ECE 307 – Techniques for Engineering Decisions

Simulation

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SIMULATION

- **Simulation provides a *systematic* approach for dealing with uncertainty by "*flipping a coin*" to deal with each uncertain event**
- **In many real world situations, simulation may be the only viable means to quantitatively deal with a problem under uncertainty**
- **Effective simulation requires implementation of appropriate approximations at many and, some-times, at possibly every stage of the problem**

SIMULATION EXAMPLE

- **The problem is concerned with the fabric purchase by a fashion designer**
- **The two choices for textile suppliers are:**

supplier 1: ***fixed price – constant* 2 *$/yd***

supplier 2: ***variable price dependent on quantity***

2.10 *$/yd for the first* 20,000 *yd* 1.90 *$/yd*

for the next* 10,000 *yd* 1.70 *$/yd for the

next* 10,000 *yd* 1.50 *$/yd thereafter

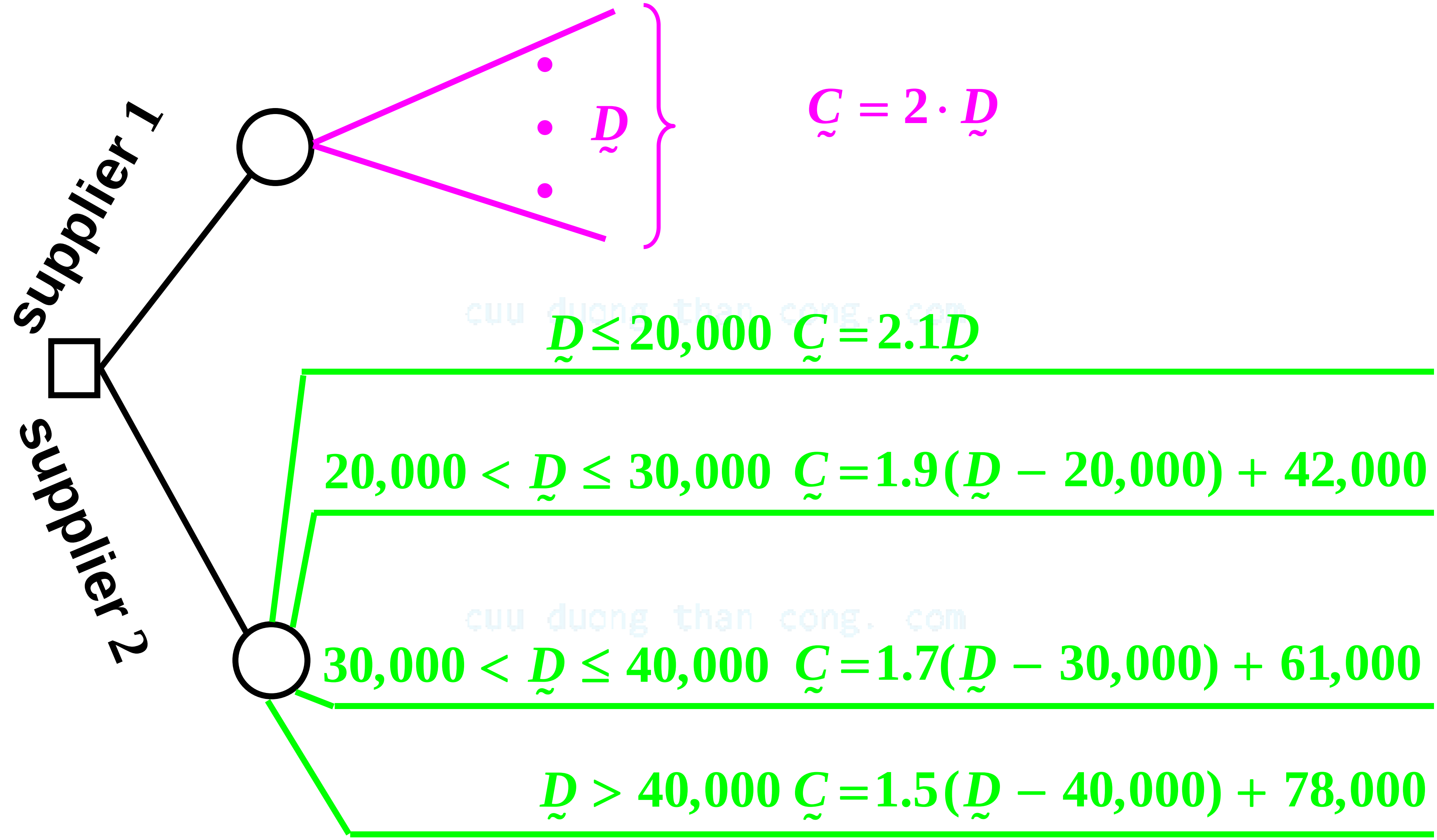
SIMULATION EXAMPLE

- The purchaser is uncertain about the demand $\underset{\sim}{D}$ but determines an appropriate model is:

$$\underset{\sim}{D} \sim \mathscr{N}(25{,}000\ yd, 5{,}000\ yd)$$

- The decision may be represented in form of the following decision branches:

SIMULATION EXAMPLE



SIMULATION EXAMPLE

- Supplier 1 has a simple linear cost function $\underset{\sim}{C}$
