Chapter 7:

Analog Modulators

References

[1] J. J. Carr, *RF Components and Circuits*, Newnes, 2002.

Analog Modulation

- **Amplitude Modulation**:
  - Standard **amplitude modulation** (AM) with carrier.
  - Suppressed-carrier **double-sideband modulation** (DSB).
  - **Single-sideband modulation** (SSB).
  - **Vestige-sideband modulation** (VSB).
- **Frequency Modulation**
- **Phase Modulation**

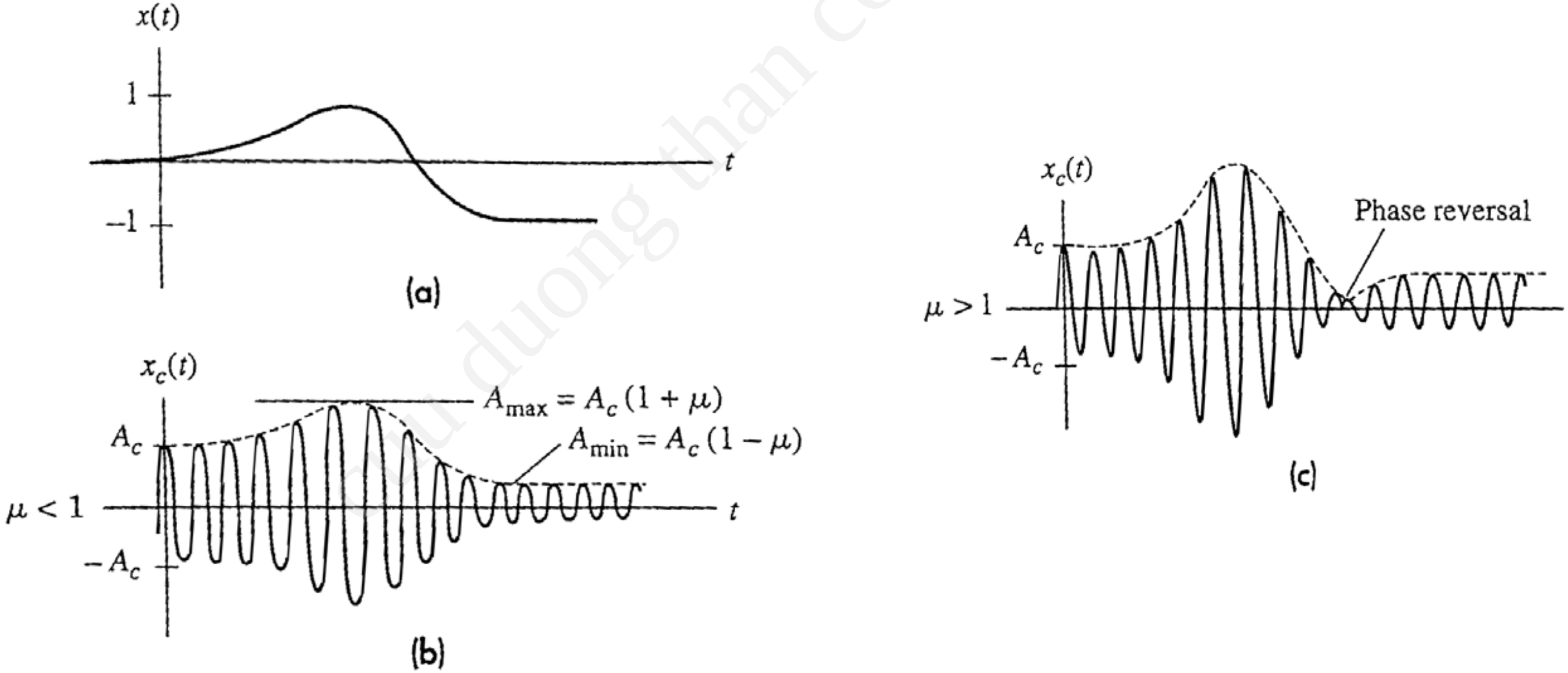
Standard AM (1)

- **AM signal and spectra**:

  The **AM signal** is: $x_c(t) = A_c[1 + \mu x(t)] \cos \omega_c t$

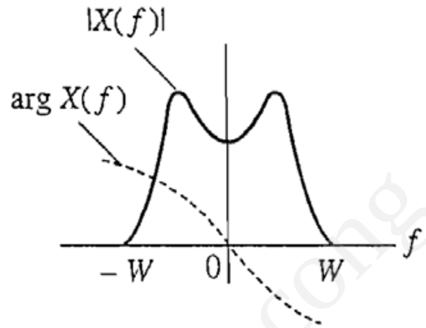
  $$= A_c \cos \omega_c t + A_c \mu x(t) \cos \omega_c t$$

  where μ is positive constant called AM modulation index, $x(t)$ is message, $A_c \cos \omega_c t$ is carrier signal.

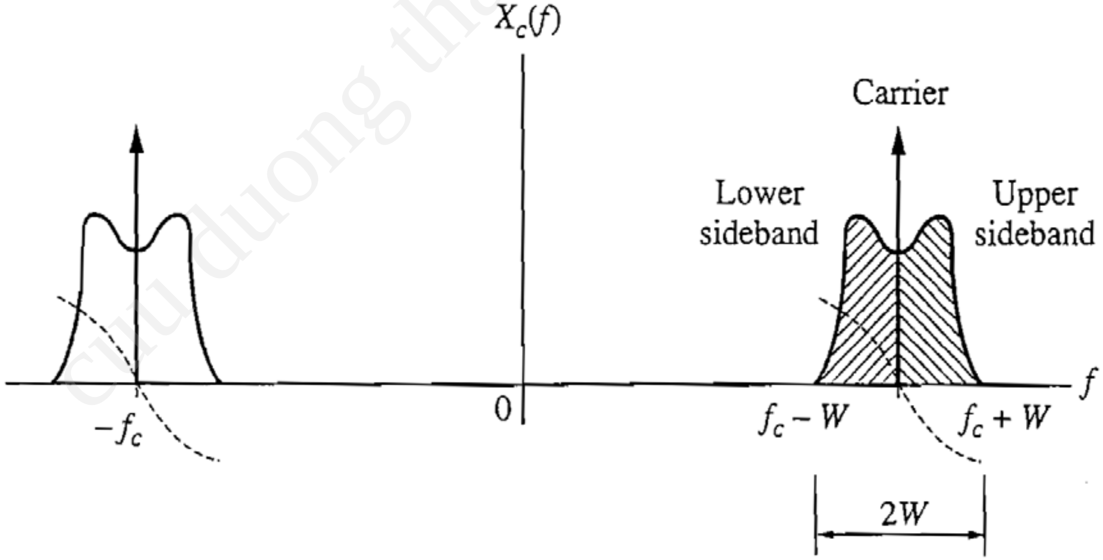


Standard AM (2)

Message spectrum with bandwidth *W*:



AM spectrum: AM bandwidth is 2*W*



Standard AM (3)

AM transmitted power:

$$S_T = P_c + 2P_{sb}$$

where P_c represents unmodulated carrier power: $P_c = \frac{1}{2}A_c^2$

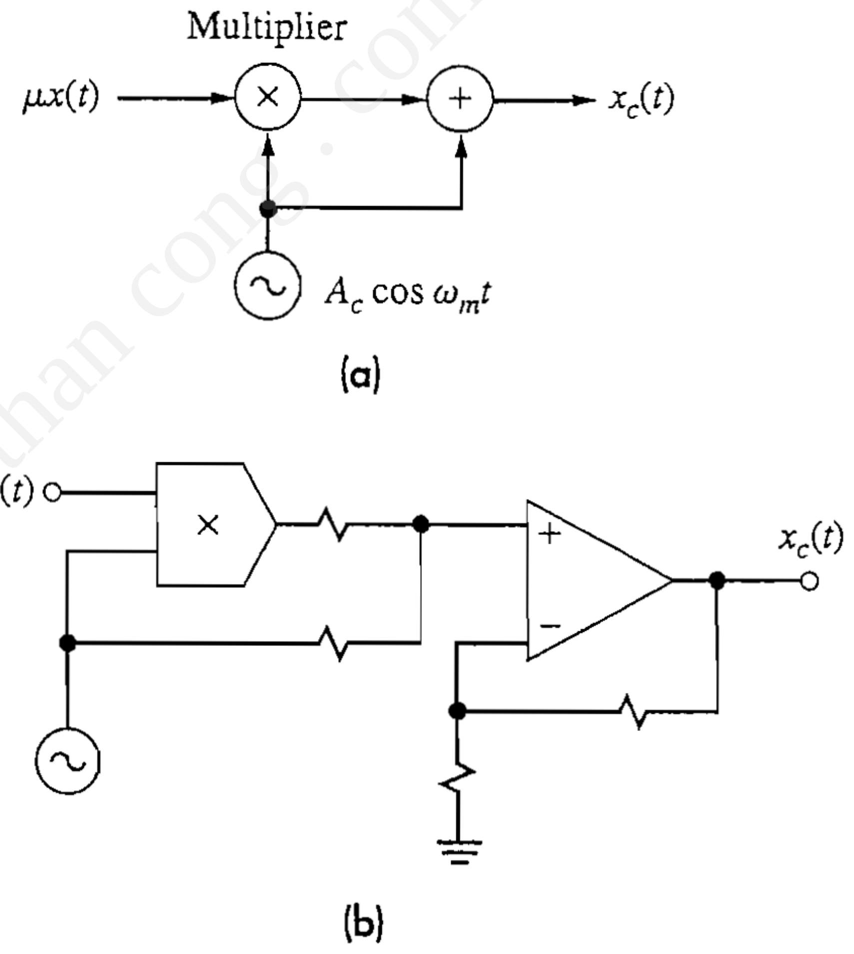
and P_{sb} represents power per sideband: $P_{sb} = \frac{1}{4}A_c^2\mu^2 S_x = \frac{1}{2}\mu^2 S_x P_c$

where S_x is average message power.

