Chương 4: KÉO – NÉN ĐÚNG TÂM

4.1. NGUYÊN LÝ SAINT-VENANT

4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

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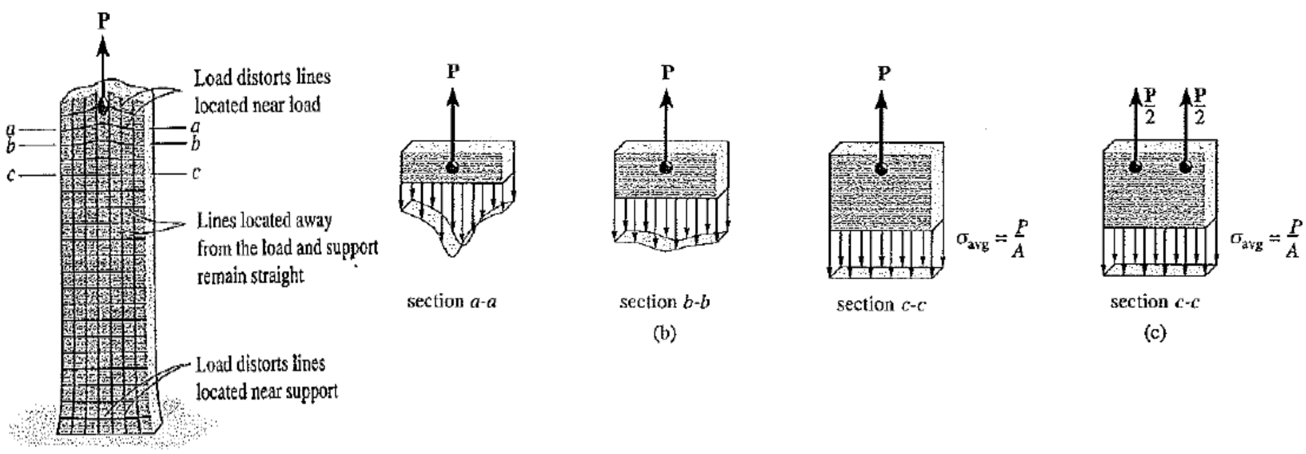
4.5. ỨNG SUẤT NHIỆT

4.1. NGUYÊN LÝ SAINT-VENANT

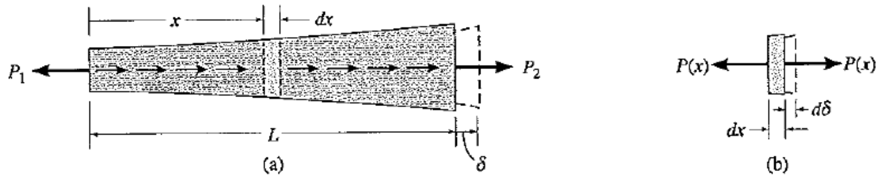
Bài toán của lý thuyết đàn hồi khi giải thường gặp khó khăn khi phải thỏa mãn hoàn toàn điều kiện biên, đặc biệt các bài toán về thanh, tấm, vỏ.

Nguyên lý Saint-Venant còn được gọi là nguyên lý về hiệu ứng cân bằng cục bộ của ngoại lực. Nguyên lý này được phát biểu như sau: "Nếu trên một phần nhỏ của vật thể có tác dụng của một hệ lực cân bằng thì ứng suất phát sinh sẽ tắt dần nhanh ở những đểm xa miền đặt lực" hoặc "Tại những điểm trên hệ vật cách xa điểm đặt lực thì trạng thái ứng suất, biến dạng của vật phụ thuộc rất ít vào cách tác dụng của lực".



4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Xét thanh chịu tải dọc trục như hình dưới:



Xét một đoạn nhỏ dx trên thanh: $\sigma = \dfrac{P(x)}{A(x)}$ and $\epsilon = \dfrac{d\delta}{dx}$

Áp dụng định luật Hooke: $\sigma = E\epsilon \quad \dfrac{P(x)}{A(x)} = E\left(\dfrac{d\delta}{dx}\right) \quad d\delta = \dfrac{P(x)\,dx}{A(x)\,E}$

Như vậy,

$$\delta = \int_0^L \frac{P(x)\,dx}{A(x)\,E}$$

δ = displacement of one point on the bar relative to another point
L = distance between the points
$P(x)$ = internal axial force at the section, located a distance x from one end
$A(x)$ = cross-sectional area of the bar, expressed as a function of x
E = modulus of elasticity for the material

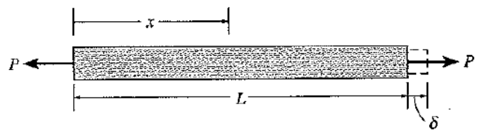
4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Giả sử rằng, thanh có tiết diện ngang không đổi, vật liệu thanh đồng nhất, ngoại lực F tác dụng dọc thanh không đổi thì nội lực dọc trong thanh là hằng số:

$$\delta = \frac{PL}{AE}$$

Trong trường hợp có nhiều lực độc lập tác dụng lên thanh, mặt cắt ngang thay đổi đột nhột, vật liệu thay đổi tại từng đoạn trên thanh thì sự chuyển vị tại đầu cuối của thanh sẽ bằng tổng các sự chuyển vị độc lập.

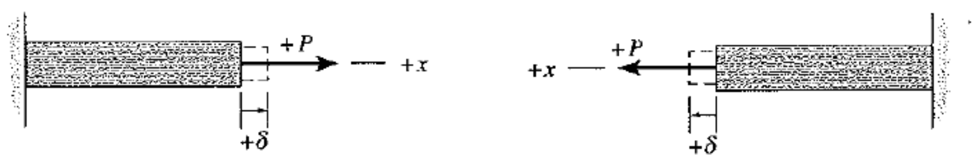
$$\delta = \sum \frac{PL}{AE}$$



Quy ước cho lực & chuyển vị:

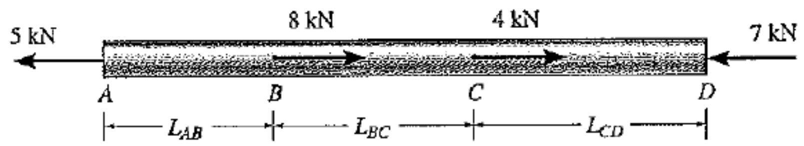
"+": lực gây ứng suất căng & làm giãn dài thanh.

