Chapter 4: Morphological Image Processing

4. Morphological Image Processing (MIP)

- **Morphology**: a branch of biology that deals with the form and structure of animals and plants.
- **Morphological image processing**: used to extract image components that are useful in the representation and description of region shape, such as:
  - Boundaries extraction
  - Skeletons
  - Convex hull
  - Morphological filtering
  - Thinning
  - Pruning

4. MIP: Basic Set Theory (1)

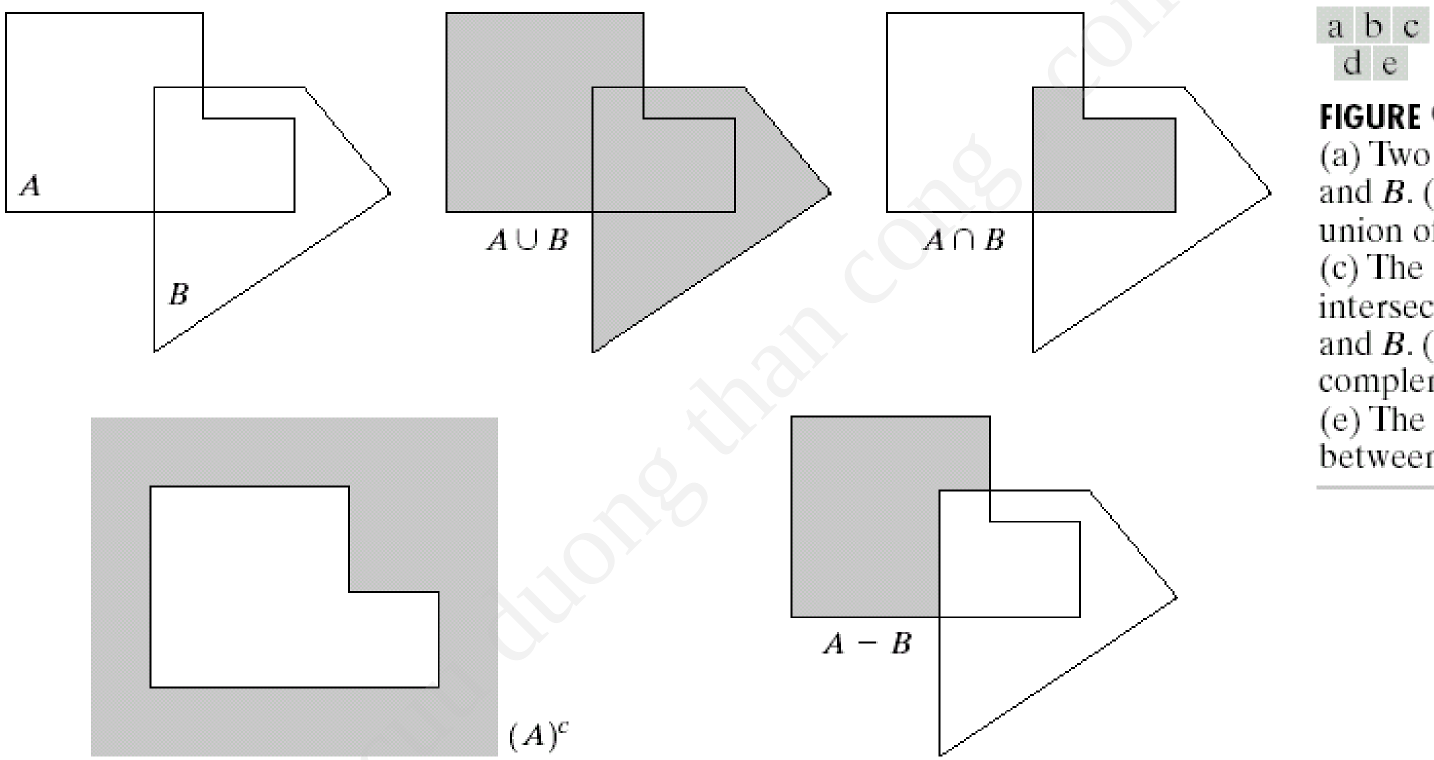


FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

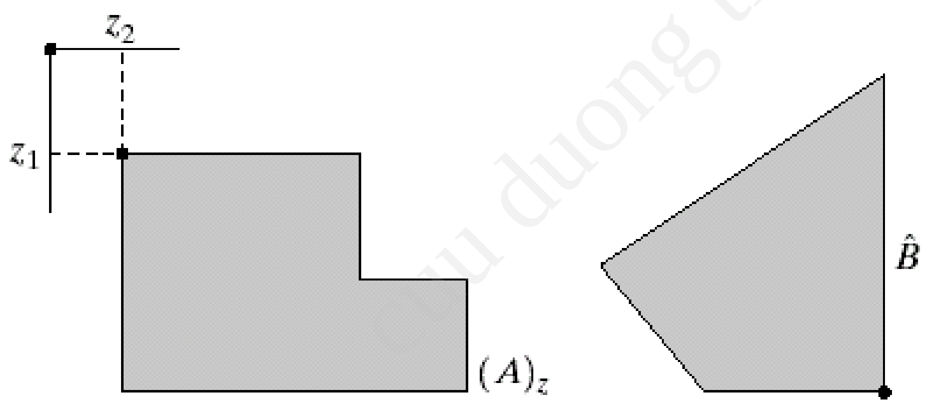
4. MIP: Basic Set Theory (2)

Reflection:

$$\hat{B} = \{w \mid w \in -b, \quad for\ b \in B\}$$

Translation:

$$(A)_z = \{c \mid c \in a + z, \quad for\ a \in A\}$$



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

4. MIP: Basic Set Theory (3)

TABLE 9.1
The three basic logical operations.

p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

4. MIP: Basic Set Theory (4)

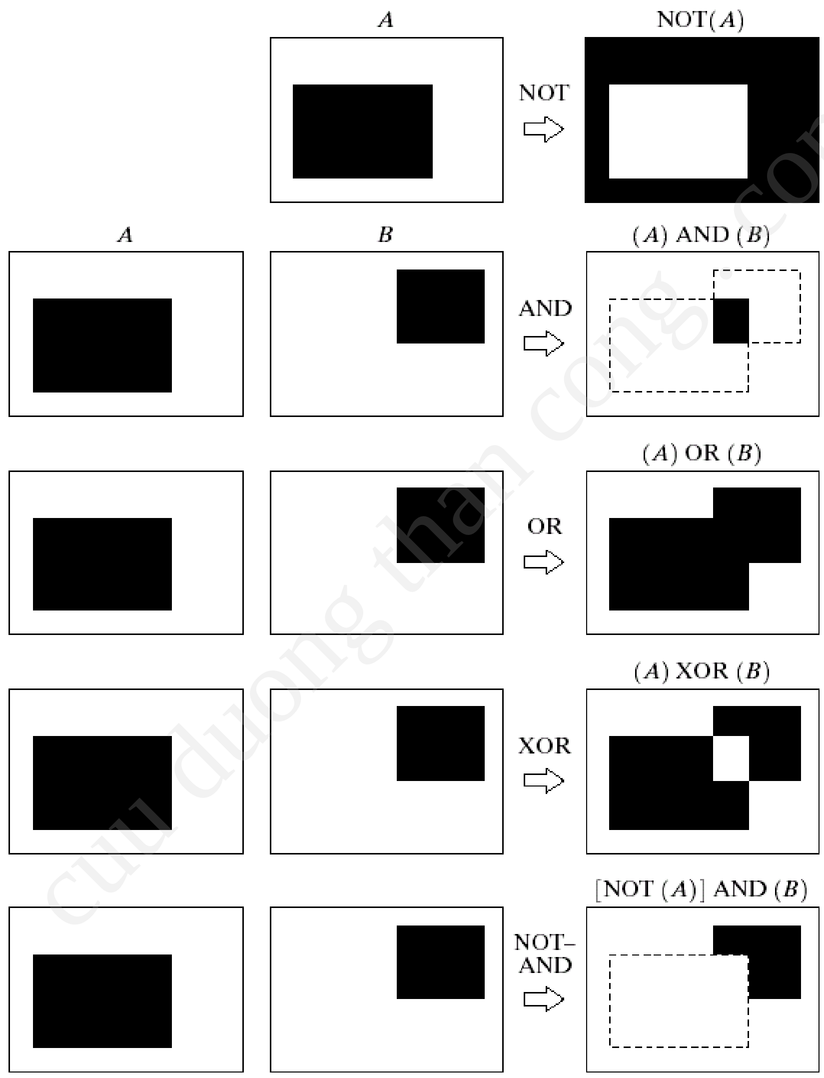


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

4. MIP: Structure Elements (1)

Structure elements (SE): Small sets or sub-images used to probe an image under study for properties of interest.

