CHAPTER ONE
STRESS

Learning objectives

1. Understanding the concept of stress.
2. Understanding the two-step analysis of relating stresses to external forces and moments.

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On January 16th, 1943 a World War II tanker S.S. Schenectady, while tied to the pier on Swan Island in Oregon, fractured just aft of the bridge and broke in two, as shown in Figure 1.1. The fracture started as a small crack in a weld and propagated rapidly overcoming the strength of the material. But what exactly is the strength? How do we analyze it? To answer these questions, we introduce the concept of *stress*. Defining this variable is the first step toward developing formulas that can be used in strength analysis and the design of structural members.

Figure 1.1   Failure of S.S. Schenectady.

Figure 1.2 shows two links of the logic that will be fully developed in Section 3.2. What motivates the construction of these two links is an idea introduced in Statics—analysis is simpler if any distributed forces in the free-body diagram are replaced by equivalent forces and moments before writing equilibrium equations (see Appendix A.6). Formulas developed in mechanics of materials relate stresses to internal forces and moments. Free-body diagrams are used to relate internal forces and moments to external forces and moments.
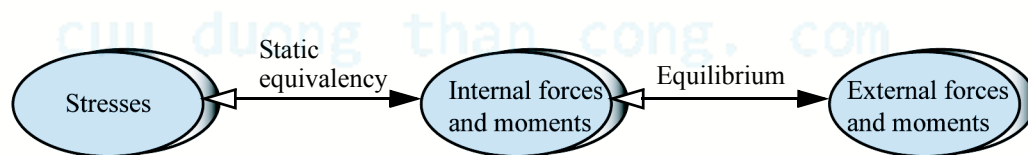

Figure 1.2   Two-step process of relating stresses to external forces and moments.

1.1   STRESS ON A SURFACE

The stress on a surface is an internally distributed force system that can be resolved into two components: normal (perpendicular) to the imaginary cut surface, called *normal* stress, and tangent (parallel) to the imaginary cut surface, called *shear* stress.

1.1.1   Normal Stress

In Figure 1.3, the cable of the chandelier and the columns supporting the building must be strong enough to support the weight of the chandelier and the weight of the building, respectively. If we make an imaginary cut and draw the free-body diagrams, we see that forces normal to the imaginary cut are needed to balance the weight. The *internal* normal force N divided by the area of the cross section A exposed by the imaginary cut gives us the average intensity of an internal normal force distribution, which we call the average **normal** stress:

$$\sigma_{av} = \frac{N}{A}$$

(1.1)

where σ is the Greek letter sigma used to designate normal stress and the subscript av emphasizes that the normal stress is an average value. We may view σ_{av} as a uniformly distributed normal force, as shown in Figure 1.3, which can be replaced by a statically equivalent internal normal force. We will develop this viewpoint further in Section 1.1.4. Notice that N is in boldface italics, as are all internal forces (and moments) in this book.
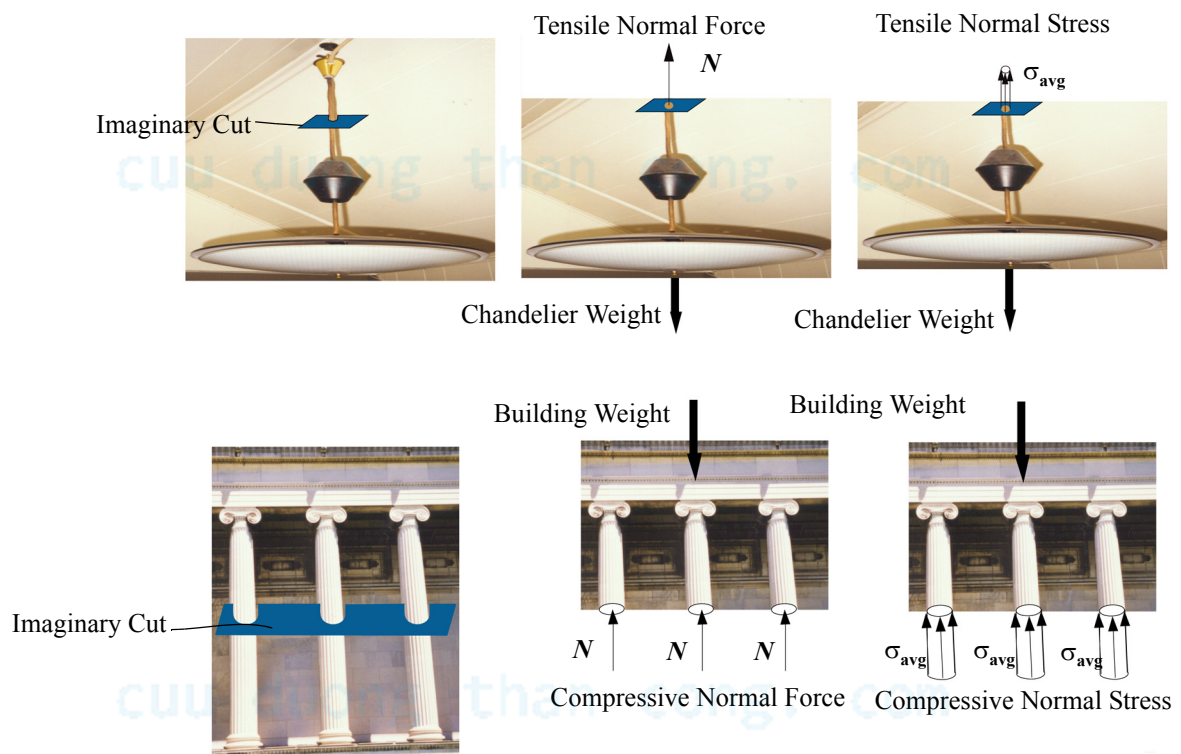

Figure 1.3   Examples of normal stress distribution.

Equation (1.1) is consistent with our intuitive understanding of strength. Consider the following two observations. (i) We know that if we keep increasing the force on a body, then the body will eventually break. Thus we expect the quantifier for strength (stress) to increase in value with the increase of force until it reaches a critical value. In other words, we expect stress to be directly proportional to force, as in Equation (1.1). (ii) If we compare two bodies that are identical in all respects except that one is thicker than the other, then we expect that the thicker body is stronger. Thus, for a given force, as the body gets thicker (larger cross-sectional area), we move away from the critical breaking value, and the value of the quantifier of strength should decrease. In other words, stress should vary inversely with the cross-sectional area, as in Equation (1.1).

Equation (1.1) shows that the unit of stress is force per unit area. Table 1.1 lists the various units of stress used in this book. It should be noted that 1 psi is equal to 6.895 kPa, or approximately 7 kPa. Alternatively, 1 kPa is equal to 0.145 psi, or

approximately 0.15 psi. Normal stress that pulls the imaginary surface away from the material is called **tensile** stress, as shown on the cable of the chandelier in Figure 1.3. Normal stress that pushes the imaginary surface into the material is called **compressive** stress, as shown on the column. In other words, tensile stress acts in the direction of the outward normal whereas compressive stress is opposite to the direction of the outward normal to the imaginary surface. Normal stress is usually reported as tensile or compressive and not as positive or negative. Thus $\sigma = 100$ MPa (T) or $\sigma = 10$ ksi (C) are the preferred ways of reporting tensile or compressive normal stresses.

TABLE 1.1   Units of stress

Abbreviation	Units	Basic Units
psi	Pounds per square inch	lb/in.^{2}
ksi	Kilopounds (kips) per square inch	10^3 lb/in.^{2}
Pa	Pascal	N/m^2
kPa	Kilopascal	10^3 N/m^2
MPa	Megapascal	10^6 N/m^2
GPa	Gigapascal	10^9 N/m^2

The normal stress acting in the direction of the axis of a slender member (rod, cable, bar, column) is called **axial** stress. The compressive normal stress that is produced when one real surface presses against another is called the **bearing** stress. Thus, the stress that exist between the base of the column and the floor is a bearing stress but the compressive stress inside the column is not a bearing stress.

An important consideration in all analyses is to know whether the calculated values of the variables are reasonable. A simple mistake, such as forgetting to convert feet to inches or millimeters to meters, can result in values of stress that are incorrect by orders of magnitude. Less dramatic errors can also be caught if one has a sense of the limiting stress values for a material. Table 1.2 shows fracture stress values for a few common materials. Fracture stress is the experimentally measured value at which a material breaks. The numbers are approximate, and $\pm$ indicates variations of the stress values in each class of material. The order of magnitude and the relative strength with respect to wood are shown to help you in acquiring a feel for the numbers.

TABLE 1.2   Fracture stress magnitudes

Material	ksi	MPa	Relative to Wood
Metals	$90 \pm 90\%$	$630 \pm 90\%$	7.0
Granite	$30 \pm 60\%$	$210 \pm 60\%$	2.5
Wood	$12 \pm 25\%$	$84 \pm 25\%$	1.0
Glass	$9 \pm 90\%$	$63 \pm 90\%$	0.89
Nylon	$8 \pm 10\%$	$56 \pm 10\%$	0.67
Rubber	$2.7 \pm 20\%$	$19 \pm 20\%$	0.18
Bones	$2 \pm 25\%$	$14 \pm 25\%$	0.16
Concrete	$6 \pm 90\%$	$42 \pm 90\%$	0.03
Adhesives	$0.3 \pm 60\%$	$2.1 \pm 60\%$	0.02

EXAMPLE 1.1

A girl whose mass is 40 kg is using a swing set. The diameter of the wire used for constructing the links of the chain is 5 mm. Determine the average normal stress in the links at the bottom of the swing, assuming that the inertial forces can be neglected.

Figure 1.4   Girl in a swing, Example 1.2.

PLAN

We make an imaginary cut through the chains, draw a free-body diagram, and find the tension T in each chain. The link is cut at two imaginary surfaces, and hence the internal normal force N is equal to $T/2$ from which we obtain the average normal stress.

SOLUTION

The cross-sectional area and the weight of the girl can be found as

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.005 \text{ m})^2}{4} = 19.6(10^{-6}) \text{ m}^2 \qquad W = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.4 \text{ N} \tag{E1}$$

Figure 1.5 shows the free body diagram after an imaginary cut is made through the chains. The tension in the chain and the normal force at each surface of the link can be found as shown in Equations (E2) and (E3).

$$T = 2N \tag{E2}$$

$$2T = 392.4 \text{ N} \qquad \text{or} \qquad 4N = 392.4 \text{ N} \qquad \text{or} \qquad N = 98.1 \text{ N} \tag{E3}$$
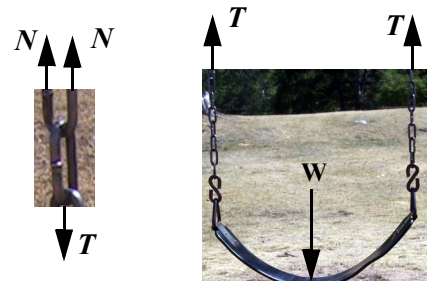
Figure 1.5   Free-body diagram of swing.

The average normal stress can be found as shown in Equation (E4).

$$\sigma_{av} = \frac{N}{A} = \frac{98.1 \text{ N}}{(19.6 \times 10^{-6} \text{ m}^2)} = 4.996 \times 10^6 \text{ N/m}^2 \tag{E4}$$

ANS. $\sigma_{av} = 5.0$ MPa (T)

COMMENTS

1. The stress calculations had two steps. First, we found the internal force by equilibrium; and second we calculated the stress from it.

2. An alternative view is to think that the total material area of the link in each chain is $2A = 39.2 \times 10^{-6} \text{ m}^2$. The internal normal force in each chain is $T = 196.2$ N thus the average normal stress is $\sigma_{av} = \frac{T}{2A} = (196.2/39.2 \times 10^{-6}) = 5 \times 10^6 \text{ N/m}^2$, as before.

1.1.2 Shear Stress

In Figure 1.6*a* the double-sided tape used for sticking a hook on the wall must have sufficient bonding strength to support the weight of the clothes hung from the hook. The free-body diagram shown is created by making an imaginary cut at the wall sur-

face. In Figure 1.6*b* the paper in the ring binder will tear out if the pull of the hand overcomes the strength of the paper. The free-body diagram shown is created by making an imaginary cut along the path of the rings as the paper is torn out. In both free-body diagrams the internal force necessary for equilibrium is parallel (tangent) to the imaginary cut surface. The *internal* shear force *V* divided by the cross sectional area *A* exposed by the imaginary cut gives us the average intensity of the internal shear force distribution, which we call the average **shear** stress:

$$\tau_{av} = \frac{V}{A}$$

(1.2)

where τ is the Greek letter tau used to designate shear stress and the subscript av emphasizes that the shear stress is an average value. We may view τ_{av} as a uniformly distributed shear force, which can be replaced by a statically equivalent internal normal force *V*. We will develop this viewpoint further in Section 1.1.4.
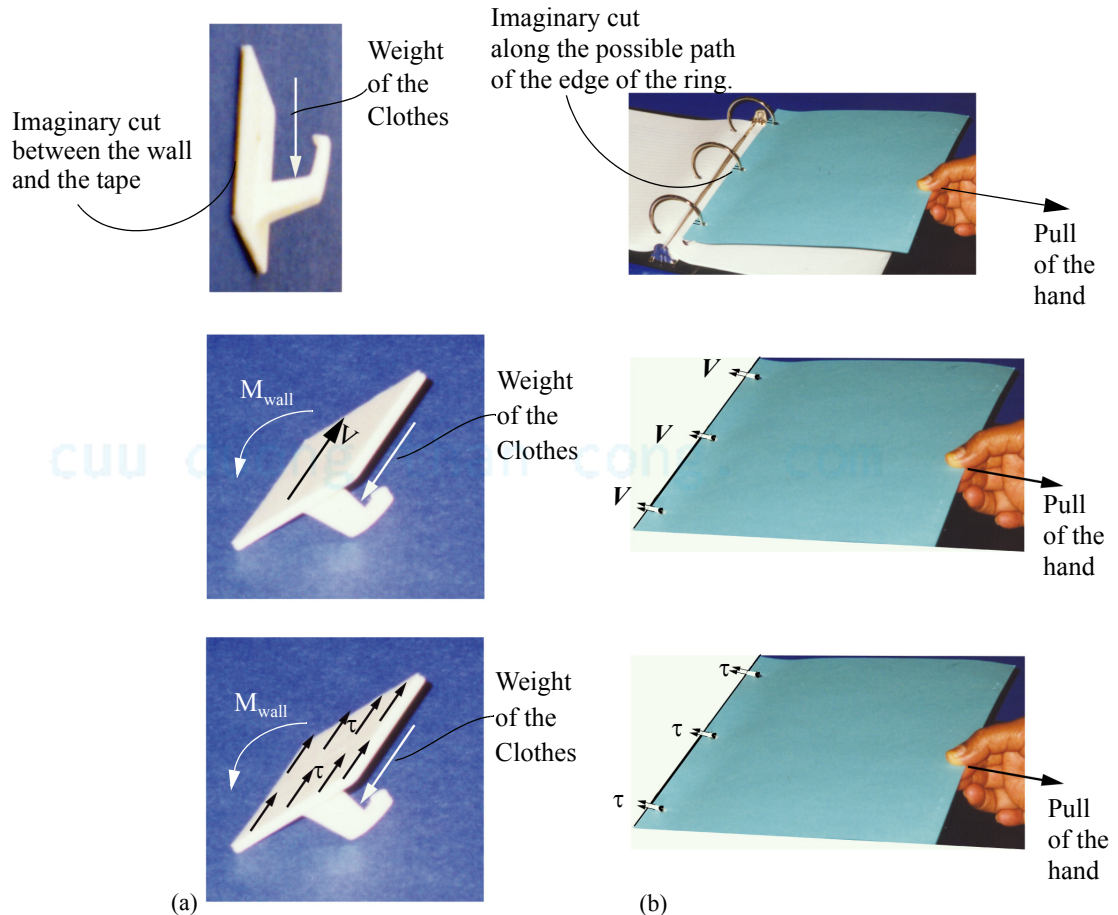
Figure 1.6   Examples of shear stress distribution.

1.1.3   Pins

Pins are one of the most common example of a structural member in which shear stress is assumed uniform on the imaginary surface perpendicular to the pin axis. Bolts, screws, nails, and rivets are often approximated as pins if the *primary* function of these mechanical fasteners is the transfer of shear forces from one member to another. However, if the *primary* function of these mechanical fasteners is to press two solid bodies into each other (seals) then these fasteners cannot be approximated as pins as the forces transferred are normal forces.

Shear pins are mechanical fuses designed to break in shear when the force being transferred exceeds a level that would damage a critical component. In a lawn mower shear pins attach the blades to the transmission shaft and break if the blades hit a large rock that may bend the transmission shaft.

Figure 1.7 shows magnified views of two types of connections at a support. Figure 1.7*a* shows pin in *single* shear as a single cut between the support and the member will break the connection. Figure 1.7*b* shows a pin in *double* shear as two cuts are needed to break the connection. For the same reaction force, the pin in double shear has a smaller shear stress.
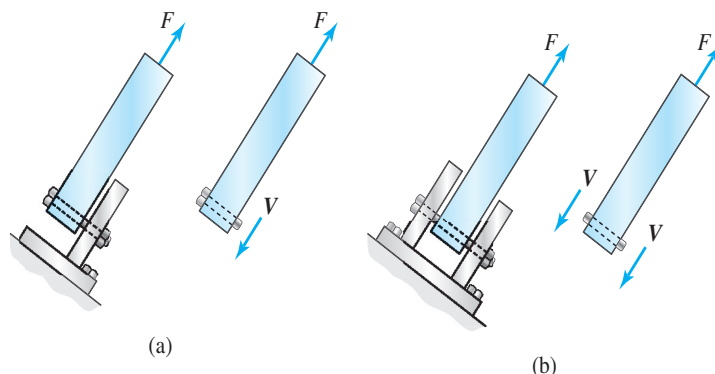

Figure 1.7   Pins in (*a*) single and (*b*) double shear.

When more than two members (forces) are acting on a pin, it is important to visualize the imaginary surface on which the shear stress is to be calculated. Figure 1.8*a* shows a magnified view of a pin connection between three members. The shear stress on the imaginary cut surface 1 will be different from that on the imaginary cut surface 2, as shown by the free-body diagrams in Figure 1.8*b*.
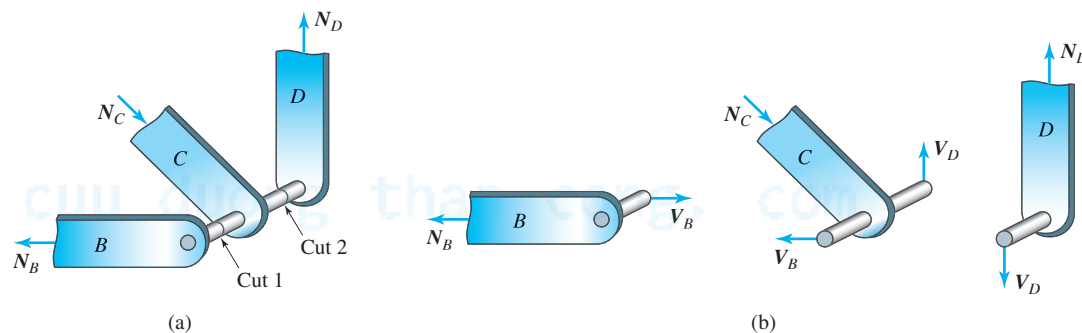
Figure 1.8   Multiple forces on a pin.