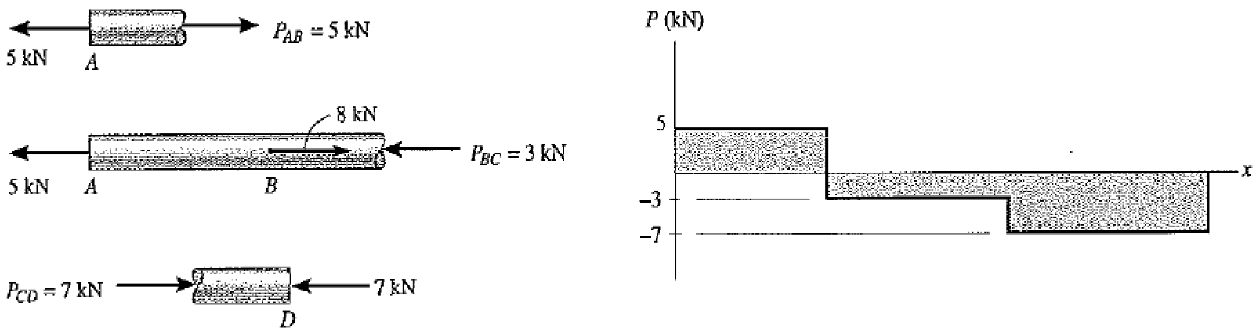
"-": lực gây ứng suất nén & làm co thanh.



4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Ví dụ: Cho sơ đồ lực tác động lên thanh như hình vẽ:





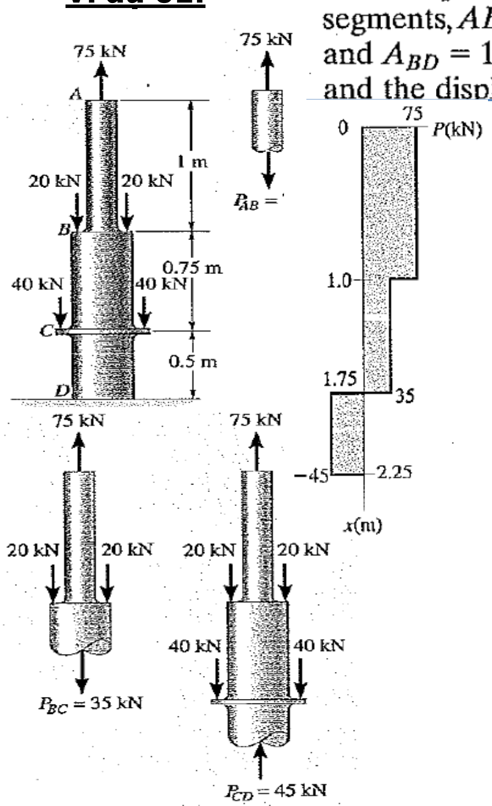
Chuyển vị tại điểm A so với điểm D trên thanh là:

$$\delta_{A/D} = \sum \frac{PL}{AE} = \frac{(5\text{ kN})\,L_{AB}}{AE} + \frac{(-3\text{ kN})\,L_{BC}}{AE} + \frac{(-7\text{ kN})\,L_{CD}}{AE}$$

4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Ví dụ 01:

The composite A-36 steel bar shown in Fig. 10–6*a* is made from two segments, *AB* and *BD*, having cross-sectional areas of A_{AB} = 600 mm^2 and A_{BD} = 1200 mm^2. Determine the vertical displacement of end *A* and the displacement of *B* relative to *C*.



Internal Force. Due to the application of the external loadings, the *internal axial forces* in regions *AB*, *BC*, and *CD* will all be *different*. These forces are obtained by applying the method of sections and the equation of vertical force equilibrium as shown in Fig. 10–6*b*. This variation is plotted in Fig. 10–6*c*.

Displacement. From Appendix B, E_{st} = 210(10^3) MPa. Using the sign convention, i.e., the internal tensile forces are positive and the compressive forces are negative, the vertical displacement of *A* relative to the *fixed* support *D* is

$$\delta_A = \sum \frac{PL}{AE} = \frac{[+75\text{ kN}](1\text{ m})(10^6)}{[600\text{ mm}^2(210)(10^3)\text{ kN/m}^2]} + \frac{[+35\text{ kN}](0.75\text{ m})(10^6)}{[1200\text{ mm}^2(210)(10^3)\text{ kN/m}^2]} + \frac{[-45\text{ kN}](0.5\text{ m})(10^6)}{[1200\text{ mm}^2(210)(10^3)\text{ kN/m}^2]}$$

$$= +0.61\text{ mm} \qquad \textit{Ans.}$$

Since the result is *positive*, the bar *elongates* and so the displacement at *A* is upward.

Applying Eq. 10–2 between points *B* and *C*, we obtain,

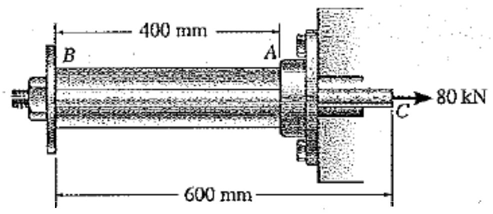
$$\delta_{B/C} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{[+35\text{ kN}](0.75\text{ m})(10^6)}{[1200\text{ mm}^2(210)(10^3)\text{ kN/m}^2]} = +0.104\text{ mm} \qquad \textit{Ans.}$$

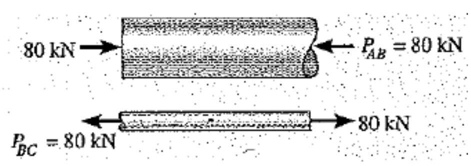
Here *B* moves away from *C*, since the segment elongates.

