APPENDIX A
STATICS REVIEW

Statics is the foundation course for mechanics of materials. This appendix briefly reviews statics from the perspective of this course. It presupposes that you are familiar with the basic concepts so if you took a course in statics some time ago, then you may need to review your statics textbook along with this brief review. Review exams at the end of this appendix can also be used for self-assessment.

A.1 TYPES OF FORCES AND MOMENTS

We classify the forces and moments into three categories: external, reaction, and internal.

A.1.1 External Forces and Moments

External forces and moments are those that are applied to the body and are often referred to as the *load* on the body. They are assumed to be known in an analysis, though sometimes we carry external forces and moments as variables. In that way we may answer questions such these: How much load can a structure support? What loads are needed to produce a given deformation?

Surface forces and moments are external forces (moments), which act on the surface and are transmitted to the body by contact. Surface forces (moments) applied at a point are called *concentrated* forces (moment or couple). Surface forces (moments) applied along a line or over a surface are called *distributed* forces (moments).

Body forces are external forces that act at every point on the body. Body forces are not transmitted by contact. Gravitational forces and electromagnetic forces are two examples of body forces. A body force has units of force per unit volume.

A.1.2 Reaction Forces and Moments

Other forces and moments are developed at the supports of a body to resist movement due to the external forces (moments). These *reaction* forces (moments) are usually not known and must be calculated before further analysis can be conducted. Three principles are used to decide whether there is a reaction force (reaction moment) at the support:

1. If a point cannot move in a given direction, then a reaction force opposite to the direction acts at that support point.
2. If a line cannot rotate about an axis in a given direction, then a reaction moment opposite to the direction acts at that support.
3. The support in isolation and not the entire body is considered in making decisions about the movement of a point or the rotation of a line at the support. Exceptions to the rule exist in three-dimensional problems, such as bodies supported by balanced hinges or balanced bearings (rollers). These types of three-dimensional problems will not be covered in this book.

Table C.1 shows several types of support that can be replaced by reaction forces and moments using the principles described above.

A.1.3 Internal Forces and Moments

A body is held together by *internal* forces. Internal forces exist irrespective of whether or not we apply external forces. The material resists changes due to applied forces and moments by increasing the internal forces. Our interest is in the resistance the

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material offers to the applied loads—that is, in the internal forces. Internal forces always exist in pairs that are equal and opposite on the two surfaces produced by an imaginary cut.

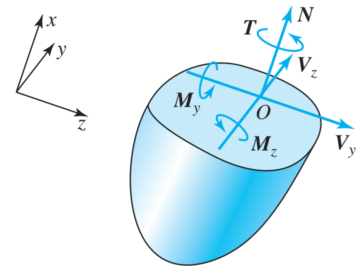

Figure A.1 Internal forces and moments.

The internal forces are shown in Figure A.1. (In this book, all internal forces and moments are printed in bold italics: $\boldsymbol{N}$ = axial force; $\boldsymbol{V}_y$, $\boldsymbol{V}_z$ = shear force; $\boldsymbol{T}$ = torque; $\boldsymbol{M}_y$, $\boldsymbol{M}_z$ = bending moment.) They are defined as follows:

- Forces that are normal to the imaginary cut surface are called **normal forces**. The normal force that points away from the surface (pulls the surface) is called **tensile force**. The normal force that points into the surface (pushes the surface) is called **compressive force**.
- The normal force acting in the direction of the axis of the body is called **axial force**.
- Forces that are tangent to the imaginary cut surface are called **shear forces**.
- Internal moments about an axis normal to the imaginary cut surface are called torsional moments or **torque**.
- Internal moments about an axis tangent to the imaginary cut are called **bending moments**.

A.2 FREE-BODY DIAGRAMS

Newton's laws are applicable only to free bodies. By "free" we mean that if a body is not in equilibrium, it will move. If there are supports, then these supports must be replaced by appropriate reaction forces and moments using the principles described in Section A.1.2. The diagram showing all the forces acting on a free body is called the **free-body diagram**.

Additional free-body diagrams may be created by making imaginary cuts for the calculation of internal quantities. Each imaginary cut will produce two additional free-body diagrams. Either of the two free-body diagrams can be used for calculating internal forces and moments.

A body is in static equilibrium if the vector sum of all forces acting on a free body and the vector sum of all moments about any point in space are zero:

$$\sum \bar{F} = 0 \qquad \sum \bar{M} = 0 \tag{A.1}$$

where $\sum$ represents summation and the overbar represents a vector quantity. In a three-dimensional Cartesian coordinate system the equilibrium equations in scalar form are

$$\begin{aligned} &\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0 \\ &\sum M_x = 0 \qquad \sum M_y = 0 \qquad \sum M_z = 0 \end{aligned} \tag{A.2}$$

Equations (A.2) imply that there are six independent equations in three dimensions. In other words, we can at most solve for six unknowns from a free-body diagram in three dimensions.

In two dimensions the sum of the forces in the z direction and the sum of the moments about the x and y axes are automatically satisfied, as all forces must lie in the x, y plane. The remaining equilibrium equations in two dimensions that have to be satisfied are

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_z = 0 \tag{A.3}$$

Equations (A.3) imply that there are three independent equations per free-body diagram in two dimensions. In other words, we can at most solve for three unknowns from a free-body diagram in two dimensions.

The following can be used to reduce the computational effort:

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- Balance the moments at a point through which the unknown forces passes. These forces do not appear in the moment equation.
- Balance the forces or moments perpendicular to the direction of an unknown force. These forces do not appear in the equation.

A structure on which the number of unknown reaction forces and moments is greater than the number of equilibrium equations (six in three dimensions and three in two dimensions) is called a **statically indeterminate structure**. Statically indeterminate problems arise when more supports than needed are used to support a structure. Extra supports may be used for safety considerations or for the purpose of increasing the stiffness of a structure. We define the following:

Degree of static redundancy = number of unknown reactions – number of equilibrium equations. (A.4)

To solve a statically indeterminate problem, we have to generate equations on the displacement or rotation at the support points. A mistake sometimes made is to take moments at many points in order to generate enough equations for the unknowns. A statically indeterminate problem cannot be solved from equilibrium equations alone. There are only three independent equations of static equilibrium in two dimensions and six independent equations of static equilibrium in three dimensions. Additional equations must come from displacements or rotation conditions at the support.

The number of equations on the displacement or rotation needed to solve a statically indeterminate problem is equal to the degree of static redundancy. There are two exceptions: (i) With symmetric structures with symmetric loadings by using the arguments of symmetry one can reduce the total number of unknown reactions. (ii) *Pin connections* do not transmit moments from one part of a structure to another. Thus it is possible that a seemingly indeterminate pin structure may be a determinate structure. We will not consider such pin-connected structures in this book.

A structural member on which there is no moment couple and forces act at two points only is called a **two-force member.** Figure A.2 shows a two-force member. By balancing the moments at either point A or B we can conclude that the resultant forces at A and B must act along the line joining the two points. Notice that the shape of the member is immaterial. Identifying two-force members by inspection can save significant computation effort.

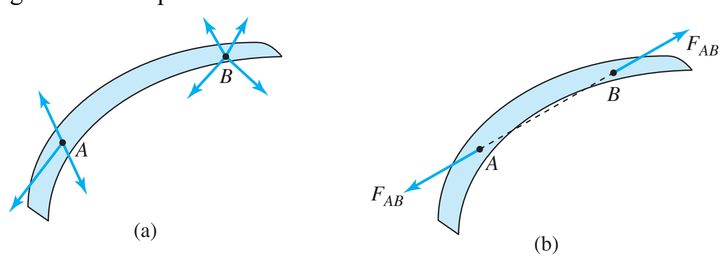

Figure A.2 Two-force member.

A.3 TRUSSES

A **truss** is a structure made up of two-force members. The method of joints and the method of sections are two methods of calculating the internal forces in truss members.

In the method of joints, a free-body diagram is created by making imaginary cuts on all members joined at the pin. If a force is directed away from the pin, then the two-force member is assumed to be in tension; and if it is directed into the pin, then the member is assumed to be in compression. By conducting force balance in two (or three) dimensions two (or three) equations per pin can be written.

In the method of sections an imaginary cut is made through the truss to produce a free-body diagram. The imaginary cut can be of any shape that will permit a quick calculation of the force in a member. Three equations in two dimensions or six equations in three dimensions can be written per free-body diagram produced from a single imaginary cut.

A **zero-force member** in a truss is a member that carries no internal force. Identifying zero-force members can save significant computation time. Zero-force members can be identified by conducting the method of joints mentally. Usually if two members are collinear at a joint *and* if there is no external force, then the zero-force member is the member that is inclined to the collinear members.

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A.4 CENTROIDS

The y and z coordinates of the centroid of the two-dimensional body shown in A.3 are defined as

$$y_c = \frac{\int_A y\,dA}{\int_A dA} \qquad z_c = \frac{\int_A z\,dA}{\int_A dA} \tag{A.5}$$

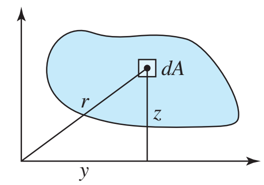

Figure A.3 Area moments.

The numerator in Equations (A.5) is referred to as the first moment of the area. If there is an axis of symmetry, then the area moment about the symmetric axis from one part of the body is canceled by the moment from the symmetric part, and hence we conclude that the centroid lies on the axis of symmetry.

Consider a coordinate system fixed to the centroid of the area. If we now consider the first moment of the area in this coordinate system and it turns out to be nonzero, then it would imply that the centroid is not located at the origin, thus contradicting our starting assumption. We therefore conclude that the first moment of the area calculated in a coordinate system fixed to the centroid of the area is zero.

The *centroid* for a composite body in which the centroids of the individual bodies are known can be calculated from Equations (A.6).

$$y_c = \frac{\sum_{i=1}^{n} y_{c_i} A_i}{\sum_{i=1}^{n} A_i} \qquad z_c = \frac{\sum_{i=1}^{n} z_{c_i} A_i}{\sum_{i=1}^{n} A_i} \tag{A.6}$$

where y_{c_i} and z_{c_i} are the known coordinates of the centroids of the area A_i. Table C.2 shows the locations of the centroids of some common shapes that will be useful in solving problems in this book.

A.5 AREA MOMENTS OF INERTIA

The *area moments of inertia*, also referred to as second area moments, are defined as

$$I_{yy} = \int_A z^2\,dA \qquad I_{zz} = \int_A y^2\,dA \qquad I_{yz} = \int_A yz\,dA \tag{A.7}$$

The polar moment of inertia is defined as in Equation (A.7) with the relation to I_{yy} and I_{zz} deduced using A.3:

$$J = \int_A r^2\,dA = I_{yy} + I_{zz} \tag{A.8}$$

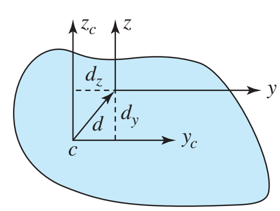

Figure A.4 Parallel-axis theorem.

If we know the area moment of inertia in a coordinate system fixed to the centroid, then we can compute the area moments about an axis parallel to the coordinate axis by the parallel-axis theorem illustrated in Figure A.4 and are given by

$$I_{yy} = I_{y_c y_c} + A d_y^2 \qquad I_{zz} = I_{z_c z_c} + A d_z^2 \qquad I_{yz} = I_{y_c z_c} + A d_y d_z \qquad J = J_c + A d^2 \tag{A.9}$$

where the subscript c refers to the axis fixed to the centroid of the body. The quantities y^2, z^2, r^2, A, d_y^2, d_z^2, and d^2 are always positive. From Equations (A.7) through (A.9) we conclude that I_{yy}, I_{zz}, and J are always positive and minimum about the axis passing through the centroid of the body. However, I_{yz} can be positive or negative, as y, z, d_y, and d_z can be positive or negative in

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Equation (A.7). If either y or z is an axis of symmetry, then the integral in I_{yz} on the positive side will cancel the integral on the negative side in Equation (A.7), and hence I_{yz} will be zero. We record the observations as follows:

- I_{yy}, I_{zz}, and J are always positive and minimum about the axis passing through the centroid of the body.
- If either the y or the z axis is an axis of symmetry, then I_{yz} will be zero.

The moment of inertia of a composite body in which we know the moments of inertia of the individual bodies about its centroid can be calculated from Equation (A.10).

$$I_{yy} = \sum_{i=1}^{n}(I_{y_{c_i}y_{c_i}} + A_i d_{y_i}^2) \qquad I_{zz} = \sum_{i=1}^{n}(I_{z_{c_i}z_{c_i}} + A_i d_{z_i}^2) \qquad I_{yz} = \sum_{i=1}^{n}(I_{y_{c_i}z_{c_i}} + A_i d_{y_i} d_{z_i}) \qquad J = \sum_{i=1}^{n}(J_{c_i} + A_i d_i^2) \qquad \text{(A.10)}$$

where $I_{y_{c_i}y_{c_i}}$, $I_{z_{c_i}z_{c_i}}$, $I_{y_{c_i}z_{c_i}}$, and J_{c_i} are the area moments of inertia about the axes passing through the centroid of the ith body. Table C.2 shows the area moments of inertia about an axis passing through the centroid of some common shapes that will be useful in solving the problems in this book.

The *radius of gyration* $\hat{r}$ about an axis is defined by

$$\hat{r} = \sqrt{\frac{I}{A}} \qquad \text{or} \qquad I = A\hat{r}^2 \qquad \text{(A.11)}$$

where I is the area moment of inertia about the same axis about which the radius of gyration $\hat{r}$ is being calculated.

A.6 STATICALLY EQUIVALENT LOAD SYSTEMS

Two systems of forces that generate the same resultant force and moment are called **statically equivalent** load systems. If one system satisfies the equilibrium, then the statically equivalent system also satisfies the equilibrium. The statically equivalent systems simplifies analysis and is often used in problems with distributed loads.

A.6.1 Distributed Force on a Line

Let $p(x)$ be a distributed force per unit length, which varies with x. We can replace this distributed force by a force and moment acting at any point or by a single force acting at point x_c, as shown in Figure A.5.

