Chapter 3:

Low Noise Amplifier (LNA)

References

[1] J. Rogers, C. Plett, *Radio Frequency Integrated Circuit Design*, Artech House, 2003.

[2] W. A. Davis, K. Agarwal, *Radio Frequency Circuit Design*, John Wiley & Sons, 2001.

[3] F. Ellinger, *RF Integrated Circuits and Technologies*, Springer Verlag, 2008.

Origin of Noise (1)

- **Resistor thermal noise:** Probably the most well known noise source is the **thermal noise** of a resistor (also called Johnson noise). It is generated by thermal energy causing random electron motion. It is **white noise** since the PSD of the noise signal is flat throughout the frequency band.

  The noise is also called **Gaussian** which means the amplitude of the noise signal has random characteristics with a **Gaussian distribution**. We are able to apply statistic measures such as the **mean square values**. The noise power is proportional to absolute temperature.

  The **thermal noise spectral density** in a resistor is given by

$$N_{\text{resistor}} = 4kTR$$

  where k is Boltzmann's constant ($\sim 1.38 \times 10-23$ J/K), T is the absolute temperature in Kelvin temperature of the resistor, and R is the value of the resistor.

Origin of Noise (2)

Noise power spectral density is expressed using volts squared per hertz (power spectral density). In order to find out how much power a resistor produces in a finite bandwidth of interest Δf, we use:

$$v_n^2 = 4kTR\Delta f$$

where v_n is the rms value of the noise voltage in the bandwidth Δf. This can also be written equivalently as a noise current rather than a noise voltage:

$$i_n^2 = \frac{4kT\Delta f}{R}$$

Maximum power is transferred to the load when R_{LOAD} is equal to R. Then v_o is equal to v_n /2. The **output power spectral density** P_o is then given by

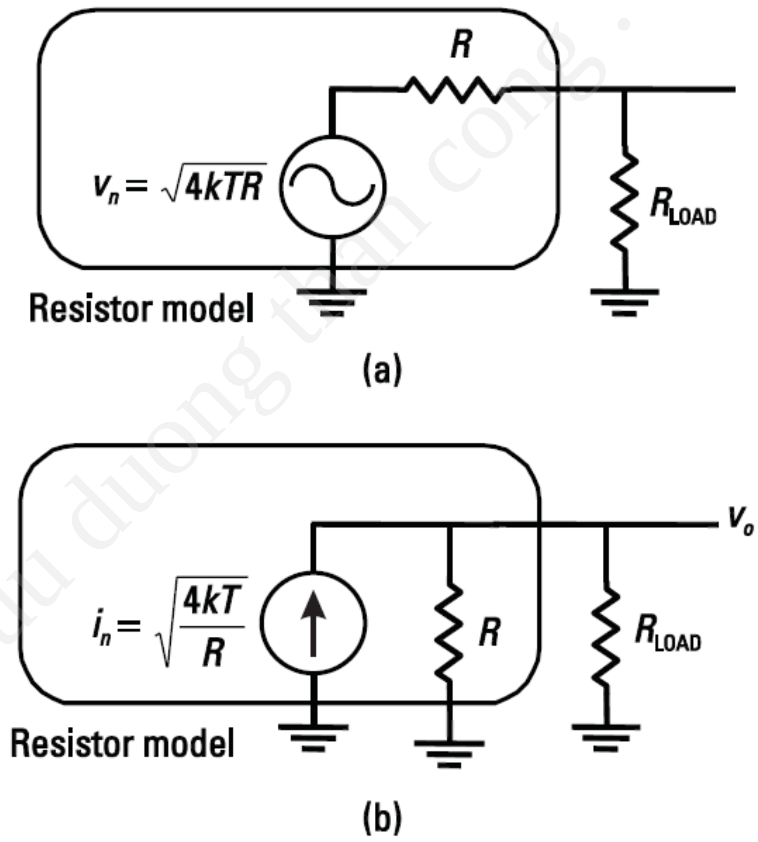
$$P_o = \frac{v_o^2}{R} = \frac{v_n^2}{4R} = kT$$

Thus, **available noise power** is kT, independent of resistor size. Note that kT is in watts per hertz, which is a power density.

Origin of Noise (3)

To get **total power out** P_{out} in watts, multiply by the bandwidth, with the result that:

$$P_{out} = kTB$$



Origin of Noise (4)

Available power from antenna: The noise from an antenna can be modeled as a resistor. Thus, the available power from an antenna is given by:

$$P_{\text{available}} = kT = 4 \times 10^{-21} \text{ W/Hz}$$

at T = 290K, or in dBm per hertz:

$$P_{\text{available}} = 10 \log_{10}\left(\frac{4 \times 10^{-21}}{1 \times 10^{-3}}\right) = -174 \text{ dBm/Hz}$$

Example: For any receiver required to receive a given signal bandwidth, the minimum detectable signal can now be determined. From $P_{out} = kTB$, the noise floor depends on the bandwidth. For example, with a bandwidth of 200 kHz, the noise floor is

$$\text{Noise floor} = kTB = 4 \times 10^{-21} \times 200{,}000 = 8 \times 10^{-16}$$

or in dBm:

$$\text{Noise floor} = -174 \text{ dBm/Hz} + 10 \log_{10}(200{,}000) = -121 \text{ dBm}$$

Origin of Noise (5)

Thus, we can now also formally define **signal-to-noise ratio (SNR)**. If the signal has a power of *S*, then the SNR is

$$\mathrm{SNR} = \frac{S}{\text{Noise floor}}.$$

Thus, if the electronics added no noise and if the detector required a SNR of 0 dB, then a signal at -121 dBm could just be detected. The minimum detectable signal in a receiver is also referred to as the **receiver sensitivity.**

However, the SNR required to detect bits reliably (e.g., bit error rate (BER) = 10^{-3}) is typically not 0 dB. Typical results for a bit error rate of 10^{-3} (for voice transmission) is about 7 dB for quadrature phase shift keying (QPSK), about 12 dB for 16 quadrature amplitude modulation (QAM), and about 17 dB for 64 QAM. For data transmission, lower BER is often required (e.g., 10^{-6}), resulting in an SNR requirement of 11 dB or more for QPSK.

Origin of Noise (6)

- **<u>Shot noise</u>:** Shot noise is generated if current flows through a potential barrier such as a *pn* junction. The square root of the shot noise current can be described by

$$i_{sh}^2 = 2qI_{dc}\Delta f$$

with q as the electron charge. As expected, the shot noise increases with DC current I_{dc} since it determines the number of available carriers.

Thus, shot noise can be minimised by reducing the DC current. However, a reduced DC current may decrease the maximum possible gain and large signal properties of transistors. Consequently, a tradeoff has to be found.

Shot noise plays an important role in BJTs since they consist of *pn* junctions (especially for the forward biased base emitter junction).

Origin of Noise (7)

Usually, the shot noise of FETs is very small since there are no relevant *pn*-junctions, and the current flowing through them is weaker than for BJTs. However, the aggressively scaling of MOSFETs can introduce a significant current from the gate to the channel, which may generate shot noise.

In contradiction to thermal noise, shot noise does not occur in an ideal resistor.

Origin of Noise (8)

- **<u>1/*f* Noise</u>:** This type of noise is also called **flicker noise**, or **excess noise**. The 1/*f* noise is due to variation in the conduction mechanism, for example, fluctuations of surface effects (such as the filling and emptying of traps) and of recombination and generation mechanisms. Typically, the power spectral density of 1/*f* noise is **inversely proportional to frequency** and is given by the following equation:

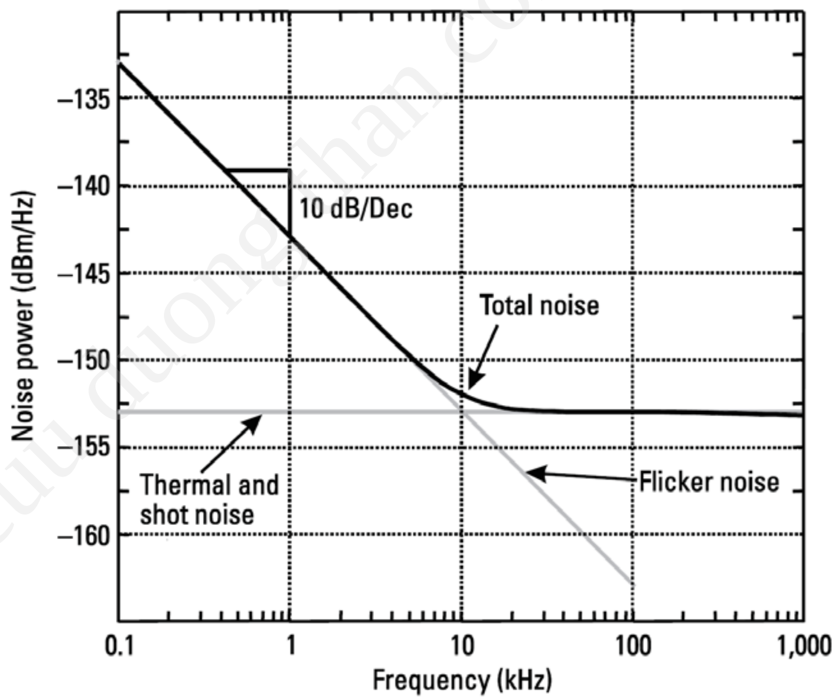
$$\overline{i_{\mathrm{bf}}^{2}} = K I_C^{m} \frac{1}{f^{\alpha}}$$

  where m is between 0.5 and 2, α is about equal to 1, and K is a process constant.

  The 1/*f* noise is dominant at low frequencies, however, beyond the corner frequency (shown as 10 kHz, see the diagram next slide), thermal noise dominates. <u>The effect of 1/*f* noise on RF circuits can usually be ignored</u>.

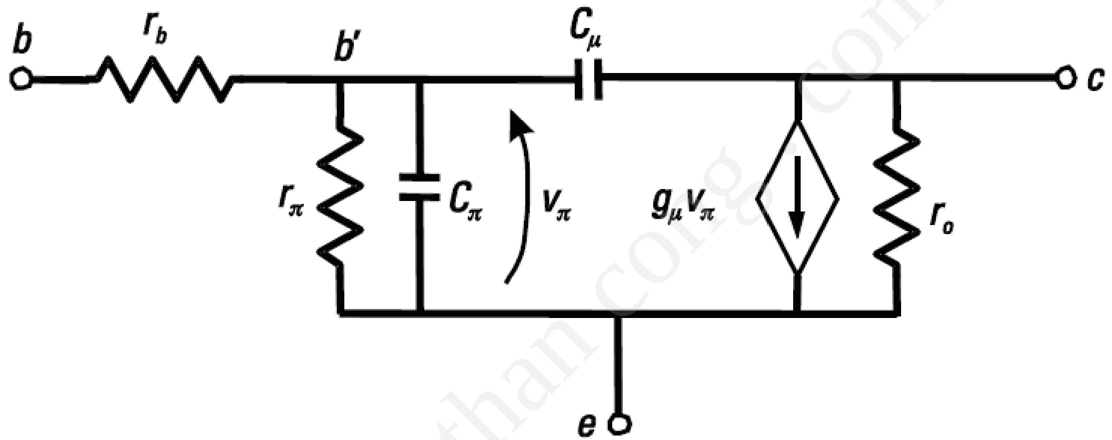
Origin of Noise (9)

An exception is in the design of oscillators, where 1/*f* noise can modulate the oscillator output signal, producing or increasing phase noise. The 1/*f* noise is also important in direct down-conversion receivers, as the output signal is close to DC. Note also that 1/*f* noise is much worse for MOS transistors, where it can be significant up to 1 MHz.



Noise in Bipolar Transistors (1)

- Small-signal equivalent circuit of BJT at hight frequencies (without noise):



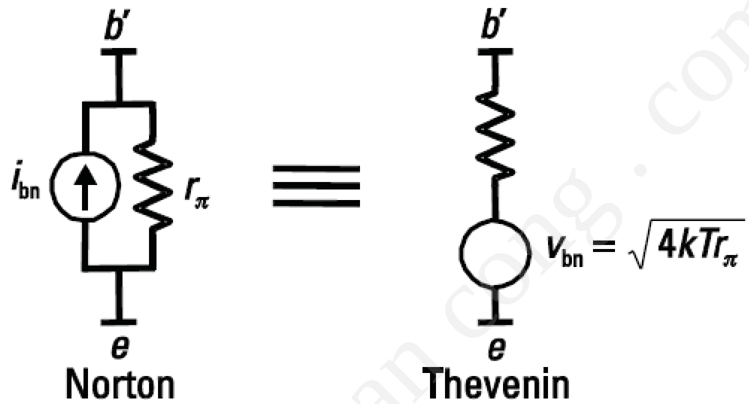
$r_b = r_{bb'}$, $r_\pi = r_{b'e}$, $C_\pi = C_{b'e}$, $C_\mu = C_{b'c}$, $g_m = \frac{i_c}{v_\pi} = \frac{I_C}{v_T} = \frac{I_c q}{kT}$

$$f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} \approx \frac{1}{2\pi r_\pi C_\pi}$$

$$f_T = \beta f_\beta = \frac{g_m}{2\pi (C_\pi + C_\mu)} \approx \frac{g_m}{2\pi C_\pi} = \frac{I_C}{2\pi C_\pi v_T} \qquad \beta = g_m r_\pi$$

Noise in Bipolar Transistors (2)

- **Base shot noise:** Consider shot noise (i_{bn} or v_{bn}) at the base of BJT.