4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Ví dụ 02:

The assembly shown in Fig. 10–7*a* consists of an aluminum tube *AB* having a cross-sectional area of 400 mm^2. A steel rod having a diameter of 10 mm is attached to a rigid collar and passes through the tube. If a tensile load of 80 kN is applied to the rod, determine the displacement of the end *C* of the rod. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.





Internal Force. The free-body diagram of the tube and rod, Fig. 10–7*b*, shows that the rod is subjected to a tension of 80 kN and the tube is subjected to a compression of 80 kN.

Displacement. We will first determine the displacement of end *C* with respect to end *B*. Working in units of newtons and meters, we have

$$\delta_{C/B} = \frac{PL}{AE} = \frac{[+80(10^3)\text{ N}](0.6\text{ m})}{\pi(0.005\text{ m})^2[200(10^9)\text{ N/m}^2]} = +0.003056\text{ m} \rightarrow$$

The positive sign indicates that end *C* moves *to the right* relative to end *B*, since the bar elongates.

The displacement of end *B* with respect to the *fixed* end *A* is

$$\delta_B = \frac{PL}{AE} = \frac{[-80(10^3)\text{ N}](0.4\text{ m})}{[400\text{ mm}^2(10^{-6})\text{ m}^2/\text{mm}^2][70(10^9)\text{ N/m}^2]}$$
$$= -0.001143\text{ m} = 0.001143\text{ m} \rightarrow$$

Here the negative sign indicates that the tube shortens, and so *B* moves to the *right* relative to *A*.

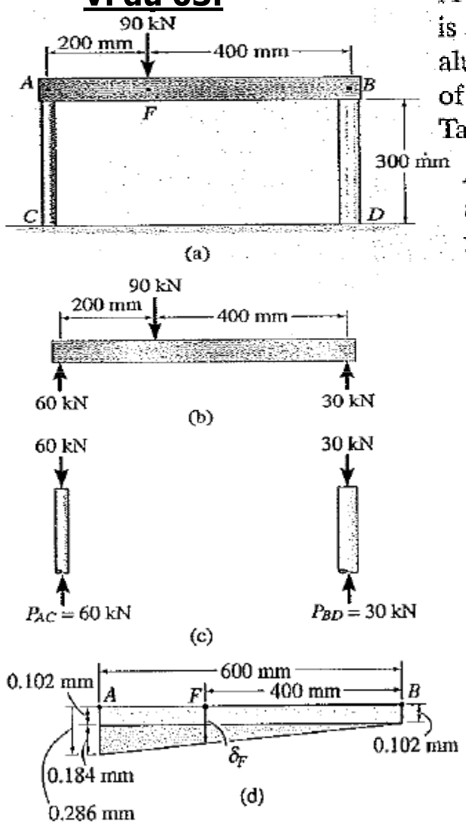
Since both displacements are to the right, the resultant displacement of *C* relative to the fixed end *A* is therefore

$$\overset{+}{\rightarrow} \qquad \delta_C = \delta_B + \delta_{C/B} = 0.001143\text{ m} + 0.003056\text{ m}$$
$$= 0.00420\text{ m} = 4.20\text{ mm} \rightarrow \qquad \textit{Ans.}$$

4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Ví dụ 03:

A *rigid beam AB* rests on the two short posts shown in Fig. 10–8*a*. *AC* is made of steel and has a diameter of 20 mm, and *BD* is made of aluminum and has a diameter of 40 mm. Determine the displacement of point *F* on *AB* if a vertical load of 90 kN is applied over this point. Take $E_{st} = 200$ GPa, $E_{al} = 70$ GPa.



Internal Force. The compressive forces acting at the top of each post are determined from the equilibrium of member *AB*, Fig. 10–8*b*. These forces are equal to the internal forces in each post, Fig. 10–8*c*.

Displacement. The displacement of the top of each post is

Post *AC*:

$$\delta_A = \frac{P_{AC}L_{AC}}{A_{AC}E_{st}} = \frac{[-60(10^3)\text{ N}](0.300\text{ m})}{\pi(0.010\text{ m})^2[200(10^9)\text{ N/m}^2]} = -286(10^{-6})\text{ m}$$
$$= 0.286\text{ mm} \downarrow$$

Post *BD*:

$$\delta_B = \frac{P_{BD}L_{BD}}{A_{BD}E_{al}} = \frac{[-30(10^3)\text{ N}](0.300\text{ m})}{\pi(0.020\text{ m})^2[70(10^9)\text{ N/m}^2]} = -102(10^{-6})\text{ m}$$
$$= 0.102\text{ mm} \downarrow$$

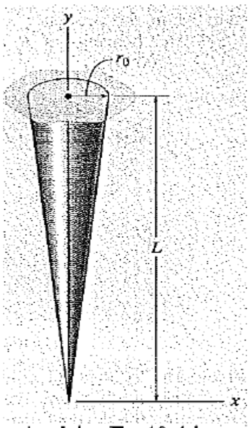
A diagram showing the centerline displacements at points *A*, *B*, and *F* on the beam is shown in Fig. 10–8*d*. By proportion of the shaded triangle, the displacement of point *F* is therefore

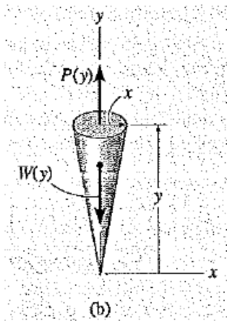
$$\delta_F = 0.102\text{ mm} + (0.184\text{ mm})\left(\frac{400\text{ mm}}{600\text{ mm}}\right) = 0.225\text{ mm} \downarrow \qquad \textit{Ans.}$$

4.2. BIẾN DẠNG ĐÀN HỒI CỦA THANH CHỊU TẢI DỌC TRỤC

Ví dụ 04:

A member is made from a material that has a specific weight γ and modulus of elasticity E. If it is formed into a *cone* having the dimensions shown in Fig. 10–9a, determine how far its end is displaced due to gravity when it is suspended in the vertical position.





