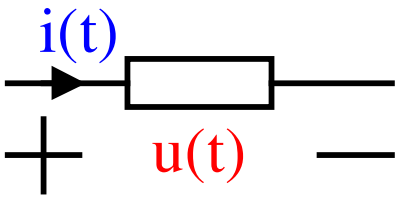
2.8 Công suất

- Xét một đoạn mạch mà dòng và áp tại xác lập điều hòa là



$$i(t) = I_m \cos(\omega t + \varphi_i)$$

$$u(t) = U_m \cos(\omega t + \varphi_u)$$

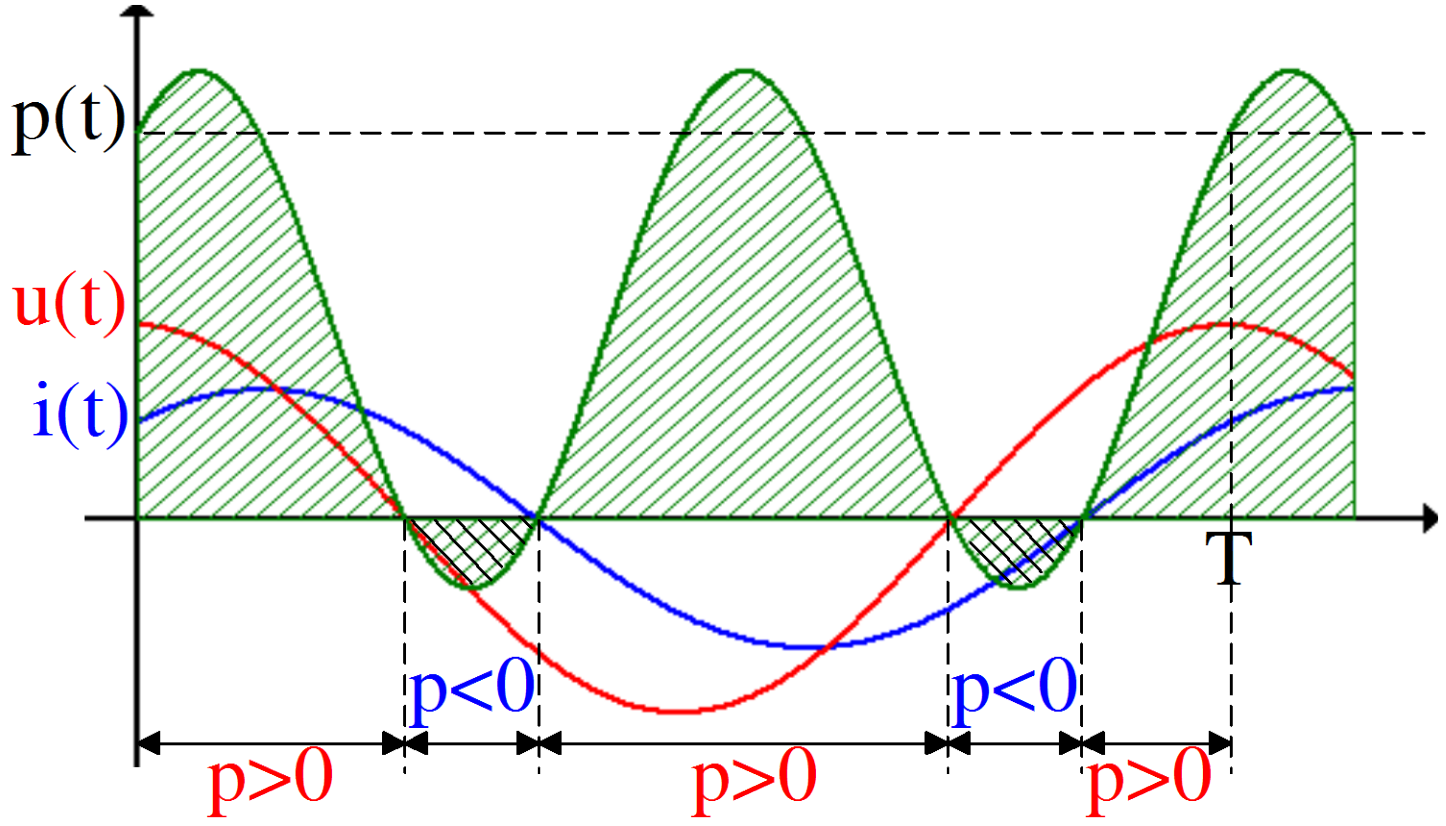
- Công suất tức thời

$$p(t) = u(t)i(t) = \frac{1}{2}U_m I_m \cos(\varphi_u - \varphi_i) + \frac{1}{2}U_m I_m \cos(2\omega t + \varphi_u + \varphi_i)$$

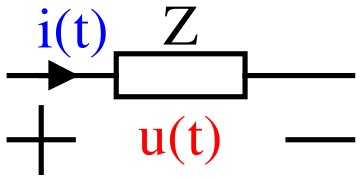
- p(t) > 0 : mạch đang nhận công suất
- p(t) < 0 : mạch đang phát công suất

2.8 Công suất

$$p(t) = \frac{1}{2} U_m I_m \cos(\varphi_u - \varphi_i) + \frac{1}{2} U_m I_m \cos(2\omega t + \varphi_u + \varphi_i)$$