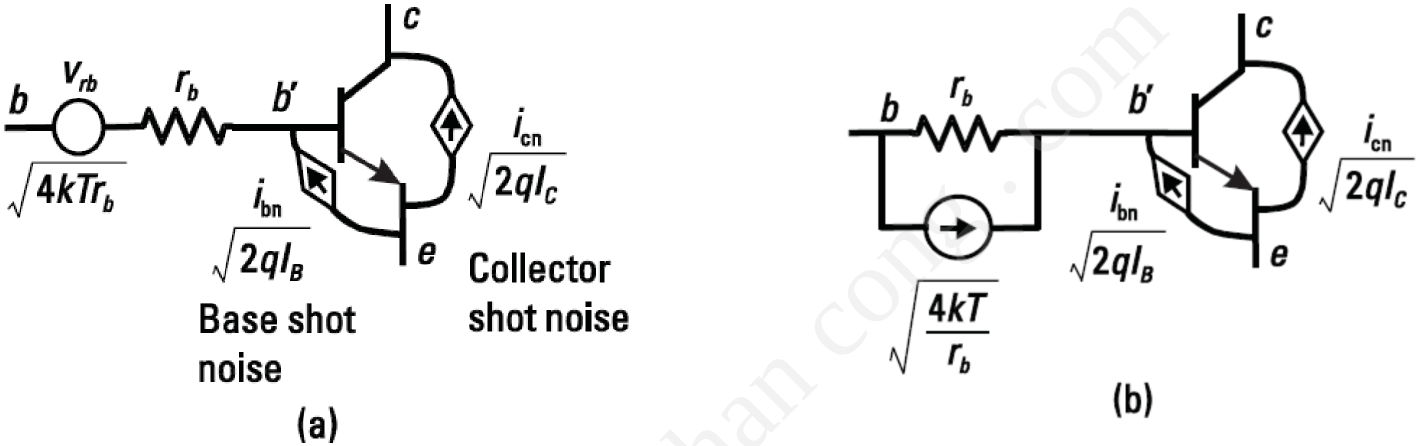
Base shot noise is related to thermal noise in the resistor r_π as

$$v_{bn} = i_{bn} \cdot r_\pi = \sqrt{2qI_B} \cdot r_\pi = \sqrt{2q\frac{I_C}{\beta}} \cdot r_\pi = \sqrt{2q\frac{I_C}{g_m r_\pi}} \cdot r_\pi$$

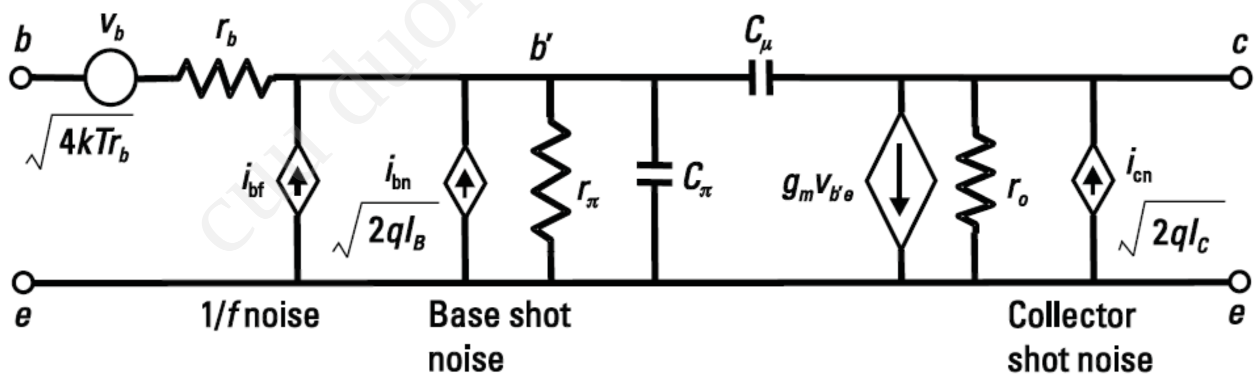
$$= \sqrt{2q\frac{I_C}{\frac{I_C q}{kT} r_\pi}} \cdot r_\pi = \sqrt{2kTr_\pi}$$

Noise in Bipolar Transistors (3)

- BJT with base shot noise, collector shot noise, and thermal noise at r_b:



- **Small-signal equivalent circuit of BJT with noise**:



Noise Figure (1)

- **Noise from the electronics** (e.g. thermal noise, shot noise…) is described by **noise factor** *F*, which is a measure of **how much the signal-to-noise ratio is degraded through the system**. We note that:

$$S_o = G \cdot S_i$$

where S_i is the input signal power, S_o is the output signal power, and G is the power gain S_o/S_i. Then, the noise factor is:

$$F = \frac{\text{SNR}_i}{\text{SNR}_o} = \frac{S_i/N_{i\,(\text{source})}}{S_o/N_{o\,(\text{total})}} = \frac{S_i/N_{i\,(\text{source})}}{(S_i \cdot G)/N_{o\,(\text{total})}} = \frac{N_{o\,(\text{total})}}{G \cdot N_{i\,(\text{source})}}$$

where $N_{o(\text{total})}$ is the total noise at the output. If $N_{o(\text{source})}$ is the noise at the output originating at the source, and $N_{o(\text{added})}$ is the noise at the output added by the electronic circuitry, then we can write:

$$N_{o\,(\text{total})} = N_{o\,(\text{source})} + N_{o\,(\text{added})}$$

Noise Figure (2)

Noise factor can be written in useful alternative form:

$$F = \frac{N_{o\,(\text{total})}}{G \cdot N_{i\,(\text{source})}} = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = \frac{N_{o\,(\text{source})} + N_{o\,(\text{added})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o\,(\text{added})}}{N_{o\,(\text{source})}}$$

This shows that the minimum possible noise factor, which occurs if the electronics adds no noise, is equal to 1.

<u>Noise figure</u> NF is related to noise factor *F* by:

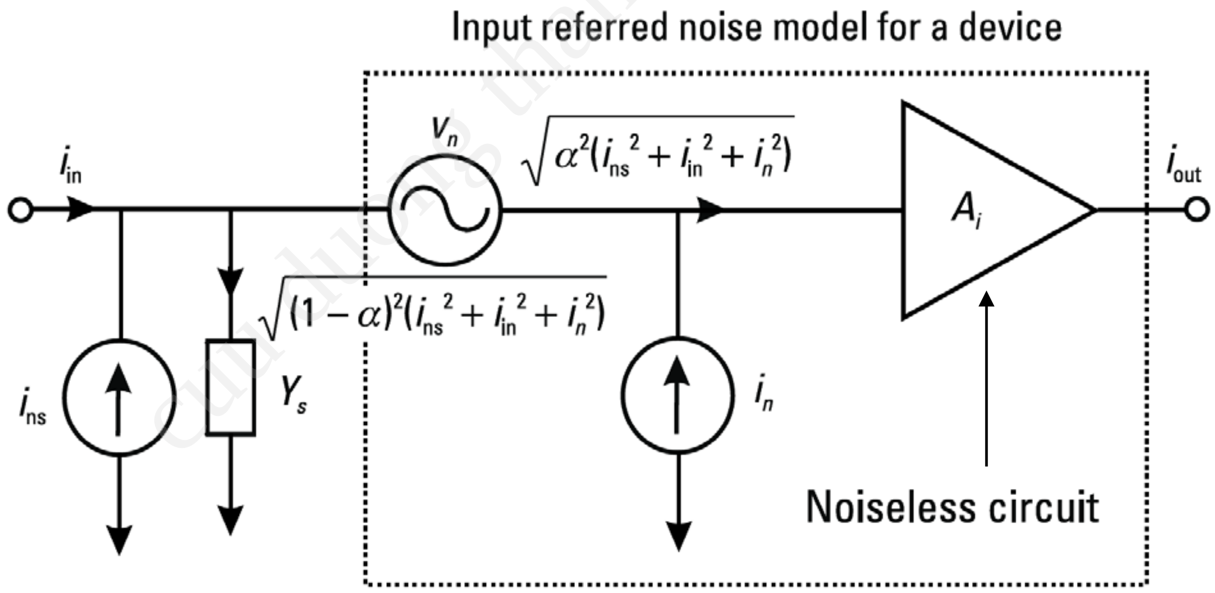
$$\text{NF} = 10\ \log_{10} F$$

Thus, an electronic system that adds no noise has a noise figure of 0 dB.

Noise Figure (3)

- **The Noise Figure of an amplifier circuit:** It is assumed that all practical amplifiers can be characterized by an **input-referred noise model**, such as the figure below, where the amplifier is characterized with current gain A_i.

  In this model, all noise sources in the circuit are lumped into a series noise voltage source v_n and a parallel current noise source i_n placed in front of a noiseless circuit.



Noise Figure (4)

If the amplifier has finite input impedance, then the input current will be split by some ratio α between the amplifier and the source admittance Y_s:

$$\mathrm{SNR}_{\mathrm{in}} = \frac{\alpha^2 i_{\mathrm{in}}^2}{\alpha^2 i_{\mathrm{ns}}^2}$$

Assuming that the input-referred noise sources are correlated, the output signal-to-noise ratio is:

$$\mathrm{SNR}_{\mathrm{out}} = \frac{\alpha^2 A_i^2\, i_{\mathrm{in}}^2}{\alpha^2 A_i^2 \left(i_{\mathrm{ns}}^2 + \left|i_n + v_n Y_s\right|^2\right)}$$

Thus, the noise factor can now be written in terms of the preceding two equations:

$$F = \frac{i_{\mathrm{ns}}^2 + \left|i_n + v_n Y_s\right|^2}{i_{\mathrm{ns}}^2} = \frac{N_{o\,(\mathrm{total})}}{N_{o\,(\mathrm{source})}}$$

Noise Figure (5)

In general, two input noise sources will not be correlated with each other, but rather the current i_n will be partially correlated with v_n and partially uncorrelated. We can expand both current and voltage into these two explicit parts:

$$i_n = i_c + i_u$$

$$v_n = v_c + v_u$$

In addition, the correlated components will be related by the ratio

$$i_c = Y_c v_c$$

where Y_c is the correlation admittance.

Thus, the noise figure is now written as

$$\mathrm{NF} = 1 + \frac{G_u + |Y_c + Y_s|^2 R_c + |Y_s|^2 R_u}{G_s}$$

Noise Figure (6)

or

$$\mathrm{NF} = 1 + \frac{G_u + [(G_c + G_s)^2 + (B_c + B_s)^2]R_c + (G_s^2 + B_s^2)R_u}{G_s}$$

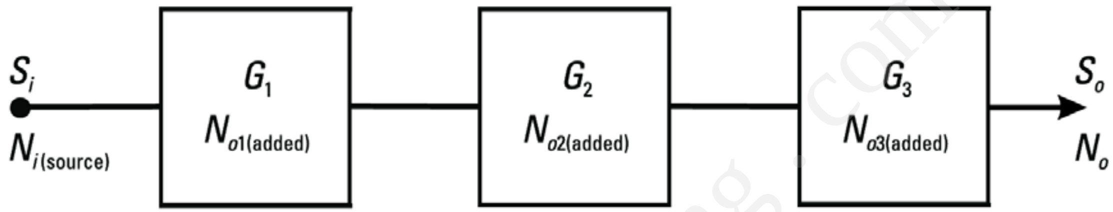
This equation can be used not only to <u>determine the noise figure</u>, but also to <u>determine the source</u> loading conditions that will <u>minimize the noise figure</u>. Differentiating with respect to G_S and B_S and setting the derivative to zero yields the following two conditions for minimum noise (G_{opt} and B_{opt}):

$$G_{\mathrm{opt}} = \sqrt{\frac{G_u + R_u\left(\dfrac{R_c B_c}{R_c + R_u}\right)^2 + G_c^2 R_c + \left(B_c - \dfrac{R_c B_c}{R_c + R_u}\right)^2 R_c}{R_c + R_u}}$$

$$B_{\mathrm{opt}} = \frac{-R_c B_c}{R_c + R_u}$$

Noise Figure (7)

- **<u>Noise Figure of stages in series</u>:** Consider three stages in series as



The output signal S_o is given by: $S_o = S_i \cdot G_i \cdot G_2 \cdot G_3$

The input noise is: $N_{i\,(\text{source})} = kT$

The total output noise is:

$$N_{o\,(\text{total})} = N_{i\,(\text{source})} G_1 G_2 G_3 + N_{o1(\text{added})} G_2 G_3 + N_{o2(\text{added})} G_3 + N_{o3(\text{added})}$$

The output noise due to the source is:

$$N_{o\,(\text{source})} = N_{i\,(\text{source})} G_1 G_2 G_3$$

Noise Figure (8)