Internal Force. The internal axial force varies along the member since it is dependent on the weight $W(y)$ of a segment of the member below any section, Fig. 10–9b. Hence, to calculate the displacement, we must use Eq. 10-1. At the section located at a distance y from its bottom end, the radius x of the cone as a function of y is determined by proportion; i.e.,

$$\frac{x}{y} = \frac{r_0}{L}; \qquad x = \frac{r_0}{L}y$$

The volume of a cone having a base of radius x and height y is

$$V = \frac{\pi}{3}yx^2 = \frac{\pi r_0^2}{3L^2}y^3$$

Since $W = \gamma V$, the internal force at the section becomes

$$+\uparrow \Sigma F_y = 0; \qquad P(y) = \frac{\gamma \pi r_0^2}{3L^2}y^3$$

Displacement. The area of the cross section is also a function of position y, Fig. 10–9b. We have

$$A(y) = \pi x^2 = \frac{\pi r_0^2}{L^2}y^2$$

Applying Eq. 10–1 between the limits of $y = 0$ and $y = L$ yields

$$\delta = \int_0^L \frac{P(y)\,dy}{A(y)\,E} = \int_0^L \frac{[(\gamma\pi r_0^2/3L^2)\,y^3]\,dy}{[(\pi r_0^2/L^2)\,y^2]\,E}$$

$$= \frac{\gamma}{3E}\int_0^L y\,dy$$

$$= \frac{\gamma L^2}{6E} \qquad \textit{Ans.}$$

As a partial check of this result, notice how the units of the terms, when canceled, give the displacement in units of length as expected.

4.3. NGUYÊN LÝ CÔNG TÁC DỤNG

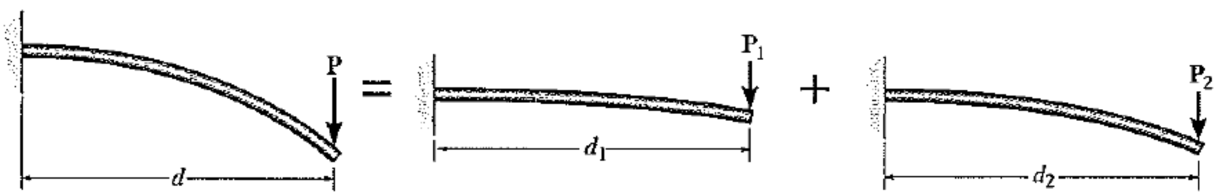
Nguyên lý công tác dụng ứng dụng trong trường hợp xác định ứng suất, chuyển vị của hệ vật khi chịu tác dụng của các tải trọng phức tạp.

Sự dịch chuyển hay ứng suất tại một điểm trên hệ vật chịu tải phức tạp được tính như sau:

- Tính ứng suất hay chuyển vị bởi từng tải trọng riêng rẽ.
- Cộng đại số các giá trị ứng suất hay chuyển vị.

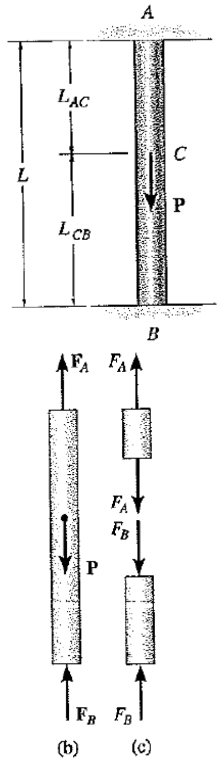
Khi áp dụng nguyên lý cộng tác dụng thì phải thỏa 02 điều kiện sau:

- Tải phải tương quan tuyến tính với ứng suất hay chuyển vị (ví dụ: $\delta = PL/AE$; $\sigma = P/A$
- Hình dạng ban đầu hay cấu hình của phần tử không thay đổi nhiều.



4.4. BÀI TOÁN SIÊU TĨNH – TRƯỜNG HỢP CHỊU TẢI DỌC TRỤC

Khi thanh chịu tải dọc trục & bị cố định 02 đầu của thanh như hình dưới thì ta gọi đó là bài toán siêu tĩnh. Khi đó, ta có phương trình cân bằng:



$$+\uparrow \Sigma F = 0;$$

$$F_B + F_A - P = 0 \qquad (1)$$

Điều kiện tương thích về biến dạng hay động học:

$$\delta_{A/B} = 0$$

Ta có công thức về chuyển vị: $\delta = PL/AE$

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0 \qquad (2)$$

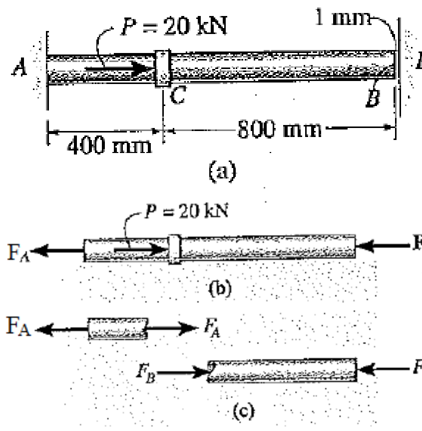
Giả sử rằng: AE là hằng. Từ (1) & (2), ta được:

$$F_A = P\left(\frac{L_{CB}}{L}\right) \quad \text{and} \quad F_B = P\left(\frac{L_{AC}}{L}\right)$$

4.4. BÀI TOÁN SIÊU TĨNH – TRƯỜNG HỢP CHỊU TẢI DỌC TRỤC

Ví dụ 01:

The steel rod shown in Fig. 10–12*a* has a diameter of 5 mm. It is attached to the fixed wall at *A*, and before it is loaded, there is a gap between the wall at *B'* and the rod of 1 mm. Determine the reactions at *A* and *B'* if the rod is subjected to an axial force of $P = 20$ kN as shown. Neglect the size of the collar at *C*. Take $E_{st} = 200$ GPa.