- Supplier 2 has a far more complicated scheme to evaluate costs: in effect, the range of the demand and the corresponding probability for $\underset{\sim}{D}$ to be in a part of the range must be known, as well as the expected value of $\underset{\sim}{D}$ for each range

SIMULATION EXAMPLE

- **We simulate the situation in the decision tree by "*drawing multiple samples from the appropriate population*"**
- **We systematically tabulate the results and evaluate the required statistics**
- **The simple algorithm for the simulation consists of just a few steps which are repeated until an appropriate sized sample is obtained**

BASIC ALGORITHM

Step 0 **:** **store the distribution** $\mathcal{N}\left(25{,}000,\ 5{,}000\right)$ **;**
determine $\bar{k}$ **, the maximum number of draws; set** $k = 0$

Step 1 **:** **if** $k > \bar{k}$ **, stop; else set** $k = k + 1$

Step 2 **:** **draw a random sample from the normal distribution** $\mathcal{N}\left(25{,}000,\ 5{,}000\right)$

Step 3 **:** **evaluate the outcomes on both branches; enter each outcome into the database and return to Step 1**

SIMULATION EXAMPLE

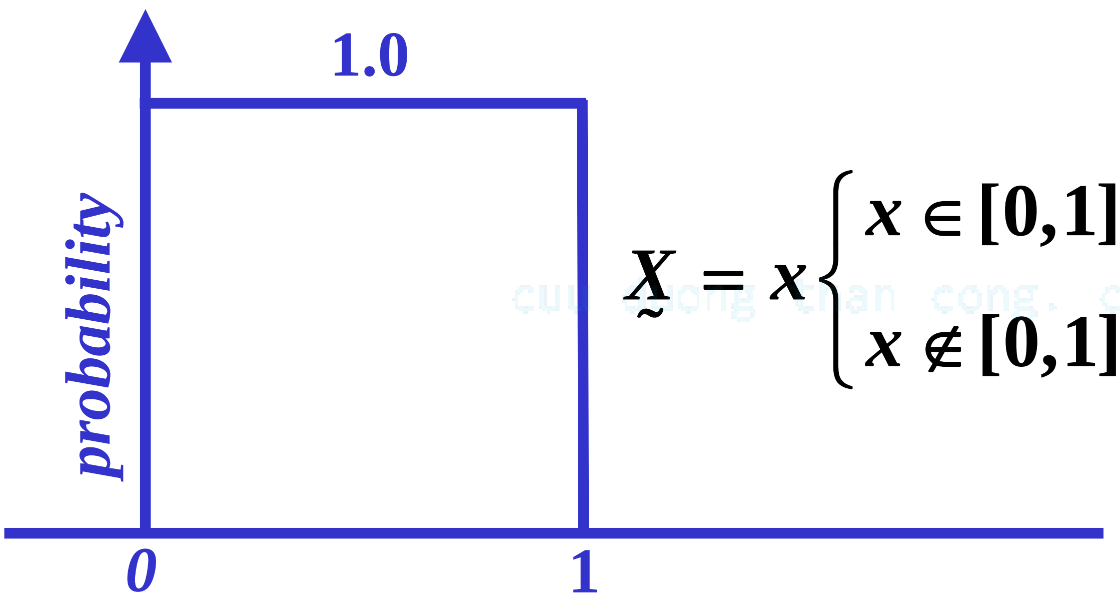
- Application of the algorithm allows the determi– nation of the histogram of the cost figures and then the evaluation of the expected costs
- For the assumed demand, for supplier 1, we have the straight forward case of

