Chapter 3

Scattering Matrix

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Trinh Xuan Dung, PhD
dung.trinh@hcmut.edu.vn

Department of Telecommunications
Faculty of Electrical and Electronics Engineering
Ho Chi Minh city University of Technology

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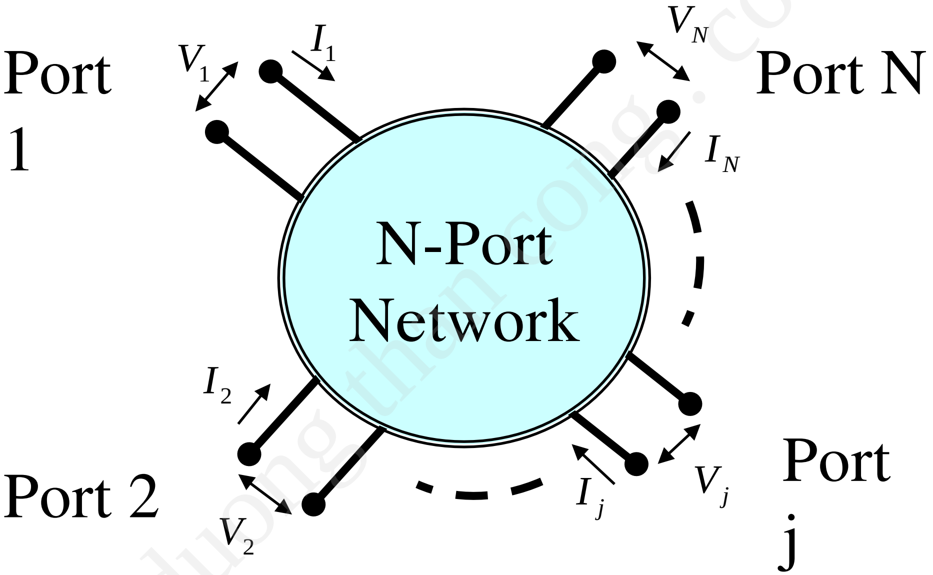

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$$[\bar{V}] = [\bar{\bar{Z}}][\bar{I}]$$

$$[\bar{I}] = [\bar{\bar{Y}}][\bar{V}]$$

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

1. Introduction

Review of [Z], [Y], [ABCD] matrixs

[Z] Matrix

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

[Y] Matrix

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

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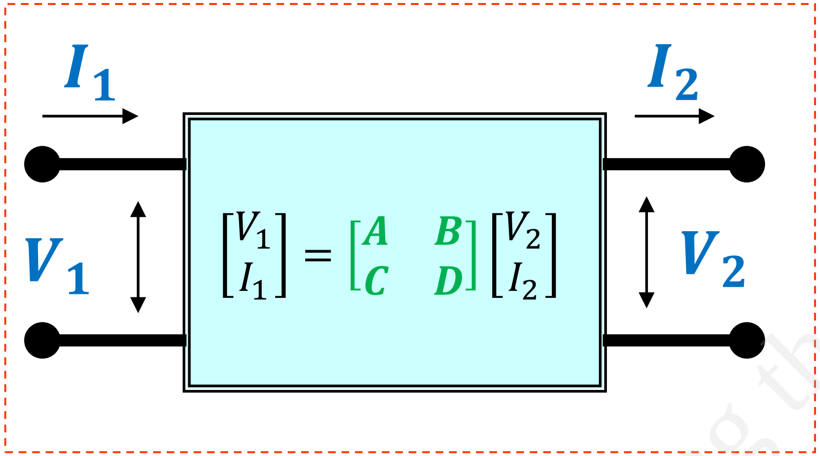
$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \quad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \quad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \quad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

1. Introduction

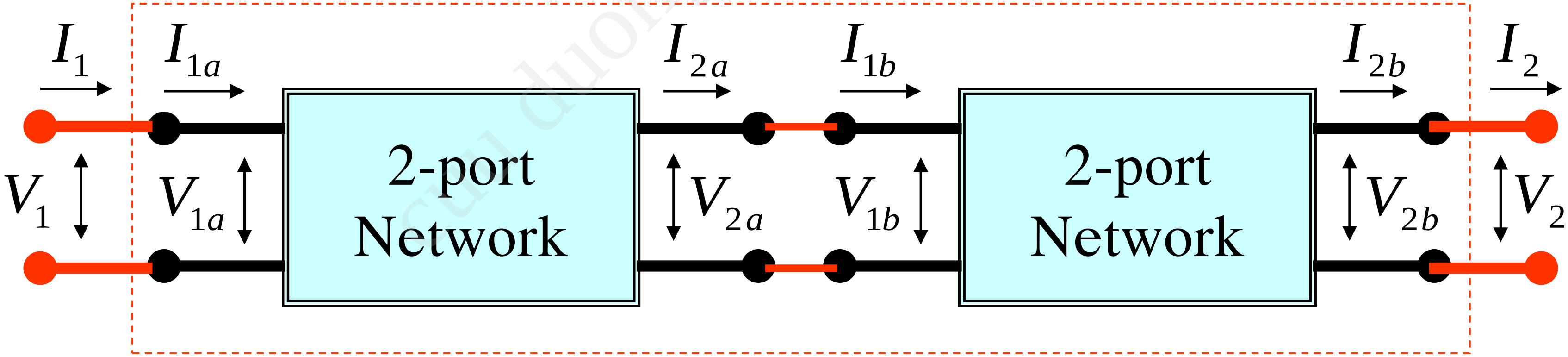
Review of [Z], [Y], [ABCD] matrixs



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2=0} \qquad B = \left. \frac{V_1}{I_2} \right|_{V_2=0}$$

$$C = \left. \frac{I_1}{V_2} \right|_{I_2=0} \qquad D = \left. \frac{I_1}{I_2} \right|_{V_2=0}$$



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_a \begin{bmatrix} A & B \\ C & D \end{bmatrix}_b \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

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1. Introduction

- ❖ [Z],[Y] or [ABCB] are not preferred to use for microwave networks.
- ❖ Open and short conditions are difficult to achieve at microwave → error in measurement.
- ❖ Open and short terminations may make active circuits unstable (oscillation) → error or cannot meas.
- ❖ Microwave circuit analysis and design are based on the transmission & reflection of waves. [Z] or [Y] do not reflect these natures.
- ❖ Impedance and admittance matrices relate the total voltages and currents at the ports.
- ✓ At microwave regime: Voltages, currents and power are represented in term of incident and reflect qualities.
- ✓ S-parameters matrix, defined in terms of traveling waves, is used instead.
- ✓ The scattering matrix represents the relation between the voltage incident waves on the ports to voltage reflected wave from the ports.
- ✓ S-parameters are measured with matched loads rather than open- or short-circuits.
- ✓ At microwave frequencies, matched loads are relatively easy to realize.
- ✓ S-parameters are measured using Vector Network Analyzer (VNA).