Equilibrium. As shown on the free-body diagram, Fig. 10–12*b*, we will assume that the force *P* is large enough to cause the rod's end *B* to contact the wall at *B'*. The problem is statically indeterminate since there are two unknowns and only one equation of equilibrium.

Equilibrium of the rod requires

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_A - F_B + 20(10^3)\text{ N} = 0 \qquad (1)$$

Compatibility. The loading causes point B to move to B', with no further displacement. Therefore the compatibility condition for the rod is

$$\delta_{B/A} = 0.001 \text{ m}$$

This displacement can be expressed in terms of the unknown reactions by using the load–displacement relationship, Eq. 10–2, applied to segments *AC* and *CB*, Fig. 10–12*c*. Working in units of newtons and meters, we have

$$\delta_{B/A} = 0.001 \text{ m} = \frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE}$$

$$0.001 \text{ m} = \frac{F_A(0.4 \text{ m})}{\pi(0.0025 \text{ m})^2[200(10^9)\text{ N/m}^2]} - \frac{F_B(0.8 \text{ m})}{\pi(0.0025 \text{ m})^2[200(10^9)\text{ N/m}^2]}$$

or

$$F_A(0.4 \text{ m}) - F_B(0.8 \text{ m}) = 3927.0 \text{ N}\cdot\text{m} \qquad (2)$$

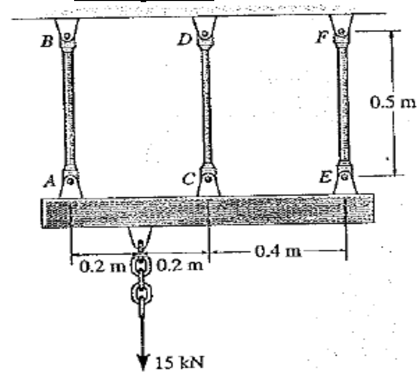
Solving Eqs. 1 and 2 yields

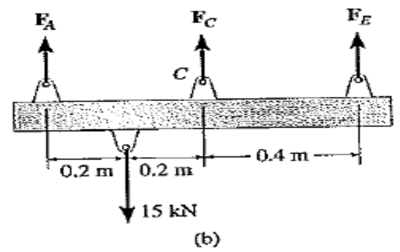
$$F_A = 16.6 \text{ kN} \qquad F_B = 3.39 \text{ kN} \qquad \textit{Ans.}$$

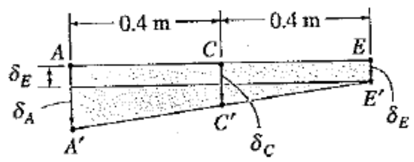
Since the answer for F_B is *positive*, indeed the end *B* contacts the wall at *B'* as originally assumed. On the other hand, if F_B were a negative quantity, the problem would be statically determinate, so that $F_B = 0$ and $F_A = 20$ kN.

4.4. BÀI TOÁN SIÊU TĨNH – TRƯỜNG HỢP CHỊU TẢI DỌC TRỤC

Ví dụ 02:







The three A-36 steel bars shown in Fig. 10–14*a* are pin connected to a *rigid* member. If the applied load on the member is 15 kN, determine the force developed in each bar. Bars *AB* and *EF* each have a cross-sectional area of 25 mm^2, and bar *CD* has a cross-sectional area of 15 mm^2.

Equilibrium. The free-body diagram of the rigid member is shown in Fig. 10–14*b*. This problem is statically indeterminate since there are three unknowns and only two available equilibrium equations. These equations are

$$+\uparrow \Sigma F_y = 0; \qquad F_A + F_C + F_E - 15\text{ kN} = 0 \qquad (1)$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -F_A(0.4\text{ m}) + 15\text{ kN}(0.2\text{ m}) + F_E(0.4\text{ m}) = 0 \qquad (2)$$

Compatibility. The applied load will cause the horizontal line *ACE* shown in Fig. 10–14*c* to move to the inclined line *A′C′E′*. The displacements of points *A*, *C*, and *E* can be related by proportional triangles. Thus the compatibility equation for these displacements is

$$\frac{\delta_A - \delta_E}{0.8\text{ m}} = \frac{\delta_C - \delta_E}{0.4\text{ m}}$$

$$\delta_C = \frac{1}{2}\delta_A + \frac{1}{2}\delta_E$$

Using the load–displacement relationship, Eq. 10–2, we have

$$\frac{F_C L}{(15\text{ mm}^2)E_{st}} = \frac{1}{2}\left[\frac{F_A L}{(25\text{ mm}^2)E_{st}}\right] + \frac{1}{2}\left[\frac{F_E L}{(25\text{ mm}^2)E_{st}}\right]$$