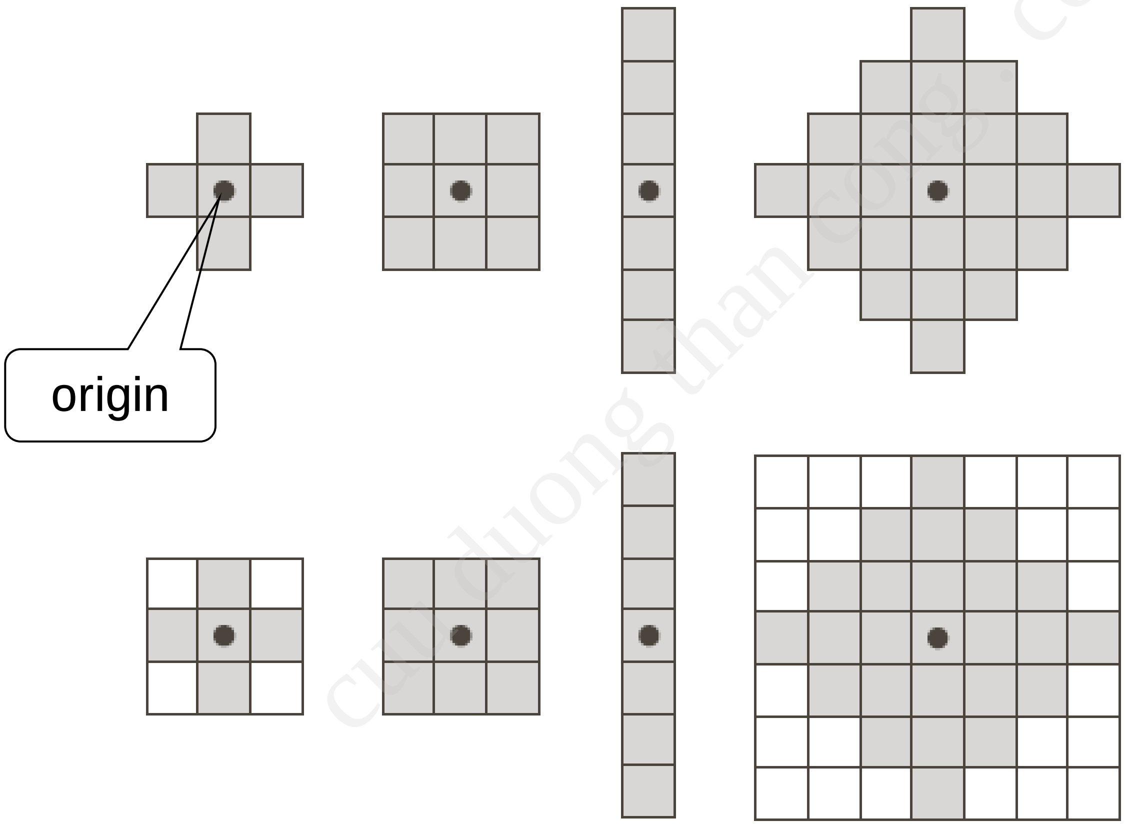


FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.

4. MIP: Structure Elements (2)

Example:

At each location of the origin of *B*, if *B* is completely contained in *A*, then the location is a member of the new set, otherwise it is not a member of the new set.