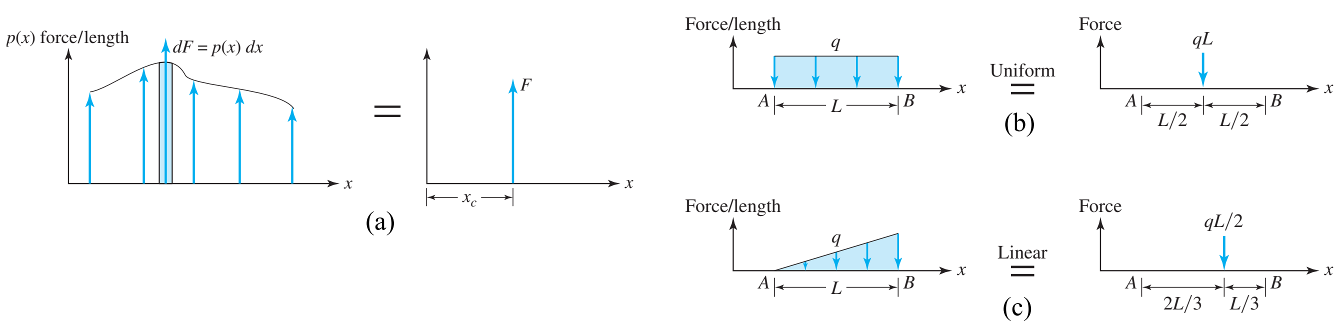

Figure A.5 Static equivalency for (a) distributed force on a line. (b) uniform distribution. (c) linear distribution.

For two systems in Figure A.5*a* to be statically equivalent, the resultant force and the resultant moment about any point (origin) must be the same.

$$F = \int_L p(x)\,dx \qquad x_c = \frac{\int_L x\,p(x)\,dx}{F} \qquad \text{(A.12)}$$

The force F is equal to the area under the curve and x_c represents the location of the centroid of the *distribution*. This is used in replacing a uniform or a linearly varying distribution by a statically equivalent force, as shown in Figure A.5*b* and *c*.

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Two statically equivalent systems are *not* identical systems. The deformation (change of shape of bodies) in two statically equivalent systems is different. The distribution of the internal forces and internal moments of two statically equivalent systems is different. The following rule must be remembered:

- The imaginary cut for the calculation of internal forces and moments must be made on the original body and not on the statically equivalent body.

A.6.2 Distributed Force on a Surface

Let $\sigma(y, z)$ be a distributed force per unit area that varies in intensity with y and z. We would like to replace it by a single force, as shown in Figure A.6.

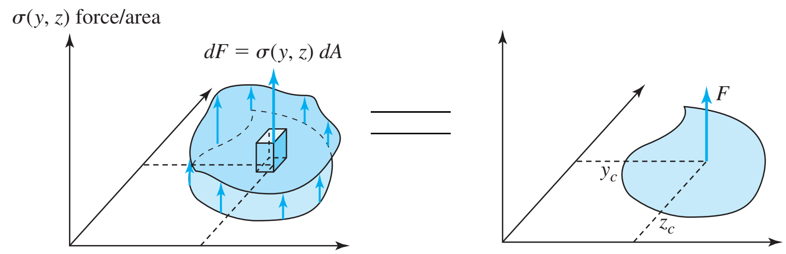

Figure A.6 Static equivalency for distributed force on a surface.

For the two systems shown in Figure A.6 to be statically equivalent load systems, the resultant force and the resultant moment about the y axis and on the z axis must be the same.

$$F = \iint_A \sigma(y,z)\, dy\, dz \qquad y_c = \frac{\iint_A y\,\sigma(y,z)\, dy\, dz}{F} \qquad z_c = \frac{\iint_A z\,\sigma(y,z)\, dy\, dz}{F} \tag{A.13}$$

The force F is equal to the volume under the curve. y_c and z_c represent the locations of the centroid of the *distribution*, which can be different from the centroid of the area on which the distributed force acts. The centroid of the area depends only on the geometry of that area. The centroid of the distribution depends on how the intensity of the distributed load $\sigma(y, z)$ varies over the area.

Figure A.7 shows a uniform and a linearly varying distributed force, which can be replaced by a single force at the centroid of the distribution. Notice that for the uniformly distributed force, the centroid of the distributed force is the same as the centroid of the rectangular area, but for the linearly varying distributed force, the centroid of the distributed force is different from the centroid of the area. If we were to place the equivalent force at the centroid of the area rather than at the centroid of distribution, then we would also need a moment at that point.

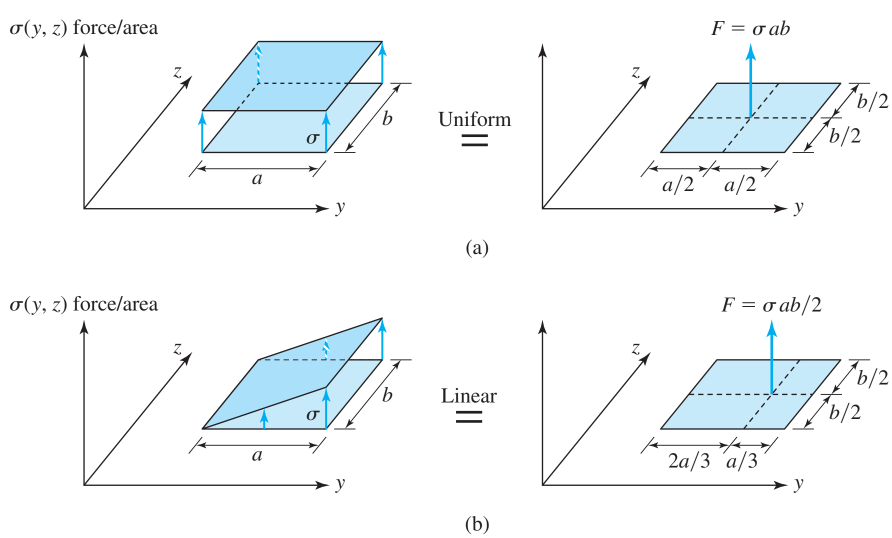

Figure A.7 Statically equivalent force for uniform and linearly distributed forces on a surface.

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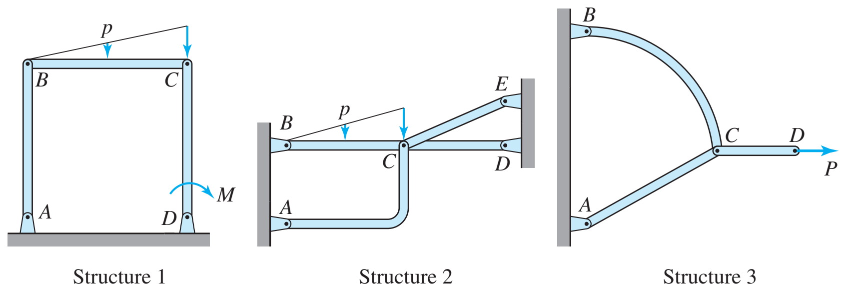
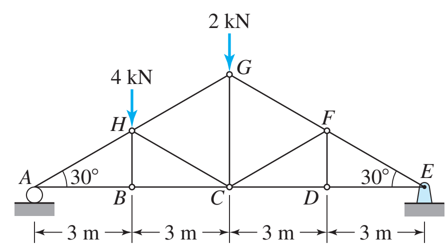
Quick Test A.1 **Time: 15 minutes/Total: 20 points**

Grade yourself using the answers and points given in Appendix G.

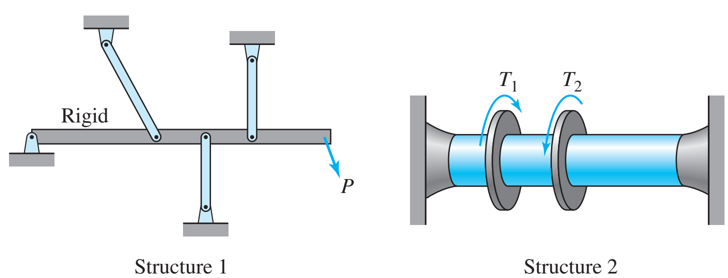
1. Three pin-connected structures are shown: (a) How many two-force members are there in each structure? (b) Which are the two-force member

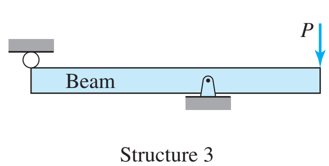


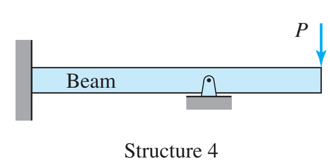
2. Identify all the zero-force members in the truss shown.



3. Determine the degree of static redundancy in each of the following structures and identify the statically determinate and indeterminate structures. Force P, and torques T_1 and T_2 are known external loads.







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STATIC REVIEW EXAM 1

To get full credit, you must draw a free-body diagram any time you use equilibrium equations to calculate forces or moments. Grade yourself using the solution and grading scheme given in Appendix D. Each question is worth 20 points.

1. Determine (a) the coordinates (y_c, z_c) of the centroid of the cross section shown in Figure R1.1; (b) the area moment of inertia about an axis passing through the centroid of the cross section and parallel to the z axis.

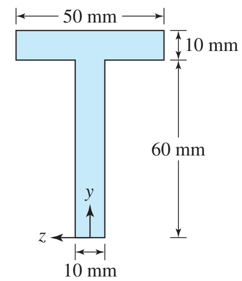

Figure R1.1

2. A linearly varying distributed load acts on a symmetric T section, as shown in Figure R1.2. Determine the force F and its location (x_F, y_F coordinates) that is statically equivalent to the distributed load.

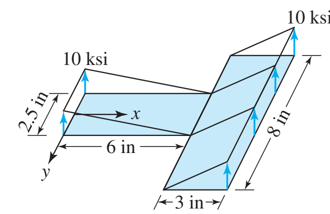

Figure R1.2

3. Find the internal axial force (indicate tension or compression) and the internal torque (magnitude and direction) acting on an imaginary cut through point E in Figure R1.3.

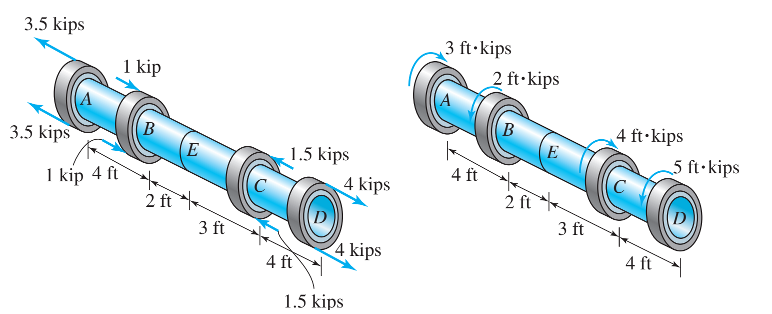

Figure R1.3

4. Determine the internal shear force and the internal bending moment acting at the section passing through A in Figure R1.4

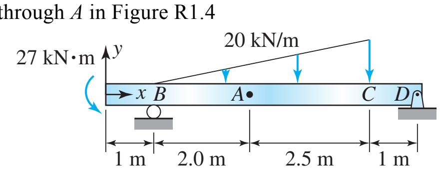

Figure R1.4

5. A system of pipes is subjected to a force P, as shown in Figure P1.5 By inspection (or by drawing a free-body diagram) identify the zero and non-zero internal forces and moments. Also indicate in the table the coordinate directions in which the internal shear forces and internal bending moments act

Internal Force/ Moment	Section *AA* (zero/nonzero)	Section *BB* (zero/nonzero)
Axial force	___________	___________
Shear force	___________ in___ direction	___________ in___ direction
Shear force	___________ in___ direction	___________ in___ direction
Torque	___________	___________
Bending moment	___________ in___ direction	___________ in___ direction
Bending moment	___________ in___ direction	___________ in___ direction

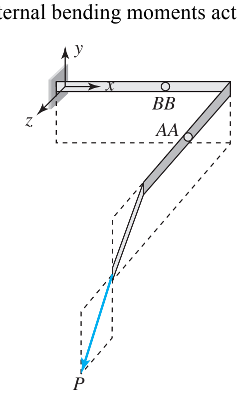

Figure R1.5

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STATIC REVIEW EXAM 2

To get full credit, you must draw a free-body diagram any time you use equilibrium equations to calculate forces or moments. Discuss the solution to this exam with your instructor.

1. Determine (a) the coordinates (y_c, z_c) of the centroid of the cross section in Figure P1.6; (b) the area moment of inertia about an axis passing through the centroid of the cross section and parallel to the z axis.

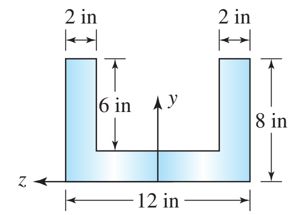

Figure R1.6

2. A distributed load acts on a symmetric C section, as shown in Figure P1.7. Determine the force F and its location (x_F, y_F coordinates) that is statically equivalent to the distributed load.

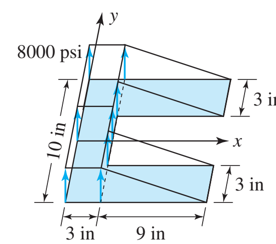

Figure R1.7

3. Find the internal axial force (indicate tension or compression) and the internal torque (magnitude and direction) acting on an imaginary cut through point E in Figure P1.8.

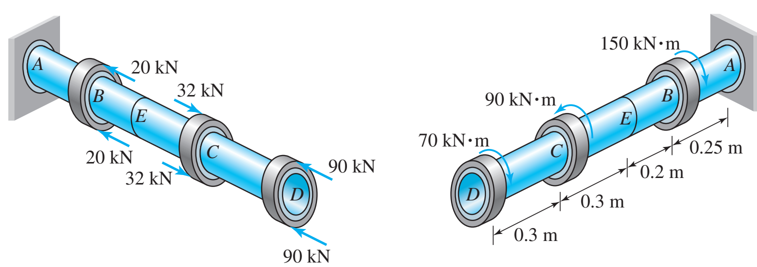

Figure R1.8

4. A simply supported beam is loaded by a uniformly distributed force of intensity 0.1 kip/in. applied at 60°, as shown in Figure P1.9. Also applied is a force F at the centroid of the beam. Neglecting the effect of beam thickness, determine at section C the internal axial force, the internal shear force, and the internal bending moment.

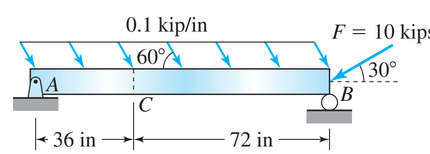

Figure R1.9

5. A system of pipes is subjected to a force P, as shown in Figure P1.10.By inspection (or by drawing a free-body diagram) identify the zero and nonzero internal forces and moments. Also indicate in the table the coordinate directions in which the internal shear forces and internal bending moments act.