Finally, the noise factor can be determined as:

$$F = \frac{N_{o\,(\text{total})}}{N_{o\,(\text{source})}} = 1 + \frac{N_{o1(\text{added})}}{N_{i\,(\text{source})}\,G_1} + \frac{N_{o2(\text{added})}}{N_{i\,(\text{source})}\,G_1\,G_2} + \frac{N_{o3(\text{added})}}{N_{i\,(\text{source})}\,G_1\,G_2\,G_3}$$

$$= F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1\,G_2}$$

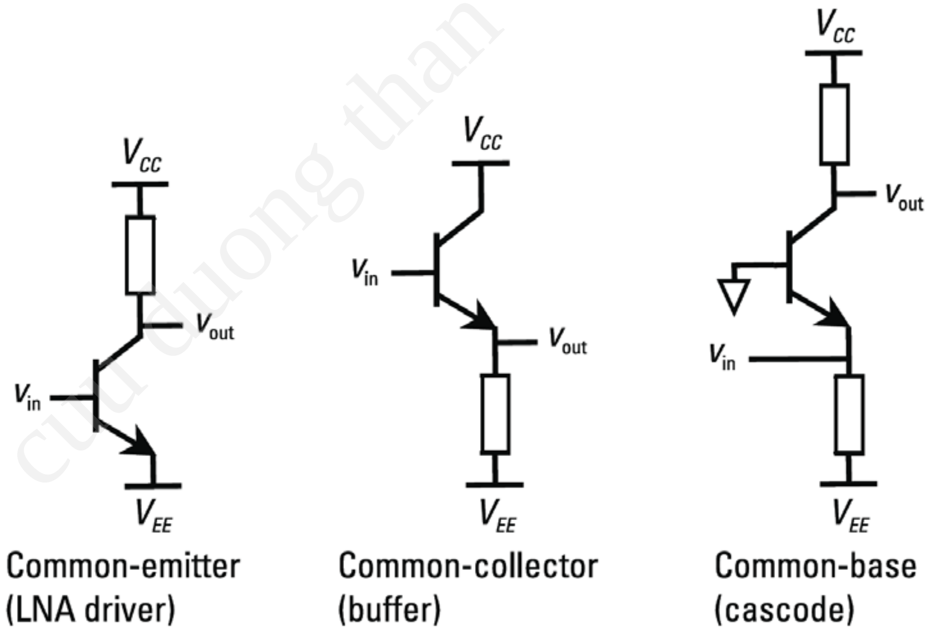
The above formula shows how the presence of gain preceding a stage causes the effective noise figure to be **reduced** compared to the measured noise figure of a stage by itself. For this reason, **we typically design systems with a low-noise amplifier at the front of the system.**

<u>Question</u>: Derive the formula for *N* stages in series?

LNA Design (1)

- **<u>Introduction</u>:** The LNA is the first block in most receiver front ends. Its job is to **amplify the signal** while **introducing a minimum amount of noise to the signal**.

  Gain can be provided by a single transistor. Since a transistor has three terminals, one terminal should be ac grounded, one serves as the input, and one is the output. There are three possibilities, as shown below:



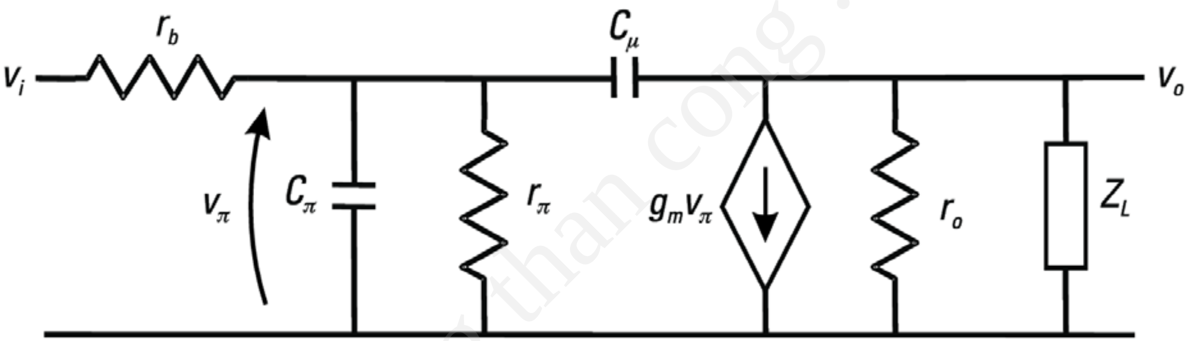
LNA Design (2)

The **common-emitter** (CE) amplifier is most often used as a **driver** for an LNA. The **common-collector** (CC), with high input impedance and low output impedance, makes an excellent **buffer** between stages or before the output driver. The **common-base** (CB) is often used as a **cascode** in combination with the common-emitter to form an LNA stage with gain to **high frequency** (as will be shown).

The **loads** of the circuits can be made either with **resistors** for **broadband operation**, or with **tuned resonators** for **narrow-band operation**.

LNA Design (3)

- **Common-Emitter (CE) amplifier (Driver):** For the analysis of the common-emitter amplifier, we replace the transistor with its small-signal model, as shown below, where Z_L represents some arbitrary load that the amplifier is driving.



At low frequency, the voltage gain of the amplifier can be given by:

$$A_{\mathrm{vo}} = \frac{v_o}{v_i} = -\frac{r_\pi}{r_b + r_\pi} g_m Z_L \approx \frac{Z_L}{r_e}$$

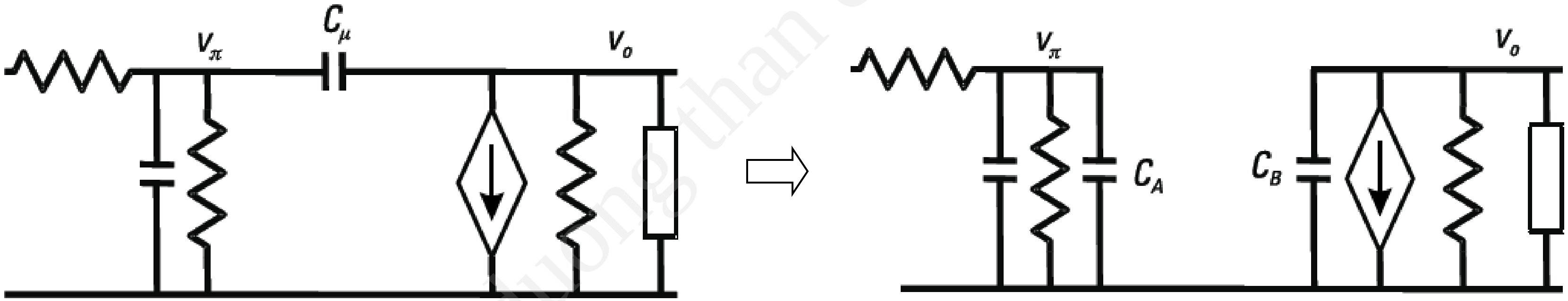
where r_e is the small-signal base-emitter diode resistance as seen from the emitter. Note that $r_\pi = \beta r_e$ and $g_m = 1/r_e$. For low frequencies, the parasitic capacitances have been ignored.

LNA Design (4)

The input impedance of the circuit at low frequencies is given by:

$$Z_{\text{in}} = r_b + r_\pi$$

At radio frequency, C_π will provide a low impedance across r_π, and C_μ will provide a feedback path. By using Miller's Theorem, C_μ can be replaced with two capacitors C_A and C_B, as illustrated in below:



where C_A and C_B are:

$$C_A = C_\mu \left(1 - \frac{v_o}{v_\pi}\right) = C_\mu (1 + g_m Z_L) \approx C_\mu g_m Z_L$$

$$C_B = C_\mu \left(1 - \frac{v_\pi}{v_o}\right) = C_\mu \left(1 + \frac{1}{g_m Z_L}\right) \approx C_\mu$$

LNA Design (5)

Normally, the reactance of C_B is much higher than Z_L, then C_B can be ignored, then we can estimate the gain at RFs as

$$A_v(f) = \frac{A_{vo}}{1 + j\dfrac{f}{f_{P1}}}$$

where

$$f_{P1} = \frac{1}{2\pi \cdot \left[r_\pi \| (r_b + R_S) \right] \left[C_\pi + C_A \right]}$$

$$A_{vo} = \frac{v_o}{v_i} = -\frac{r_\pi}{r_b + r_\pi} g_m Z_L \approx \frac{Z_L}{r_e}$$

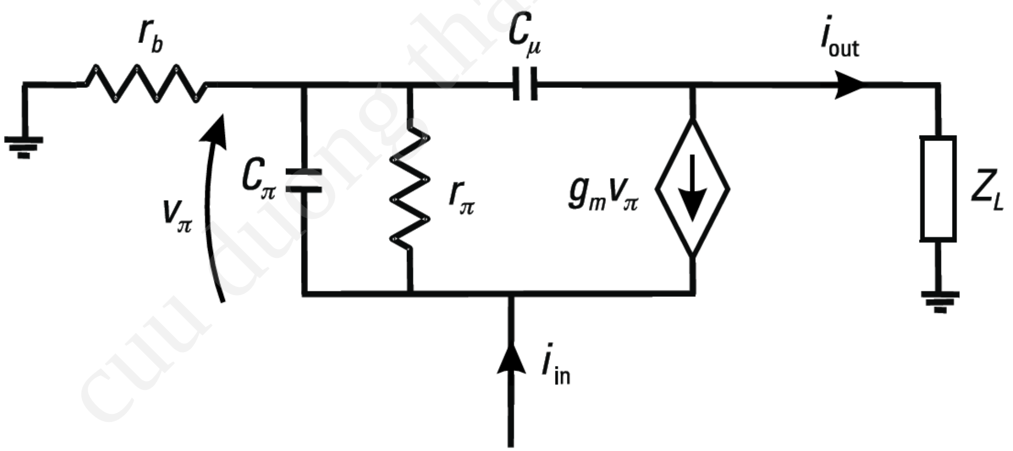
where R_S is the resistance of the source driving the amplifier.

The frequency f_β, where the current gain is reduced by 3 dB, and the unity current gain frequency f_T, are given by:

$$f_\beta = \frac{1}{2\pi \cdot r_\pi (C_\pi + C_\mu)} \qquad f_T = \frac{g_m}{2\pi \cdot (C_\pi + C_\mu)}$$

LNA Design (6)

- **<u>Common-Base (CB) amplifier (Cascode):</u>** The CB amplifier is often combined with the CE amplifier to form an LNA (but it can be used by itself as well). Since it has low input impedance when it is driven from a current source, it can pass current through it with near unity gain up to a very high frequency. Therefore, with an appropriate choice of impedance levels, it can also provide voltage gain. The small-signal model for the CB amplifier is shown in below (ignoring output impedance r_o).



LNA Design (7)

The current gain (ignoring C_μ and r_o) for this stage can be found to be:

$$\frac{i_{\text{out}}}{i_{\text{in}}} \approx \frac{1}{1 + j\omega C_\pi r_e} \approx \frac{1}{1 + j\dfrac{\omega}{\omega_T}}$$

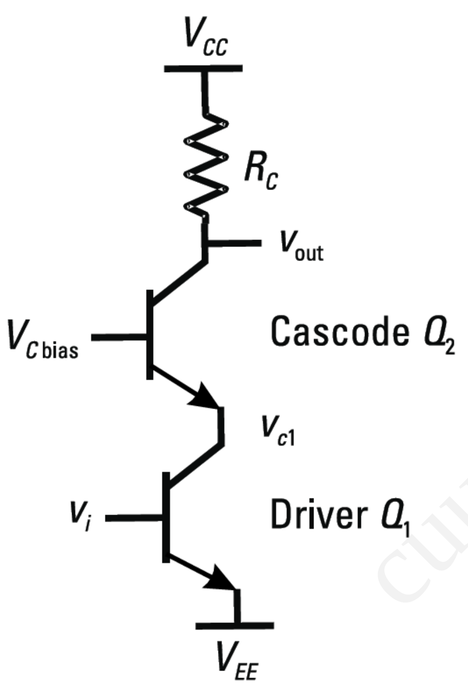
where $\omega_T = 2\pi f_T$. At frequencies below ω_T , the current gain for the stage is 1.

Note that the **pole** in this equation (**CB amplifier**) is usually at a **much higher frequency** than the **pole** in the **CE amplifier**, since $r_e < r_b + R_S$.

The **input impedance** of this stage is **low** and is equal to $1/g_m$ at low frequencies. At the pole frequency, the capacitor will start to dominate and the impedance will drop.

LNA Design (8)

The CB amplifier can be used in combination with the CE amplifier to form a cascode LNA, as shown in figure below. In this case, the current i_{c1} through Q_1 is about the same as the current i_{c2} through Q_2, since the CB amplifier has a current gain of approximately 1. Then, $i_{c1} \approx i_{c2} = g_{m1}v_i$.

