CHAPTER 10
DYNAMIC LOADING

I/ INTRODUCTION

- In previous chapters, the loads are static or pseudo-static

- Static Loads:
  - ✓ are applied slowly, gradually increasing from zero to its maximum value
  - ✓ l'acceleration is absent or very small so that it can be neglected

- Dynamic Loads :
  - ✓ may be applied very suddenly, thus causing vibrations of the structures
  - ✓ may change in magnitude as time elapses

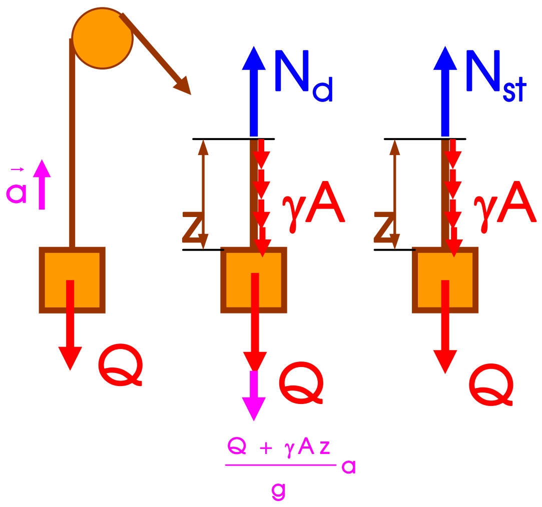
- Types of Dynamic Loads
  - ✓ impact loads : applied in short time (when two objects collide, a falling object strikes a structure,...)
  - ✓ cyclical loads : caused by rotating machinery

- Study Cases :

- ✓ Solids subjects to translational movement at a constant acceleration
- ✓ Solids subjects to rotational movement at a constant rotationary speed
- ✓ Vibration of solids
- ✓ Solids subjects to impact loads

II/ BODIES SUBJECT TO UNIFORMLY ACCELERATED MOTION

PROBLEM : a body of weight Q is pulled by a wire of section A, having a specific weight γ at an acceleration $\vec{a}$. Compute the force induced in the wire and then verify the strength condition.



Solution

✓ Dynamic Equilibrium Equation :

$$N_d - (Q + \gamma Az) = \left(\frac{Q + \gamma Az}{g}\right)a$$

$$\rightarrow N_d = (Q + \gamma Az)\left(1 + \frac{a}{g}\right) = N_{st}.k_d$$

with $N_{st} = Q + \gamma Az$ & $k_d = 1 + \frac{a}{g}$

✓ Dynamic Stress:

$$\sigma_d = \frac{N_d}{A} = \frac{N_{st}k_d}{A} = \sigma_{st}.k_d$$

with $\sigma_{st} = \frac{N_{st}}{A}$: static stress

✓ Strength Condition:

$$\max|\sigma_d| = \max|\sigma_{st}|.k_d \leq [\sigma]$$

■ REMARKS :

✓ When $a > 0 \rightarrow k_d > 1$

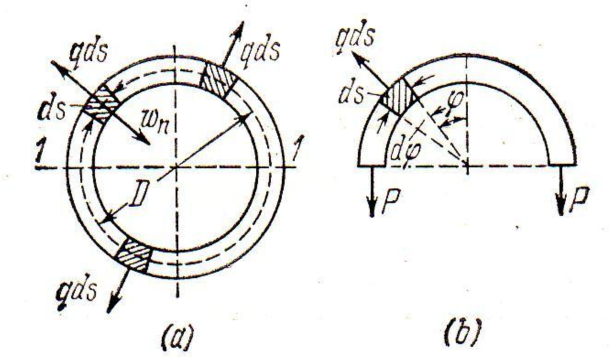
$\Rightarrow \sigma_d > \sigma_{st}$: dynamic action

✓ When $a < 0 \rightarrow k_d < 1$

$\Rightarrow \sigma_d > \sigma_{st}$: static action

III/ BODY SUBJECTS TO ROTATIONARY MOTION

PROBLEM : Determine stresses in a Rotating Ring (Flywheel Rim) at a high speed ω



A – Cross-sectional area of the ring

γ – specific weight of its material

n – number of revolutions/unit time

ω – angular velocity of rotation

D – mean diameter of the ring

■ Calculating inertia forces of an element of length ds :