Internal Force/Moment	Section *AA* (zero/nonzero)	Section *BB* (zero/nonzero)
Axial force	________	________
Shear force	________ in___ direction	________ in___ direction
Shear force	________ in___ direction	________ in___ direction
Torque	________	________
Bending moment	________ in___ direction	________ in___ direction
Bending moment	________ in___ direction	________ in___ direction

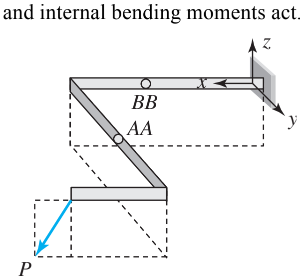

Figure R1.10

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POINTS TO REMEMBER

- If a point cannot move in a given direction, then a reaction force opposite to the direction acts at that support point.
- If a line cannot rotate about an axis in a given direction, then a reaction moment opposite to the direction acts at that support.
- The support in isolation and not the entire body is considered in making decisions about the reaction at the support.
- Forces that are normal to the imaginary cut surface are called *normal forces*.
- The normal force that points away from the surface (pulls the surface) is called *tensile force*.
- The normal force that points into the surface (pushes the surface) is called *compressive force*.
- The normal force acting in the direction of the axis of the body is called *axial force*.
- Forces that are tangent to the imaginary cut surface are called *shear forces*.
- The internal moment about an axis normal to the imaginary cut surface is called torsional moment or *torque*.
- Internal moments about axes tangent to the imaginary cut are called *bending moments*.
- Calculation of internal forces or moments requires drawing a free-body diagram after making an imaginary cut.
- There are six independent equations in three dimensions and three independent equations in two dimensions per free-body diagram.
- A structure on which the number of unknown reaction forces and moments is greater than the number of equilibrium equations (6 in 3-D and 3 in 2-D) is called a statically *indeterminate* structure.
- Degree of static redundancy = number of unknown reactions − number of equilibrium equations.
- The number of equations on displacement and/or rotation we need to solve a statically indeterminate problem is equal to the degree of static redundancy.
- A structural member on which there is no moment couple and forces act at two points only is called a *two-force member.*
- The centroid lies on the axis of symmetry.
- The first moment of the area calculated in a coordinate system fixed to the centroid of the area is zero.
- I_{yy}, I_{zz}, and J are always positive and minimum about the axis passing through the centroid of the body.
- I_{yz} can be positive or negative.
- If either the y or the z axis is an axis of symmetry, then I_{yz} will be zero.
- Two systems that generate the same resultant force and moment are called *statically equivalent load systems*.
- The imaginary cut for the calculation of internal forces and moments must be made on the original body and not on the statically equivalent body.

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APPENDIX B
ALGORITHMS FOR NUMERICAL METHODS

This appendix describes simple numerical techniques for evaluating the value of an integral, determining a root of a nonlinear equation, and finding constants of a polynomial by the least-squares method. Algorithms are given that can be programmed in any language. Also shown are methods of solving the same problems using a spreadsheet.

B.1 NUMERICAL INTEGRATION

We seek to numerically evaluate the integral

$$I = \int_a^b f(x)\,dx \tag{B.1}$$

where the function $f(x)$ and the limits a and b are assumed known.

This integral represents the area underneath the curve $f(x)$ in the interval defined by $x = a$ and $x = b$. The interval between a and b can be subdivided into N parts, as shown in Figure A.8. In each of the subintervals the function can be approximated by a straight-line segment. The area under the curve in each subinterval is the area of a trapezoid. Thus in the ith interval the area is $(\Delta x_i)[f(x_i) + f(x_{i-1})]/2$. By summing all the areas we obtain an approximate value of the total area represented by the integral in Equation (A.1),

$$I \cong \sum_{i=1}^{N} (\Delta x_i)\,\frac{f(x_i) + f(x_{i-1})}{2} \tag{B.2}$$

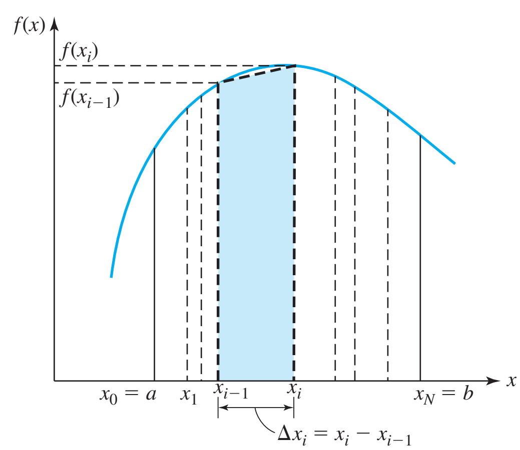

Figure A.8 Numerical integration by trapezoidal rule

By increasing the value of N in Equation (B.2) we can improve the accuracy in our approximation of the integral. More sophisticated numerical integration schemes such as Gauss quadrature may be needed with increased complexity of the function $f(x)$. For the functions that will be seen in this book, integration by the trapezoidal rule given by Equation (B.2) will give adequate accuracy.

B.1.1 Algorithm for Numerical Integration

Following are the steps in the algorithm for computing numerically the value of an integral of a function, assuming that the function value $f(x_i)$ is known at $N + 1$ points x_i, where i varies from 0 to N.

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1. Read the value of N.
2. Read the values of x_i and $f(x_i)$ for $i = 0$ to N.
3. Initialize $I = 0$.
4. For $i = 1$ to N, calculate $I = I + (x_i - x_{i-1})[f(x_i) + f(x_{i-1})]/2$.
5. Print the value of I.

B.1.2 Use of a Spreadsheet for Numerical Integration

Figure A.9 shows a sample spreadsheet that can be used to evaluate an integral numerically by the trapezoidal rule given by Equation (B.2). The data x_i and $f(x_i)$ can be either typed or imported into columns A and B of the spreadsheet, starting at row 2. In cells A2 and B2 are the values of x_0 and $f(x_0)$, and in cells A3 and B3 are the values of x_1 and $f(x_1)$. Using these values, the first term ($i = 1$) of the summation in Equation (B.2) can be found, as shown in cell C2. In a similar manner the second term of the summation in Equation (B.2) can be found and added to the result of the first term in cell C2. On copying the formula of cell C3, the spreadsheet automatically updates the column and row entries. Thus in all but the last entry we add one term of the summation at a time to the result of the previous row and obtain the final result.

	A	B	C	D	
1	x_i	$f(x_i)$	I		
2	•	•	=(A3−A2)*(B3+B2)/2		
3	•	•	=C2+(A4−A3)*(B4+B3)/2		
4	•	•	Copy formula in cell C3		
5	•	•			
6	•	•			
7	•	•			

← Comment row

Figure A.9 Numerical integration algorithm on a spreadsheet.

B.2 ROOT OF A FUNCTION

We seek the value of x in a function that satisfies the equation

$$f(x) = 0 \tag{B.3}$$

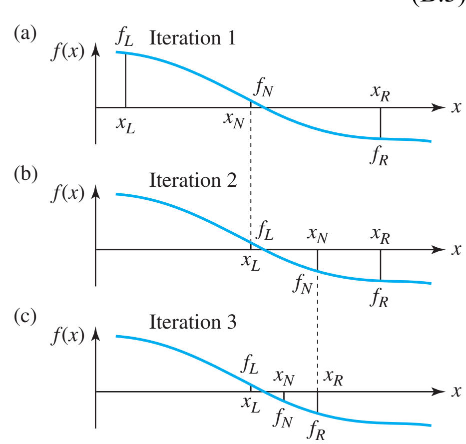

Figure A.10 Roots of an equation by halving the interval.

We are trying to find that value of x at which $f(x)$ crosses the x axis. Suppose we can find two values of x for which the function $f(x)$ has different signs. Then we know that the root of Equation (B.3) will be bracketed by these values. Let the two values of x that bracket the root from the left and the right be represented by x_L and x_R. Let the corresponding function values be $f_L = f(x_L)$

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and $f_R = f(x_R)$, as shown in Figure A.10*a*. We can find the mean value $x_N = (x_L + x_R)/2$ and calculate the function value $f_N = f(x_N)$. We compare the sign of f_N to those of f_L and f_R and replace the one with the same sign, as elaborated below.

In iteration 1, f_N has the same sign as f_L; hence in iteration 2 we make the x_L value as x_N and the f_L value as f_N. In so doing we ensure that the root of the equation is still bracketed by x_L and x_R, but the size of the interval bracketing the root has been halved. On repeating the process in iteration 2, we find the mean value x_N, and the corresponding value f_N has the same sign as f_R. Thus for iteration 3, x_R and f_R are replaced by x_N and f_N found in iteration 2. In each iteration the root is bracketed by an interval that is half the interval in the previous iteration. When f_N reaches a small enough value, the iteration is stopped and x_N is the approximate root of Equation (B.3). This iterative technique for finding the root is called *half interval method* or *bisection method.*

B.2.1 Algorithm for Finding the Root of an Equation

The steps in the algorithm for computing the root of Equation (B.3) numerically are listed here. The computation of $f(x)$ should be done in a subprogram, which is not shown in the algorithm. It is assumed that the x_L and x_R values that bracket the root are known, but the algorithm checks to ensure that the root is bracketed by x_L and x_R. Note that if two functions have the same sign, then the product will yield a positive value.

1. Read the values of x_L and x_R.
2. Calculate $f_L = f(x_L)$ and $f_R = f(x_R)$.
3. If the product $f_L f_R > 0$, print "root of equation not bracketed" and stop.
4. Calculate $x_N = (x_L + x_R)/2$ and $f_N = f(x_N)$.
5. If the absolute value of f_N is less than 0.0001 (or a user-specified small number), then go to step 8.
6. If the product $f_L f_N > 0$, then replace x_L by x_N, and f_L by f_N. Go to step 4.
7. If the product $f_R f_N > 0$, then replace x_R by x_N, and f_R by f_N. Go to step 4.
8. Print the value of x_N as the root of the equation and stop.

B.2.2 Use of a Spreadsheet for Finding the Root of a Function

Finding the roots of a function on a spreadsheet can be done without the algorithm described. The method is in essence a digital equivalent to making a plot to find the value of x where the function $f(x)$ crosses the x axis.

To demonstrate the use of a spreadsheet for finding the root of a function, consider the function $f(x) = x^2 - 28.54x + 88.5$. We guess that the root is likely to be a value of x between 0 and 10.

Trial 1: In cell A2 of Figure A.11*a* we enter our starting guess as $x = 0$. In cell A3 we increment the value of cell A2 by 1, then copy the formula in the next nine cells (copying into more cells will not be incorrect or cause problems). In cell B2 we write our formula for finding $f(x)$ and then copy it into the cells below. The results of this trial are shown in Figure A.11*b*. We note that the function value changes sign between $x = 3$ and $x = 4$ in trial 1.

Trial 2: Based on our results of trial 1, we set $x = 3$ as our starting guess in cell D2. In cell D3 we increment the value of cell D2 by 0.1 and then copy the formula into the cells below. We copy the formula for $f(x)$ from cell B2 into the column starting at cell E2. The results of this trial are given in Figure A.11*b*. The function changes sign between $x = 3.5$ and $x = 3.6$.

Trial 3: Based on our results of trial 2, we set $x = 3.5$ as our starting guess in cell G2. In cell G3 we increment the value of cell G2 by 0.01 and then copy the formula into the cells below. We copy the formula for $f(x)$ from cell B2 into the column starting at cell H2. The results of this trial are given in Figure A.11*b*. The function value is nearly zero at $x = 3.54$, which gives us our root of the function.

The starting value and the increments in x are all educated guesses that will not be difficult to make for the problems in this book. If there are multiple roots, these too can be determined and, based on the problem, the correct root chosen.

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(a)

	A	B	C	D	E	F	G	H
	Trial 1			Trial 2			Trial 3	
1	x	$f(x)$		x	$f(x)$		x	$f(x)$
2	0	=A2*A2−28.54*A2+88.5		3	Copy formula from cell B2		3.5	Copy formula from cell B2
3	=A2+1	Copy formula from cell B2		=D2+0.1			=G2+0.01	
4	Copy formula from cell A3			Copy formula from cell D3			Copy formula from cell D3	
5								
6								
7								

(b)

Trial 1 x	Trial 1 $f(x)$	Trial 2 x	Trial 2 $f(x)$	Trial 3 x	Trial 3 $f(x)$
0	88.5	3	11.88	3.5	0.86
1	60.96	3.1	9.636	3.51	0.6447
2	35.42	3.2	7.412	3.52	0.4296
$\boxed{3}$	11.88	3.3	5.208	3.53	0.2147
4	−9.66	3.4	3.024	$\boxed{3.54}$	2.84217E-14
5	−29.2	$\boxed{3.5}$	0.86	3.55	−0.2145
6	−46.74	3.6	−1.284	3.56	−0.4288
7	−62.28	3.7	−3.408	3.57	−0.6429
8	−75.82	3.8	−5.512	3.58	−0.8568
9	−87.36	3.9	−7.596	3.59	−1.0705
10	−96.9	4	−9.66	3.6	−1.284

Figure A.11 Roots of an equation using spreadsheet.

B.3 DETERMINING COEFFICIENTS OF A POLYNOMIAL

We assume that at N points x_i we know the values of a function f_i. Often the values of x_i and f_i are known from an experiment. We would like to approximate the function by the quadratic function

$$f(x) = a_0 + a_1 x + a_2 x^2 \tag{B.4}$$

If $N = 3$, then there is a unique solution to the values of a_0, a_1, and a_2. However, if $N > 3$, then we are trying find the coefficients a_0, a_1, and a_2 such that the error of approximation is minimized. One such method of defining and minimizing the error in approximation is the least-squares method elaborated next.

If we substitute $x = x_i$ in Equation (B.4), the value of the function $f(x_i)$ may be different than the value f_i. This difference is the error e_i, which can be written as

$$e_i = f_i - f(x_i) = f_i - (a_0 + a_1 x_i + a_2 x_i^2) \tag{B.5}$$

In the least-squares method an error E is defined as $E = \sum_{i=1}^{N} e_i^2$. This error E is then minimized with respect to the coefficients a_0, a_1, and a_2 and to generate a set of linear algebraic equations. These equations are then solved to obtain the coefficients.