$$E\{\underset{\sim}{C}\} = 2 \cdot E\{\underset{\sim}{D}\} = 50,000 \quad \text{and} \quad \sigma_{\underset{\sim}{C}} = 10,000$$

  and the use of the algorithm may be bypassed
- For the supplier 2, the algorithm is applied for the random $\bar{k}$ draws

RANDOM DRAWS

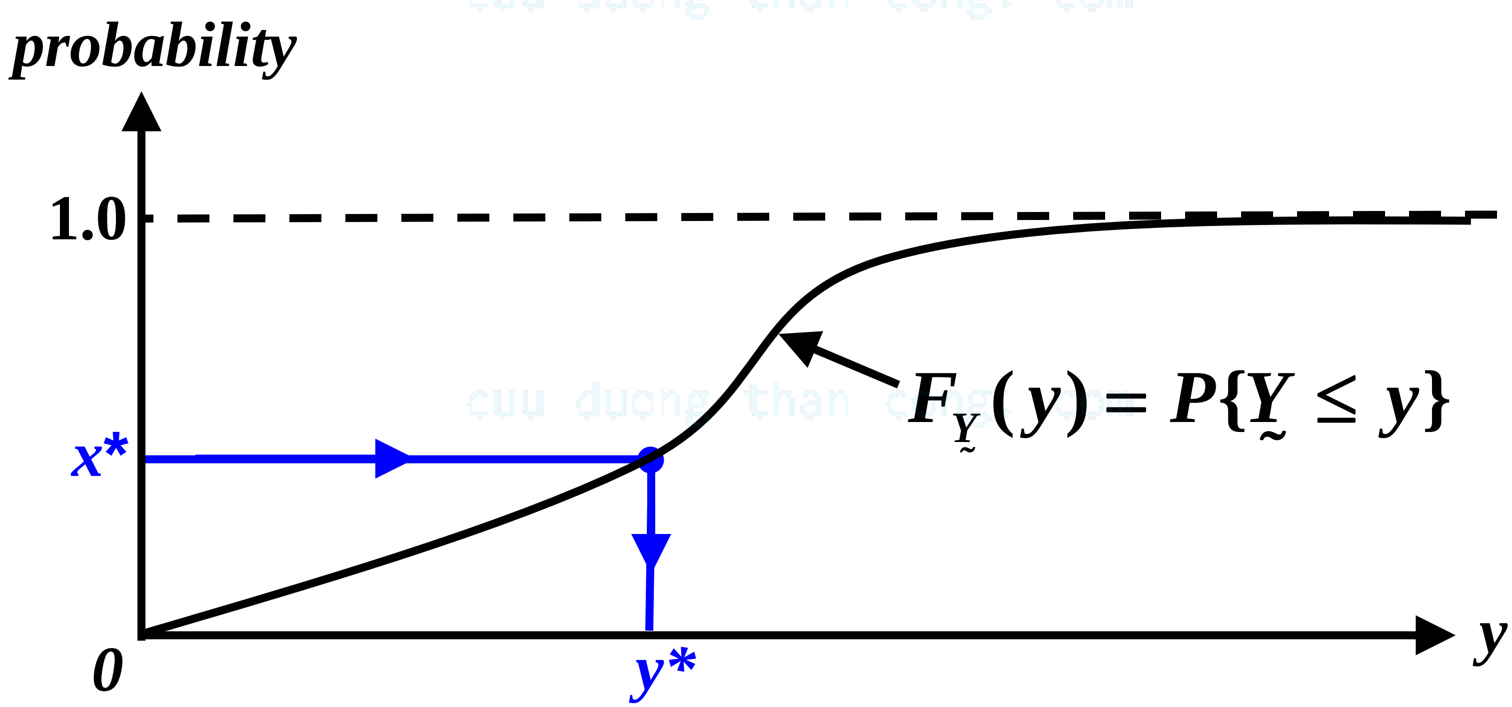
- A key issue is the generation of random draws for which we need a random number generator
- One possibility is to use a uniformly distributed *r.v.* between *0* and 1