✓ ω = constant → ang.accélé. $\alpha = \dfrac{d\omega}{dt} = 0$

→ Tangential accélération: $w_t = \alpha D/2 = 0$

→ Normal acceleration: $w_n = \omega^2 D/2$

✓ Intensity of the inertia force/unit length

$$q = \frac{\gamma A}{g} w_n = \frac{\gamma A D}{2g}\omega^2$$

■ Calculating the internal force and stress

✓ Isolate a circle half of the ring;

✓ Equilibrium of this circle half:

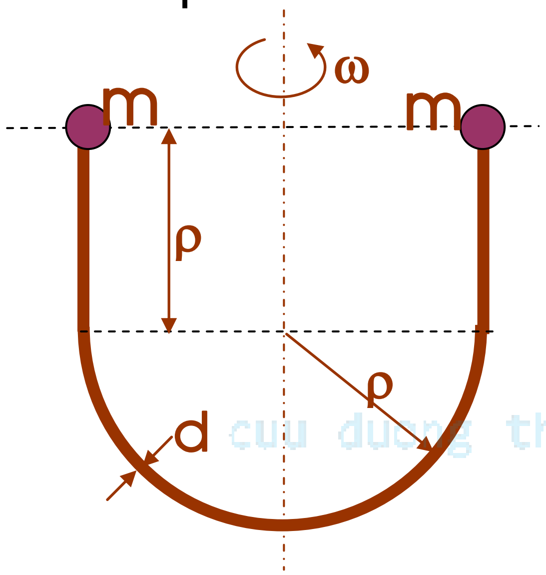
$$\sum F_y = 0 \rightarrow 2P.A - \int_{-\pi/2}^{\pi/2} q.ds.\sin\varphi = 0$$

$$\text{with } ds = \frac{D}{2}d\varphi \rightarrow P = \frac{Dq}{2} = \frac{\gamma A D^2\omega^2}{4g}$$

✓ Dynamic Stress :

$$\sigma_d = \frac{P}{A} = \frac{\gamma D^2 \omega^2}{4g} = \frac{\gamma R^2 \omega^2}{g}$$

■ Example :

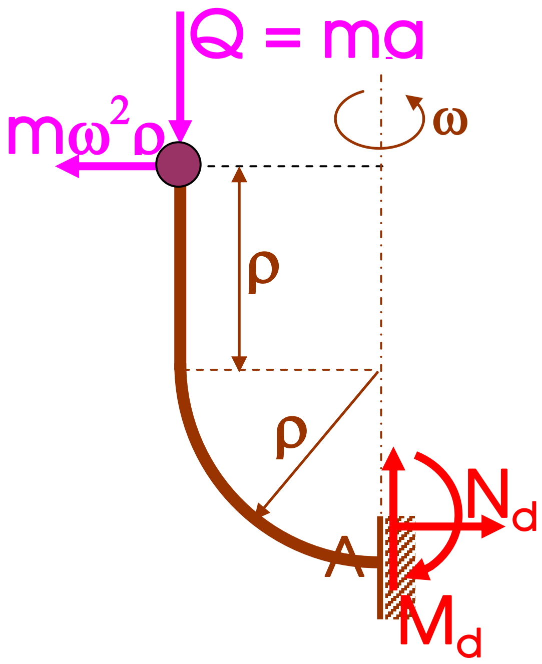


Data:
m = 1 kg
r = 0.1m
d = 0.01m
[σ] = 160 MPa

Calculate the allowable number of revolutions per unit time. Neglect the specific weight of the material and the change in distance between two masses.

SOLUTION

✓ Calculating Scheme



✓ Critical Section at A

N_d = mw2r;

M_d = mr(g+2w2r)

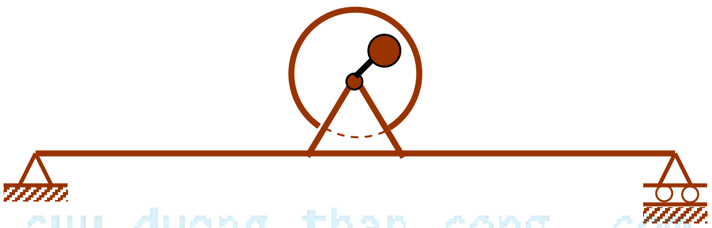
$$\max \sigma_d = \frac{m\omega^2\rho}{A} + \frac{m\rho(g + 2\omega^2\rho)}{W} \le [\sigma]$$

$$\rightarrow \omega = \frac{\pi n}{30} \le \sqrt{\frac{[\sigma] - \dfrac{mg\rho}{W}}{m\rho\left(\dfrac{1}{A} + \dfrac{2\rho}{W}\right)}} \rightarrow n \approx \frac{30}{\pi}\sqrt{750} = 270 \text{ r.p.m.}$$