EXAMPLE 1.2

Two possible configurations for the assembly of a joint in a machine are to be evaluated. The magnified view of the two configurations with the forces in the members are shown in Figure 1.9. The diameter of the pin is 1 in. Determine which joint assembly is preferred by calculating the maximum shear stress in the pin for each case.
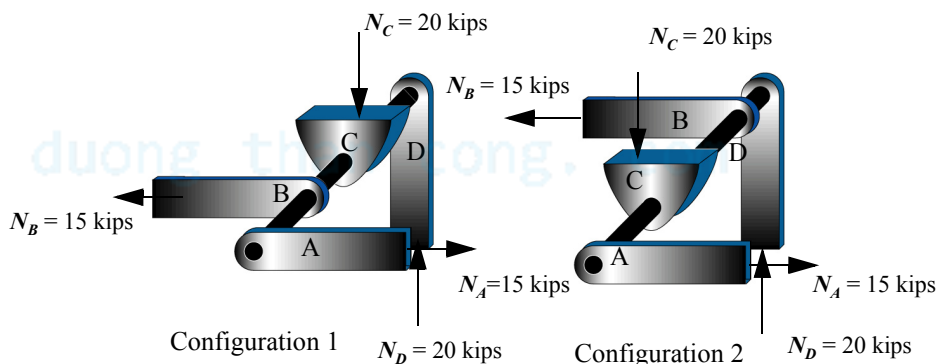

Figure 1.9   Forces on a joint and different joining configurations.

PLAN

We make imaginary cuts between individual members for the two configurations and draw free-body diagrams to determine the shear force at each cut. We calculate and compare the shear stresses to determine the maximum shear stress in each configuration.

SOLUTION

The area of the pin is $A = \pi(0.5 \text{ in.})^2 = 0.7854 \text{ in.}^2$. Making imaginary cuts between members we can draw the free-body diagrams and calculate the internal shear force at the imaginary cut, as shown in Figure 1.10.
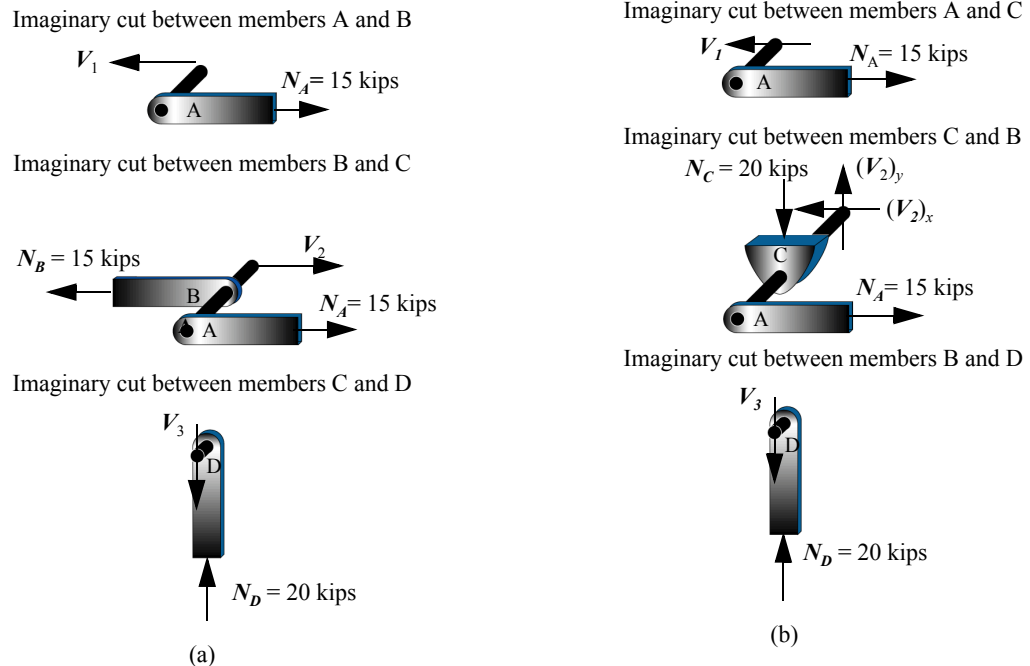
Imaginary cut between members A and B

Imaginary cut between members A and C

Imaginary cut between members B and C

Imaginary cut between members C and B

Imaginary cut between members C and D

Imaginary cut between members B and D

(a)

(b)

Figure 1.10   Free-body diagrams. (a) Configuration 1. (b) Configuration 2.

Configuration 1: From the free-body diagrams in Figure 1.10a

$$V_1 = 15 \text{ kips} \qquad V_2 = 0 \qquad V_3 = 20 \text{ kips} \tag{E1}$$

We see that the maximum shear force exists between members C and D. Thus the maximum shear stress is

$$\tau_{max} = V_3/A = 25.46 \text{ ksi.} \tag{E2}$$

Configuration 2: From the free-body diagrams in Figure 1.10b

$$V_1 = 15 \text{ kips} \qquad (V_2)_x = 15 \text{ kips} \qquad (V_2)_y = 20 \text{ kips} \qquad V_2 = \sqrt{15^2 + 20^2} = 25 \text{ kips.} \qquad V_3 = 20 \text{ kips} \tag{E3}$$

The maximum shear force exists the between C and B. Thus the maximum shear stress is

$$\tau_{max} = V_2/A = 31.8 \text{ ksi.} \tag{E4}$$

Comparing Equations (E2) and (E3) we conclude

ANS.   The configuration 1 is preferred, as it will result in smaller shear stres

COMMENTS

1. Once more note the two steps: we first calculated the internal shear force by equilibrium and then calculated the shear stress from it.
2. The problem emphasizes the importance of visualizing the imaginary cut surface in the calculation of stresses.
3. A simple change in an assembly sequence can cause a joint to fail. This observation is true any time more than two members are joined together. Gusset plates are often used at the joints such as in bridge shown in Figure 1.11 to eliminate the problems associated with an assembly sequence.

Gusset plate

Figure 1.11   Use of gusset plates at joints in a bridge truss.

EXAMPLE 1.3

All members of the truss shown in Figure 1.12 have a cross-sectional area of 500 mm^2 and all pins have a diameter of 20 mm. Determine:
(a) The axial stresses in members *BC* and *DE*, (b) The shear stress in the pin at *A*, assuming the pin is in double shear.
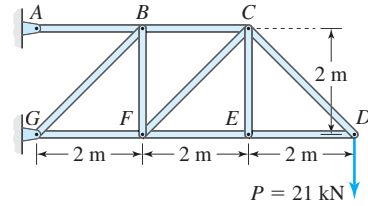

Figure 1.12   Truss.

$P = 21$ kN

PLAN

(a) The free-body diagram of joint D can be used to find the internal axial force in member DE. The free body diagram drawn after an imaginary cut through BC, CF, and EF can be used to find the internal force in member BC. (b) The free-body diagram of the entire truss can be used to find the support reaction at A, from which the shear stress in the pin at A can be found.

SOLUTION

The cross-sectional areas of pins and members can be calculated as in Equation (E1)

$$A_p = \frac{\pi(0.02 \text{ m})^2}{4} = 314.2(10^{-6})\text{m}^2 \qquad A_m = 500(10^{-6})\text{m}^2 \tag{E1}$$

(a) Figure 1.13*a* shows the free-body diagram of joint *D*. The internal axial force N_{DE} can be found using equilibrium equations as shown in Equation (E3).

$$N_{DC} \sin 45^o - 21 \text{ kN} = 0 \qquad \text{or} \qquad N_{DC} = 29.7 \text{ kN} \tag{E2}$$

$$-N_{DE} - N_{DC} \cos 45^o = 0 \qquad \text{or} \qquad N_{DE} = -21 \text{ kN} \tag{E3}$$
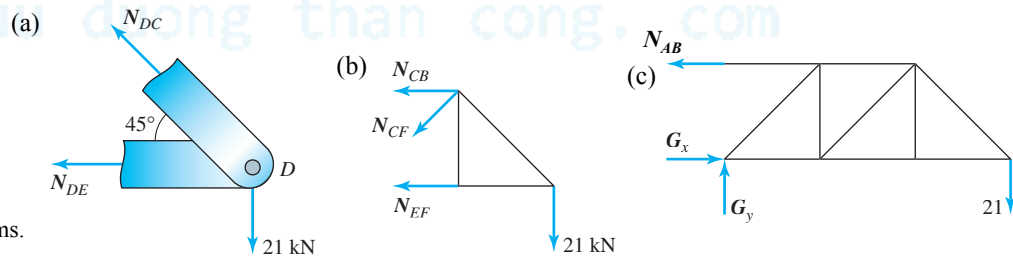

Figure 1.13   Free-body diagrams.

The axial stress in member *DE* can be found as shown in Equation (E4).

$$\sigma_{DE} = \frac{N_{DE}}{A_m} = \frac{[-21(10^3) \text{ N}]}{[500(10^{-6}) \text{ m}^2]} = -42(10^6) \text{ N/m}^2 \tag{E4}$$

ANS. $\sigma_{DE} = 42$ MPa (C)

Figure 1.13*b* shows the free-body diagram after an imaginary cut is made through members *CB*, *CF*, and *EF*. By taking the moment about point *F* we can find the internal axial force in member *CB* as shown in Equation (E5).

$$N_{CB}(2 \text{ m}) - (21 \text{ kN})(4 \text{ m}) = 0 \qquad \text{or} \qquad N_{CB} = 42 \text{ kN} \tag{E5}$$

The axial stress in member *CB* can be found as shown in Equation (E6).

$$\sigma_{CB} = \frac{N_{CB}}{A_m} = 84(10^6) \text{ N/m}^2 \tag{E6}$$

ANS. $\sigma_{CD} = 84$ MPa (T)

(b) Figure 1.13*c* shows the free-body diagram of the entire truss.
By moment equilibrium about point *G* we obtain

$$N_{AB}(2 \text{ m}) - 21 \text{ kN}(6 \text{ m}) = 0 \qquad \text{or} \qquad N_{AB} = 63 \text{ kN} \tag{E7}$$

The shear force in the pin will be half the force of N$_{AB}$ as it is in double shear. We obtain the shear stress in the pin as

$$\tau_A = \frac{N_{AB}/2}{A_p} = \frac{31.5(10^3) \text{ N}}{314.2(10^{-6})\text{m}^2} = 100(10^6) \text{ N/m}^2 \tag{E8}$$

ANS. $\tau_A = 100$ MPa

COMMENTS

1. We calculated the internal forces in each member before calculating the axial stresses, emphasizing the two steps Figure 1.2 of relating stresses to external forces.
2. In part (a) we could have solved for the force in *BC* by noting that *EC* is a zero force member and by drawing the free-body diagram of joint *C*.

PROBLEM SET 1.1

Tensile stress

1.1 In a tug of war, each person shown in Figure P1.1 exerts a force of 200 lb. If the effective diameter of the rope is $\frac{1}{2}$ in., determine the axial stress in the rope.
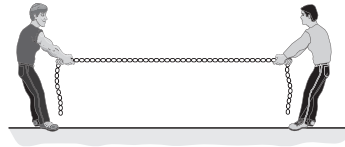
Figure P1.1

1.2 A weight is being raised using a cable and a pulley, as shown in Figure P1.2. If the weight $W = 200$ lb, determine the axial stress assuming: (a) the cable diameter is $\frac{1}{8}$ in. (b) the cable diameter is $\frac{1}{4}$ in.
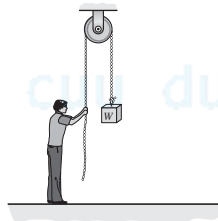
Figure P1.2

1.3 The cable in Figure P1.2 has a diameter of $\frac{1}{5}$ in. If the maximum stress in the cable must be limited to 4 ksi (T), what is the maximum weight that can be lifted?

1.4 The weight $W = 250$ lb in Figure P1.2. If the maximum stress in the cable must be limited to 5 ksi (T), determine the minimum diameter of the cable to the nearest $\frac{1}{16}$ in.

1.5 A 6-kg light shown in Figure P1.5 is hanging from the ceiling by wires of 0.75-mm diameter. Determine the tensile stress in wires *AB* and *BC*.
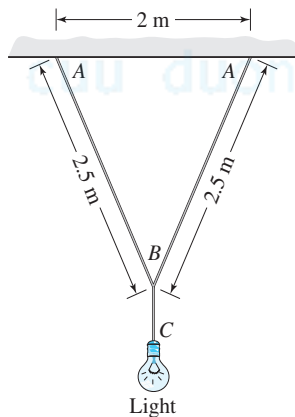

Figure P1.5

1.6 An 8-kg light shown in Figure P1.5 is hanging from the ceiling by wires. If the tensile stress in the wires cannot exceed 50 MPa, determine the minimum diameter of the wire, to the nearest tenth of a millimeter.

1.7   Wires of 0.5-mm diameter are to be used for hanging lights such as the one shown in Figure P1.5. If the tensile stress in the wires cannot exceed 80 MPa, determine the maximum mass of the light that can be hung using these wires.

1.8   A 3 kg picture is hung using a wire of 3 mm diameter, as shown in Figure P1.8. What is the average normal stress in the wires?
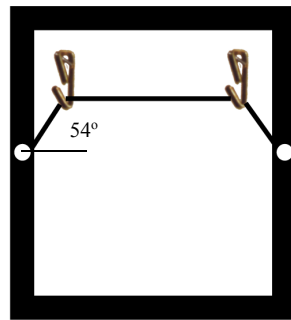

Figure P1.8

1.9   A 5 kg picture is hung using a wire, as shown in Figure P1.8. If the tensile stress in the wires cannot exceed 10MPa, determine the minimum required diameter of the wire to the nearest millimeter.

1.10   Wires of 16-mil diameter are used for hanging a picture, as shown in Figure P1.8. If the tensile stress in the wire cannot exceed 750 psi, determine the maximum weight of the picture that can be hung using these wires. 1 mil = $\frac{1}{1000}$ in.

1.11   A board is raised to lean against the left wall using a cable and pulley, as shown in Figure P1.11. Determine the axial stress in the cable in terms of the length L of the board, the specific weight γ per unit length of the board, the cable diameter d, and the angles θ and α, shown in Figure P1.11.
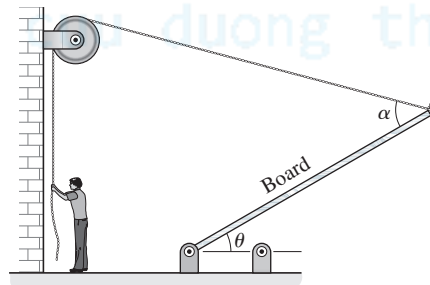

Figure P1.11

Compressive and bearing stresses

1.12   A hollow circular column supporting a building is attached to a metal plate and bolted into the concrete foundation, as shown in Figure P1.12. The column outside diameter is 100 mm and an inside diameter is 75 mm. The metal plate dimensions are 200 mm × 200 mm × 10 mm. The load P is estimated at 800 kN. Determine: (a) the compressive stress in the column; (b) the average bearing stress between the metal plate and the concrete.
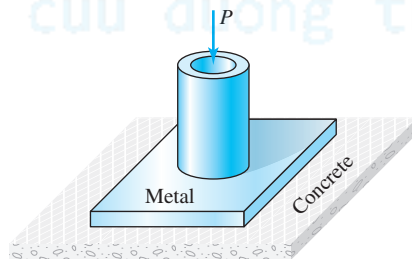

Figure P1.12

1.13   A hollow circular column supporting a building is attached to a metal plate and bolted into the concrete foundation, as shown in Figure P1.12. The column outside diameter is 4 in. and an inside diameter is 3.5 in. The metal plate dimensions are 10 in. × 10 in. × 0.75 in. If the allowable average compressive stress in the column is 30 ksi and the allowable average bearing stress in concrete is 2 ksi, determine the maximum load P that can be applied to the column.

1.14 A hollow square column supporting a building is attached to a metal plate and bolted into the concrete foundation, as shown in Figure P1.14. The column has outside dimensions of 120 mm × 120 mm and a thickness of 10 mm. The load P is estimated at 600 kN. The metal plate dimensions are 250 mm × 250 mm × 15 mm. Determine: (a) the compressive stress in the column; (b) the average bearing stress between the metal plate and the concrete.
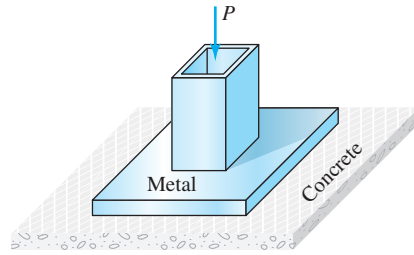

Figure P1.14
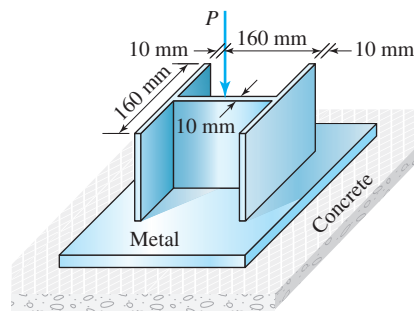
1.15 A column with the cross section shown in Figure P1.15 supports a building. The column is attached to a metal plate and bolted into the concrete foundation. The load P is estimated at 750 kN. The metal plate dimensions are 300 mm × 300 mm × 20 mm. Determine: (a) the compressive stress in the column; (b) the average bearing stress between the metal plate and the concrete

FigureP1.15

1.16 A 70-kg person is standing on a bathroom scale that has dimensions of 150 mm × 100 mm × 40 mm (Figures P1.16). Determine the bearing stress between the scale and the floor. Assume the weight of the scale is negligible.

Figure P1.16

1.17 A 30-ft-tall brick chimney has an outside diameter of 3 ft and a wall thickness of 4 in. (Figure P1.17). If the specific weight of the bricks is 80 lb/ft^3, determine the average bearing stress at the base of the chimney.
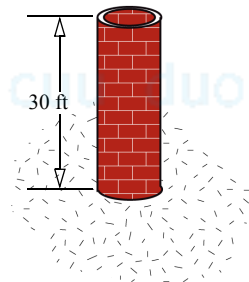

Figure P1.17

1.18 Determine the average bearing stress at the bottom of the block shown in Figure P1.18 in terms of the specific weight γ and the length dimensions a and h.
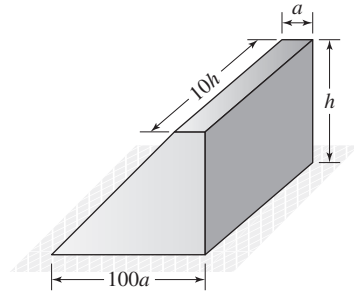

Figure P1.18

1.19 The Washington Monument is an obelisk with a hollow rectangular cross section that tapers along its length. An approximation of the monument geometry is shown in Figure P1.19. The thickness at the base is 4.5 m and at top it is 2.5 m. The monument is constructed from marble and granite. Using a specific weight of 28 kN/m³ for these materials, determine the average bearing stress at the base of the monument.
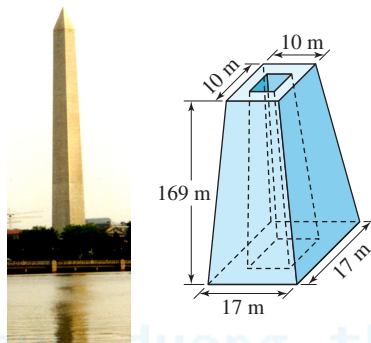

Figure P1.19

1.20 Show that the average compressive stress due to weight on a cross section at a distance x from the top of the wall in Figure P1.20b is half that of wall in Figure P1.20a, thus confirming the wisdom of ancient Egyptians in building inward-sloping walls for the pyramids. (Hint: Using γ the specific weight of wall material, H the height of the wall, t the thickness of the wall, and L the length of the wall, calculate the average compressive stress at any cross section at a distance x from the top for the two walls.).
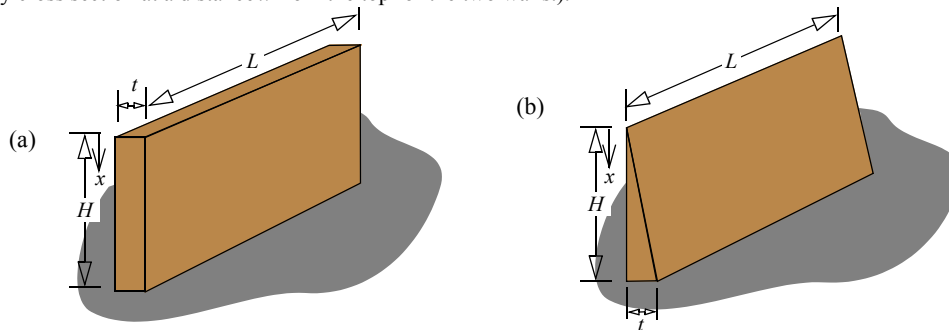

Figure P1.20   (a) Straight wall (b) Inward sloping tapered wall.