$$\underset{\sim}{X} = x \begin{cases} x \in [0,1] \ \textit{with probability} \ \ 1 \\ x \notin [0,1] \ \textit{with probability} \ \ 0 \end{cases}$$

SIMULATION EXAMPLE

- We draw a random value of x, say x^*, and work through the *c.d.f.* $F_{\underset{\sim}{Y}}(y)$ to get the value y^* of the *r.v.* $\underset{\sim}{Y}$ *with* $F_{\underset{\sim}{Y}}(y^*) = x^*$



SOFT PRETZEL EXAMPLE

- The market size is unknown but we assume that the market size is a normal with

$$\underset{\sim}{S} \sim \mathscr{N}\left(100{,}000\,,10{,}000\right)$$

- We are interested in determining the fraction $\underset{\sim}{F}$ of the new market the new company can capture

- We model the distribution of $\underset{\sim}{F}$ with a discrete distribution

SOFT PRETZEL EXAMPLE

$\underset{\sim}{F} = x\%$	$P\{\underset{\sim}{F} = x\}$
16	0.15
19	0.35
25	0.35
28	0.15

SOFT PRETZEL EXAMPLE

- Sales price of a pretzel is *$* 0.50
- Variable costs $\underset{\sim}{V}$ are represented by a uniformly distributed *r.v.* in the range [0.08 , 0.12] *$/pretzel*
- Fixed costs $\underset{\sim}{C}$ are also random
- The contributions to profits are given by

$$\underset{\sim}{\pi} = (\underset{\sim}{S} \cdot \underset{\sim}{F}) \cdot (0.5 - \underset{\sim}{V}) - \underset{\sim}{C}$$

and may be evaluated via simulation

MANUFACTURING CASE STUDY

- **The selection of one of two manufacturing *processes* based on net present value (*NPV*) using a 3 – year horizon (current year plus next two years) and a 10% discount rate**

- **The *process* is used to manufacture a product sold at 8 *$/unit***

PROCESS 1 DESCRIPTION

- This *process* uses the current machinery for manufacturing
- The annual fixed costs are $\$12{,}000$
- The yearly variable costs are represented by the *r.v.*

$$\underset{\sim}{V}_i \sim \mathcal{N}(4, 0.4) \qquad\qquad i = 0,1,2$$

PROCESS 1 DESCRIPTION

- The failure of a machine in the *process* is random and the number failures $\underset{\sim}{Z}_i$ in year $i = 0,1,2$ is a *r.v.* with

$$\underset{\sim}{Z}_i \sim Poisson(m = 4) \qquad\qquad i = 0,1,2$$

- Each failure incurs costs of $ 8,000
- Total costs are the sum of $\underset{\sim}{V}_i$ and $8,000\ \underset{\sim}{Z}_i$

PROCESS 1: UNCERTAINTY IN THE SALES FORECAST

current year $i = 0$		*next year* $i = 1$		*year after* $i = 2$	
d_0	$P\left\{ \underset{\sim}{D}_0 = d_0 \right\}$	d_1	$P\left\{ \underset{\sim}{D}_1 = d_1 \right\}$	d_2	$P\left\{ \underset{\sim}{D}_2 = d_2 \right\}$
11,000	0.2	8,000	0.2	4,000	0.1
16,000	0.6	19,000	0.4	21,000	0.5
21,000	0.2	27,000	0.4	37,000	0.4