Minimizing E implies setting the first derivative of E with respect to the coefficients equal to zero, as follows. In these equations all summations are performed for $i = 1$ to N.

$$\frac{\partial E}{\partial a_0} = 0 \quad \text{or} \quad \sum 2e_i \frac{\partial e_i}{\partial a_0} = 0 \quad \text{or} \quad \sum 2[f_i - (a_0 + a_1 x_i + a_2 x_i^2)][-1] = 0 \tag{B.6}$$

$$\frac{\partial E}{\partial a_1} = 0 \quad \text{or} \quad \sum 2e_i \frac{\partial e_i}{\partial a_1} = 0 \quad \text{or} \quad \sum 2[f_i - (a_0 + a_1 x_i + a_2 x_i^2)][-x_i] = 0 \tag{B.7}$$

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$$\frac{\partial E}{\partial a_2} = 0 \quad \text{or} \quad \sum 2e_i \frac{\partial e_i}{\partial a_2} = 0 \quad \text{or} \quad \sum 2[f_i - (a_0 + a_1 x_i + a_2 x_i^2)][-x_i^2] = 0 \tag{B.8}$$

The equations on the right can be rearranged and written in matrix form,

$$\begin{bmatrix} N & \sum x_i & \sum x_i^2 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} \sum f_i \\ \sum x_i f_i \\ \sum x_i^2 f_i \end{Bmatrix} \quad \text{or} \quad \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \end{Bmatrix} = \begin{Bmatrix} r_1 \\ r_2 \\ r_3 \end{Bmatrix} \tag{B.9}$$

The coefficients of the b matrix and the r vector can be determined by comparison to the matrix form of the equations on the left. The coefficients a_0, a_1, and a_2 can be determined by Cramer's rule. Let D represent the determinant of the b matrix. By Cramer's rule, we replace the first column in the matrix of b's by the right-hand side, find the determinant of the constructed matrix, and divide by D. Thus, the coefficients a_0, a_1, and a_2 can be written as

$$a_0 = \frac{\begin{vmatrix} r_1 & b_{12} & b_{13} \\ r_2 & b_{22} & b_{23} \\ r_3 & b_{32} & b_{33} \end{vmatrix}}{D} \qquad a_1 = \frac{\begin{vmatrix} b_{11} & r_1 & b_{13} \\ b_{21} & r_2 & b_{23} \\ b_{31} & r_3 & b_{33} \end{vmatrix}}{D} \qquad a_2 = \frac{\begin{vmatrix} b_{11} & b_{12} & r_1 \\ b_{21} & b_{22} & r_2 \\ b_{31} & b_{32} & r_3 \end{vmatrix}}{D} \tag{B.10}$$

Evaluating the determinant by expanding about the r elements, we obtain the values of a_0, a_1, and a_2

$$D = b_{11}C_{11} + b_{12}C_{12} + b_{13}C_{13} \tag{B.11}$$

$$a_0 = [C_{11}r_1 + C_{12}r_2 + C_{13}r_3]/D \tag{B.12}$$

$$a_1 = [C_{21}r_1 + C_{22}r_2 + C_{23}r_3]/D \tag{B.13}$$

$$a_2 = [C_{31}r_1 + C_{32}r_2 + C_{33}r_3]/D \tag{B.14}$$

where

$$\begin{aligned} C_{11} &= b_{22}b_{33} - b_{23}b_{32} & C_{12} &= C_{21} = -[b_{21}b_{33} - b_{23}b_{31}] \\ C_{13} &= C_{31} = b_{21}b_{32} - b_{22}b_{31} & C_{22} &= b_{11}b_{33} - b_{13}b_{31} \\ C_{23} &= C_{32} = -[b_{11}b_{23} - b_{13}b_{21}] & C_{33} &= b_{11}b_{22} - b_{12}b_{21} \end{aligned} \tag{B.15}$$

It is not difficult to extend these equations to higher order polynomials. However, a numerical method for solving the algebraic equations will be needed as the size of the b matrix grows. For problems in this book a quadratic representation of the function is adequate.

B.3.1 Algorithm for Finding Polynomial Coefficients

The steps in the algorithm for computing the coefficients of a quadratic function numerically by the least-squares method are listed here. It is assumed that x_i and f_i, are known values at N points.

1. Read the value of N.
2. Read the values of x_i and f_i for $i = 1$ to N.
3. Initialize the matrix coefficients b and r to zero.
4. Set $b_{11} = N$.
5. For $i = 1$ to N, execute the following computations:

$$b_{12} = b_{12} + x_i \qquad b_{13} = b_{13} + x_i^2 \qquad b_{23} = b_{23} + x_i^3 \qquad b_{33} = b_{33} + x_i^4$$

$$r_1 = r_1 + f_i \qquad r_2 = r_2 + x_i f_i \qquad r_3 = r_3 + x_i^2 f_i$$

6. Set $b_{21} = b_{12}$, $b_{22} = b_{13}$, $b_{31} = b_{13}$, $b_{32} = b_{23}$.
7. Determine D using Equation (B.11).
8. Determine the coefficients a_0, a_1, and a_2 using Equations (B.12), (B.13), and (B.14).

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B.3.2 Use of a Spreadsheet for Finding Polynomial Coefficients

Figure A.12 shows a sample spreadsheet that can be used to evaluate the coefficients in a quadratic polynomial numerically. The data x_i and $f(x_i)$ can be either typed or imported into columns A and B of the spreadsheet, starting at row 2. In cells C2 through G2, the various quantities shown in row 1 can be found and the formulas copied to the rows below. We assume that the data fill up to row 50, that is, $N = 49$. In cell A51 the sum of the cells between cells A2 and A50 can be found using the summation command in the spreadsheet. By copying the formula to cells B51 through G51, the remaining sums in Equation (B.9) can be found. The coefficients in the b matrix and the right-hand-side r vector in Equation (B.9) can be identified as shown in comment row 52. The formulas in Equations (B.11) through (B.14) in terms of cell numbers can be entered in row 53, and D, a_0, a_1, and a_2 can be found.

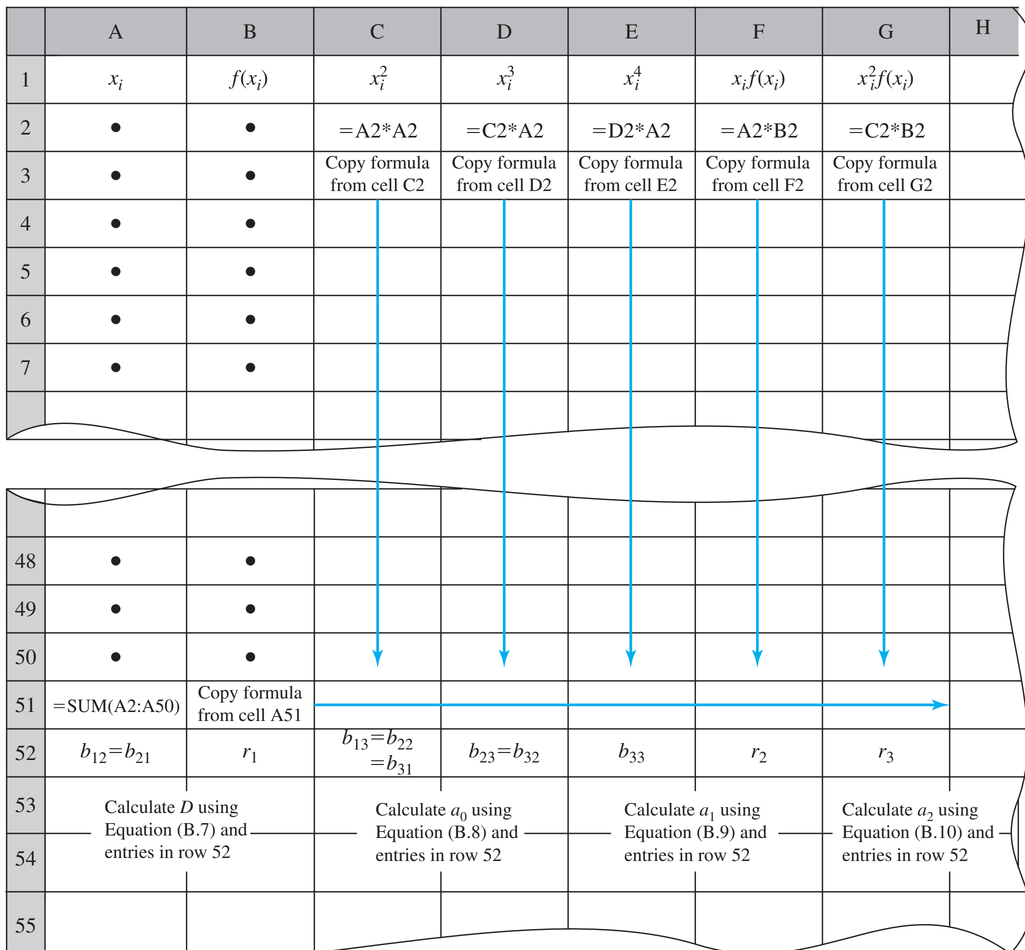

Figure A.12 Numerical evaluation of coefficients in a quadratic function on a spreadsheet.

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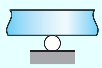
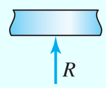
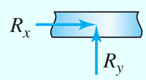
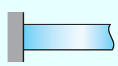
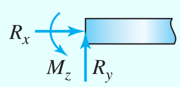
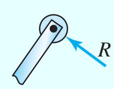
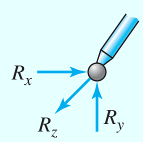
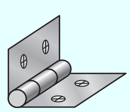
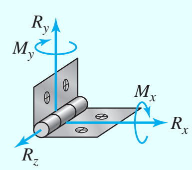
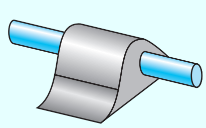
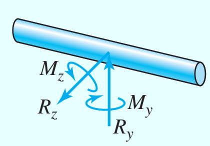
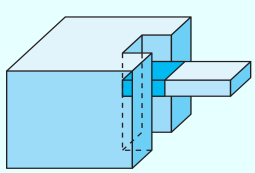
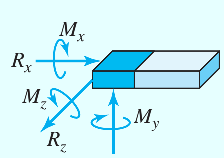
APPENDIX C
REFERENCE INFORMATION

C.1 SUPPORT REACTIONS

TABLE C.1 **Reactions at the support**

Type of Support	Reactions	Comments
 Roller on smooth surface		Only downward translation is prevented. Hence the reaction force is upward.
 Smooth pin		Translation in the horizontal and vertical directions is prevented. Hence the reaction forces R_x and R_y can be in the directions shown, or opposite.
 Fixed support		Beside translation in the horizontal and vertical directions, rotation about the *z* axis is prevented. Hence the reactions R_x and R_y and M_z can be in the directions shown, or opposite.
 Roller in smooth slot		Translation perpendicular to slot is prevented. The reaction force R can be in the direction shown, or opposite.
 Ball and socket		Translation in all directions is prevented. The reaction forces can be in the directions shown, or opposite.
 Hinge		Except for rotation about the hinge axis, translation and rotation are prevented in all directions. Hence the reaction forces and moments can be in the directions shown, or opposite.
 Journal bearing		Translation and rotation are prevented in all directions, except in the direction of the shaft axis. Hence the reaction forces and moments can be in the directions shown, or opposite.
 Smooth slot		Translation in the *z* direction and rotation about any axis are prevented. Hence the reaction force R_z and reaction moments can be in the directions shown, or opposite. Translation in the *x* direction into the slot is prevented but not out of it. Hence the reaction force R_x should be in the direction shown.

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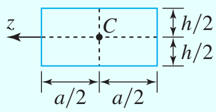
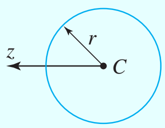
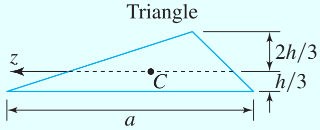
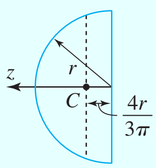
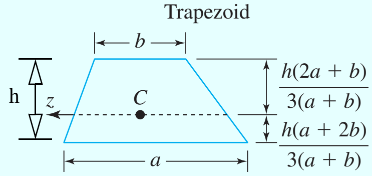
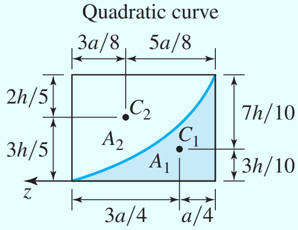
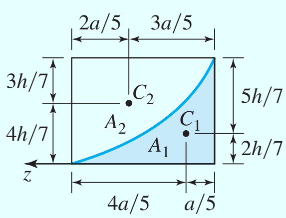
C.2 GEOMETRIC PROPERTIES OF COMMON SHAPES

TABLE C.2 **Areas, centroids, and second area moments of inertia**

Shapes[a]	**Areas**	**Second Area Moments of Inertia**
Rectangle 	$A = ah$	$I_{zz} = \frac{1}{12}ah^3$
Circle 	$A = \pi r^2$	$I_{zz} = \frac{1}{4}\pi r^4 \qquad J = \frac{1}{2}\pi r^4$
Triangle 	$A = \frac{ah}{2}$	$I_{zz} = \frac{1}{36}ah^3$
Semicircle 	$A = \frac{\pi r^2}{2}$	$I_{zz} = \frac{1}{8}\pi r^4$
Trapezoid 	$A = \frac{h(a+b)}{2}$	$I_{zz} = \frac{h^3(a^2+4ab+b^2)}{36(a+b)}$
Quadratic curve 	$A_1 = \frac{ah}{3}$ $A_2 = \frac{2ah}{3}$	$(I_{zz})_1 = \frac{1}{21}ah^3$ $(I_{zz})_2 = \frac{2}{7}ah^3$
Cubic curve 	$A_1 = \frac{ah}{4}$ $A_2 = \frac{3ah}{4}$	$(I_{zz})_1 = \frac{1}{30}ah^3$ $(I_{zz})_2 = \frac{3}{10}ah^3$

a. *C*-location of centroid.