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1. Introduction

Vector Network Analyzer (VNA)

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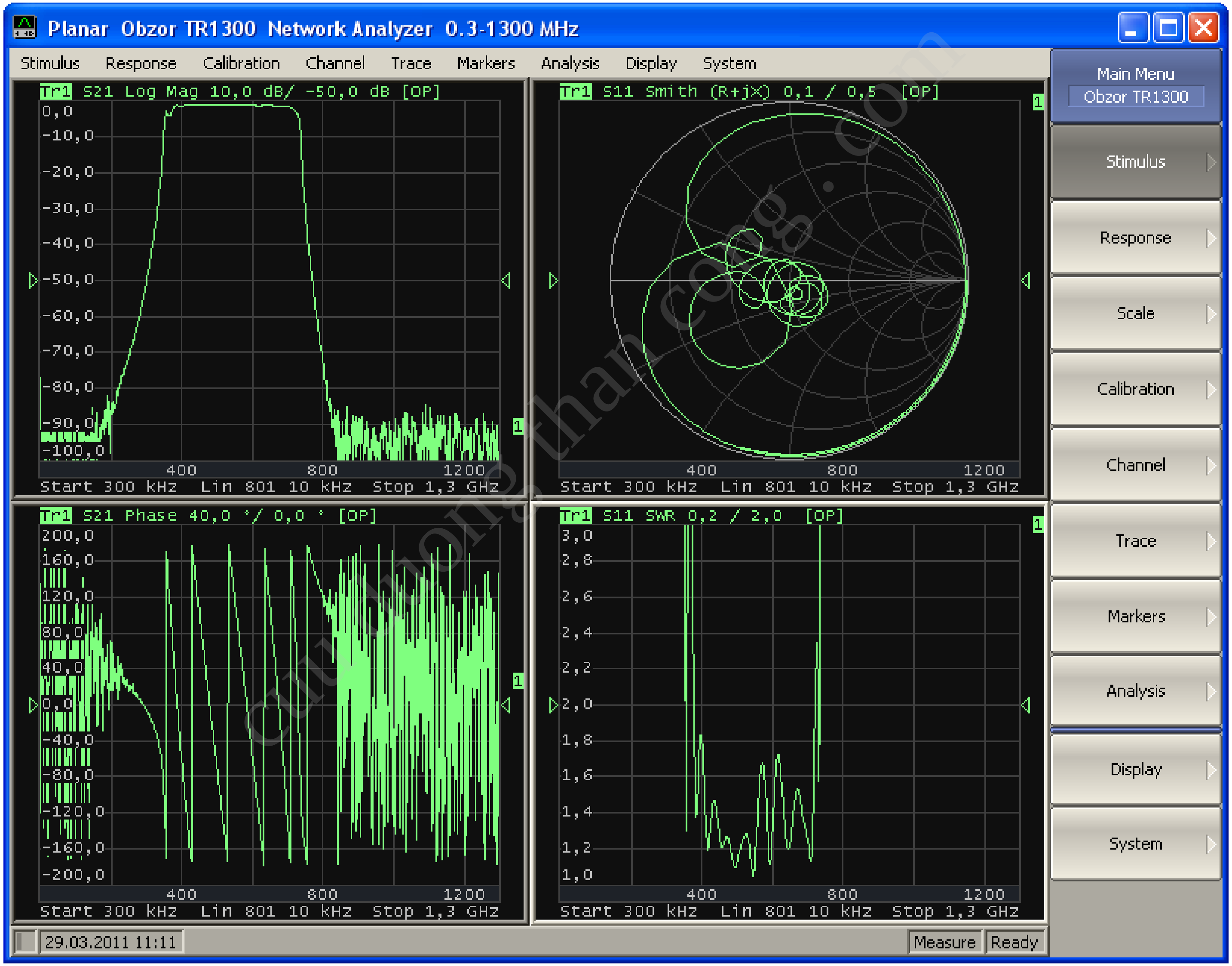
1. Introduction



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2. S Parameter Definition

- V_n^+: the **incident** signal voltage (wave) entering **port n**.
- V_n^-: the **reflected** signal voltage (wave) from **port n**.
- The scattering matrix, or **[S]** matrix, is defined in relation to these incident and reflected voltage waves as:

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$$\boxed{S_{ij} = \left.\frac{V_i^-}{V_j^+}\right|_{V_k^+=0, k\neq j}}$$

- The incident waves on all ports = 0 → **all ports are terminated in matched loads**.

- S_{ii} reflection coefficient looking into port i when all other ports are matched.
- S_{ij} transmission coefficient looking from port j to port i when all other ports are matched.

