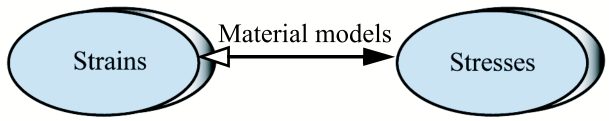
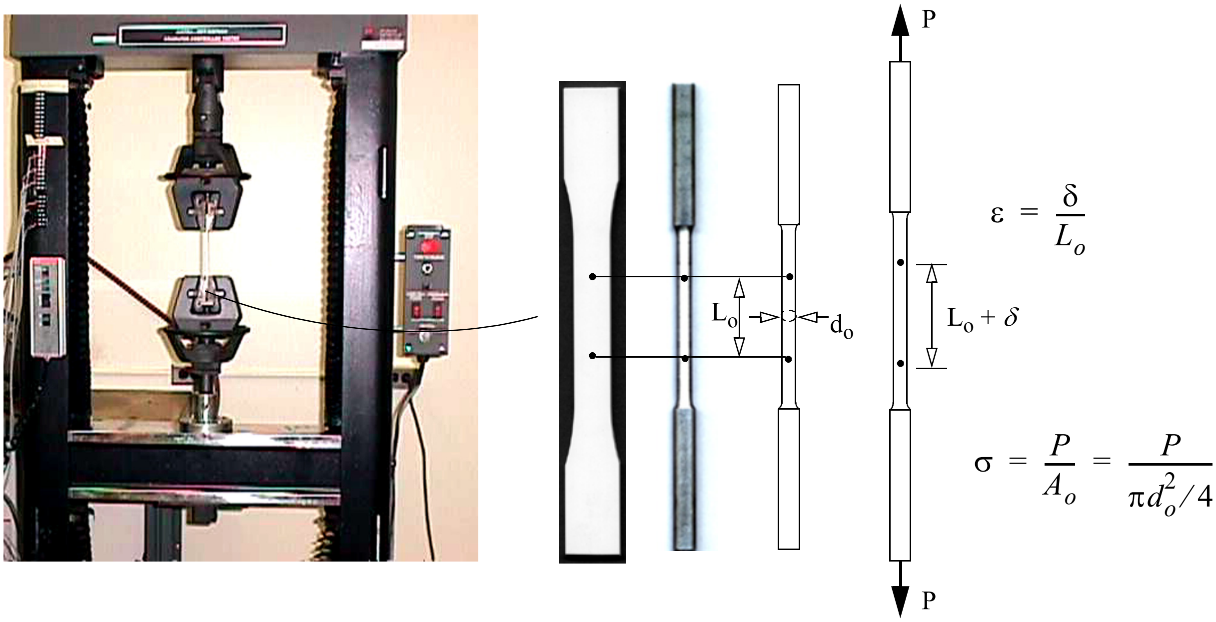
Mechanical Properties of Materials