IV/ VIBRATION OF ELASTIC SYSTEMS

■ In the two first problems, the accelerations do not change in direction

▪ If the beam is loaded by a motor having eccentricity in rotation, the force of inertia of the rotating load will give rise to stresses and deformations in the beam which periodically change their sign. The beam will begin to vibrate with a period which is equal to the period of rotation of the load. These vibrations are known as *forced vibration* (or *excited vibration)*



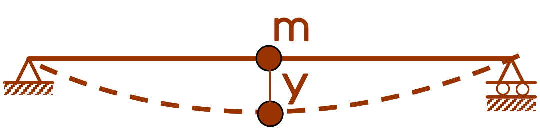
▪ Types of vibrations :

✓ Natural Vibration (or Free vibration)

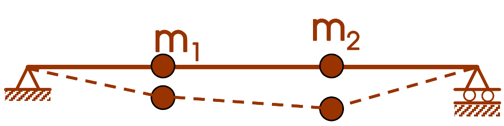
✓ Forced Vibration

▪ Degree of Freedom (d.o.f.) = number of indépendent parameters for determining the position of a system

Example: Neglecting the proper weight of the beam, the system (a) has 1 d.o.f. and the system (b) has 2 d.o.f.s



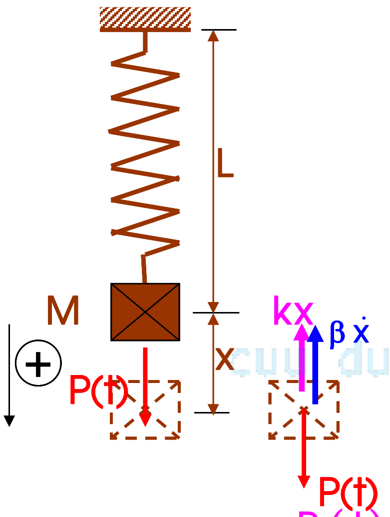
a/ one-d.o.f. system



b/ two-d.o.f. system

VIBRATION OF ONE-DEGREE-OF-FREEDOM SYSTEM

■ General Differential Equation of vibration (P/t vi phân dao động tổng quát)



✓ Exciting Force (Lực kích thích): P(t)

✓ Elastic Force (Lực đàn hồi) : F = - kx

✓ Damping Force of medium (Lực cản của môi trường) : $R = -\beta \dot{x}$

$$P(t) - kx - \beta \dot{x} = M\ddot{x}$$

$$\ddot{x} + 2\alpha \dot{x} + \omega^2 x = \frac{1}{M} P(t)$$

$$\text{with} \quad 2\alpha = \frac{\beta}{M}; \quad \omega^2 = \frac{k}{M} = \frac{kg}{Mg} = \frac{g}{x_t}$$

■ Free Oscillation without damping force (Dao động tự do không có lực cản)

✓ Motion Differential Equation: $\ddot{x} + \omega^2 x = 0$

✓ General Solution: $x = A\cos\omega t + B\sin\omega t$

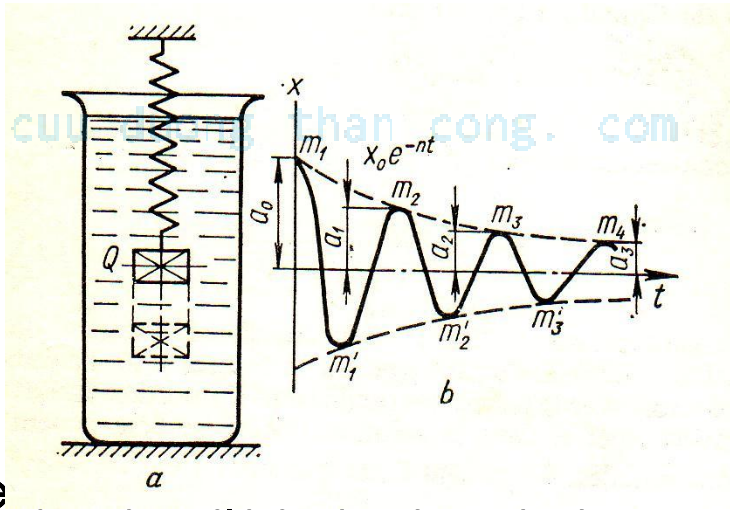
✓ Angular Frequency of Natural Vibration :

$$\omega = \sqrt{\frac{g}{x_t}}$$

✓ Period of Natural Vibration :

