REVIEW

Stress

- Stress on a surface is an internal distributed force system.
- The relationship of external forces (and moments) to internal forces and the relationship of internal forces and moments to the stress distribution are two ditinct ideas.

Stress at a point:

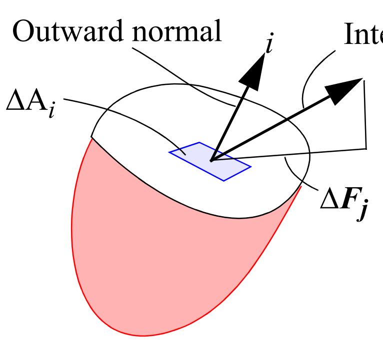
---- Is an internal quantity.

---- Needs two directions and a magnitude to specify it.(2nd order tensor).

---- Stress is a symmetric tensor.

---- Has units of force per unit area.

---- Sign is determined by the direction of the internal force and the direction of the outward normal of the imaginary cut surface.



$$\sigma_{ij} = \lim_{\Delta A_i \to 0}\left(\frac{\Delta F_j}{\Delta A_i}\right)$$

direction of outward normal to the imaginary cut surface. (i)

direction of the internal force component. (j)

Stresses on various planes passing through a point in two dimension can be found by the:

---- Wedge method (equilibrium method). --- Convert stresses to forces and use equilibrium equations to determine the unknowns.

---- Mohr's circle.----A point on the circle represents a unique plane and the co-ordinates of the point represent the normal and shear stress on the plane.

Prefixes on stress: Normal stress, Shear stress, Principal stress, Maximum shear stress, Maximum in-plane shear stress, axial stress, flextural stresses, torsional shearing stress, maximum normal stress, yield stress, ultimate stress etc.

Strain

- Measure of relative movement of two points on the body. (deformation).
- Elongations are positive normal strains.
- Decrease from right angle results in positive shear strains.
- Small Strain (< 0.01) approximation simplifies calculations.
- In small strain In normal small strain, the deformation in the orignal direction only is required.

Strain at a point:

---- Needs magnitude and two direction to specify it.

---- Tensor normal strain=Engineering normal strain; Tensor shear strain=Engineering shear strain/2;

Is related to the first partial derivative of deformation. (3) Strain is a symmetric tensor. In 3-D: 6 components are needed to specify strain at a point. In 2-D: 3 components are needed to specify strain at a point. (4) Strains in different coordinate systems can be found using Mohr's circle for strains.

Strain gages measure only normal strains. Gages are usually stuck on free surfaces (plane stress).

Generalized Hooke’s Law for Isotropic Material:

- Valid for any orthogonal coordinate system

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy} + \sigma_{zz}) \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz}) \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad G = \frac{E}{2(1+\nu)}$$

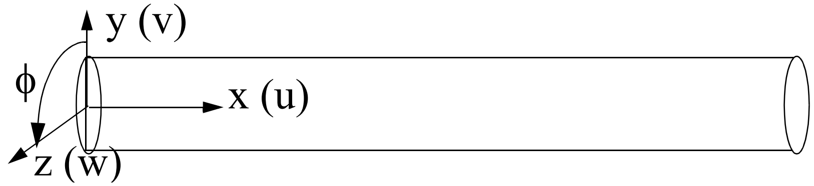
$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

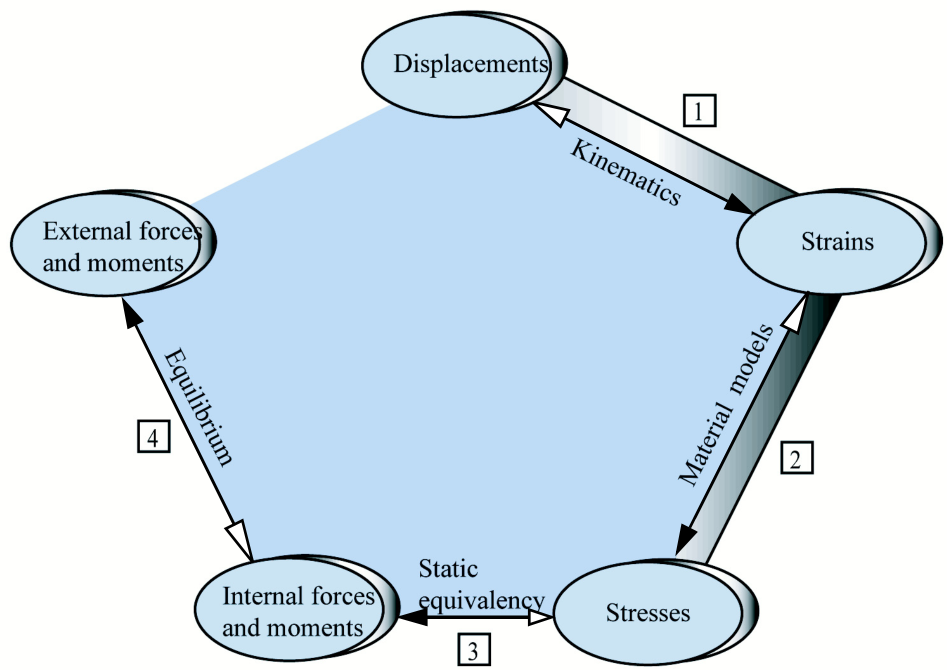
- Plane Stress: Stresses with subscript z are zero.
- Plane Strain: Strains with subscript z are zero.
- The state of stress and the strain affects the third principal stress and strain and are important in the caluculation of maximum shear stress and strain.