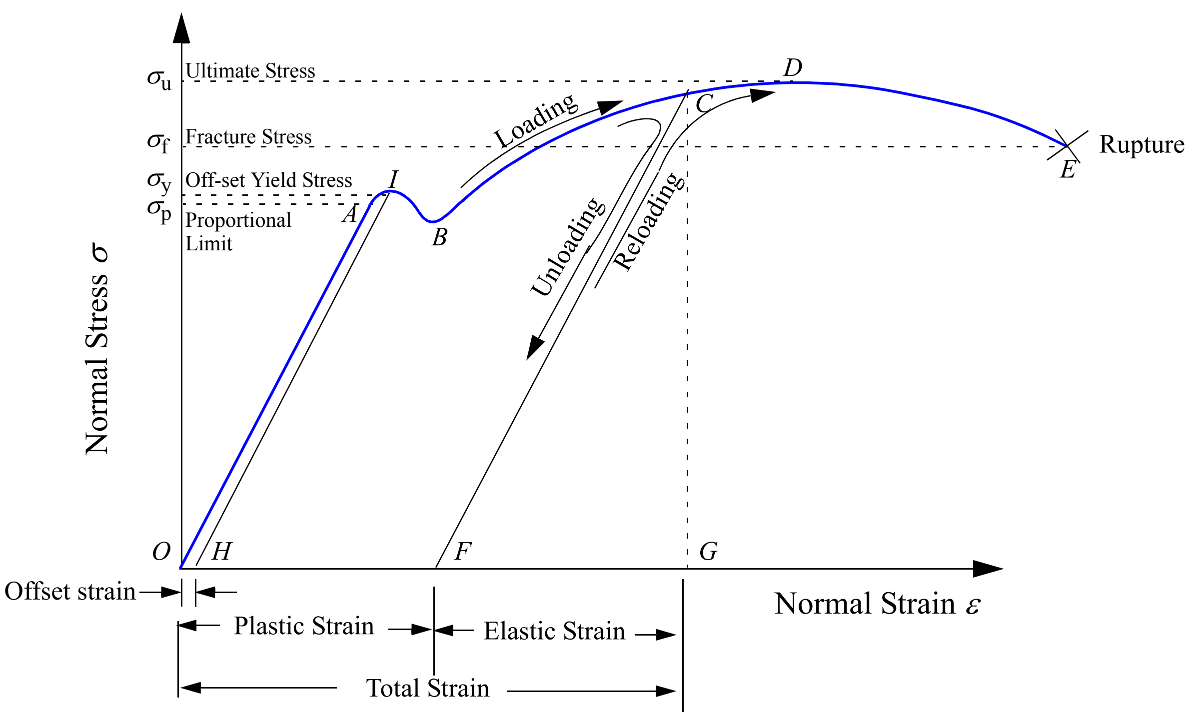
Learning objectives

- Understand the qualitative and quantitative description of mechanical properties of materials.
- Learn the logic of relating deformation to external forces.

Tension Test



$$\varepsilon = \frac{\delta}{L_o} \qquad \sigma = \frac{P}{A_o} = \frac{P}{\pi d_o^2/4}$$

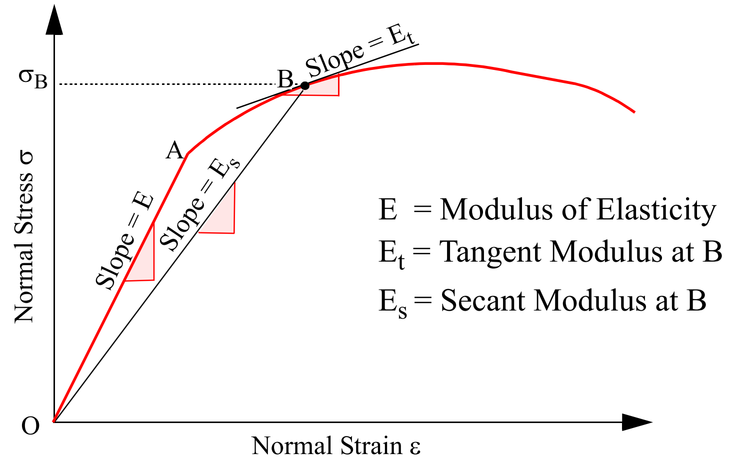


ıre 3.1 Stress–strain curve.

Definitions

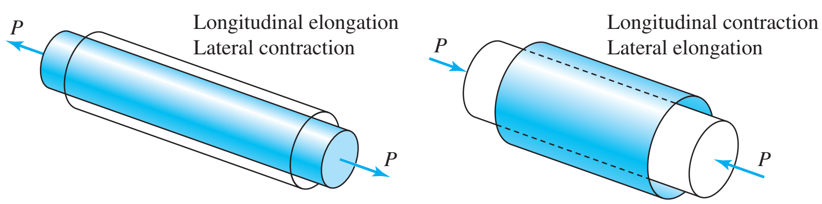
- The point up to which the stress and strain are linearly related is called the proportional limit.
- The largest stress in the stress strain curve is called the ultimate stress.
- The stress at the point of rupture is called the fracture or rupture stress.
- The region of the stress-strain curve in which the material returns to the undeformed state when applied forces are removed is called the elastic region.
- The region in which the material deforms permanently is called the plastic region.
- The point demarcating the elastic from the plastic region is called the yield point. The stress at yield point is called the yield stress.
- The permanent strain when stresses are zero is called the plastic strain.
- The off-set yield stress is a stress that would produce a plastic strain corresponding to the specified off-set strain.
- A material that can undergo large plastic deformation before fracture is called a ductile material.
- A material that exhibits little or no plastic deformation at failure is called a brittle material.
- Hardness is the resistance to indentation.
- The raising of the yield point with increasing strain is called strain hardening.
- The sudden decrease in the area of cross-section after ultimate stress is called necking.