If f_c >>W and $\mu \leq 1$, then the message can be extracted from $x_c(t)$ by a simple envelope detector.

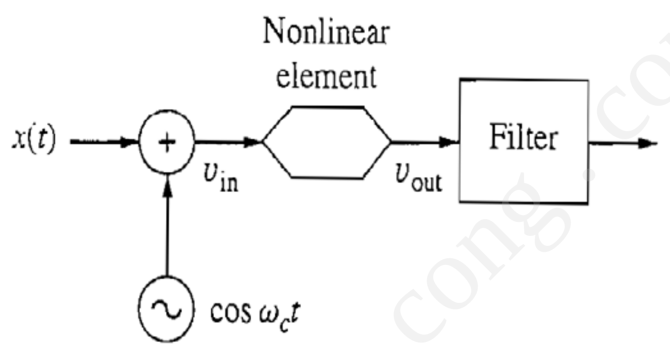
❑ **Product modulator for AM:**

- Using Multiplier:

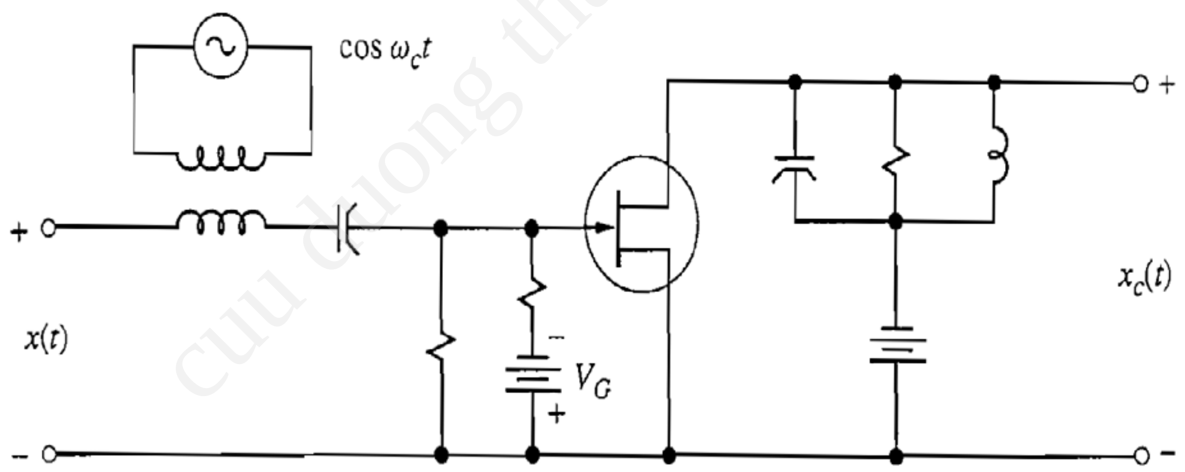


Standard AM (5)

- Using square-law element:



(a)



(b)

Standard AM (6)

We assumed that the nonlinear element approximates the square law transfer curve:

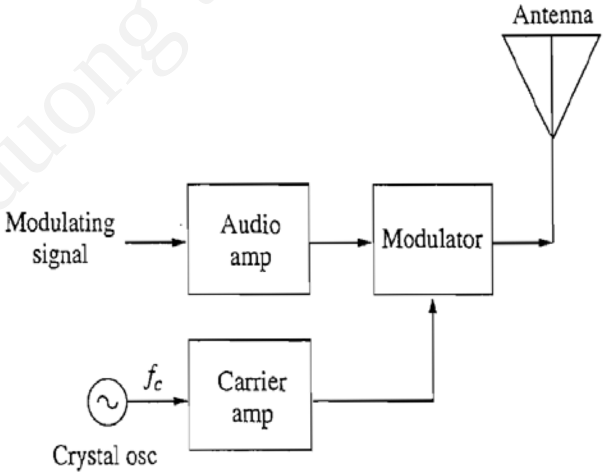
$$v_{out} = a_1 v_{in} + a_2 v_{in}^2$$

Thus, with

$$v_{in}(t) = x(t) + \cos \omega_c t$$

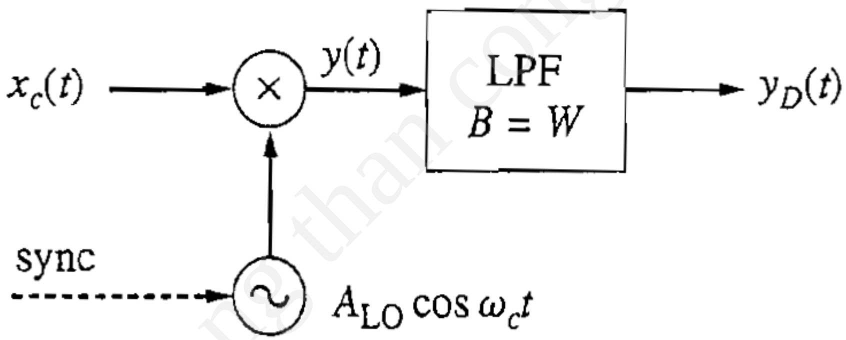
$$v_{out}(t) = a_1 x(t) + a_2 x^2(t) + a_2 \cos^2 \omega_c t + a_1 \left[1 + \frac{2a_2}{a_1} x(t)\right] \cos \omega_c t$$

- High-level modulation using class C amplifier:



Standard AM (7)

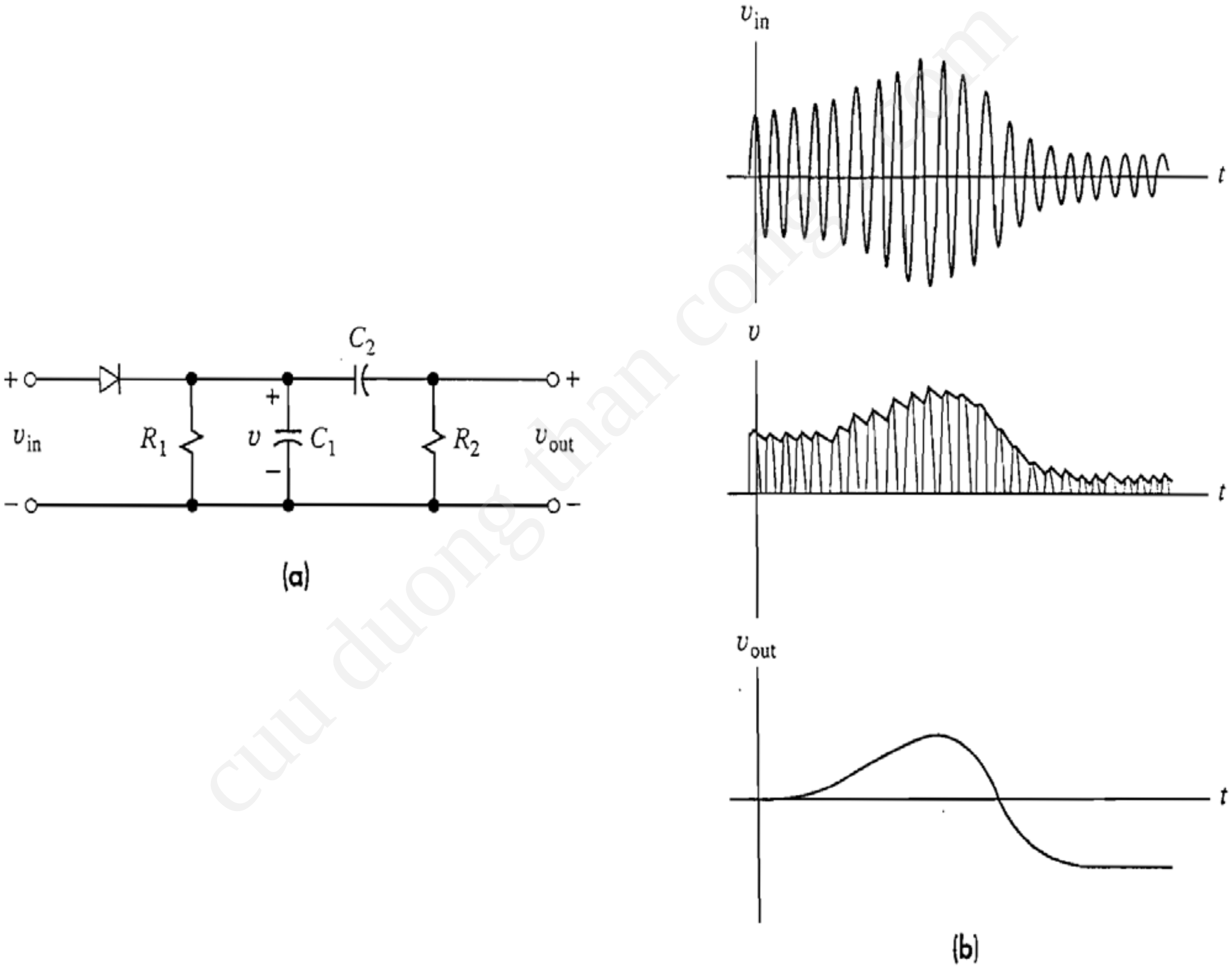
- **AM demodulation**:
  - Synchronous detection (coherent detection): The oscillator of demodulator is exactly synchronized (both phase and frequency) with the carrier.