Origin of *B* visits every element of *A*

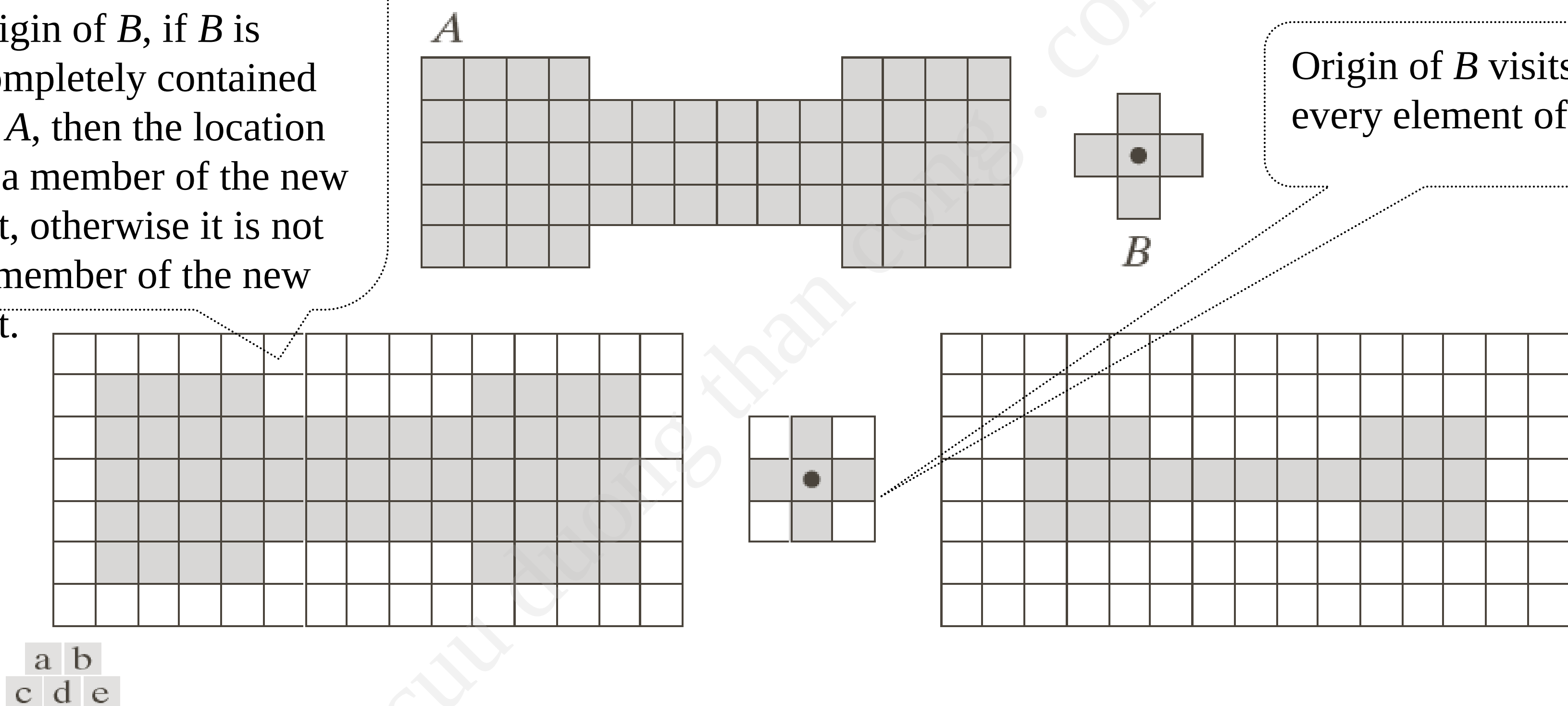


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

4. MIP: Structure Elements (3)

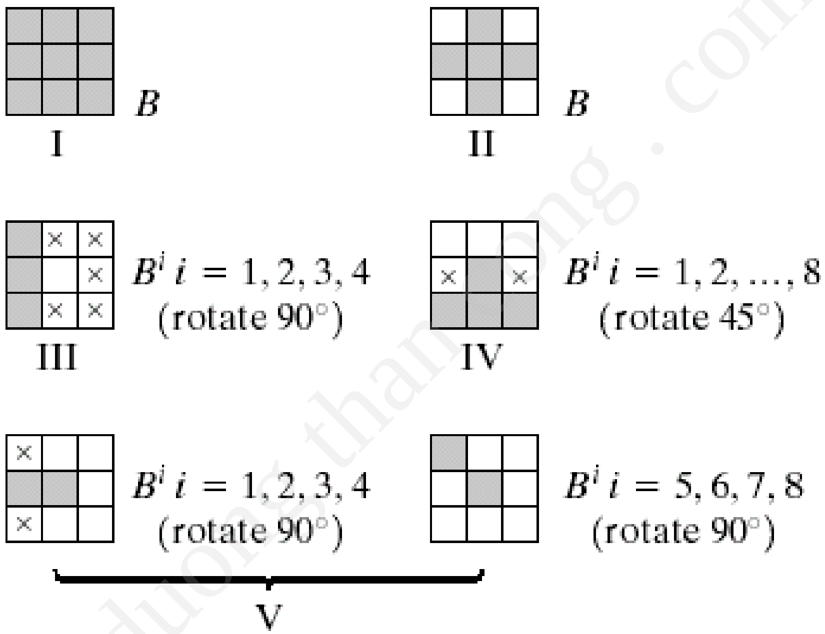


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

4. MIP: Dilation (1)

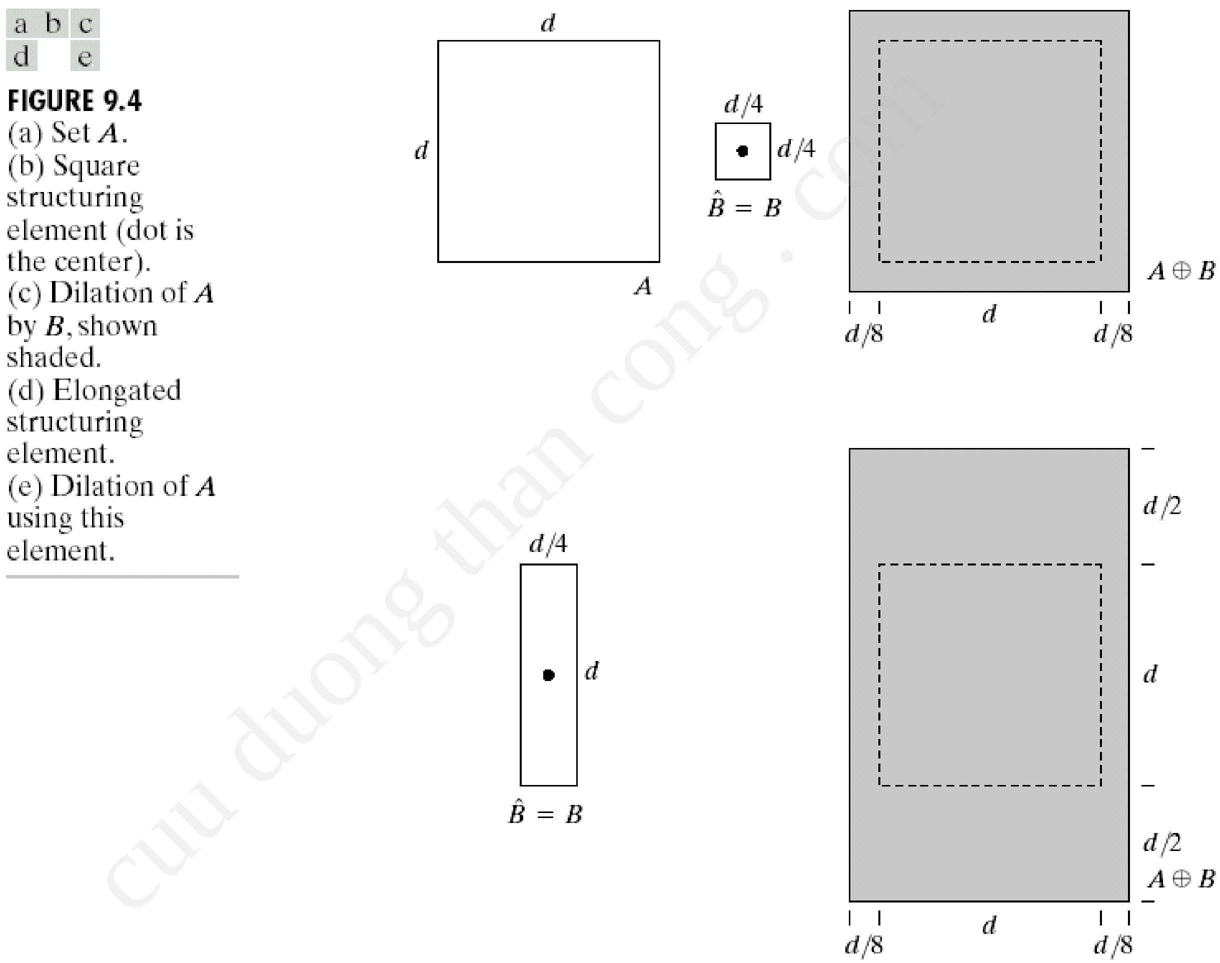
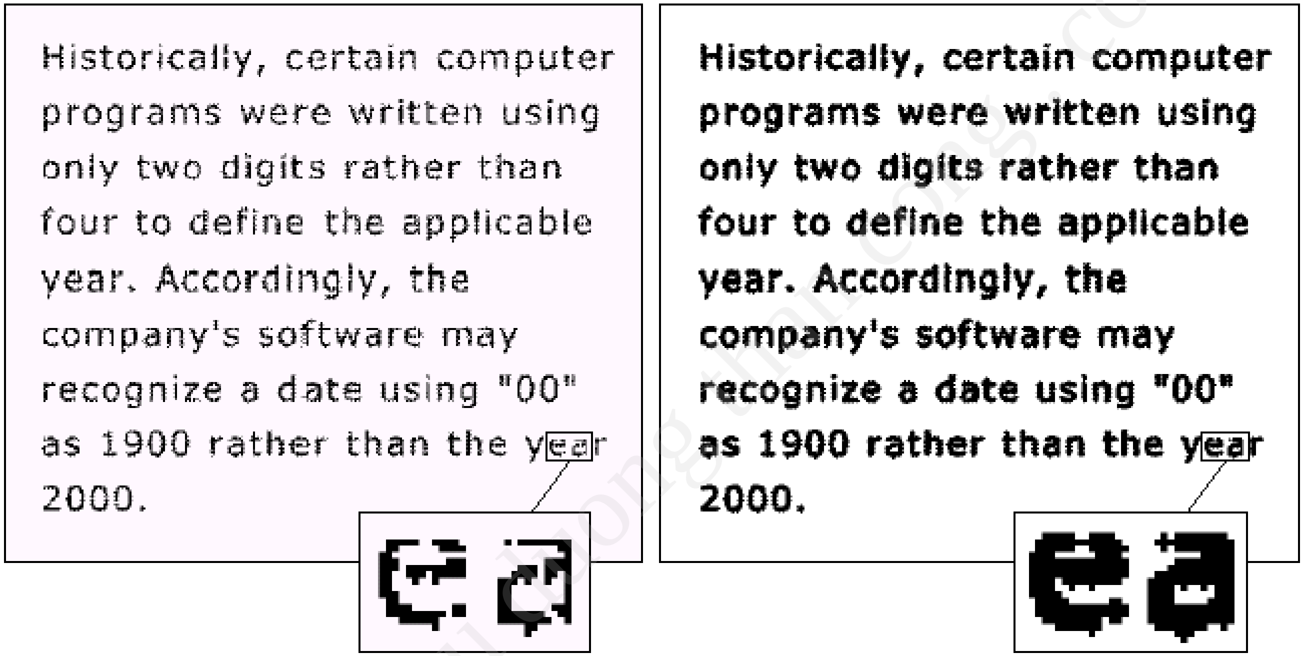


FIGURE 9.4 (a) Set A. (b) Square structuring element (dot is the center). (c) Dilation of A by B, shown shaded. (d) Elongated structuring element. (e) Dilation of A using this element.

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \Phi\}$$

4. MIP: Dilation (2)

Example of Dilation: Bridging gaps



0	1	0
1	1	1
0	1	0

a c
b

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.

4. MIP: Erosion (1)

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

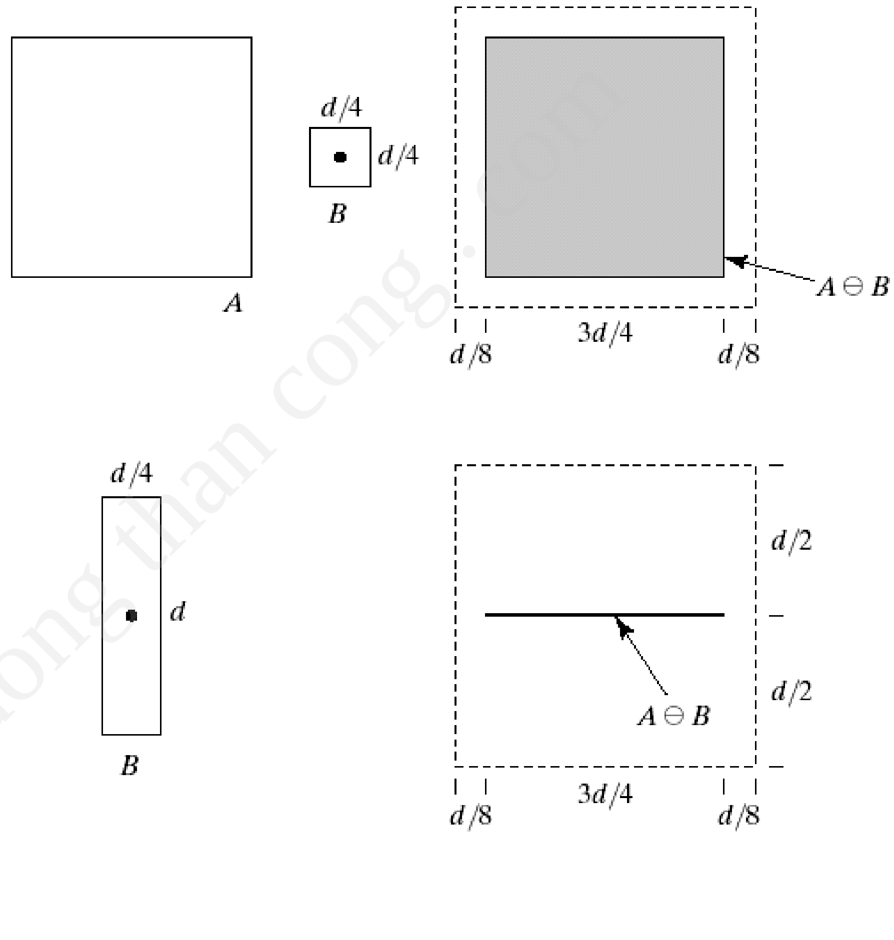


FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.

4. MIP: Erosion (2)

Example of Erosion: Eliminating irrelevant detail

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Structuring element $B = 13 \times 13$ pixels of gray level 1

4. MIP: Duality of Dilation and Erosion

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A\}^c$$

$$(A \ominus B)^c = \{z \mid (B)_z \subseteq A^c = \Phi\}^c$$

$$= \{z \mid (B)_z \subseteq A^c \neq \Phi\}$$

$$= A^c \oplus \hat{B}$$

4. MIP: Opening

Opening generally smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions.