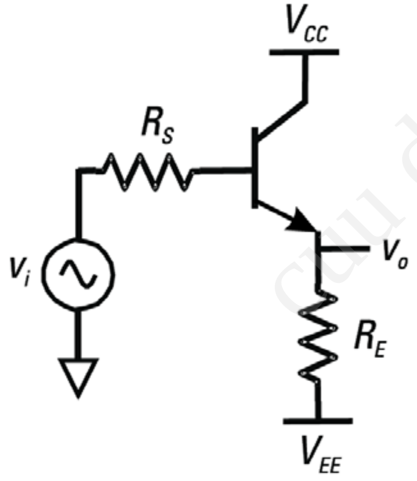


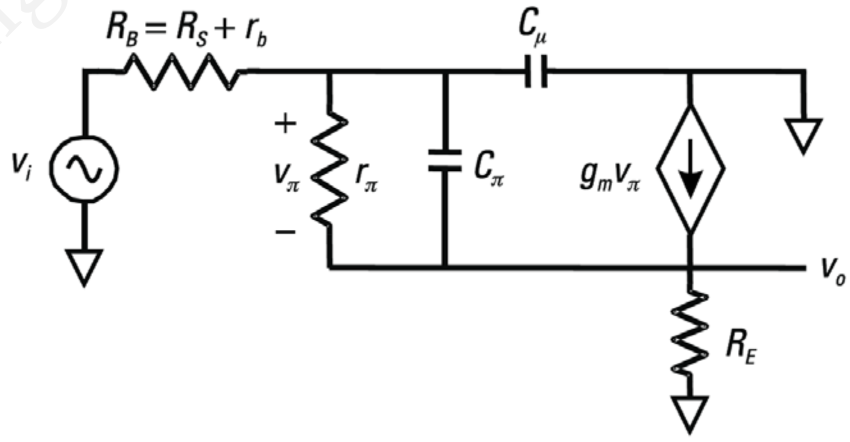
For the case where $R_S + r_b << r_\pi$ and $v_o/v_i \approx -g_m R_C$, the gain is the same as for the CE amplifier. However, the cascade transistor reduces the feedback of $C_{\mu 1}$, resulting in increased bandwidth. The **new estimate for the pole frequency** in the CE amplifier (with cascode transistor) is:

$$f_{P1} = \frac{1}{2\pi\left[r_{\pi 1} \| (r_{b1} + R_S)\right]\left[C_\pi + 2C_\mu\right]} \approx \left(1 + \frac{r_{\pi 1}}{R_S + r_{b1}}\right)\frac{f_T}{\beta}$$

LNA Design (9)

- **Common-Collector (CC) amplifier (Emitter Follower):** The CC amplifier is a very useful general-purpose amplifier. It has a **voltage gain that is close to 1**, but has a **high input impedance** and a **low output impedance**. Thus, it makes a very good **buffer stage** or output stage. The CC amplifier and its small-signal model are shown in figure below. The resistor R_E may represent a resistor or the output resistance of a current source. Note that the Miller effect is not a problem in this amplifier, since the collector is grounded. Since C_μ is typically much less than C_π, it can be left out of the analysis with little impact on the gain.





LNA Design (10)

The voltage gain of this amplifier is given by

$$A_v(s) = A_{vo}\left(\frac{1 - s/z_1}{1 - s/p_1}\right)$$

A_{vo} is the gain at low frequency and is given by

$$A_{vo} = \frac{g_m R_E + \dfrac{R_E}{r_\pi}}{1 + g_m R_E + \dfrac{(R_B + R_E)}{r_\pi}} \approx \frac{g_m R_E}{1 + g_m R_E} \approx 1$$

The pole and zero are given by

$$f_{z1} \approx \frac{g_m}{C_\pi} = -\omega_T \quad \text{and} \quad f_{p1} \approx -\frac{1}{C_\pi R_A}$$

where $R_A = r_\pi \left\| \dfrac{R_B + R_E}{1 + g_m R_E} \right.$

if $g_m R_E >> 1$, then $R_A \approx \dfrac{R_B + R_E}{g_m R_E}$ and $f_{p1} \approx -\dfrac{R_E}{R_E + R_B}\omega_T$

LNA Design (11)

The input impedance of this amplifier can also be determined. If $g_m R_E >> 1$, then we can use the small-signal circuit to find Z_{in}. The input impedance, again ignoring C_μ, is given by

$$Z_{\text{in}} = Z_\pi + R_E(1 + g_m Z_\pi)$$

where $Z_\pi = r_\pi \,||\, C_\pi$

Likewise, the output impedance can be found and is given by

$$Z_{\text{out}} = \frac{r_\pi + R_B + sC_\pi r_\pi R_B}{1 + g_m r_\pi + sC_\pi r_\pi}$$

Provided that $r_\pi > R_B$ and $\omega C_\pi r_\pi > r_\pi$ at the frequency of interest, the output impedance simplifies to

$$Z_{\text{out}} \approx r_e \frac{1 + g_m R_B j\omega/\omega_T}{1 + j\omega/\omega_T}$$

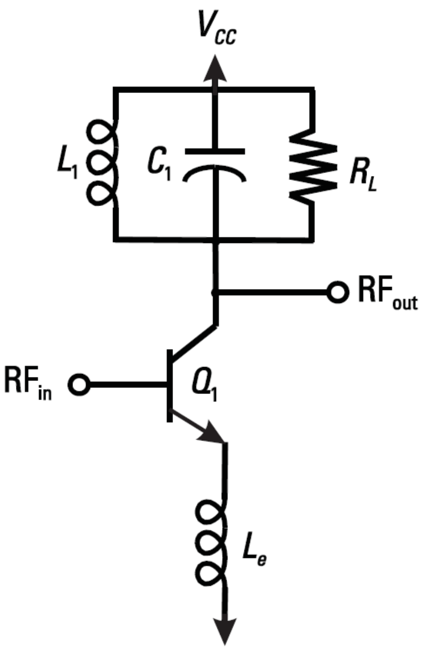
- At low frequencies, this further simplifies to $Z_{\text{out}} \approx r_e \approx \dfrac{1}{g_m}$

LNA Design (12)

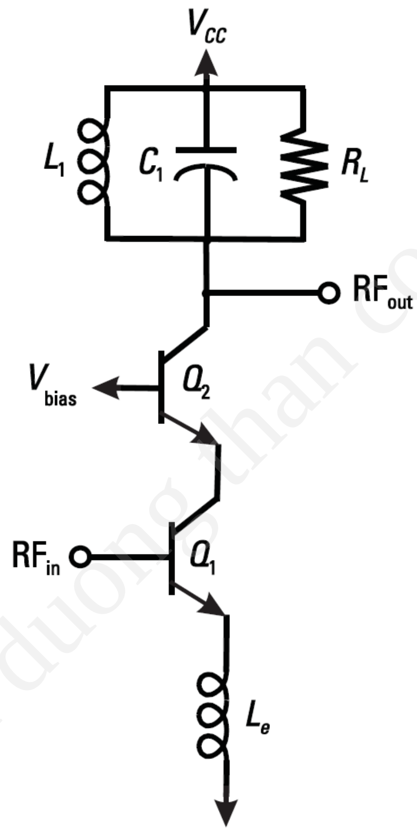
- At higher frequencies, if $r_e > R_B$ (recalling that $R_B = R_S + r_b$), for example, at low current levels, then $|Z_{out}|$ decreases with frequency, and so the output impedance is capacitive. However, if $r_e < R_B$, then $|Z_{out}|$ increases for higher frequency and the output impedance can be inductive. In this case, if the circuit is driving a capacitive load, the inductive component can produce resonance or even instability.

LNA Design (13)

❑ **<u>Common-Emitter with series feedback (emitter degeneration):</u>**



Common-emitter tuned LNA



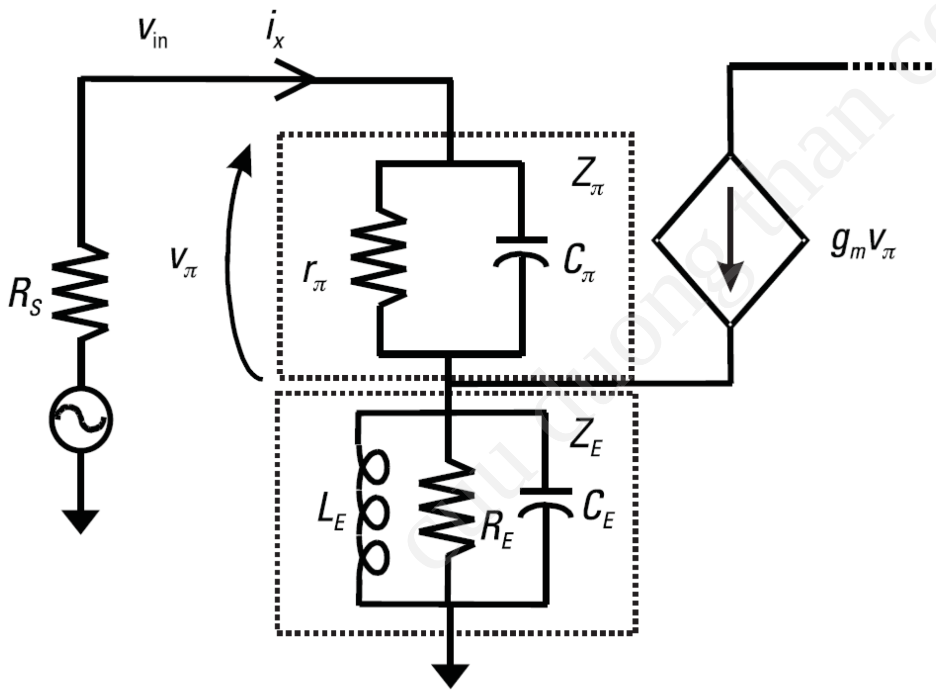
Cascode tuned LNA

Most common-emitter and cascode LNAs employ the use of **degeneration** (usually in the form of an <u>inductor</u> in narrowband applications) as shown in figure (left). The purpose of degeneration is to provide a means to transform the real part of the impedance seen looking into the base to a higher impedance for **matching purposes**.

LNA Design (14)

The **gain** of either amplifier at the resonance frequency of the LC parallel circuit in the collector, ignoring the effect of C_μ, is given by

$$\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{-g_m R_L}{\left(1 + \frac{Z_E}{Z_\pi} + g_m Z_E\right)} \approx -\frac{R_L}{Z_E}$$



where Z_E is the impedance of the emitter degeneration. Here it is assumed that the impedance in the emitter is a complex impedance.

LNA Design (15)

If the input impedance is matched to R_S (which would require an input series inductor), then the gain can be written out in terms of source resistance and f_T; v_{out} in terms of i_x can be given by

$$v_{out} = -g_m v_\pi R_L = -g_m i_x Z_\pi R_L$$

Noting that $i_x = v_{in} / R_S$:

$$\frac{v_{out}}{v_{in}} = \frac{-g_m Z_\pi R_L}{R_S}$$

Assuming that Z_π is primarily capacitive at the frequency of interest, then:

$$\left|\frac{v_{out}}{v_{in}}\right| = \frac{g_m R_L}{R_S \omega_o C_\pi} = \frac{R_L \omega_T}{R_S \omega_o}$$

where ω_o is the frequency of interest.

The **input impedance** has the same form as the common-collector amplifier and is also given by

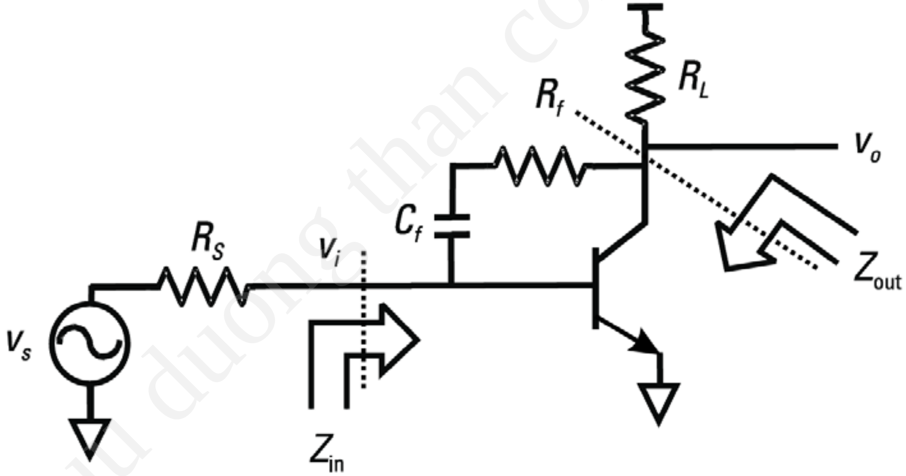
$$Z_{in} = Z_\pi + Z_E(1 + g_m Z_\pi)$$

LNA Design (16)

Of particular interest is the product of Z_E and Z_π. If the emitter impedance is **inductive**, then when this is reflected into the base, it will become a real resistance. Thus, placing an inductor in the emitter tends to **raise the input impedance** of the circuit, so it is very useful for matching purposes. (Conversely, placing a **capacitor** in the emitter will tend to **reduce the input impedance** of the circuit and can even make it negative.)

LNA Design (17)

- **<u>Common-Emitter with shunt feedback</u>:** Applying shunt feedback to a common-emitter amplifier is a good basic building block for broadband amplifiers. This technique allows the amplifier to be **matched over a broad bandwidth** while having **minimal impact on the noise figure** of the stage. A basic common-emitter amplifier with shunt feedback is shown below:



Resistor R_f forms the feedback and capacitor C_f is added to allow for independent biasing of the base and collector. C_f can normally be chosen so that it is large enough to be a short circuit over the frequency of interest.

