ECE 307 – Techniques for Engineering Decisions

Value-at-Risk or *VaR*

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INTRODUCTION TO FUTURES

- Commodity traders trade important commodities such as foodstuff, livestock, metals, fuel, and electricity using financial instruments known as *forward contracts*
- Standardized forward contracts are known as futures

INTRODUCTION TO FUTURES

- Futures have finite lives and are primarily used for hedging commodity price-fluctuation risks or for taking advantage of price movements, rather than for the buying or the selling of the actual cash commodity

- The buyer of the futures contract agrees on a fixed purchase price to buy the underlying

INTRODUCTION TO FUTURES

commodity from the seller at the expiration of the contract; the seller of the futures contract agrees to sell the underlying commodity to the buyer at expiration at the fixed sales price

- As time passes, the contract's price changes relative to the fixed price at which the trade was initiated

- This creates profits or losses for the trader

INTRODUCTION TO FUTURES

- The word "contract" is used because a futures contract requires delivery of the commodity in a stated month in the future unless the contract is liquidated before it expires
- However, in most cases, delivery never takes place
- Instead, both the buyer and the seller, usually liquidate their positions before the contract expires; the buyer sells futures and the seller buys futures

COMMODITY PORTFOLIOS

- Traders usually hold portfolios of commodities; a collection of different commodities, each bought at a certain price, with different terms and conditions
- This is done in order to diversify the portfolio and mitigate the overall risk
- The value of a portfolio, at any given point in time, is determined by the summation of the individual values of each of the commodities in the 'basket'

MARKET UNCERTAINTIES

- We consider the purchase of a portfolio $\underset{\sim}{P}$ at a certain time $t = 0$ for the overall price p_0
- The value of the portfolio at any time t is p_t
- This portfolio is exposed to the various sources of uncertainty to which the market for each commodity is subjected and consequently its value will fluctuate

PERFORMANCE PREDICTION

- On any given trading day $t = T$, the fixed portfolio may either incur a loss or a gain or remain unchanged with respect to its value at $t = T - 1$
- We wish to study what the worst *performance* of the portfolio may be from the day $t = T - 1$ to the day $t = T$ and how to systematically measure the performance

PERFORMANCE PREDICTION

- At $t = T$, we cannot lose more than the overall value p_T of the portfolio and this statement is true with a probability of 1

- In other words, with a probability of 1, the loss must be less than or equal to p_T

PORTFOLIO VALUE AND RETURNS

- We evaluate the change δ_t in the portfolio close value p_t from $t = T - 1$ to $t = T$ as:

$$\delta_T = p_T - p_{T-1}$$

- We define the rate of return r_t of the portfolio from $t = T - 1$ to $t = T$ in terms of δ_T to be

$$r_T = \frac{\delta_T}{p_{T-1}}$$

PORTFOLIO VALUE AND RETURNS

- **The value of r_t for each observation is the change in the portfolio value from day $t = T - 1$ to day $t = T$**
- **The value of r_t must lie in the interval $[-1, \infty)$**
- **A non-positive value of r_t means there is a loss in the portfolio value from $t = T - 1$ to $t = T$**

DATA COLLECTION

- Suppose that we have the set of data for r_T
- We are sampling from a population, the realizations of the random variable $\underset{\sim}{p}$ with values $\{ p_0, p_1, \dots, p_{T-1}, p_T, \dots \}$
- We use $\underset{\sim}{P}$ to define $\underset{\sim}{\Delta}$ and $\underset{\sim}{R}$
- The sample values of $\underset{\sim}{R}$ are $\{ r_1, r_2, \dots, r_{T-1}, r_T, \dots \}$

DATA COLLECTION

date	*close price*	*loss/gain*	*percent loss/gain*
3/5/2007	$42.15	-$0.33	-0.78%
3/2/2007	$42.48	-$0.65	-1.51%
3/1/2007	$43.13	-$0.20	-0.46%
2/28/2007	$43.33	$0.14	0.32%
2/27/2007	$43.19	-$1.85	-4.11%
.	.	.	.
.	.	.	.
.	.	.	.
3/18/1999	$105.12	$2.00	1.94%
3/17/1999	$103.12	-$0.75	-0.72%
3/16/1999	$103.87	$0.87	0.84%
3/15/1999	$103.00	$2.88	2.88%
3/12/1999	$100.12	-$2.50	-2.44%
3/11/1999	$102.62	$0.50	0.49%