$$F_C = 0.3F_A + 0.3F_E \qquad (3)$$

Solving Eqs. 1–3 simultaneously yields

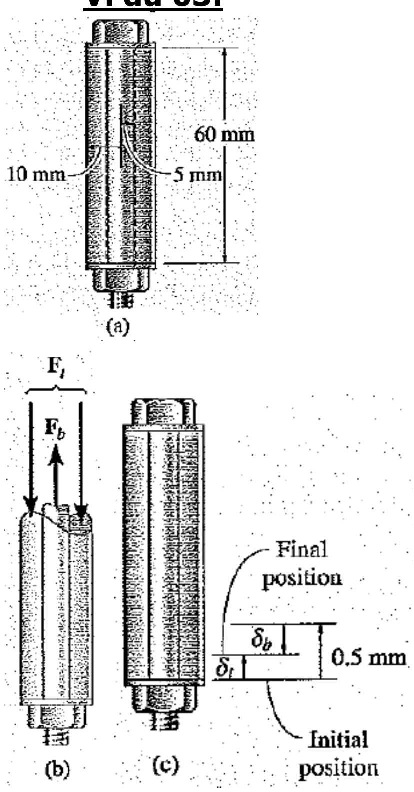
$F_A = 9.52$ kN **Ans.**

$F_C = 3.46$ kN **Ans.**

$F_E = 2.02$ kN **Ans.**

4.4. BÀI TOÁN SIÊU TĨNH – TRƯỜNG HỢP CHỊU TẢI DỌC TRỤC

Ví dụ 03:



The bolt shown in Fig. 10–15*a* is made of 2014-T6 aluminum alloy and is tightened so it compresses a cylindrical tube made of Am 1004-T61 magnesium alloy. The tube has an outer radius of 10 mm, and it is assumed that both the inner radius of the tube and the radius of the bolt are 5 mm. The washers at the top and bottom of the tube are considered to be rigid and have a negligible thickness. Initially the nut is hand-tightened slightly; then, using a wrench, the nut is further tightened one-half turn. If the bolt has 20 threads per 20 mm, determine the stress in the bolt.

Equilibrium. The free-body diagram of a section of the bolt and the tube, Fig. 10–15*b*, is considered in order to relate the force in the bolt F_b to that in the tube, F_t. Equilibrium requires

$$+\uparrow \Sigma F_y = 0; \qquad F_b - F_t = 0 \qquad (1)$$

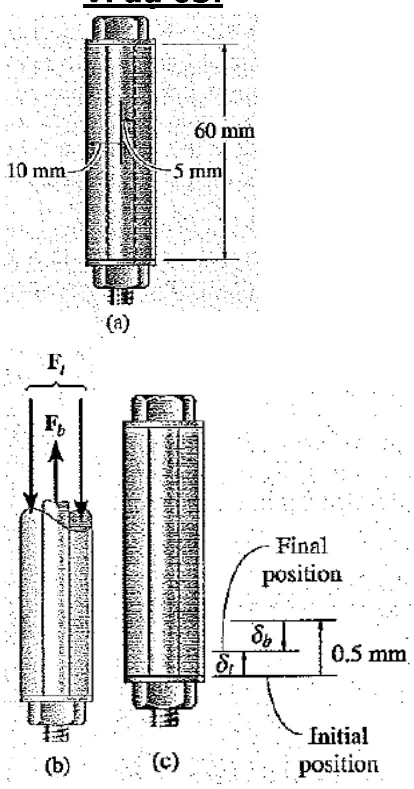
The problem is statically indeterminate since there are two unknowns in this equation.

Compatibility. When the nut is tightened on the bolt, the tube will shorten δ_t, and the bolt will *elongate* δ_b, Fig. 10-15c. Since the nut undergoes one-half turn, it advances a distance of $\frac{1}{2}(\frac{20}{20}\text{ mm}) = 0.5$ mm along the bolt. Thus, the compatibility of these displacements requires

$$+\uparrow \qquad \delta_t = 0.5\text{ mm} - \delta_b$$

4.4. BÀI TOÁN SIÊU TĨNH – TRƯỜNG HỢP CHỊU TẢI DỌC TRỤC

Ví dụ 03:



Taking the module of elasticity E_{Am} = 45 GPa, E_{al} = 75 GPa, and applying Eq. 10–2, yields

$$\frac{F_t\,(60\text{ mm})}{\pi[(10\text{ mm})^2-(5\text{ mm})^2][45(10^3)\text{ MPa}]}=0.5\text{ mm}-\frac{F_b\,(60\text{ mm})}{\pi(5\text{ mm})^2[75(10^3)\text{ MPa}]}$$

$$5F_t = 125\pi(1125) - 9F_b \qquad (2)$$

Solving Eqs. 1 and 2 simultaneously, we get

$$F_b = F_t = 31556\text{ N} = 31.56\text{ kN}$$

The stresses in the bolt and tube are therefore

$$\sigma_b = \frac{F_b}{A_b} = \frac{31556\text{ N}}{\pi(5\text{ mm})^2} = 401.8\text{ N/mm}^2 = 401.8\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_s = \frac{F_t}{A_t} = \frac{31556\text{ N}}{\pi[(10\text{ mm})^2-(5\text{ mm})^2]} = 133.9\text{ N/mm}^2 = 133.9\text{ MPa}$$

These stresses are less than the reported yield stress for each material, $(\sigma_Y)_{al} = 414$ MPa and $(\sigma_Y)_{mg} = 152$ MPa (see Appendix B), and therefore this "elastic" analysis is valid.

4.5. ỨNG SUẤT NHIỆT

Sự giản nở hay co lại của phân tử do yếu tố nhiệt tỷ lệ tuyến tính với nhiệt độ.

$$\delta_T = \alpha \Delta T L$$

α = a property of the material, referred to as the ***linear coefficient of thermal expansion***. The units measure strain per degree of temperature. They are 1/°C (Celsius) or 1/°K (Kelvin) in the SI system. Typical values are given in Appendix B

ΔT = the algebraic change in temperature of the member

L = the original length of the member

δ_T = the algebraic change in length of the member

Sự chênh lệch nhiệt độ là hàm phụ thuộc vào vị trí của phần tử:

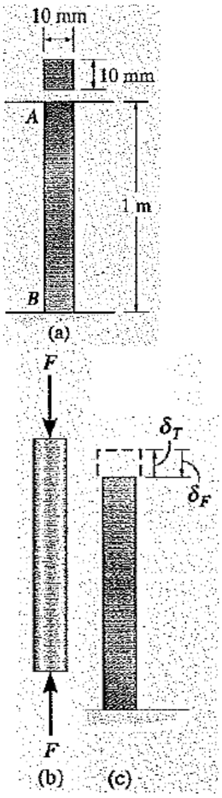
$$\Delta T = \Delta T(x)$$

Khi đó, độ dãn dài do ứng suất nhiệt là:

$$\delta_T = \int_0^L \alpha\, \Delta T\, dx$$

4.5. ỨNG SUẤT NHIỆT

Ví dụ 01:



The A-36 steel bar shown in Fig. 10–16 is constrained to just fit between two fixed supports when $T_1 = 30°C$. If the temperature is raised to $T_2 = 60°C$, determine the average normal thermal stress developed in the bar.