1.21 The Great pyramid of Giza shown in Figure 1.14d has a base of 757.7 ft x 757.7 ft and a height of 480.9 ft. Assume an average specific weight of $\gamma = 75$ lb/ft³. Determine (a) the bearing stress at the base of the pyramid. (b) the average compressive stress at mid height.

1.22 The Bent pyramid shown in Figure 1.14c has a base of 188 m x 188 m. The initial slopes of the sides is 54°27'44". After a certain height the slope is 43°22'. The total height of the pyramid is 105 m. Assume an average mass density of 1200 kg/ m³. Determine the bearing stress at the base of the pyramid.

1.23 A steel bolt of 25 mm diameter passes through an aluminum sleeve of thickness 4 mm and outside diameter of 48 mm as shown in Figure

P1.23. Determine the average normal stress in the sleeve if in the assembled position the bolt has an average normal stress of 100 MPa (T).
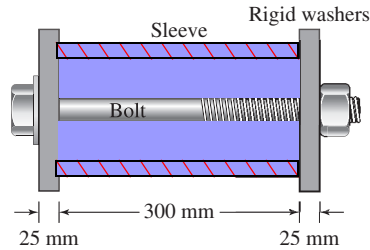
Figure P1.23

Shear stress

1.24   The device shown in Figure P1.24 is used for determining the shear strength of the wood. The dimensions of the wood block are 6 in. × 8 in. × 2 in. If the force required to break the wood block is 15 kips, determine the average shear strength of the wood.
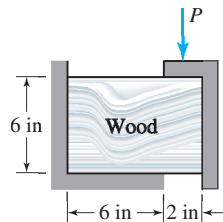
Figure P1.24

1.25   The dimensions of the wood block in Figure P1.24 are 6 in. × 8 in. × 1.5 in. Estimate the force P that should be applied to break the block if the average shear strength of the wood is 1.2 ksi.

1.26   The punch and die arrangement shown schematically in Figure P1.26 is used to punch out thin plate objects of different shapes. The cross section of the punch and die shown in Figure P1.26 is a circle of 1-in. diameter. A force $P = 6$ kips is applied to the punch. If the plate thickness $t = \frac{1}{8}$ in., determine the average shear stress in the plate along the path of the punch.
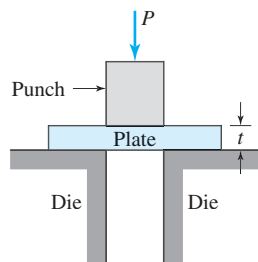
Figure P1.26

1.27   The cross section of the punch and die shown in Figure P1.26 is a square of 10 mm × 10 mm. The plate shown has a thickness $t = 3$ mm and an average shear strength of 200 MPa. Determine the average force P needed to drive the punch through the plate.

1.28   The schematic of a punch and die for punching washers is shown in Figure P1.28. Determine the force P needed to punch out washers, in terms of the plate thickness t, the average plate shear strength τ, and the inner and outer diameters of the washers d_i and d_o
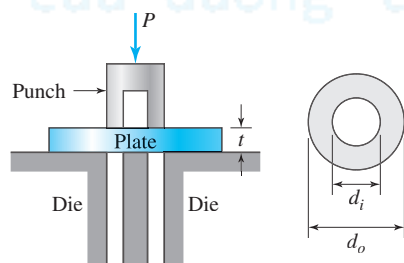
Figure P1.28

1.29   The magnified view of a pin joint in a truss are shown in Figure P1.29. The diameter of the pin is 25 mm. Determine the maximum transverse shear stress in the pin.
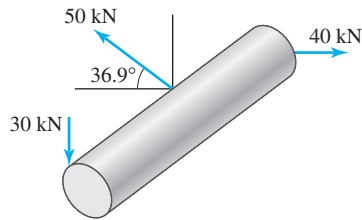
Figure P1.29

Normal and shear stresses

1.30   A weight W = 200 lb. is being raised using a cable and a pulley, as shown in Figure P1.30. The cable effective diameter is $\frac{1}{4}$ in. and the pin in the pulley has a diameter of $\frac{3}{8}$ in. Determine the axial stress in the cable and the shear stress in the pin, assuming the pin is in double shear.
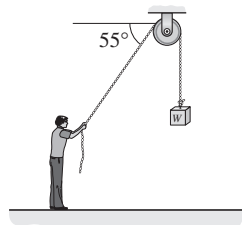
Figure P1.30

1.31   The cable in Figure P1.30 has a diameter of $\frac{1}{5}$ in. and the pin in the pulley has a diameter of $\frac{3}{8}$ in. If the maximum normal stress in the cable must be limited to 4 ksi (T) and the maximum shear stress in the pin is to be limited to 2 ksi, determine the maximum weight that can be lifted to the nearest lb. The pin is in double shear.

1.32   The manufacturer of the plastic carrier for drywall panels shown in Figure P1.32 prescribes a maximum load P of 200 lb. If the cross-sectional areas at sections AA and BB are 1.3 in.^{2} and 0.3 in.^{2}, respectively, determine the average shear stress at section AA and the average normal stress at section BB at the maximum load P.
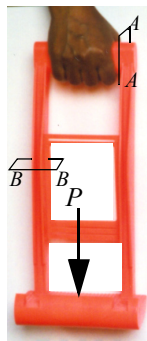
Figure P1.32

1.33   A bolt passing through a piece of wood is shown in Figure P1.33. Determine: (a) the axial stress in the bolt; (b) the average shear stress in the bolt head; (c) the average bearing stress between the bolt head and the wood; (d) the average shear stress in the wood.
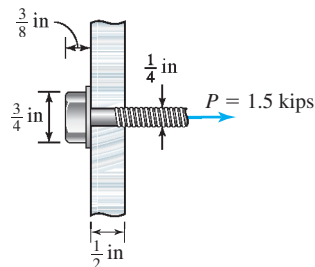
Figure P1.33

1.34 A load of P = 10 kips is transferred by the riveted joint shown in Figure P1.34. Determine (a) the average shear stress in the rivet. (b) the largest average normal stress in the members attached (c) the largest average bearing stress between the pins and members.
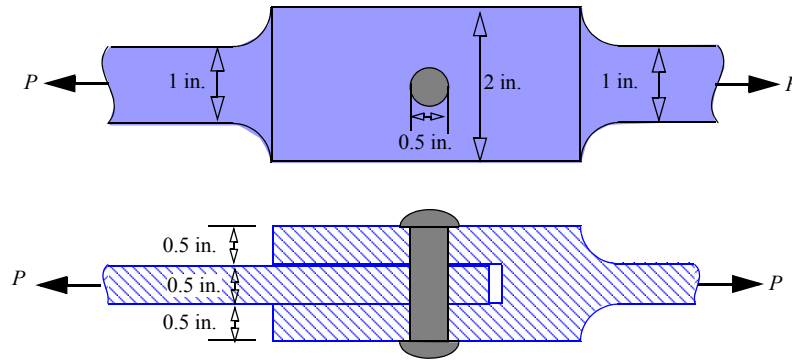

Figure P1.34

1.35 A joint in a wooden structure is shown in Figure P1.35. The dimension $h = 4\frac{3}{8}$ in. and $d = 1\frac{1}{8}$ in. Determine the average normal stress on plane BEF and average shear stress on plane BCD. Assume plane BEF and the horizontal plane at AB are a smooth surfaces.
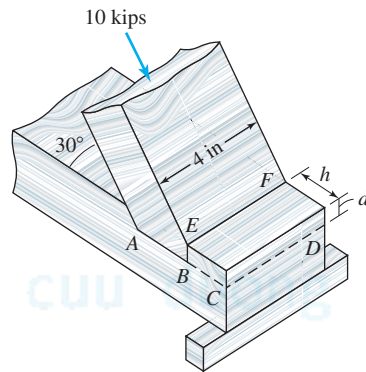

Figure P1.35

1.36 A metal plate welded to an I-beam is securely fastened to the foundation wall using four bolts of 1/2 in. diameter as shown in Figure P1.36. If P = 12 kips determine the normal and shear stress in each bolt. Assume the load is equally distributed among the four bolts.
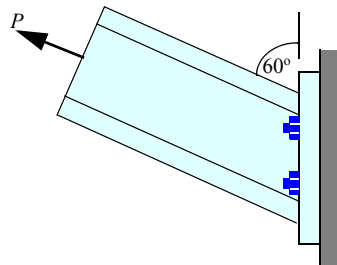

Figure P1.36

1.37 A metal plate welded to an I-beam is securely fastened to the foundation wall using four bolts as shown Figure P1.36. The allowable normal stress in the bolts is 100 MPa and the allowable shear stress is 70 MPa. Assume the load is equally distributed among the four bolts. If the beam load P= 50 kN, determine the minimum diameter to the nearest millimeter of the bolts.

1.38 A metal plate welded to an I-beam is securely fastened to the foundation wall using four bolts of 1/2 in. diameter as shown Figure P1.36. The allowable normal stress in the bolts is 15 ksi and the allowable shear stress is 12 ksi. Assume the load is equally distributed among the four bolts. Determine the maximum load P to the nearest pound the beam can support.

1.39 An adhesively bonded joint in wood is fabricated as shown in Figure P1.39. The length of the overlap is L= 4 in. and the thickness of the wood is 3/8 in. Determine the average shear stress in the adhesive.
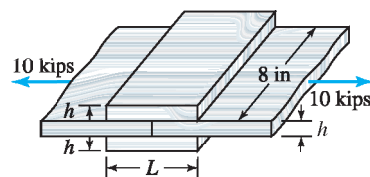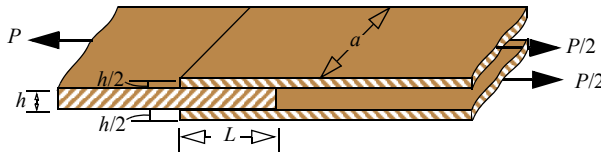

Figure P1.39

August, 2012

1.40   A double lap joint adhesively bonds three pieces of wood as shown in Figure P1.40. The joints transmits a force of P= 20 kips and has the following dimensions: L = 3 in., a = 8 in. and h = 2 in. Determine the maximum average normal stress in the wood and the average shear stress in the adhesive.

Figure P1.40

1.41   The wood in the double lap joint of Figure P1.40 has a strength of 15 MPa in tension and the strength of the adhesive in shear is 2 MPa. The joint has the following dimensions: L = 75 mm, a =200 mm, and h = 50 mm. Determine the maximum force P the joint can transfer.

1.42   A wooden dowel of diameter d = 20 mm is used for constructing the double lap joint in Figure P1.42. The wooden members have a strength of 10 MPa in tension, the bearing stress between the dowel and the members is to be limited to 18 MPa, the shear strength of the dowel is 25 MPa. The joint has the following dimensions: L = 75 mm, a =200 mm, and h = 50 mm. Determine the maximum force P the joint can transfer.
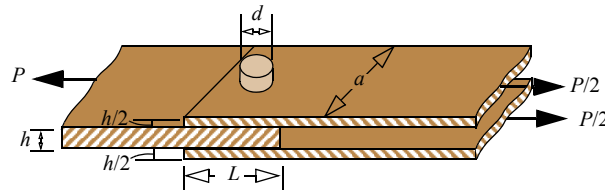
Figure P1.42

1.43   A couple is using the chair lift shown in Figure P1.43 to see the Fall colors in Michigan's Upper Peninsula. The pipes of the chair frame are 1/16 in. thick. Assuming each person weighs 180 lb, determine the average normal stress at section AA and BB and average shear stress at section CC assuming the chair is moving at a constant speed.

Figure P1.43

1.44   The axial force P = 12 kips acts on a rectangular member, as shown in Figure P1.44. Determine the average normal and shear stresses on the inclined plane AA.

Figure P1.44

1.45   A wooden axial member has a cross section of 2 in.× 4 in. The member was glued along line AA and transmits a force of P = 80 kips as shown in Figure P1.45. Determine the average normal and shear stress on plane AA.
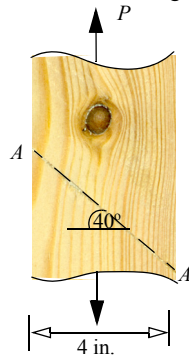
Figure P1.45

1.46   Two rectangular bars of 10-mm thickness are loaded as shown in Figure P1.46. If the normal stress on plane AA is 180 MPa (C), determine the force F_1 and the normal and shear stresses on plane BB.
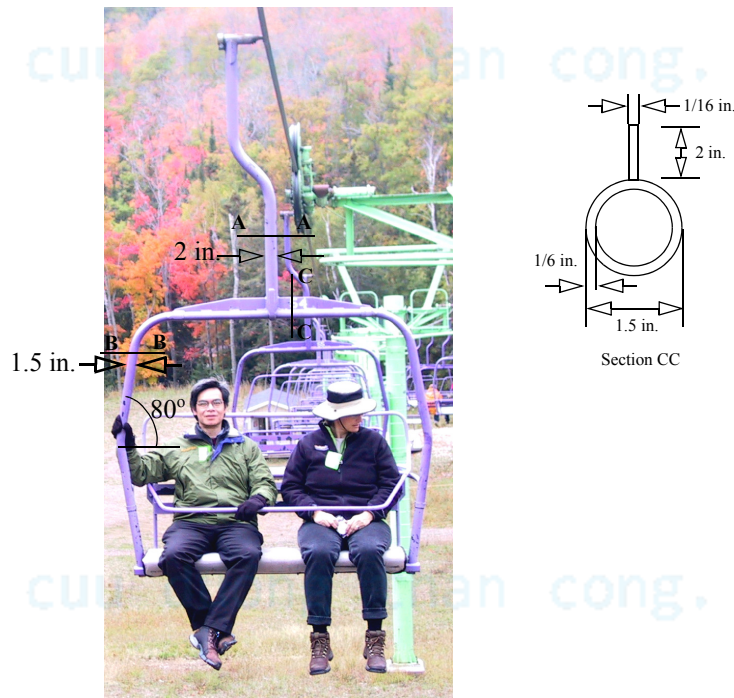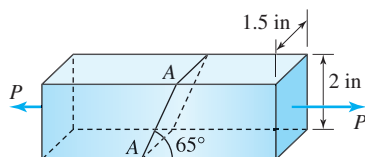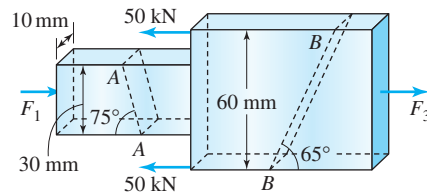
Figure P1.46

1.47   A butt joint is created by welding two plates to transmits a force of P = 250 kN as shown in Figure P1.47. Determine the average normal and shear stress on the plane AA of the weld.
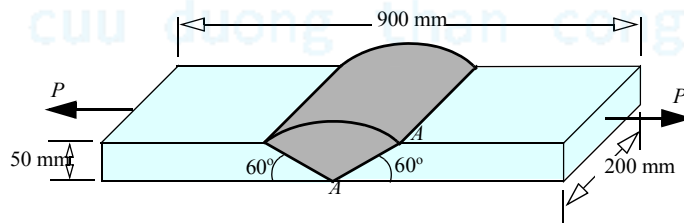
Figure P1.47

1.48   A square tube of 1/4 in thickness is welded along the seam and used for transmitting a force of P = 20 kips as shown in Figure P1.48. Determine average normal and shear stress on the plane AA of the weld.

Figure P1.48

1.49   (a) In terms of P, a, b, and θ determine the average normal and shear stresses on the inclined plane AA shown in Figure P1.49. (b) Plot the normal and shear stresses as a function of θ and determine the maximum values of the normal and shear stresses. (c) At what angles of the inclined plane do the maximum normal and maximum shear stresses occurs.

Figure P1.49

1.50   An axial load is applied to a 1-in-diameter circular rod (Figure P1.50). The shear stress on section *AA* was found to be 20 ksi. The section *AA* is at 45° to the axis of the rod. Determine the applied force *P* and the average normal stress acting on section *AA*.

Figure P1.50
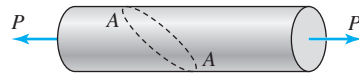
1.51   A simplified model of a child's arm lifting a weight is shown in Figure P1.51. The cross-sectional area of the biceps muscle is estimated as 2 in². Determine the average normal stress in the muscle and the average shear force at the elbow joint *A*.
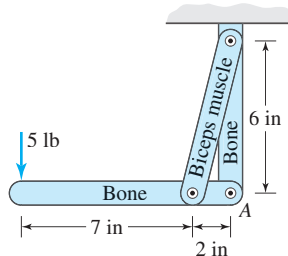
Figure P1.51

1.52   Figure P1.52 shows a truss and the sequence of assembly of members at pins H, G, and F. All members of the truss have cross-sectional areas of 250 mm² and all pins have diameters of 15 mm. Determine (a) the axial stresses in members *HA*, *HB*, *HG*, and *HC*. (b) the maximum shear stress in pin *H*.
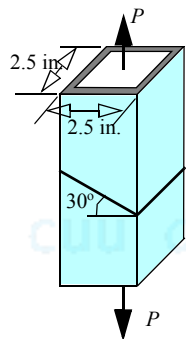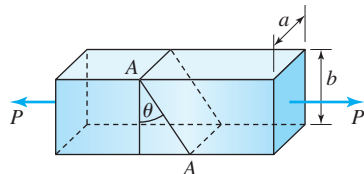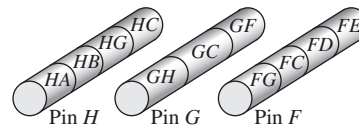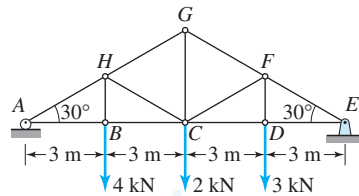
Figure P1.52

1.53   Figure P1.52 shows a truss and the sequence of assembly of members at pins H, G, and F. All members of the truss have cross-sectional areas of 250 mm² and all pins have diameters of 15 mm. Determine (a) the axial stresses in members *FG, FC, FD,* and *FE*. (b) the maximum shear stress in pin *F*.