$\underset{\sim}{P}$

$\underset{\sim}{R}$

$\underset{\sim}{\Delta}$

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$r_{3/1/2007}$

$\delta_{3/1/2007}$

$p_{3/1/2007}$

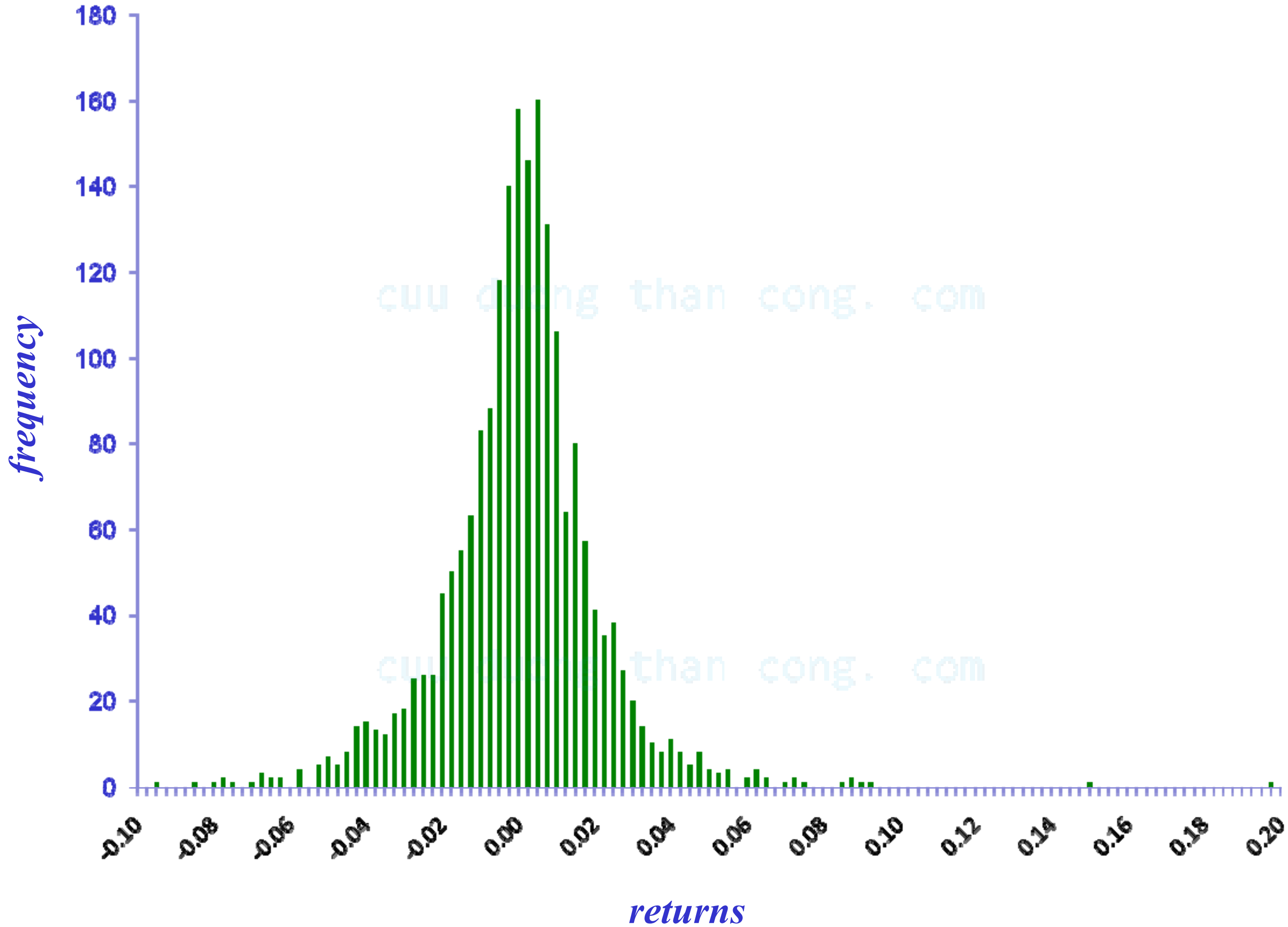
DATA COLLECTION

- We can use the historical values of $\underset{\sim}{R}$ to construct a probability distribution function
- The first step is to determine the frequency of $\underset{\sim}{R}$ taking on values in certain intervals; for this purpose, we discretize $\underset{\sim}{R}$ and define ‘buckets’ in which we drop the realized values of $\underset{\sim}{R}$
- The number of values in each bucket represents the frequency of $\underset{\sim}{R}$ taking on a value in that bucket

BUCKETS AND FREQUENCY

buckets	*frequency*
-10.00 %	*0*
-9.75 %	*0*
-9.50 %	1
-9.25 %	*0*
.	.
.	.
-0.50 %	118
-0.25 %	140
0.00 %	158
0.25 %	146
0.50 %	160
.	.
.	.
19.25 %	*0*
19.50 %	*0*
19.75 %	1
20.00 %	*0*