Equilibrium. The free-body diagram of the bar is shown in Fig. 10–16*b*. Since there is no external load, the force at A is equal but opposite to the force acting at B; that is,

$$+\uparrow \Sigma F_y = 0; \qquad F_A = F_B = F$$

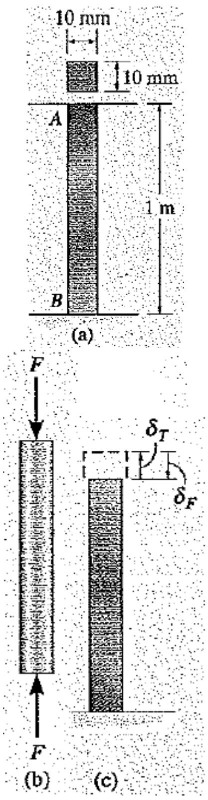
The problem is statically indeterminate since this force cannot be determined from equilibrium.

Compatibility. Since $\delta_{A/B} = 0$, the thermal displacement δ_T at A that would occur, Fig. 10–16*c*, is counteracted by the force **F** that would be required to push the bar δ_F back to its original position. The compatibility condition at A becomes

$$+\uparrow \qquad \delta_{A/B} = 0 = \delta_T - \delta_F$$

4.5. ỨNG SUẤT NHIỆT

Ví dụ 01:



The A-36 steel bar shown in Fig. 10–16 is constrained to just fit between two fixed supports when $T_1 = 30°C$. If the temperature is raised to $T_2 = 60°C$, determine the average normal thermal stress developed in the bar.

Applying the thermal and load–displacement relationships, we have

$$0 = \alpha \Delta T L - \frac{FL}{AE}$$

Thus, from the data in Appendix B,

$$F = \alpha \Delta T A E$$
$$= [12(10^{-6})/°C](60°C - 30°C)(0.010\ m)^2[200(10^6)\ kPa]$$
$$= 7.2\ kN$$

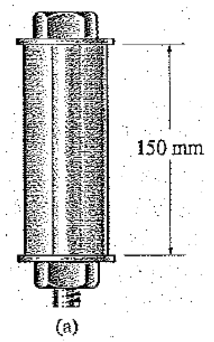
From the magnitude of **F**, it should be apparent that changes in temperature can cause large reaction forces in statically indeterminate members.

Since **F** also represents the internal axial force within the bar, the average normal compressive stress is thus

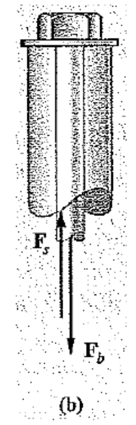
$$\sigma = \frac{F}{A} = \frac{7.2(10^{-3})\ MN}{(0.01\ m)^2} = 72\ MPa \qquad \textit{Ans.}$$

4.5. ỨNG SUẤT NHIỆT

Ví dụ 02:



(a)



(b)

A 2014-T6 aluminum tube having a cross-sectional area of 600 mm^2 is used as a sleeve for an A-36 steel bolt having a cross-sectional area of 400 mm^2, Fig. 10–17*a*. When the temperature is $T_1 = 15°C$, the nut holds the assembly in a snug position such that the axial force in the bolt is negligible. If the temperature increases to $T_2 = 80°C$, determine the average normal stress in the bolt and sleeve.

Equilibrium. A free-body diagram of a sectioned segment of the assembly is shown in Fig. 10–17*b*. The forces F_b and F_s are produced since the sleeve has a higher coefficient of thermal expansion than the bolt, and therefore the sleeve will expand more when the temperature is increased. The problem is statically indeterminate since these forces cannot be determined from equilibrium. However, it is required that

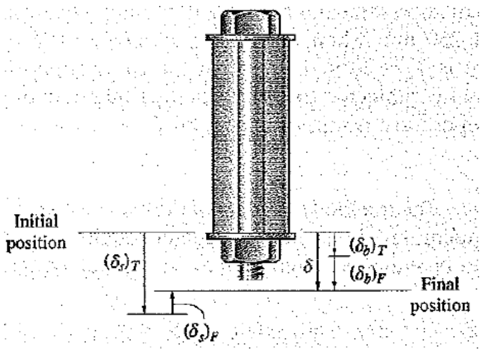
$$+\uparrow \Sigma F_y = 0; \qquad F_s = F_b \qquad (1)$$

Compatibility. The temperature increase causes the sleeve and bolt to expand $(\delta_s)_T$ and $(\delta_b)_T$, Fig. 10–17*c*. However, the redundant forces F_b and F_s elongate the bolt and shorten the sleeve. Consequently, the end of the assembly reaches a final position, which is not the same as the initial position. Hence, the compatibility condition becomes

$$+\downarrow \qquad \delta = (\delta_b)_T + (\delta_b)_F = (\delta_s)_T - (\delta_s)_F$$

4.5. ỨNG SUẤT NHIỆT

Ví dụ 02:



Applying Eqs. 10–2 and 10–4, and using the mechanical properties from Appendix B, we have

$$[12(10^{-6})/°C](80°C - 15°C)(0.150\text{ m}) + \frac{F_b(0.150\text{ m})}{(400\text{ mm}^2)(10^{-6}\text{ m}^2/\text{mm}^2)[200(10^9)\text{ N/m}^2]}$$

$$= [23(10^{-6})/°C](80°C - 15°C)(0.150\text{ m}) - \frac{F_s(0.150\text{ m})}{600\text{ mm}^2(10^{-6}\text{ m}^2/\text{mm}^2)[73.1(10^9)\text{ N/m}^2]}$$

Using Eq. 1 and solving gives

$$F_s = F_b = 20.26\text{ kN}$$

The average normal stress in the bolt and sleeve is therefore

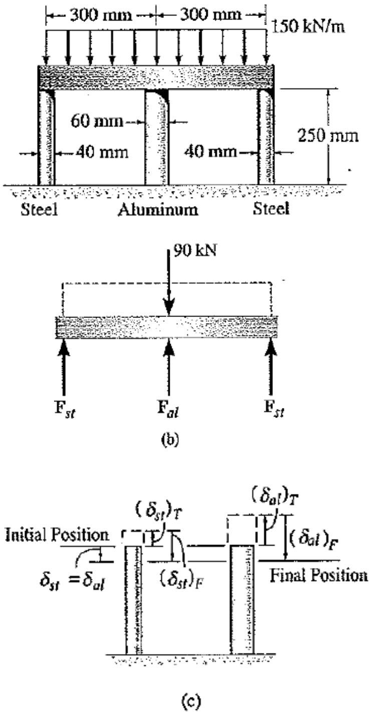
$$\sigma_b = \frac{20.26\text{ kN}}{400\text{ mm}^2(10^{-6}\text{ m}^2/\text{mm}^2)} = 50.6\text{ MPa} \qquad \textit{Ans.}$$

$$\sigma_s = \frac{20.26\text{ kN}}{600\text{ mm}^2(10^{-6}\text{ m}^2/\text{mm}^2)} = 33.8\text{ MPa} \qquad \textit{Ans.}$$

Since linear–elastic material behavior was assumed in this analysis, the calculated stresses should be checked to make sure that they do not exceed the proportional limits for the material.

4.5. ỨNG SUẤT NHIỆT

Ví dụ 03:



The rigid bar shown in Fig. 10–18*a* is fixed to the top of the three posts made of A-36 steel and 2014-T6 aluminum. The posts each have a length of 250 mm when no load is applied to the bar, and the temperature is $T_1 = 20°C$. Determine the force supported by each post if the bar is subjected to a uniform distributed load of 150 kN/m and the temperature is raised to $T_2 = 80°C$.

Equilibrium. The free-body diagram of the bar is shown in Fig. 10–18*b*. Moment equilibrium about the bar's center requires the forces in the steel posts to be equal. Summing forces on the free-body diagram, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2F_{st} + F_{al} - 90(10^3)\text{ N} = 0 \qquad (1)$$

Compatibility. Due to load, geometry, and material symmetry, the top of each post is displaced by an equal amount. Hence,

$$+\downarrow \qquad \delta_{st} = \delta_{al} \qquad (2)$$

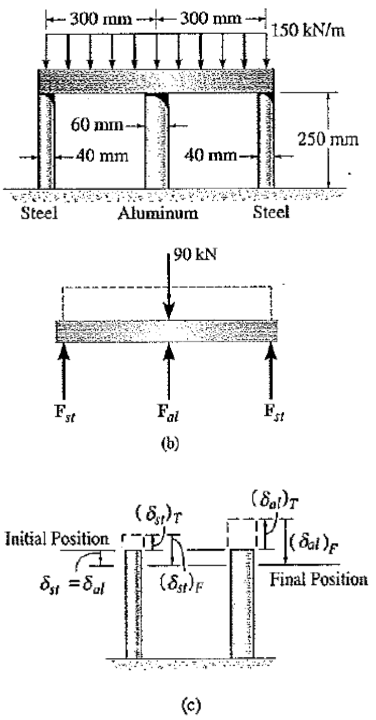
The final position of the top of each post is equal to its displacement caused by the temperature increase, plus its displacement caused by the internal axial compressive force, Fig. 10–18*c*. Thus, for a steel and aluminum post, we have

$$+\downarrow \qquad \delta_{st} = -(\delta_{st})_T + (\delta_{st})_F$$

$$+\downarrow \qquad \delta_{al} = -(\delta_{al})_T + (\delta_{al})_F$$

4.5. ỨNG SUẤT NHIỆT

Ví dụ 03:



Applying Eq. 2 gives

$$-(\delta_{st})_T + (\delta_{st})_F = -(\delta_{al})_T + (\delta_{al})_F$$

Using Eqs. 10–2 and 10–4 and the material properties in Appendix B, we get

$$-[12(10^{-6})/°C](80°C - 20°C)(0.250\text{ m}) + \frac{F_{st}(0.250\text{ m})}{\pi(0.020\text{ m})^2[200(10^9)\text{ N/m}^2]}$$

$$= -[23(10^{-6})/°C](80°C - 20°C)(0.250\text{ m}) + \frac{F_{al}(0.250\text{ m})}{\pi(0.03\text{ m})^2[73.1(10^9)\text{ N/m}^2]}$$

$$F_{st} = 1.216F_{al} - 165.9(10^3) \qquad (3)$$

To be *consistent*, all numerical data has been expressed in terms of newtons, meters, and degrees Celsius. Solving Eqs. 1 and 3 simultaneously yields

$$F_{st} = -16.4\text{ kN} \qquad F_{al} = 123\text{ kN} \qquad \textit{Ans.}$$

The negative value for F_{st} indicates that this force acts opposite to that shown in Fig. 10–18*b*. In other words, the steel posts are in tension and the aluminum post is in compression.