1.54   Figure P1.52 shows a truss and the sequence of assembly of members at pins H, G, and F. All members of the truss have cross-sectional areas of 200 mm² and all pins have diameters of 10 mm. Determine (a) the axial stresses in members *GH, GC,* and *GF* of the truss shown in Figure P1.52. (b) the maximum shear stress in pin *G*

1.55   The pin at *C* in Figure P1.55 is has a diameter of $\frac{1}{2}$ in. and is in double shear. The cross-sectional areas of members *AB* and *BC* are 2 in.² and 2.5 in.², respectively. Determine the axial stress in member *AB* and the shear stress in pin *C*.
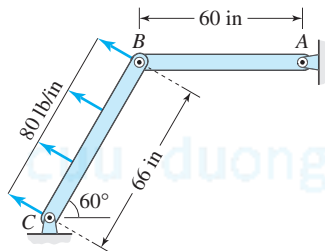
Figure P1.55

1.56   All pins shown in Figure P1.56 are in single shear and have diameters of 40 mm. All members have square cross sections and the surface at E is smooth. Determine the maximum shear stresses in the pins and the axial stress in member *BD*.
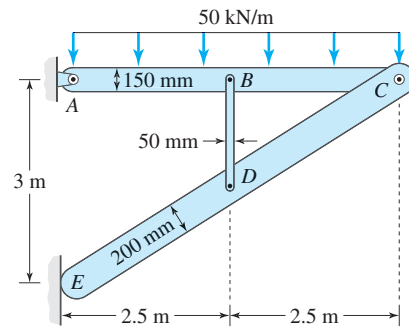
Figure P1.56

1.57   A student athlete is lifting weight W = 36 lbs as shown in Figure P1.57a. The weight of the athlete is W_A = 140 lb. A model of the student pelvis and legs is shown in Figure P1.57b. The weight of legs and pelvis W_L = 32 lb acts at the center of gravity G. Determine the normal stress in the erector spinae muscle that supports the trunk if the average muscle area at the time of lifting the weight is 1.75 in^2.
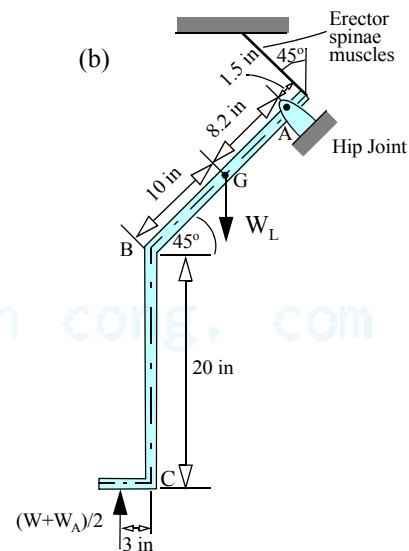
Figure P1.57

1.58   A student is exercising his shoulder muscles using a W = 15 lb dumbbell as shown in Figure P1.58a. The model of the student arm is shown in Figure P1.58b. The weight of the arm of W_A = 9 lb acts at the center of gravity G. Determine the average normal stress in the deltoid muscle if the average area of the muscle is 0.75 in^2 at the time the weight is in the horizontal position.
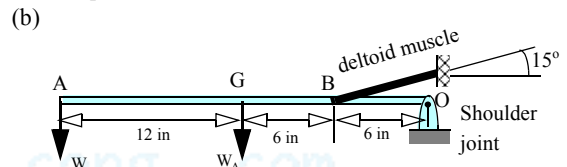
Figure P1.58

Design problems

1.59   The bottom screw in the hook shown in Figure P1.59 supports 60% of the load *P* while the remaining 40% of *P* is carried by the top screw. The shear strength of the screws is 50 MPa. Develop a table for the maximum load *P* that the hook can support for screw diameters that vary from 1 mm to 5 mm in steps of 1 mm.
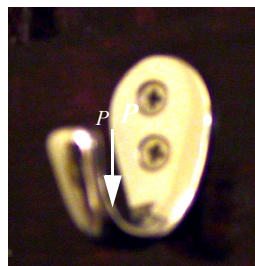
Figure P1.59

1.60   Determine the maximum force P that can be transferred by the riveted joint shown in Figure P1.34 if the limits apply: maximum normal stress in the attached members can be 30 ksi., maximum bearing stress between the pins and members can be 15 ksi, and the maximum shear stress in the rivet can be 20 ksi.

1.61   A tire swing is suspended using three chains, as shown in Figure P1.61. Each chain makes an angle of 12° with the vertical. The chain is made from links as shown. For design purposes assume that more than one person may use the swing, and hence the swing is to be designed to carry a weight of 500 lb. If the maximum average normal stress in the links is not to exceed 10 ksi, determine to the nearest $\frac{1}{16}$ in. the diameter of the wire that should be used for constructing the links.

Figure P1.61

1.62   Two cast-iron pipes are held together by a bolt, as shown in Figure P1.62. The outer diameters of the two pipes are 50 mm and 70 mm and the wall thickness of each pipe is 10 mm. The diameter of the bolt is 15 mm. What is the maximum force *P* this assembly can transmit if the maximum permissible stresses in the bolt and the cast iron are 200 MPa in shear and 150 MPa in tension, respectively.

P   *P*
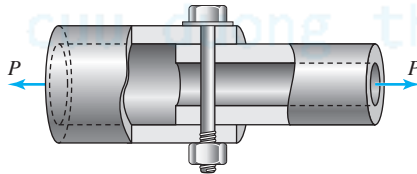
Figure P1.62

1.63   A normal stress of 20 ksi is to be transferred from one plate to another by riveting a plate on top, as shown in Figure P1.63. The shear strength of the $\frac{1}{2}$ in. rivets used is 40 ksi. Assuming all rivets carry equal shear stress, determine the minimum even number of rivets that must be used.

$\sigma = 20$ ksi

1 in
1 in
← 6 in →   1 in $\sigma = 20$ ksi
8 in
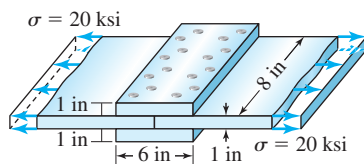
Figure P1.63

1.64   Two possible joining configurations are to be evaluated. The forces on joint in a truss were calculated and a magnified view is shown Figure P1.64. The pin diameter is 20 mm. Determine which joint assembly is better by calculating the maximum shear stress in the pin for each case.

$N_C = 50$ kN

$N_D = 30$ kN

$N_A = 32.68$ kN

$N_B = 67.32$ kN

30°   30°

Configuration 1

$N_C = 50$ kN $N_B = 67.32$ kN

$N_A = 32.68$ kN

$N_D = 30$ kN

30°

30°

Configuration 2

Figure P1.64

1.65   Truss analysis showed the forces at joint A given in Figure P1.65. Determine the sequence in which the three members at joint A should be assembled so that the shear stress in the pin is minimum.

Figure P1.65

1.66   An 8 in × 8 in reinforced concrete bar needs to be designed to carry a compressive axial force of 235 kips. The reinforcement is done using $\frac{1}{2}$-in. round steel bars. Assuming the normal stress in concrete to be a uniform maximum value of 3 ksi and in steel bars to be a uniform value of 20 ksi, determine the minimum number of iron bars that are needed.

1.67   A wooden axial member has a cross section of 2 in. × 4 in. The member was glued along line AA, as shown in Figure P1.45. Determine the maximum force P that can be applied to the repaired axial member if the maximum normal stress in the glue cannot exceed 800 psi and the maximum shear stress in the glue cannot exceed 350 psi.

1.68   An adhesively bonded joint in wood is fabricated as shown in Figure P1.68. The length of the bonded region $L = 5$ in. Determine the maximum force P the joint can support if the shear strength of the adhesive is 300 psi and the wood strength is 6 ksi in tension.
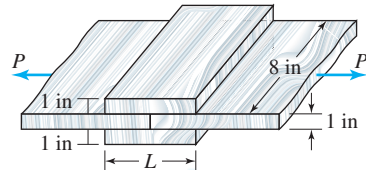
Figure P1.68

1.69   The joint in Figure P1.68 is to support a force $P = 25$ kips. What should be the length L of the bonded region if the adhesive strength in shear is 300 psi?

1.70   The normal stress in the members of the truss shown in Figure P1.70 is to be limited to 160 MPa in tension or compression. All members have circular cross sections. The shear stress in the pins is to be limited to 250 MPa. Determine (a) the minimum diameters to the nearest millimeter of members ED, EG, and EF. (b) the minimum diameter of pin E to the nearest millimeter and the sequence of assembly of members ED, EG, and EF.
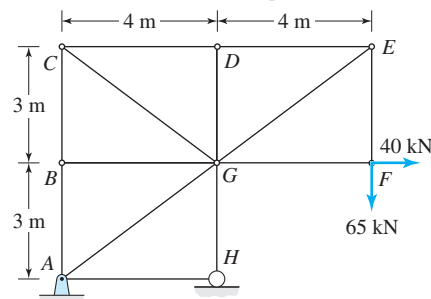
Figure P1.70

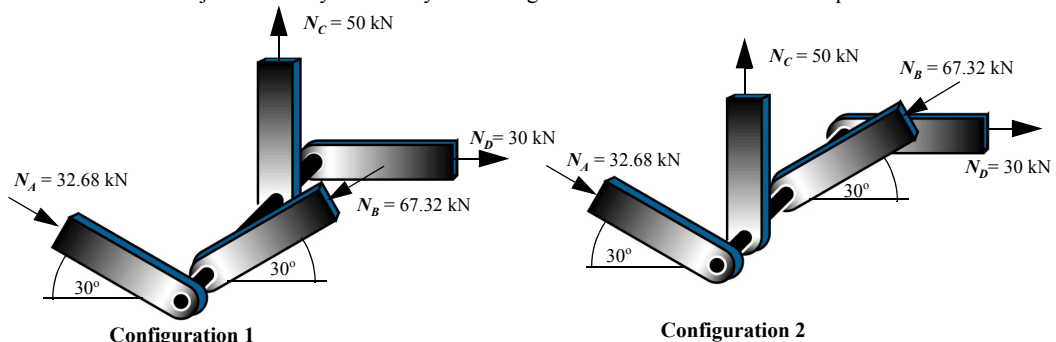**1.71**   The normal stress in the members of the truss shown in Figure P1.70 is to be limited to 160 MPa in tension or compression. All members have circular cross sections. The shear stress in the pins is to be limited to 250 MPa. Determine (a) the minimum diameters to the nearest millimeter of members CG, CD, and CB. (b) the minimum diameter of pin C to the nearest millimeter and the sequence of assembly of members CG, CD, and CB.

Stretch yourself

1.72   Truss analysis showed the forces at joint A given in Figure P1.72. Determine the sequence in which the four members at joint A should be assembled to minimize the shear stress in the pin.
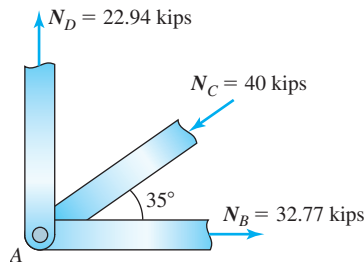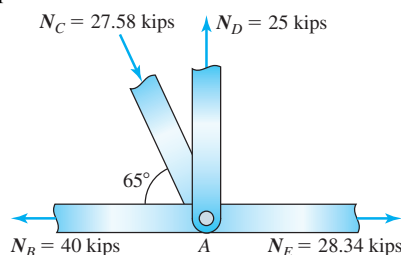

Figure P1.72

MoM in Action: Pyramids

The pyramids of Egypt are a remarkable engineering feat. The size, grandeur, and age of the pyramids excites the human imagination. Science fiction authors create stories about aliens building them. Pyramid design, however, is a story about human engineering in a design process that incorporates an intuitive understanding of material strength.
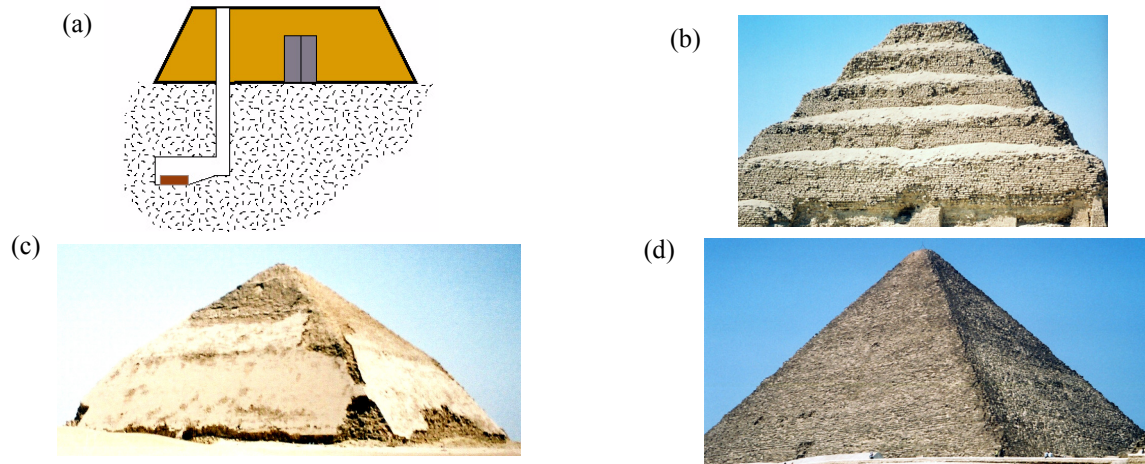
Figure 1.14   Pyramids of Egypt (a) Mastaba (b) Step pyramid (c) Bent pyramid (d) Great pyramid of Giza.

Before pyramids were built, Egyptians kings and nobles were buried in tombs called *mastaba (*Figure 1.14*a)*. Mastaba have underground chambers that are blocked off by dropping heavy stones down vertical shafts. On top of these underground burial chambers are rectangular structures with inward-sloping, tapered brick mud walls. The ancient Egyptians had learned by experience that inward-sloping walls that taper towards the top do not crumble as quickly as straight walls of uniform thickness (see problem 1.20).

Imhotep, the world's first renowned engineer-architect, took many of the design elements of *mastaba* to a very large scale in building the world's first Step pyramid (Figure 1.14*b*) for his pharaoh Djozer (2667-2648 BCE). By building it on a bedrock, Imhotep intuitively understood the importance of bearing stresses which were not properly accounted for in building of the leaning tower of Pisa 4000 years later. The Step pyramid rose in six steps to a height of 197 ft with a base of 397 ft x 358 ft. A 92-ft deep central shaft was dug beneath the base for the granite burial chamber. The slopes of the faces of the Step pyramid varied from 72º to 78º. Several pharaohs after Djozer tried to build their own step pyramids but were unsuccessful.

The next development in pyramid design took place in the reign of pharaoh Sneferu (2613-2589 BCE). Sneferu architects started by building a step pyramid but abandoned it because he wanted a pyramid with smooth sides. The pyramid shown in Figure 1.14*c* was started with a base of 617 ft x 617 ft and an initial slope of approximately 54º. Signs of structural problem convinced the builders to change the slope to 43º resulting in the unique bent shape seen in Figure 1.14*c* (see problem 1.22). Sneferu then ordered a third pyramid built. This pyramid had an initial slope of 43º, stood on a base of 722 ft x722 ft, rose to a height of 345 ft, and had smooth sides. This experience was used by architects in the reign of Khufu (2589-2566 BCE) to build the largest pyramid in the world called the Great pyramid of Giza (Figure 1.14*d*). The Great Pyramid (see problem 1.21) stands on a base of 756.7 ft x 756.7 ft and has a height of 481 ft.

The ancient Egyptians did not have a formal definition of stress, but they had an intuitive understanding of the concept of strength. As happens often in engineering design they were able to design and construct pyramids through trial and error. Modern engineering can reduce this costly and time consuming process by developing rigorous methodologies and formulas. In this book we will be developing formulas for strength and stiffness design of structures and machine elements.

1.1.4   Internally Distributed Force Systems

In Sections 1.1.1 and 1.1.2 the normal stress and the shear stress were introduced as the average intensity of an internal normal and shear force distribution, respectively. But what if there are internal moments at a cross section? Would there be normal and shear stresses at such sections? How are the normal and shear stresses related to internal moments? To answer these questions and to get a better understanding of the character of normal stress and shear stress, we now consider an alternative and more fundamental view.
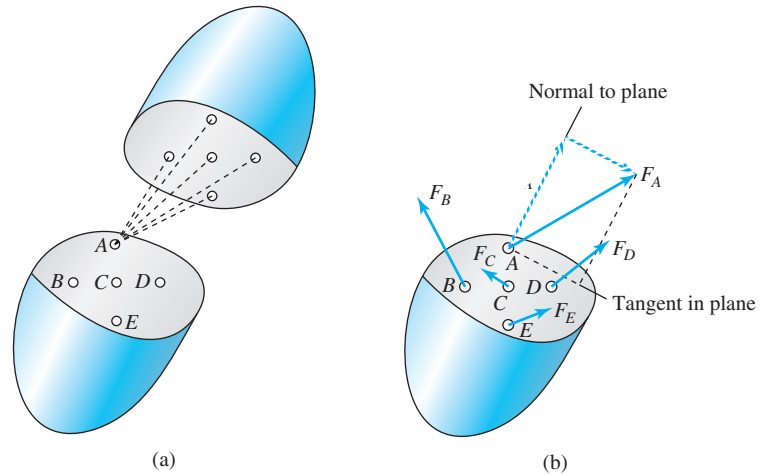

Figure 1.15   Internal forces between particles on two sides of an imaginary cut. (*a*) Forces between particles in a body, shown on particle *A*. (*b*) Resultant force on each particle.

The forces of attraction and repulsion between two particles (atoms or molecules) in a body are assumed to act along the line that joins the two particles.[1] The forces vary inversely as an exponent of the radial distance separating the two particles. Thus every particle exerts a force on every other particle, as shown symbolically in Figure 1.15*a* on an imaginary surface of a body. These forces between particles hold the body together and are referred to as *internal forces*. The shape of the body changes when we apply external forces thus changing distance between particles and hence changing the forces between the particles (internal forces). The body breaks when the change in the internal forces exceeds some characteristic material value. Thus the strength of the material can be characterized by the measure of *change in the intensity* of internal forces. This measure of change in the intensity of internal forces is what we call stress.

In Figure 1.15*b* we replace all forces that are exerted on any single particle by the resultants of these forces on that particle. The magnitude and direction of these resultant forces will vary with the location of the particle (point) implying that this is an internal distributed force system. The intensity of internally distributed forces on an imaginary cut surface of a body is called the **stress on the surface**. The internally distributed forces (stress on a surface) can be resolved into normal (perpendicular to the surface) and tangential (parallel to the surface) distribution. The intensity of an internally distributed force that is normal to the surface of an imaginary cut is called the **normal** stress on the surface. The intensity of an internally distributed force that is parallel to the surface of an imaginary cut surface is called the **shear** stress on the surface.

Normal stress on a surface may be viewed as the internal forces that develop due to the material resistance to the pulling apart or pushing together of two adjoining planes of an imaginary cut. Like pressure, normal stress is always perpendicular to the surface of the imaginary cut. But unlike pressure, which can only be compressive, normal stress can be tensile.

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[1]Forces that act along the line joining two particles are called *central* forces. The concept of central forces started with Newton's universal gravitation law, which states: "the force between two particles is inversely proportional to the square of the radial distance between two particles and acts along the line joining the two particles." At atomic levels the central forces do not vary with the square of the radial distance but with an exponent, which is a power of 8 or 10.

Shear stress on a surface may be viewed as the internal forces that develop due to the material resistance to the sliding of two adjoining planes along the imaginary cut. Like friction, shear stresses act tangent to the plane in the direction opposite to the impending motion of the surface. But unlike friction, shear stress is not related to the normal forces (stresses).
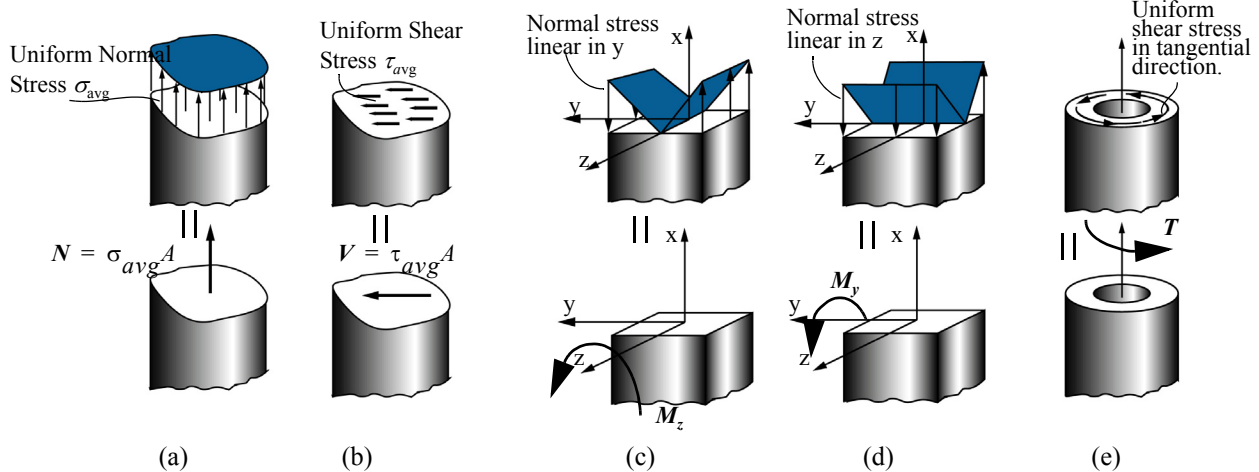

Figure 1.16   Static equivalency.