Công suất tác dụng & công suất phản kháng



$$i(t) = I_m \cos(\omega t + \varphi_i)$$

$$u(t) = U_m \cos(\omega t + \varphi_u)$$

$$\varphi = \varphi_u - \varphi_i ; \quad Z = |Z| \angle \varphi$$

- P (Active Power) [W]

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)dt = \frac{1}{2}U_m I_m \cos\varphi \ [W]$$

$$P = UI\cos\varphi = \frac{1}{2}\mathrm{Re}\left\{\dot{U}_m \overset{*}{I}_m\right\}$$

$$P = \frac{1}{2} I_m^2 \,\mathrm{Re}\{Z\}$$

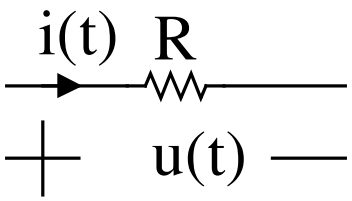
Công suất tác dụng & công suất phản kháng

- P (Active Power) [W]
- Q (Reactive Power) [VAr]

P (Active Power) [W]	Q (Reactive Power) [VAr]
$P = \frac{1}{2} U_m I_m \cos\varphi$	$Q = \frac{1}{2} U_m I_m \sin\varphi$
$P = UI \cos\varphi$	$Q = UI \sin\varphi$
$P = \frac{1}{2} \mathrm{Re}\left\{ \dot{U}_m \, I_m^* \right\}$	$Q = \frac{1}{2} \mathrm{Im}\left\{ \dot{U}_m \, I_m^* \right\}$
$P = \frac{1}{2} I_m^2 \, \mathrm{Re}\{Z\}$	$Q = \frac{1}{2} I_m^2 \, \mathrm{Im}\{Z\}$