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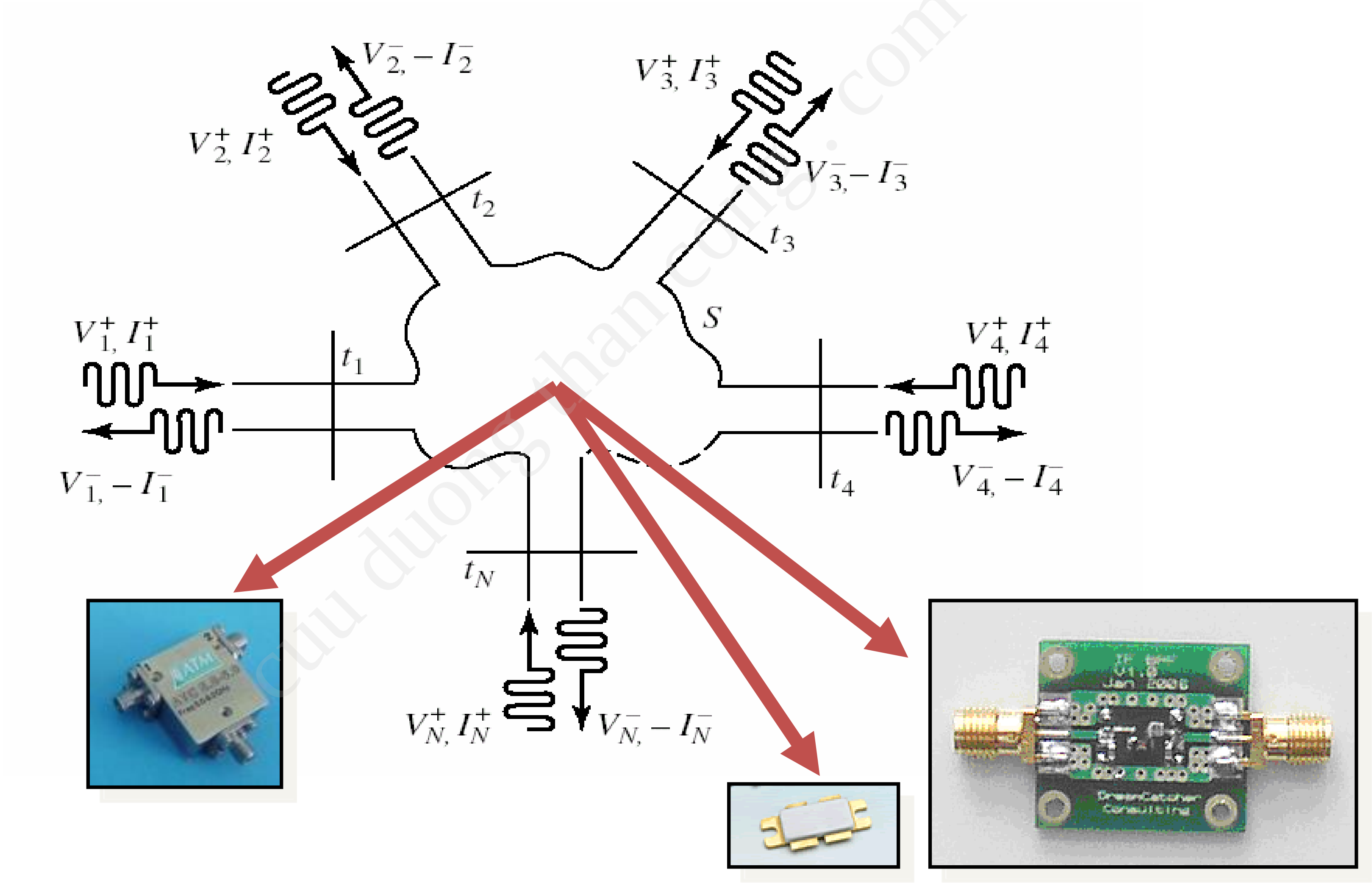
2. S Parameter Definition



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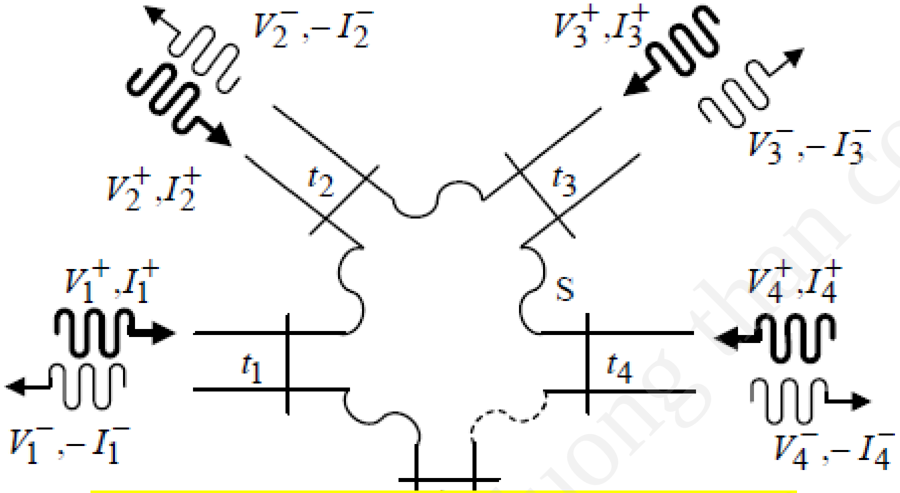
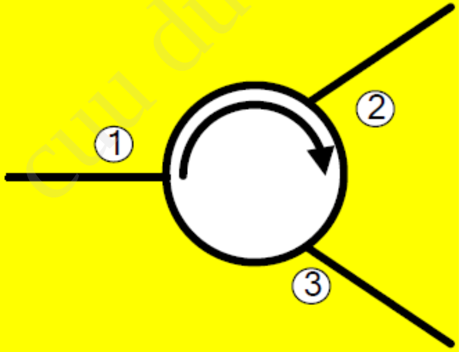
2. S Parameter Definition

- **N-Port Network**

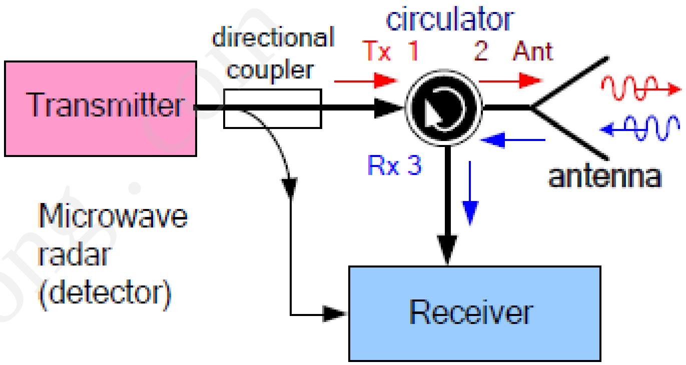


$$[S]=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix}$$

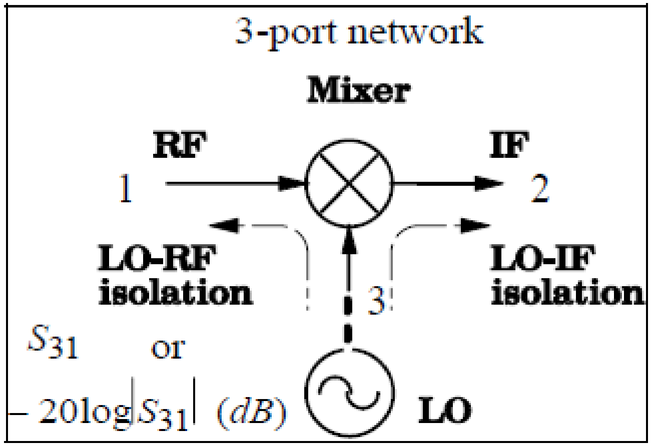


$S_{21}=S_{32}=1 \Rightarrow$ signal flowing $1\rightarrow 2$ & $2\rightarrow 3$

$S_{31}=S_{23}=0 \Rightarrow$ no signal flowing $1\rightarrow 3$ & $3\rightarrow 2$



3-port microwave circulator to let the radar use the same freq. for Tx & Rx (Tx- & Rx-port isolation S_{31} is important for receiving sensitivity)



S_{31} or $-20\log|S_{31}|$ (dB)