Now that we have established that the stress on a surface is an internally distributed force system, we are in a position to answer the questions raised at the beginning of the section. If the normal and shear stresses are constant in magnitude and direction across the cross section, as shown in Figure 1.16a and b, then these can be replaced by statically equivalent normal and shear forces. [We obtain the equivalent forms of Equations (1.1) and (1.2).] But if either the magnitude or the direction of the normal and shear stresses changes across the cross section, then internal bending moments M_y, M_z and the internal torque T may be necessary for static equivalency, as shown in Figure 1.16c, d, and e. Figure 1.16 shows some of the stress distributions we will see in this book. But how do we deduce the variation of stress on a surface when stress is an internal quantity that cannot be measured directly? The theories in this book that answer this question were developed over a long period of time using experimental observations, intuition, and logical deduction in an iterative manner. Assumptions have to be made regarding loading, geometry, and material properties of the structural member in the development of the theory. If the theoretical predictions do not conform to experimental observations, then assumptions have to be modified to include added complexities until the theoretical predictions are consistent with experimental observations. In Section 3.2, we will see the logic whose two links are shown in Figure 1.2. This logic with assumptions regarding loading, geometry, and material properties will be used to develop the simplified theories in Chapters 4 through 6.

EXAMPLE 1.4

Figure 1.17 shows a fiber pull-out test that is conducted to determine the shear strength of the interface between the fiber and the resin matrix in a composite material (see Section 3.12.3). Assuming a uniform shear stress τ at the interface, derive a formula for the shear stress in terms of the applied force P, the length of fiber L, and the fiber diameter D.
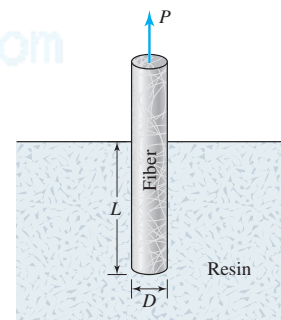

Figure 1.17   Fiber pull-out test.

PLAN

The shear stress is acting on the cylindrical surface area of the embedded fiber. The shear stress is uniform and hence can be replaced by an equivalent shear force V, which we can equate to P.

SOLUTION

Figure 1.18*a* shows the cylindrical surface of the fiber with the uniform shear stress on the surface. The surface area A is equal to the circumference multiplied by the fiber length L as shown by the Equation (E1).
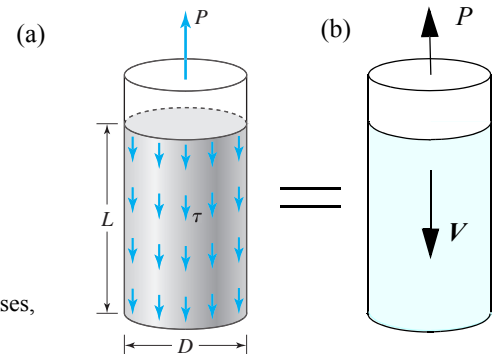
$$A = \pi D L \tag{E1}$$

(a)   (b)

Figure 1.18   Free body diagrams of the fiber in Example 1.4 (a) with shear stresses, (b) with equivalent internal shear force.

The shear force is the shear stress multiplied by the surface area,

$$V = \tau A = (\pi D L)\tau \tag{E2}$$

By equilibrium of forces in Figure 1.18*b*

$$V = P \qquad \text{or} \qquad (\pi D L)\tau = P \tag{E3}$$

ANS. $\tau = P/(\pi D L)$

COMMENTS

1. First, we replaced an internal distributed force system (shear stress) by an equivalent shear force. Second, we related the internal shear force to external force by equilibrium.
2. In the preceding test it is implicitly assumed that the strength of the fiber is greater than the interface strength. Otherwise the fiber would break before it gets pulled out.
3. In a test the force P is increased slowly until the fiber is pulled out. The pull-out force is recorded, and the shear strength can be calculated.
4. Suppose we have determined the shear strength from our formula for specific dimensions D and L of the fiber. Now we should be able to predict the force P that a fiber with different dimensions would support. If on conducting the test the experimental value of P is significantly different from the value predicted, then our assumption of uniform shear stress in the interface is most likely incorrect.

EXAMPLE 1.5

Figure 1.19 shows a test to determine the shear strength of an adhesive. A torque (a moment along the axis) is applied to two thin cylinders joined together with the adhesive. Assuming a uniform shear stress τ in the adhesive, develop a formula for the adhesive shear stress τ in terms of the applied torque T_{ext}, the cylinder radius R, and the cylinder thickness t.

Figure 1.19   Adhesive shear strength test.
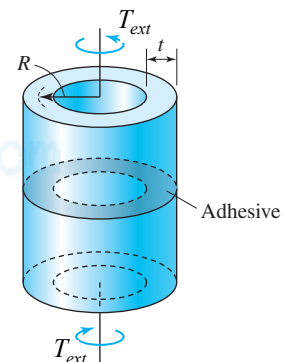
PLAN

A free body diagram can be constructed after making an imaginary cut through the adhesive layer. On a differential area the internal shear force can be found and the moment from the internal shear force on the differential area obtained. By integrating we can find the total internal moment acting on the adhesive, which we can equate to the applied external moment T_{ext}.

SOLUTION

We make an imaginary cut through the adhesive and draw the shear stress in the tangential direction, as shown in Figure 1.20a.
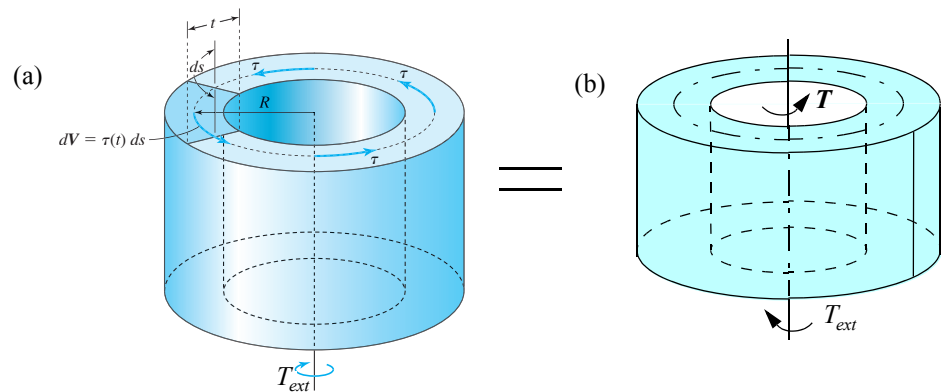
Figure 1.20   Free-body diagrams in Example 1.5 (a) with shear stress, (b) with equivalent internal torque.

The differential area is the differential arc length ds multiplied by the thickness t.
The differential tangential shear force dV is the shear stress multiplied by the differential area.
The differential internal torque (moment) is the moment arm R multiplied by dV, that is, $dT = RdV = R\tau t ds$
Noting that $[ds = Rd\theta]$, we obtain the total internal torque by integrating over the entire circumference.

$$T = \int R\,dV = \int R(\tau t)R\,d\theta = \int_0^{2\pi} \tau t R^2\,d\theta = \tau t R^2 (2\pi) \tag{E1}$$

By equilibrium of moment in Figure 1.20b

$$T = T_{ext} \qquad \text{or} \qquad 2\pi R^2 t \tau = T_{ext} \tag{E2}$$

ANS. $\tau = \dfrac{T_{ext}}{2\pi R^2 t}$

COMMENTS

1. By recording the value of the torque at which the top half of the cylinder separates from the bottom half, we can calculate the shear strength of the adhesive.

2. The assumption of uniform shear stress can only be justified for thin cylinders. In Chapter 5 we will see that shear stress for thicker cylinders varies linearly in the radial direction.

3. First, we replaced an internal distributed force system (shear stress) by an equivalent internal torque. Second, we related the internal torque to external torque by equilibrium.

EXAMPLE 1.6

Figure 1.21 shows a drill being used to make a $L = 12$-in.-deep hole for placing explosive charges in a granite rock. The shear strength of the granite is $\tau = 5$ ksi. Determine the minimum torque T that must be applied to a drill of radius $R = 1$-in., assuming a uniform shear stress along the length of the drill. Neglect the taper at the end.

Figure 1.21   Torque on a drill.

PLAN

The imaginary cut surface is the surface of the hole in the granite. The shear stress on the surface of the hole would act like a distributed frictional force on the cylindrical surface of the drill bit. We can find the moment from this frictional force and relate it to the applied torque.

SOLUTION

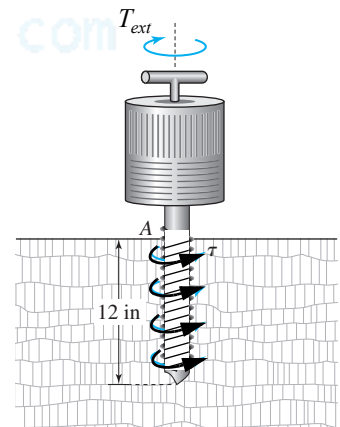The shear stress acts tangential to the cylindrical surface of the drill bit, as shown in Figure 1.22a.
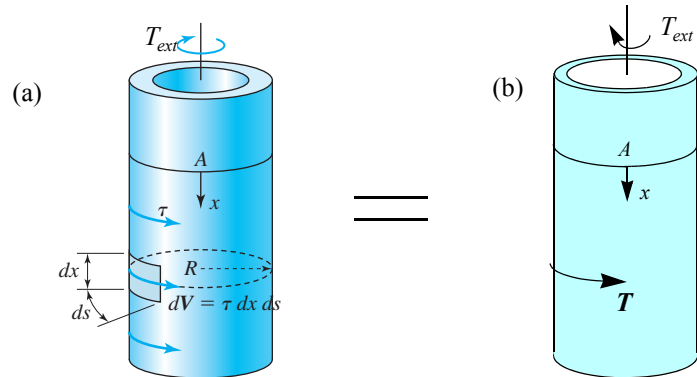
Figure 1.22   Free body diagram of drill bit in Example 1.6 (a) with shear stress, (b) with equivalent internal torque.

Multiplying the shear stress by the differential surface area $ds\,dx$ we obtain the differential tangential shear force dV.
Multiplying dV by the moment arm R, we obtain the internal torque $dT = R\,dV = R\tau\,ds\,dx$, which is due to the shear stress over the differential surface area.
Integrating over the circumference $ds = R\,d\theta$ and the length of the drill, we obtain the total internal torque.

$$T = \int R\,dV = \int_0^L \int_0^{2\pi} R(\tau)R\,d\theta\,dx = \tau R^2 \int_0^L \int_0^{2\pi} d\theta\,dx = \tau R^2 \int_0^L 2\pi\,dx = 2\pi = 2\pi\tau R^2 L \text{ or}$$

$$T = 2\pi(5 \text{ ksi})(1 \text{ in.})^2(12 \text{ in.}) = 120\pi \text{ in.}\cdot\text{kips} \qquad (E1)$$

By equilibrium of moment in Figure 1.22b

$$T_{ext} = T \qquad (E2)$$

ANS. $T_{ext} = 377 \text{ in.}\cdot\text{kips}$

COMMENTS

1. In this example and in Example 1.4 shear stress acted on the outside cylindrical surface. In Example 1.4 we replaced the shear stresses by just an internal shear force, whereas in this example we replaced the shear stresses by an internal torque. The difference comes from the direction of the shear stress.

2. In Example 1.5 and in this example the surfaces on which the shear stresses are acting are different. Yet in both examples we replaced the shear stresses by the equivalent internal torque.

3. The two preceding comments emphasize that before we can define which internal force or which internal moment is statically equivalent to the internal stress distribution, we must specify the direction of stress and the orientation of the surface on which the stress is acting. We shall develop this concept further in Section 1.2.

Consolidate your knowledge

1.   In your own words describe stress on a surface.

QUICK TEST 1.1

Time: 15 minutes Total: 20 points

Answer true or false and justify each answer in one sentence. Grade yourself with the answers given in Appendix E. Give yourself one point for each correct answer (true or false) and one point for every correct explanation.

1. You can measure stress directly with an instrument the way you measure temperature with a thermometer.
2. There can be only one normal stress component acting on the surface of an imaginary cut.
3. If a shear stress component on the left surface of an imaginary cut is upward, then on the right surface it will be downward.
4. If a normal stress component puts the left surface of an imaginary cut in tension, then the right surface will be in compression.
5. The correct way of reporting shear stress is $\tau = 70$ kips.
6. The correct way of reporting positive axial stress is $\sigma = +15$ MPa.
7. 1 GPa equals 10^6 Pa.
8. 1 psi is approximately equal to 7 Pa.
9. A common failure stress value for metals is 10,000 Pa.
10. Stress on a surface is the same as pressure on a surface as both quantities have the same units.

PROBLEM SET 1.2

Internally Distributed Force Systems

1.73 The post shown in Figure P1.73 has a rectangular cross section of 2 in. × 4 in. The length L of the post buried in the ground is 12 in. and the average shear strength of the soil is 2 psi. Determine the force P needed to pull the post out of the ground.
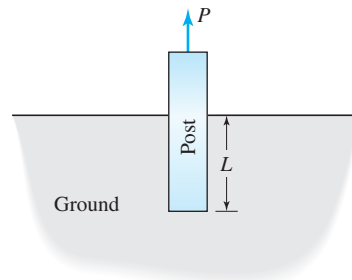
Figure P1.73

1.74 The post shown in Figure P1.73 has a circular cross section of 100-mm diameter. The length L of the post buried in the ground is 400 mm. It took a force of 1250 N to pull the post out of the ground. What was the average shear strength of the soil?

1.75 The cross section of the post shown in Figure P1.73 is an equilateral triangle with each side of dimension a. If the average shear strength of the soil is τ, determine the force P needed to pull the post out of the ground in terms of τ, L, and a.

1.76 A force P = 10 lb is applied to the handle of a hammer in an effort to pull a nail out of the wood, as shown in Figure P1.76. The nail has a diameter of $\frac{1}{8}$ in. and is buried in wood to a depth of 2 in. Determine the average shear stress acting on the nail.
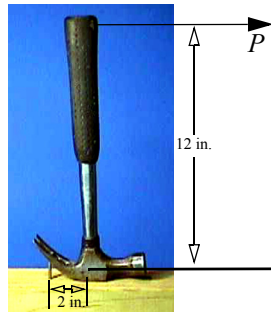
Figure P1.76

1.77 Two cast-iron pipes are adhesively bonded together over a length of 200 mm as shown in Figure P1.77. The outer diameters of the two pipes are 50 mm and 70 mm, and the wall thickness of each pipe is 10 mm. The two pipes separated while transmitting a force of 100 kN. What was the average shear stress in the adhesive just before the two pipes separated?
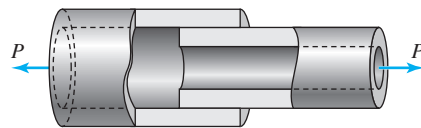
Figure P1.77

1.78 Two cast-iron pipes are adhesively bonded together over a length of 200 mm (Figure P1.78). The outer diameters of the two pipes are 50 mm and 70 mm, and the wall thickness of each pipe is 10 mm. The two pipes separated while transmitting a torque of 2 kN · m. What was the average shear stress in the adhesive just before the two pipes separated?
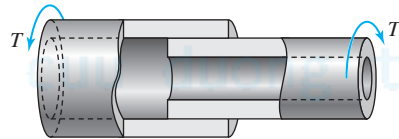
Figure P1.78

1.79 Two cast-iron pipes are held together by a bolt, as shown in Figure P1.79. The outer diameters of the two pipes are 50 mm and 70 mm, and the wall thickness of each pipe is 10 mm. The diameter of the bolt is 15 mm. The bolt broke while transmitting a torque of 2 kN·m. On what surface(s) did the bolt break? What was the average shear stress in the bolt on the surface where it broke?
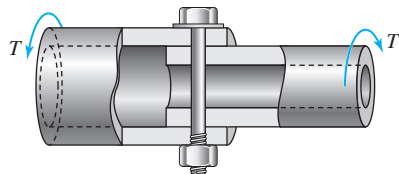
Figure P1.79

1.80 The can lid in Figure P1.80a gets punched on two sides AB and AC of an equilateral triangle ABC. Figure P1.80b is the top view showing relative location of the points. The thickness of the lid is t = 1/64 in. and the lid material can at most support a shear stress of 1800 psi. Assume a uniform shear stress during punching and point D acts like a pin joint. Use a= 1/2 in., b = 3 in. and c =1/4 in. Determine the minimum force F that must be applied to the can opener.
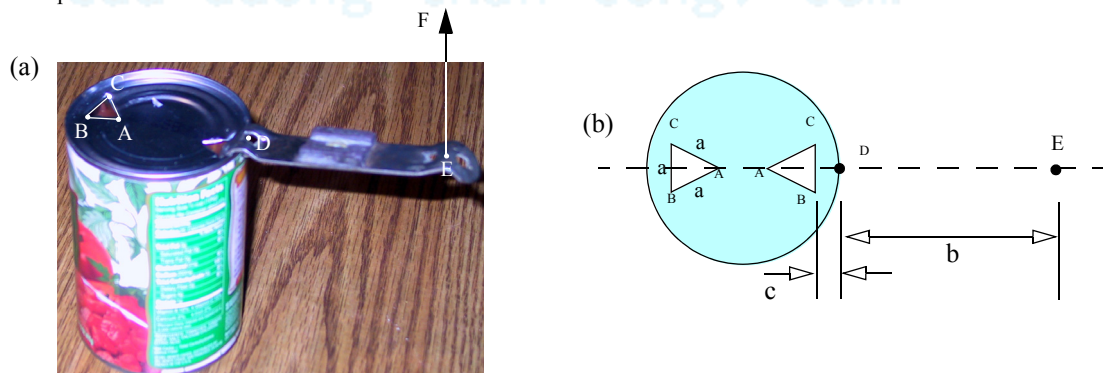
Figure P1.80

1.81 It is proposed to use $\frac{1}{2}$-in. diameter bolts in a 10-in.-diameter coupling for transferring a torque of 100 in. · kips from one 4-in.-diameter shaft onto another (Figure P1.81). The maximum average shear stress in the bolts is to be limited to 20 ksi. How many bolts are needed, and at what radius should the bolts be placed on the coupling? (Note there are multiple answers.)
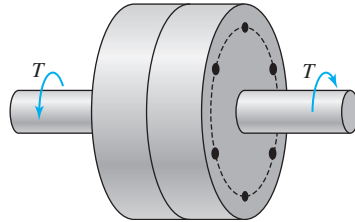
Figure P1.81

1.82 A human hand can comfortably apply a torsional moment of 15 in.·lb (Figure P1.82). (a) What should be the breaking shear strength of a seal between the lid and the bottle, assuming the lid has a diameter of $1\frac{1}{2}$ in. and a height of $\frac{1}{2}$ in.? (b) If the same sealing strength as in part (a) is used on a lid that is 1 in. in diameter and $\frac{1}{2}$ in. in height, what would be the torque needed to open the bottle?

Figure P1.82

1.83 The torsional moment on the lid is applied by hand exerting a force F on the handle of bottle opener as shown in Figure P1.83. Assume the average shear strength of the bond between the lid and the bottle is 10 psi. Determine the minimum force F needed to open the bottle. Use t = 3/8 in. d = 2 1/2 in. and a = 4 in.
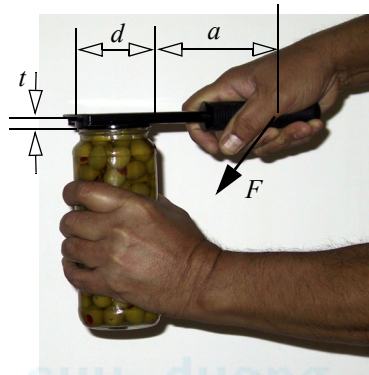
Figure P1.83

1.2 STRESS AT A POINT

The breaking of a structure starts at the point where the internal force intensity—that is, where stress exceeds some material characteristic value. This implies that we need to refine our definition of stress on a surface to that of *Stress at a Point*. But an infinite number of planes (surfaces) can pass through a point. Which imaginary surface do we shrink to zero? And when we shrink the surface area to zero, which equation should we use, (1.1) or (1.2)? Both difficulties can be addressed by assigning an orientation to the imaginary surface and to the internal force on this surface. We label these directions with subscripts for the stress components, in the same way that subscripts x, y, and z describe the components of vectors.