Công suất trên các phần tử mạch

- Điện trở



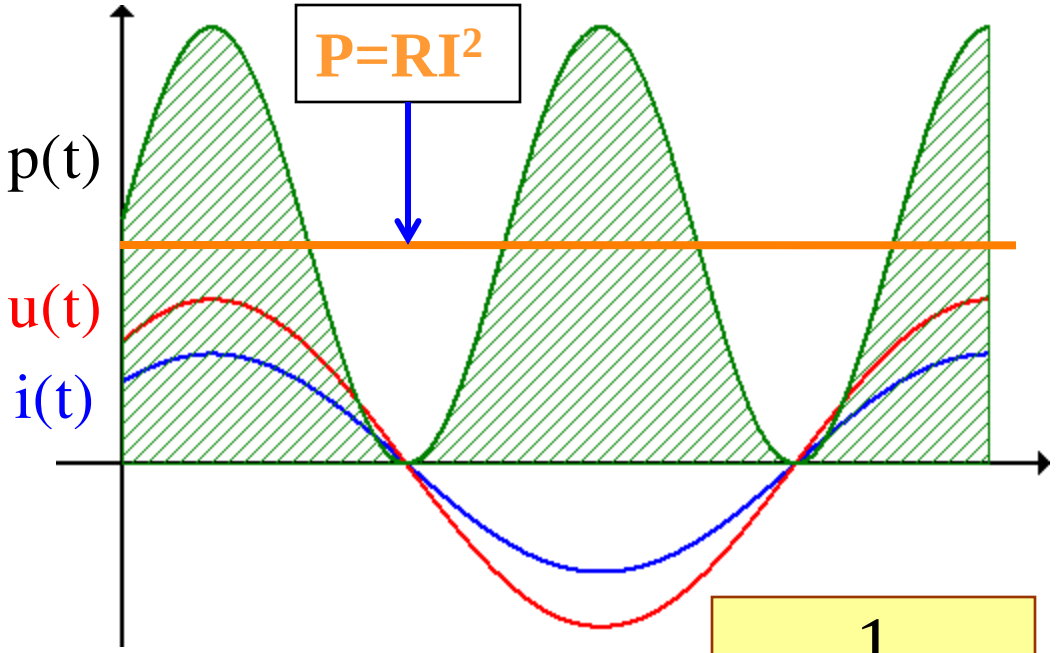
$$i = I_m \cos(\omega t + \psi)$$

$$u = Ri$$

$$p(t) = u(t)i(t) = Ri^2$$

$$p(t) = RI_m^2 \cos^2(\omega t + \psi)$$

$$p(t) = \frac{1}{2} RI_m^2 \left[1 + \cos(2\omega t + 2\psi)\right]$$



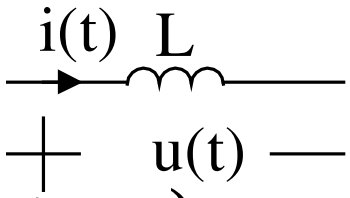
$$P = \frac{1}{2} RI_m^2$$

$$P = RI^2$$

$$Q = 0$$

Công suất trên các phần tử mạch

- Điện cảm



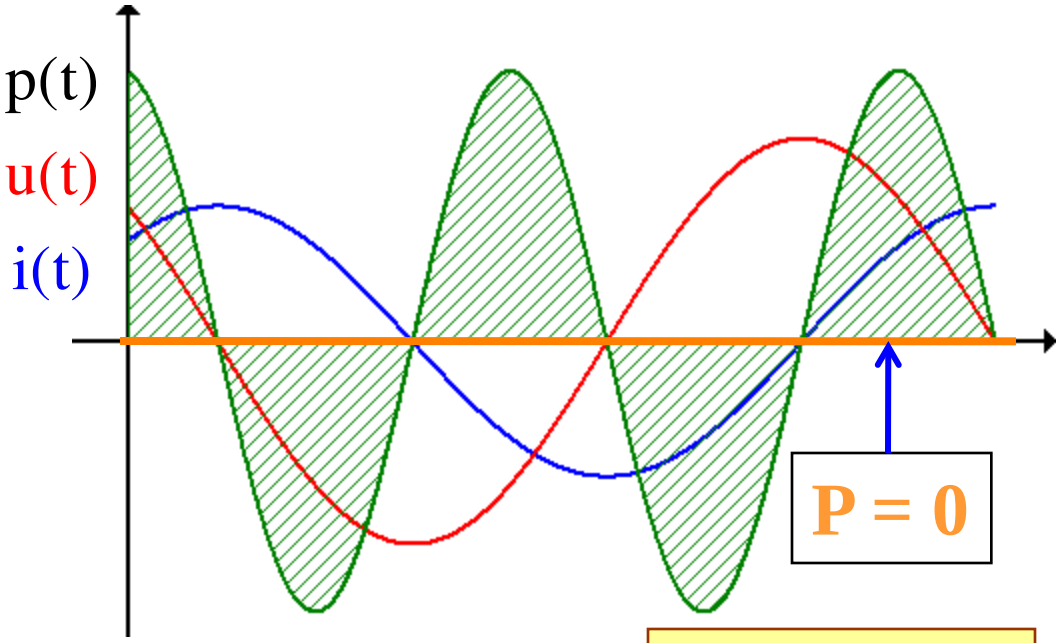
$$i = I_m \cos(\omega t + \psi)$$

$$u = L\frac{di}{dt}$$

$$p(t) = u(t)i(t) = Li\frac{di}{dt}$$

$$p(t) = -\omega L I_m^2 \cos(\omega t + \psi)\sin(\omega t + \psi)$$

$$p(t) = -\frac{1}{2} X_L I_m^2 \sin(2\omega t + 2\psi)$$



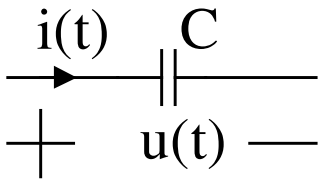
$$P = 0$$

$$Q = \frac{1}{2}\omega L I_m^2$$

$$Q = \omega L I^2$$

Công suất trên các phần tử mạch

- Điện dung



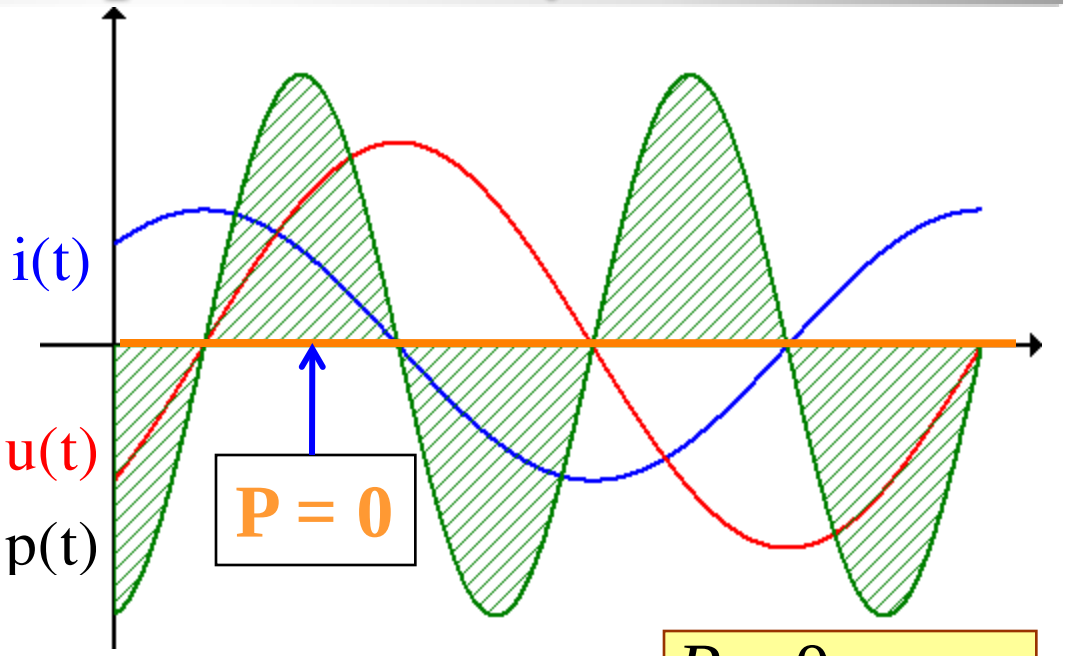
$$i = I_m \cos(\omega t + \psi)$$

$$u = \frac{1}{C}\int i dt$$

$$p(t) = u(t)i(t) = \frac{i}{C}\int i dt$$

$$p(t) = \frac{1}{\omega C} I_m^2 \cos(\omega t + \psi)\sin(\omega t + \psi)$$

$$p(t) = -\frac{1}{2} X_C I_m^2 \sin(2\omega t + 2\psi)$$



$$P = 0$$

$$Q = \frac{-1}{2\omega C} I_m^2$$

$$Q = \frac{-1}{\omega C} I^2$$

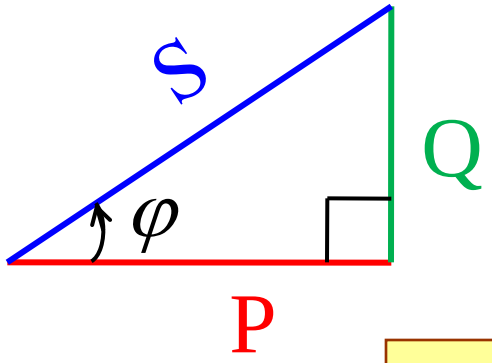
Công suất biểu kiến (Apparent Power)