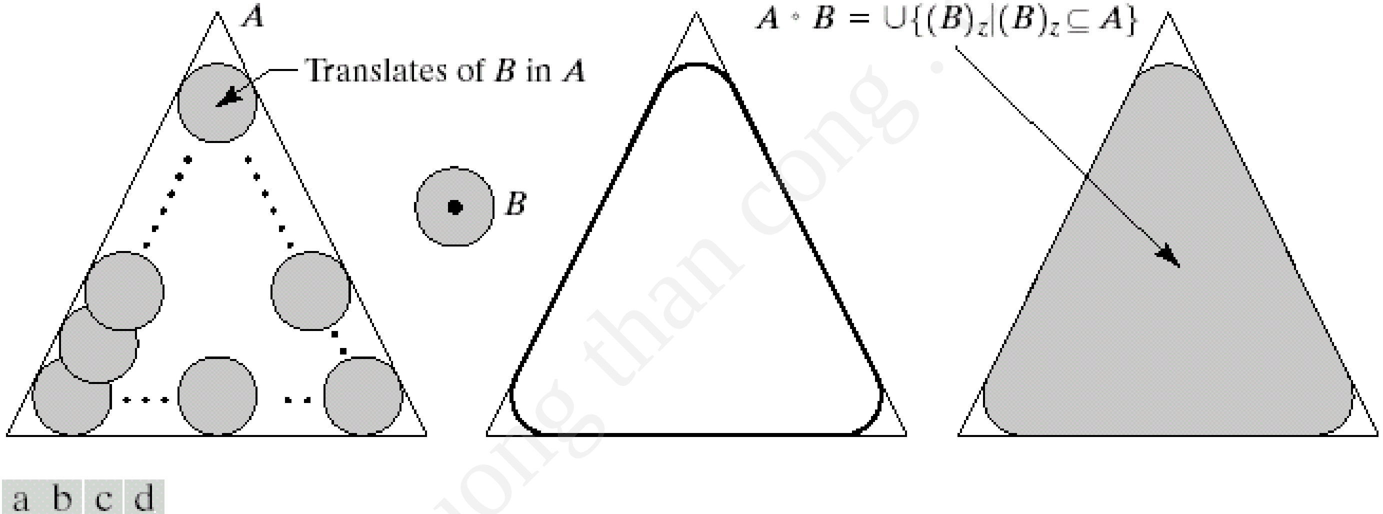


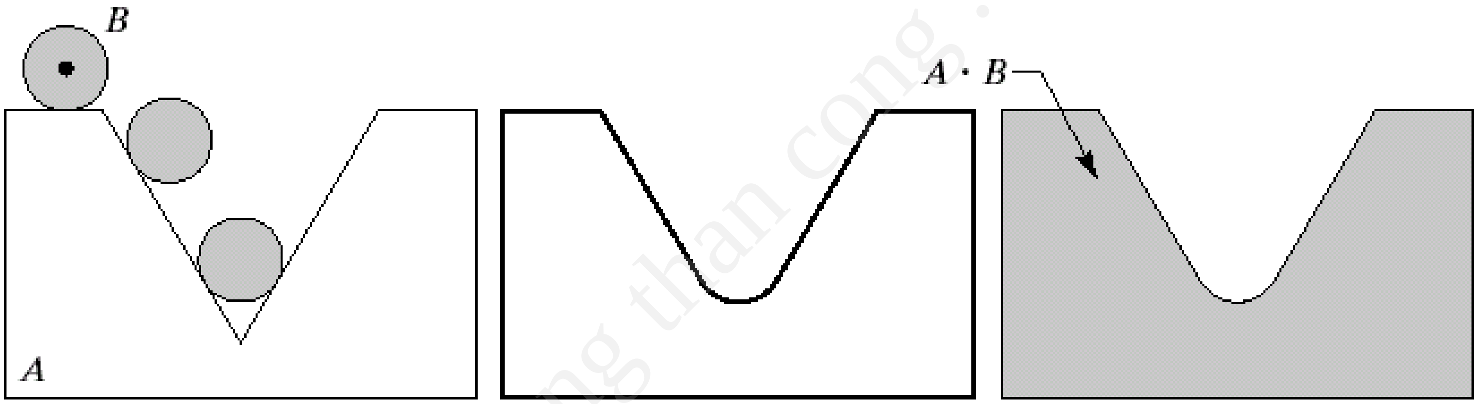
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \cup\{(B)_z \mid (B)_z \subseteq A\}$$

4. MIP: Closing

Closing tends to smooth sections of contours but it generates fuses narrow breaks and long thin gulfs, eliminates small holes, and fills gaps in the contour.



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

$$A \bullet B = (A \oplus B) \ominus B$$

4. MIP: Duality of Opening and Closing

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

Opening

(i) $A \circ B$ is a subset (subimage) of A

(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$

(iii) $(A \circ B) \circ B = A \circ B$

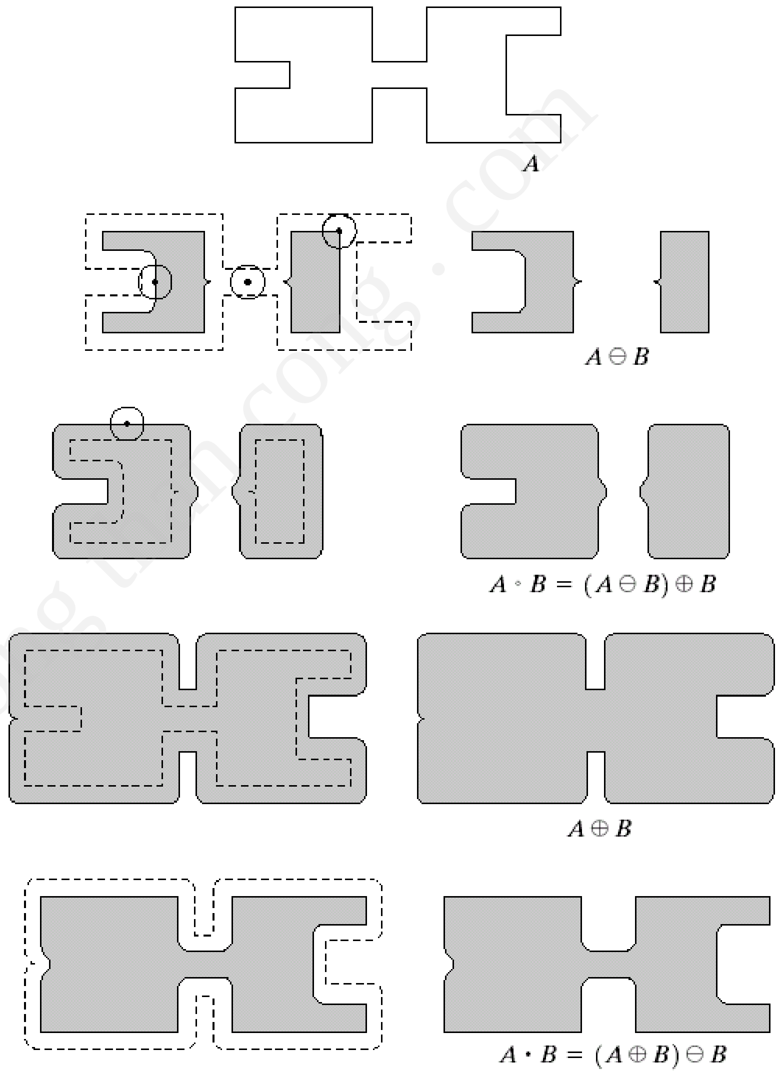
Closing

(i) A is a subset (subimage) of $A \bullet B$

(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$

(iii) $(A \bullet B) \bullet B = A \bullet B$

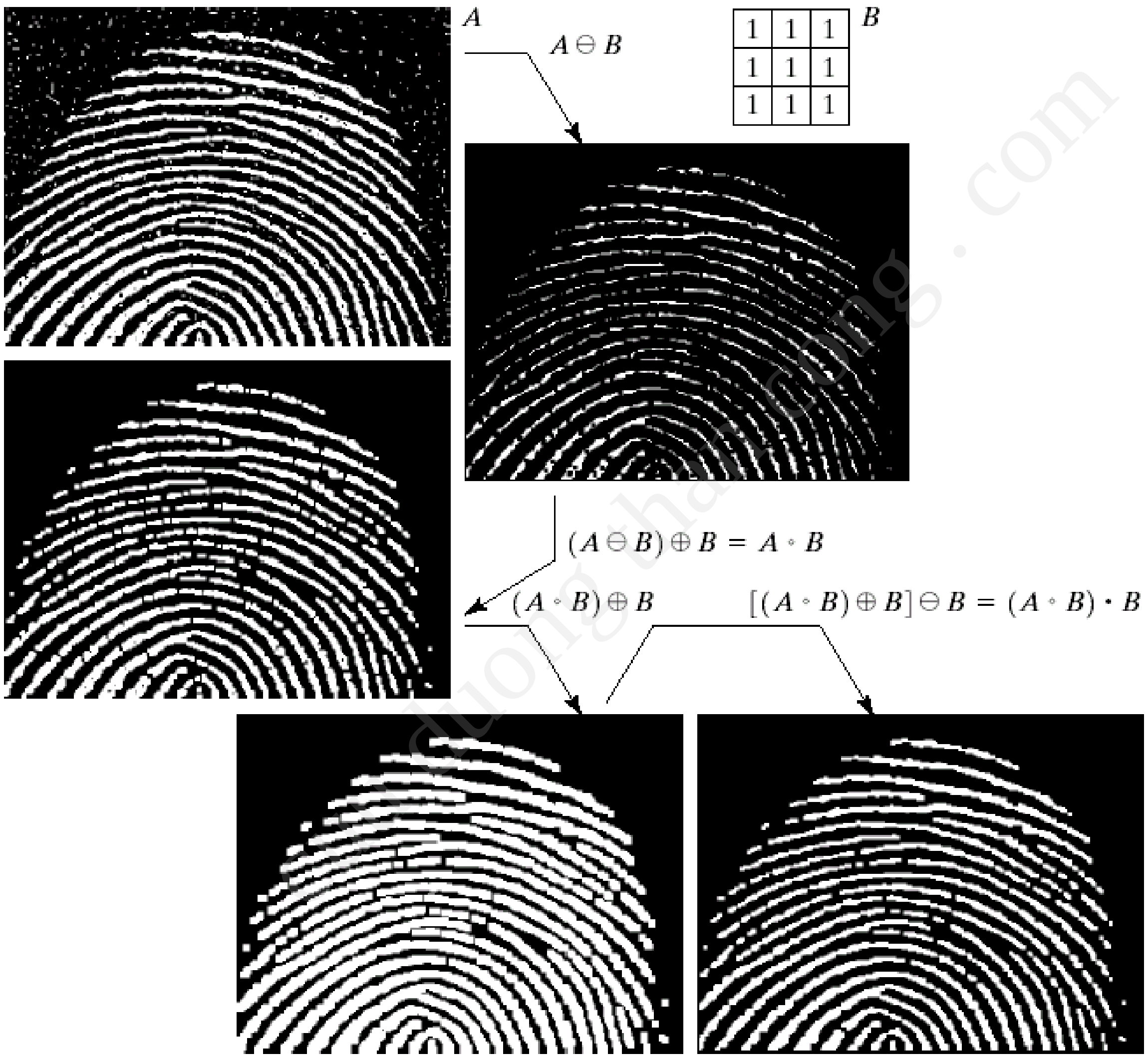
4. MIP: Example of Opening & Closing (1)



a
b c
d e
f g
h i

FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

4. MIP: Example of Opening & Closing (2)



a b
d c
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

4. MIP: Hit-or-Miss Transformation

if B denotes the set composed of D and its background,the match (or set of matches) of B in A, denoted $A * B$,