Figure 1.23 shows a body cut by an imaginary plane that has an outward normal in the i direction. On this surface we have a differential area ΔA_i on which a resultant force acts. $\Delta \boldsymbol{F}_j$ is the component of the force in the j direction. A component

of average stress is $\Delta \boldsymbol{F}_j / \Delta A_i$. If we shrink ΔA_i to zero we get the definition of a stress component at a point as shown by the Equation (1.3).

$$\sigma_{ij} = \lim_{\Delta A_i \to 0} \left(\frac{\Delta \boldsymbol{F}_j}{\Delta A_i} \right)$$

direction of outward normal to imaginary cut surface

direction of internal force component

(1.3)

Outward normal
i
Internal force
ΔA_i
$\Delta \boldsymbol{F}_j$
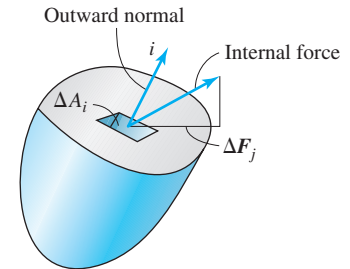
Figure 1.23   Stress at a point.

Now when we look at a stress component, the first subscript tells us the orientation of the imaginary surface and the second the direction of the internal force.

In three dimensions each subscript i and j can refer to an x, y, or z direction. In other words, there are nine possible combinations of the two subscripts. This is shown in the stress matrix in Equation (1.4). The diagonal elements in the stress matrix are the normal stresses and all off-diagonal elements represent the shear stresses.

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

(1.4)

To specify the stress at a point, we need a magnitude and two directions. In three dimensions we need nine components of stress, and in two dimensions we need four components of stress to completely define stress at a point. Table 1.3 shows the number of components needed to specify a scalar, a vector, and stress. Now force, moment, velocity, and acceleration are all different quantities, but they all are called vectors. In a similar manner, stress belongs to a category called *tensors*. More specifically, stress is a **second-order tensor,**[2] where 'second order' refers to the exponent in the last row. In this terminology, a vector is a tensor of order 1, and a scalar is a tensor of order 0.

TABLE 1.3   **Comparison of number of components**

Quantity	One Dimension	Two Dimensions	Three Dimensions
Scalar	$1 = 1^0$	$1 = 2^0$	$1 = 3^0$
Vector	$1 = 1^1$	$2 = 2^1$	$3 = 3^1$
Stress	$1 = 1^2$	$4 = 2^2$	$9 = 3^2$

1.2.1   Sign convention

To obtain the sign of a stress component in Equation (1.3) we establish the following sign convention.

> **Sign Convention:** Differential area ΔA_i will be considered positive if the outward normal to the surface is in the positive i direction. If the outward normal is in the negative i direction, then ΔA_i will be considered negative.

We can now deduce the sign for stress. A stress component can be positive in two ways. Both the numerator and the denominator are positive or both the numerator and the denominator are negative in Equation (1.3). Alternatively, if numerator and the

---

[2]To be labeled as tensor, a quantity must also satisfy certain coordinate transformation properties, which will be discussed briefly in Chapter 8.

denominator in Equation (1.3) have: the same sign the stress component is *positive*; if they have opposite signs the stress component is *negative*.

We conclude this section with the following points to remember.

- Stress is an internal quantity that has units of force per unit area.
- A stress component at a *point* is specified by a magnitude and two directions. Stress at a point is a second-order tensor.
- Stress on a *surface* is specified by a magnitude and only one direction. Stress on a surface thus is a vector.
- The first subscript on stress gives the direction of the outward normal of the imaginary cut surface. The second subscript gives the direction of the internal force.
- The sign of a stress component is determined from the direction of the internal force and the direction of the outward normal to the imaginary cut surface.

1.3   STRESS ELEMENTS

The previous section showed that stress at a point is an abstract quantity. Stress on a surface, however, is easier to visualize as the intensity of a distributed force on a surface. A **stress element** is an imaginary object that helps us visualize stress at a point by constructing surfaces that have outward normals in the coordinate directions. In Cartesian coordinates the stress element is a cube; in cylindrical or a spherical coordinates the stress element is a fragment of a cylinder or a sphere, respectively. We start our discussion with the construction of a stress element in Cartesian coordinates to emphasize the basic construction principles. We can use a similar process to draw stress elements in cylindrical and spherical coordinate systems as demonstrated in Example 1.9.

1.3.1   Construction of a Stress Element for Axial Stress

Suppose we wish to visualize a positive stress component σ_{xx} at a point that may be generated in an bar under axial forces shown in Figure 1.24*a*. Around this point imagine an object that has sides with outward normals in the coordinate direction. The cube has six surfaces with outward normals that are either in the positive or in the negative coordinate direction, as shown in Figure 1.24. The first subscript of σ_{xx} tells us it must be on the surface that has an outward normal in the *x* direction. Thus, the two surfaces on which σ_{xx} will be shown are at *A* and *B*.
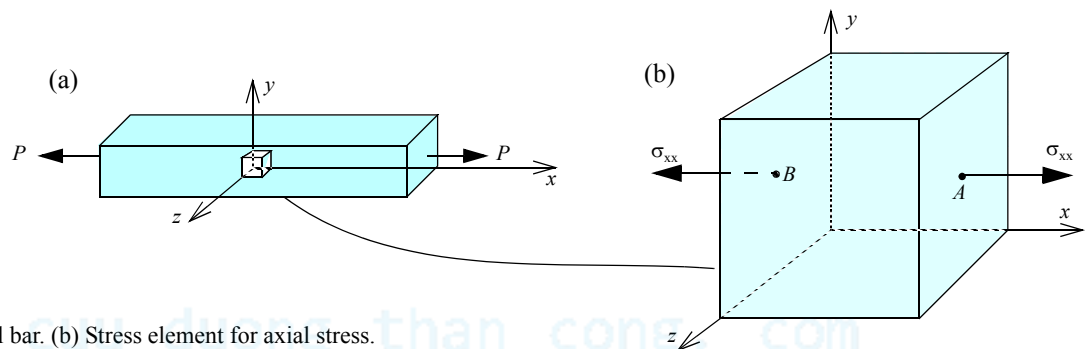
Figure 1.24   (a) Axial bar. (b) Stress element for axial stress.

The direction of the outward normal on surface *A* is in the positive *x* direction [the denominator is positive in Equation (1.3)]. For the stress component to be positive on surface *A*, the force must be in the positive *x* direction [the numerator must be positive in Equation (1.3)], as shown in Figure 1.24*b*.

The direction of the outward normal on surface *B* is in the negative *x* direction [the denominator is negative in Equation (1.3)]. For the stress component to be positive on surface *B*, the force must be in the negative *x* direction [the numerator must be negative in Equation (1.3)], as shown in Figure 1.24*b*.

The positive stress component σ_{xx} are pulling the cube in opposite directions; that is, the cube is in tension due to a positive normal stress component. We can use this information to draw normal stresses in place of subscripts. A tensile normal stress will pull the surface away from the interior of the element and a compressive normal stress will push the surface into the element. As mentioned earlier, normal stresses are usually reported as tension or compression and not as positive or negative.

It should be emphasized that the single arrow used to show the stress component does not imply that the stress component is a force. Showing the stress components as distributed forces on surfaces *A* and *B* as in Figure 1.25 is visually more accurate

but very tedious to draw every time we need to visualize stress. We will show stress components using single arrows as in Figure 1.24, but visualize them as shown in Figure 1.25.
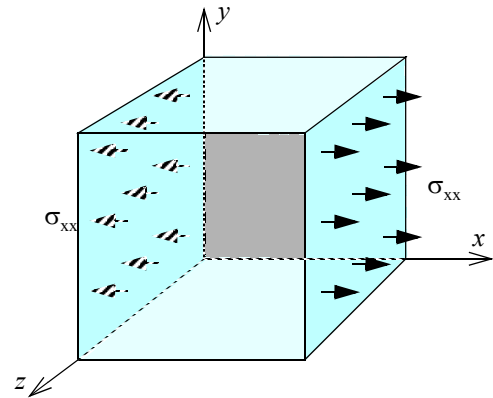
Figure 1.25   Stress components are distributed forces on a surface.

1.3.2   Construction of a Stress Element for Plane Stress

Plane stress is one of the two types of two-dimensional simplifications used in mechanics of materials. In Chapter 2 we will study the other type, plane strain. In Chapter 3 we will study the difference between the two types. By two dimensional we imply that one of the coordinates does not play a role in the description of the problem. If we choose z to be the coordinate, we set all stresses with subscript z to zero to get

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{1.5}$$

We assume that the stress components in Equation (1.5) are positive. Let us consider the first row. The first subscript gives us the direction of the outward normal, which is the x direction. Surfaces A and B in Figure 1.26a have outward normals in the x direction, and it is on these surfaces that the stress component of the first row will be shown.
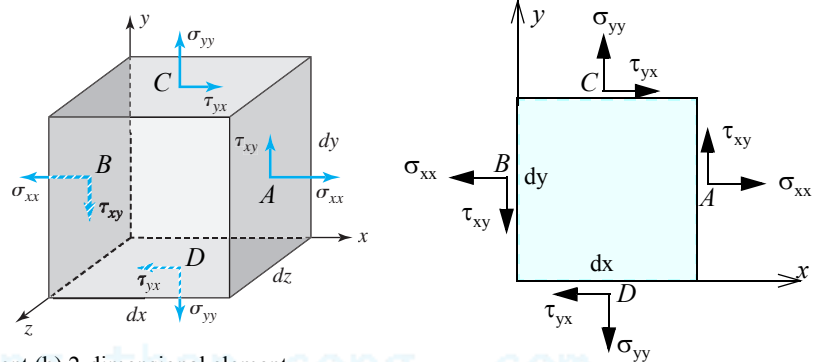
Figure 1.26   Plane stress: (a) 3-dimensional element (b) 2-dimensional element.

The direction of the outward normal on surface A is in the positive x direction [the denominator is positive in Equation (1.3)]. For the stress component to be positive on surface A, the force must be in the positive direction [the numerator must be positive in Equation (1.3)], as shown in Figure 1.26a.

The direction of the outward normal on surface B is in the negative x direction [the denominator is negative in Equation (1.3)]. For the stress component to be positive on surface B, the force must be in the negative direction [the numerator must be negative in Equation (1.3)], as shown in Figure 1.26a.

Now consider row 2 in the stress matrix in Equation (1.5). From the first subscript we know that the normal to the surface is in the y direction. Surface C has an outward normal in the positive y direction, therefore all forces on surface C are in the positive direction of the second subscript, as shown in Figure 1.26a. Surface D has an outward normal in the negative y direction, therefore all forces on surface D are in the negative direction of the second subscript, as shown in Figure 1.26a.

We note that the plane with outward normal in the z direction is stress-free. Stress-free surfaces are also called **free surfaces**, and these surfaces play an important role in stress analysis.

Figure 1.26b shows the two-dimensional representation of the stress element that will be seen looking down the z axis.

1.4 SYMMETRIC SHEAR STRESSES

If a body is in equilibrium, then all points on the body are in equilibrium. Is the stress element that represents a point on the body in equilibrium? To answer this question we need to convert the stresses into forces by multiplying by the surface area. We take a simple problem of plane stress and assume that the cube in Figure 1.26 has lengths of *dx*, *dy*, and *dz* in the coordinate directions. We draw a two-dimensional picture of the stress cube after multiplying each stress component by the surface area and get the force diagram of Figure 1.27.[3]
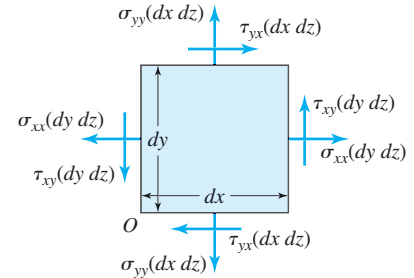
Figure 1.27   Force diagram for plane stress.

In Figure 1.27 we note that the equations of force equilibrium are satisfied by the assumed state of stress at a point. We consider the moment about point *O* and obtain

$$(\tau_{xy} \, dy \, dz)dx \; = \; (\tau_{yx} \, dx \, dz)dy \qquad\qquad (1.6)$$

We cancel the differential volume (dx dy dz) on both sides to obtain

$$\boxed{\tau_{xy} \; = \; \tau_{yx}} \qquad\qquad (1.7a)$$

In a similar manner we can show that

$$\boxed{\tau_{yz} \; = \; \tau_{zy}} \qquad\qquad (1.7b)$$

$$\boxed{\tau_{zx} \; = \; \tau_{xz}} \qquad\qquad (1.7c)$$

Equations (1.7a) through (1.7c) emphasize that shear stress is symmetric. The symmetry of shear stress implies that in *three dimensions* there are only *six independent* stress components out of the nine components necessary to specify stress at a point. In *two dimensions* there are only *three independent* stress components out of the four components necessary to specify stress at a point. In Figure 1.26 notice that the shear stress components τ_{xy} and τ_{yx} point either toward the corners or away from the corners. This observation can be used in drawing the symmetric pair of shear stresses after drawing the shear stress on one of the surfaces of the stress cube.

EXAMPLE 1.7

Show the non-zero stress components on the surfaces of the two cubes shown in different coordinate systems in Figure 1.28.

$$\begin{bmatrix} \sigma_{xx} = 80\text{MPa(T)} & \tau_{xy} = 30\text{MPa} & 0 \\ \tau_{yx} = 30\text{MPa} & \sigma_{yy} = 40\text{MPa(C)} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
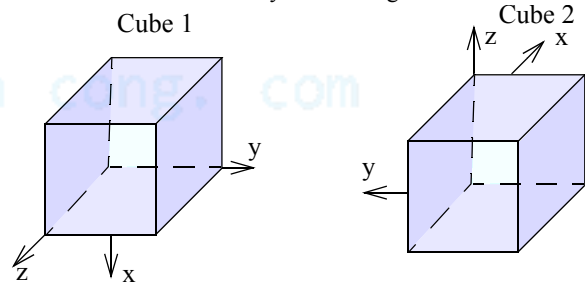
Figure 1.28   Cubes in different coordinate systems in Example 1.7.

PLAN

We can identify the surface with the outward normal in the direction of the first subscript. Using the sign convention and Equation (1.3) we draw the force in the direction of the second subscript.

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[3]Figure 1.27 is only valid if we assume that the stresses are varying very slowly with the *x* and *y* coordinates. If this were not true, we would have to account for the increase in stresses over a differential element. But a more rigorous analysis will also reveal that shear stresses are symmetric, see Problem 1.105.

SOLUTION

Cube 1: The first subscript of σ_{xx} and τ_{xy} shows that the outward normal is in the x direction; hence these components will be shown on surfaces C and D in Figure 1.29a.

Figure 1.29   Solution of Example 1.7.(a) Cube 1. (b) Cube 2.

The outward normal on surface C is in the negative x direction; hence the denominator in Equation (1.3) is negative. Therefore on Figure 1.29a:
- The internal force has to be in the negative x direction to produce a positive (tensile) σ_{xx}.
- The internal force has to be in the negative y direction to produce a positive τ_{xy}.

The outward normal on surface D is in the positive x direction; hence the denominator in Equation (1.3) is positive. Therefore on Figure 1.29a:
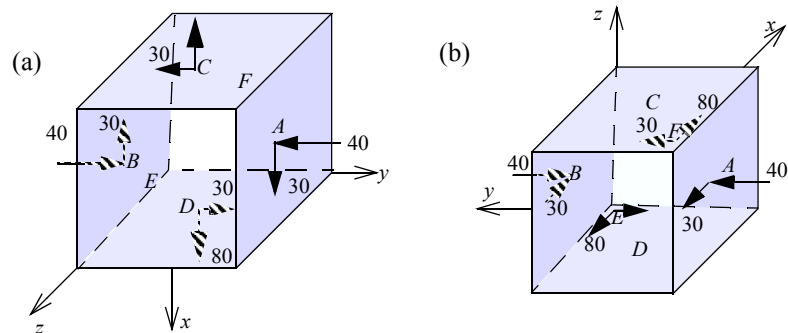- The internal force has to be in the positive x direction to produce a positive (tensile) σ_{xx}.
- The internal force has to be in the positive y direction to produce a positive τ_{xy}.

The first subscript of τ_{yx} and σ_{yy} shows that the outward normal is in the y direction; hence this component will be shown on surfaces A and B.

The outward normal on surface A is in the positive y direction; hence the denominator in Equation (1.3) is positive. Therefore on Figure 1.29a:
- The internal force has to be in the positive x direction to produce a positive τ_{yx}.
- The internal force has to be in the negative y direction to produce a negative (compressive) σ_{yy}.

The outward normal on surface B is in the negative y direction; hence the denominator in Equation (1.3) is negative. Therefore on Figure 1.29a:
- The internal force has to be in the negative x direction to produce a positive τ_{yx}.
- The internal force has to be in the positive y direction to produce a negative (compressive) σ_{yy}.

Cube 2: The first subscript of σ_{xx} and τ_{xy} shows that the outward normal is in the x direction; hence these components will be shown on surfaces E and F.

The outward normal on surface E is in the negative x direction; hence the denominator in Equation (1.3) is negative. Therefore on Figure 1.29a:
- The internal force has to be in the negative x direction to produce a positive (tensile) σ_{xx}.
- The internal force has to be in the negative y direction to produce a positive τ_{xy}.

The outward normal on surface F is in the positive x direction; hence the denominator in Equation (1.3) is positive. Therefore on Figure 1.29a:
- The internal force has to be in the positive x direction to produce a positive (tensile) σ_{xx}.
- The internal force has to be in the positive y direction to produce a positive τ_{xy}.

The first subscript of τ_{yx} and σ_{yy} shows that the outward normal is in the y direction; hence this component will be shown on surfaces A and B.

The outward normal on surface A is in the negative y direction; hence the denominator in Equation (1.3) is negative. Therefore on Figure 1.29a:
- The internal force has to be in the negative x direction to produce a positive τ_{yx}.
- The internal force has to be in the positive y direction to produce a negative (compressive) σ_{yy}.

The outward normal on surface B is in the positive y direction; hence the denominator in Equation (1.3) is positive. Therefore on Figure 1.29a:
- The internal force has to be in the positive x direction to produce a positive τ_{yx}.
- The internal force has to be in the negative y direction to produce a negative (compressive) σ_{yy}.

COMMENTS

1. Figure 1.30 shows the two-dimensional representations of stress cubes shown in Figure 1.29. These two-dimensional representations are easier to draw but it must be kept in mind that the point is in three-dimensional space with surfaces with outward normals in the z direction being stress free.
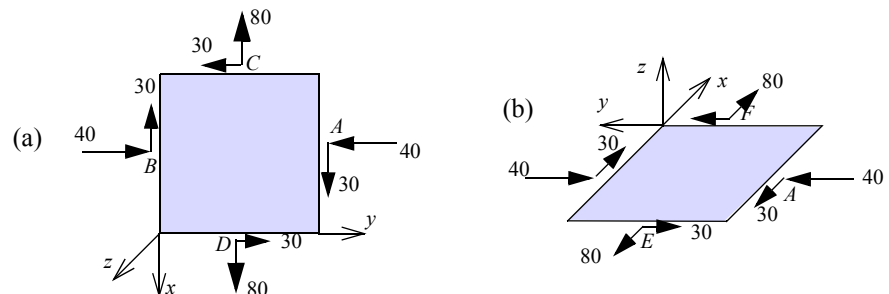

Figure 1.30   Two-dimensional depiction of the solution of Example 1.7.(a) Cube 1. (b) Cube 2.

2. We note that σ_{xx} pulls the surfaces outwards and σ_{yy} pushes the surfaces inwards in Figures 1.29 and 1.30 as these are tensile and compressive stresses, respectively. Hence, we can use this information to draw these stress components without using the subscripts.

3. The shear stress τ_{xx} and τ_{yx} either point towards the corner or away from the corner as seen in Figures 1.29 and 1.30. Using this information we can draw the shear stress on the appropriate surfaces after obtaining the direction on one surface using subscripts.