$* \; f_{RF} \neq f_{LO}$ & usually $\gg f_{IF}$

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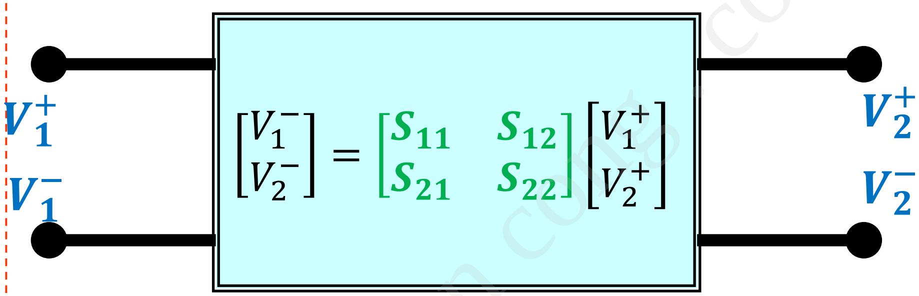
2. S Parameter Definition

- For a two port microwave network:



$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

- V_1^+: the **incident** signal voltage (wave) entering the **input port 1**.
- V_1^-: the **reflected** signal voltage (wave) from the **input port 1**.
- V_2^+: the **incident** signal voltage (wave) entering the **output port 2**.
- V_2^-: the **reflected** signal voltage (wave) from the **output port 2**.

○ S_{11}: **input reflection coefficient** $= V_1^- / V_1^+ \|_{V_2^+=0} = \Gamma_{in}$

○ S_{21}: forward transmission coefficient $= V_2^- / V_1^+ \|_{V_2^+=0} = Forward\ Gain$

○ S_{12}: reversed transmission coefficient $= V_1^- / V_2^+ \|_{V_1^+=0} = Reverse\ Gain, Isolation$

○ S_{22}: **output reflection coefficient** $= V_2^- / V_2^+ \|_{V_1^+=0} = \Gamma_{out}$

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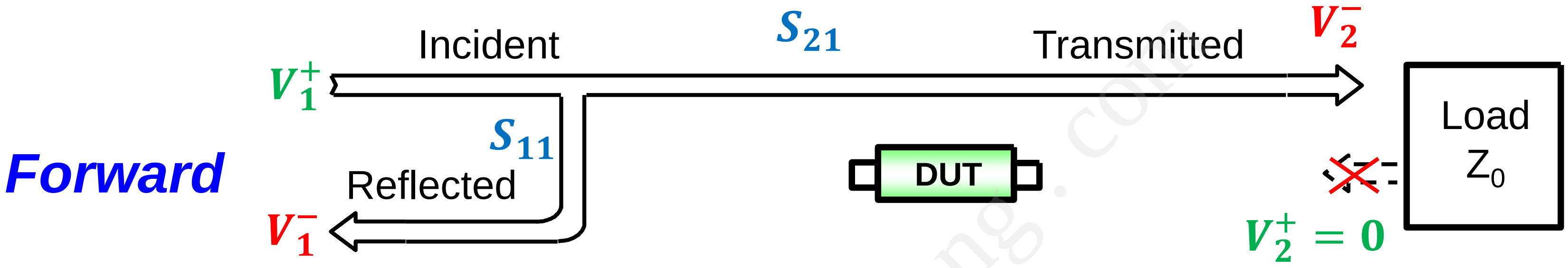
2. S Parameter Definition

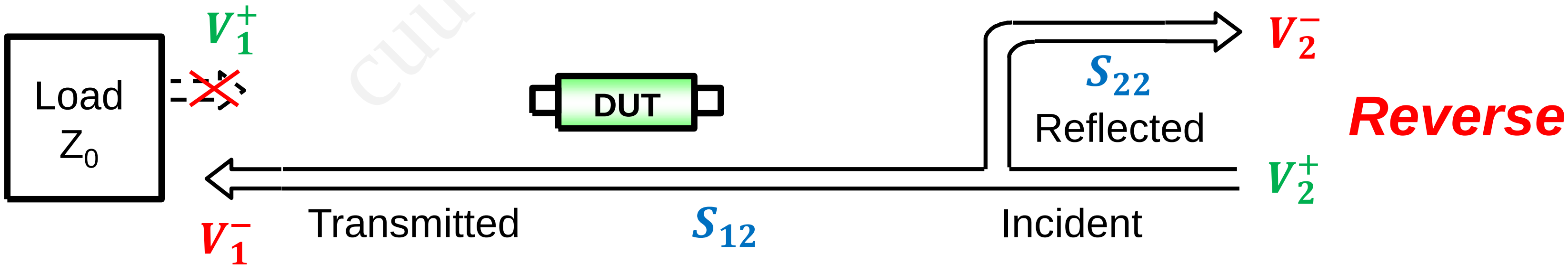


$$S_{11} = \frac{\mathit{Reflected}}{\mathit{Incident}} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$$

$$S_{21} = \frac{\mathit{Transmitted}}{\mathit{Incident}} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$$

$$S_{22} = \frac{\mathit{Reflected}}{\mathit{Incident}} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0}$$

$$S_{12} = \frac{\mathit{Transmitted}}{\mathit{Incident}} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0}$$



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3. S Matrix Properties

❖ Reciprocal Network

- In the case of reciprocal networks, it can be shown that: $[S] = [S]^T$

where $[S]^T$ indicates the transpose of $[S]$. In other words, the equation is symmetric about the main diagonal.

❖ Lossless Network

- If the network is lossless, no real power can be delivered to the network

$$P_{avg} = \frac{1}{2}\Re e\{[V]^T[I]^*\} = \frac{1}{2Z_0}\Re e\left\{\left([V^+]^T + [V^-]^T\right)\left([V^+]^* - [V^-]^*\right)\right\}$$

$$= \frac{1}{2Z_0}\Re e\left\{\left([V^+]^T[V^+]^* - [V^+]^T[V^-]^* + [V^-]^T[V^+]^* - [V^-]^T[V^-]^*\right)\right\}$$

$$= \frac{1}{2Z_0}\left(\underbrace{[V^+]^T[V^+]^*}_{\text{Total Incident Power}} - \underbrace{[V^-]^T[V^-]^*}_{\text{Total Reflected Power}}\right) = 0$$

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3. S Matrix Properties

- **Lossless Network:** $[V^+]^T[V^+]^* = [V^-]^T[V^-]^*$
- Using: $[V^-] = [S][V^+] \leftrightarrow [V^+]^T[V^+]^* = [V^+]^T[S]^T[S]^*[V^+]^*$
- So that for non-zero: $[S]^T[S]^* = [U]$ or $[S]^* = \{[S]^T\}^{-1}$
- Expanding the above equation, we obtain:

$$\begin{bmatrix} S_{11} & S_{21} & \dots & S_{N1} \\ S_{12} & S_{22} & \dots & S_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ S_{1N} & S_{2N} & \dots & S_{NN} \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & \dots & S_{1N}^* \\ S_{21}^* & S_{22}^* & \dots & S_{2N}^* \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1}^* & S_{N2}^* & \dots & S_{NN}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

- This condition is equivalent to:

$$\sum_{k=1}^{N} S_{ki}S_{ki}^* = 1, for\ i = 1, \dots, N. \qquad \sum_{k=1}^{N} S_{ki}S_{kj}^* = 0, for\ i \neq j.$$

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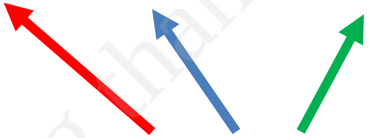
3. S Matrix Properties

- In other words, each column of the scattering matrix will have a magnitude equal to one

$$\begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix}$$



Matrix columns

while the inner product of dissimilar columns must be **zero**.