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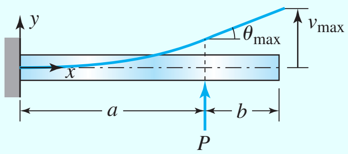
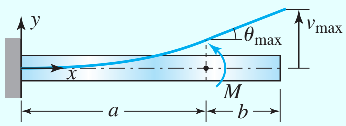
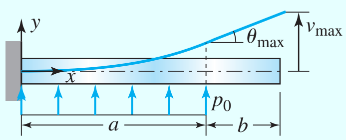
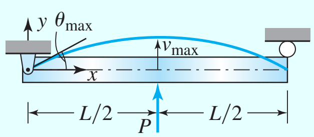
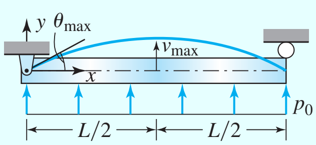
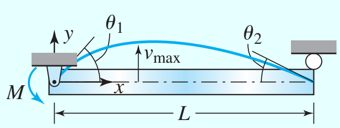
C.3 FORMULAS FOR DEFLECTION AND SLOPES OF BEAMS

TABLE C.3 Deflections and slopes of beams[a]

Case	Beam and Loading	Maximum Deflection and Slope	Elastic Curve
1		$v_{max} = \frac{Pa^2}{6EI}(2a+3b)$ $\theta_{max} = \frac{Pa^2}{2EI}$	$v = \frac{Px^2}{6EI}(3a-x)$ for $0 \le x \le a$ $v = \frac{Pa^2}{6EI}(3x-a)$ for $x \ge a$
2		$v_{max} = \frac{Ma(a+2b)}{2EI}$ $\theta_{max} = \frac{Ma}{EI}$	$v = \frac{Mx^2}{2EI}$ for $0 \le x \le a$ $v = \frac{Ma}{2EI}(2x-a)$ for $x \ge a$
3		$v_{max} = \frac{p_0 a^3(3a+4b)}{24EI}$ $\theta_{max} = \frac{p_0 a^3}{6EI}$	$v = \frac{p_0 x^2}{24EI}(x^2 - 4ax + 6a^2)$ for $0 \le x \le a$ $v = \frac{p_0 a^3}{24EI}(4x-a)$ for $x \ge a$
4		$v_{max} = \frac{PL^3}{48EI}$ $\theta_{max} = \frac{PL^2}{16EI}$	$v = \frac{Px}{48EI}(3L^2 - 4x^2)$ for $0 \le x \le \frac{L}{2}$
5		$v_{max} = \frac{5p_0 L^4}{384EI}$ $\theta_{max} = \frac{p_0 L^3}{24EI}$	$v = \frac{p_0 x}{24EI}(x^3 - 2Lx^2 + L^3)$
6		$v_{max} = \frac{ML^2}{9\sqrt{3}EI}$ @ $x = 0.4226L$ $\theta_1 = \frac{ML}{3EI}$ $\theta_2 = \frac{ML}{6EI}$	$v = \frac{Mx}{6EIL}(x^2 - 3Lx + 2L^2)$

[a] These equations can be used for composite beams by replacing the bending rigidity EI by the sum of bending rigidities $\sum E_i I_i$.

C.4 CHARTS OF STRESS CONCENTRATION FACTORS

The stress concentration factor charts given in this section are approximate. For more accurate values the reader should consult a handbook. From Equation (3.25), the stress concentration factor is defined as

$$\boxed{K = \frac{\sigma_{max}}{\sigma_{nom}} \quad \text{or} \quad K = \frac{\tau_{max}}{\tau_{nom}}} \tag{C.1}$$

where σ_{max} and τ_{max} are the maximum normal and shear stress, respectively; σ_{nom} and τ_{nom} are the nominal normal and shear stress obtained from elementary theories.

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C.4.1 Finite Plate with a Central Hole

Figure A.13 shows two stress concentration factors that differ because of the cross-sectional area used in the calculation of the nominal stress. If the gross cross-sectional area Ht of the plate is used, then we obtain the nominal stress

$$(\sigma_{\text{nom}})_{\text{gross}} = \frac{P}{Ht} \tag{C.2a}$$

and the top line in Figure A.13 should be used for the stress concentration factor. If the net area at the hole $(H - d)t$ is used, then we obtain the nominal stress

$$(\sigma_{\text{nom}})_{\text{net}} = \frac{P}{(H-d)t} \tag{C.2b}$$

and the bottom line in Figure A.13 should be used for the stress concentration factor. The two stress concentration factors are related as

$$K_{\text{net}} = \left(1 - \frac{d}{H}\right)K_{\text{gross}} \tag{C.2c}$$

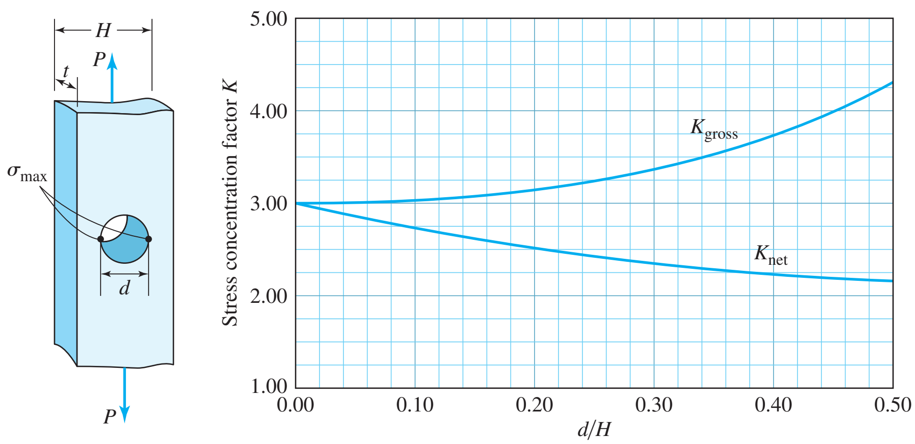

Figure A.13 Stress concentration factor for plate with a central hole.

C.4.2 Stepped axial circular bars with shoulder fillet

The maximum axial stress in a stepped circular bar with shoulder fillet will depend on the values of the diameters D and d of the two circular bars and the radius of the fillet r. From these three variables we can create two nondimensional variables D/d and r/d for showing the variation of the stress concentration factor, as illustrated in Figure A.14. The maximum nominal axial stress will be in the smaller diameter bar and, from Equation (4.8), is given by

$$\sigma_{\text{nom}} = \frac{4P}{\pi d^2} \tag{C.3}$$

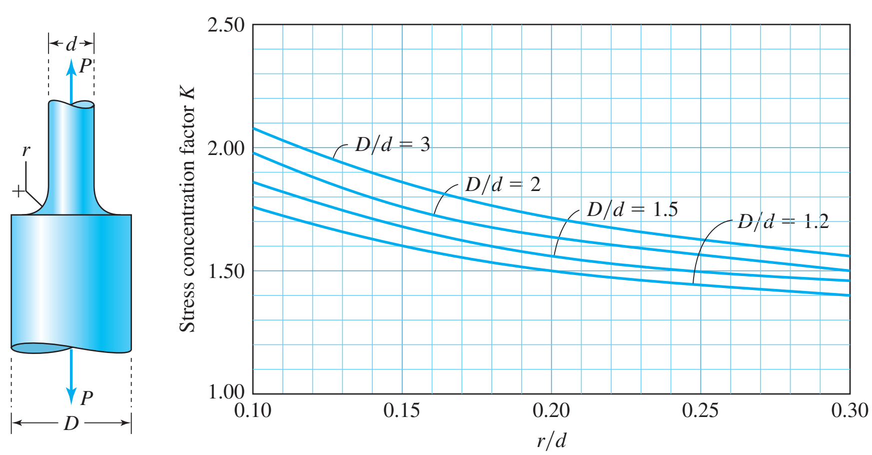

Figure A.14 Stress concentration factor for stepped axial circular bars with shoulder fillet.

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C.4.3 Stepped circular shafts with shoulder fillet in torsion

The maximum shear stress in a stepped circular shaft with shoulder fillet will depend on the values of the diameters D and d of the two circular shafts and the radius of the fillet r. From these three variables we can create two nondimensional variables D/d and r/d to show the variation of the stress concentration factor, as illustrated in Figure A.15. The maximum nominal shear stress will be on the outer surface of the smaller diameter bar and, from Equation (5.10), is given by

$$\tau_{\text{nom}} = \frac{16T}{\pi d^3} \tag{C.4}$$

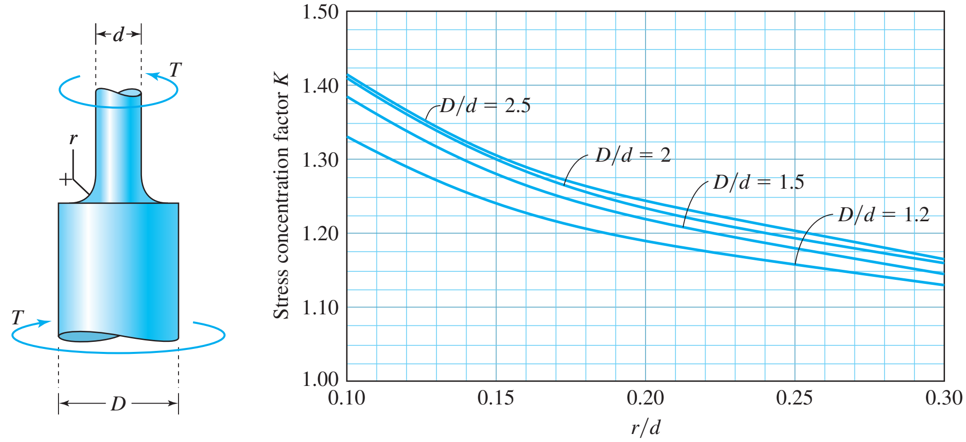

Figure A.15 Stress concentration factor for stepped circular shaft with shoulder fillet.

C.4.4 Stepped circular beam with shoulder fillet in bending

The maximum bending normal stress in a stepped circular beam with shoulder fillet will depend on the values of the diameters D and d of the two circular shafts and the radius of the fillet r. From these three variables we can create two nondimensional variables D/d and r/d to show the variation of the stress concentration factor, as illustrated in Figure A.16. The maximum nominal bending normal stress will be on the outer surface in the smaller diameter bar and, from Equation (6.12), is given by

$$\sigma_{\text{nom}} = \frac{32M}{\pi d^3} \tag{C.5}$$

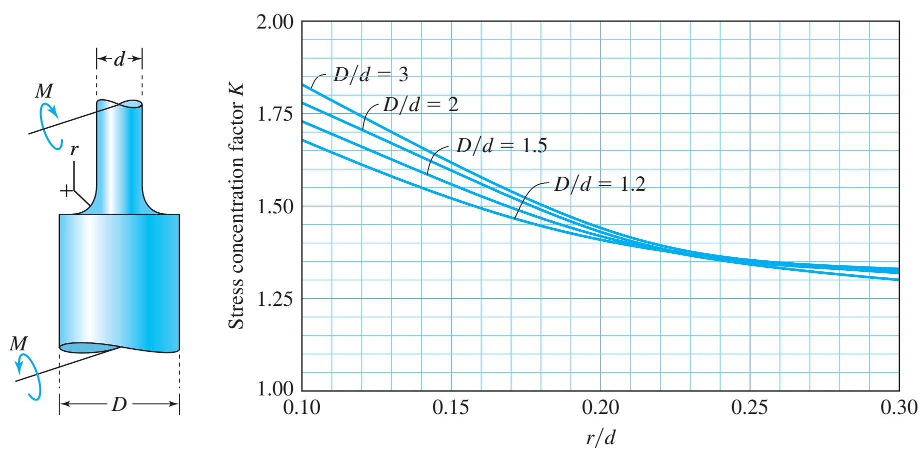

Figure A.16 Stress concentration factor for stepped circular beam with shoulder fillet.

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C.5 PROPERTIES OF SELECTED MATERIALS

Material properties depend on many variables and vary widely. The properties given here are approximate mean values. Elastic strength may be represented by yield stress, proportional limit, or offset yield stress. Both elastic strength and ultimate strength refer to tensile strength unless stated otherwise.

TABLE C.4 **Material properties in U.S. customary units**

Material	Specific Weight (lb/in.^{3})	Modulus of Elasticity E (ksi)	Poisson's Ratio ν	Coefficient of Thermal Expansion α (μ/°F)	Elastic Strength (ksi)	Ultimate Strength (ksi)	Ductility (% elongation)
Aluminum	0.100	10,000	0.25	12.5	40	45	17
Bronze	0.320	15,000	0.34	9.4	20	50	20
Concrete	0.087	4000	0.15	6.0		2*	
Copper	0.316	15,000	0.35	9.8	12	35	35
Cast iron	0.266	25,000	0.25	6.0	25*	50*	
Glass	0.095	7500	0.20	4.5		10	
Plastic	0.035	400	0.4	50		9	50
Rock	0.098	8000	0.25	4	12*	78*	
Rubber	0.041	0.3	0.5	90	0.5	2	300
Steel	0.284	30,000	0.28	6.6	30	90	30
Titanium	0.162	14,000	0.33	5.3	135	155	13
Wood	0.02	1800	0.30			5*	

*Compressive strength.

TABLE C.5 **Material properties in metric units**

Material	Density (mg/m^3)	Modulus of Elasticity E (GPa)	Poisson's Ratio ν	Coefficient of Thermal Expansion α (μ/°C)	Elastic Strength (MPa)	Ultimate Strength (MPa)	Ductility (% elongation)
Aluminum	2.77	70	0.25	12.5	280	315	17
Bronze	8.86	105	0.34	9.4	140	350	20
Concrete	2.41	28	0.15	6.0		14*	
Copper	8.75	105	0.35	9.8	84	245	35
Cast iron	7.37	175	0.25	6.0	175*	350*	
Glass	2.63	52.5	0.20	4.5		70	0
Plastic	0.97	2.8	0.4	50		63	50
Rock	2.72	56	0.25	4	84*	546*	
Rubber	1.14	2.1	0.5	90	3.5	14	300
Steel	7.87	210	0.28	6.6	210	630	30
Titanium	4.49	98	0.33	5.3	945	1185	13
Wood	0.55	12.6	0.30			35*	

*Compressive strength.