1.5*   CONSTRUCTION OF A STRESS ELEMENT IN 3-DIMENSION

We once more visualize a cube with outward normals in the coordinate direction around the point we wish to show our stress components. The cube has six surfaces with outward normals that are either in the positive or in the negative coordinate direction, as shown in Figure 1.31. In other words, we have now accounted for the first subscript in our stress definition. We know that force is in the positive or negative direction of the second subscript. We use our sign convention to show the stress in the direction of the force on each of the six surfaces.
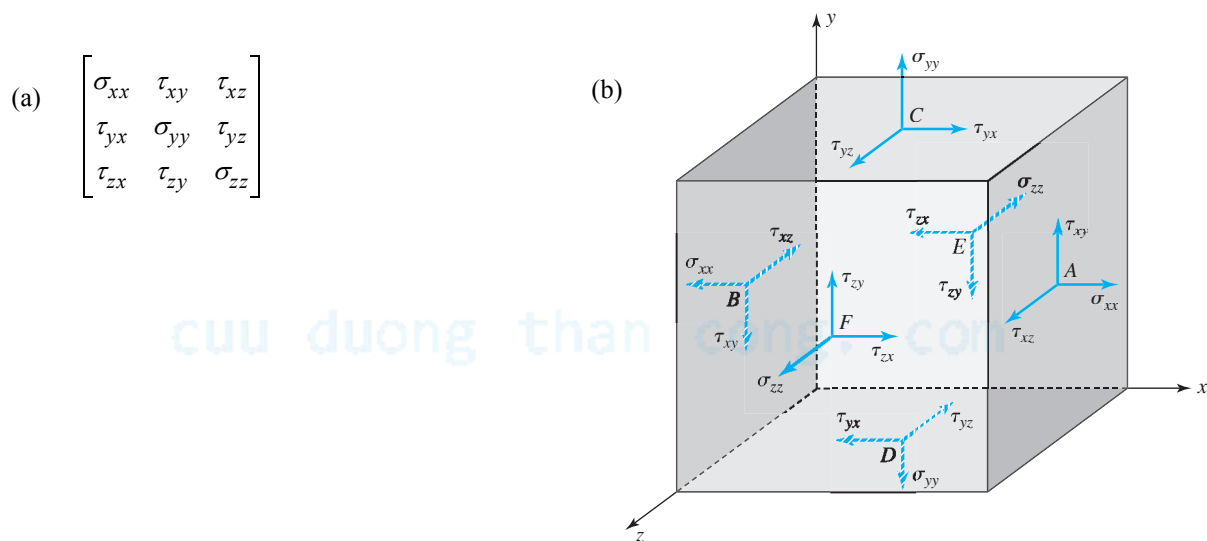
(a)
$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

Figure 1.31   Stress cube showing all positive stress components in three dimensions.

To demonstrate the construction of the stress element we will assume that all nine stress components in the stress matrix shown in Figure 1.31a are positive. Let us consider the first row. The first subscript gives us the direction of the outward normal, which is the *x* direction. Surfaces *A* and *B* in Figure 1.31b have outward normals in the *x* direction, and it is on these surfaces that the stress component of the first row will be shown.

The direction of the outward normal on surface *A* is in the positive *x* direction [the denominator is positive in Equation (1.3)]. For the stress component to be positive on surface *A*, the force must be in the positive direction [the numerator must be positive in Equation (1.3)], as shown in Figure 1.31b.

The direction of the outward normal on surface *B* is in the negative *x* direction [the denominator is negative in Equation (1.3)]. For the stress component to be positive on surface *B*, the force must be in the negative direction [the numerator must be negative in Equation (1.3)], as shown in Figure 1.31b.

Now consider row 2 in the stress matrix in Figure 1.31a. From the first subscript we know that the normal to the surface is in the *y* direction. Surface *C* has an outward normal in the positive *y* direction, therefore all forces on surface *C* are in the positive direction of the second subscript, as shown in Figure 1.31b. Surface *D* has an outward normal in the negative *y* direction, therefore all forces on surface *D* are in the negative direction of the second subscript, as shown in Figure 1.31b.

By the same logic, the components of row 3 in the stress matrix are shown on surfaces *E* and *F* in Figure 1.31b.

EXAMPLE 1.8

Show the nonzero stress components on surfaces *A*, *B*, and *C* of the two cubes shown in different coordinate systems in Figure 1.32.

$$\begin{bmatrix} \sigma_{xx} = 80 \text{ MPa (T)} & \tau_{xy} = 30 \text{ MPa} & \tau_{xz} = -70 \text{ MPa} \\ \tau_{yx} = 30 \text{ MPa} & \sigma_{yy} = 0 & \tau_{yz} = 0 \\ \tau_{zx} = -70 \text{ MPa} & \tau_{zy} = 0 & \sigma_{zz} = 40 \text{ MPa (C)} \end{bmatrix}$$
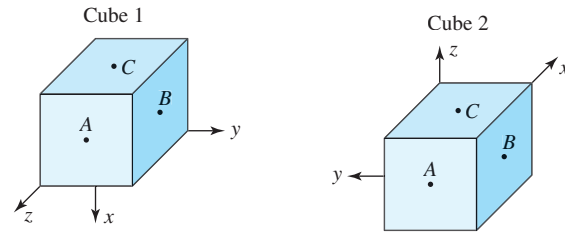

Figure 1.32   Cubes in different coordinate systems.

PLAN

We can identify the surface with the outward normal in the direction of the first subscript. Using the sign convention and Equation (1.3) we draw the force in the direction of the second subscript.

SOLUTION

Cube 1: The first subscript of σ_{xx}, τ_{xy}, and τ_{xz} shows that the outward normal is in the *x* direction; hence these components will be shown on surface *C*. The outward normal on surface *C* is in the negative *x* direction; hence the denominator in Equation (1.3) is negative. Therefore on Figure 1.33*a:*
* The internal force has to be in the negative *x* direction to produce a positive (tensile) σ_{xx}.
* The internal force has to be in the negative *y* direction to produce a positive τ_{xy}.
* The internal force has to be in the positive *z* direction to produce a negative τ_{xz}.

The first subscript of τ_{yx} shows that the outward normal is in the *y* direction; hence this component will be shown on surface *B*. The outward normal on surface *B* is in the positive *y* direction; hence the denominator in Equation (1.3) is positive. Therefore on Figure 1.33*a:*
* The internal force has to be in the positive *x* direction to produce a positive τ_{yx}.

The first subscript of τ_{zx}, σ_{zz} shows that the outward normal is in the *z* direction; hence these components will be shown on surface *A*. The outward normal on surface *A* is in the positive *z* direction; hence the denominator in Equation (1.3) is positive. Therefore on Figure 1.33*a:*
* The internal force has to be in the negative *x* direction to produce a negative τ_{zx}.
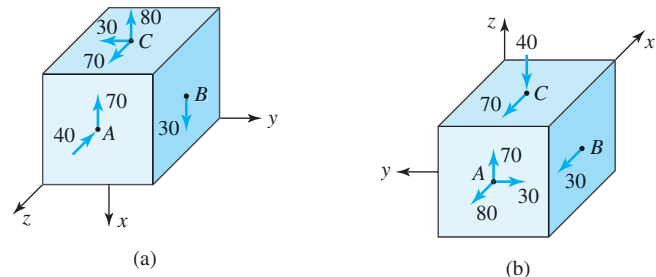* The internal force has to be in the negative *z* direction to produce a negative (compressive) σ_{zz}.

Figure 1.33   Solution of Example 1.8.

(a)   (b)

Cube 2: The first subscript of σ_{xx}, τ_{xy}, and τ_{xz} shows that the outward normal is in the *x* direction; hence these components will be shown on surface *A*. The outward normal on surface *A* is in the negative *x* direction; hence the denominator in Equation (1.3) is negative. Therefore in Figure 1.33*b:*
* The internal force has to be in the negative *x* direction to produce a positive (tensile) σ_{xx}.
* The internal force has to be in the negative *y* direction to produce a positive τ_{xy}.
* The internal force has to be in the positive *z* direction to produce a negative τ_{xz}.

The first subscript of τ_{yx} shows that the outward normal is in the *y* direction; hence this component will be shown on surface *B*. The outward normal on surface *B* is in the negative *y* direction; hence the denominator in Equation (1.3) is negative. Therefore in Figure 1.33*b:*
* The internal force has to be in the negative *y* direction to produce a positive τ_{yx}.

The first subscript of τ_{zx}, σ_{zz} shows that the outward normal is in the *z* direction; hence these components will be shown on surface *C*. The outward normal on surface *C* is in the positive *z* direction; hence the denominator in Equation (1.3) is positive. Therefore in Figure 1.33*b:*
* The internal force has to be in the negative *x* direction to produce a negative τ_{zx}.
* The internal force has to be in the negative *z* direction to produce a negative (compressive) σ_{zz}.

COMMENTS

1. In drawing the normal stresses we could have made use of the fact that σ_{xx} is tensile and hence pulls the surface outward. σ_{zz} is compressive and hence pushes the surface inward. This is a quicker way of getting the directions of these stress components than the arguments based on signs and subscripts.

2. Once we have drawn τ_{xy} and τ_{xz} using the subscripts, we could draw τ_{yx} and τ_{zx} using the observation that the pair of symmetric shear stresses point towards or away from the corner formed by the two adjoining surfaces, thus saving some effort in the construction of the stress element.

EXAMPLE 1.9

Show the following positive stress components on a stress element drawn in the spherical coordinate system shown in Figure 1.34.

$$\begin{bmatrix} \sigma_{rr} & \tau_{r\theta} & \tau_{r\phi} \\ \tau_{\theta r} & \sigma_{\theta\theta} & \tau_{\theta\phi} \\ \tau_{\phi r} & \tau_{\phi\theta} & \sigma_{\phi\phi} \end{bmatrix}$$
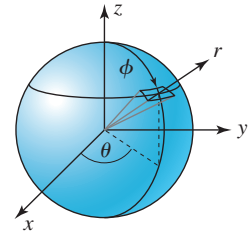

Figure 1.34   Stresses in spherical coordinates.

PLAN

We construct a stress element with surfaces that have outward normals in the r, θ, and ϕ directions. The first subscript will identify the surface on which the row of stress components is to be shown. The second subscript then will show the direction of the stress component on the surface.

SOLUTION

We draw a stress element with lines in the directions of r, θ, and ϕ, as shown in Figure 1.35.
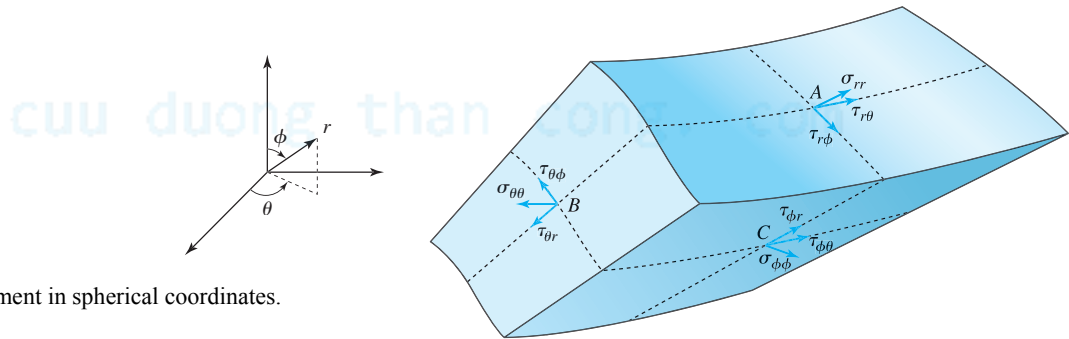

Figure 1.35   Stress element in spherical coordinates.

The stresses σ_{rr}, $\tau_{r\theta}$, and $\tau_{r\phi}$ will be on surface A in Figure 1.35. The outward normal on surface A is in the positive r direction. Thus the forces have to be in the positive r, θ, and ϕ directions to result in positive σ_{rr}, $\tau_{r\theta}$, and $\tau_{r\phi}$.

The stresses $\tau_{\theta r}$, $\sigma_{\theta\theta}$, and $\tau_{\theta\phi}$ will be on surface B in Figure 1.35. The outward normal on surface B is in the negative θ direction. Thus the forces have to be in the negative r, θ, and ϕ directions to result in positive $\tau_{\theta r}$, $\sigma_{\theta\theta}$, and $\tau_{\theta\phi}$.

The stresses $\tau_{\phi r}$, $\tau_{\phi\theta}$, and $\sigma_{\phi\phi}$ will be on surface C in Figure 1.35. The outward normal on surface C is in the positive ϕ direction. Thus the forces have to be in the positive r, θ, and ϕ directions to result in positive $\tau_{\phi r}$, $\tau_{\phi\theta}$, and $\sigma_{\phi\phi}$.

COMMENT

1. This example demonstrates that use of subscripts in determining the direction of stress components follows the same procedure as in cartesian coordinates even though the stress element is a fragment of a sphere.

Consolidate your knowledge

1.   In your own words describe stress at a point and how it differs from stress on a surface
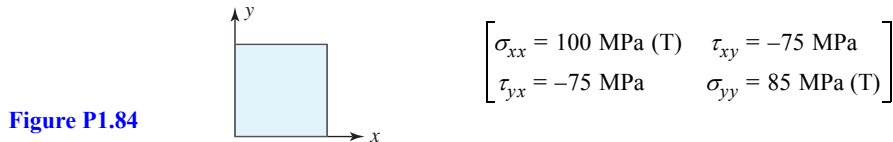
QUICK TEST 1.2   **Time: 15 minutes/Total: 20 points**

Answer true or false and justify each answer in one sentence. Grade yourself with the answers given in Appendix E. Give yourself one point for every correct answer (true or false) and one point for every correct explanation.
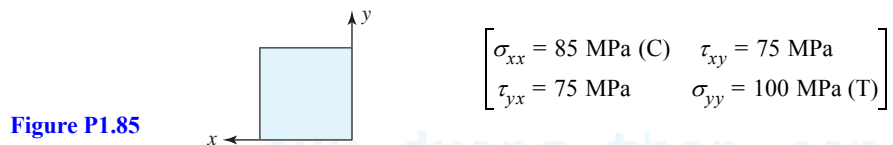
1. Stress at a point is a vector like stress on a surface.
2. In three dimensions stress has nine components.
3. In three dimensions stress has six independent components.
4. At a point in plane stress there are three independent stress components.
5. At a point in plane stress there are always six zero stress components.
6. If the shear stress on the left surface of an imaginary cut is upward and defined as positive, then on the right surface of the imaginary cut it is downward and negative.
7. A stress element can be drawn to any scale.
8. A stress element can be drawn at any orientation.
9. Stress components are opposite in direction on the two surfaces of an imaginary cut.
10. Stress components have opposite signs on the two surfaces of an imaginary cut.

PROBLEM SET 1.3
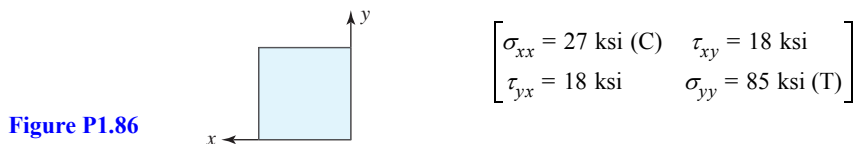
Plane Stress: Cartesian Coordinates

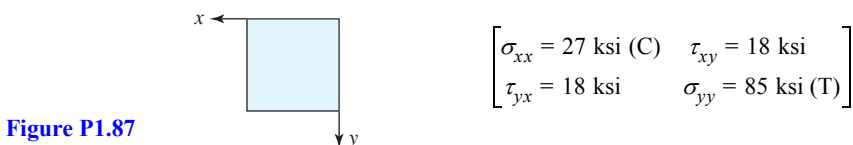**1.84** Show the stress components of a point in plane stress on the square in Figure P1.84.

Figure P1.84

$$\begin{bmatrix} \sigma_{xx} = 100 \text{ MPa (T)} & \tau_{xy} = -75 \text{ MPa} \\ \tau_{yx} = -75 \text{ MPa} & \sigma_{yy} = 85 \text{ MPa (T)} \end{bmatrix}$$

1.85 Show the stress components of a point in plane stress on the square in Figure P1.85.

Figure P1.85

$$\begin{bmatrix} \sigma_{xx} = 85 \text{ MPa (C)} & \tau_{xy} = 75 \text{ MPa} \\ \tau_{yx} = 75 \text{ MPa} & \sigma_{yy} = 100 \text{ MPa (T)} \end{bmatrix}$$

1.86 Show the stress components of a point in plane stress on the square in Figure P1.86.

Figure P1.86

$$\begin{bmatrix} \sigma_{xx} = 27 \text{ ksi (C)} & \tau_{xy} = 18 \text{ ksi} \\ \tau_{yx} = 18 \text{ ksi} & \sigma_{yy} = 85 \text{ ksi (T)} \end{bmatrix}$$

1.87 Show the stress components of a point in plane stress on the square in Figure P1.87.

Figure P1.87

$$\begin{bmatrix} \sigma_{xx} = 27 \text{ ksi (C)} & \tau_{xy} = 18 \text{ ksi} \\ \tau_{yx} = 18 \text{ ksi} & \sigma_{yy} = 85 \text{ ksi (T)} \end{bmatrix}$$

1.88 Show the nonzero stress components on the *A*, *B*, and *C* faces of the cube in Figure P1.88.
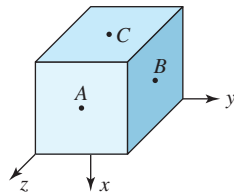
Figure P1.88

$$\begin{bmatrix} \sigma_{xx} = 70 \text{ MPa (T)} & \tau_{xy} = -40 \text{ MPa} & \tau_{xz} = 0 \\ \tau_{yx} = -40 \text{ MPa} & \sigma_{yy} = 85 \text{ MPa (C)} & \tau_{yz} = 0 \\ \tau_{zx} = 0 & \tau_{zy} = 0 & \sigma_{zz} = 0 \end{bmatrix}$$

1.89 Show the nonzero stress components on the *A*, *B*, and *C* faces of the cube in Figure P1.89.
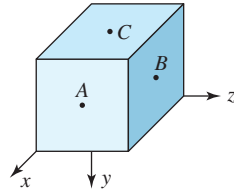
Figure P1.89

$$\begin{bmatrix} \sigma_{xx} = 70 \text{ MPa (T)} & \tau_{xy} = -40 \text{ MPa} & \tau_{xz} = 0 \\ \tau_{yx} = -40 \text{ MPa} & \sigma_{yy} = 85 \text{ MPa (C)} & \tau_{yz} = 0 \\ \tau_{zx} = 0 & \tau_{zy} = 0 & \sigma_{zz} = 0 \end{bmatrix}$$

Plane Stress: Polar Coordinates

1.90 Show the stress components of a point in plane stress on the stress element in polar coordinates in Figure P1.90.
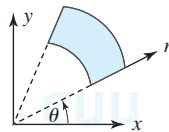
Figure P1.90

$$\begin{bmatrix} \sigma_{rr} = 125 \text{ MPa (T)} & \tau_{r\theta} = -65 \text{ MPa} \\ \tau_{\theta r} = -65 \text{ MPa} & \sigma_{\theta\theta} = 90 \text{ MPa (C)} \end{bmatrix}$$

1.91 Show the stress components of a point in plane stress on the stress element in polar coordinates in Figure P1.91.
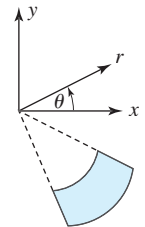
Figure P1.91

$$\begin{bmatrix} \sigma_{rr} = 125 \text{ MPa (T)} & \tau_{r\theta} = -65 \text{ MPa} \\ \tau_{\theta r} = -65 \text{ MPa} & \sigma_{\theta\theta} = 90 \text{ MPa (C)} \end{bmatrix}$$

1.92 Show the stress components of a point in plane stress on the stress element in polar coordinates in Figure P1.92.
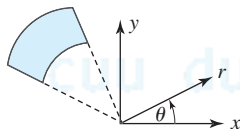
Figure P1.92

$$\begin{bmatrix} \sigma_{rr} = 18 \text{ ksi (T)} & \tau_{r\theta} = -12 \text{ ksi} \\ \tau_{\theta r} = -12 \text{ ksi} & \sigma_{\theta\theta} = 25 \text{ ksi (C)} \end{bmatrix}$$