- Example of a unitary scattering matrix of a lossless device:

$$\begin{bmatrix} 0 & {}^{1}/_{2} & j\,{}^{\sqrt{3}}/_{2} & 0 \\ {}^{1}/_{2} & 0 & 0 & j\,{}^{\sqrt{3}}/_{2} \\ j\,{}^{\sqrt{3}}/_{2} & 0 & 0 & {}^{1}/_{2} \\ 0 & j\,{}^{\sqrt{3}}/_{2} & {}^{1}/_{2} & 0 \end{bmatrix}$$

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3. S Matrix Properties

Example 1: The S matrix of a component is: $S = \begin{bmatrix} 0.1 & j0.8 \\ j0.8 & 0.2 \end{bmatrix}$

a. Is the network reciprocal?
b. Is the network lossless?

a. The network is reciprocal because $[S]^T = [S]$

b. The network is lossy.

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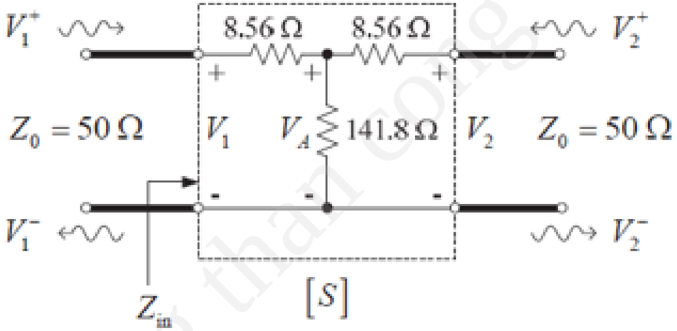
3. S Matrix Properties

Example 3: Given the following T network. Compute the scattering matrix of the network?



- The network is reciprocal because $[S]^T = [S]$

$$S_{22} = S_{11} = \Gamma_{11} = V_1^- / V_1^+ \Big|_{V_2^+=0} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = 0$$

$$S_{21} = S_{12} = V_2^- / V_1^+ \Big|_{V_2^+=0} = \frac{1}{\sqrt{2}}$$

- The T network is a 3dB attenuator!

30dB 5W RF Attenuator DC-18GHz

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3. S Matrix Properties

Example 4: The two port network has the following scattering matrix:

$$S = \begin{bmatrix} 0.15\angle 0^0 & 0.85\angle -45^0 \\ 0.85\angle 45^0 & 0.2\angle 0^0 \end{bmatrix}$$

a. Determine if the network is reciprocal and lossless?
b. If port 2 is terminated with a matched load, what is the return loss seen at port 1?
c. If port 2 is terminated with a short circuit, what is the return loss seen at port 1?

a. The network is NOT reciprocal because $[S]^T \neq [S]$

The network is lossy because: $|S_{11}|^2 + |S_{21}|^2 = 0.745 \neq 1$

b. Return loss seen at port 1: $RL = -20log_{10}|\Gamma_{11}| = 16.5(dB)$

c. $\Gamma_{11} = V_1^- / V_1^+ = S_{11} - S_{12}\frac{V_2^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}} = -0.452$

Return loss seen at port 1: $RL = -20log_{10}|\Gamma_{11}| = 6.9(dB)$

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4. Time Average Power

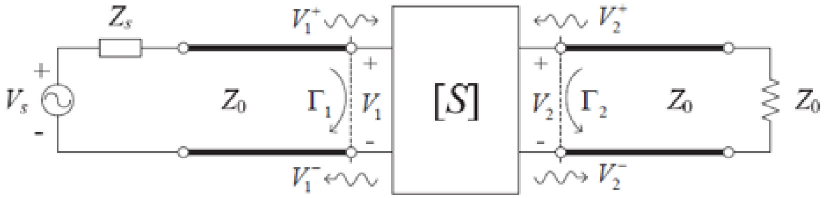
- At port 1, the voltage is: $V_1 = V_1^+ + V_1^-$
- The total time average power at that port is comprised of the two terms:

$$P_{inc} = \frac{|V_1^+|^2}{2Z_0} \qquad P_{ref} = \frac{|V_1^-|^2}{2Z_0}$$

- Since port 2 is matched, then: $V_2^+ = 0$. Consequently:

$$P_{trans}\Big|_{V_2^+=0} = \frac{|V_2^-|^2}{2Z_0}$$

- **At port 1:** the Input Return Loss is defined as: $\frac{P_{ref}}{P_{inc}}\Big|_{V_2^+=0} = \frac{|V_1^-|^2}{|V_1^+|^2} = |S_{11}|^2$
- **At port 2:** $\frac{P_{trans}}{P_{inc}}\Big|_{V_2^+=0} = \frac{|V_2^-|^2}{|V_1^+|^2} = |S_{21}|^2$



Assume ΔL=0 !