FREQUENCY VS. RETURNS DISTRIBUTION

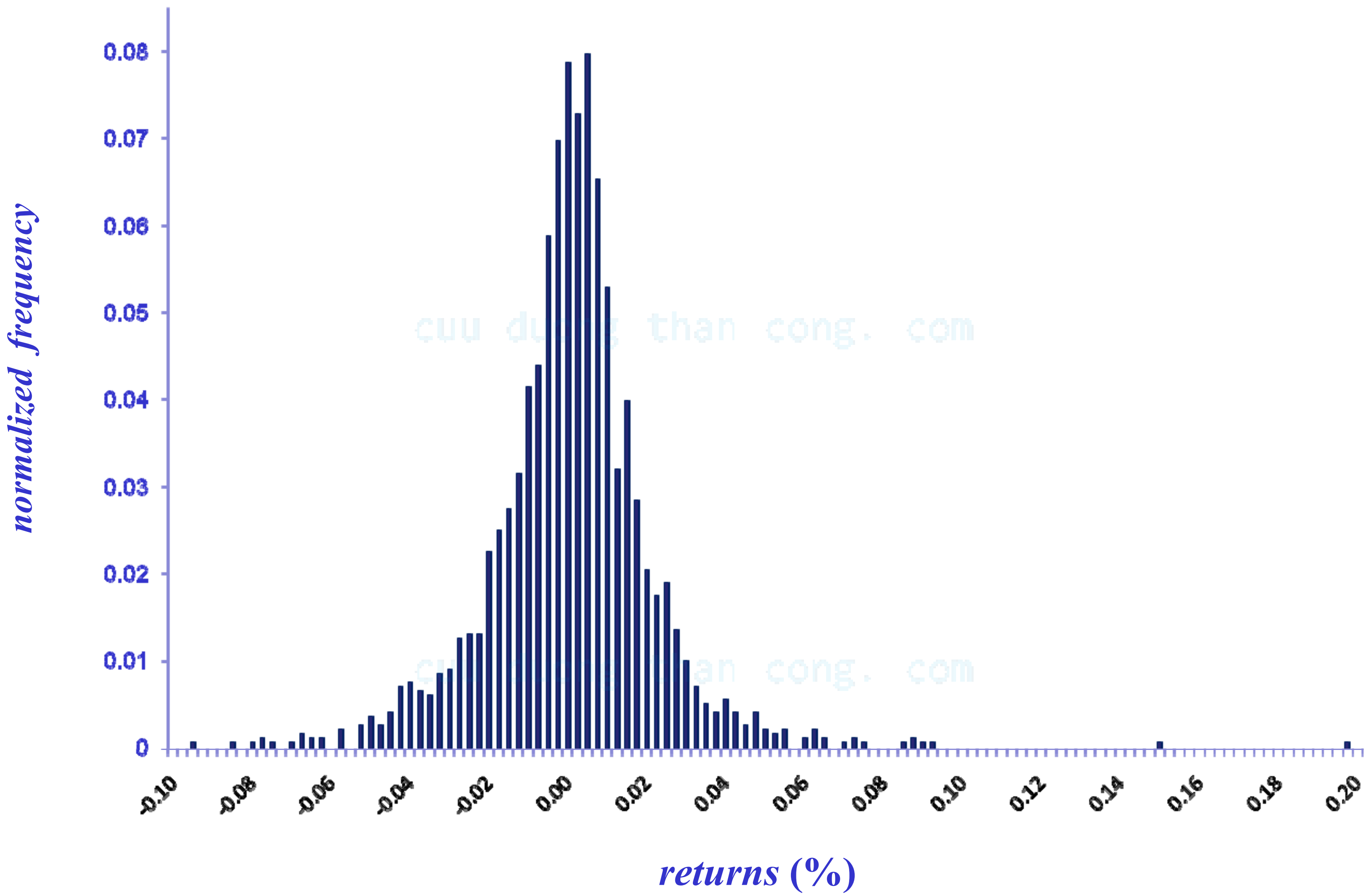


NORMALIZATION

- We normalize these frequencies using the total number of observations and interpret the normalized quantities as the values of a discrete probability mass distribution function