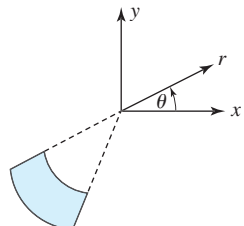**1.93** Show the stress components of a point in plane stress on the stress element in polar coordinates in Figure P1.93.
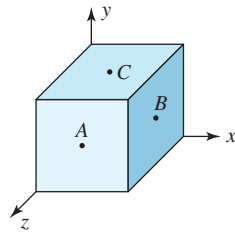
$$\begin{bmatrix} \sigma_{rr} = 25 \text{ ksi (C)} & \tau_{r\theta} = 12 \text{ ksi} \\ \tau_{\theta r} = 12 \text{ ksi} & \sigma_{\theta\theta} = 18 \text{ ksi (T)} \end{bmatrix}$$

Figure P1.93

Stress Element in 3-dimensions

1.94   Show the nonzero stress components on the A, B, and C faces of the cube in Figure P1.94.

$$\begin{bmatrix} \sigma_{xx} = 100 \text{ MPa (T)} & \tau_{xy} = 200 \text{ MPa} & \tau_{xz} = -125 \text{ MPa} \\ \tau_{yx} = 200 \text{ MPa} & \sigma_{yy} = 175 \text{ MPa (C)} & \tau_{yz} = 225 \text{ MPa} \\ \tau_{zx} = -125 \text{ MPa} & \tau_{zy} = 225 \text{ MPa} & \sigma_{zz} = 150 \text{ MPa (T)} \end{bmatrix}$$
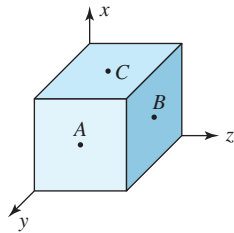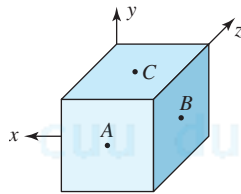
Figure P1.94

1.95   Show the nonzero stress components on the A, B, and C faces of the cube in Figure P1.95.

$$\begin{bmatrix} \sigma_{xx} = 90 \text{ MPa (T)} & \tau_{xy} = 200 \text{ MPa} & \tau_{xz} = 0 \\ \tau_{yx} = 200 \text{ MPa} & \sigma_{yy} = 175 \text{ MPa (T)} & \tau_{yz} = -225 \text{ MPa} \\ \tau_{zx} = 0 & \tau_{zy} = -225 \text{ MPa} & \sigma_{zz} = 150 \text{ MPa (C)} \end{bmatrix}$$
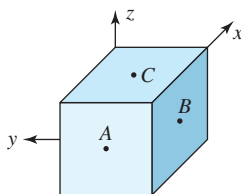
Figure P1.95

1.96   Show the nonzero stress components on the A, B, and C faces of the cube in Figure P1.96.

$$\begin{bmatrix} \sigma_{xx} = 0 & \tau_{xy} = 15 \text{ksi} & \tau_{xz} = 0 \\ \tau_{yx} = 15 \text{ ksi} & \sigma_{yy} = 10 \text{ ksi (T)} & \tau_{yz} = -25 \text{ ksi} \\ \tau_{zx} = 0 & \tau_{zy} = -25 \text{ ksi} & \sigma_{zz} = 20 \text{ ksi (C)} \end{bmatrix}$$

Figure P1.96

1.97   Show the nonzero stress components on the A, B, and C faces of the cube in Figure P1.97.

$$\begin{bmatrix} \sigma_{xx} = 0 & \tau_{xy} = -15 \text{ ksi} & \tau_{xz} = 0 \\ \tau_{yx} = -15 \text{ ksi} & \sigma_{yy} = 10 \text{ ksi (C)} & \tau_{yz} = 25 \text{ ksi} \\ \tau_{zx} = 0 & \tau_{zy} = 25 \text{ ksi} & \sigma_{zz} = 20 \text{ ksi (T)} \end{bmatrix}$$
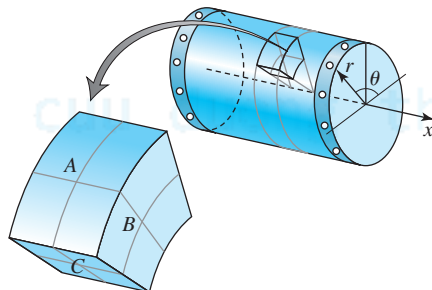
Figure P1.97

1.98   Show the nonzero stress components in the r, θ, and x cylindrical coordinate system on the A, B, and C faces of the stress elements shown in Figures P1.98.

$$\begin{bmatrix} \sigma_{rr} = 150 \text{ MPa (T)} & \tau_{r\theta} = -100 \text{ MPa} & \tau_{rx} = 125 \text{ MPa} \\ \tau_{\theta r} = -100 \text{ MPa} & \sigma_{\theta\theta} = 160 \text{ MPa (C)} & \tau_{\theta x} = 165 \text{ MPa} \\ \tau_{xr} = 125 \text{ MPa} & \tau_{x\theta} = 165 \text{ MPa} & \sigma_{xx} = 145 \text{ MPa (C)} \end{bmatrix}$$
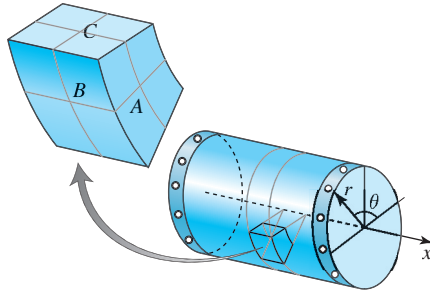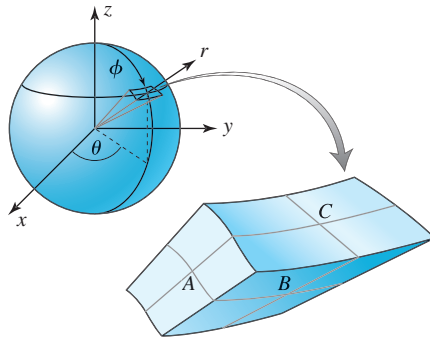
Figure P1.98

1.99   Show the nonzero stress components in the r, θ, and x cylindrical coordinate system on the A, B, and C faces of the stress elements shown in P1.99.

$$\begin{bmatrix} \sigma_{rr} = 10 \text{ ksi (C)} & \tau_{r\theta} = 22 \text{ ksi} & \tau_{rx} = 32 \text{ ksi} \\ \tau_{\theta r} = 22 \text{ ksi} & \sigma_{\theta\theta} = 0 & \tau_{\theta x} = 25 \text{ ksi} \\ \tau_{xr} = 32 \text{ ksi} & \tau_{x\theta} = 25 \text{ ksi} & \sigma_{xx} = 20 \text{ ksi (T)} \end{bmatrix}$$
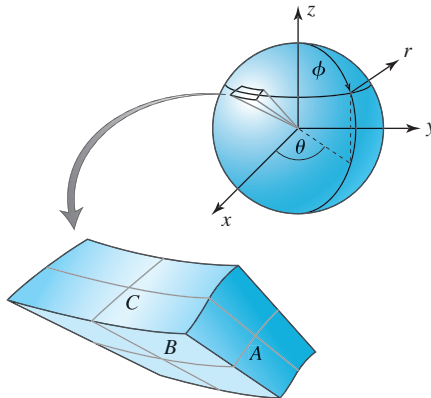
Figure P1.99

1.100   Show the nonzero stress components in the r, θ, and ϕ spherical coordinate system on the A, B, and C faces of the stress elements shown in Figure P1.100.

$$\begin{bmatrix} \sigma_{rr} = 150 \text{ MPa (T)} & \tau_{r\theta} = 100 \text{ MPa} & \tau_{r\phi} = 125 \text{ MPa} \\ \tau_{\theta r} = 100 \text{ MPa} & \sigma_{\theta\theta} = 160 \text{ MPa (C)} & \tau_{\theta\phi} = -175 \text{ MPa} \\ \tau_{\phi r} = 125 \text{ MPa} & \tau_{\phi\theta} = -175 \text{ MPa} & \sigma_{\phi\phi} = 135 \text{ MPa (C)} \end{bmatrix}$$

Figure P1.100

1.101   Show the nonzero stress components in the r, θ, and ϕ spherical coordinate system on the A, B, and C faces of the stress elements shown in P1.101.

$$\begin{bmatrix} \sigma_{rr} = 0 & \tau_{r\theta} = -18 \text{ ksi} & \tau_{r\phi} = 0 \\ \tau_{\theta r} = -18 \text{ ksi} & \sigma_{\theta\theta} = 10 \text{ ksi (C)} & \tau_{\theta\phi} = 25 \text{ ksi} \\ \tau_{\phi r} = 0 & \tau_{\phi\theta} = 25 \text{ ksi} & \sigma_{\phi\phi} = 20 \text{ ksi (T)} \end{bmatrix}$$

Figure P1.101

Stretch yourself

1.102   Show that the normal stress σ_{xx} on a surface can be replaced by the equivalent internal normal force N and internal bending moments M_y and M_z as shown in Figure P1.102 and given by the equations (1.8a) through (1.8c).
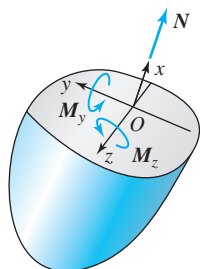
$$N = \int_A \sigma_{xx}\, dA \tag{1.8a}$$

$$M_y = -\int_A z\sigma_{xx}\, dA \tag{1.8b}$$

$$M_z = -\int_A y\sigma_{xx}\, dA \tag{1.8c}$$

Figure P1.102

1.103 The normal stress on a cross section is given by $\sigma_{xx} = a + by$, where y is measured from the centroid of the cross section. If A is the cross-sectional area, I_{zz} is the area moment of inertia about the z axis, and N and M_z are the internal axial force and the internal bending moment given by Equations (1.8a) and (1.8c), respectively, prove the result in Equation (1.8).

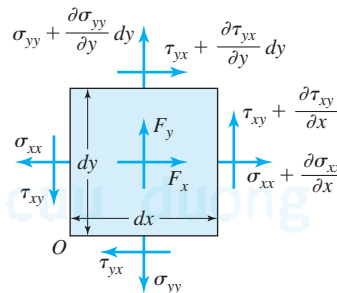$$\sigma_{xx} = \frac{N}{A} - \left(\frac{M_z}{I_{zz}}\right)y \tag{1.8}$$

We will encounter Equation (1.8) in combined axial and symmetric bending problems in later Chapter 10.

1.104 The normal stress on a cross section is given by $\sigma_{xx} = a + by + cz$, where y and z are measured from the centroid of the cross section. Using Equations (1.8a), (1.8b), and (1.8c) prove the result of Equation (1.9).

$$\sigma_{xx} = \frac{N}{A} - \left(\frac{M_z I_{yy} - M_y I_{yz}}{I_{yy}I_{zz} - I_{yz}^2}\right)y - \left(\frac{M_y I_{zz} - M_z I_{yz}}{I_{yy}I_{zz} - I_{yz}^2}\right)z \tag{1.9}$$

where I_{yy}, I_{zz}, and I_{yz} are the area moment of inertias. Equation (1.9) is used in the unsymmetrical bending of beams. Note that if either y or z is an axis of symmetry, then $I_{yz} = 0$. In such a case Equation (1.9) simplifies considerably.

1.105 An infinitesimal element in plane stress is shown in Figure P1.105. F_x and F_y are the body forces acting at the point and have the dimensions of force per unit volume. By converting stresses into forces and writing equilibrium equations obtain the results in Equations (1.10a) through (1.10c).

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + F_x = 0 \tag{1.10a}$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + F_y = 0 \tag{1.10b}$$

$$\tau_{xy} = \tau_{yx} \tag{1.10c}$$

Figure P1.105

1.6* CONCEPT CONNECTOR

Formulating the concept of stress took 500 years of struggle, briefly described in Section 1.6.1. In hindsight, the long evolution of quantifier of the strength is not surprising, because stress is not a single idea. It is a package of ideas that may be repackaged in many ways, depending on the needs of the analysis. Our chapter dealt with only one such package, called *Cauchy's stress,* which is used most in engineering design and analysis.

1.6.1 History: The Concept of Stress

The first formal treatment of strength is seen in the notes of the inventor and artist Leonardo da Vinci (1452–1519). Leonardo conducted several experiments on the strength of structural materials. His notes on "testing the strength of iron wires of various lengths" includes a sketch of how to measure the strength of wire experimentally. We now recognize that the dependence of the strength of a material on its length is due to the variations in manufacturing defects along the length.

The first indication of a concept of stress is found in Galileo Galilei (1564–1642). Galileo was born in Pisa and became a professor of mathematics at the age of twenty-five. For his belief in the Copernican theory on the motion of planets, which contradicted the interpretation of scriptures at that time, Galileo was put under house arrest for the last eight years of his life. During that period he wrote *Two New Sciences,* which lays out his contributions to the field of mechanics. Here he discusses the strength of a cantilever beam bending under the action of its own weight. Galileo viewed strength as the resistance to fracture, concluding that the strength of a bar depends on its cross-sectional area but is independent of its length. We will discuss Galileo's work on beam bending in Section 6.7.

The first person to differentiate between normal stress and shear stress was Charles-Augustin Coulomb (1736–1806) born in Angoulême. He was honored by the French Academy of Sciences in 1781 for his memoir *Theorie des machines simples,* in

which he discussed friction between bodies. The theory of dry friction is named after him. Given the similarities between shear stress and friction, it seems only natural that Coulomb would be the first to differentiate between normal and shear stress. We will see other works of Coulomb when we come to failure theory and on the torsion of circular shafts.

Claude Louis Navier (1785–1857) initiated the mathematical development of the concept of stress starting with Newton's concept of a central force—one that acts along a line between two particles. His approach led to a controversy that took eighty years to resolve, as we shall see in Section 3.12.
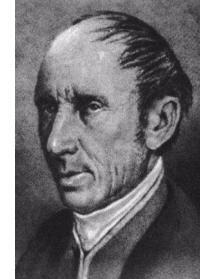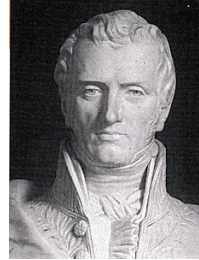
Figure 1.36   Pioneers of stress concept.

Claude Louis Navier   Augustin Cauchy.

Augustin Cauchy (1789–1857) brought the concept of stress to the form we studied it in this chapter (Figure 1.36). Forced to leave Paris, his birthplace, during the French Revolution, he took refuge in the village of Arcueil along with many other mathematicians and scientists of the period. At the age of twenty-one Cauchy worked engineering at the port of Cherbourg, which must have enhanced his understanding of the hydrodynamic concept of pressure. Pressure acts always normal to a surface, but Cauchy assumed that on an internal surface it acts at an angle hence he reasoned it can be resolved into components, the normal stress and shear stress. Combining this idea with his natural mathematical abilities, Cauchy developed what is now called Cauchy's stress. We shall see Cauchy's genius again in chapters on strain, material properties, and stress and strain transformation.

We have seen that unlike force, which is indivisible into more elementary ideas, stress is a package of ideas. Other packages will contain related but different elementary ideas. If instead of the cross-sectional area of an undeformed body, we use the cross-sectional area of a deformed body, then we get *true stress*. If we use the cross-sectional area of a deformed body and take the component of this area in the undeformed configuration, then we get *Kirchhoff's stress*. Still other stress measures are used in nonlinear analysis.

The English physicist James Clerk Maxwell (1831–1879) recognized the fact that the symmetry of shear stress given by Equations (1.7a) through (1.7c) is a consequence of there being no body moments. If a body moment is present, as in electromagnetic fields, then shear stresses will not be symmetric.

In Figure 1.15 we replaced the internal forces on a particle by a resultant force but no moment, because we assumed a central force between two particles. Woldemar Voigt (1850–1919), a German scientist who worked extensively with crystals is credited with introducing the stress tensor. Voigt recognized that in some cases a couple vector should be included when representing the interaction between particles by equivalent internal loads. If stress analysis is conducted at a very small scale, as the frontier research in nanostructures, then the moment transmitted by bonds between molecules may need to be included. The term *couple stress* is sometimes used to indicate the presence of a couple vector.

As history makes clear, stress has many definition. We choose the definition depending on the problem at hand and the information we are seeking. Most engineering analysis is linear and deals with large bodies, for which Cauchy's stress gives very good results. Cauchy's stress is thus sometimes referred to as engineering stress. Unless stated otherwise, *stress* always means Cauchy's stress in mechanics of materials and in this book.

1.7   CHAPTER CONNECTOR

In this chapter we have established the linkage between stresses, internal forces and moments, and external forces and moments. We have seen that to replace stresses by internal forces and internal moments requires knowledge of how the stress varies at each point on the surface. Although we can deduce simple stress behavior on a cross section, we would like to have other alternatives, in particular ones in which the danger of assuming physically impossible deformations is eliminated. This can be achieved if we can establish a relationship between stresses and deformations. Before we can discuss this relationship we need to understand the measure of deformation, which is the subject of Chapter 2. We will relate stresses and strains in

Chapter 3. In Section 3.2 we will synthesize the links introduced in Chapters 1, 2, and 3 into a logic that is used in mechanics of materials. We will use the logic to obtain simplified theories of one-dimensional structure members in Chapters 4, 5, 6, and 7.

All analyses in mechanics are conducted in a coordinate system, which is chosen for simplification whenever possible. Thus the stresses we obtain are in a given coordinate system. Now, our motivation for learning about stress is to define a measure of strength. Thus we can conclude that a material will fail when the stress at a point reaches some critical maximum value. There is no reason to expect that the stresses will be maximum in the arbitrarily chosen coordinate system. To determine the maximum stress at a point thus implies that we establish a relationship between stresses in different coordinate systems, as we shall do in Chapter 8.

We have seen that the concept of stress is a difficult one. If this concept is to be internalized so that an intuitive understanding is developed, then it is imperative that a discipline be developed to visualize the imaginary surface on which the stress is being considered.

POINTS AND FORMULAS TO REMEMBER

- Stress is an internal quantity.
- The internally distributed force on an imaginary cut surface of a body is called stress on a surface.
- Stress has units of force per unit area.
- 1 psi is equal to 6.95 kPa, or approximately 7 kPa. 1 kPa is equal to 0.145 psi, or approximately 0.15 psi.
- The internally distributed force that is normal (perpendicular) to the surface of an imaginary cut is called normal stress on a surface.
- Normal stress is usually reported as tensile or compressive and not as positive or negative.
- Average stress on a surface:

$$\sigma_{av} = N/A \quad (1.1) \qquad \tau_{av} = V/A \quad (1.2)$$

- where σ_{av} is the average normal stress, τ_{av} is the average shear stress, N is the internal normal force, V is the internal shear force, and A is the cross-sectional area of the imaginary cut on which N and V act.
- The relationship of external forces (and moments) to internal forces and the relationship of internal forces to stress distributions are two distinct ideas.
- Stress at a point:

$$\sigma_{ij} = \lim_{\Delta A_i \to 0} \left(\frac{\Delta F_j}{\Delta A_i} \right) \quad (1.3)$$

- where i is the direction of the outward normal to the imaginary cut surface, and j is the outward normal to the direction of the internal force.
- Stress at a point needs a magnitude and two directions to specify it, i.e., stress at a point is a second-order tensor.
- The first subscript on stress denotes the direction of the outward normal of the imaginary cut surface. The second subscript denotes the direction of the internal force.
- The sign of a stress component is determined from the direction of the internal force and the direction of the outward normal to the imaginary cut surface.
- Stress element is an imaginary object that helps us visualize stress at a point by constructing surfaces that have outward normals in the coordinate directions.

$$\tau_{xy} = \tau_{yx} \quad (1.7a) \qquad \tau_{yz} = \tau_{zy} \quad (1.7b) \qquad \tau_{zx} = \tau_{xz} \quad (1.7c)$$

- Shear stress is symmetric.
- In three dimensions there are nine stress components, but only six are independent.
- In two dimensions there are four stress components, but only three are independent.
- The pair of symmetric shear stress components point either toward the corner or away from the corner on a stress element.
- A point on a free surface is said to be in plane stress.