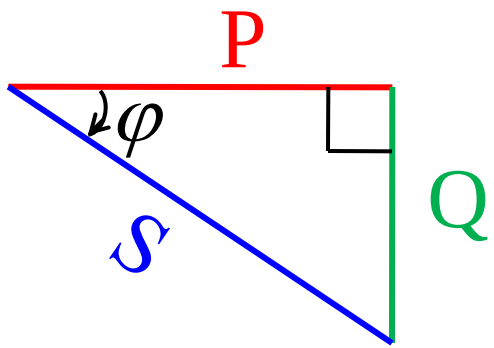
- Định nghĩa $$S = UI = \frac{1}{2} U_m I_m \quad [VA]$$
- Các cách tính khác

$$P = UI\cos\varphi$$

$$Q = UI\sin\varphi$$

$$S = \sqrt{P^2 + Q^2}$$





❖ Công suất phức

$$\tilde{S} = \frac{1}{2} \dot{U}_m \dot{I}_m^* = \dot{U} \dot{I}^* \quad [VA]$$

$$P = \mathrm{Re}\left\{\tilde{S}\right\}$$

$$Q = \mathrm{Im}\left\{\tilde{S}\right\}$$

Nguyên lý cân bằng công suất

- Phát biểu

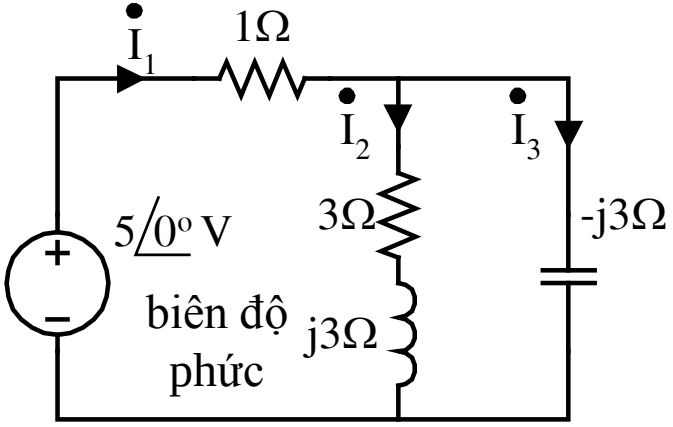
$$\sum P_{send} = \sum P_{receive} \Leftrightarrow \sum EI_E \cos\varphi_E + \sum JU_J \cos\varphi_J = \sum RI_R^2$$

$$\sum Q_{send} = \sum Q_{receive} \Leftrightarrow \sum EI_E \sin\varphi_E + \sum JU_J \sin\varphi_J = \sum XI_X^2$$

$$\Rightarrow \sum \tilde{S}_{send} = \sum \tilde{S}_{receive}$$

$$\sum \dot{E}\,\dot{I}_E^{*} + \sum \dot{U}_J\,\dot{J}^{*} = \sum RI_R^2 + \sum XI_X^2$$

Ví dụ



- Tính **P ? Q ?**
- Nghiệm lại nguyên lý cân bằng công suất

$$\dot{I}_1 = 1\angle 36,87^0$$
$$\dot{I}_2 = 1\angle -53,13^0$$
$$\dot{I}_3 = \sqrt{2}\angle 81,87^0$$

Giải

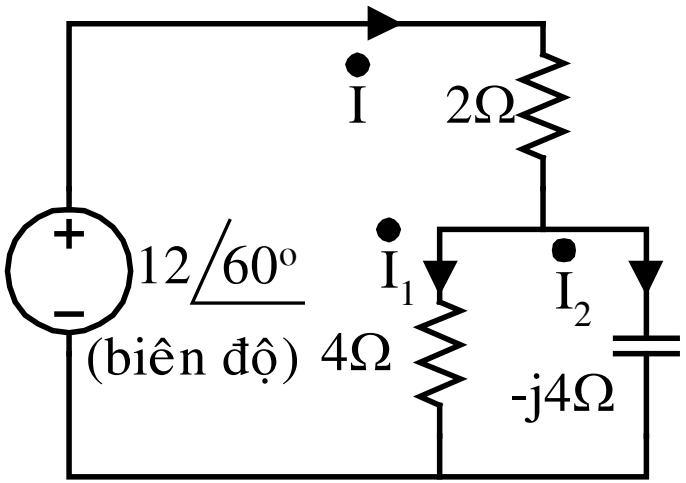
$$R_1 = 1\Omega : P_{1\Omega} = \tfrac{1}{2}.1.1^2 = 0,5[W], Q_{1\Omega} = 0$$
$$R_2 = 3\Omega : P_{3\Omega} = \tfrac{1}{2}.3.1^2 = 1,5[W], Q_{3\Omega} = 0$$
$$x_L = j3\Omega : P_L = 0, Q_L = \tfrac{1}{2}.3.1^2 = 1,5\ [VAr]$$
$$x_C = -j3\Omega : P_C = 0, Q_C = -\tfrac{1}{2}.3.(\sqrt{2})^2 = -3[VAr]$$
$$\dot{E} = 5 : \tilde{S} = \tfrac{1}{2}\dot{E}\,\dot{I}^* = \tfrac{1}{2}5.1\angle 36,87^o = 2 - j1,5\ \ [VA]$$

$$P_{1\Omega} + P_{3\Omega} + P_L + P_C = 0,5 + 1,5 + 0 + 0 = 2$$
$$Q_{1\Omega} + Q_{3\Omega} + Q_L + Q_C = 0 + 0 + 1,5 - 3 = -1,5$$