Using PLL for carrier synchronization.

Standard AM (8)

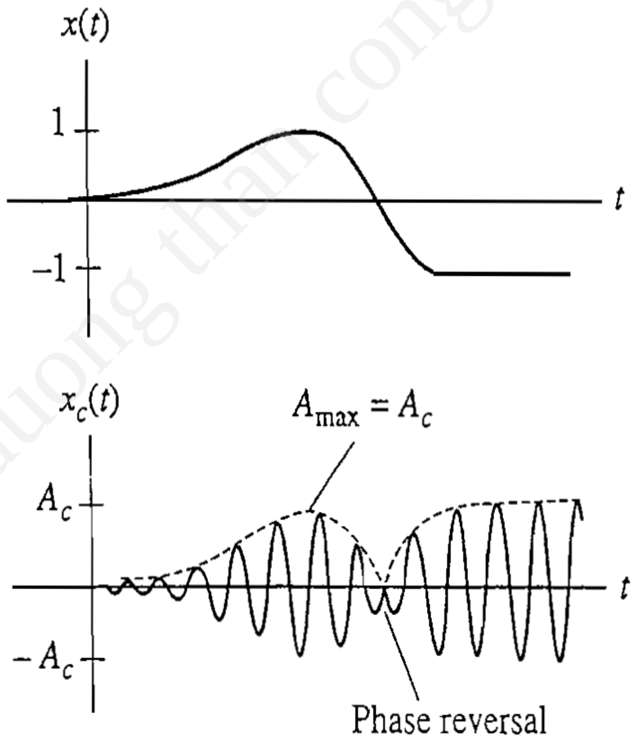
- <u>Envelop detection</u>:



(Suppressed-Carrier) DSB (1)

- **<u>DSB signal and spectra</u>**: The wasted carrier power in AM can be eliminated by setting $\mu = 1$ and suppressing the carrier-frequency component. The resulting DSB signal is:

$$x_c(t) = A_c x(t) \cos \omega_c t$$

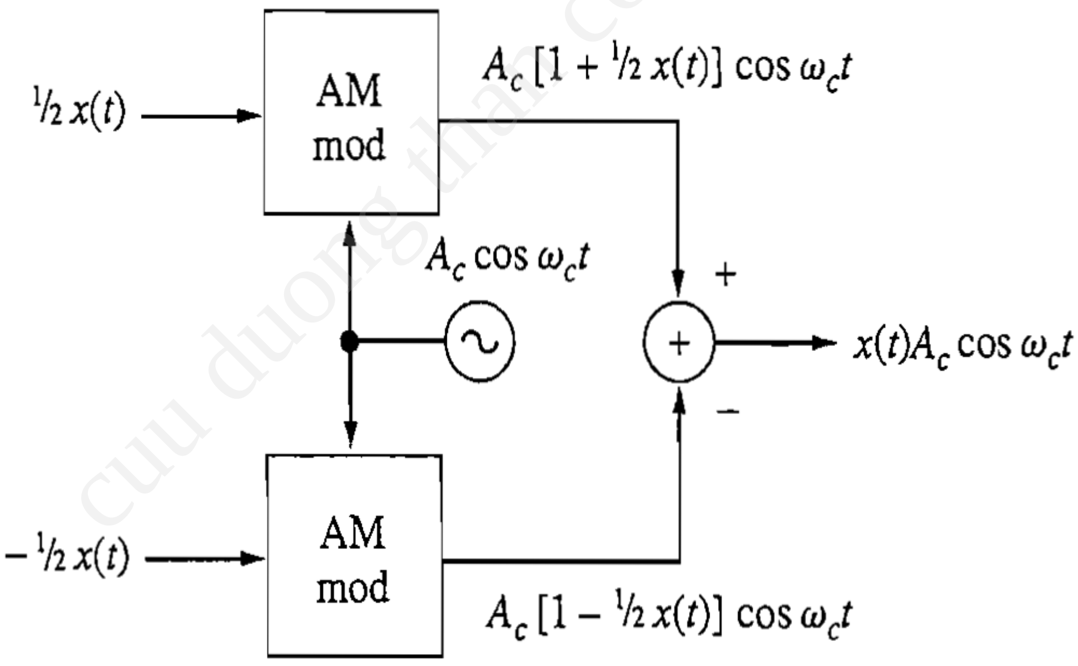


(Suppressed-Carrier) DSB (2)

DSB transmitted power:

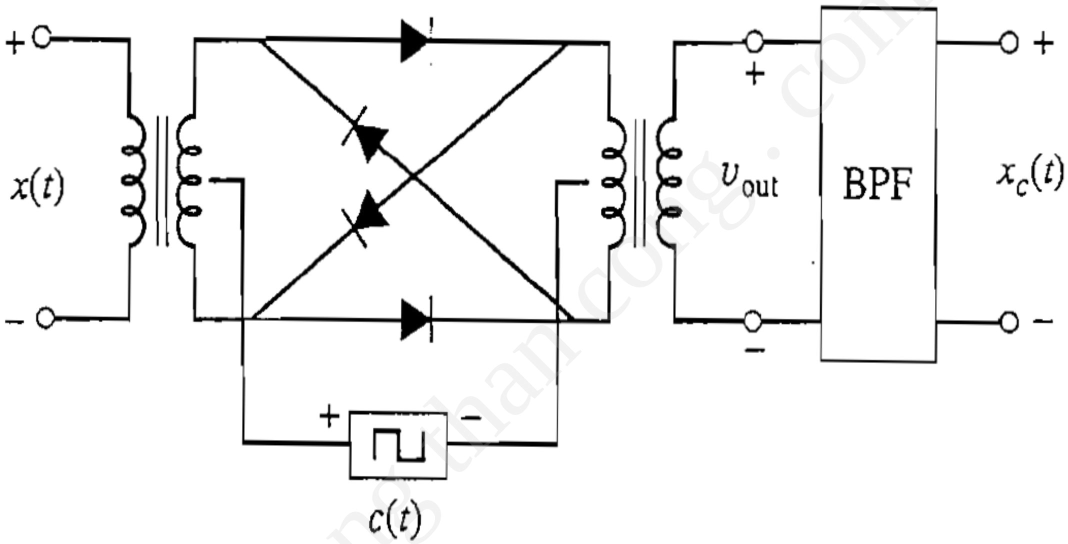
$$S_T = 2P_{sb} = \frac{1}{2}A_c^2 S_x$$

- **Product DSB modulator**:
  - Balanced modulator:



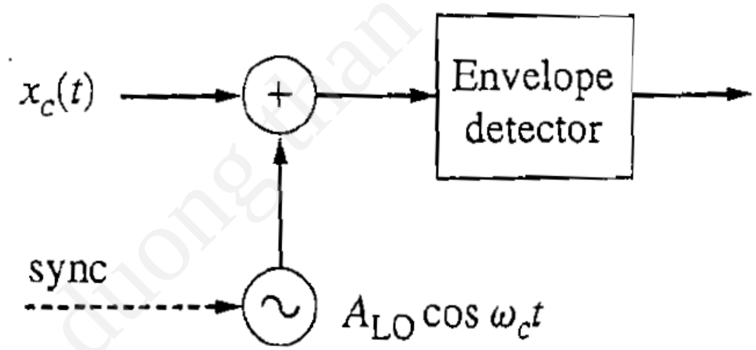
(Suppressed-Carrier) DSB (3)

- Ring modulator:



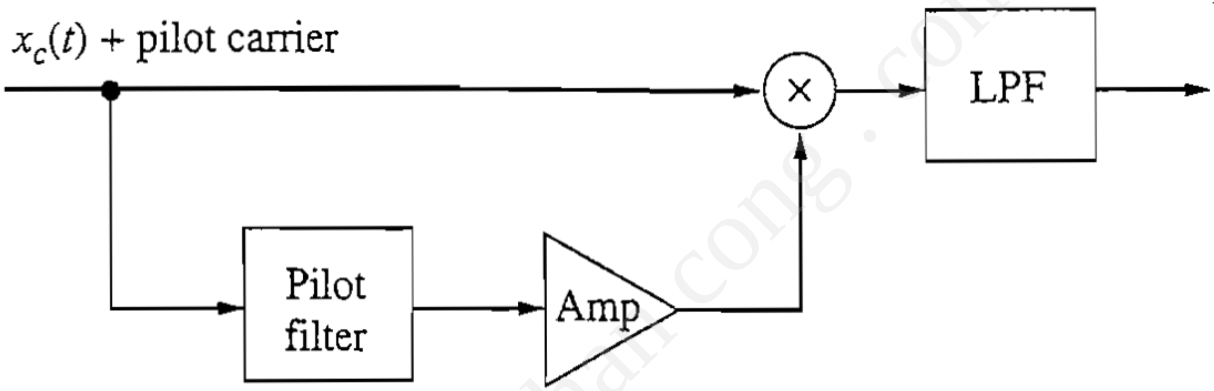
(Suppressed-Carrier) DSB (4)

- **<u>DSB demodulator</u>**:
  - <u>Synchronous detection</u> (coherent detection): As AM demodulator. Required carrier synchronization.
  - <u>Envelope reconstruction</u> (for suppressed carrier modulation): Required <u>carrier synchronization</u>.