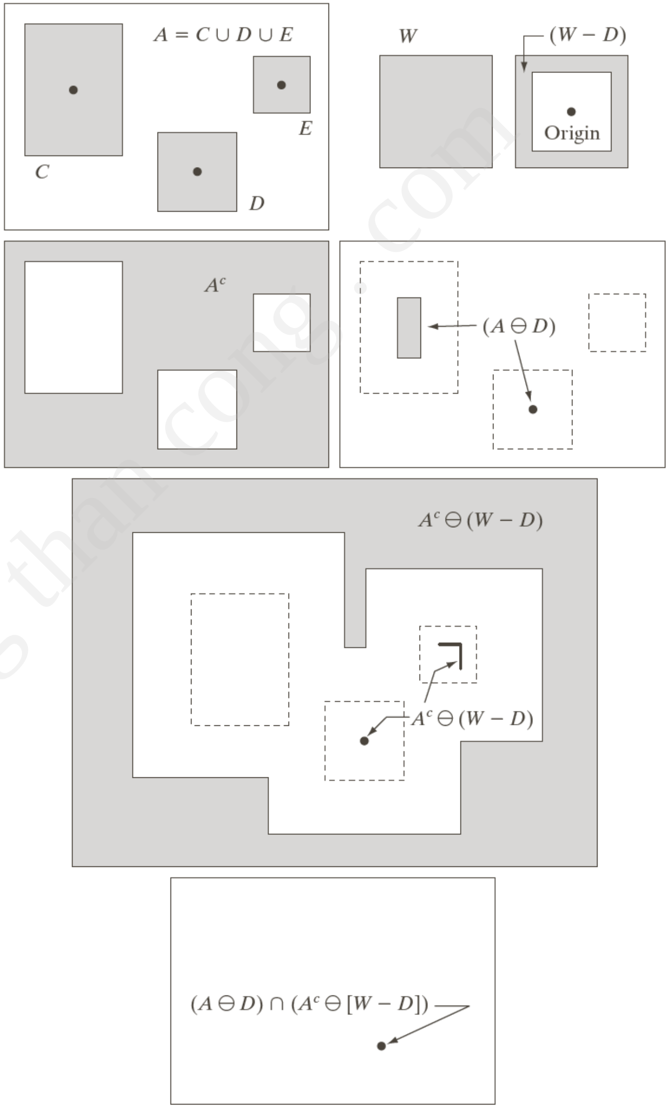
$$A * B = (A - D) \cap \left[A^c - (W - D) \right]$$

$$B = (B_1, B_2)$$

B_1 : object

B_2 : background

$$A * B = (A - B_1) \cap (A^c - B_2)$$

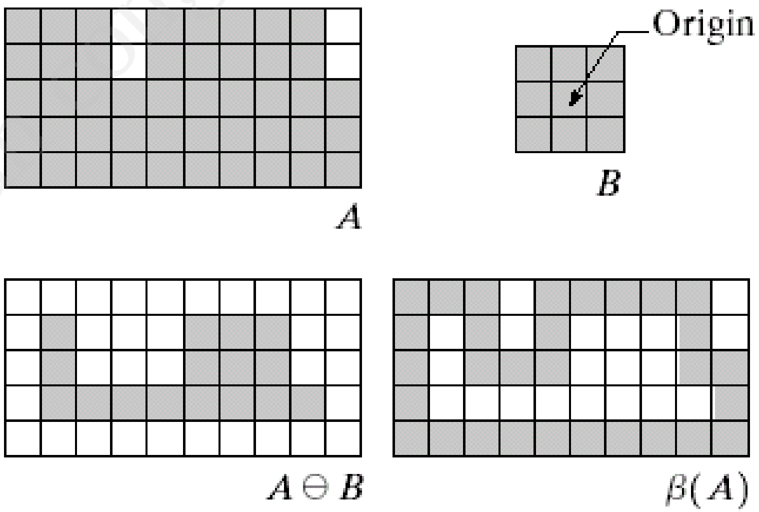


a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of D with respect to W, $(W - D)$. (c) Complement of A. (d) Erosion of A by D. (e) Erosion of A^c by $(W - D)$. (f) Intersection of (d) and (e), showing the location of the origin of D, as desired. The dots indicate the origins of C, D, and E.

4. MIP: Boundary Extraction (1)

Boundary extraction: The boundary of a set *A*, can be obtained by first eroding *A* by *B* and then performing the set difference between *A* and its erosion.



a b
c d

FIGURE 9.13 (a) Set *A*. (b) Structuring element *B*. (c) *A* eroded by *B*. (d) Boundary, given by the set difference between *A* and its erosion.

$$\beta(A) = A - (A \ominus B)$$

4. MIP: Boundary Extraction (2)

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

4. MIP: Hole Filling (1)

Hole (region) filling: A hole may be defined as a background region surrounded by a connected border of foreground pixels.

Let A denote a set whose elements are 8-connected boundaries, each boundary enclosing a background region (i.e., a hole). Given a point in each hole, the objective is to fill all the holes with 1s.

1. Forming an array X_0 of 0s (the same size as the array containing A), except the locations in X_0 corresponding to the given point in each hole, which we set to 1.

2. $X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$

Stop the iteration if $X_k = X_{k-1}$

4. MIP: Hole Filling (2)

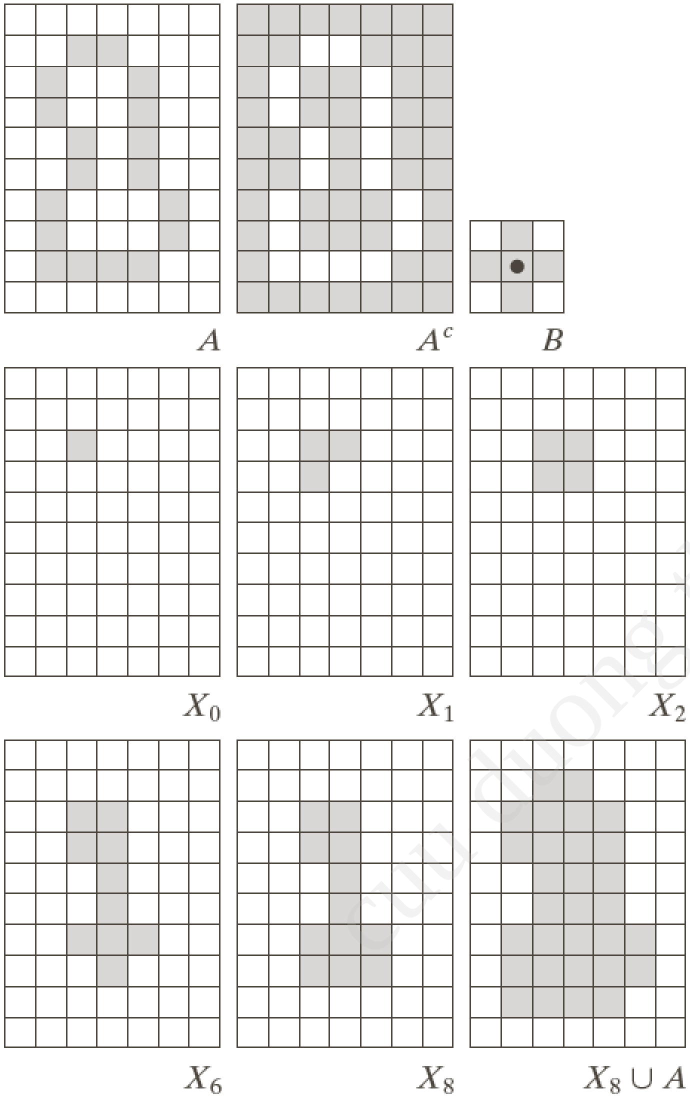
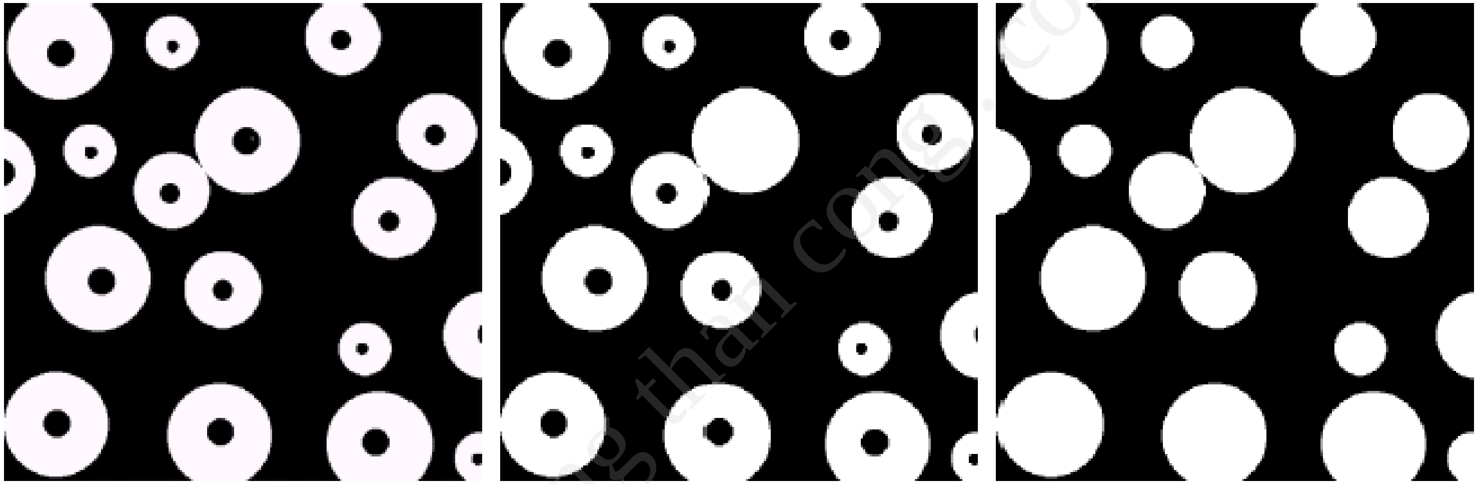