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{x_t/g}$$

■ Natural Oscillation of single degree-of-freedom systems in case of resisting force being proportional to speed



✓ Diffe

$$\ddot{x} + 2\alpha\dot{x} + \omega^2 x = 0$$

✓ Putting: $\omega_1^2 = \omega^2 - \alpha^2$

✓ General Solution:

$$x = e^{-\alpha t}(A\sin\omega_1 t + B\cos\omega_1 t)$$

✓ Period of vibration:

$$T = \frac{2\pi}{\omega_1} = \frac{2\pi}{\sqrt{\omega^2 - \alpha^2}}$$

■ Forced Oscillations under harmonic loads

✓ Differential Equation of Motion:

$$\ddot{x} + 2\alpha\dot{x} + \omega^2 x = \frac{1}{M} P_0 \sin\Omega t$$

✓ General Solution: $x(t) = x_1(t) + x_2(t)$

with $x_1(t) = e^{-\alpha t}(A \sin\omega_1 t + B\cos\omega_1 t)$

$$x_2(t) = C_1 \sin\Omega t + C_2 \cos\Omega t$$

t □ , $x_1 \to 0$, $x(t) \to C_1 \sin\Omega t + C_2 \cos\Omega t$

$$C_1 = \frac{P_0}{M}\left[\frac{\omega^2 - \Omega^2}{\left(\omega^2 - \Omega^2\right)^2 + 4\alpha^2\Omega^2}\right]$$

$$C_2 = -\frac{P_0}{M}\left[\frac{2\alpha\Omega}{\left(\omega^2 - \Omega^2\right)^2 + 4\alpha^2\Omega^2}\right]$$

Putting :
$$\delta = \frac{1}{M\omega^2}$$

$$x(t) = A_1 \sin(\Omega t - \psi)$$
$$= \frac{P_0\delta}{\sqrt{\left(1 - \frac{\Omega^2}{\omega^2}\right)^2 + \frac{4\alpha^2\Omega^2}{\omega^4}}} \sin(\Omega t - \psi)$$

✓ Amplitude of excited oscillations:

$$A_1 = \frac{P_0\delta}{\sqrt{\left(1 - \frac{\Omega^2}{\omega^2}\right)^2 + \frac{4\alpha^2\Omega^2}{\omega^4}}}$$

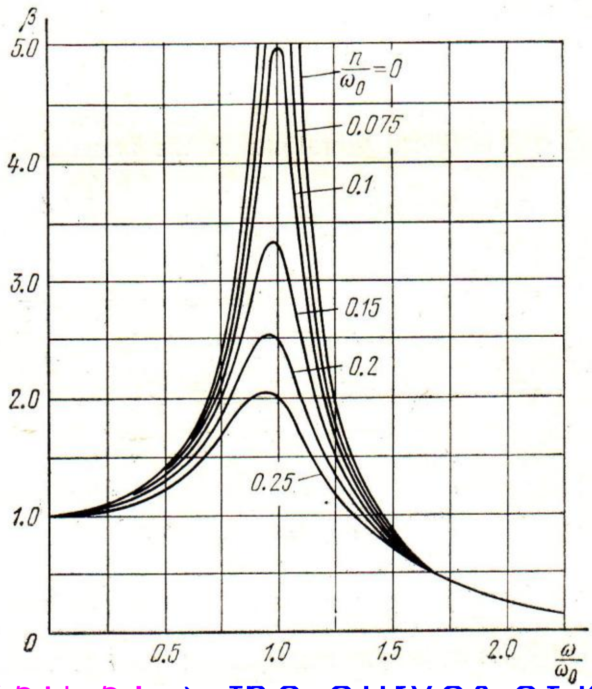
✓ Static Displacement due to P_0:

$$\Delta_{st} = P_0\delta$$

✓ Dynamic Coefficient:

$$k_d = \frac{1}{\sqrt{\left(1 - \frac{\Omega^2}{\omega^2}\right)^2 + \frac{4\alpha^2\Omega^2}{\omega^4}}}$$