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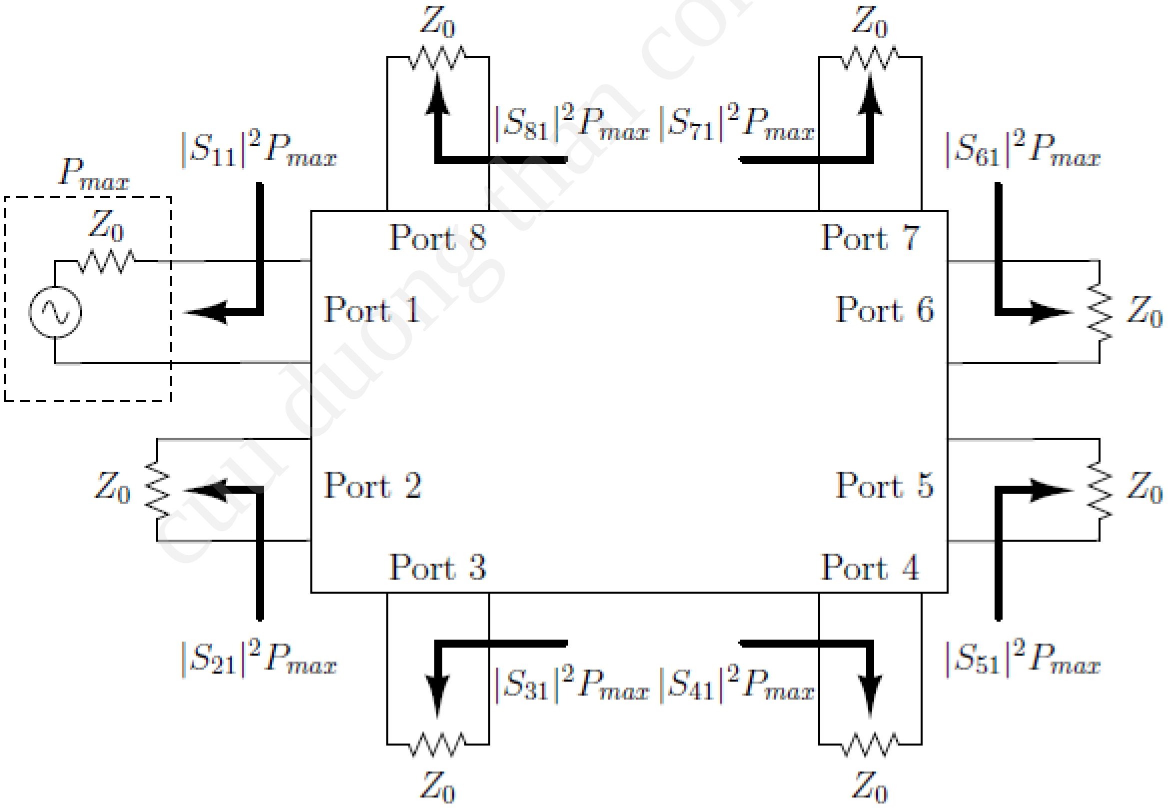
4. Time Average Power

- When all ports have equal characteristic impedance, the power delivered to the load is simply given by:

$$P_{out} = P_{max}|S_{21}|^2$$



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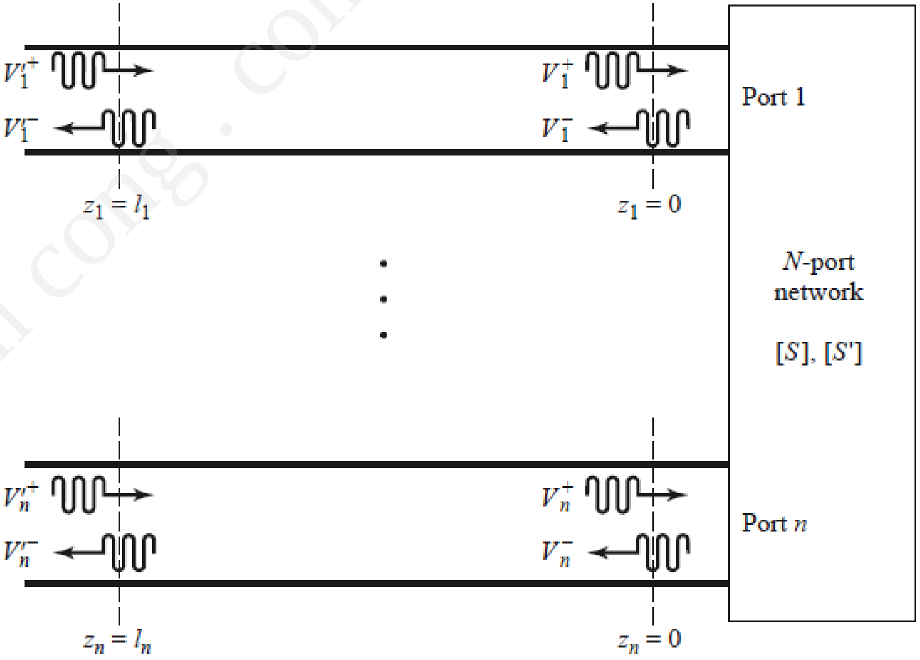
4. Time Average Power

❖ A shift in reference planes



- In terms of the incident and reflected port voltages we have:

$$[V^-] = [S][V^+]$$

$$[V'^-] = [S'][V'^+]$$

- From the theory of travelling waves on lossless T.L, we relate:

$$V_n'^+ = V_n^+ e^{j\beta_n l_n} \qquad V_n'^- = V_n^- e^{-j\beta_n l_n}$$

- This gives: $$[V^-] = \begin{bmatrix} e^{-j\theta_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & e^{-j\theta_N} \end{bmatrix} [V^+] = [S'][V'^+]$$

- This equivalent to: $S'_{nn} = e^{-j2\theta_N} S_{nn}$

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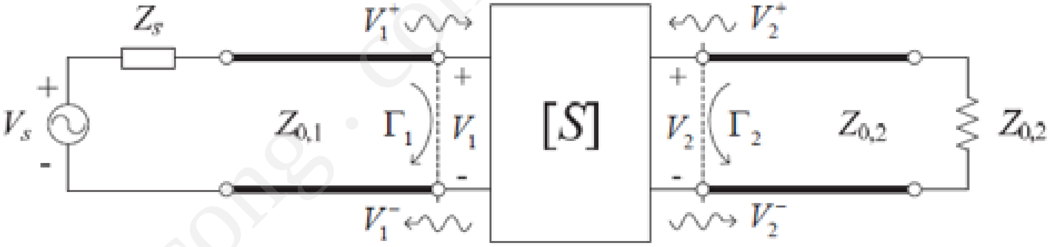
5. Generalized Parameters

- If the **characteristic impedances are different** for some ports, it becomes necessary to redefine the scattering parameters so that $|S_{ij}|^2$ still relates to the time average power flow.