FIGURE 9.15 Hole filling. (a) Set A (shown shaded). (b) Complement of A. (c) Structuring element B. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1,2,3,\ldots$$

4. MIP: Hole Filling (3)



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

4. MIP: Extraction of Connected Components (1)

Extraction of connected components: Central to many automated image analysis applications.

$$X_k = (X_{k-1} + B) \cap A$$

B : structuring element

until $X_k = X_{k\text{-}1}$

4. MIP: Extraction of Connected Components (2)

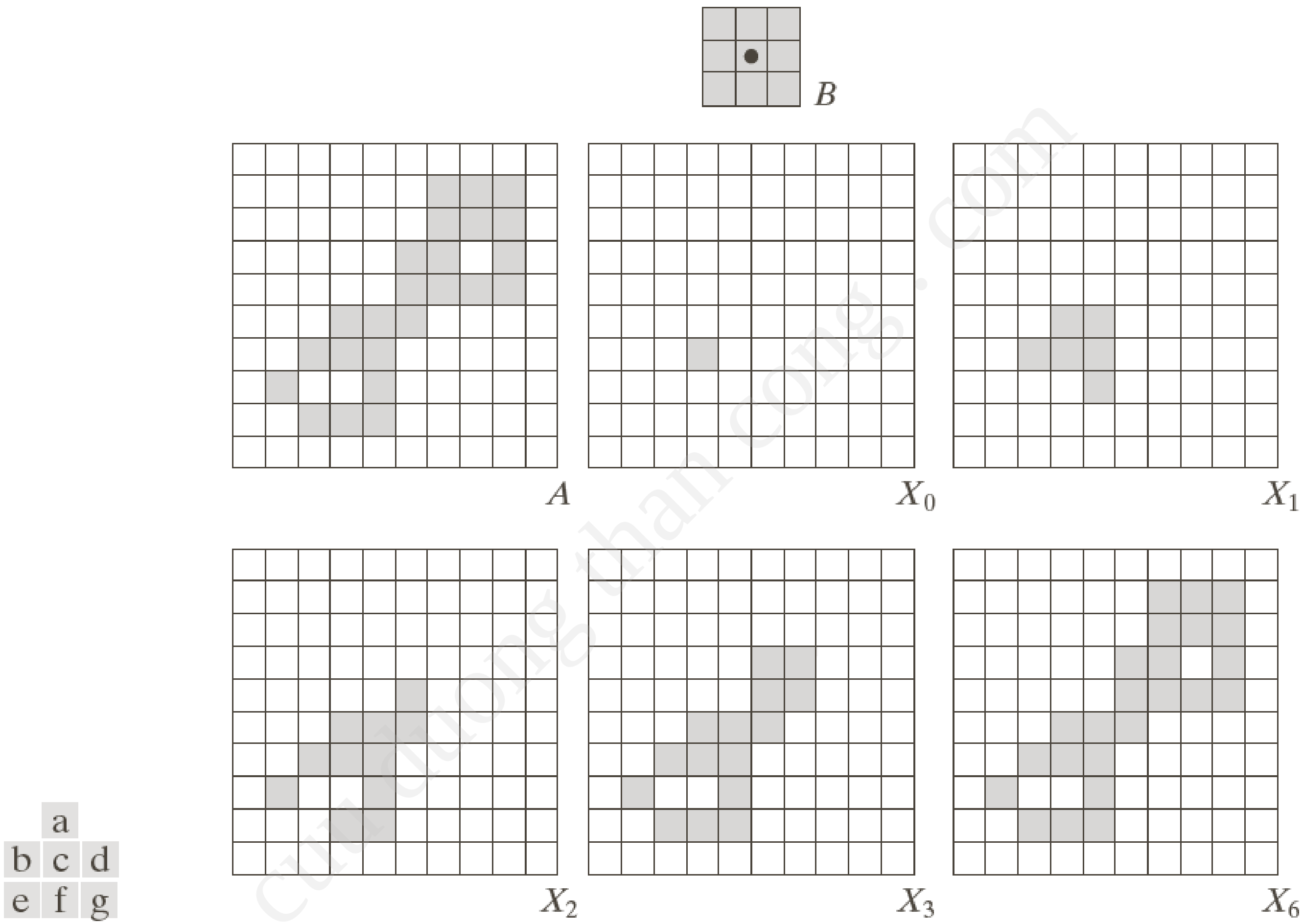
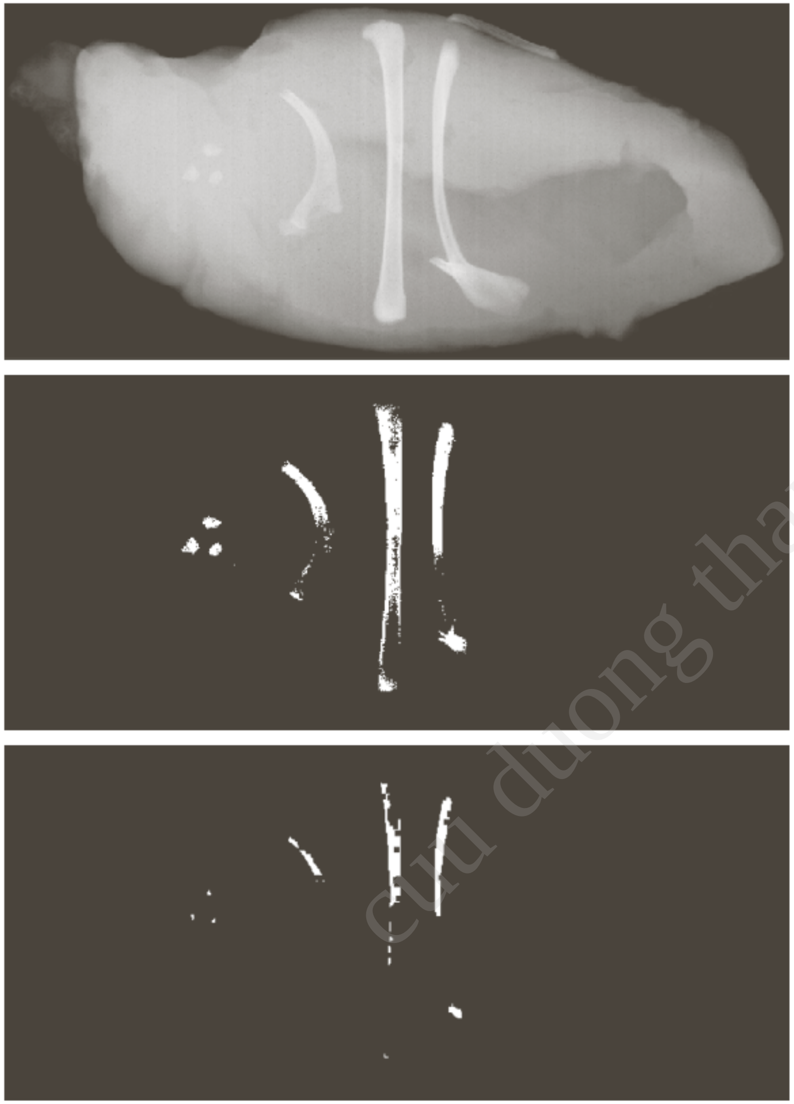


FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).

4. MIP: Extraction of Connected Components (3)



Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1s.
(d) Number of pixels in the connected components of (c).
(Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Convex Hull

A set *A* is said to be **convex** if the straight line segment joining any two points in *A* lies entirely within *A*.

The **convex hull** H or of an arbitrary set *S* is the smallest convex set containing *S*.

4. MIP: Convex Hull (2)

Let B^i, $i = 1,\ 2,\ 3,\ 4$, represent the four structuring elements.

The procedure consists of implementing the equation:

$$X_k^i = (X_{k-1} * B^i) \cup A$$

$$i = 1, 2, 3, 4 \ \text{ and } \ k = 1, 2, 3, \ldots$$

with $X_0^i = A$.