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C.6 GEOMETRIC PROPERTIES OF STRUCTURAL STEEL MEMBERS

TABLE C.6 **Wide-flange sections (FPS units)**

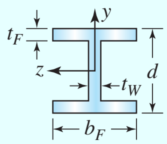				Flange		z Axis			y Axis		
Designation (in. × lb/ft)	**Depth d (in.)**	**Area A (in.^{2})**	**Web Thickness t_W (in.)**	**Width b_F (in.)**	**Thickness t_F (in.)**	**I_{zz} (in.^{4})**	**S_z (in.^{3})**	**r_z (in.)**	**I_{yy} (in.^{4})**	**S_y (in.^{3})**	**r_y (in.)**
W12 × 35	12.50	10.3	0.300	6.560	0.520	285.0	45.6	5.25	24.5	7.47	1.54
W12 × 30	12.34	8.79	0.260	6.520	0.440	238	38.6	5.21	20.3	6.24	1.52
W10 × 30	10.47	8.84	0.300	5.81	0.510	170	32.4	4.38	16.7	5.75	1.37
W10 × 22	10.17	6.49	0.240	5.75	0.360	118	23.2	4.27	11.4	3.97	1.33
W8 × 18	8.14	5.26	0.230	5.250	0.330	61.9	15.2	3.43	7.97	3.04	1.23
W8 × 15	8.11	4.44	0.245	4.015	0.315	48	11.8	3.29	3.41	1.70	0.876
W6 × 20	6.20	5.87	0.260	6.020	0.365	41.4	13.4	2.66	13.3	4.41	1.50
W6 × 16	6.28	4.74	0.260	4.03	0.405	32.1	10.2	2.60	4.43	2.20	0.967

TABLE C.7 **Wide-flange sections (metric units)**

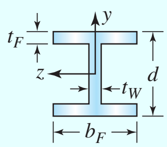				Flange		z Axis			y Axis		
Designation (mm × kg/m)	**Depth d (mm)**	**Area A (mm^2)**	**Web Thickness t_W (mm)**	**Width b_F (mm)**	**Thickness t_F (mm)**	**I_{zz} (10^6 mm^4)**	**S_z (10^3 mm^3)**	**r_z (mm)**	**I_{yy} (10^6 mm^4)**	**S_y (10^3 mm^3)**	**r_y (mm)**
W310 × 52	317	6650	7.6	167	13.2	118.6	748	133.4	10.20	122.2	39.1
W310 × 44.5	313	5670	6.6	166	11.2	99.1	633	132.3	8.45	101.8	38.6
W250 × 44.8	266	5700	7.6	148	13.0	70.8	532	111.3	6.95	93.9	34.8
W250 × 32.7	258	4190	6.1	146	9.1	49.1	381	108.5	4.75	65.1	33.8
W200 × 26.6	207	3390	5.8	133	8.4	25.8	249	87.1	3.32	49.9	31.2
W200 × 22.5	206	2860	6.2	102	8.0	20.0	194.2	83.6	1.419	27.8	22.3
W150 × 29.8	157	3790	6.6	153	9.3	17.23	219	67.6	5.54	72.4	28.1
W150 × 24	160	3060	6.6	102	10.3	13.36	167	66	1.844	36.2	24.6

TABLE C.8 **S shapes (FPS units)**

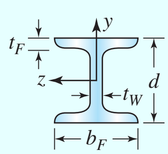				Flange		z Axis			y Axis		
Designation (in. × lb/ft)	**Depth d (in.)**	**Area A (in.^{2})**	**Web Thickness t_W (in.)**	**Width b_F (in.)**	**Thickness t_F (in.)**	**I_{zz} (in.^{4})**	**S_z (in.^{3})**	**r_z (in.)**	**I_{yy} (in.^{4})**	**S_y (in.^{3})**	**r_y (in.)**
S12 × 35	12	10.3	0.428	5.078	0.544	229	38.4	4.72	9.87	3.89	0.98
S12 × 31.8	12	9.35	0.350	5.000	0.544	218	36.4	4.83	9.36	3.74	1.0
S10 × 35	10	10.3	0.594	4.944	0.491	147	29.4	3.78	8.36	3.38	0.901
S10 × 25.4	10	7.46	0.311	4.661	0.491	124	24.7	4.07	6.79	2.91	0.954
S8 × 23	8	6.77	0.411	4.171	0.426	64.9	16.2	3.10	4.31	2.07	.798
S8 × 18.4	8	5.41	0.271	4.001	0.426	57.6	14.4	3.26	3.73	1.86	0.831
S7 × 20	7	5.88	0.450	3.860	0.392	42.4	12.1	2.69	3.17	1.64	0.734
S7 × 15.3	7	4.50	0.252	3.662	0.392	36.9	10.5	2.86	2.64	1.44	0.766

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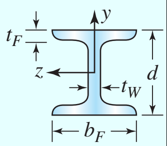
TABLE C.9 **S shapes (metric units)**



Designation (mm × kg/m)	Depth d (mm)	Area A (mm^2)	Web Thickness t_W (mm)	Flange Width b_F (mm)	Flange Thickness t_F (mm)	z Axis I_{zz} (10^6 mm^4)	z Axis S_z (10^3 mm^3)	z Axis r_z (mm)	y Axis I_{yy} (10^6 mm^4)	y Axis S_y (10^3 mm^3)	y Axis r_y (mm)
S310 × 52	305	6640	10.9	129	13.8	95.3	625	119.9	4.11	63.7	24.9
S310 × 47.3	305	6032	8.9	127	13.8	90.7	595	122.7	3.90	61.4	25.4
S250 × 52	254	6640	15.1	126	12.5	61.2	482	96.0	3.48	55.2	22.9
S250 × 37.8	254	4806	7.9	118	12.5	51.6	406	103.4	2.83	48.0	24.2
S200 × 34	203	4368	11.2	106	10.8	27.0	266	78.7	1.794	33.8	20.3
S200 × 27.4	203	3484	6.9	102	10.8	24	236	82.8	1.553	30.4	21.1
S180 × 30	178	3794	11.4	97	10.0	17.65	198.3	68.3	1.319	27.2	18.64
S180 × 22.8	178	2890	6.4	92	10.0	15.28	171.7	72.6	1.099	23.9	19.45

C.7 GLOSSARY

The terms used in this book are given in alphabetic order. The third column gives the chapter number in which the term is first introduced, with A representing Appendix A.

Term	Definition	Chapter
Anisotropic material:	A material that has a stress-strain relationships that changes with orientation of the coordinate system at a point.	3
Axial force diagram:	A plot of the internal axial force ***N*** versus *x*.	4
Axial force:	Normal force acting on a surface in the direction of the axis of the body.	A
Axial member:	A long straight body on which the forces are applied along the longitudinal axis.	4
Axial rigidity:	The product of modulus of elasticity (E) and cross-sectional area (A).	4
Axial stress:	The normal stress acting in the direction of the axis of a slender member.	1
Axial template:	An infinitesimal segment of an axial bar constructed by making an imaginary cuts on either side of a supposed external axial force.	4
Axisymmetric body:	A body whose geometry, material properties, and loading are symmetric with respect to an axis.	1
Bauschinger effect:	Material breaking at stress levels smaller than the ultimate stress due to load cycle reversal in the plastic region.	3
Beam template:	An infinitesimal segment of a beam constructed by making an imaginary cuts on either side of a supposed external force or moment.	6
Beam:	A long structural member on which loads are applied perpendicular to the axis.	6
Bearing stress:	The compressive normal stress that is produced when one surface presses against another.	1
Bending moment:	Moments about an axis tangent to a surface of a body.	A
Bending rigidity:	The product of modulus of elasticity (E) and the second area moment of inertia (I_{zz}) about the bending axis.	6
Bifurcation point:	The point at which more then one equilibrium configuration may exist.	11
Body forces:	External forces that act at every point on the body.	A

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Term	Definition	Chapter
Boundary-value problem:	The mathematical statement listing of all the differential equations and all the necessary conditions to solve them.	7
Brittle material:	A material that exhibits little or no plastic deformation at failure.	3
Buckling load:	The force (or moment) at which buckling occurs. Also called critical load.	11
Buckling modes:	The deformed shape at buckling load.	11
Buckling:	An instability of equilibrium in structures that occurs from compressive loads or stresses.	11
Centroid:	An imaginary point on a body about which the first area moment is zero.	A
Characteristic equation:	The equation whose roots are the eigenvalues of the problem.	11
Columns:	Axial members that support compressive axial loads.	11
Compatibility equations:	Geometric relationships between the deformations or strains.	4
Complimentary strain energy density:	Complimentary strain energy per unit volume. It is the area between the stress axis and the stress-strain curve at a given value of stress or strain.	3
Complimentary strain energy:	Energy stored in a body due to forces acting on it.	3
Compressive stress:	Normal stress that pushes the imaginary surface into the rest of the material.	1
Concentrated forces (moment).	Surface forces (moments) applied at a point.	A
Continuity conditions:	Conditions that ensure continuity of deformations.	7
Critical load:	The force (or moment) at which buckling occurs. Also called buckling load.	11
Critical slenderness ratio:	The slenderness ratio at which material failure and buckling failure can occur simultaneously. Separates the long from the short columns.	11
Deformation:	The relative movement of a point with respect to another point on the body.	2
Degree of freedom:	The minimum number of displacements / rotations that are necessary to describe the deformed geometry.	4
Degree of static redundancy:	The number of unknown reactions minus the number of equilibrium equations.	A
Delta function:	A function that is zero everywhere except in a small interval where it tends to infinity in such a manner that the area under the curve is 1. It is also called the *Dirac* delta function.	7
Discontinuity functions:	A class of functions that are zero before a point and are non-zero after a point or are singular at the point.	7
Displacement:	The total movement of a point on a body with respect to fixed reference coordinates.	2
Distributed forces (moments):	Surface forces (moments) applied along a line or over a surface.	A
Ductile material:	A material that can undergo a large plastic deformation before fracture.	3
Eccentric loading:	Compressive axial force that is applied at a point that is not on the axis of the column.	11
Elastic curve:	Curve describing the deflection of the beam.	7
Elastic region:	The region of the stress-strain curve in which the material returns to the undeformed state when applied forces are removed.	3
Elastic-plastic boundary:	The set of points forming the boundary between the elastic and plastic regions.	3
Endurance limit:	The highest stress level for which the material would not fail under cyclic loading. Also called fatigue strength.	3
Eulerian strain:	Strain computed from deformation by using the final deformed geometry as the reference geometry.	2
Failure envelope:	The surface (or curve) that separates the acceptable design space from the unacceptable values of the variables affecting design.	10

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Term	Definition	Chapter
Failure theory:	A statement on the relationship of the stress components to the characteristic value of material failure	10
Failure:	A component or a structure does not perform the function for which it was designed.	3
Fatigue strength:	The highest stress level for which the material would not fail under cyclic loading. Also called endurance limit.	3
Fatigue:	Failure due to cyclic loading at stress levels significantly lower than the static ultimate stress.	3
Finite element method:	A numerical method used in stress analysis in which the body is divided into elements of finite size.	4
Flexibility coefficient:	The coefficient multiplying internal forces / moments in an algebraic equation.	4
Flexibility matrix:	The matrix multiplying the unknown internal forces / moments in a set of algebraic equations.	4
Fracture stress:	The stress at the point where material breaks.	3
Free surface:	A surface on which there are no forces. Alternatively, a surface that is stress free.	1
Free-body diagram:	A diagram showing all the forces acting on a free body.	A
Gage length:	Length between two marks on a tension test specimen in the central region of uniform axial stress.	3
Generalized Hooke's law:	The equations relating stresses and strains in three dimensions.	3
Global coordinate system:	A fixed reference coordinate system in which the entire problem is described.	8
Hardness:	The resistance of material to indentation and scratches.	3
Homogeneous material:	A material that has same the material properties at all points in the body.	3
Hooke's law:	Equation relating normal stress and strain in the linear region of a tension test.	3
In-plane maximum shear strain:	The maximum shear strain in coordinate systems that can be obtained by rotating about the z axis.	9
In-plane maximum shear stress:	The maximum shear stress on a plane that can be obtained by rotating about the z axis.	8
Isotropic material:	A material that has a stress-strain relationships independent of the orientation of the coordinate system at a point.	3
Lagrangian strain:	Strain computed from deformation by using the original undeformed geometry as the reference geometry.	2
Load cells:	Any device that measures, controls, or applies a force or moment.	9
Loads:	External forces and moments that are applied to the body.	A
Local buckling:	Buckling that occurs in thin plates or shells due to compressive stresses.	11
Local coordinate system:	A coordinate system that can be fixed at any point on the body and has an orientation that is defined with respect to the global coordinate system.	8
Maximum normal stress theory:	A material will fail when the maximum normal stress at a point exceeds the ultimate normal stress obtained from a uniaxial tension test.	10
Maximum octahedral shear stress theory:	A material will fail when the maximum octahedral shear stress exceeds the octahedral shear stress at the yield obtained from a uniaxial tensile test.	10
Maximum shear strain:	The maximum shear strain at a point in any coordinate system.	9
Maximum shear stress theory:	A material will fail when the maximum shear stress exceeds the shear stress at yield that is obtained from a uniaxial tensile test.	10
Maximum shear stress:	The maximum shear stress at a point that acts on any plane passing through the point.	8
Method of joints:	Analysis is conducted by making imaginary cuts through all the members a the joint.	A

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Term	Definition	Chapter
Method of sections:	Analysis is conducted by making an imaginary cut (section) through a member or a structure.	A
Modulus of elasticity:	The slope of the normal stress-strain line in the linear region of a tension test. Also called Young's modulus.	3
Modulus of resilience:	Strain energy density at the yield point.	3
Modulus of rigidity:	Same as shear modulus of elasticity.	3
Modulus of toughness:	The strain energy density at rupture.	3
Mohr's failure theory:	A material will fail if a stress state is on the envelope that is tangent to the three Mohr's circles corresponding to uniaxial ultimate stress in tension, uniaxial ultimate stress in compression, and pure shear.	10
Moment diagram:	A plot of the internal bending moment M_z versus x.	6
Monotonic functions:	Functions that either continuously increases or decreases.	7
Necking:	The sudden decrease in cross-sectional area after ultimate stress.	3
Negative normal strains:	Normal strains from contraction of a line.	2
Negative shear strain:	Shear strain due to a increase of angle between orthogonal lines.	2
Neutral axis:	The line on the cross section where the bending normal stress is zero.	6
Nominal stress:	The stress predicted by theoretical models away from the regions of stress concentration.	3
Normal stress:	Internal distributed forces that are normal to an imaginary cut surface.	1
Offset yield stress:	Stress that would produce a plastic strain corresponding to the specified offset strain.	3
Pitch:	The distance between two adjoining peaks on the threads of a bolt. It is the distance moved by the nut in one full rotation.	4
Plane stress:	A state of stress in which all stress components on the z-plane are zero.	1
Plastic region:	The region in which the material deforms permanently.	3
Plastic strain:	The permanent strain when stresses are zero.	3
Poisson's ratio:	The negative ratio of lateral normal strain to longitudinal normal strain.	3
Positive normal strains:	Normal strains from elongation of a line.	2
Positive shear strain:	Shear strain due to a decrease of angle between orthogonal lines.	2
Principal angle 1:	Angle principal direction one makes with the global coordinate direction x. Counterclockwise rotation from the x axis is defined as positive.	8
Principal angles:	The angles the principal directions makes with the global coordinate system.	8
Principal angles:	The angles the principal axes make with the global coordinate system.	9
Principal axes for strain:	The coordinate axes in which the shear strain is zero.	9
Principal axis for stress:	The normal direction to the principal planes. Also referred to as the principal direction.	8
Principal direction:	The normal direction to the principal planes. Also referred to as the principal axis.	8
Principal planes:	Planes on which the shear stresses are zero.	8
Principal strain 1:	The greatest principal strain.	9
Principal strains:	Normal strains in the principal directions.	9
Principal stress 1:	The greatest principal stress.	8
Principal stress element:	A properly oriented wedge constructed from the principal planes and the plane of maximum shear stress showing all the stresses acting on the respective planes.	8