- We then construct the cumulative distribution function from this data, and interpret the results with respect to the returns

NORMALIZED FREQUENCY DISTRIBUTION



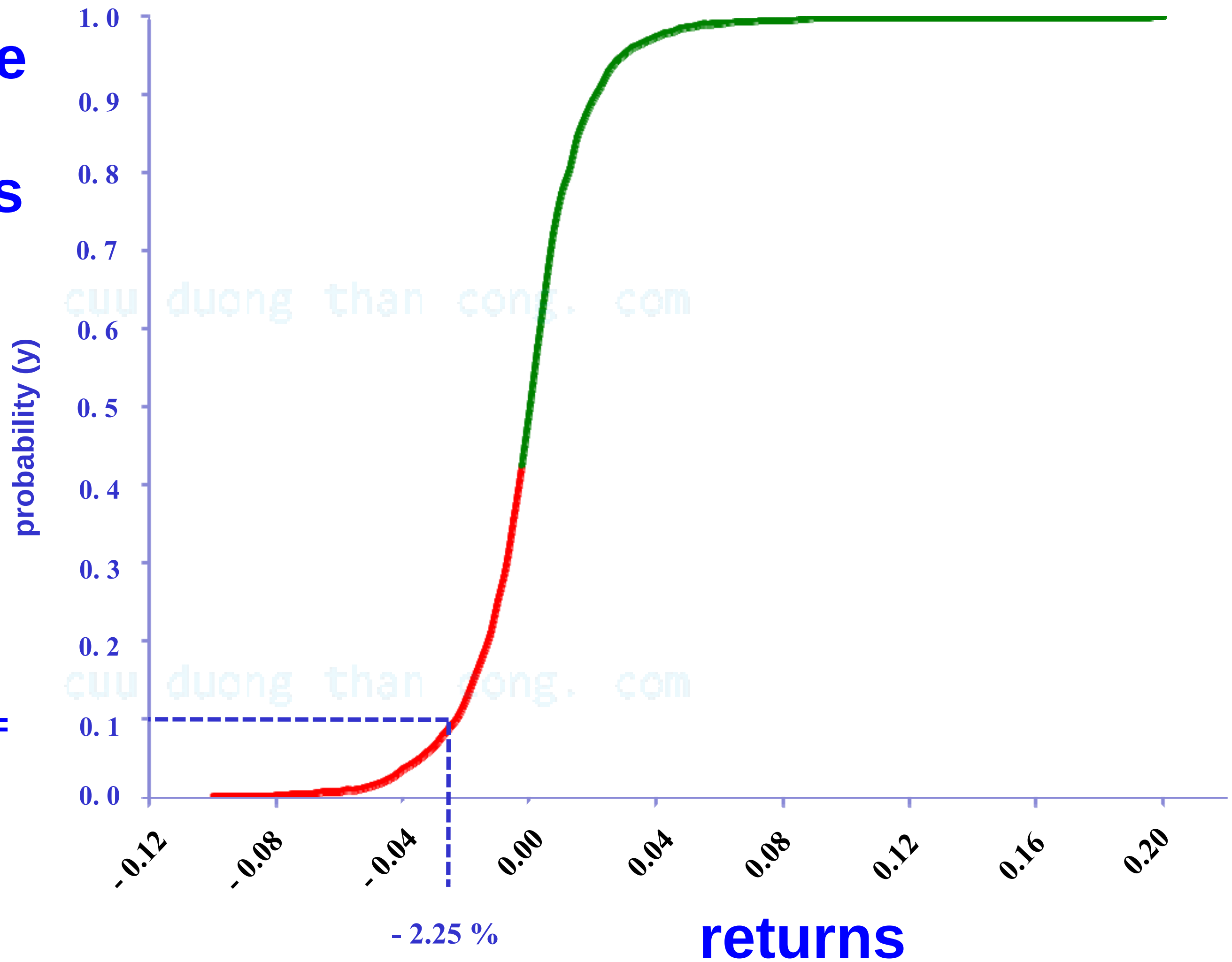
CUMULATIVE DISTRIBUTION FUNCTION (*CDF*)