Nghiệm đúng

Ví dụ

Nghiệm lại nguyên lý cân bằng công suất trong mạch phức biên độ



$$\dot{I} = \frac{12\angle 60^o}{2 + 4//(-j4)} = \frac{12\angle 60^o}{4 - j2} = 2,68\angle 86,6^o \ [A]$$

$$\dot{I}_1 = 2,68\angle 86,6^o . \frac{-j4}{4 - j4} = 1,9\angle 41,6^o \ [A]$$

$$\dot{I}_2 = 2,68\angle 86,6^o . \frac{4}{4 - j4} = 1,9\angle 131,6^o \ [A]$$

$P_{(2\Omega)} = ½\ 2.2,68^2 = 7,18$ [W]

$P_{(4\Omega)} = ½\ 4.1,9^2 = 7,2$ [W]

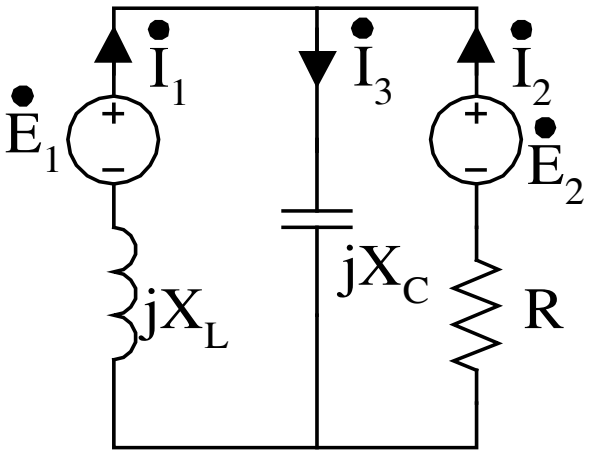
$Q_{(-j4\Omega)} = -½\ 4.1,9^2 = -7,2$ [Var]

$$\tilde{S} = \frac{1}{2}\left\{12\angle 60^o . 2,68\angle -86,6^o\right\}$$

$$\tilde{S} = 14,38 - j7,2$$

$$\Rightarrow \quad \mathrm{Re}\{\tilde{S}\} = P_{2\Omega} + P_{4\Omega} \qquad \mathrm{Im}\{\tilde{S}\} = Q_{-j4\Omega}$$

Ví dụ



- Cho biết $X_L = 10\Omega$

  $\dot{I}_1 = 12,7\angle -78,45^o \quad (RMS)$

  $\dot{E}_1 : \tilde{S}_1 = 259 + j1250 \ [VA]$

  $\dot{E}_2 : \tilde{S}_2 = 375 + j125 \ [VA]$

- Tìm $\dot{E}_1 ? \dot{E}_2 ? \dot{I}_2 ? \dot{I}_3 ? X_C ? R ?$

$$\varphi_1 = \arg \dot{E}_1 - \arg \dot{I}_1 \longrightarrow \varphi_1 = \tan^{-1} 1250/259 = 78,30^o$$

$$E_1 = 259/(12,7 \cos \varphi_1) = 100,5$$

$$\arg \dot{E}_1 = \varphi_1 - 78,45^o = -0,15^o$$

$$\boxed{\dot{E}_1 = 100,5\angle -0,15^o}$$

Ví dụ

$$\dot{U}_{ab} = \dot{E}_1 - jx_L\dot{I}_1 = 35,12\angle -133^o$$

$$Q_C = Q_{E1} + Q_{E2} - Q_L$$

$$Q_C = 1250 + 125 - 10.12,7^2 = -237,9$$

$$\boxed{x_C = U_{ab}^2 / Q_C = -5,18\Omega}$$

$$\dot{I}_3 = \dot{U}_{ab} \big/ (jx_C) = 6,78\angle -43^o$$

$$\dot{I}_2 = \dot{I}_3 - \dot{I}_1 = 8,19\angle 72,8^o$$

$$P_R = P_{E1} + P_{E2} = 634W$$

$$\boxed{R = 634/(8,19)^2 = 9,45\Omega}$$

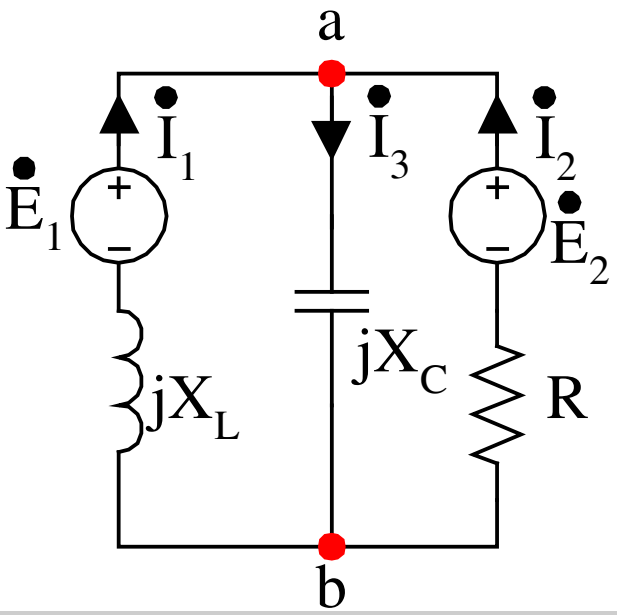
$$\boxed{\dot{E}_2 = \dot{U}_{ab} + R\dot{I}_2 = 48,21\angle 91^o}$$