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Term	Definition	Chapter
Principal stress:	Normal stress on a principal plane. Also referred to as maximum or minimum normal stress at a point.	8
Proportional limit:	The point up to which stress and strain are related linearly.	3
Ramp function:	A function whose value is zero before a point and is a linear function after the point.	7
Reaction forces:	Forces developed at the supports that resist translation in a direction.	A
Reaction moment:	Moments developed at the support that resist rotation about an axis.	A
Rupture stress:	The stress at the point where material breaks.	3
Secant modulus:	The slope of the line that joins the origin to the point on the normal stress-strain curve at a given stress value.	3
Second order tensor:	A quantity that requires two directions and obeys certain coordinate transformation properties.	1
Section modulus:	The ratio of the second area moment of inertia about bending axis to the maximum distance from the neutral axis.	6
Shaft:	A long structural member that transmits torque from one plane to another parallel plane.	5
Shear flow:	The product of thickness and tangential shear stress along the center line of a thin cross section.	6
Shear force diagram:	A plot of the internal shear force V_y versus x.	6
Shear force:	Tangential force acting on a surface of a body.	A
Shear modulus of elasticity:	The slope of the shear stress-strain line in the linear region of a torsion test. Also called modulus of rigidity.	3
Shear stress:	Internal distributed forces that are parallel to an imaginary cut surface.	1
Singularity functions:	A class of functions that are zero everywhere except in a small region where they tend towards infinity.	7
Slenderness Ratio:	The ratio of the effective column length to the radius of gyration of the cross section about the buckling axis.	11
SN curve:	A plot of stress versus the number of cycles to failure.	3
Snap buckling:	A structure suddenly jumping (snapping) from one equilibrium position to another very different equilibrium position.	11
Statically equivalent load systems:	Two systems of forces that generate the same resultant force and moment.	A
Statically indeterminate structure:	A structure on which the number of unknown reaction forces and moments is greater than the number of equilibrium equations.	A
Step function:	A function whose value is zero before a point and equal to 1 after the point.	7
Stiffness coefficient:	The coefficient multiplying displacements / rotations in an algebraic equation.	4
Stiffness matrix:	The matrix multiplying the unknown displacements / rotations in a set of algebraic equations.	4
Strain energy density:	Strain energy per unit volume. It is the area under the stress-strain curve at a given value of stress or strain.	3
Strain energy:	Energy stored in a body due to deformation.	3
Strain hardening:	The increase of yield point each time the stress value exceeds the yield stress.	3
Stress concentration:	Large stress gradients in a small region.	3
Stress element:	An imaginary object representing a point that has surfaces with outward normals in the coordinate directions.	1
Tangent modulus:	The slope of the tangent drawn to the normal stress-strain curve at a given stress value.	3

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Term	Definition	Chapter
Tensile stress:	Normal stress that pulls the imaginary surface away from the rest of the material.	1
Tension test:	A test conducted to determine mechanical properties by applying tensile forces on a specimen.	3
Thin body:	The thickness of the body is an order of magnitude (factor of 10) smaller than the other dimensions.	1
Timoshenko beam:	Beam in which shear is accounted for by dropping the assumption that planes originally perpendicular remain perpendicular.	6
Torque Diagram:	A plot of the internal torque $\boldsymbol{T}$ versus x.	5
Torque:	Moment about an axis normal to a surface of a body.	A
Torsion template:	An infinitesimal segment of a shaft constructed by making an imaginary cuts on either side of a supposed external torque.	5
Torsional rigidity:	The product of shear modulus of elasticity (G) and the polar moment of inertia (J) of a shaft.	5
Truss:	A structure made up of two-force members.	A
Two-force member:	A structural member on which there is no moment couple and forces act at two points only.	A
Ultimate stress:	The largest stress in the stress-strain curve.	3
Warping:	Axial deformation of shaft cross section due to torque.	5
Yield point:	The point demarcating the elastic from the plastic region.	3
Yield stress:	The stress at yield point.	3
Young's modulus:	Same as modulus of elasticity.	3
Zero-force member:	A two-force member that carries no internal force.	A

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C.8 CONVERSION FACTORS BETWEEN U.S. CUSTOMARY SYSTEM (USCS) AND THE STANDARD INTERNATIONAL (SI) SYSTEM

Quantity	USCS to SI	SI to USCS
Length	1 in = 25.400 mm 1 ft = 0.3048	1 m = 39.37 in 1 m = 3.281 ft
Area	1 in^2 = 645.2 mm^2 1 ft^2 = 0.0929 m^2	1 mm^2 = $1.550(10^{-3})$ in^2 1 m^2 = 10.76 ft^2
Volume	1 in^3 = $16.39(10^3)$ mm^3 1 ft^3 = 0.028 m^3	1 mm^3 = $61.02(10^{-6})$ in^3 1 m^3 = 35.31 ft^3
Area Moment of Inertia	1 in^4 = $0.4162(10^6)$ mm^4	1 m^4 = $2.402(10^{-6})$ in^4
Mass	1 slug = 14.59 kg	1 kg = 0.06852 slugs
Force	1 lb = 4.448 N 1 kip = 4.448 kN	1 N = 0.2248 lb 1 kN = 0.2248 kip
Moment	1 in-lb = 0.1130 N-m 1 ft-lb = 1.356 N-m	1 N-m = 8.851 in-lb 1 N-m = 0.7376 ft-lb
Force per unit length	1 lb/ft = 14.59 N/m	1 N/m = 0.06852 lb/ft
Pressure; Stress	1 psi = 6.895 kPa 1 ksi = 6.895 MPa 1 lb/ft^2 = 47.88 Pa	1 kPa = 0.1450 psi 1 MPa = 0.1450 ksi 1 kPa = 20.89 lb/ft^2
Work; Energy	1 lb.ft = 1.356 J	1 J = 0.7376 lb.ft
Power	1 lb.ft/s = 1.356 W 1 hp = 745.7 W	1 W = 0.7376 lb.ft/s 1 kW = 1.341 hp

C.9 SI PREFIXES

Prefix Word	Prefix Symbol	Multiplication Factor
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

C.10 GREEK ALPHABET

Lowercase	Uppercase	Pronunciation	Lowercase	Uppercase	Pronunciation
α	A	Alpha	ν	N	Nu
β	B	Beta	ξ	ξ	Xi
γ	Γ	Gamma	o	O	Omicron
δ	Δ	Delta	π	Π	Pi
ε	E	Epsilon	ρ	P	Rho
ζ	Z	Zeta	σ	Σ	Sigma
η	H	Eta	τ	T	Tau
θ	Θ	Theta	υ	Y	Upsilon
ι	I	Iota	ϕ	Φ	Phi
κ	K	Kappa	χ	X	Chi
λ	Λ	Lambda	ψ	Ψ	Psi
μ	M	Mu	ω	Ω	Omega

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APPENDIX D
SOLUTIONS TO STATIC REVIEW EXAM

D.1 REVIEW EXAM 1

1. As the y axis is the axis of symmetry, the centroid will lie on the y axis. Thus $z_c = 0$. 1 point

Equations (A.6) and (A.9) can be used to find the y coordinate of the centroid and the area moment of inertia. Figure A.17 and Table D.4 show the calculations.

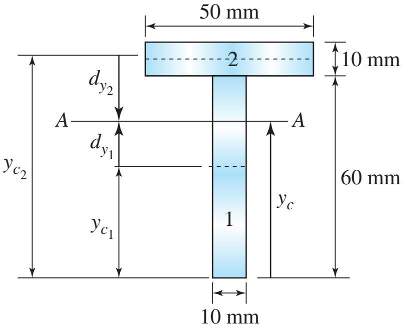

Figure A.17

TABLE D.4 **Calculation of centroid and area moment of inertia.**

	Centroids			Area moment of inertia		
Section	y_{c_i} (mm)	A_i (mm^2)	$y_{c_i}A_i$ (mm^3)	$d_{z_i} = y_c - y_{c_i}$ (mm)	$I_{z_iz_i} = \frac{1}{12}a_ib_i^3$ (mm^4)	$I_{z_iz_i} + A_id_{z_i}^2$ (mm^4)
1	30	60 x 10 = 600	18,000	15.9	10 x 60^3/12 = 180 ¥ 10^3	331.7 x 10^3
2	65	50 x 10 = 500	32,500	19.1	50 x 10^3/12 = 4.2 ¥ 10^3	186.6 x 10^3
Total		1100	50,500			

1 point for each correct entry for a total of 7 points. 1 point for each correct entry. 2 points for each correct entry. 1 point for each correct entry.

From Equations (A.6) and (A.9)we obtain

1 point for each correct answer with units.

$$y_c = \frac{50{,}500}{1100} = 45.9 \text{ mm} \qquad I_{AA} = (331.7 + 186.6)(10^3) = 518.3(10^3)\text{ mm}^4$$

2. We can replace each linear loading by an equivalent force, as shown in Figure A.7, then replace it by a single force. Using Figure A.18 we obtain

3 points/force for correct calculation.

$$F_1 = \frac{10 \times 2.5 \times 6}{2} = 75 \text{ kips} \qquad F_2 = \frac{10 \times 8 \times 3}{2} = 120 \text{ kips}$$

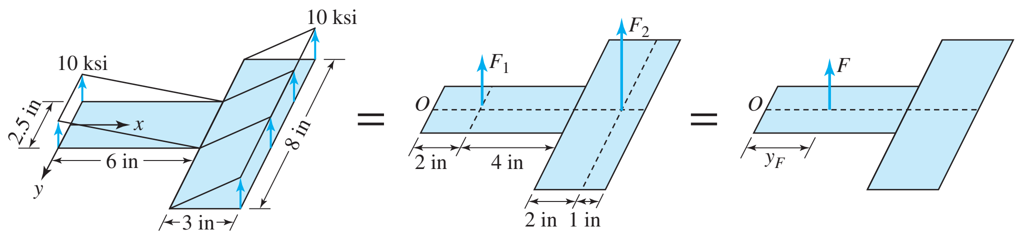

Figure A.18

For correct location of forces F_1 and F_2 3 points/force.

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The resultant forces for the two systems on the right in Figure A.18 must be the same. We thus obtain

2 points for correct answer
1 point for correct units.

$$F = F_1 + F_2 = 75 + 120 = 195 \text{ kips}$$

The resultant moments about any point (point O) for the two systems on the right of Figure A.18 must also be the same. We obtain

$$2F_1 + 8F_2 = y_F F \qquad \text{or} \qquad y_F = \frac{150 + 960}{195} = 5.69 \text{ in}$$

1 point for each correct entry in this equation

1 point for correct answer
1 point for correct units

3. We make an imaginary cut at E and draw the free-body diagrams shown in Figures A.19 and A.20.

Internal axial force calculations (Figure A.19).

Either FBD is acceptable.
For drawing: 7 kip force at A or 8 kips at D--2 marks
2 kips at B or 3 kips at C ----2 marks
Normal force at E---2 marks

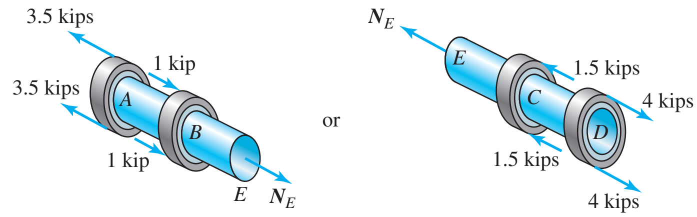

Figure A.19

1 point for correct equation

$$N_E - 7 + 2 = 0 \qquad \text{or} \qquad N_E - 8 + 3 = 0$$

1 point for correct answer
1 point for correct units
1 point for reporting tension

$$N_E = 5 \text{ kips (T)}$$

Internal torque calculations (Figure A.20)

Either FBD is acceptable.
For drawing: torque at A or D--2 marks
torque at B or at C ----2 marks
torque at E (either direction) ---2 marks

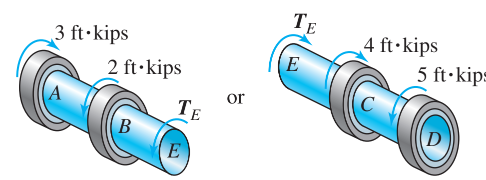

Figure A.20

2 points for correct equation

$$T_E - 3 + 2 = 0 \qquad \text{or} \qquad T_E - 5 + 4 = 0$$

1 point for correct answer
1 point for correct units

$$T_E = 1 \text{ ft·kips}$$

4. We can draw the free-body diagram of the entire beam as shown in Figure A.21*a*.

1 mark for each force or moment and 1 mark for correct location of F
Total 5 marks.

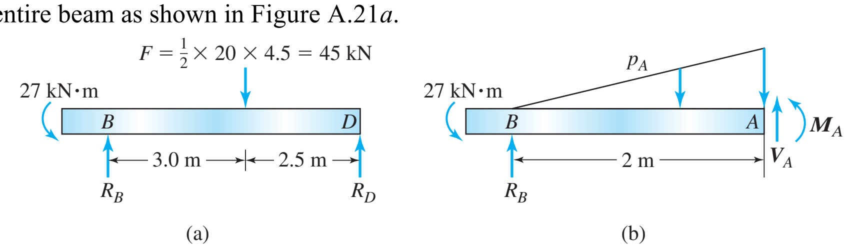

Figure A.21

By balancing the moment at D we find the reaction at B,

1 point for each term in the equation for a total of 3 points.

$$5.5R_B - 27 - 45 \times 2.5 = 0 \qquad \text{or} \qquad R_B = 25.36 \text{ kN}$$

We can then make an imaginary cut at A on the original beam and draw the free-body diagram in Figure A.21*b*. We can find the intensity of the distributed force at A by similar triangles (Figure A.22*a*),

$$\frac{p_A}{2} = \frac{20}{4.5} \qquad \text{or} \qquad p_A = 8.89 \text{ kN/m}$$

We can then replace the distributed force on the beam that is cut at A and draw the free-body diagram shown in Figure A.22*b*.