- The time average incident, reflected and transmitted power are, respectively

$$P_{inc} = \frac{|V_1^+|^2}{2Z_{0,1}} \qquad P_{ref} = \frac{|V_1^-|^2}{2Z_{0,1}} \qquad P_{trans}\Big|_{V_2^+=0} = \frac{|V_2^-|^2}{2Z_{0,2}}$$

- Consequently, $|S_{11}|^2 = \frac{P_{ref}}{P_{inc}}\Big|_{V_2^+=0} = \frac{|V_1^-|^2}{|V_1^+|^2}$

$$|S_{21}|^2 = \frac{P_{trans}}{P_{inc}}\Big|_{V_2^+=0} = \frac{|V_2^-|^2 / Z_{0,2}}{|V_1^+|^2 / Z_{0,1}}$$

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5. Generalized Parameters

- To preserve the useful interpretation of $\left|S_{ij}\right|^2$ as a relative time average power flow with matched ports, we need to redefine the S parameters when the ports are not equal.
- The "wave amplitude" toward port n is defined as

$$a_n \equiv {V_n^+}/{\sqrt{Z_{0,n}}}$$

- The "wave amplitude" away from port n is defined as

$$b_n \equiv {V_n^-}/{\sqrt{Z_{0,n}}}$$

- Then

$$S_{ij} = {b_i}/{a_j}\Big|_{a_k=0,k\neq j}$$

- And it can be shown that:

$$a_n = \frac{1}{2\sqrt{Z_{0,n}}}\left(V_n + Z_{0,n}I_n\right)$$

$$b_n = \frac{1}{2\sqrt{Z_{0,n}}}\left(V_n - Z_{0,n}I_n\right)$$

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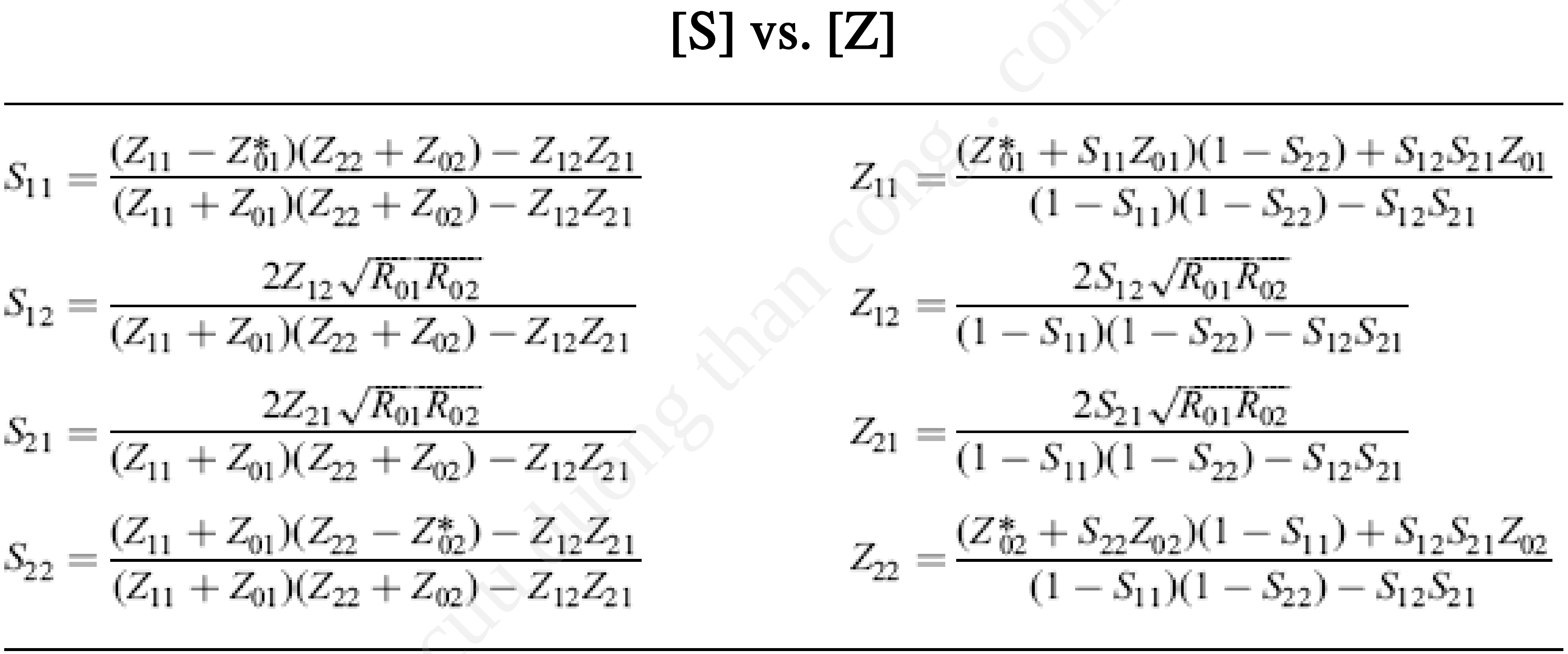
6. S Parameter Conversion



[S] vs. [Z]

$$S_{11} = \frac{(Z_{11} - Z_{01}^*)(Z_{22} + Z_{02}) - Z_{12}Z_{21}}{(Z_{11} + Z_{01})(Z_{22} + Z_{02}) - Z_{12}Z_{21}}$$

$$Z_{11} = \frac{(Z_{01}^* + S_{11}Z_{01})(1 - S_{22}) + S_{12}S_{21}Z_{01}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$$

$$S_{12} = \frac{2Z_{12}\sqrt{R_{01}R_{02}}}{(Z_{11} + Z_{01})(Z_{22} + Z_{02}) - Z_{12}Z_{21}}$$

$$Z_{12} = \frac{2S_{12}\sqrt{R_{01}R_{02}}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$$

$$S_{21} = \frac{2Z_{21}\sqrt{R_{01}R_{02}}}{(Z_{11} + Z_{01})(Z_{22} + Z_{02}) - Z_{12}Z_{21}}$$

$$Z_{21} = \frac{2S_{21}\sqrt{R_{01}R_{02}}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$$

$$S_{22} = \frac{(Z_{11} + Z_{01})(Z_{22} - Z_{02}^*) - Z_{12}Z_{21}}{(Z_{11} + Z_{01})(Z_{22} + Z_{02}) - Z_{12}Z_{21}}$$

$$Z_{22} = \frac{(Z_{02}^* + S_{22}Z_{02})(1 - S_{11}) + S_{12}S_{21}Z_{02}}{(1 - S_{11})(1 - S_{22}) - S_{12}S_{21}}$$