Material Constants

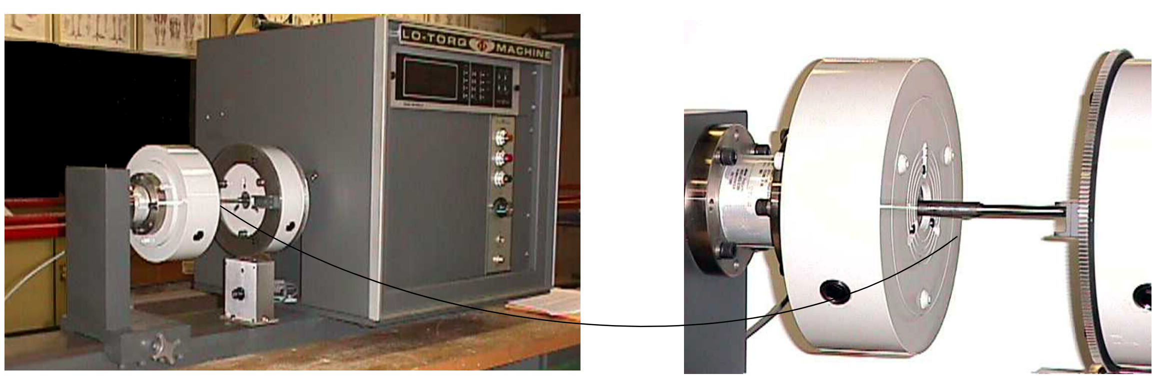


$$\sigma = E\varepsilon$$ ----------------Hooke's Law

- E Young's Modulus or Modulus of Elasticity



- Poisson's ratio: $\nu = -\left(\dfrac{\varepsilon_{lateral}}{\varepsilon_{longitudnal}}\right)$

