# ECE 307 – Techniques for Engineering Decisions

## Value of Information



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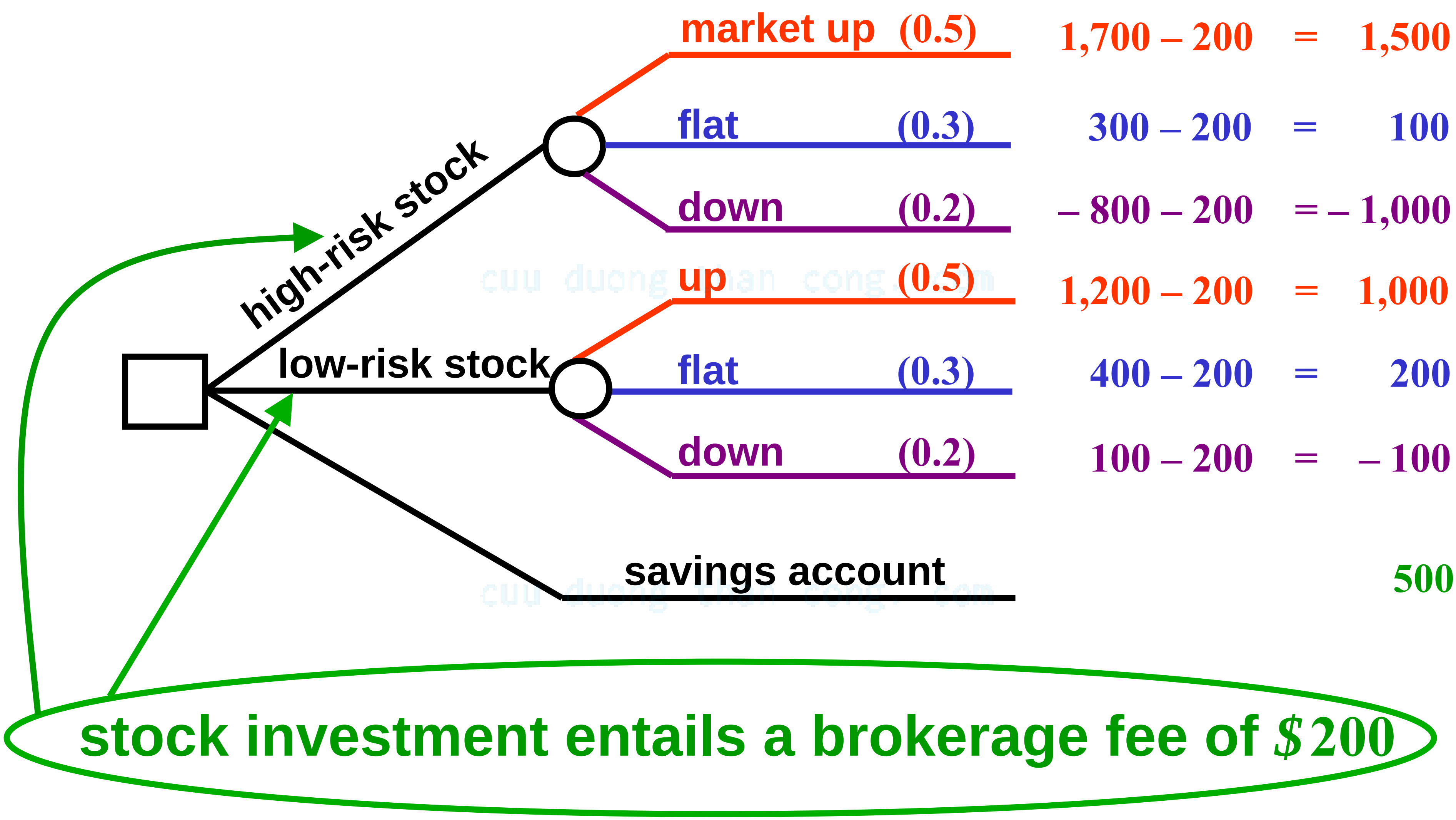
# VALUE OF INFORMATION

- While we cannot do away with uncertainty, there is always a desire to attempt to reduce the uncertainty about future outcomes
- The reduction in uncertainty about future outcomes may give us choices that improve chances for a good outcome
- We focus on the principles behind information valuation



# SIMPLE INVESTMENT EXAMPLE



# NOTION OF PERFECT INFORMATION

- **We say that an expert's information is perfect if it is always correct; we think of an expert as essentially a *clairvoyant***
- **We can place a value on information in a decision problem by measuring the expected value of info ( *EVI* )**

# NOTION OF PERFECT INFORMATION

- **We consider the role of *perfect information* in the simple investment example**
- **In this decision problem, the optimal policy is to invest in high – risk stock since it has the highest returns**
- **Suppose an expert predicts that the market goes up: this implies the investor still chooses the high – risk stock investment and consequently the *perfect information* of the expert appears to have no value**

# NOTION OF PERFECT INFORMATION

- **On the other hand, suppose the expert predicts a market decrease or a flat market: under this information, the investor's choice is the savings account and the *perfect information* has value because it leads to a *changed* outcome with im-proved results then would be the case otherwise**