LNA Design (18)

Ignoring the Miller effect and assuming C_f is a short circuit ($1/\omega C_f << R_f$), the **gain** is given by

$$A_v = \frac{v_o}{v_i} = \frac{\frac{R_L}{R_F} - g_m R_L}{1 + \frac{R_L}{R_f}} \approx \frac{-g_m R_L}{1 + \frac{R_L}{R_f}}$$

Thus, we see that in this case the **gain without feedback** ($-g_m R_L$) is **reduced** by the presence of feedback.

The **input impedance** of this stage is also changed dramatically by the presence of feedback. Ignoring C_μ, the input impedance can be computed to be

$$Z_{\text{in}} = \frac{Z_\pi (R_f + R_L)}{R_f + R_L + Z_\pi (1 + g_m R_L)} \approx R_f \| Z_\pi \| \frac{R_f + R_L}{g_m R_L} \approx \frac{R_f + R_L}{g_m R_L}$$

Compared to the amplifier without feedback, the **input impedance** for the **shunt feedback amplifier** has **less variation** over frequency.

LNA Design (19)

Similarly, the **output impedance** can be determined as

$$Z_{\text{out}} = \frac{R_f}{1 + Z_{\text{ip}}\left(g_m - \frac{1}{R_f}\right)} \approx \frac{R_f}{1 + g_m Z_{\text{ip}}}$$

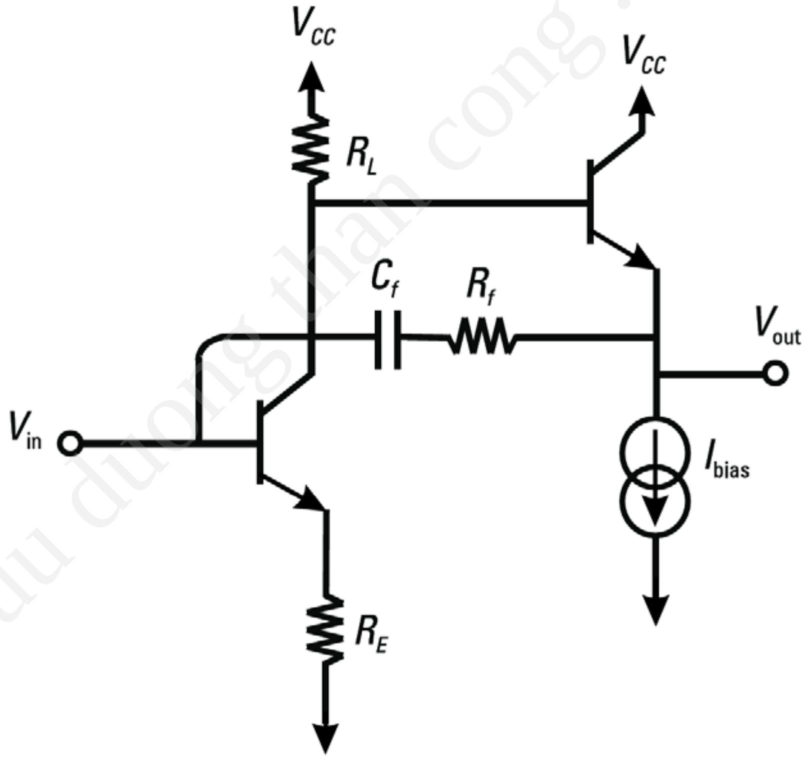
where $Z_{ip} = R_S \parallel R_f \parallel Z_\pi$.

With this type of amplifier, it is sometimes advantageous to couple it with an **output buffer** (see figure next slide). The output buffer provides some inductance to the input, which tends to make for a **better match**. The presence of the buffer does change the previously developed formulas somewhat. If the buffer is assumed to be lossless, the input impedance now becomes:

$$Z_{\text{in}} = Z_\pi \left(1 + \frac{Z_\pi}{R_f} + \frac{g_m R_L Z_\pi}{R_f}\right)^{-1} = \frac{R_f}{1 + g_m R_L + \frac{R_f}{Z_\pi}} \approx \frac{R_f}{g_m R_L}$$

LNA Design (20)

With the **addition of a buffer**, the **voltage gain** is **no longer affected by the feedback**, so it is approximately that of a common emitter amplifier given by $[R_L/(R_E + 1/g_m)]$ minus the loss in the buffer.

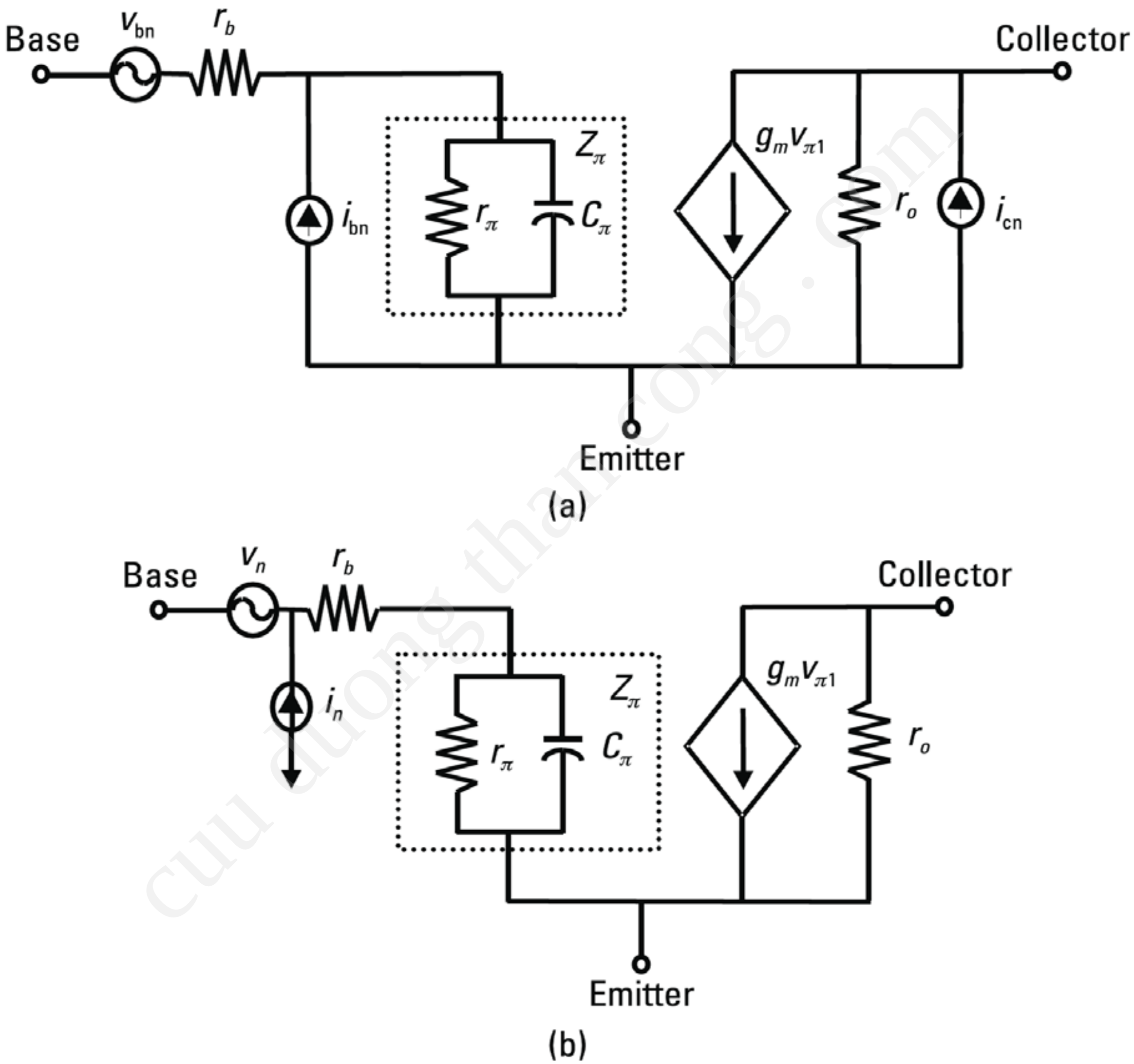


LNA Design (21)

- **<u>Noise in amplifiers</u>:** When the signal is first received by the radio, it can be quite weak and can be in the presence of a great deal of interference. The LNA is the first part of the radio to process the signal, and it is therefore essential that it **amplify the signal** while **adding a minimal amount of additional noise** to it. Thus, one of the most important considerations when designing an LNA is the **amount of noise present** in the circuit.

  For an actual LNA, then all the noise sources must be written in terms of these **two input-referred noise sources**, as shown in figure next slide. Starting with the model shown in figure (a), and assuming that the emitter is grounded with base input and collector output, the model may be determined with some analysis.

LNA Design (22)



LNA Design (23)

Two input-referred noise sources can be found as

$$\overline{v_n^2} = \frac{2qI_c}{g_m^2} + 4kTr_b$$

$$\overline{i_n^2} = 2qI_B + \frac{2qI_C}{g_m^2} Y_\pi^2$$

<u>**Noise Figure of the CE amplifier**</u>**:** Now that the equivalent input-referred noise sources have been derived, it can be applied to find the noise figure and the optimum source impedance for noise in terms of transistor parameters.

The <u>noise figure</u> can be written in terms of circuit parameters:

$$\mathrm{NF} = 1 + \frac{\dfrac{I_C}{2v_T\beta} + \left[G_S^2 + (\omega C_\pi)^2\right]\dfrac{v_T}{2I_C} + G_S^2 r_b}{G_S}$$

Here it is assumed that the source resistance has no reactive component.

LNA Design (24)

Then, the optimum source admittance $Y_{opt} = G_{opt} + jB_{opt}$ can be found as

$$G_{opt} = \sqrt{\frac{\dfrac{I_C}{2v_T\beta} + r_b\left(\dfrac{-\dfrac{v_T}{2I_C}(\omega C_\pi)}{\dfrac{v_T}{2I_C} + r_b}\right)^2 + \dfrac{v_T}{2I_C}\left(\omega C_\pi - \dfrac{\dfrac{v_T}{2I_C}(\omega C_\pi)}{\dfrac{v_T}{2I_C} + r_b}\right)^2}{\dfrac{v_T}{2I_C} + r_b}}$$

$$B_{opt} = \frac{-\dfrac{v_T}{2I_C}(\omega C_\pi)}{\dfrac{v_T}{2I_C} + r_b}$$

LNA Design (25)

Relationship between Noise Figure and Bias Current:

Noise due to the base resistance is in series with the input voltage, so it sees the full amplifier gain. The output noise due to base resistance (thermal noise) is given by

$$v_{\text{no}, r_b} \approx \sqrt{4kTr_b} \cdot g_{m1} R_L$$

Collector shot noise is in parallel with collector signal current and is directly sent to the output load resistor:

$$v_{\text{no}, I_C} \approx \sqrt{2qI_C} R_L$$

Note that this output voltage is proportional to the square root of the collector current, and therefore, to improve the noise figure due to collector shot noise, we increase the current.

LNA Design (26)

Base shot noise can be converted to input voltage by considering the impedance on the base. If Z_{eq} is the impedance on the base (formed by a combination of matching, base resistance, source resistance, and transistor input impedance), then

$$v_{no, I_B} \approx \sqrt{\frac{2qI_C}{\beta}} Z_{eq} g_m R_L$$

Note that this output voltage is proportional to the collector current. Therefore, to improve the noise figure due to base shot noise, we decrease the current, because the signal-to-noise ratio (in voltage terms) is inversely proportional to the square root of the collector current.

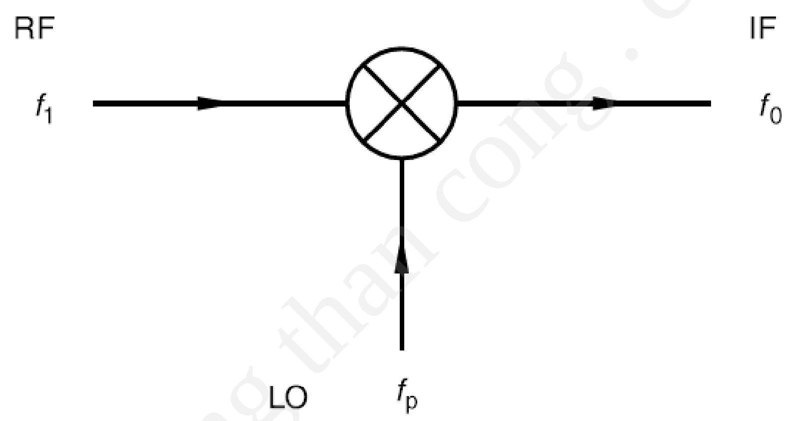
Therefore, at low currents, collector shot noise will dominate and noise figure will improve with increasing current. However, the effect of base shot noise also increases and will eventually dominate. Thus, there will be some optimum level to which the collector current can be increased, beyond which the noise figure will start to degrade again.

Chapter 4:

RF Mixer
(Frequency Converter)

Nonlinear Device Characteristics (1)

- A typical **mixer** is a **three-port circuit** that accepts two signals at two different frequencies and produces at the third port a signal that is the **sum** or **difference** of the two input frequencies.