$$\boxed{\dot{E}_1 = 100,5\angle -0,15^o}$$

$$\dot{I}_1 = 12,7\angle -78,45^o \quad (RMS)$$

$$\dot{E}_1 : \tilde{S}_1 = 259 + j1250 \ [VA]$$

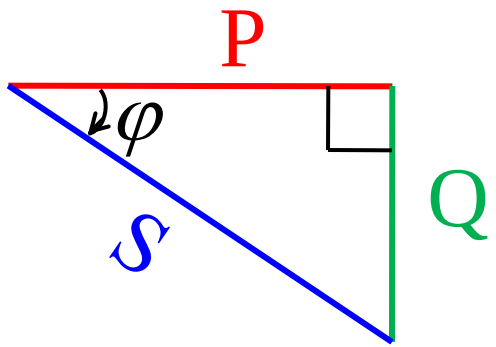
$$\dot{E}_2 : \tilde{S}_2 = 375 + j125 \ [VA]$$



2.9 Hệ số công suất & cách hiệu chỉnh

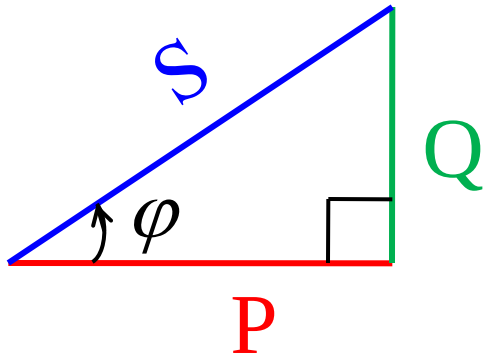
- Hệ số công suất (Power Factor)

$$\cos\varphi = \frac{P}{S}$$



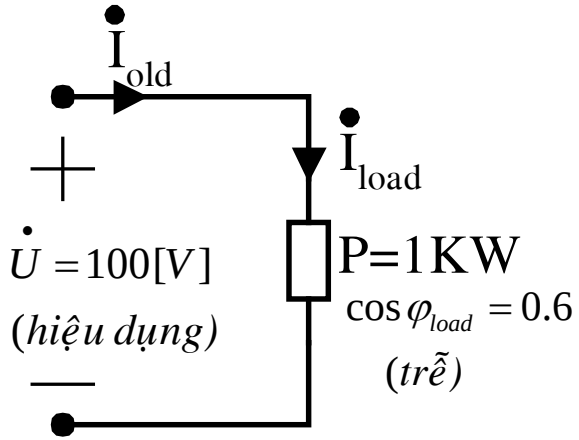
- Cosφ sớm , vượt (leading) : nhánh dung φ < 0

- Cosφ trễ , chậm (lagging) : nhánh cảm φ > 0



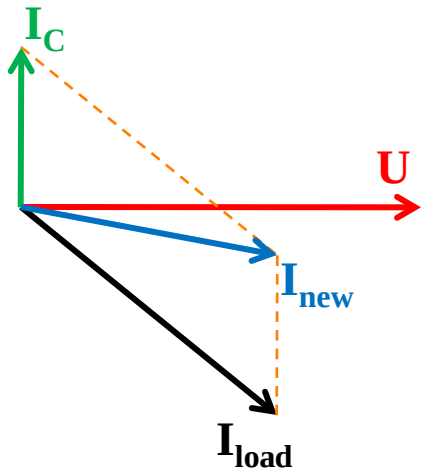
Hiệu chỉnh hệ số công suất

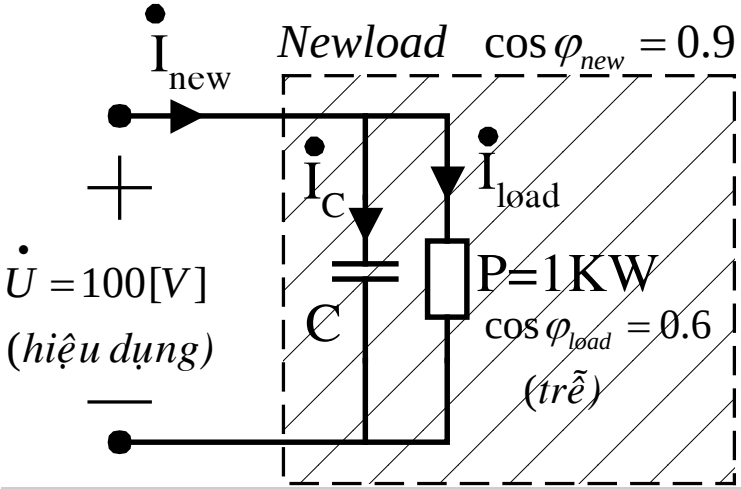
- Sự cần thiết



$$I_{old} = I_{load} = \frac{P_{load}}{U\cos\varphi_{load}}$$

$$= \frac{1000}{100.0,6} = 16,67[A]$$





$$I_{new} = \frac{P_{load} + P_C}{U\cos\varphi_{new}} = \frac{P_{load}}{U\cos\varphi_{new}}$$

$$= \frac{1000}{100.0,9} = 11,11[A]$$

Hiệu chỉnh hệ số công suất

- Tải ban đầu

$$\tilde{S}_{old} = P_{old} + jQ_{old} \quad \rightarrow \quad Q_{old} = P_{old}\, tg\varphi_{old}$$