PROCESS 2: DESCRIPTION

- *Process* 2 involves an investment of $ 60,000 paid in cash to buy new equipment and doing away with the worthless current machinery; the fixed costs of $ 12,000 per year remain unchanged
- The yearly variable costs $\underset{\sim}{V}_i$

$$\underset{\sim}{V}_i \sim \mathcal{N}\left(\$3.50, \$1.0\right) \qquad i = 0, 1, 2$$

- The number of machine failures $\underset{\sim}{Z}_i$ for year

$$\underset{\sim}{Z}_i \sim \text{Poisson}\left(m = 3\right) \qquad i = 0, 1, 2$$

and the costs per failure are $ 6,000

PROCESS 2: SALES FORECAST

current year $i = 0$		*next year* $i = 1$		*year after* $i = 2$	
d_0	$P\{\underset{\sim}{D}_0 = d_0\}$	d_1	$P\{\underset{\sim}{D}_1 = d_1\}$	d_2	$P\{\underset{\sim}{D}_2 = d_2\}$
14,000	0.3	12,000	0.36	9,000	0.4
19,000	0.4	23,000	0.36	26,000	0.1
24,000	0.3	31,000	0.28	42,000	0.5

NET PROFITS

- The net profits $\underset{\sim}{\pi}_i$ each year are a function

$$\underset{\sim}{\pi}_i = f\left(\underset{\sim}{D}_i, \underset{\sim}{V}_i, \underset{\sim}{Z}_i\right) \qquad\qquad i = 0, 1, 2$$

- While for each *process* the determination of $F_{\underset{\sim}{\pi}_i}(\cdot)$ requires the evaluation of all the possible out-comes; both $E\left\{\underset{\sim}{\pi}_i\right\}$ and $var\left\{\underset{\sim}{\pi}_i\right\}$ may be estimated by simulation by drawing an appropriate number of samples from the underlying distribution

NPV

- The *NPV* of these profits needs to be assessed and expressed in terms of year *0* dollars
- The profits are collected at the end of each year or equivalently the beginning of the following year
- We use the $i = 10\%$ discount factor to express the $var\{\tilde{\pi}_i\}$ in year *0* (current) dollars

NPV

- We can evaluate for *processes* 1 and 2 the profits for each year; we use superscript to denote the *process*

$$\textit{process } 1: \quad \underset{\sim}{\pi}_i^1 = 8\underset{\sim}{D}_i - \underset{\sim}{D}_i \underset{\sim}{V}_i - 8{,}000 \underset{\sim}{Z}_i - 12{,}000$$

$$i = 0, 1, 2$$

$$\textit{process } 2: \quad \underset{\sim}{\pi}_i^2 = 8\underset{\sim}{D}_i - \underset{\sim}{D}_i \underset{\sim}{V}_i - 6{,}000 \underset{\sim}{Z}_i - 12{,}000$$

and we also need to account for the *$* 60,000 *investment* in year *0* for *process* 2

NPV

- The *NPV* evaluation then is stated as the *r.v.*

$$\underset{\sim}{\Pi}^{1} = \sum_{i=0}^{2} \underset{\sim}{\pi}_{i}^{1} (1.1)^{-(i+1)}$$

and

$$\underset{\sim}{\Pi}^{2} = -60,000 + \sum_{i=0}^{2} \underset{\sim}{\pi}_{i}^{2} (1.1)^{-(i+1)}$$