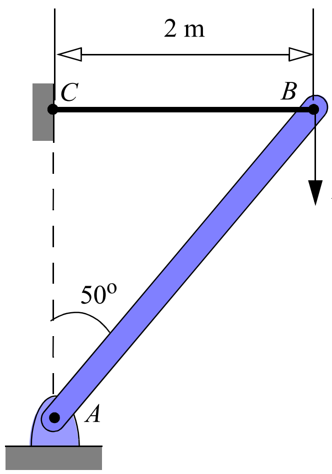


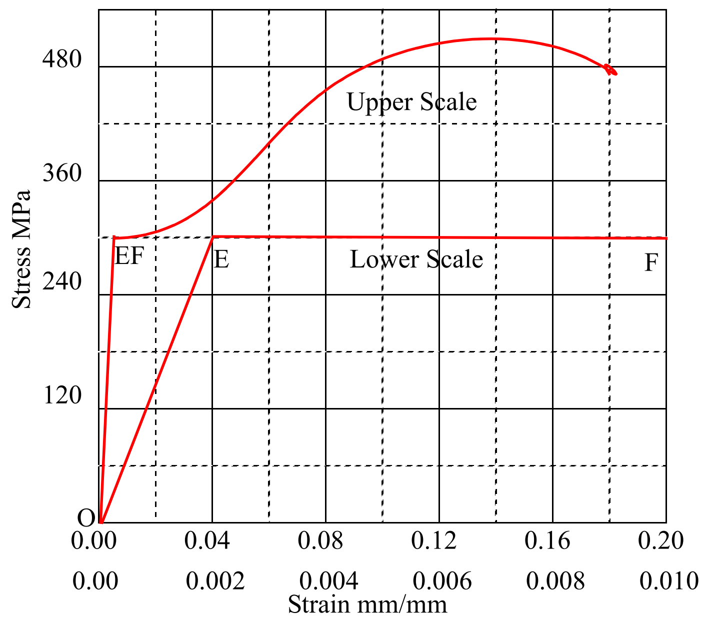
$$\tau = G\gamma$$

G is called the Shear Modulus of Elasticity or the Modulus of Rigidity

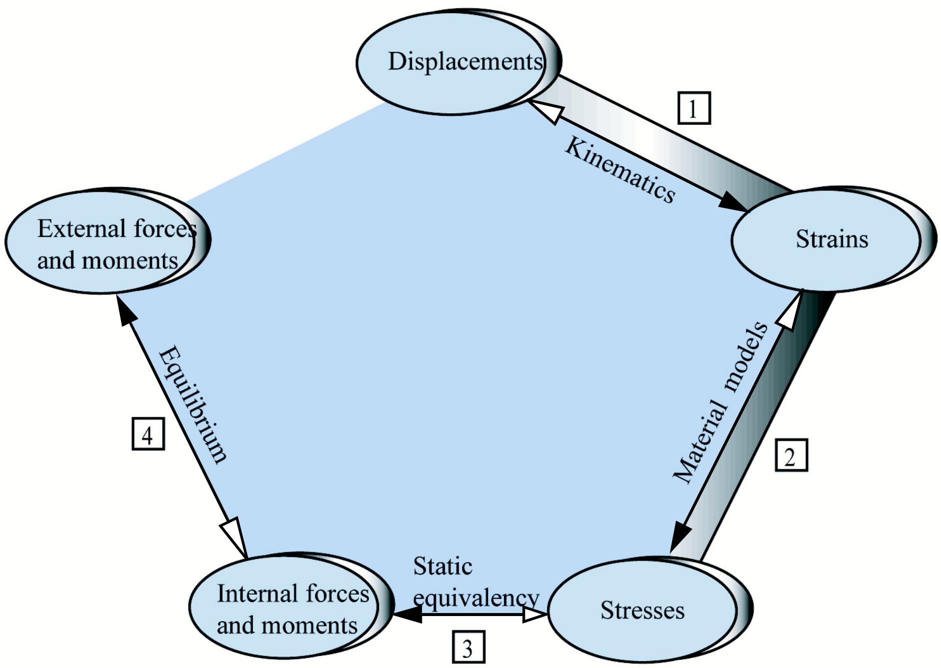
C3.1 An aluminum rectangular bar has a cross-section of 25 mm x 50 mm and a length of 500 mm. The Modulus of Elasticity of E = 70 GPa and a Poisson's ratio of $\nu = 0.25$. Determine the percentage change in the volume of the bar when an axial force of 300 kN is applied to the bar.

C3.2 A rigid bar *AB* of negligible weight is supported by cable of diameter 5 mm, as shown. The cable is made from a material that has a stress- strain curve shown. (a) Determine the extension of the cable when $P = 10$ kN. (b) What is the permanent deformation in *BC* when the load P is removed?





Logic in structural analysis



C3.3 A roller slides in a slot by the amount $\delta_P = 0.25$ mm in the direction of the force F. Both bars have an area of cross-section of $A = 100\ mm^2$ and a Modulus of Elasticity E = 200 GPa. Bar AP and BP have lengths of $L_{AP}= 200$ mm and $L_{BP}= 250$ mm respectively. Determine the applied force F.