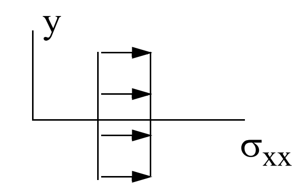
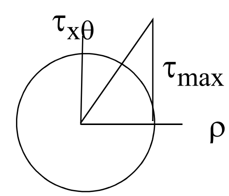
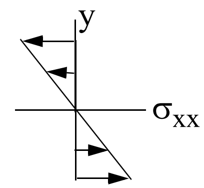
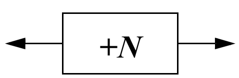
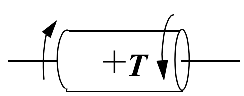
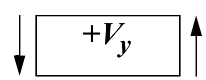
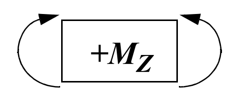
Factor of Safety

$$K = \frac{\text{Failure producing value of -----}}{\text{Estimated value of -------------}} \quad \text{[Load, Stress, Displacement]}$$

Sudden changes in geometry, loading or material properties causes stress concentration. The effect of these sudden changes dies out rapidly as one moves away from the region of sudden changes (Saint Venant's Principle)





	Axial (Rods)	Torsion (Shafts)	Bending (Beams)
Displacements	$u(x, y, z) = u(x) \quad v = 0 \quad w = 0$	$u = 0 \quad v = 0 \quad w = 0$ $\phi(x, y, z) = \phi(x)$	$u(x, y, z) = -y\frac{dv}{dx} \quad v = v(x) \quad w = 0$
Strains	$\varepsilon_{xx} = \frac{du}{dx}$	$\gamma_{x\theta} = \rho\frac{d\phi}{dx}$	$\varepsilon_{xx} = -y\frac{d^2v}{dx^2}$
Stresses	$\sigma_{xx} = E\varepsilon_{xx} = E\frac{du}{dx}$ 	$\tau_{x\theta} = G\gamma_{x\theta} = \rho\frac{d\phi}{dx}$ 	$\sigma_{xx} = E\varepsilon_{xx} = -Ey\frac{d^2v}{dx^2} \qquad \tau_{xy} \neq 0 \ll \sigma_{xx}$ 
Internal Forces & Moments	$\boldsymbol{N} = \int_A \sigma_{xx} dA$	$\boldsymbol{T} = \int_A \rho\tau_{x\theta} dA$	$\boldsymbol{N} = \int_A \sigma_{xx} dA = 0 \Rightarrow \int_A y dA = 0$ $\boldsymbol{M_z} = -\int_A y\sigma_{xx} dA \quad \boldsymbol{V_y} = \int_A \tau_{xy} dA$
Sign Convention			 
Stress Formulas	$\sigma_{xx} = \frac{\boldsymbol{N}}{A}$	$\tau_{x\theta} = \frac{\boldsymbol{T}\rho}{J}$	$\sigma_{xx} = -\left(\frac{\boldsymbol{M_z} y}{I_{zz}}\right) \qquad \tau_{xs} = -\left(\frac{\boldsymbol{V_y} Q_z}{I_{zz} t_z}\right)$
Deformation Formulas	$\frac{du}{dx} = \frac{\boldsymbol{N}}{EA}$ $u_2 - u_1 = \frac{\boldsymbol{N}(x_2 - x_1)}{EA}$ EA = Axial Rigidity	$\frac{d\phi}{dx} = \frac{\boldsymbol{T}}{GJ}$ $\phi_2 - \phi_1 = \frac{\boldsymbol{T}(x_2 - x_1)}{GJ}$ GJ = Torsional Rigidity	$\frac{d^2v}{dx^2} = \frac{\boldsymbol{M_z}}{EI_{zz}}$ $v = \int\left[\int \frac{\boldsymbol{M_z}}{EI} dx\right] dx + C_1 x + C_2$ EI_{zz} = Bending Rigidity

Buckling:

- Bending due to *compressive* axial forces is called buckling.
- It is sudden and catastrophic.
- Buckling occurs about the axis of *minimum* area moment of inertia.
- Euler Buckling Load P_{cr} can be calculated from: $P_{cr} = \frac{\pi^2 EI}{L^2}$
- Slenderness ratio is defined as L/r where L is length of column and r is radius of gyration.