this *CDF* gives the cumulative values of probability

$$P\{ \underset{\sim}{R} \leq r\} = y$$

example:

$$P\{ \underset{\sim}{R} \leq -2.25\ \%\} = 0.1$$



INTERPRETING THE *CDF*

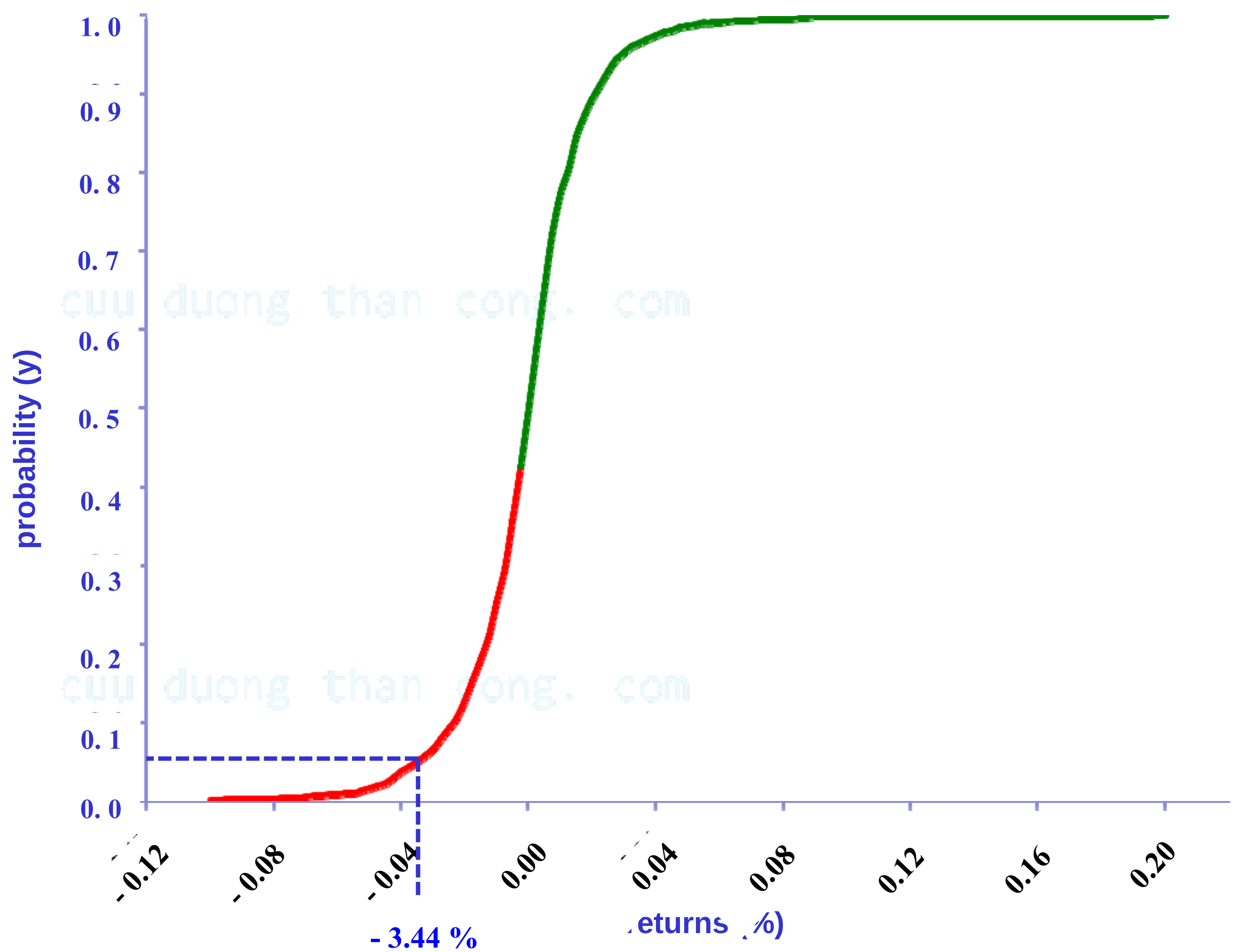
- We consider the data set to be a representative of the distribution of the population of trading days
- In the previous example, "the probability that $\underset{\sim}{R}$ is less than or equal to - 2.25 % is 0.1"
- By treating the complement of the probability value (0.1) as a "confidence level" (0.9), the above may be restated as "with a confidence level of 0.9, $\underset{\sim}{R}$ *will exceed* - 2.25 %"