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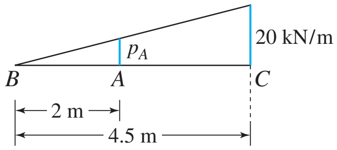
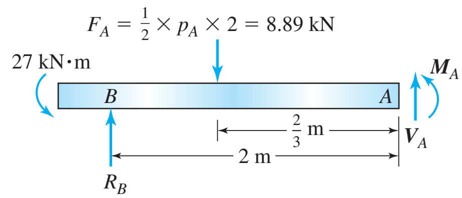
1 mark for correct value of F_A
1 mark for correct location of F_A

3 marks for correct calculation of p_A.





2 marks for showing V_A and M_A irrespective of direction.

Figure A.22 (a) (b)

Balancing the forces in the y direction we obtain

$$V_A + 25.36 - 8.89 = 0 \quad \text{or} \quad V_A = -16.5 \text{ kN}$$

1 point for correct equation correponding to your direction of V_A

1 point for correct answer

Balancing moments about point A, we obtain

$$M_A + 27 - 25.36 \times 2 + 8.89 \times 2/3 = 0 \quad \text{or} \quad M_A = 17.8 \text{ kN·m}$$

2 point for correct equation correponding to your direction of M_A

1 point for correct answer

5. By inspection we can write the following answers.

Internal Force/Moment	Section *AA* (zero/nonzero)	Section *BB* (zero/nonzero)
Axial force	Nonzero	Zero
Shear force	Nonzero in y direction	Nonzero in y direction
Shear force	Zero in x direction	Nonzero in z direction
Torque	Zero	Nonzero
Bending moment	Nonzero in x direction	Nonzero in y direction
Bending moment	Zero in y direction	Nonzero in z direction

1 point for each correct zero/non-zero entry.

1 point for each correct direction. Total 20 points.

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APPENDIX E
ANSWERS TO QUICK TESTS

QUICK TEST 1.1

1. False. Stress is an internal quantity that can only be inferred but cannot be measured directly.

2. True. A surface has a unique normal, and normal stress is the internally distributed force in the direction of the normal.

3. True. Shear stress is an internally distributed force, and internal forces are equal and opposite in direction on the two surfaces produced by an imaginary cut.

4. False. Tension implies that the normal stress pulls the imaginary surface outward, which will result in opposite directions for the stresses on the two surfaces produced by the imaginary cut.

5. False. kips are units of force, not stress.

6. False. The normal stress should be reported as tension.

7. False. 1 GPa equals 10^9 Pa.

8. False. 1 psi nearly equals 7 kPa not 7 Pa.

9. False. Failure stress values are in millions of pascals for metals.

10. False. Pressure on a surface is always normal to the surface and compressive. Stress on a surface can be normal or tangential to it, and the normal component can be tensile or compressive.

QUICK TEST 1.2

1. False. Stress at a point is a second-order tensor.

2. True. Each of the two subscripts can have three values, resulting in nine possible combinations.

3. True. The remaining three components can be found from the symmetry of shear stresses.

4. True. The fourth component can be found from the symmetry of shear stresses.

5. False. A point in plane stress has four nonzero components; thus only five components are zero in general.

6. False. The sign of stress incorporates both the direction of the force and the direction of the imaginary surface.

7. True. A stress element is an imaginary object representing a point.

8. False. The normals of the surface of a stress element have to be in the direction of the coordinate system in which the stress at a point is defined.

9. True. Stress is an internally distributed force system that is equal and in opposite directions on the two surfaces of an imaginary cut.

10. False. The sign of stress incorporates the direction of the force and the direction of the imaginary surface. Alternatively, the sign of stress at a point is independent of the orientation of the imaginary cut surface.

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QUICK TEST 2.1

1. Displacement is the movement of a point with respect to a fixed coordinate system, whereas deformation is the relative movement of a point with respect to another point on the body.
2. The reference geometry is the original undeformed geometry in Lagrangian strain and the deformed geometry in Eulerian strain.
3. The value of normal strain is 0.3/100 = 0.003.
4. The value of normal strain is $2000 \times 10^{-6} = 0.002$.
5. Positive shear strain corresponds to a decrease in the angle from right angle.
6. The strain will be positive as it corresponds to extension and is independent of the orientation of the rod.
7. No. We have defined small strain to correspond to strains less than 1%.
8. There are nine strain nonzero components in three dimensions.
9. There are four nonzero strain components in plane strain.
10. There are only three independent strain components in plane strain, as the fourth strain component can be determined from the symmetry of shear strains.

QUICK TEST 3.1

1. The modulus of elasticity has units of pascals or newtons per square meter. For metals it is usually gigapascals (GPa). Poisson's ratio has no units as it is dimensionless.
2. Offset yield stress is the stress value corresponding to a plastic strain equal to a specified offset strain.
3. Strain hardening is the increase in yield stress that occurs whenever yield stress is exceeded.
4. Necking is the sudden decrease in cross-sectional area after the ultimate stress.
5. Proportional limit defines the end of the *linear* region, whereas yield point defines the end of the *elastic* region.
6. A brittle material exhibits little plastic deformation before rupture, whereas a ductile material can undergo large plastic deformation before rupture.
7. A linear material behavior implies that stress and strain be linearly related. An elastic material behavior implies that when the loads are removed, the material returns to the undeformed state but the stress–strain relationship can be nonlinear, such as in rubber.
8. Strain energy is the energy due to deformation in a volume of material, whereas strain energy density is the strain energy per unit volume.
9. The modulus of resilience is a measure of recoverable energy and represents the strain energy density at yield point. The modulus of toughness is a measure of total energy that a material can absorb through elastic as well as plastic deformation and represents the strain energy density at ultimate stress.
10. A strong material has a high ultimate stress, whereas a tough material may not have high ultimate stress but has a large strain energy density at ultimate stress.

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QUICK TEST 3.2

1. In an isotropic material the stress–strain relationship is the same in all directions but can differ at different points. In a homogeneous material the stress–strain relationship is the same at all points provided the directions are the same.

or

In an isotropic material the material constants are independent of the orientation of the coordinate system but can change with the coordinate locations. In a homogeneous material the material constants are independent of the locations of the coordinates but can change with the orientation of the coordinate system.

2. There are only two independent material constants in an isotropic linear elastic material.

3. 21 material constants are needed to specify the most general linear elastic anisotropic material.

4. There are three independent stress components in plane stress problems.

5. There are three independent strain components in plane stress problems.

6. There are five nonzero strain components in plane stress problems.

7. There are three independent strain components in plane strain problems.

8. There are three independent stress components in plane strain problems.

9. There are five nonzero stress components in plane strain problems.

10. For most materials E is greater than G as Poisson's ratio is greater than zero and $G = E/2(1 + \nu)$. In composites, however, Poisson's ratio can be negative; in such a case E will be less than G.

QUICK TEST 4.1

1. True. Material models do not affect the kinematic equation of a uniform strain.

2. False. Stress is uniform over each material but changes as the modulus of elasticity changes with the material in a nonhomogeneous cross section.

3. True. In the formulas A is the value of a cross-sectional area at a given value of x.

4. False. The formula is only valid if $\boldsymbol{N}$, E, and A do not change between x_1 and x_2. For a tapered bar A is changing with x.

5. True. The formula does not depend on external load. External loads affect the value of $\boldsymbol{N}$ but not the relationship of $\boldsymbol{N}$ to σ_{xx}.

6. False. The formula is valid only if $\boldsymbol{N}$, E, and A do not change between x_1 and x_2. For a segment with distributed load, $\boldsymbol{N}$ changes with x.

7. False. The equation represents static equivalency of $\boldsymbol{N}$ and σ_{xx}, which is independent of material models.

8. True. The equation represents static equivalency of $\boldsymbol{N}$ and σ_{xx} over the entire cross section and is independent of material models.

9. True. The uniform axial stress distribution for a homogeneous cross section is represented by an equivalent internal force acting at the centroid which will be also collinear with external forces. Thus no moment will be necessary for equilibrium.

10. True. The equilibrium of a segment created by making an imaginary cut just to the left and just to the right of the section where an external load is applied shows the jump in internal forces.

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QUICK TEST 5.1

1. True. Torsional shear strain for circular shafts varies linearly.
2. True. The shear strain variation is independent of material behavior across the cross section.
3. False. If the shear modulus of a material on the inside is significantly greater than that of the material on the outside, then it is possible for the shear stress on the outer edge of the inside material to be higher than that at the outermost surface.
4. True. The shear stress value depends on the J at the section containing the point and not on the taper.
5. False. The formula is obtained assuming that J is constant between x_1 and x_2.
6. True. The shear stress value depends on the $\boldsymbol{T}$ at the section. The equilibrium equation relating $\boldsymbol{T}$ to external torque is a separate equation.
7. False. The formula is obtained assuming that $\boldsymbol{T}$ is constant between x_1 and x_2, but in the presence of distributed torque, $\boldsymbol{T}$ is a function of x.
8. False. The equation represents static equivalency and is independent of material models.
9. True. Same reasoning as in question 8.
10. True. Equilibrium equations require that the difference between internal torques on either side of the applied torque equal the value of the applied torque.

QUICK TEST 6.1

1. True. Bending normal strain varies linearly and is zero at the centroid of the cross section. If we know the strain at another point, the equation of a straight line can be found.
2. True. Bending normal stress varies linearly and is zero at the centroid and maximum at the point farthest from the centroid. Knowing the stress at two points on a cross section, the equation of a straight line can be found.
3. False. The larger moment of inertia is about the axis parallel to the 2-in. side, which requires that the bending forces be parallel to the 4-in. side.
4. True. The stresses are smallest near the centroid. Alternatively, the loss in moment of inertia is minimum when the hole is at the centroid.
5. False. y is measured from the centroid of the beam cross section.
6. True. The formula is valid at any cross section of the beam. I_{zz} has to be found at the section where the stress is being evaluated.
7. False. The equations are independent of the material model and are obtained from static equivalency principles, and the bending normal stress distribution is such that the net axial force on a cross section is zero.
8. True. The equation is independent of the material model and is obtained from the static equivalency principle.
9. True. The equilibrium of forces requires that the internal shear force jump by the value of the applied transverse force as one crosses the applied force from left to right.
10. True. The equilibrium of moments requires that the internal moment jump by the value of the applied moment as one crosses the applied moment from left to right.

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QUICK TEST 8.1

1. $\theta = 115°$ or $295°$ or $-65°$
2. $\theta = 245°$ or $65°$ or $-115°$
3. $\theta = 155°$ or $-25°$ or $335°$
4. $\sigma_1 = 5$ ksi (T)
5. $\sigma_1 = 5$ ksi (C)
6. $\tau_{max} = 10$ ksi
7. $\tau_{max} = 12.5$ ksi
8. $\tau_{max} = 10$ ksi
9. $\theta_1 = 55°$ or $-125°$
10. $\theta_1 = -35°$

QUICK TEST 8.2

1. *D*
2. *A*
3. *E*
4. 12° ccw or 168° cw
5. 102° ccw or 78° cw
6. 78° ccw or 102° cw
7. *D*
8. *A*
9. *B*
10. $\sigma = 30$ MPa (T), $\tau = -40$ MPa

QUICK TEST 8.3

1. False. There are always three principal stresses. In two-dimensional problems the third principal stress is not independent and can be found from the other two.
2. True.
3. False. Material may affect the state of stress, but the principal stresses are unique for a given state of stress at a point.
4. False. The unique value of the principal stress depends only on the state of stress at the point and not on how these stresses are measured or described.
5. False. Planes of maximum shear stress are always at 45° to the principal planes, and not 90°.
6. True.
7. True.
8. False. Depends on the value of the third principal stress.
9. False. Each plane is represented by a single point on Mohr's circle.
10. False. Each point on Mohr's circle represents a single plane

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QUICK TEST 9.1

1. *D*
2. *C*
3. *B*
4. *C*
5. *D*
6. 108° ccw or 72° cw
7. 18° ccw or 162° cw
8. $\varepsilon_1 = 1300\ \mu$, $\gamma_{max} = 2000\ \mu$
9. $\varepsilon_1 = 2300\ \mu$, $\gamma_{max} = 2300\ \mu$
10. $\varepsilon_1 = -300\ \mu$, $\gamma_{max} = 2300\ \mu$

QUICK TEST 9.2

1. (a) $\varepsilon_{yy} = 800\ \mu$; (b) $\varepsilon_{yy} = 800\ \mu$
2. $\theta = +115°$ or $-65°$
3. $\theta = +155°$ or $-25°$
4. $\theta = +25°$ or $-155°$
5. $\gamma_{max} = 2100\ \mu$
6. $\gamma_{max} = 3100\ \mu$
7. $\gamma_{max} = 1700\ \mu$
8. False. There are always three principal strains. In two-dimensional problems the third principal strain is not independent and can be found from the other two.
9. False. The unique value of principal strains depends only on the state of strain at the point and not on how these strains are measured or described.
10. False. Only for isotropic materials are the principal coordinates for stresses and strains the same, but for any anisotropic materials the principal coordinates for stresses and strains are different.

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QUICK TEST 11.1

1. False. Only compressive axial forces can cause column buckling.

2. True.

3. False. There are infinite buckling loads. The addition of supports changes the buckling mode to the next higher critical buckling load.

4. True.

5. False. The critical buckling load does not change with the addition of uniform transverse distributed forces, but the increase in normal stress may cause the column to fail at lower loads.

6. False. Springs and elastic supports in the middle increase the critical buckling load.

7. False. The critical buckling load does not change with eccentricity, but an increase in normal stress causes the column to fail at lower loads with increasing eccentricity.

8. False. The critical buckling load decreases with increasing slenderness ratio.

9. True.

10. True.

QUICK TEST A.1

1. Structure 1: One; *AB*.
Structure 2: Three; *AC*, *CD*, *CE*.
Structure 3: Three; *AC*, *BC*, *CD*.

2. *DF*, *CF*, *HB*.

3. Structure 1: Two; indeterminate.
Structure 2: One; indeterminate.
Structure 3: Zero; determinate.
Structure 4: One; indeterminate.

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