- **In worst case conditions: regardless of the information, we take the same decision as**



# NOTION OF PERFECT INFORMATION

without the information and consequently $EVI = 0$; the interpretation is that we are equally well off without an expert

- Cases in which we have information and in which we change the optimal decision: these lead to $EVI > 0$ since we make a decision with an improved outcome using the available information

# *EVI* ASSESSMENT

- It follows that the value of information is always nonnegative, $EVI \geq 0$

- In fact, with *perfect information*, there is no uncertainty and the *expected value of perfect information EVPI* provides an upper bound for *EVI*

$$EVPI \geq EVI$$

# INVESTMENT EXAMPLE: COMPUTATION OF *EVPI*

- **Absent any expert information, a value – maximizing investor selects the high – risk stock investment**



- **The introduction of an expert or clairvoyant brings in *perfect information* since there is perfect knowledge of what the market will do before the investor makes his decision and the investor's decision is based on this information**

# COMPUTATION OF *EVPI*

- We use a decision tree approach to compute *EVPI* by **reversing the decision and uncertainty order:** we view the value of information in an *a priori* sense and define

$$EVPI = E\left\{decision\ with\ perfect\ information\right\} - E\left\{decision\ without\ information\right\}$$

# COMPUTATION OF *EVPI*

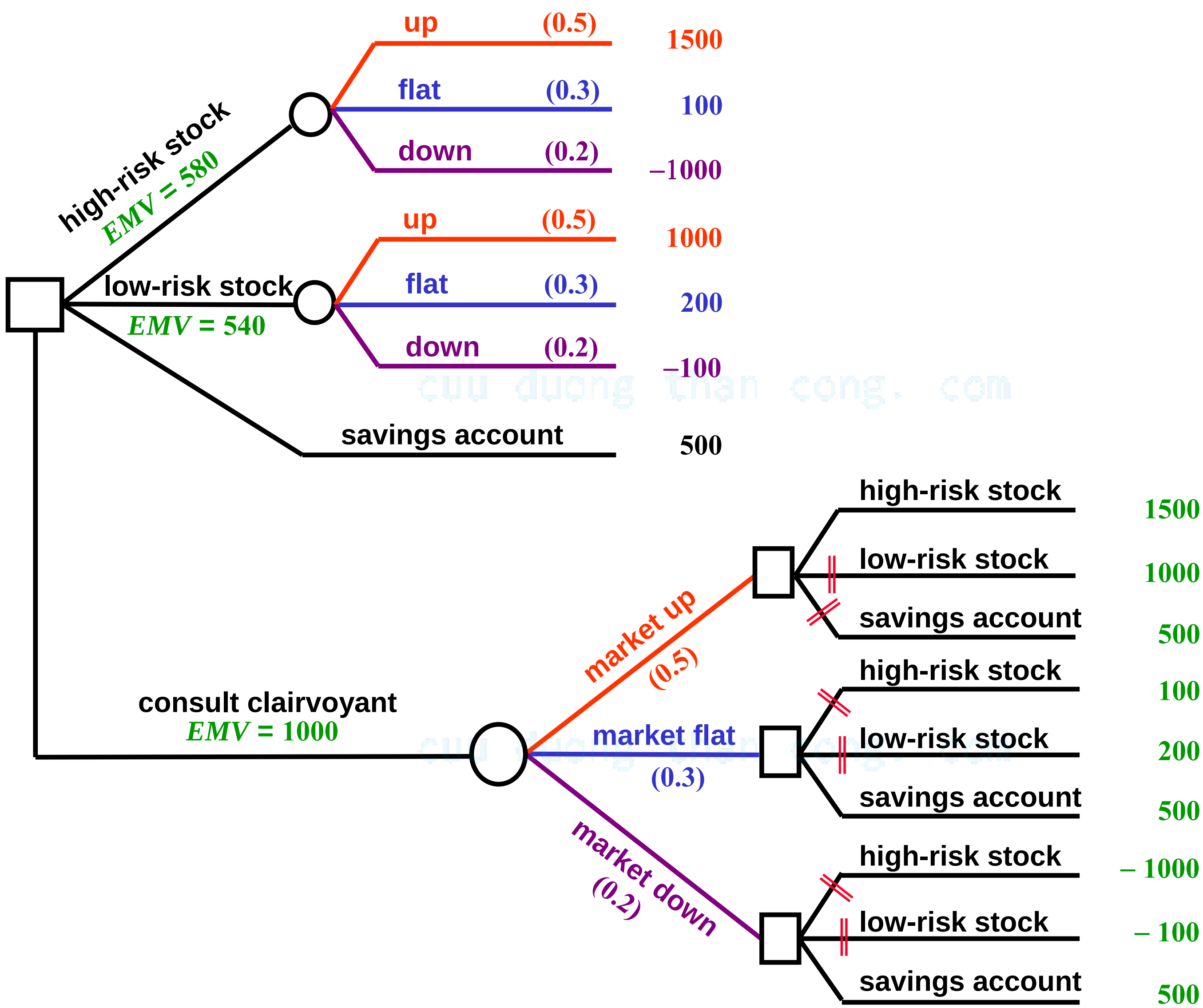
- For the investment problem,

$$EVPI = 1{,}000 - 580 = 420$$



- We may view *EVPI* to represent the maximum amount that the investor should be willing to pay the expert for the *perfect information* resulting in the improved outcome