When the procedure converges, or $X_k^i = X_{k-1}^i$, let $D^i = X_k^i$,

the convex hull of A is

$$C(A) = \bigcup_{i=1}^{4} D^i$$

4. MIP: Convex Hull (3)

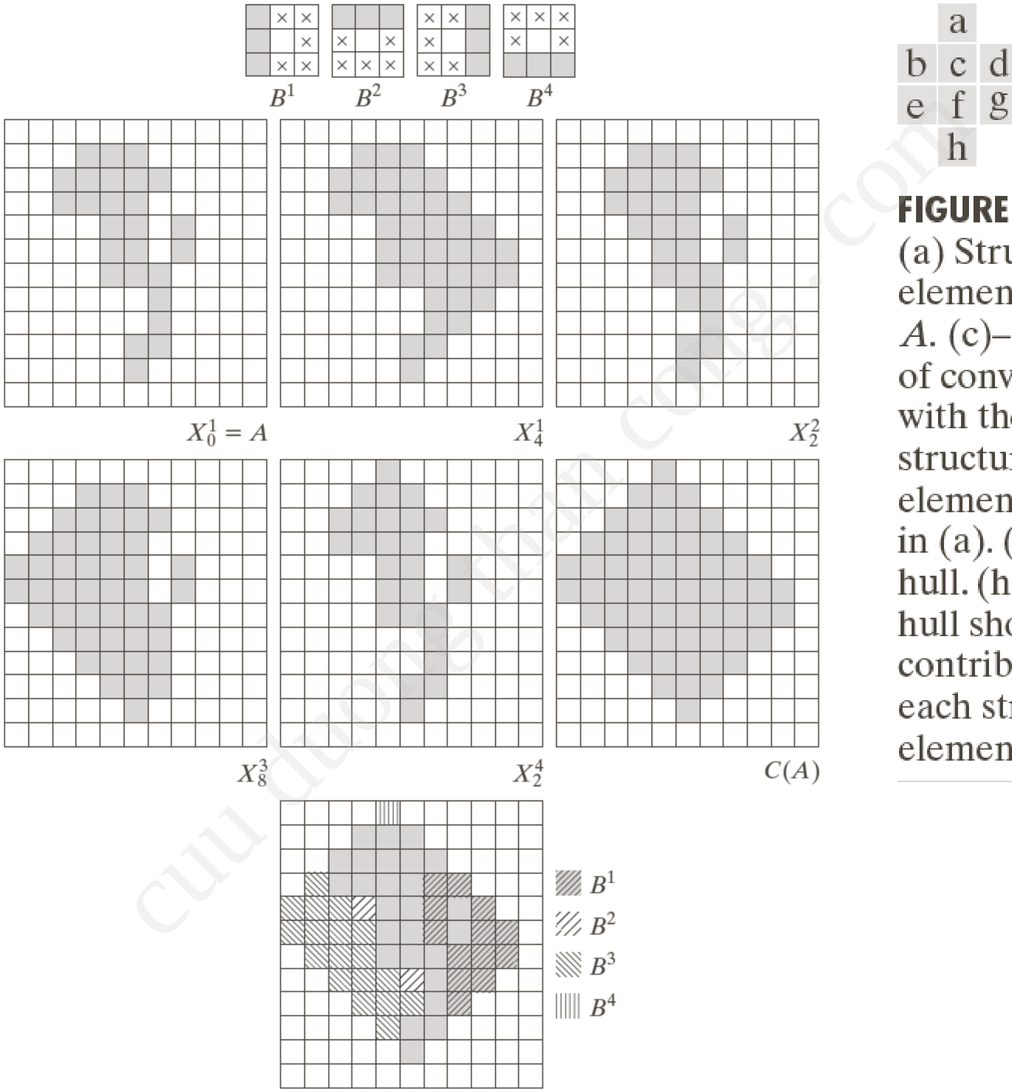


FIGURE 9.19 (a) Structuring elements. (b) Set *A*. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

4. MIP: Convex Hull (4)

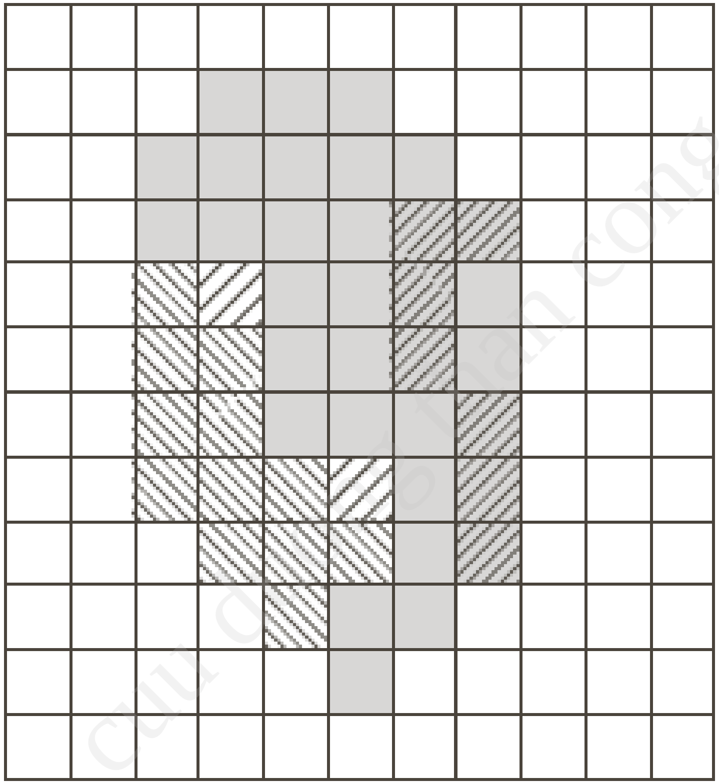


FIGURE 9.20 Result of limiting growth of the convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

4. MIP: Thinning (1)

The **thinning** of a set *A* by a structuring element *B*, defined:

$$A \otimes B = A - (A * B)$$
$$= A \cap (A * B)^c$$

A more useful expression for thinning *A* symmetrically is based on a sequence of structuring elements:

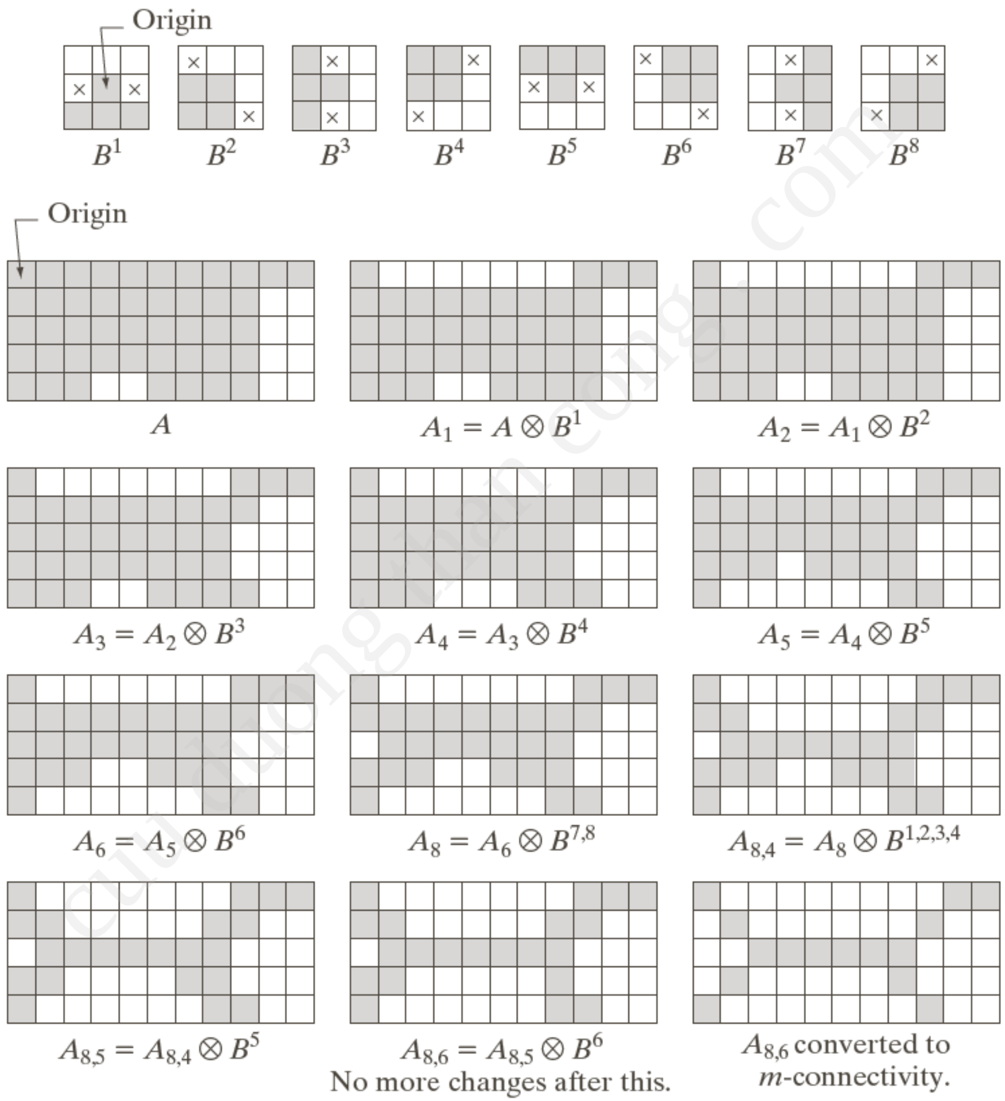
$$\{B\} = \{B^1, B^2, B^3, ..., B^n\}$$

where B^i is a rotated version of $B^{i\text{-}1}$

The thinning of *A* by a sequence of structuring element {*B*}

$$A \otimes \{B\} = ((...((A \otimes B^1) \otimes B^2)...) \otimes B^n)$$

4. MIP: Thinning (2)



4. MIP: Thickening (1)

The **thickening** is defined by the expression

$$A \odot B = A \cup \left(A \circledast B \right)$$

The thickening of A by a sequence of structuring element $\{B\}$

$$A \odot \{B\} = ((\ldots((A \odot B^1) \odot B^2)\ldots) \odot B^n)$$

In practice, the usual procedure is to thin the background of the set and then complement the result.