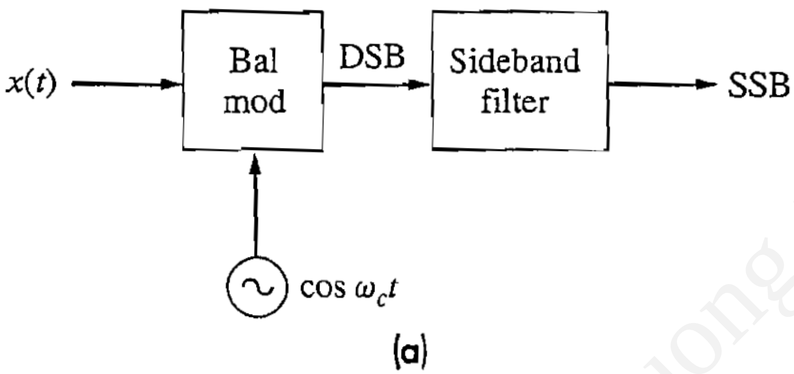
(Suppressed-Carrier) DSB (5)

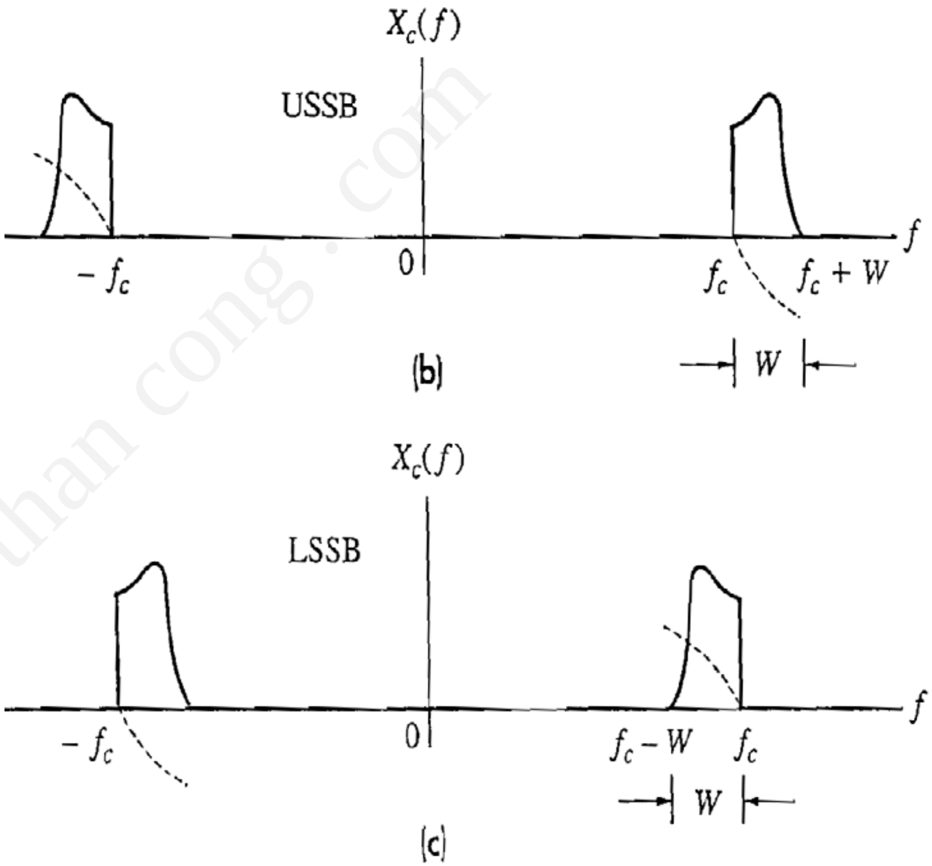
- Using pilot carrier:



(Suppressed-Carrier) SSB (1)

- **SSB signals and spectra:**

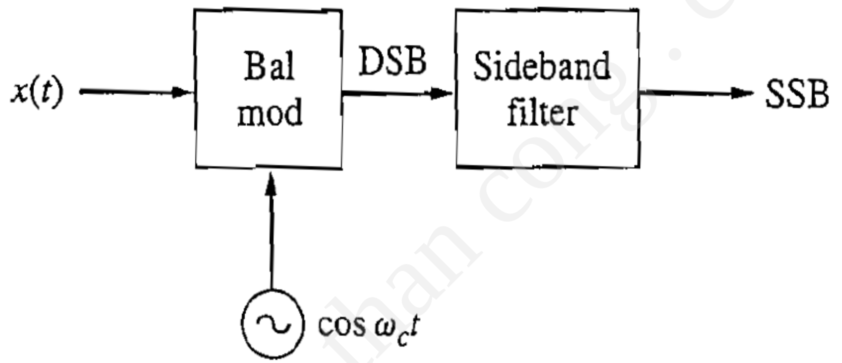




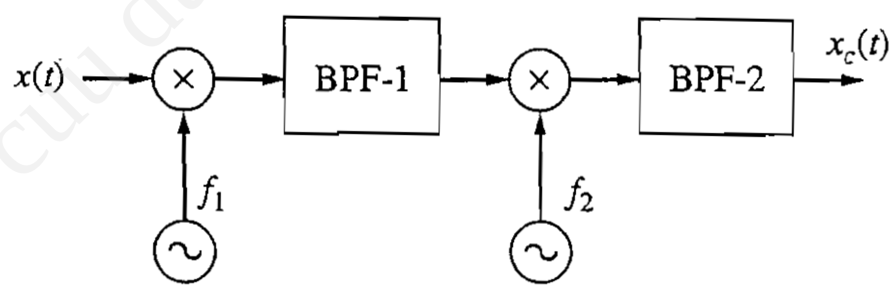
SSB bandwidth and power: $B_T = W \qquad S_T = P_{sb} = \frac{1}{4}A_c^2 S_x$

(Suppressed-Carrier) SSB (2)

- **SSB modulator**:
  - Based on DSB and filter:

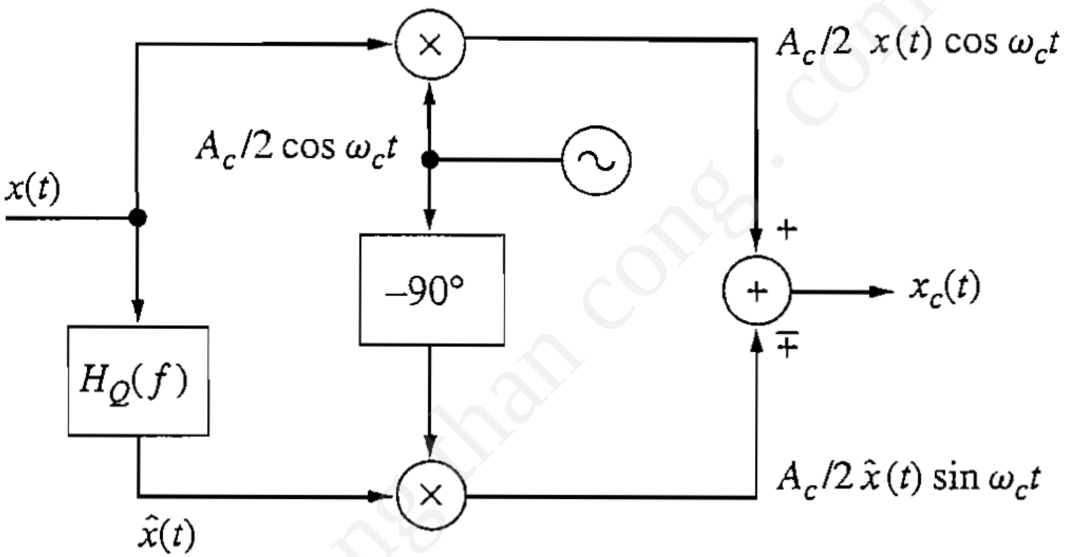


Two-step SSB modulator:



(Suppressed-Carrier) SSB (3)