# COMPUTATION OF *EVPI*



# EXPECTED VALUE OF IMPERFECT INFORMATION

- **In practice, we cannot obtain *perfect information*; rather, the information is *imperfect* since there are no clairvoyants**



- **We evaluate the expected value of *imperfect information, EVII***
- **For example we engage an economist to fore–cast the future stock market trends; his forecasts constitute *imperfect information***

# EXPECTED VALUE OF IMPERFECT INFORMATION

*conditioning event*

| *economist's prediction* | *true market state* | | |
|---|---|---|---|
| | *up* | *flat* | *down* |
| "*up*" | 0.8 | 0.15 | 0.2 |
| "*flat*" | 0.1 | 0.7 | 0.2 |
| "*down*" | 0.1 | 0.15 | 0.6 |

conditional probabilities

*P{ "flat"| market is flat }*

# *EVII* ASSESSMENT

- We use the decision tree approach to compute *EVII*
- For the decision tree, we evaluate probabilities using Bayes' theorem
- For the imperfect information, we define

$$\underset{\sim}{M} = \begin{matrix}\text{market}\\ \text{performance}\end{matrix} = \begin{cases} \text{up} & \text{with probability } 0.5 \\ \text{flat} & \text{with probability } 0.3 \\ \text{down} & \text{with probability } 0.2 \end{cases}$$



# *EVII* ASSESSMENT

and the forecast *r.v.*

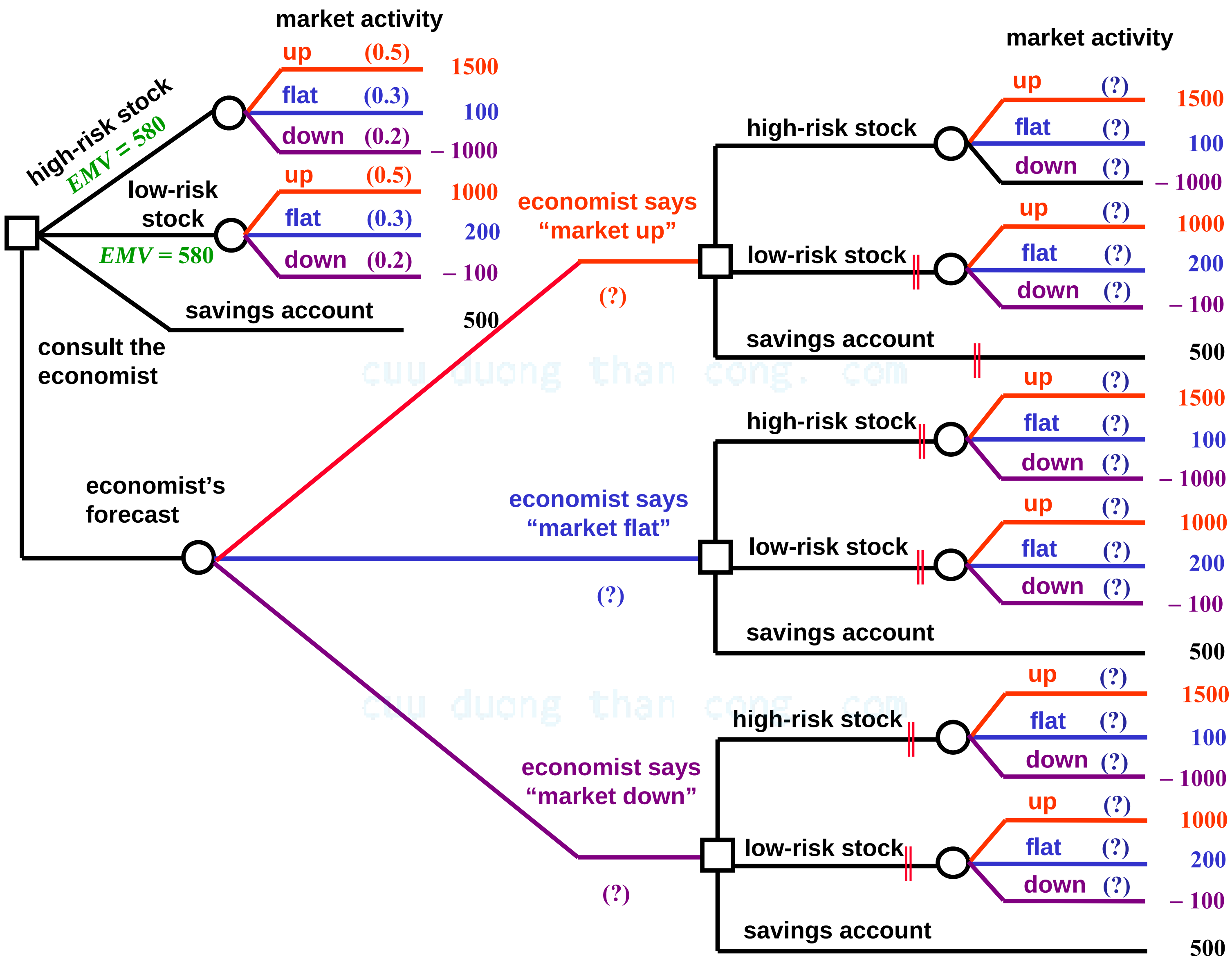
$$\underset{\sim}{F} = \begin{cases} \text{"up"} \\ \text{"flat"} \\ \text{"down"} \end{cases}$$