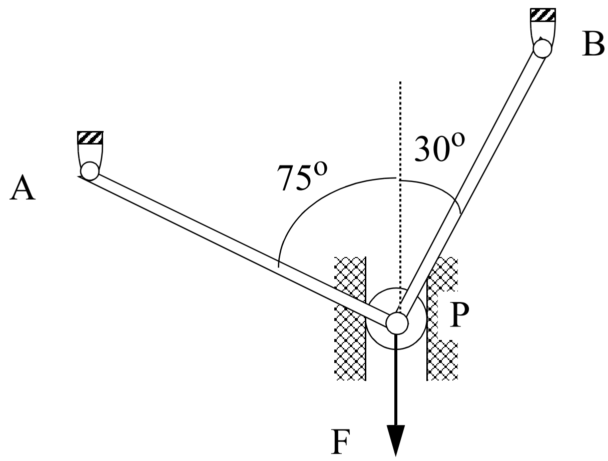


Fig. C3.3

C3.4 A gap of 0.004 in. exists between a rigid bar and bar *A* before a force *F* is applied (Figure P3.4). The rigid bar is hinged at point *C*. Due to force *F* the strain in bar *A* was found to be –500 μin/in. The lengths of bars *A* and *B* are 30 in. and 50 in., respectively. Both bars have cross-sectional areas *A* = 1 in.^{2} and a modulus of elasticity *E* = 30,000 ksi. Determine the applied force *F*.

Fig. C3.4

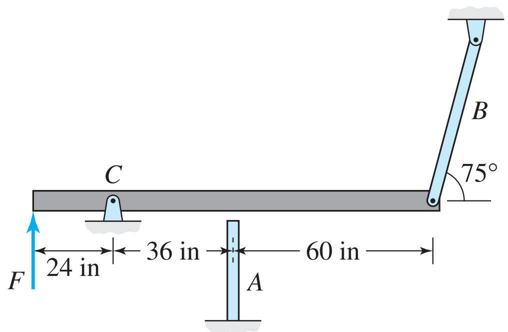


Fig. C3.4

Isotropy and Homogeneity

Linear relationship between stress and strain components:

$$\varepsilon_{xx} = C_{11}\sigma_{xx} + C_{12}\sigma_{yy} + C_{13}\sigma_{zz} + C_{14}\tau_{yz} + C_{15}\tau_{zx} + C_{16}\tau_{xy}$$
$$\varepsilon_{yy} = C_{21}\sigma_{xx} + C_{22}\sigma_{yy} + C_{23}\sigma_{zz} + C_{24}\tau_{yz} + C_{25}\tau_{zx} + C_{26}\tau_{xy}$$
$$\varepsilon_{zz} = C_{31}\sigma_{xx} + C_{32}\sigma_{yy} + C_{33}\sigma_{zz} + C_{34}\tau_{yz} + C_{35}\tau_{zx} + C_{36}\tau_{xy}$$
$$\gamma_{yz} = C_{41}\sigma_{xx} + C_{42}\sigma_{yy} + C_{43}\sigma_{zz} + C_{44}\tau_{yz} + C_{45}\tau_{zx} + C_{46}\tau_{xy}$$
$$\gamma_{zx} = C_{51}\sigma_{xx} + C_{52}\sigma_{yy} + C_{53}\sigma_{zz} + C_{54}\tau_{yz} + C_{55}\tau_{zx} + C_{56}\tau_{xy}$$
$$\gamma_{xy} = C_{61}\sigma_{xx} + C_{62}\sigma_{yy} + C_{63}\sigma_{zz} + C_{64}\tau_{yz} + C_{65}\tau_{zx} + C_{66}\tau_{xy}$$

- An isotropic material has a stress-strain relationships that are independent of the orientation of the coordinate system at a point.
- A material is said to be homogenous if the material properties are the same at all points in the body. Alternatively, if the material constants C_{ij} are functions of the coordinates x, y, or z, then the material is called non-homogenous.

For Isotropic Materials: $G = \dfrac{E}{2(1+\nu)}$

Generalized Hooke's Law for Isotropic Materials

- The relationship between stresses and strains in three-dimensions is called the Generalized Hooke's Law.

$$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]/E$$
$$\varepsilon_{yy} = [\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})]/E$$
$$\varepsilon_{zz} = [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})]/E$$

$$\gamma_{xy} = \tau_{xy}/G$$
$$\gamma_{yz} = \tau_{yz}/G$$
$$\gamma_{zx} = \tau_{zx}/G$$

$$G = \frac{E}{2(1+\nu)}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{Bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix}\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{Bmatrix}$$