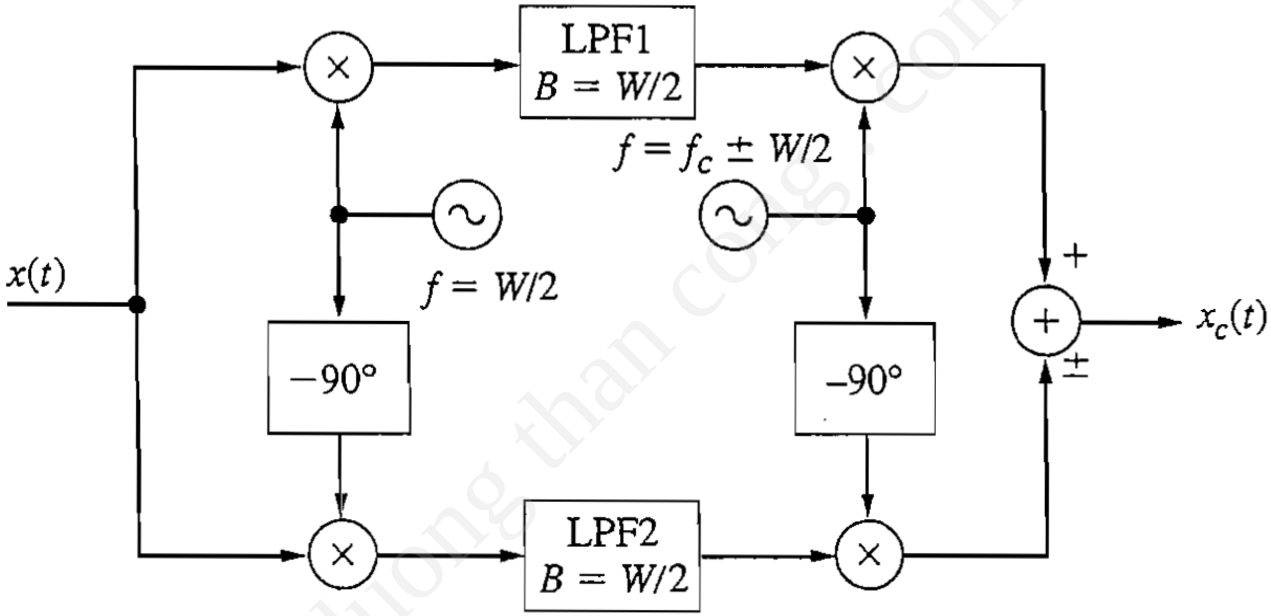
- Phase-shift method:



$H_Q(f)$: 90° phase shifter

(Suppressed-Carrier) SSB (4)

- Weaver method:

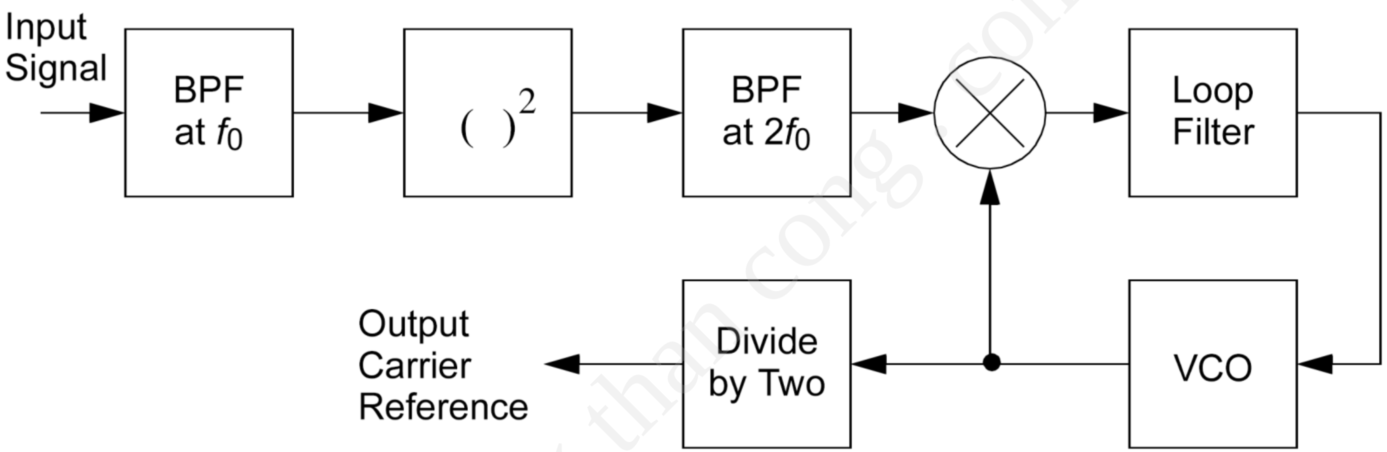


(Suppressed-Carrier) SSB (5)

- **<u>SSB demodulator</u>**: Similar to DSB demodulator.

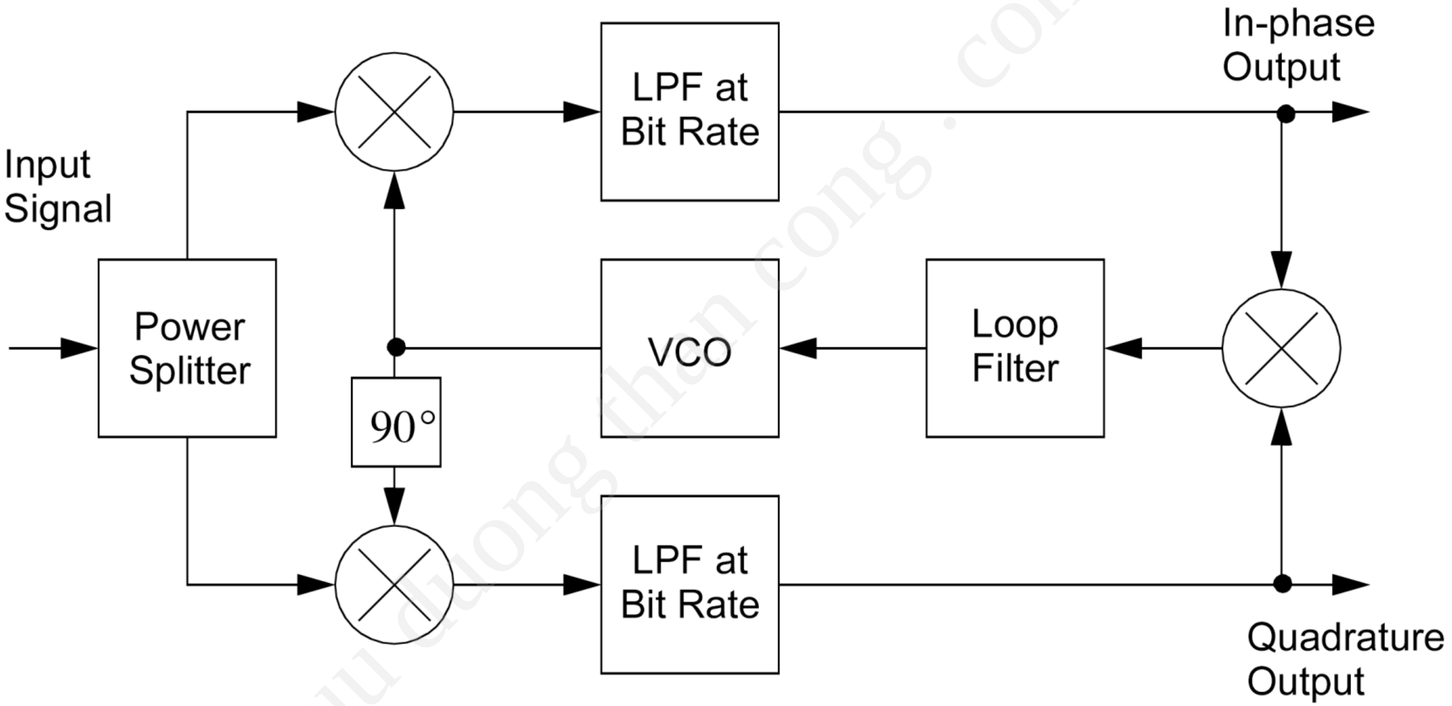
Carrier Synchronization for Coherent Detection (1)

- **Squaring loop:**



Carrier Synchronization for Coherent Detection (2)

❑ **Costas loop:**



PM/ FM (1)

- **PM and FM signals:** Considering a signal with time-varying phase:

$$x_c(t) = A_c \cos\left[\omega_c t + \phi(t)\right]$$

The **total instantaneous angle**:

$$\theta_c(t) \triangleq \omega_c t + \phi(t)$$

Therefore:

$$x_c(t) = A_c \cos\theta_c(t) = A_c \operatorname{Re}\left[e^{j\theta_c(t)}\right]$$

If $\theta_c(t)$ contains the message information $x(t)$, then it is **exponential modulation** (or **angle modulation**).