Production of a new frequency or frequencies requires a **nonlinear device**. The two most common semiconductor nonlinear characteristics are:

- the form $e^{qV(t)/kT}$ as found in *pn* junction diodes or BJTs.
- the form $I_{DSS}(1 - V(t)/V_T)^2$ as found in FETs.

Nonlinear Device Characteristics (2)

- Consider a pn junction nonlinearity that is excited by two signals (plus a DC term):

$$V(t) = V_{\mathrm{dc}} + V_{\mathrm{p}} \cos \omega_{\mathrm{p}} t + V_1 \cos \omega_1 t$$

The device current is then of the form:

$$I(t) = I_{\mathrm{s}} e^{V_{\mathrm{dc}}/V_{\mathrm{T}}} [e^{V_{\mathrm{p}} \cos \omega_{\mathrm{p}} t} \cdot e^{V_1 \cos \omega_1 t}]$$

where the thermal voltage, V_T, is defined as kT/q, k is Boltzmann's constant, T is the absolute temperature, and q is the magnitude of the electronic charge.

Nonlinear Device Characteristics (3)

Using Bessel function:

$$e^{z\cos\theta} = I_0(z) + 2\sum_{n=1}^{\infty} I_n(z)\cos n\theta$$

then

$$I(t) = I_s e^{V_{dc}/V_T}\left[I_0(V_p) + 2\sum_{n=1}^{\infty} I_n(V_p)\cos n\omega_p t\right]$$

$$\times\left[I_0(V_1) + 2\sum_{m=1}^{\infty} I_n(V_1)\cos m\omega_1 t\right]$$

$$= I_{dc}e^{V_{dc}/V_T} I_0(V_p)I_0(V_1)$$

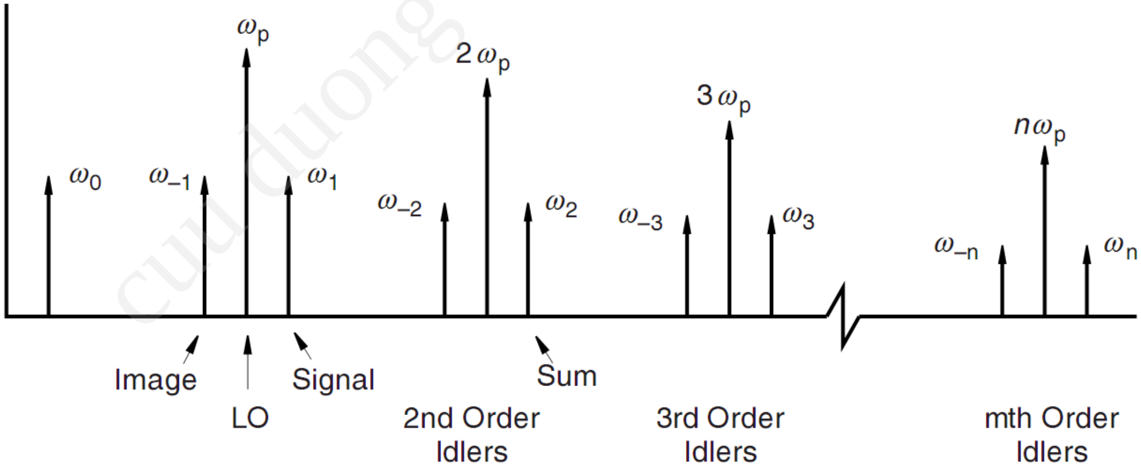
$$+ 2I_{dc}e^{V_{dc}/V_T}\left(I_0(V_1)\sum_{n=1}^{\infty} I_n(V_p)\cos n\omega_p t + I_0(V_p)\sum_{m=1}^{\infty} I_m(V_1)\cos m\omega_1 t\right)$$

$$+ 4I_{dc}e^{V_{dc}/V_T}\left[\sum_{n=1}^{\infty} I_n(V_p)\cos n\omega_p t\right]\cdot\left[\sum_{m=1}^{\infty} I_m(V_1)\cos m\omega_1 t\right]$$

Nonlinear Device Characteristics (4)

The basic result is a set of frequencies $n\omega_p + m\omega_1$ where n and m can take on any integer. The usual desired output for a receiver is the **intermediate frequency** (IF), ω_0. The frequencies of primary interest are given the following names:

ω_p	Local oscillator (pump) frequency
$\omega_0 = \omega_1 - \omega_p$	Intermediate frequency
ω_1	RF signal frequency
$\omega_{-1} = -\omega_p + \omega_0$	Image frequency
$\omega_2 = 2\omega_p + \omega_0$	Sum frequency



Nonlinear Device Characteristics (5)

- In the FET type of nonlinearity, when the mixer is excited by

$$V(t) = V_{dc} + V_p \cos\omega_p t + V_1 \cos\omega_1 t$$

the output current is

$$\frac{I(t)}{I_{DSS}} = \left(1 - \frac{V_p}{V_T}\cos\omega_p t - \frac{V_1}{V_T}\cos\omega_1 t\right)^2$$

$$= 1 - 2\left(\frac{V_p}{V_T}\cos\omega_p t + \frac{V_1}{V_T}\cos\omega_1 t\right) + \frac{V_p^2}{2V_T^2}(1 + \cos 2\omega_p t)$$

$$+ \frac{V_1^2}{2V_T^2}(1 + \cos 2\omega_1 t)$$

$$+ \frac{V_p V_1}{V_T^2}\left(\cos(\omega_p + \omega_1)t + \cos(\omega_p - \omega_1)t\right)$$

Figures of Merit for Mixers (1)

The **quality of a mixer** depends on a number of different **mixer parameters** which of course must fit the application under consideration:

- The first of these is **conversion loss**, *L*. This is the ratio of the delivered output power to the input available power:

$$L = \frac{\text{output IF power delivered to the load, } P_0}{\text{available RF input signal power, } P_1}$$

Clearly, the conversion loss is dependent on the load of the input RF circuit as well as the output impedance of the mixer at the IF port. The conversion loss for a typical diode mixer is between 6 and 7 dB.

- The **noise figure** is a measure of the noise added by the mixer itself to the RF input signal as it gets converted to the output IF.

Figures of Merit for Mixers (2)

- The **isolation** is the amount of local oscillator power that leaks into either the IF or the RF ports. For double-balanced mixers this value typically lies in the 15 to 20 dB range.

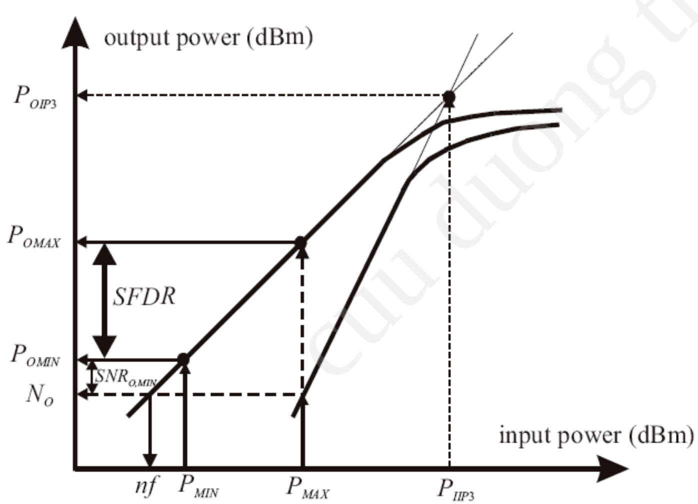
- The **conversion compression** is the RF input power, above which the RF input in terms of the IF output deviates from linearity by a given amount. For example, the **1-dB compression point** occurs when the conversion loss increases by 1 dB above the conversion loss in the low-power linear range. A typical value of 1.0 dB compression occurs when the RF power is +7 dBm and the LO is +13 dBm.

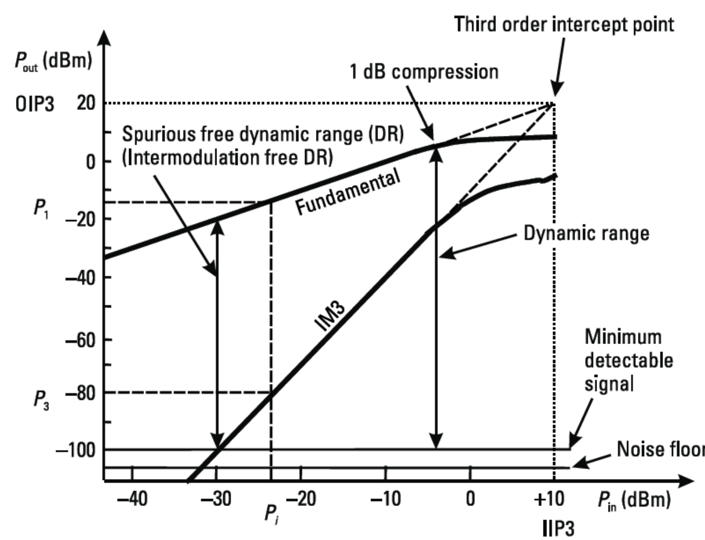
- The **LO drive power** is the required LO power level needed to make the mixer operate in optimum. For a double-balanced mixer, this is typically +6 dBm to +20 dBm.

Figures of Merit for Mixers (3)

- The **<u>dynamic range</u>** is the maximum RF input power range for the mixer. The maximum amplitude is limited by the conversion compression, and the minimum amplitude is limited by the noise figure.

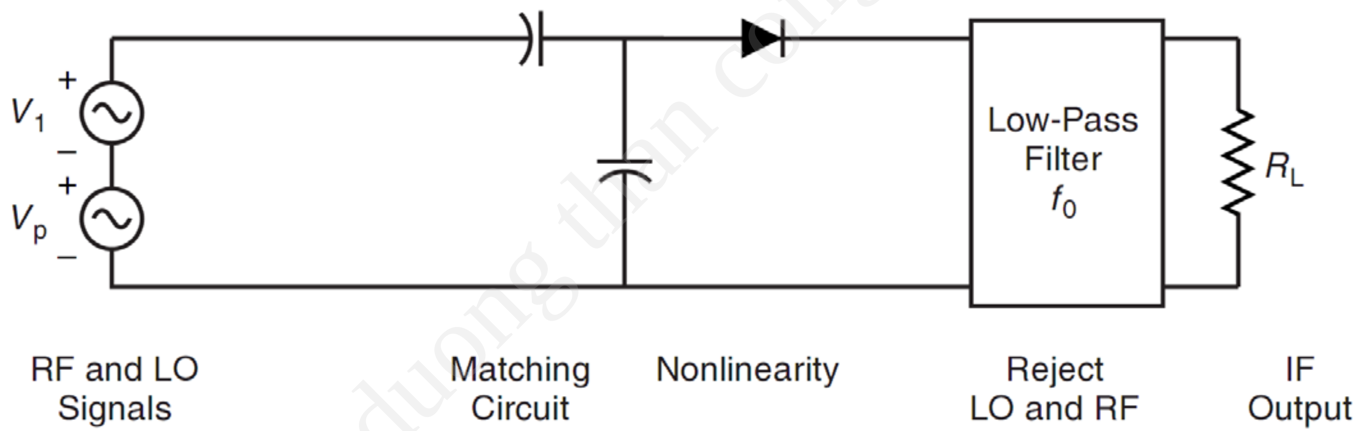
  The input intercept point is the RF input power at which the output power levels of the undesired intermodulation products (e.g. IM3) and the desired IF output would be equal. It conducts to the definition of **spurious free dynamic range (SFDR)** (See Chapter 1)





Single-Ended Mixers (1)

The **single-ended mixer** in below figure shows that the RF input signal and the local oscillator signal enter the mixer at the same point. Some degree of isolation between the two is achieved by using a directional coupler in which the RF signal enters the direct port and the local oscillator enters through the coupled port.



The amplitude of the local oscillator is large enough to turn the diode on and off during each cycle. Indeed, the LO power is so large as to cause clipping of the LO voltage, thereby approximating a square wave. The small RF signal is then presented with alternately a short or open circuit at the LO rate.

Single-Ended Mixers (2)

It is this turning on and off of the RF frequency that produces the set of frequencies:

$$|n f_{\mathrm{p}} \pm f_1|$$

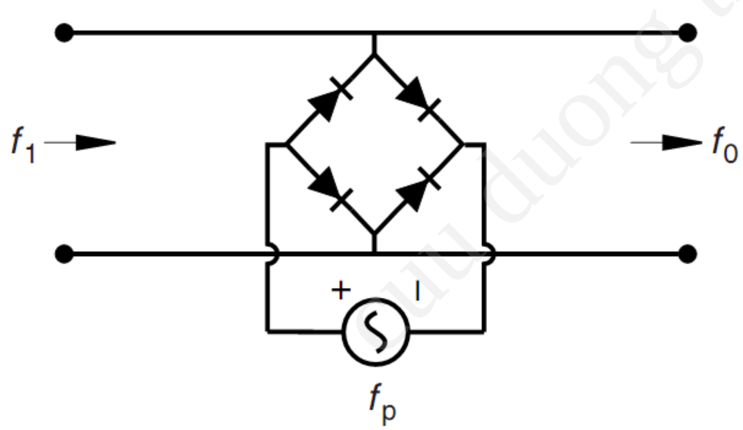
The one of most interest in the standard receiver is $f_0 = f_p - f_1$.