- Simulation is used to evaluate

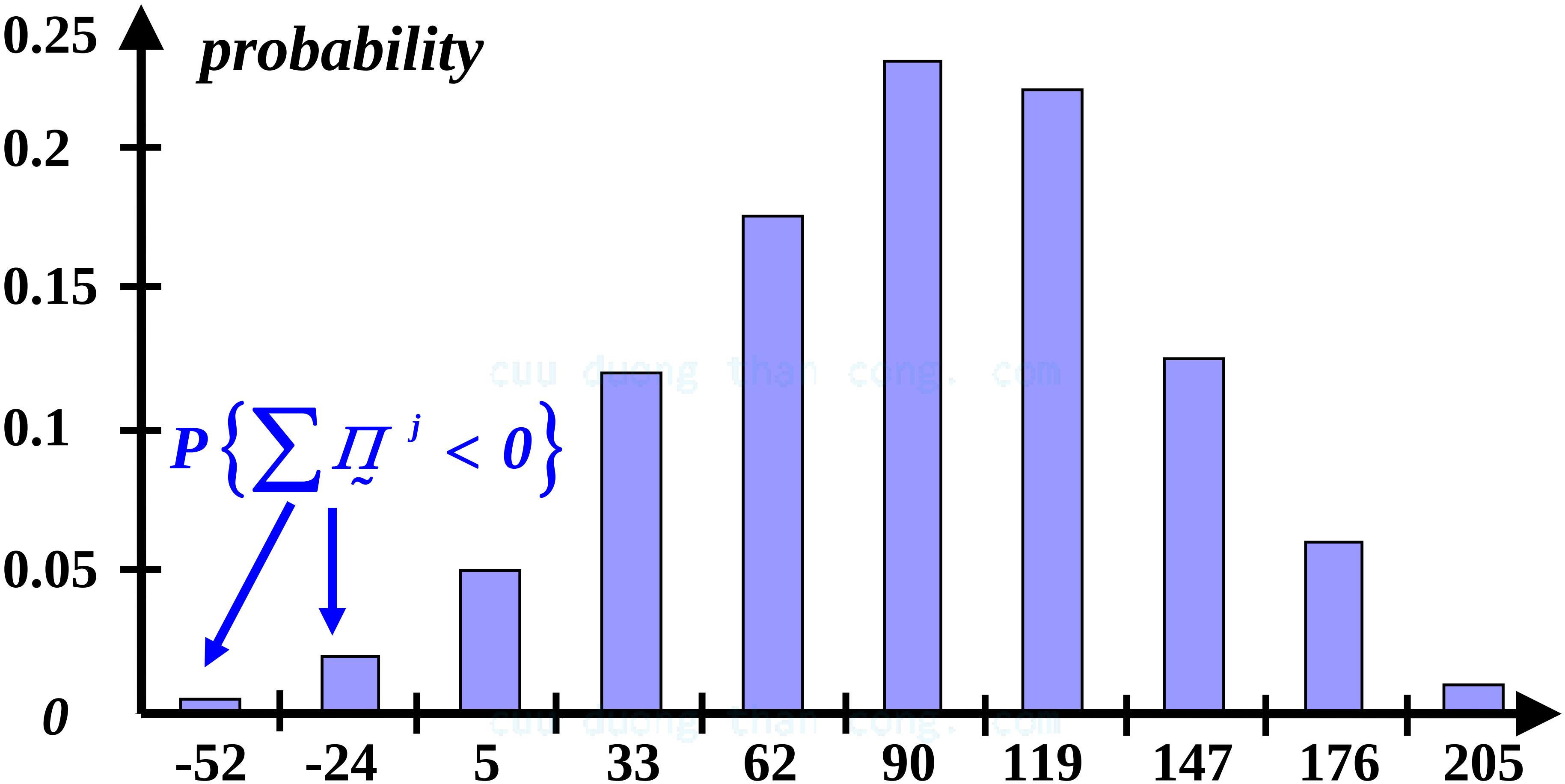
$$NPV^{1} = E\left\{ \underset{\sim}{\Pi}^{1} \right\} \qquad NPV^{2} = E\left\{ \underset{\sim}{\Pi}^{2} \right\}$$

SIMULATION RESULTS

- For a 1,000 replications we obtain

process j	*mean* (\$)	*standard deviation* (\$)	$P\left\{\sum \underset{\sim}{\Pi}^{j} < 0\right\}$
1	**91,160**	**46,970**	**0.029**
2	**110,150**	**72,300**	**0.046**