UNDERSTANDING THE *CDF*

- In general, for any confidence level $(1-y)$, the information provided by the *CDF* allows us to determine the value r that $\underset{\sim}{R}$ exceeds based on the observations in the collected data
- For example, with a 0.95 confidence level, it follows from the *CDF* that $\underset{\sim}{R}$ exceeds $-3.44\ \%$
- We can interpret this to mean that with a confidence level of 0.95 we don't expect to lose more than $3.44\ \%$ in the worst case

CUMULATIVE DISTRIBUTION FUNCTION (*CDF*)

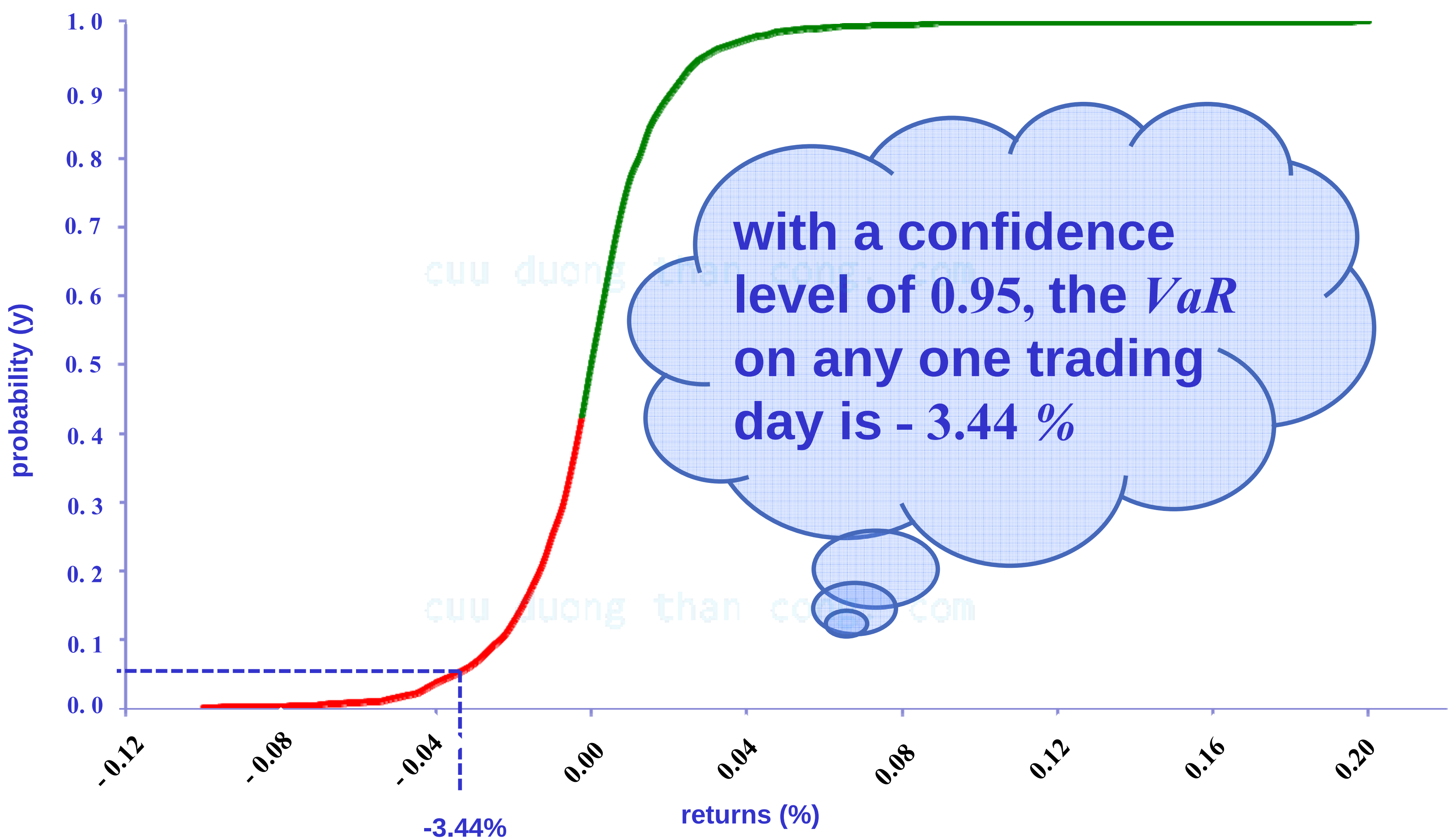
with a confidence level of 95 % we don't expect to lose more than 3.44 % in the worst case



VALUE-AT-RISK (*VaR*)

- Terminology: "With a confidence level of 0.95, the *VaR* on any one trading day is - 3.44 %" means that with a 0.95 percent confidence level, the return over two days cannot be below - 3.44 %
- A negative *VaR*, say $v < 0$, means that the *losses* on any one day cannot be greater than - v %
- *VaR* is a measure, of the return which would be exceeded based on the observations available for the given time period, with the specified confidence level

CUMULATIVE DISTRIBUTION FUNCTION (CDF)



VALUE-AT-RISK (*VaR*)

- *VaR* is usually expressed as a percentage value of the portfolio
- *VaR* answers the fundamental question facing a risk manager – on any given day, how much can we lose at the specified confidence level?
- The entire procedure can be extended to determine returns over any time period (e.g., two days, a week, or a month, etc.) and *VaR* can therefore be calculated for any such period

VALUE-AT-RISK (*VaR*)

- *VaR* is commonly used by banks, security firms and companies that are involved in trading energy and other commodities

- *VaR* is able to measure risk as it happens and is an important consideration when firms make trading or hedging decisions

ASSIGNMENT

- Pick any 5 stocks. Compose a 100-stock portfolio equally weighted (20 shares each) from each of the 5 stocks

- Obtain historic stock price data starting 1st January, 2002 (http://finance.yahoo.com)
- Calculate $\underset{\sim}{\Delta}$ and $\underset{\sim}{R}$ for each $\underset{\sim}{P}$ observation: assume that all dividends are reinvested to purchase more stock (fractional amounts, if necessary)

ASSIGNMENT

- Plot the Normalized Frequency Distribution and Cumulative Distribution Function for the data
- Compute the VaR for the confidence levels $95\ \%$ and $99\ \%$
- Interpret what these values mean specific to your chosen portfolio