→ $A_1 = k_d.\Delta_{st}$



If $\frac{\Omega}{\omega} \notin [0.75;1.5] \rightarrow$ the curves of k_d coincide

$$\alpha \rightarrow 0 \Rightarrow k_d = \frac{1}{\left|1 - \left(\frac{\Omega}{\omega}\right)^2\right|}$$

✓ Discussion :

- When $\frac{\Omega}{\omega} \approx 1 \Leftrightarrow \Omega \approx \omega$ and $\alpha = 0$

$k_d \rightarrow \infty$: resonance (sự cộng hưởng)

- Resonance domain (miền cộng hưởng)

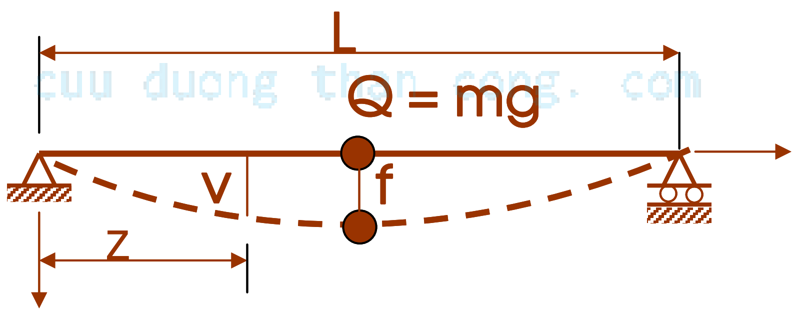
$$0.75 \leq \frac{\Omega}{\omega} \leq 1.25$$

■ The Effect of Mass of the Elastic System on vibrations

✓ Principle : «Reducing the mass of the system to one mass m′ superposed at the position of m, and studying the oscillation of the single d.o.f. system having the mass (m+m′)».

✓ Base of the mass reduction: «Two systems having the same kinetic energies will be equivalent in frequency»

■ Example : A simply supported beam, having a mass m at the middle of its span. Study the natural vibration of the mass m by taking into account the mass of the beam, which is assumed to be small compared to m.



SOLUTION

✓ Vertical Displacement of a section at z :

$$v = f\frac{3L^2z - 4z^3}{L^3}$$

where f – displacement at m position

✓ Rate at this section: $\frac{dv}{dt} = \frac{3L^2z - 4z^3}{L^3} \cdot \frac{df}{dt}$

✓ Kinetic Energie of the mass dm at this section:

$$dT = \frac{1}{2}\frac{\gamma A dz}{g}\left(\frac{3L^2z - 4z^3}{L^3}\right)^2\left(\frac{df}{dt}\right)^2$$

✓ Kinetic Energie of the whole beam:

$$T = 2\int_0^{L/2}\frac{1}{2}\frac{\gamma A}{g}\left(\frac{3L^2z - 4z^3}{L^3}\right)^2\left(\frac{df}{dt}\right)^2 dz$$

$$= \frac{1}{2} \cdot \frac{17}{35} \cdot \frac{\gamma A L}{g} \cdot \left(\frac{df}{dt}\right)^2$$

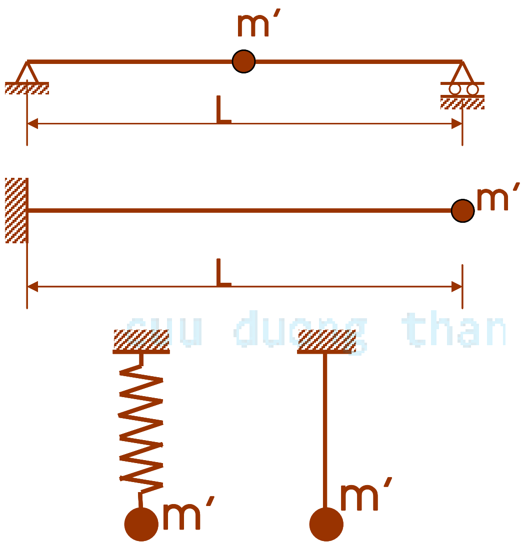
✓ Kinetic Energie of the equivalent mass m':

$$T = \frac{1}{2}.m'\left(\frac{df}{dt}\right)^2 \quad \text{avec} \quad m' = \frac{17}{35}\frac{\gamma A}{g}$$

■ Reduced Mass of the beam is :

$$m' = \mu \frac{\gamma A}{L}$$

where μ - the coefficient of mass réduction



$$\mu = \frac{17}{35}$$

$$\mu = \frac{33}{140}$$

$$\mu = \frac{1}{3}$$

<u>Home work</u> : A motor having a weight of Q = 12000 N, is put at the middle of the two-beams system of I profil $N^0$24a having 3 meter length. Calculate the natural frequency of the beam in two cases : (a) neglecting the beam weight ; (b) taking into account the beams weight.