SIMULATION RESULTS

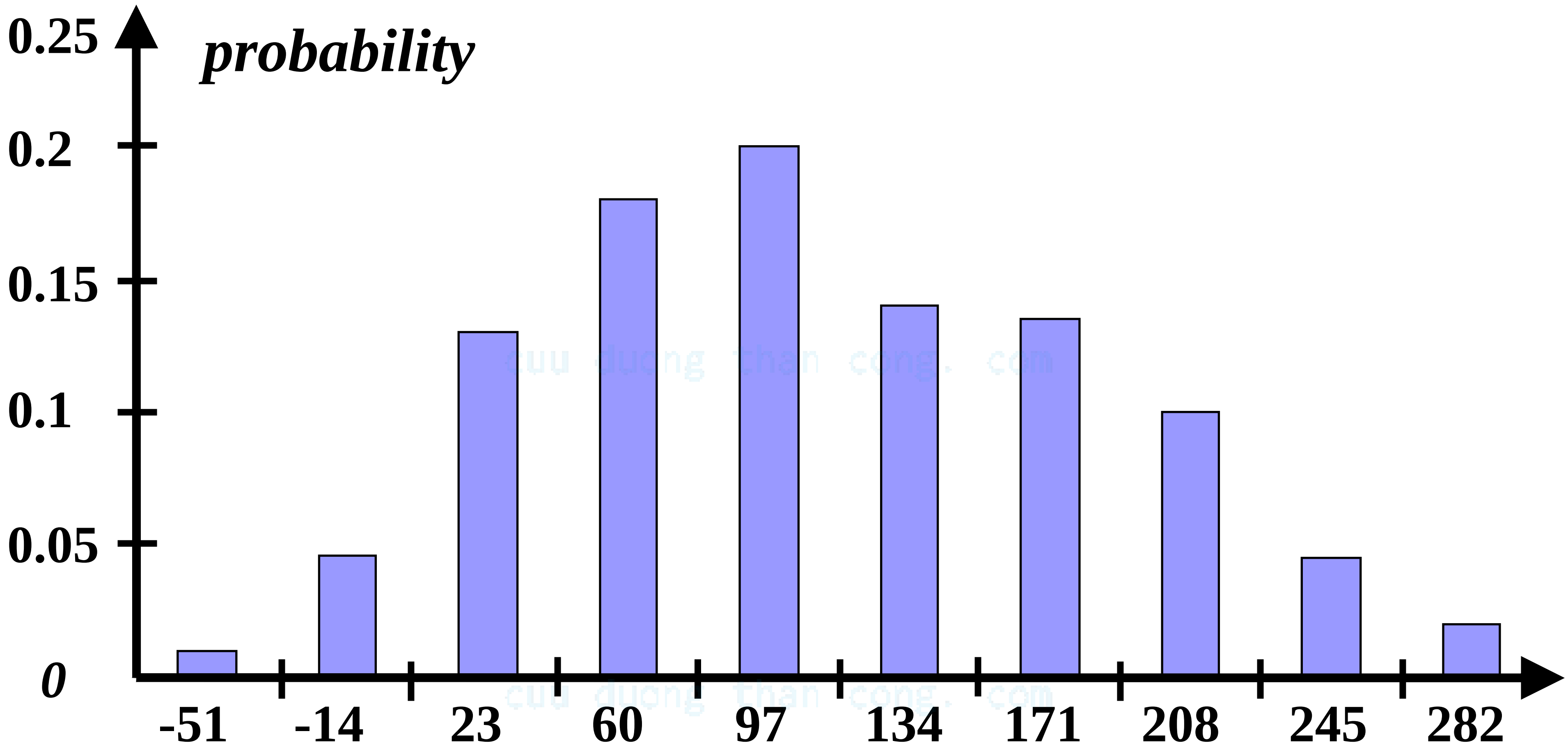


$$NPV^{1} = 91{,}160$$

$$\sigma = 46{,}970$$

$$P\left\{\sum \underset{\sim}{\Pi}^{1} < 0\right\} = 0.029$$

SIMULATION RESULTS



$$\overline{NPV}^{2} = 110,150$$

$$\sigma = 72,300$$

$$P\left\{\sum \underset{\sim}{\Pi}^{2} < 0\right\} = 0.046$$

*c.d.f.*s OF THE TWO PROCESSES