The disadvantages of the single-ended mixer are a high-noise figure, a large number of frequencies generated because of the nonlinear diode, a lack of isolation between the RF and LO signals, and large LO currents in the IF circuit. The RF to LO isolation problem can be very important, since the LO can leak back out of the RF port and be radiated through the receiver antenna. The LO currents in the IF circuit would have to be filtered out with a low-pass filter that has sufficient attenuation at the LO frequency to meet system specifications.

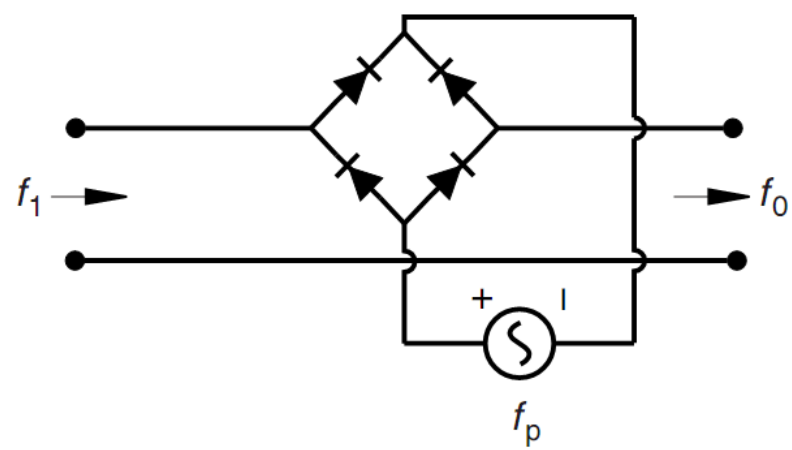
The advantage is that requiring lower LO power than the other types of mixers.

Single-Balanced Mixers (1)

- The **single-balanced** (or **simply balanced**) **mixer** has either two or four diodes as shown in four following figures. In all of these cases, when the LO voltage has a large positive value, all the diodes are shorted. When the LO voltage has a large negative value, all the diodes are open. In either case, the LO power cannot reach the IF load nor the RF load because of circuit symmetry. However, the incoming RF voltage sees alternately a path to the IF load and a blockage to the IF load. The block may either be an open circuit to the IF load or a short circuit to ground.

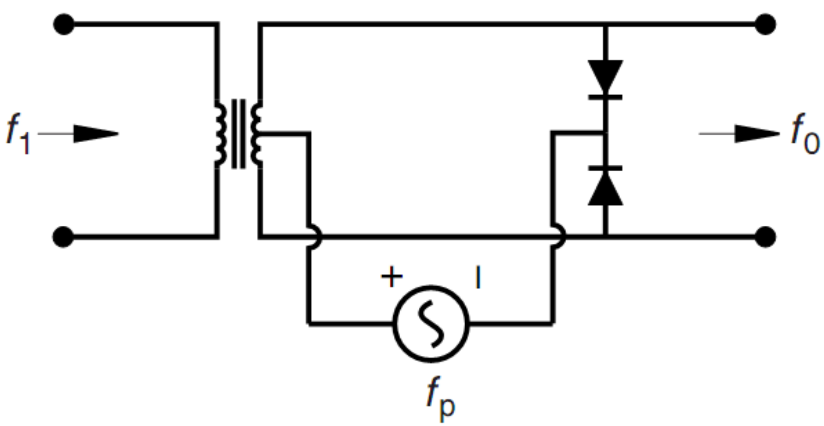


(a)

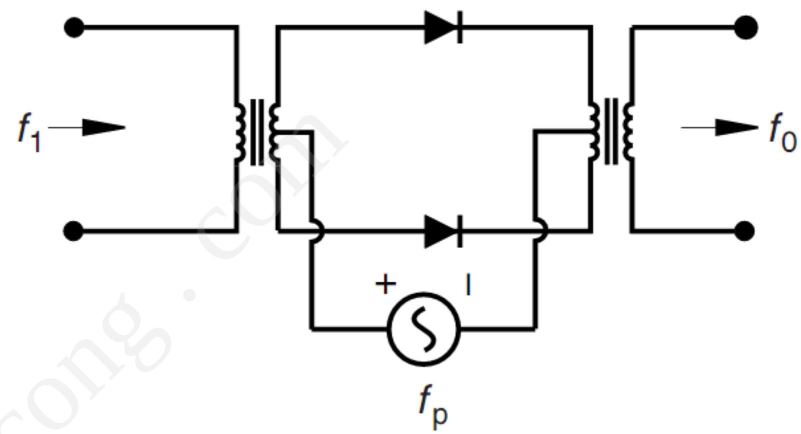


(b)

Single-Balanced Mixers (2)

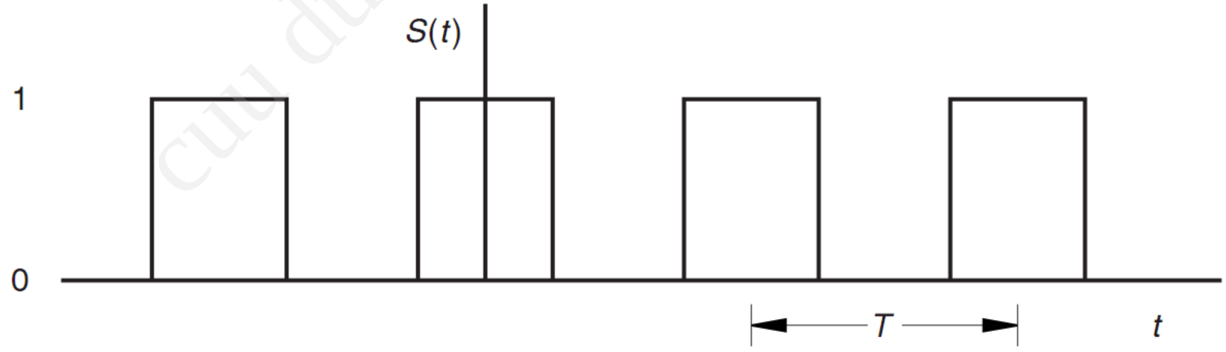


(c)



(d)

It is assumed that the LO voltage is much greater than the RF voltage, so $V_p >> V_1$. The LO voltage can be approximated as a square wave with period $T = 1/f_p$ that modulates the incoming RF signal:



Single-Balanced Mixers (3)

Fourier analysis of the square wave results in a switching function designated by $S(t)$:

$$S(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi/2} \cos n\omega_\text{p} t$$

If the input RF signal is expressed as $V_1 \cos \omega_1 t$, then the output voltage is this multiplied by the switching function:

$$\begin{aligned} V_0 &= V_1 \cos \omega_1 t \cdot S(t) \\ &= V_1 \cos \omega_1 t \left(\frac{1}{2} + \sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi/2} \cos n\omega_\text{p} t \right) \end{aligned}$$

Clearly, the RF input signal voltage will be present in the IF circuit. However, only the odd harmonics of the local oscillator voltage will effect the IF load. Thus the spurious voltages appearing in the IF circuit are:

$$f_1,\ f_\text{p} + f_1,\ 3f_\text{p} \pm f_1,\ 5f_\text{p} \pm f_1, \ldots$$

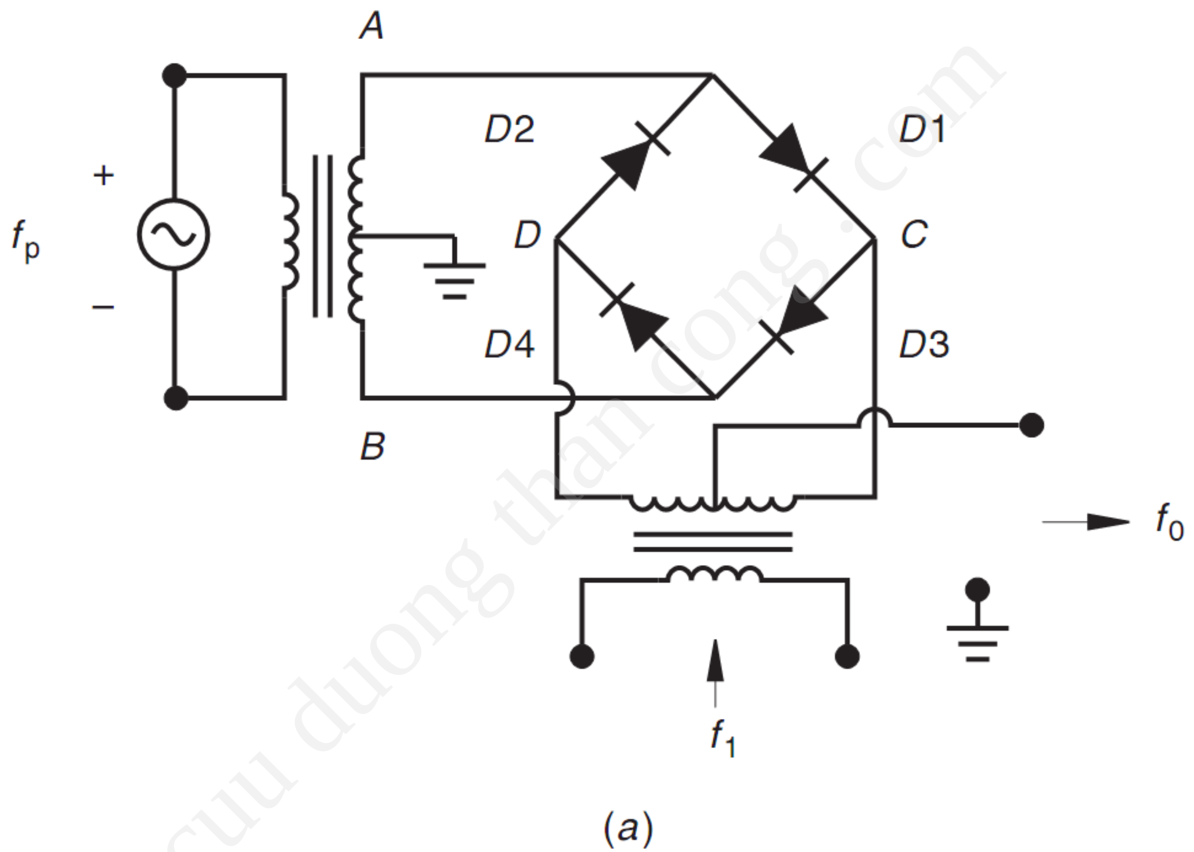
and all even harmonics of f_p are suppressed (or balanced out).

Double-Balanced Mixers (1)

- The **double-balanced mixer** is capable of isolating both the RF input voltage and the LO voltage from the IF load. The slight additional cost of some extra diodes and a balun is usually outweighed by the improved intermodulation suppression, improved dynamic range, low conversion loss, and low noise.

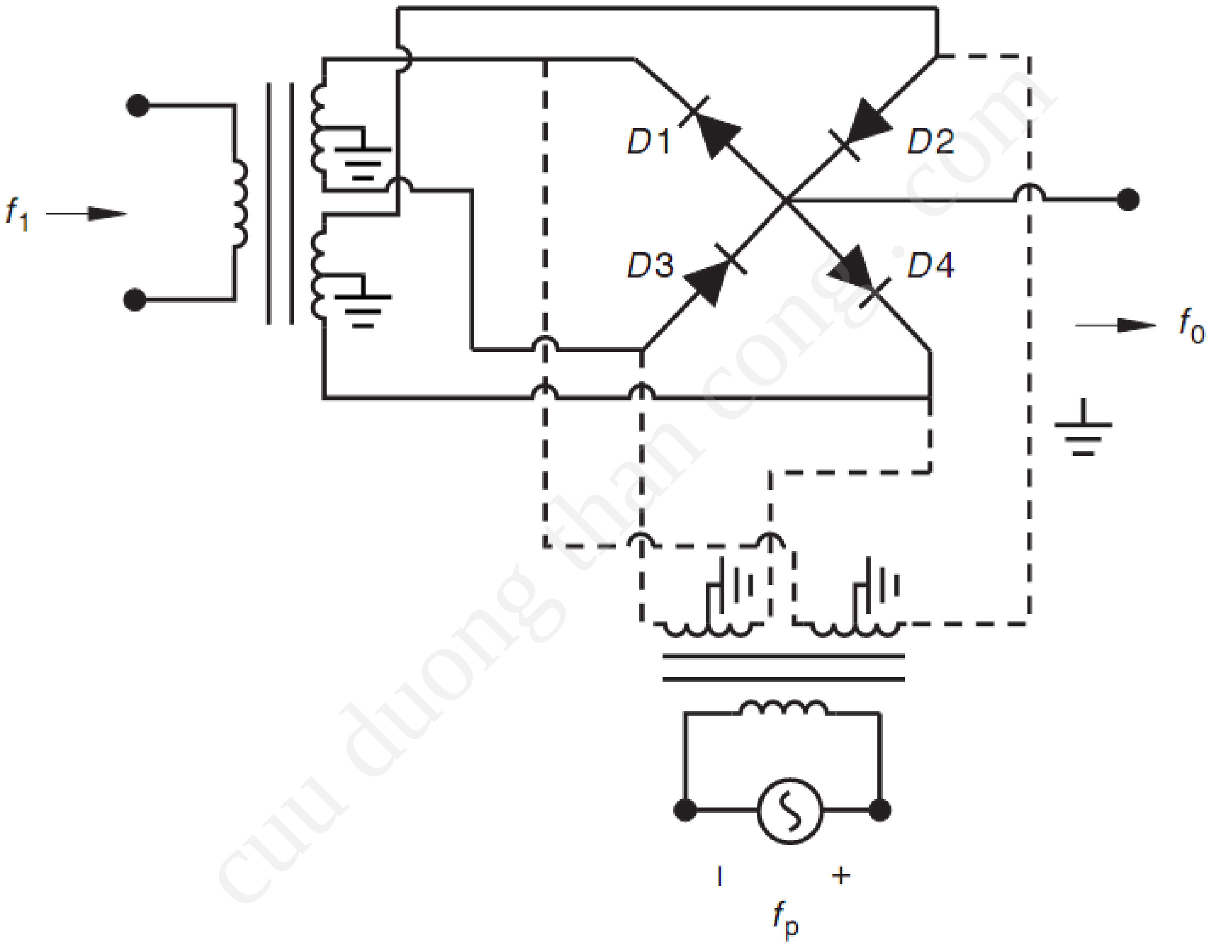
  The two most widely used double balanced mixers for the RF and microwave band are the **"ring" mixer** (figure (a)) and the **"star" mixer** (figure (b)) depicted in below:

Double-Balanced Mixers (2)



(See pp. 231-232, [2])

Double-Balanced Mixers (3)



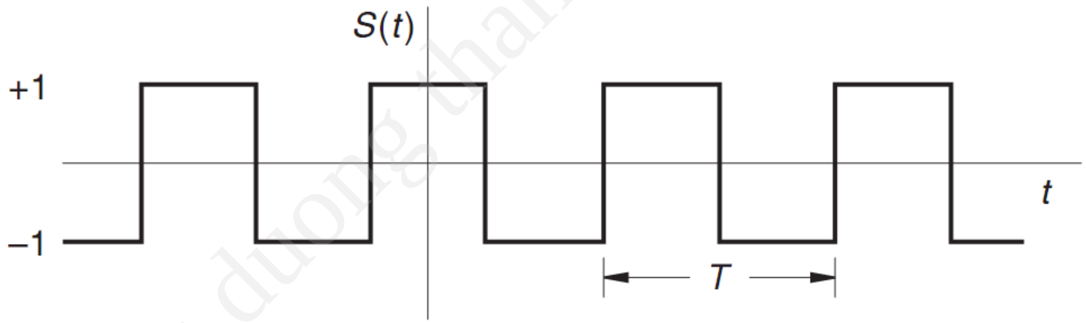
(b)

(See pp. 231-232, [2])

Double-Balanced Mixers (4)

In the single-balanced mixer all the diodes were either turned on or turned off, depending on the instantaneous polarity of the local oscillator voltage. In the double-balanced mixer half the diodes are on and half off at any given time, according to the local oscillator polarity. Thus the path from the RF signal port with frequency f_1 to the IF port, f_0, reverses polarity at the rate of $1/f_p$.

In both these cases (ring and star mixers) the switching function is shown as:



Fourier analysis provides the following time domain representation of the switching function:

$$S(t) = 2\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi/2} \cos n\omega_p t$$

Double-Balanced Mixers (5)

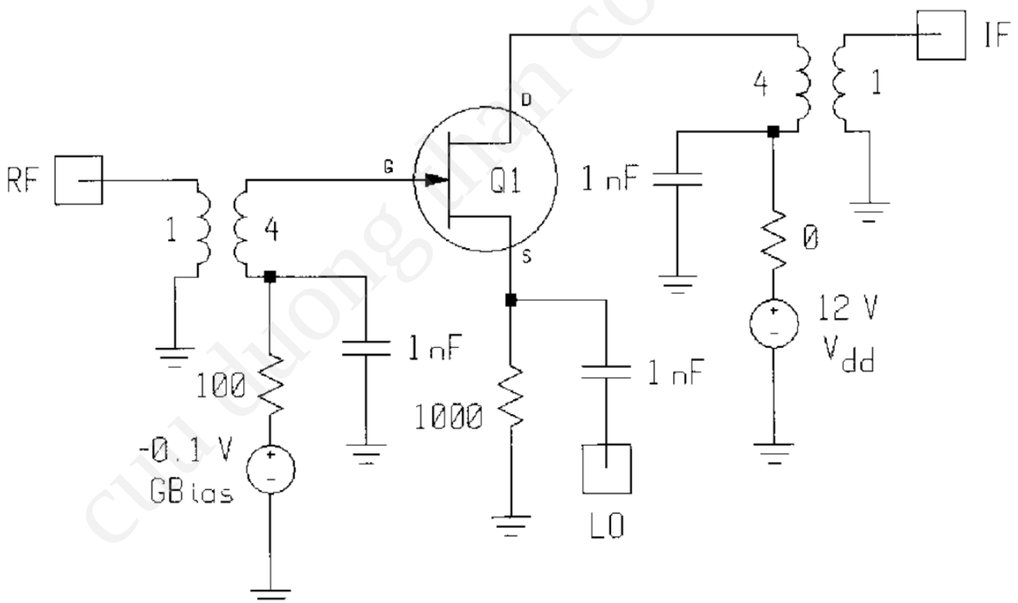
The IF voltage is found as before for the single-balanced mixer:

$$V_0 = V_1 \cos\omega_1 t \cdot S(t)$$
$$= 2V_1 \cos\omega_1 t \left(\sum_{n=1}^{\infty} \frac{\sin(n\pi/2)}{n\pi/2} \cos n\omega_p t \right)$$

Clearly, there is no RF signal nor LO voltage seen in the IF circuit, nor any even harmonics of the LO voltage.

Double-Balanced Transistor Mixers (1)

- Transistors can also be used as the mixing element in all three types of mixers described above. These are called **active mixers** because they provide the possibility of conversion gain that the diode mixers are not capable of doing. They produce approximately the same values of port isolation and suppression of even harmonic distortion as the diode mixers.



Example of single-ended mixer using JFET

Double-Balanced Transistor Mixers (2)

- An alternative design is based on the **Gilbert cell multiplier** in below figure, where

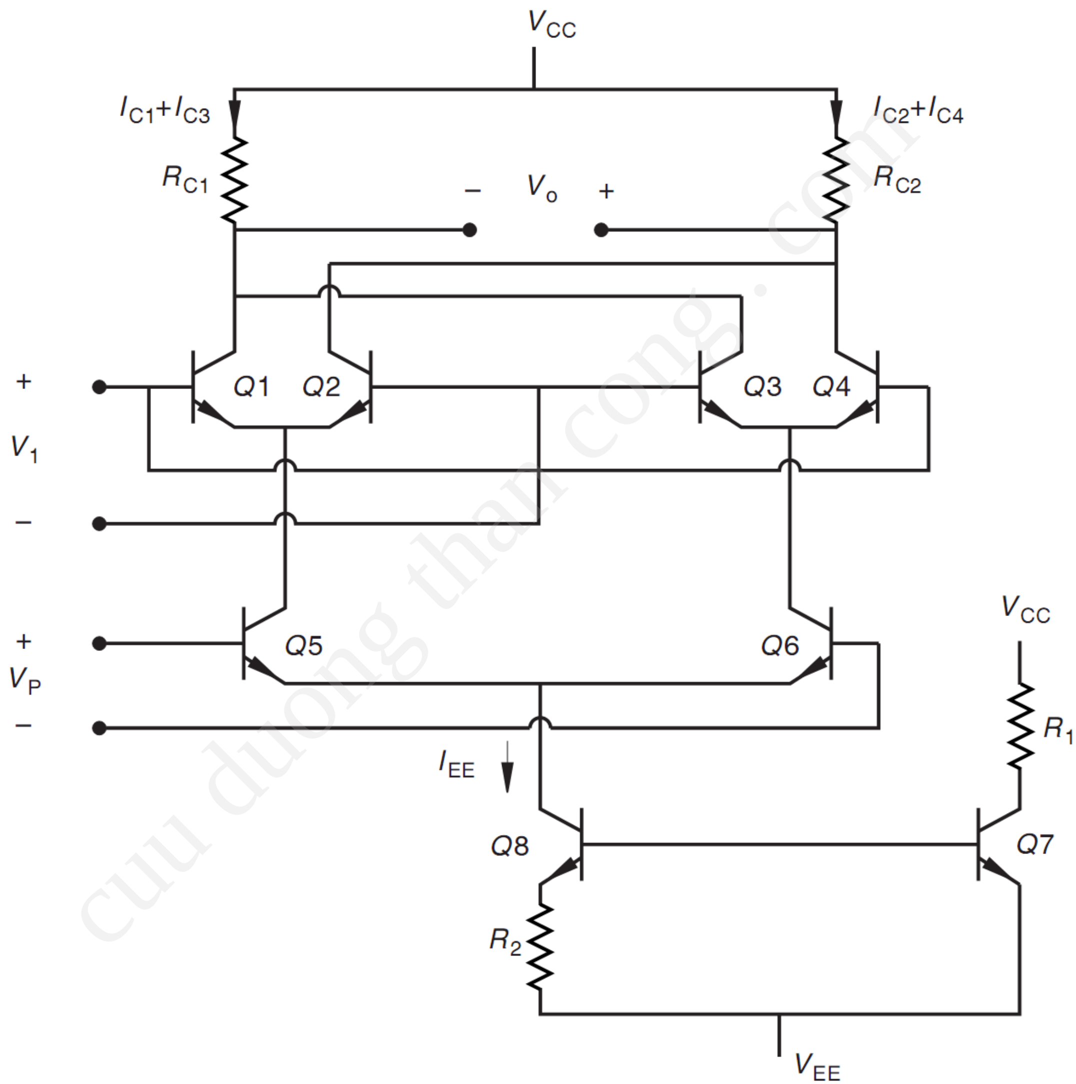
$$I_{C5} = I_{C1} + I_{C2}$$

The ratio of the diode equations with negligible saturation current gives a second relationship:

$$\frac{I_{C1}}{I_{C2}} = \frac{e^{V_{BE1}/V_T}}{e^{V_{BE2}/V_T}} = e^{V_1/V_T}$$

Combining of these two equations gives an expression for I_{C1}.

Double-Balanced Transistor Mixers (3)



Double-Balanced Transistor Mixers (4)

In the same way, the currents for Q2, Q3, and Q4 are found:

$$I_{C1} = \frac{I_{C5}}{1 + e^{-V_1/V_T}}$$

$$I_{C2} = \frac{I_{C5}}{1 + e^{V_1/V_T}}$$

$$I_{C3} = \frac{I_{C6}}{1 + e^{V_1/V_T}}$$

$$I_{C4} = \frac{I_{C6}}{1 + e^{-V_1/V_T}}$$

For Q5and Q6 the collector currents are:

$$I_{C5} = \frac{I_{EE}}{1 + e^{-V_2/V_T}}$$

$$I_{C6} = \frac{I_{EE}}{1 + e^{V_2/V_T}}$$

Double-Balanced Transistor Mixers (5)

The output voltage is proportional to the difference of the currents through the collector resistors:

$$
\begin{aligned}
V_O &= [(I_{C1} + I_{C3}) - (I_{C2} + I_{C4})]R \\
&= [(I_{C1} - I_{C4}) - (I_{C2} - I_{C3})]R \\
&= \frac{R(I_{C5} - I_{C6})}{1 + e^{-V_1/V_T}} - \frac{R(I_{C5} - I_{C6})}{1 + e^{V_1/V_T}} \\
&= \frac{I_{EE}R}{1 + e^{-V_1/V_T}} \left(\frac{1}{1 + e^{-V_2/V_T}} - \frac{1}{1 + e^{V_2/V_T}} \right) \\
&\quad - \frac{I_{EE}R}{1 + e^{V_1/V_T}} \left(\frac{1}{1 + e^{-V_2/V_T}} - \frac{1}{1 + e^{V_2/V_T}} \right) \\
&= \frac{I_{EE}R}{1 + e^{-V_1/V_T}} \left(\frac{e^{V_2/2V_T}}{e^{V_2/2V_T} + e^{-V_2/2V_T}} - \frac{e^{-V_2/V_T}}{e^{-V_2/2V_T} + e^{V_2/2V_T}} \right) \\
&\quad - \frac{I_{EE}R}{1 + e^{V_1/V_T}} \left(\frac{e^{V_2/2V_T}}{e^{V_2/2V_T} + e^{-V_2/2V_T}} - \frac{e^{-V_2/2V_T}}{e^{-V_2/2V_T} + e^{V_2/2V_T}} \right) \\
&= I_{EE}R \tanh\left(\frac{V_2}{2V_T}\right) \tanh\left(\frac{V_1}{2V_T}\right)
\end{aligned}
$$

Double-Balanced Transistor Mixers (6)

Since $\tanh(x) \approx x$ for $x << 1$, the multiplication between the two input voltages V_1 and V_2 will occur as long as $V_i << 2V_T$, where $i = 1, 2$. At the other extreme, when $x >> 1$, $\tanh(x) \approx 1$.