without the knowledge of the corresponding

probabilities of the two *r.v.*s

# *EVII* COMPUTATION: INCOMPLETE DECISION TREE



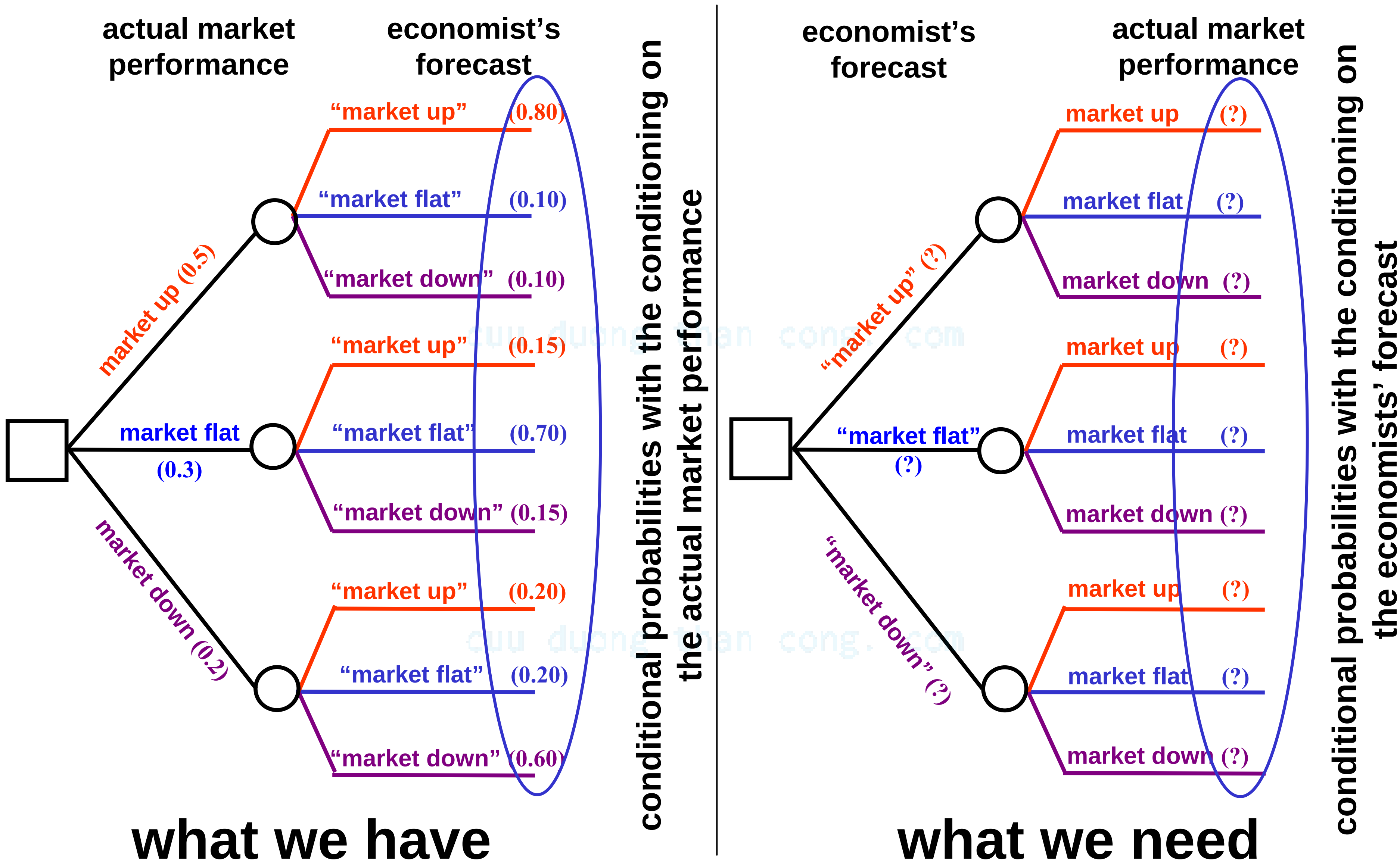
# COMPUTATION OF REVERSE CONDITIONAL PROBABILITIES

$$P\{\underset{\sim}{M} = \text{down} \mid \underset{\sim}{F} = \text{"up"}\} = \frac{P\{\underset{\sim}{F} = \text{"up"} \mid \underset{\sim}{M} = \text{down}\}\, P\{\underset{\sim}{M} = \text{down}\}}{\left[\begin{array}{l} P\{\underset{\sim}{F} = \text{"up"} \mid \underset{\sim}{M} = \text{down}\}\, P\{\underset{\sim}{M} = \text{down}\} + \\ P\{\underset{\sim}{F} = \text{"up"} \mid \underset{\sim}{M} = \text{down}\}\, P\{\underset{\sim}{M} = \text{up}\} + \\ P\{\underset{\sim}{F} = \text{"up"} \mid \underset{\sim}{M} = \text{flat}\}\, P\{\underset{\sim}{M} = \text{flat}\} \end{array}\right]}$$

$$P\{\underset{\sim}{F} = \text{"up"}\} = \frac{0.2(0.2)}{0.2(0.2) + 0.5(0.3) + 0.8(0.5)}$$