- Sau hiệu chỉnh, thêm vào điện kháng X đối nghịch tính tải

$$P_{new} = P_{old} + P_X = P_{old} \Longrightarrow Q_{new} = P_{old}\, tg(\pm \arccos \varphi_{new})$$

+ :*lagging*
- :*leading*

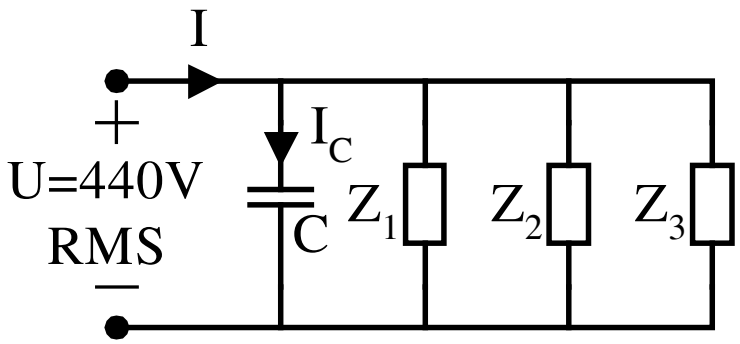
$$\Delta Q = Q_{new} - Q_{old}$$

- Phần tử kháng cần cho hiệu chỉnh

$$L = \frac{U^2}{\omega \Delta Q} \; [H]$$

$$C = \frac{-\Delta Q}{\omega U^2} \; [F]$$

Ví dụ



f = 50 Hz
Z_1: S_1 = 10KVA, cosφ$_1$ = 0,7 (lead)
Z_2: P_2 = 15KW, cosφ$_2$= 0,5 (lag)
Z_3: P_3 = 5KW, tải trở
☆ I ? cosφ ?
🕐 C ? I_{new} ? để cosφ = 0,9 (lag)

$$P_1 = 10000.0,7 = 7000[W]$$

$$Q_1 = 10000.\sin(-\cos^{-1}0,7) = -7141[VAr]$$

$$Q_2 = 15000.\tan(\cos^{-1}0,5) = 25981[VAr]$$

$$P_3 = 5000(W), Q_3 = 0$$

$$P = P_1 + P_2 + P_3 = 27[KW]$$

$$Q_{old} = Q_1 + Q_2 + Q_3 = 18840[VAr]$$

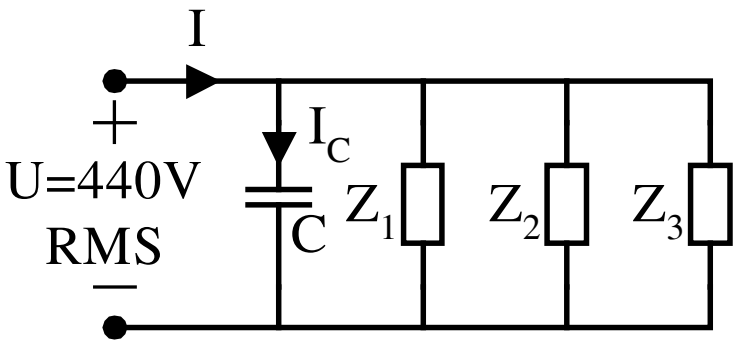
$$S = \sqrt{P^2 + Q^2} = 32923\,[VA]$$

$\Longrightarrow$

$$I = S/U = 74,8[A]$$

$$\cos\varphi = P/S = 0,82$$

Ví dụ



f = 50 Hz

Z_1: S_1 = 10KVA, cosφ$_1$ = 0,7 (lead)

Z_2: P_2 = 15KW, cosφ$_2$= 0,5 (lag)

Z_3: P_3 = 5KW, tải trở

☆ I ? cosφ ?

🕐 C ? I_{new} ? để cosφ = 0,9 (lag)