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6. S Parameter Conversion

[S] vs. [Y]

$$S_{11} = \frac{(1 - Y_{11}Z_{01}^{*})(1 + Y_{22}Z_{02}) + Y_{12}Y_{21}Z_{01}^{*}Z_{02}}{(1 + Y_{11}Z_{01})(1 + Y_{22}Z_{02}) - Y_{12}Y_{21}Z_{01}Z_{02}} \qquad Y_{11} = \frac{(1 - S_{11})(Z_{02}^{*} + S_{22}Z_{02}) + S_{12}S_{21}Z_{02}}{(Z_{01}^{*} + S_{11}Z_{01})(Z_{02}^{*} + S_{22}Z_{02}) - S_{12}S_{21}Z_{01}Z_{02}}$$

$$S_{12} = \frac{-2Y_{12}\sqrt{R_{01}R_{02}}}{(1 + Y_{11}Z_{01})(1 + Y_{22}Z_{02}) - Y_{12}Y_{21}Z_{01}Z_{02}} \qquad Y_{12} = \frac{-2S_{12}\sqrt{R_{01}R_{02}}}{(Z_{01}^{*} + S_{11}Z_{01})(Z_{02}^{*} + S_{22}Z_{02}) - S_{12}S_{21}Z_{01}Z_{02}}$$

$$S_{21} = \frac{-2Y_{21}\sqrt{R_{01}R_{02}}}{(1 + Y_{11}Z_{01})(1 + Y_{22}Z_{02}) - Y_{12}Y_{21}Z_{01}Z_{02}} \qquad Y_{21} = \frac{-2S_{21}\sqrt{R_{01}R_{02}}}{(Z_{01}^{*} + S_{11}Z_{01})(Z_{02}^{*} + S_{22}Z_{02}) - S_{12}S_{21}Z_{01}Z_{02}}$$

$$S_{22} = \frac{(1 + Y_{11}Z_{01})(1 - Y_{22}Z_{02}^{*}) + Y_{12}Y_{21}Z_{01}^{*}Z_{02}^{*}}{(1 + Y_{11}Z_{01})(1 + Y_{22}Z_{02}) - Y_{12}Y_{21}Z_{01}Z_{02}} \qquad Y_{22} = \frac{(Z_{01}^{*} + S_{11}Z_{01})(1 - S_{22}) + S_{12}S_{21}Z_{01}}{(Z_{01}^{*} + S_{11}Z_{01})(Z_{02}^{*} + S_{22}Z_{02}) - S_{12}S_{21}Z_{01}Z_{02}}$$

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6. S Parameter Conversion

[S] vs. [ABCD]

$$A = (1 + S_{11} - S_{22} - \Delta S)\sqrt{Z_{01} / Z_{02}} / 2S_{21}$$

$$B = (1 + S_{11} + S_{22} + \Delta S)\sqrt{Z_{01}.Z_{02}} / 2S_{21}$$

$$C = (1 - S_{11} - S_{22} - \Delta S) / 2S_{21}\sqrt{Z_{01}.Z_{02}}$$

$$D = (1 - S_{11} + S_{22} - \Delta S)\sqrt{Z_{02} / Z_{01}} / 2S_{21}$$

$$\Delta S = S_{11}S_{12} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02} + B - CZ_{01}Z_{02} - DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{12} = \frac{2(AD - BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02} + B - CZ_{01}Z_{02} + DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

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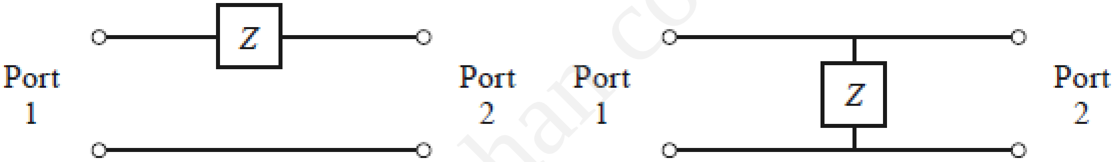
Exercises

Exercise 1: Find the scattering parameters for the series and shunt loads shown below. Show that $S_{12} = 1 - S_{11}$ for the series case and $S_{12} = 1 + S_{11}$ for the shunt case. Assume the characteristic impedance Z_0



Exercise 2: A lossless, reciprocal 3-port device has S-parameters of $S_{11} = {}^1/_2$, $S_{31} = {}^1/_{\sqrt{2}}$, and $S_{33} = 0$. It is likewise known that all scattering parameters are real. Find the remaining 6 scattering parameters

$$S = \begin{bmatrix} {}^1/_2 & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ {}^1/_{\sqrt{2}} & S_{32} & 0 \end{bmatrix}$$

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Exercises

Exercise 3: A 3 port network is characterized at certain frequency by the scattering matrix:

$$S = \begin{bmatrix} 0 & 0.2 & 0.5 \\ 0.5 & 0 & 0.2 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

A matched load is attached at port 2 while a short circuit has been placed at port 3.

a. Find the reflection coefficient at port 1?

b. Find the transmission coefficient from port 1 to port 2?

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Exercises

Exercise 4: A four-port network has the scattering matrix shown as follows.

(a) Is this network lossless? Is this network reciprocal?

(b) What is the return loss at port 1 when all other ports are terminated with matched loads?

(c) What is the insertion loss and phase delay between ports 2 and 4 when all other ports are terminated with matched loads?

(d) What is the reflection coefficient seen at port 1 if a short circuit is placed at the terminal plane of port 3 and all other ports are terminated with matched loads?

$$[S] = \begin{bmatrix} 0.178\angle 90^\circ & 0.6\angle 45^\circ & 0.4\angle 45^\circ & 0 \\ 0.6\angle 45^\circ & 0 & 0 & 0.3\angle -45^\circ \\ 0.4\angle 45^\circ & 0 & 0 & 0.5\angle -45^\circ \\ 0 & 0.3\angle -45^\circ & 0.5\angle -45^\circ & 0 \end{bmatrix}$$

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Exercises

Exercise 5: A four-port network has the scattering matrix shown as follows. If ports 3 and 4 are connected with a lossless matched transmission line with an electrical length of 45◦, find the resulting insertion loss and phase delay between ports 1 and 2.

$$[S] = \begin{bmatrix} 0.2\angle 50^\circ & 0 & 0 & 0.4\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.7\angle -45^\circ & 0 \\ 0 & 0.7\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.4\angle -45^\circ & 0 & 0 & 0.5\angle 45^\circ \end{bmatrix}$$

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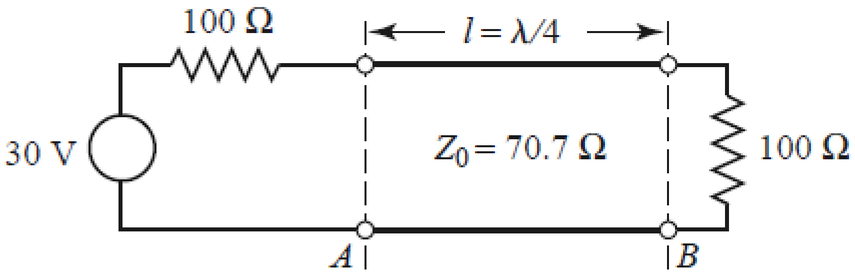
Exercise 6: At reference plane A, for the circuit shown below, choose an appropriate reference impedance, find the power wave amplitudes, and compute the power delivered to the load. Repeat this procedure for reference plane B. Assume the transmission line is lossless.



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Q&A

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