The phase modulation (PM) is defined by

$$\phi(t) \triangleq \phi_\Delta x(t) \qquad \phi_\Delta \leq 180^\circ$$

so that

$$x_c(t) = A_c \cos\left[\omega_c t + \phi_\Delta x(t)\right]$$

PM/ FM (2)

The constant ϕ_Δ ($\phi_\Delta \leq 180^o$) represents the maximum phase shift (or PM index, or phase deviation) produced by $x(t)$.

In the case of **frequency modulation** (**FM**), the instantaneous frequency is defined as:

$$f(t) \triangleq f_c + f_\Delta x(t) \qquad f_\Delta < f_c$$

The constant frequency f_Δ called frequency deviation, represents the maximum shift of $f(t)$ relative to the carrier frequency f_c

By definition:

$$f(t) \triangleq \frac{1}{2\pi}\dot{\theta}_c(t) = f_c + \frac{1}{2\pi}\dot{\phi}(t)$$

where $\dot{\phi}(t) = d\phi(t)/dt$

Then, the FM signal has $\dot{\phi}(t) = 2\pi f_\Delta x(t)$ and integration yields the PM:

PM/ FM (3)

$$\phi(t) = 2\pi f_\Delta \int_{t_0}^{t} x(\lambda)\, d\lambda + \phi(t_0) \qquad t \geq t_0$$

If t_0 is selected such that $\phi(t_0) = 0$, then

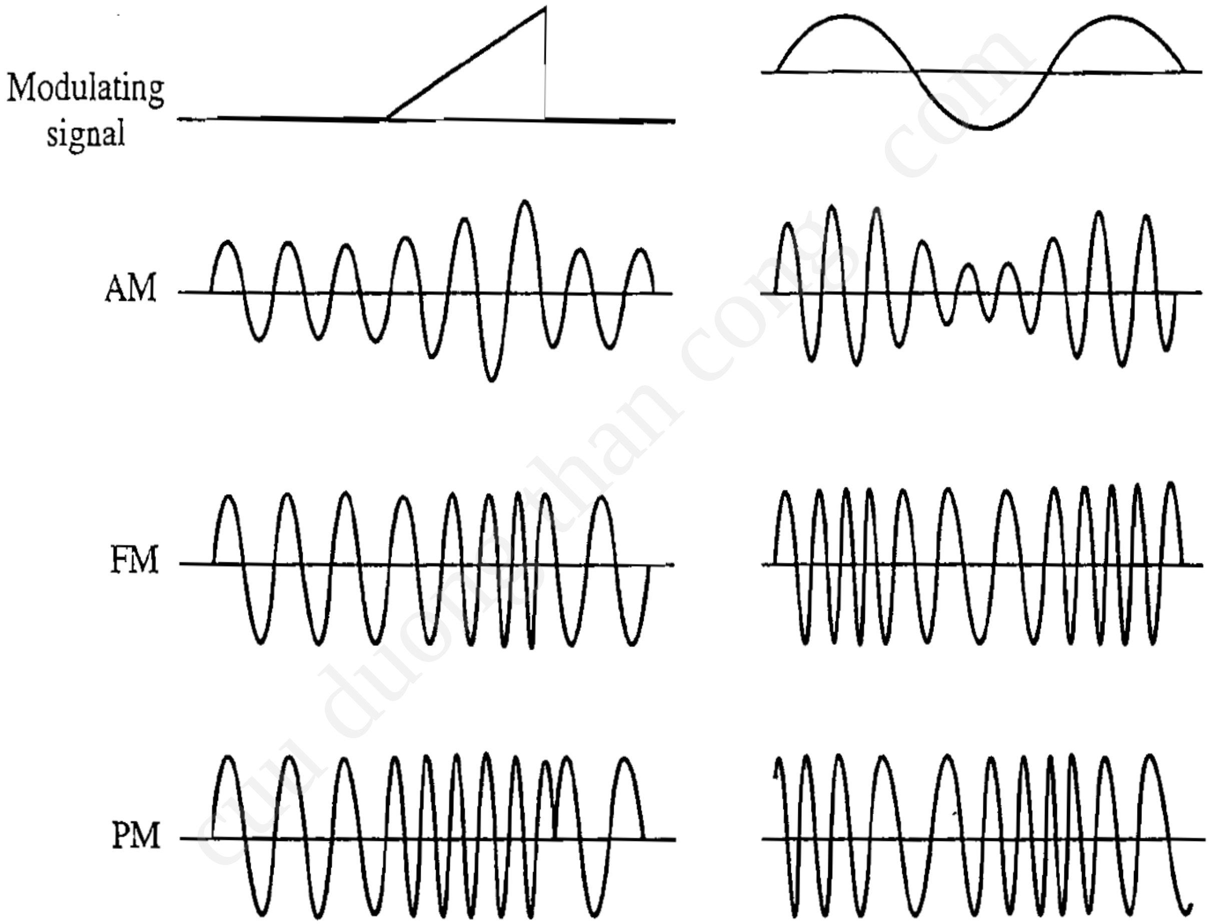
$$\phi(t) = 2\pi f_\Delta \int^{t} x(\lambda)\, d\lambda$$

Therefore, The FM signal is written as:

$$x_c(t) = A_c \cos\left[\omega_c t + 2\pi f_\Delta \int^{t} x(\lambda)\, d\lambda\right]$$

	Instantaneous phase $\phi(t)$	Instantaneous frequency $f(t)$
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi}\phi_\Delta \dot{x}(t)$
FM	$2\pi f_\Delta \int^{t} x(\lambda)\, d\lambda$	$f_c + f_\Delta x(t)$

PM/ FM (4)



PM/ FM (5)

- **Average transmitted power (for PM and FM):** <u>regardless</u> of message $x(t)$:

$$S_T = \frac{1}{2}A_c^2$$

- **Narrowband PM and FM:** Representing the exponential modulation as:

$$x_c(t) = x_{ci}(t)\cos\omega_c t - x_{cq}(t)\sin\omega_c t$$

where

$$x_{ci}(t) = A_c\cos\phi(t) = A_c\left[1 - \frac{1}{2!}\phi^2(t) + \cdots\right]$$

$$x_{cq}(t) = A_c\sin\phi(t) = A_c\left[\phi(t) - \frac{1}{3!}\phi^3(t) + \cdots\right]$$

If $|\phi(t)| \ll 1$ rad

so that

$$x_{ci}(t) \approx A_c \qquad x_{cq}(t) \approx A_c\phi(t)$$

PM/ FM (6)

Then the spectrum of $x_c(t)$ in term of the message spectrum is:

$$X_c(f) = \frac{1}{2}A_c\delta(f - f_c) + \frac{j}{2}A_c\Phi(f - f_c) \qquad f > 0$$

where

$$\Phi(f) = \mathscr{F}[\phi(t)] = \begin{cases} \phi_\Delta X(f) & \text{PM} \\ -jf_\Delta X(f)/f & \text{FM} \end{cases}$$

Therefore, we conclude that if the message has bandwidth of $W << f_c$, then the <u>bandwidth of the signal after narrowband modulation</u> is $2W$.

❑ **<u>Tone modulation</u>** (single frequency modulation): Consider the tone message as:

$$x(t) = \begin{cases} A_m \sin \omega_m t & \text{PM} \\ A_m \cos \omega_m t & \text{FM} \end{cases}$$

Then $\phi(t) = \beta \sin \omega_m t$, where $\beta \triangleq \begin{cases} \phi_\Delta A_m & \text{PM} \\ (A_m/f_m)f_\Delta & \text{FM} \end{cases}$

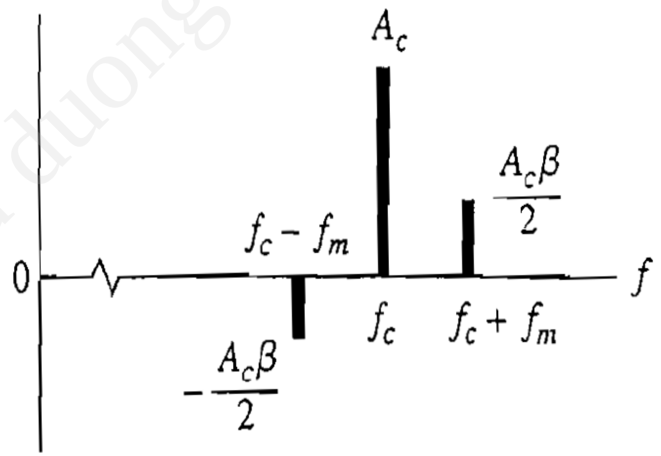
PM/ FM (7)

The parameter β is the modulation index for PM and FM in the case of tone modulation.