**we *flip* the probabilities in this way**

# *EVII* COMPUTATION: FLIPPING THE PROBABILITY TREE



# POSTERIOR PROBABILITIES

| *economist's prediction* | *posterior probability for:* | | |
|---|---|---|---|
| | *market up* | *market flat* | *market down* |
| "*up*" | **0.8247** | **0.0928** | **0.0825** |
| "*flat*" | **0.1667** | **0.7000** | **0.1333** |
| "*down*" | **0.2325** | **0.2093** | **0.5581** |

**conditional probabilities on economists forecast**

# *EVII* COMPUTATION

- We use conditional probabilities in the table to build the posterior probabilities
- For example



$$P\left\{\text{market up} \middle| \text{economist predicts "up"}\right\} = 0.8247$$

- We then compute

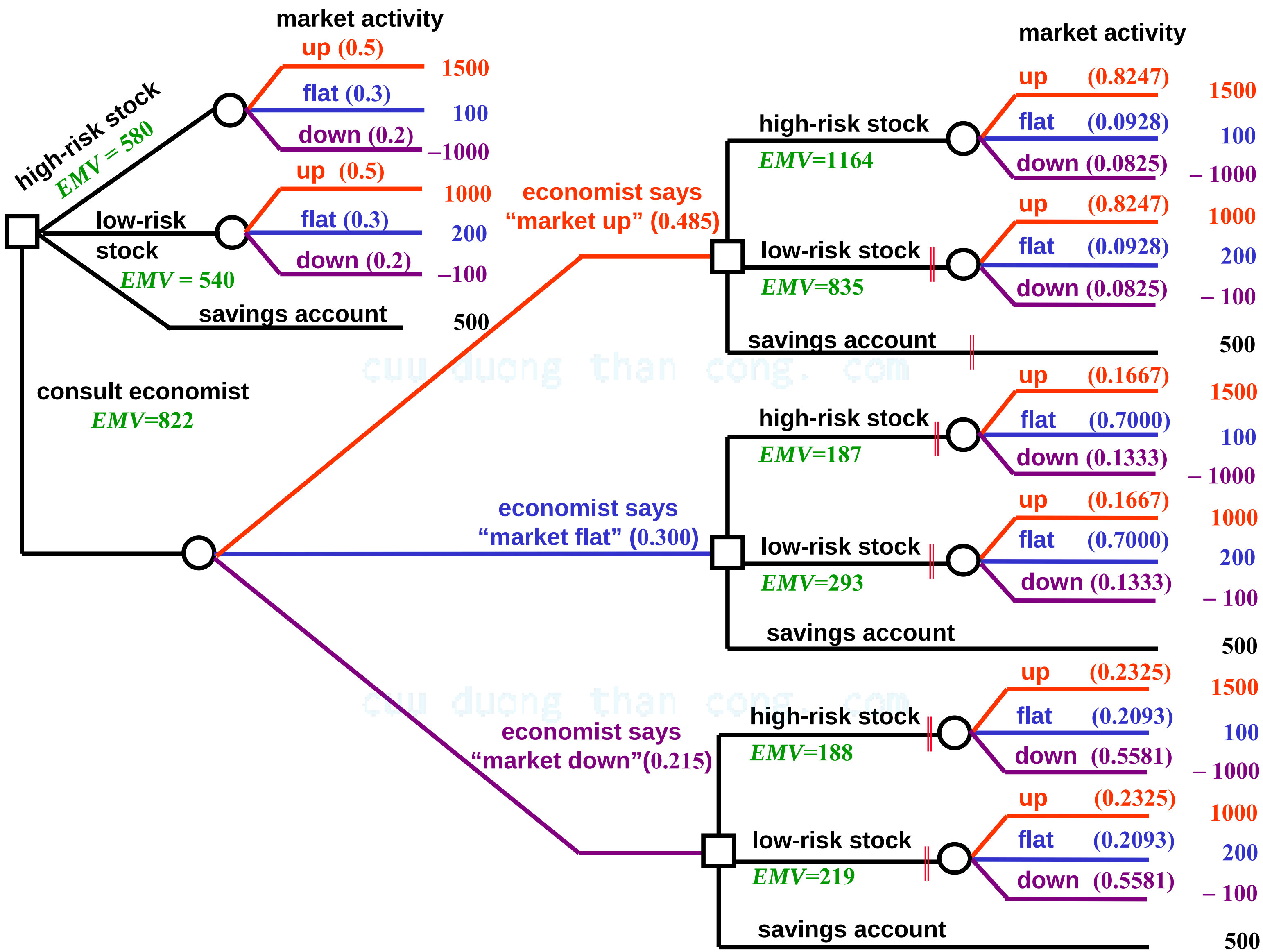
$$P\left\{\underset{\sim}{F} = \text{"up"}\right\} = 0.485$$



