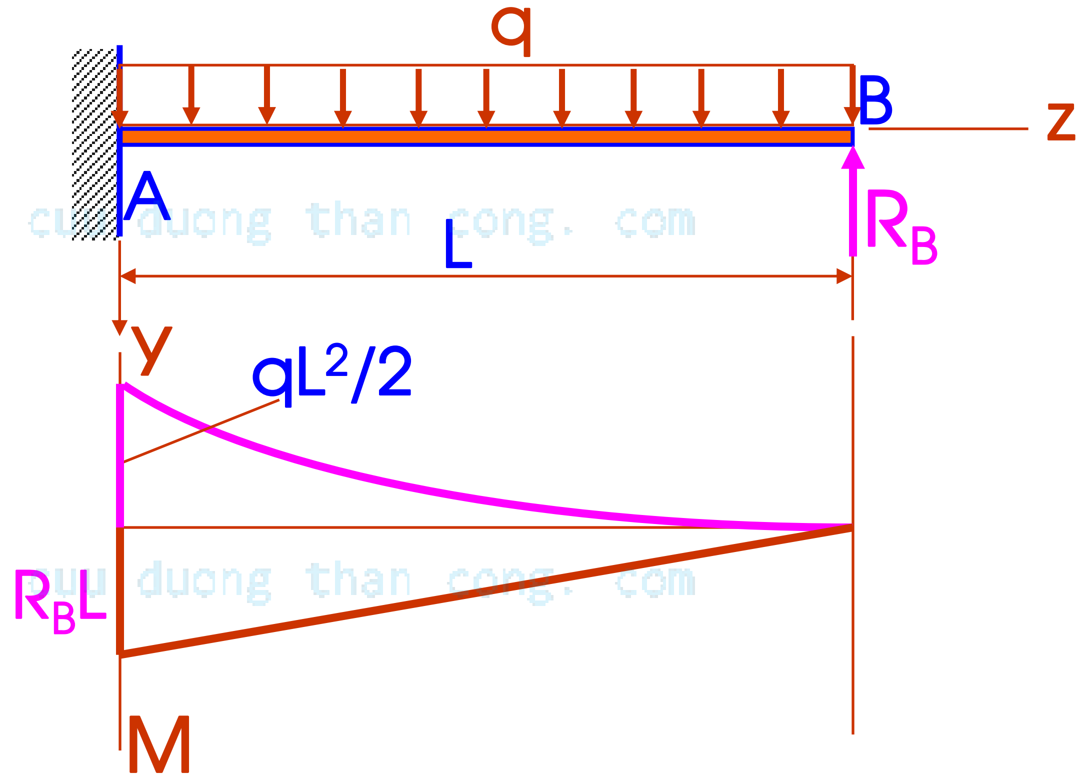
- Example 1: A propped cantiliver beam with a uniform load of intensity q. Determine the reactions R_A, R_B and M_A



VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH) (continued)

a/ Primary structure

b/ M diagram



VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH) (continued)

- Moment-Area Method:

  ✓ Considering the reaction R_B as the redundant and removed → the primary structure is a cantiliver beam, Fig.a (Xem R_B là ẩn lực thừa, HCB là dầm công xon)

  ✓ Bending moment diagram → Fig.b (biểu đồ momen uốn → Fig. b)

  ✓ The tangent to the elastic curve at A passes through point B → $\Delta_{ba} = 0$

VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH) (continued)

- ✓ From the theorem 2, the first moment of the area of M/EI between A & B, taken w/r to point B must be equal zero

$$\frac{L}{2}\left(\frac{R_B L}{EI}\right)\left(\frac{2L}{3}\right)-\frac{L}{3}\left(\frac{qL^2}{2}\right)\left(\frac{3L}{4}\right)=0$$

$$\rightarrow \quad R_B=\frac{3}{8}qL$$

- ✓ $\rightarrow \quad R_A=\frac{5qL}{8} \qquad M_A=\frac{qL^2}{8}$

VI/ STATICALLY INDETERMINATE BEAMS (DẦM SIÊU TĨNH) (continued)