- **Narrowband PM/FM tone modulation** requires $\beta << 1$, then after modulation we obtain:

$$x_c(t) \approx A_c \cos \omega_c t - A_c \beta \sin \omega_m t \sin \omega_c t$$

$$\approx A_c \cos \omega_c t - \frac{A_c \beta}{2} \cos (\omega_c - \omega_m)t + \frac{A_c \beta}{2} \cos (\omega_c + \omega_m)t$$



PM/ FM (8)

- For arbitrary β (**wideband PM/FM tone modulation**):

$$x_c(t) = A_c[\cos\phi(t)\cos\omega_c t - \sin\phi(t)\sin\omega_c t]$$

$$= A_c[\cos(\beta\sin\omega_m t)\cos\omega_c t - \sin(\beta\sin\omega_m t)\sin\omega_c t]$$

using

$$\cos(\beta\sin\omega_m t) = J_0(\beta) + \sum_{n\ \text{even}}^{\infty} 2J_n(\beta)\cos n\omega_m t$$

$$\sin(\beta\sin\omega_m t) = \sum_{n\ \text{odd}}^{\infty} 2J_n(\beta)\sin n\omega_m t$$

$$J_n(\beta) \triangleq \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin\lambda - n\lambda)}\,d\lambda$$

The coefficient $J_n(\beta)$ are Bessel function of first kind.

PM/ FM (9)

Finally, we obtain:

$$x_c(t) = A_c J_0(\beta) \cos \omega_c t$$

$$+ \sum_{n \text{ odd}}^{\infty} A_c J_n(\beta)[\cos(\omega_c + n\omega_m)t - \cos(\omega_c - n\omega_m)t]$$

$$+ \sum_{n \text{ even}}^{\infty} A_c J_n(\beta)[\cos(\omega_c + n\omega_m)t + \cos(\omega_c - n\omega_m)t]$$

Using the property: $J_{-n}(\beta) = (-1)^n J_n(\beta)$

Then, we obtain the compact form:

$$x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t$$

PM/ FM (10)

Selected values of $J_n(\beta)$

n	$J_n(0.1)$	$J_n(0.2)$	$J_n(0.5)$	$J_n(1.0)$	$J_n(2.0)$	$J_n(5.0)$	$J_n(10)$	n
0	1.00	0.99	0.94	0.77	0.22	−0.18	−0.25	0
1	0.05	0.10	0.24	0.44	0.58	−0.33	0.04	1
2			0.03	0.11	0.35	0.05	0.25	2
3				0.02	0.13	0.36	0.06	3
4					0.03	0.39	−0.22	4
5						0.26	−0.23	5
6						0.13	−0.01	6
7						0.05	0.22	7
8						0.02	0.32	8
9							0.29	9
10							0.21	10
11							0.12	11
12							0.06	12
13							0.03	13
14							0.01	14

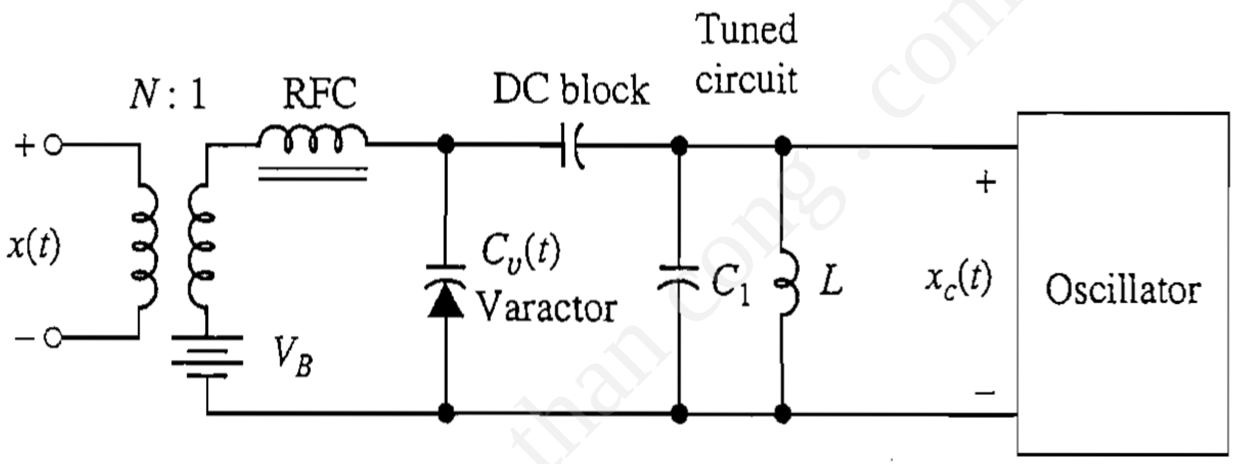
PM/ FM (11)

- **Bandwidth of PM/FM tone modulation**: According to Carson's rule:

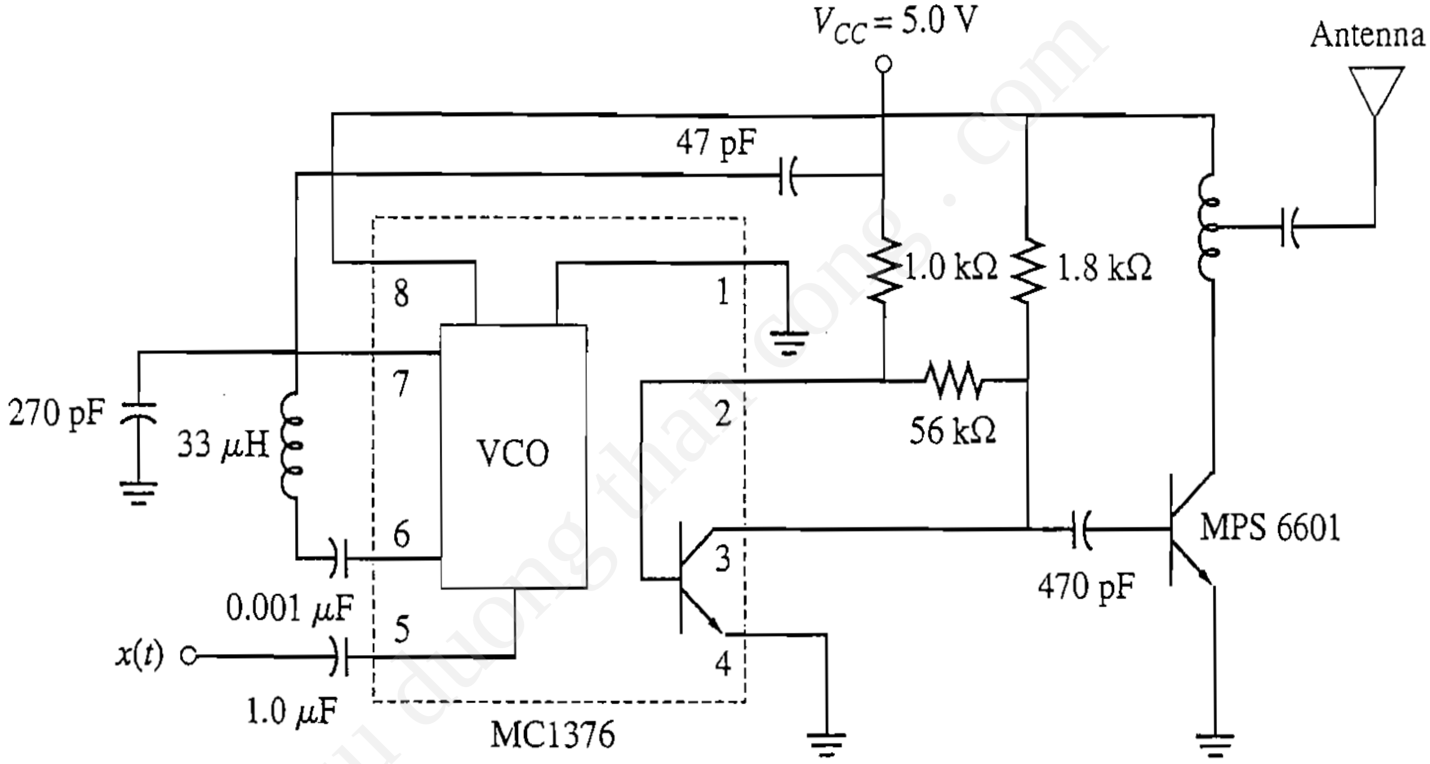
$$B_T \approx 2(f_\Delta + W) = 2(D+1)W$$

where $D \triangleq \dfrac{f_\Delta}{W}$

- **<u>Direct FM and VCO</u>:**



PM/ FM (13)

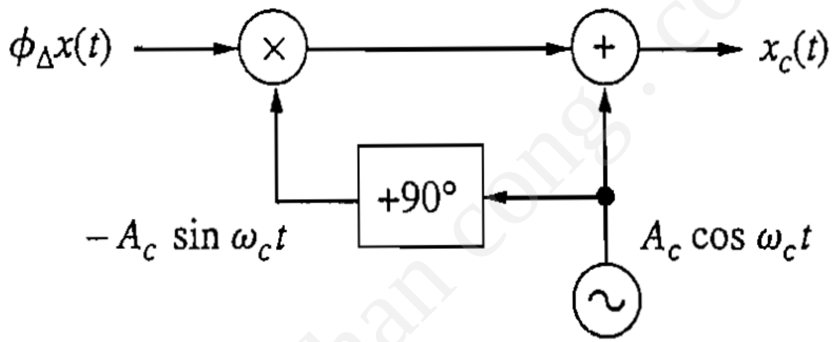


V_{CC} = 5.0 V
Antenna
47 pF
1.0 kΩ
1.8 kΩ
8
1
7
270 pF
33 μH
2
56 kΩ
VCO
6
3
MPS 6601
0.001 μF
470 pF
5
x(t)
4
1.0 μF
MC1376