CALCULATING STRUCTURES SUBJECT TO IMPACT LOADS (tải trọng va chạm)

■ INTRODUCTION:

✓ Impact Load occurs when the rate of considered element or other bodies connected to it, changes suddenly.

✓ Examples :

- Un heavy weight falling down on the head of a pile (vertical hit)
- a car strikes a barrier (horizontal hit)

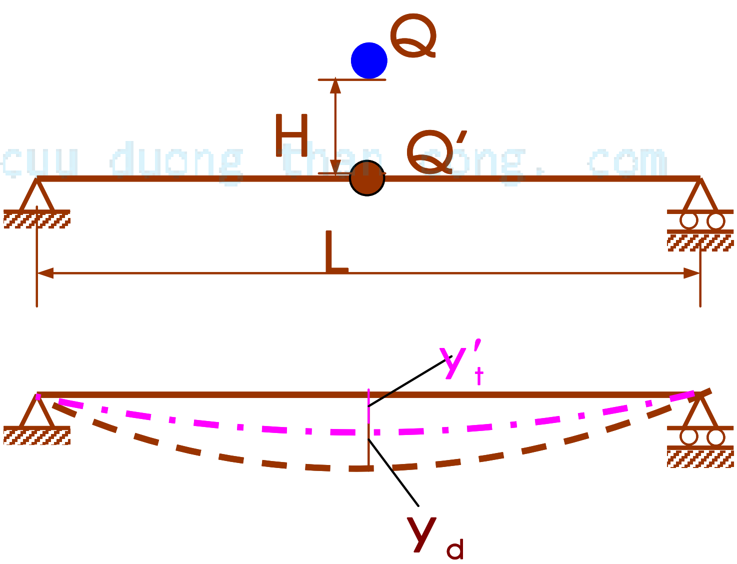
■ HYPOTHESES

✓ Energy conservation Law: The kinetic energy of the colliding object will be transferred wholly into the elastic potential energy of the collided object, assuming no loss of energy

✓ The distribution of stresses and strains is the same as in static loading

✓ The effect of impact on strains or stresses is evaluated through the impact factor (hệ số động) or dynamic coefficient

■ Vertical hit:



✓ Problem:

A weight Q of mass m falls on another weight Q′ of mass m′ existing on a beam from a height H. Calculate the maximum displacement

✓ Solution :

• Energy absorbed during the impact process

✈Before impact, Q/g has a rate of v_0.

✈After impact, (Q + Q′)/g being sticked together → v

✈Theorem on quantum conservation :
(Định lý bảo toàn động lượng)

$$\frac{Q}{g}v_0 = \frac{Q+Q'}{g}v \quad \rightarrow \quad v = \frac{Q}{Q+Q'}v_0$$

✈ Kinetic Energy (Động năng)

$$K = \frac{1}{2}.\frac{Q+Q'}{g}\left(\frac{Qv_0}{Q+Q'}\right)^2 = \frac{1}{2}\frac{Q^2}{g(Q+Q')}v_0^2$$

✈ Potential Energy (Thế năng):

$$T = (Q + Q')\,y_d$$

✈ Total Energy during the impact:

$$A = K + T = \frac{1}{2}\frac{Q^2}{g(Q+Q')}v_0^2 + (Q + Q')\,y_d$$

• The Total Absorbed Energy will transform in Potential Energie of Deformation :

✈ Before impact, Q' moves y_t'

$$\rightarrow \qquad U_1 = \frac{1}{2}Q'y_t' = \frac{1}{2}\frac{(y_t')^2}{\delta}$$

✈ After impact, $(Q + Q')$ move $y_đ$

$$U_2 = \frac{1}{2}\frac{(y_t' + y_d)^2}{\delta}$$

✈ Potential Energy accumulated during the impact process :

$$U = U_2 - U_1 = \frac{1}{2}\left\{\frac{(y_d + y_t')^2}{\delta} - \frac{(y_t')^2}{\delta}\right\}$$