$$P\left\{\underset{\sim}{F} = \text{"flat"}\right\} = 0.300$$

$$P\left\{\underset{\sim}{F} = \text{"down"}\right\} = 0.215$$

# EXPECTED VALUE OF IMPERFECT INFORMATION



# *EVII* COMPUTATION

- The expected mean value for the decision made with the economist information is

$$EMV\,|_{economist} = 1{,}164(0.485) + 500(0.515) = 822$$

- The expected mean value without information is 580

- Consequently,

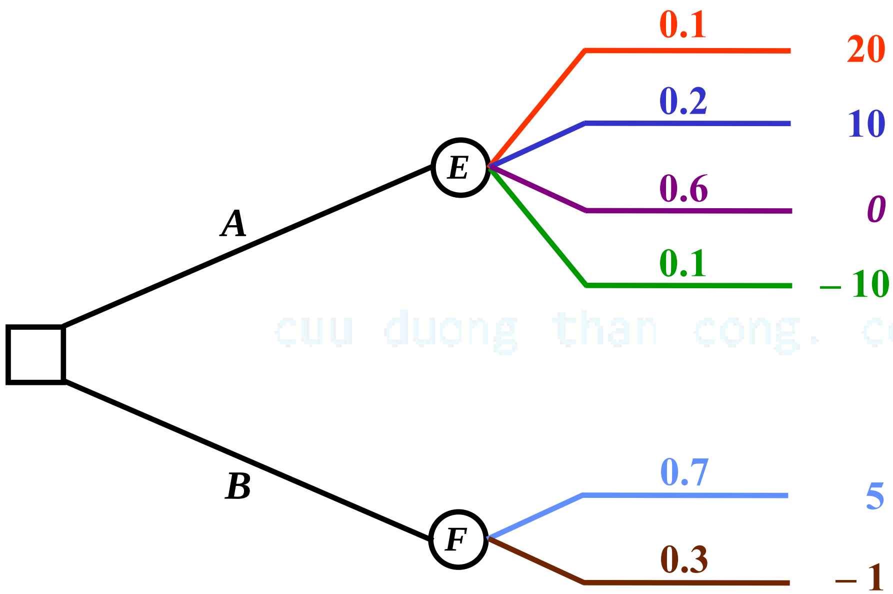
$$EVII = 822 - 580 = 242$$

- This value represents the upper limit on the worth of the economist's forecast

# EXAMPLE OF VALUE OF INFORMATION

- We consider the following decision tree



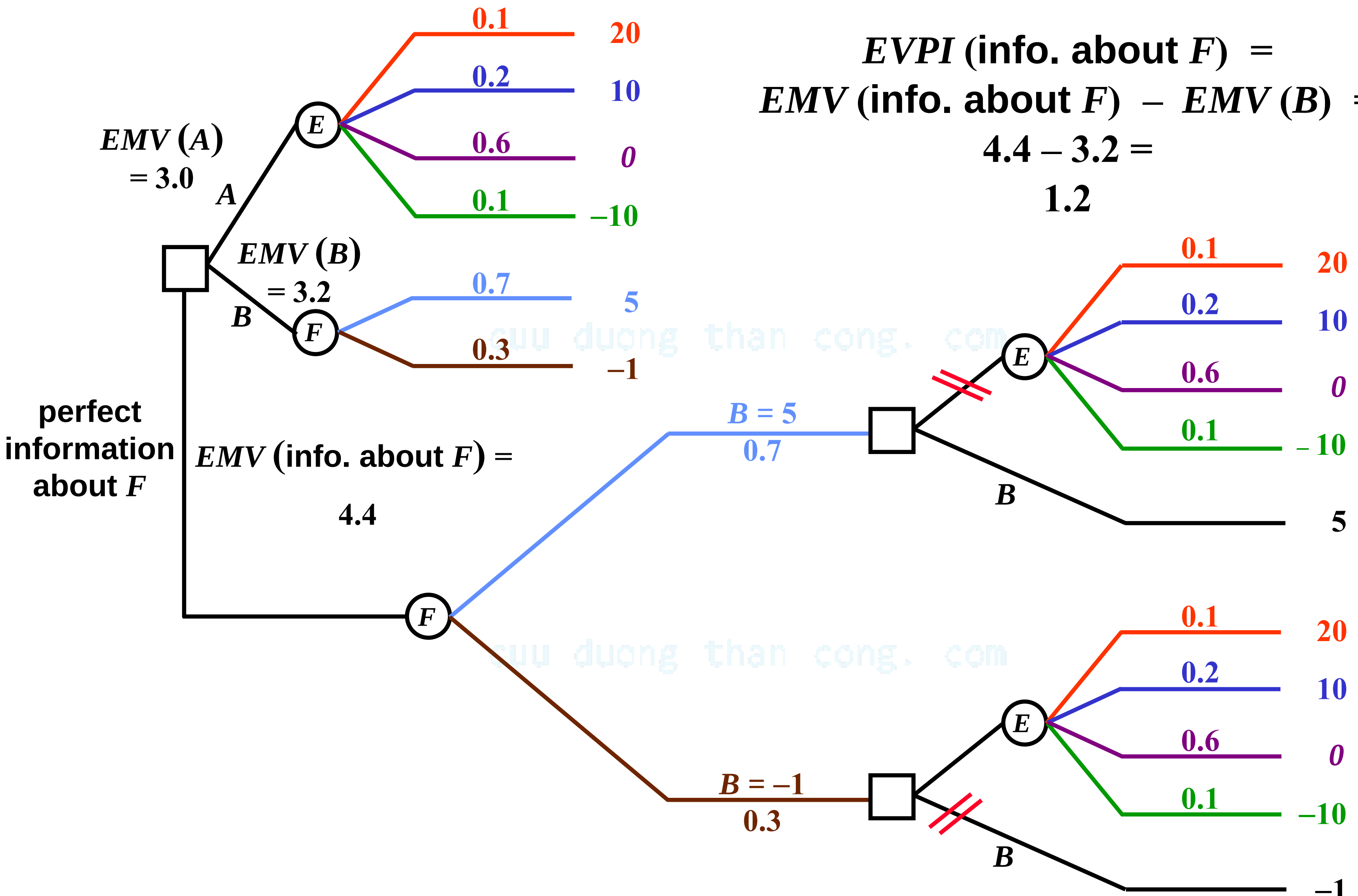

with the events at $E$ and $F$ as independent



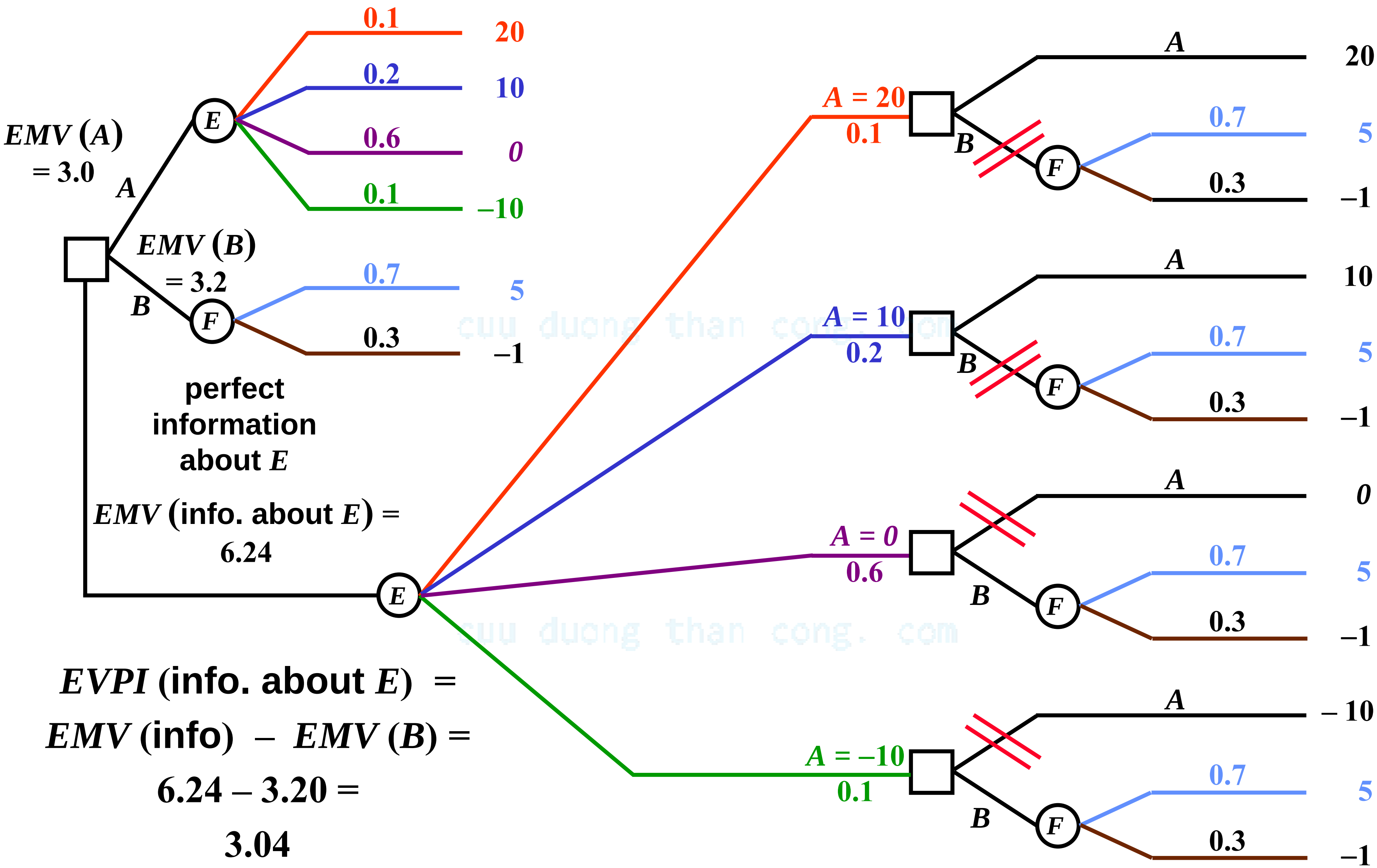
- We perform a number of valuations of $EVPI$ for this simple decision problem