$$P = P_1 + P_2 + P_3 = 27[KW]$$

$$Q_{new} = P.tg(\cos^{-1} 0,9)$$

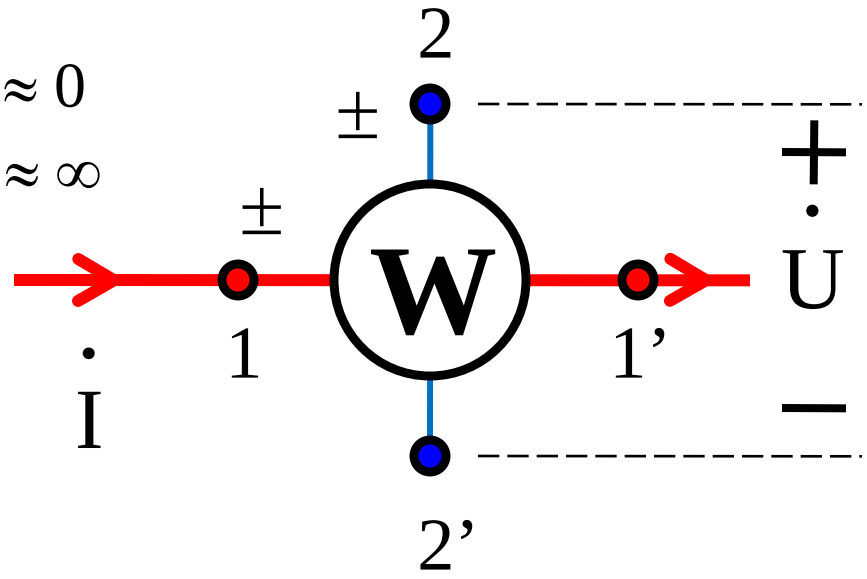
$$Q_{new} = 13077\,[VAr]$$

$$C = \frac{-\Delta Q}{2\pi f U^2} = \frac{-(Q_{new} - Q_{old})}{2\pi f U^2} = 94,8\,[\mu F]$$

$$I_{new} = P/(0,9U) = 68,2\,[A]$$

Đo công suất

- Watt kế:
  - Nội trở cuộn dòng điện : $R_{11'} \approx 0$
  - Nội trở cuộn điện áp : $R_{22'} \approx \infty$
  - Cực cùng tên : $*$, $\pm$, $\bullet$
    (giúp xác định hướng truyền công suất)



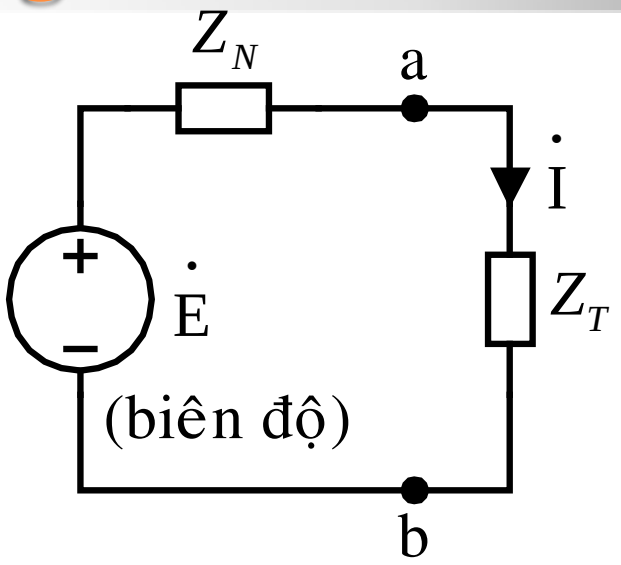
- Số chỉ:

$$P = \frac{1}{2} U_m I_m \cos(\varphi_u - \varphi_i) = UI \cos(\varphi_u - \varphi_i)$$

$$= \frac{1}{2} \mathrm{Re}\left\{ \dot{U}_m \, \dot{I}_m^* \right\} = \mathrm{Re}\left\{ \dot{U} \, \dot{I}^* \right\}$$

2.10 Phối hợp trở kháng

- Xét mạch
  - $Z_N = R_N + jX_N$
  - $Z_T = R_T + jX_T$
- Vấn đề :
  - Chọn tải nào thì công suất tải nhận từ nguồn là lớn nhất ?

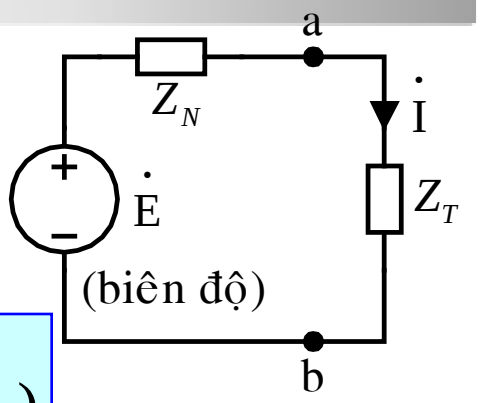


$$\dot{I} = \frac{\dot{E}}{Z_N + Z_T} = \frac{E_m \angle \varphi_E}{(R_N + R_T) + j(X_N + X_T)} = I_m \angle \varphi_I$$

$$P_Z = \frac{1}{2} I_m^2 \,\mathrm{Re}\{Z_T\} = \frac{1}{2} \frac{E_m^2}{(R_N + R_T)^2 + (X_N + X_T)^2} R_T$$

2.10 Phối hợp trở kháng

$$P_Z = \frac{1}{2}\frac{E_m^2}{(R_N + R_T)^2 + (X_N + X_T)^2} R_T$$



- Tải tùy ý $\rightarrow$ chọn $(X_N + X_T) = 0$

$$P_Z = \frac{1}{2} E_m^2 \frac{R_T}{(R_N + R_T)^2} \rightarrow P_{\max} = \frac{1}{8}\frac{E_m^2}{R_T} \quad (khi\ R_T = R_N)$$

  - $\Rightarrow$ Phối hợp trở kháng : $Z_T = Z_N^*$

- Tải có argument φ cố định $R_T = |Z_T|\cos\varphi\ ;\ Z_T = |Z_T|\sin\varphi$

$$P_Z = \frac{1}{2}\frac{E_m^2}{(R_N + |Z_T|\cos\varphi)^2 + (X_N + |Z_T|\sin\varphi)^2} |Z_T|\cos\varphi$$

  - $\Rightarrow$ Phối hợp mô đun : $|Z_T| = \sqrt{R_N^2 + X_N^2} = |Z_N|$