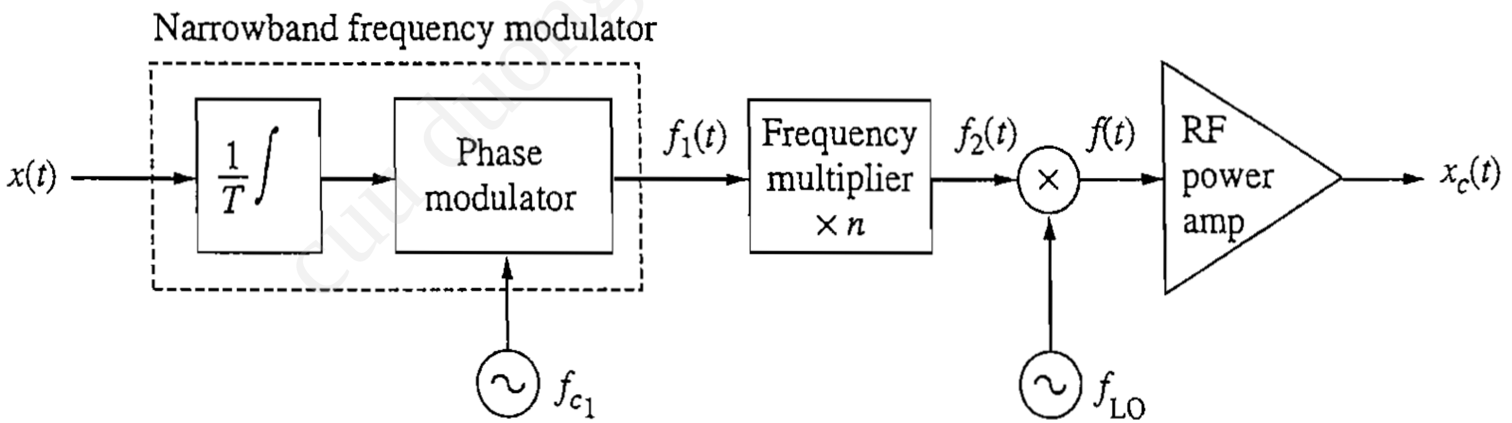
PM/ FM (14)

❑ Phase modulators and indirect FM:

- **Narrowband phase modulator:**

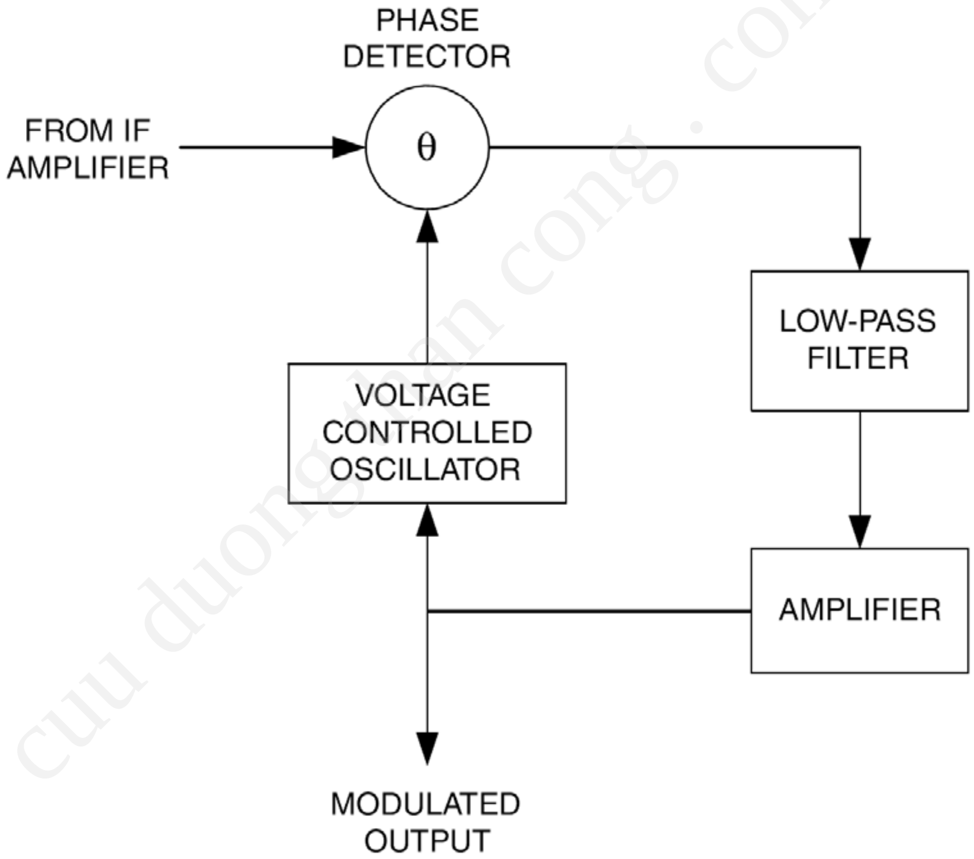


- **Indirect FM transmitter:**

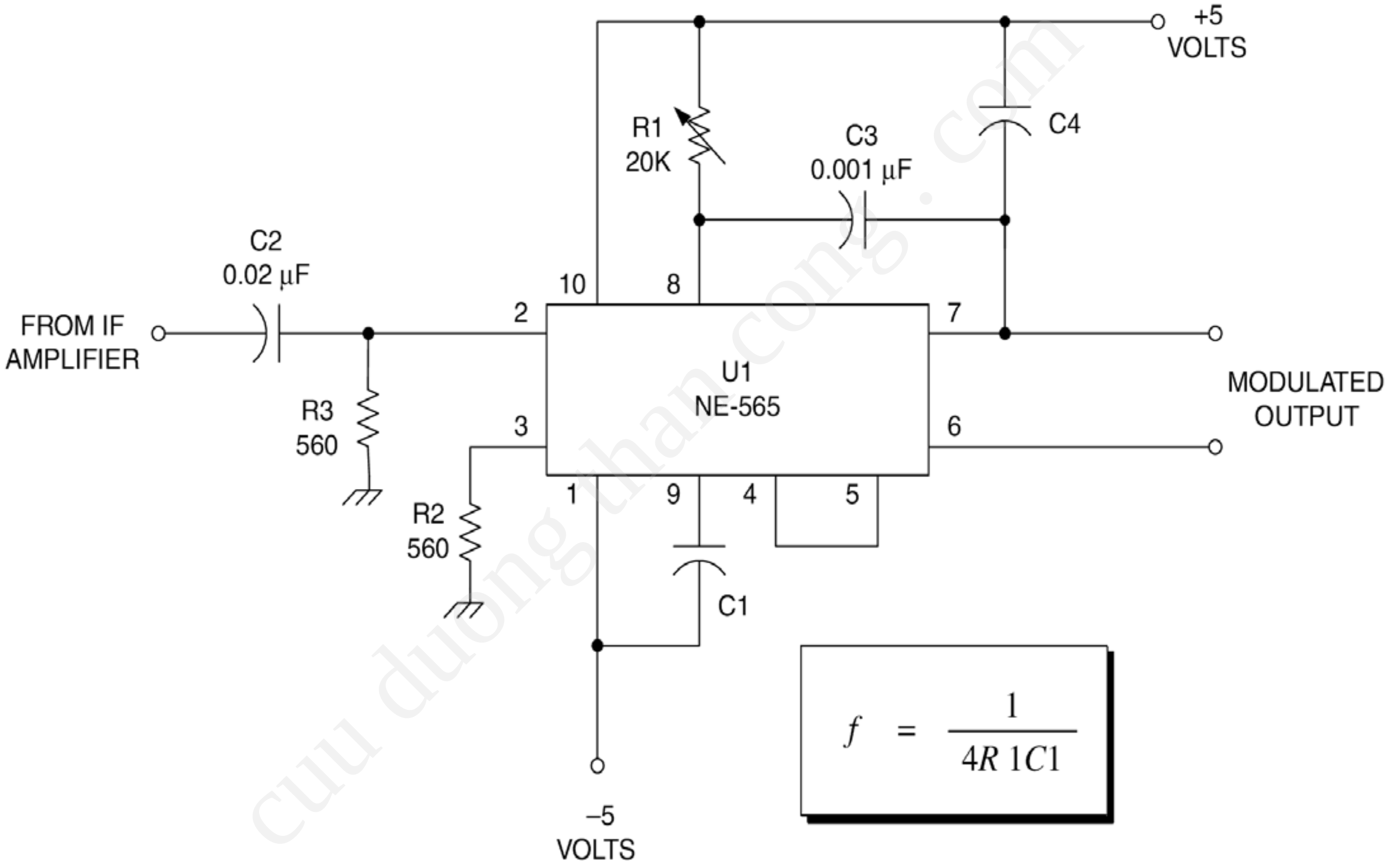


PM/ FM (15)

- **FM/PM detector (demodulator) using PLL:**



PM/ FM (16)



$$f = \frac{1}{4R1C1}$$