# *EVPI* FOR *F* ONLY

*EVPI* (info. about *F*) =
*EMV* (info. about *F*) – *EMV* (*B*) =
4.4 – 3.2 =
1.2

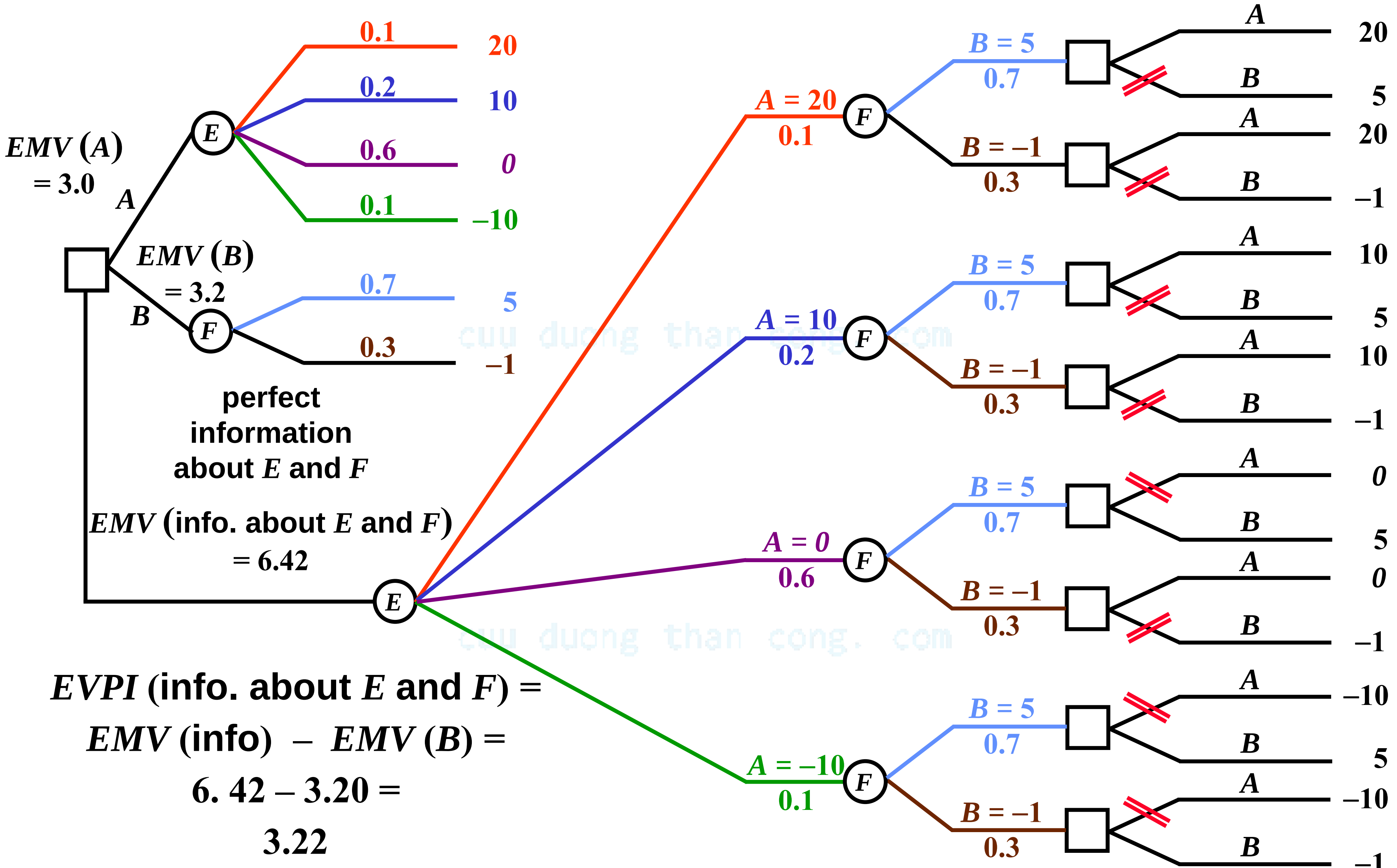


# *EVPI* FOR *E* ONLY




*EVPI* (info. about *E*) = *EMV* (info) – *EMV* (*B*) = 6.24 – 3.20 = 3.04

# *EVPI* FOR *E* AND *F* ONLY




*EVPI* (info. about *E* and *F*) =

*EMV* (info) – *EMV* (*B*) =

6. 42 – 3.20 =

3.22