Plane Stress and Plane Strain

Plane Stress $\longrightarrow$

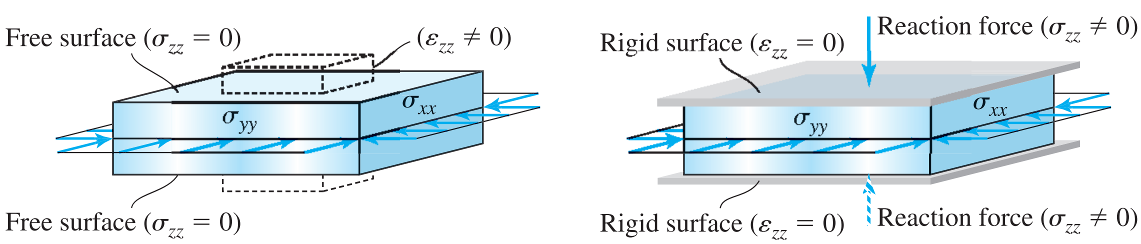
$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Generalized Hooke's Law}} \begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) \end{bmatrix}$$

Plane Strain $\longrightarrow$

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Generalized Hooke's Law}} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \end{bmatrix}$$

Plane Stress

Plane Strain



(a) (b)

C3.5 A 2in x 2 in square with a circle inscribed is stressed as shown Fig. C3.5. The plate material has a Modulus of Elasticity of E = 10,000 ksi and a Poisson's ratio $\nu = 0.25$. Determine the major and minor axis of the ellipse formed due to deformation assuming (a) plane stress. (b) plane strain.

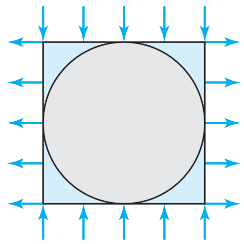


Fig. C3.5

Class Problem 1

The stress components at a point are as given.
Determine ε_{xx} assuming (a) Plane stress (b) Plane strain

$$\sigma_{xx} = 100 \quad MPa(T)$$
$$\sigma_{yy} = 200 \quad MPa(C)$$
$$\tau_{xy} = -125 \quad MPa$$

$$E = 200 \quad GPa$$
$$\nu = 0.25$$

Failure and factor of safety

- Failure implies that a component or a structure does not perform the function it was designed for.

$$K_{safety} = \frac{Failure\ \ producing\ \ value}{Computed(allowable)value} \qquad 3.1$$

C3.6 An adhesively bonded joint in wood is fabricated as shown. For a factor of safety of 1.25, determine the minimum overlap length L and dimension h to the nearest 1/8th inch. The shear strength of adhesive is 400 psi and the wood strength is 6 ksi in tension.

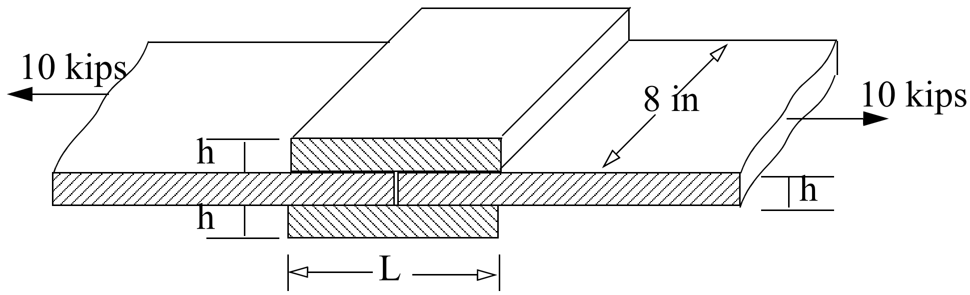


Fig. C3.6

Common Limitations to Theories in Chapters 4-7

- The length of the member is significantly greater (approximately 10 times) then the greatest dimension in the cross-section.
- We are away from regions of stress concentration, where displacements and stresses can be three-dimensional.
- The variation of external loads or changes in the cross-sectional area is gradual except in regions of stress concentration.
- The external loads are such that the axial, torsion and bending problems can be studied individually.