4. MIP: Thickening (2)

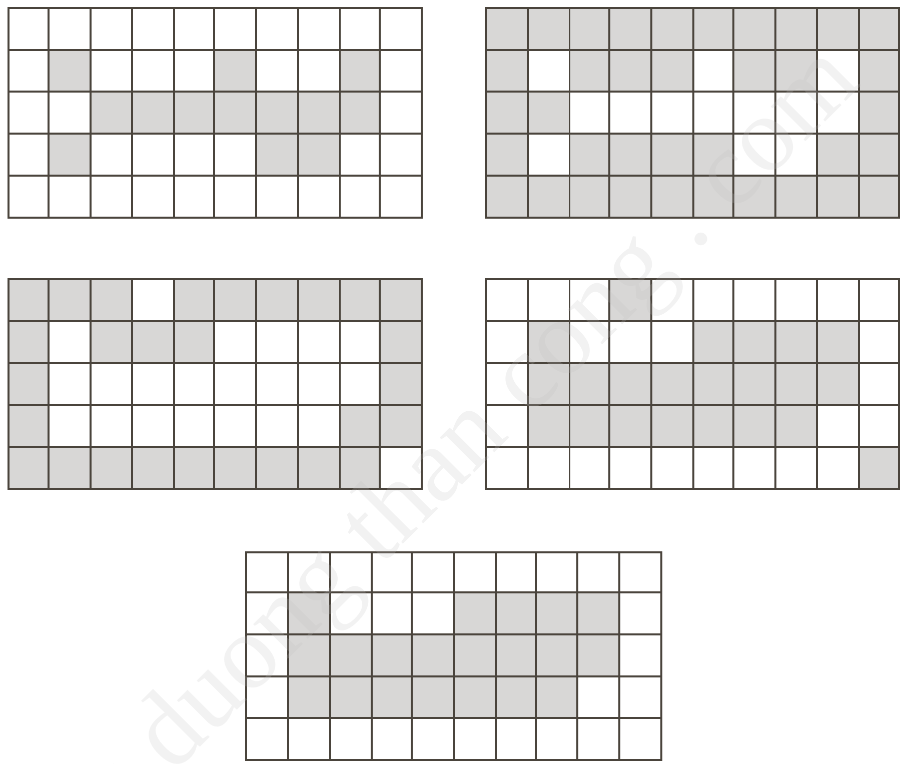


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

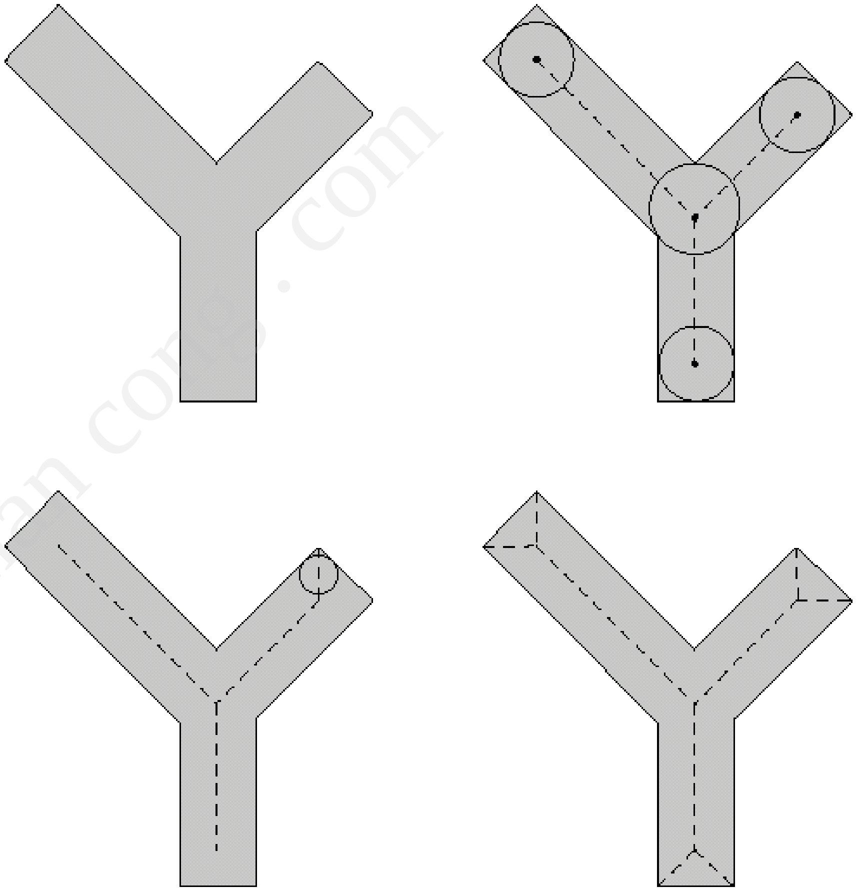
4. MIP: Skeleton (1)

A **skeleton**, $S(A)$ of a set A has the following properties:

a. if z is a point of $S(A)$ and $(D)_z$ is the largest disk centered at z and contained in A, one cannot find a larger disk containing $(D)_z$ and included in A. The disk $(D)_z$ is called a maximum disk.

b. The disk $(D)_z$ touches the boundary of A at two or more different places.

4. MIP: Skeleton (2)



a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$K = \max\{k \mid (A \ominus kB) \neq \Phi\}$$

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

4. MIP: Skeleton (3)

The skeleton of A can be expressed in terms of erosion and openings.

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

with $K = \max\{k \mid A \ominus kB \neq \phi\};$

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structuring element, and

$$(A \ominus kB) = ((..((A \ominus B) \ominus B) \ominus ...) \ominus B)$$

k successive erosions of A.

4. MIP: Skeleton (4)

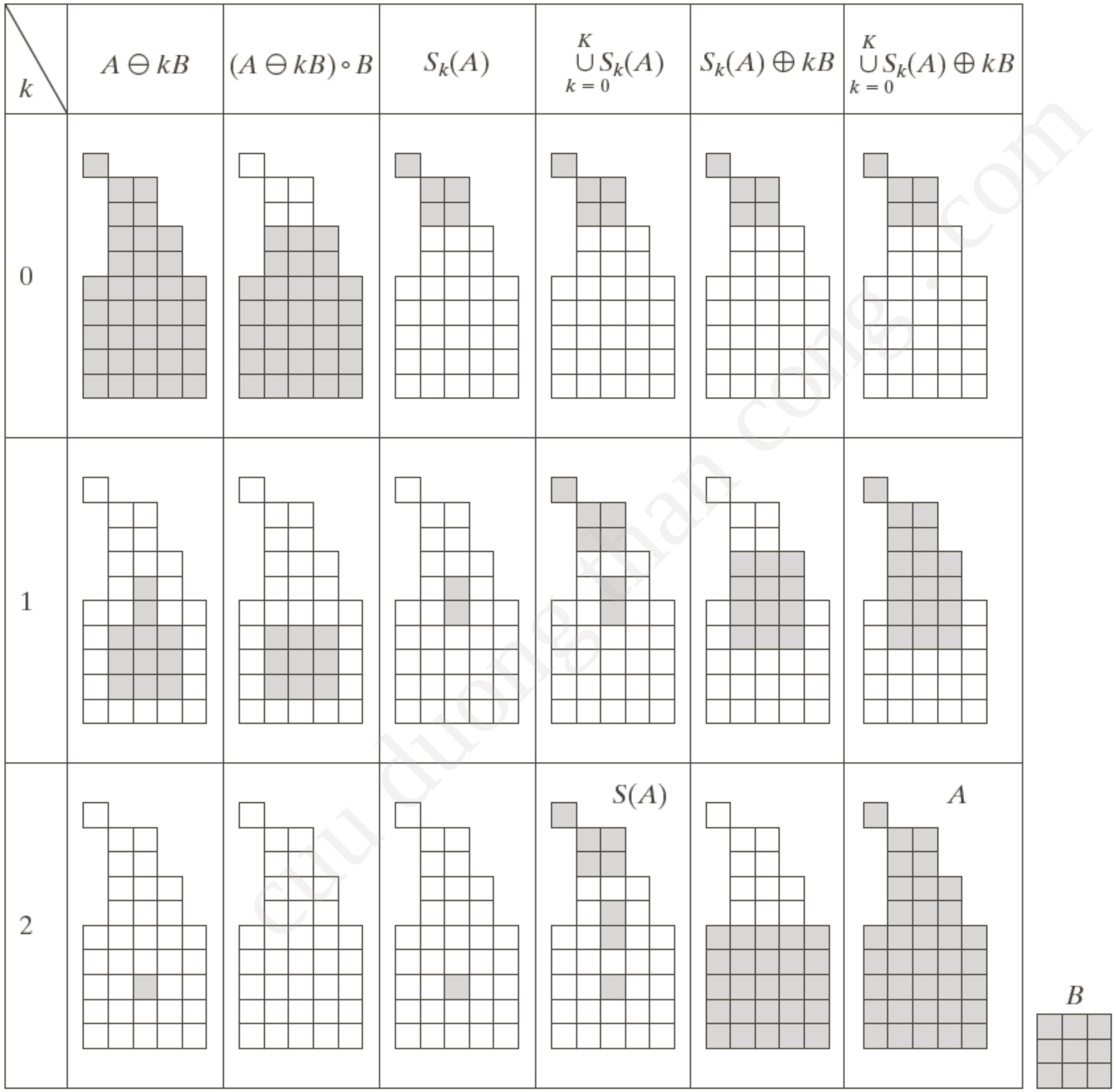


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

4. MIP: Skeleton (5)

A can be reconstructed from the subsets by using

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

where $S_k(A) \oplus kB$ denotes k successive dilations of A.

$$(S_k(A) \oplus kB) = ((\ldots((S_k(A) \oplus B) \oplus B)\ldots \oplus B)$$

4. MIP: Pruning (1)

a. Thinning and skeletonizing tend to leave parasitic components.

b. **Pruning** methods are essential complement to thinning and skeletonizing procedures.

4. MIP: Pruning (2)

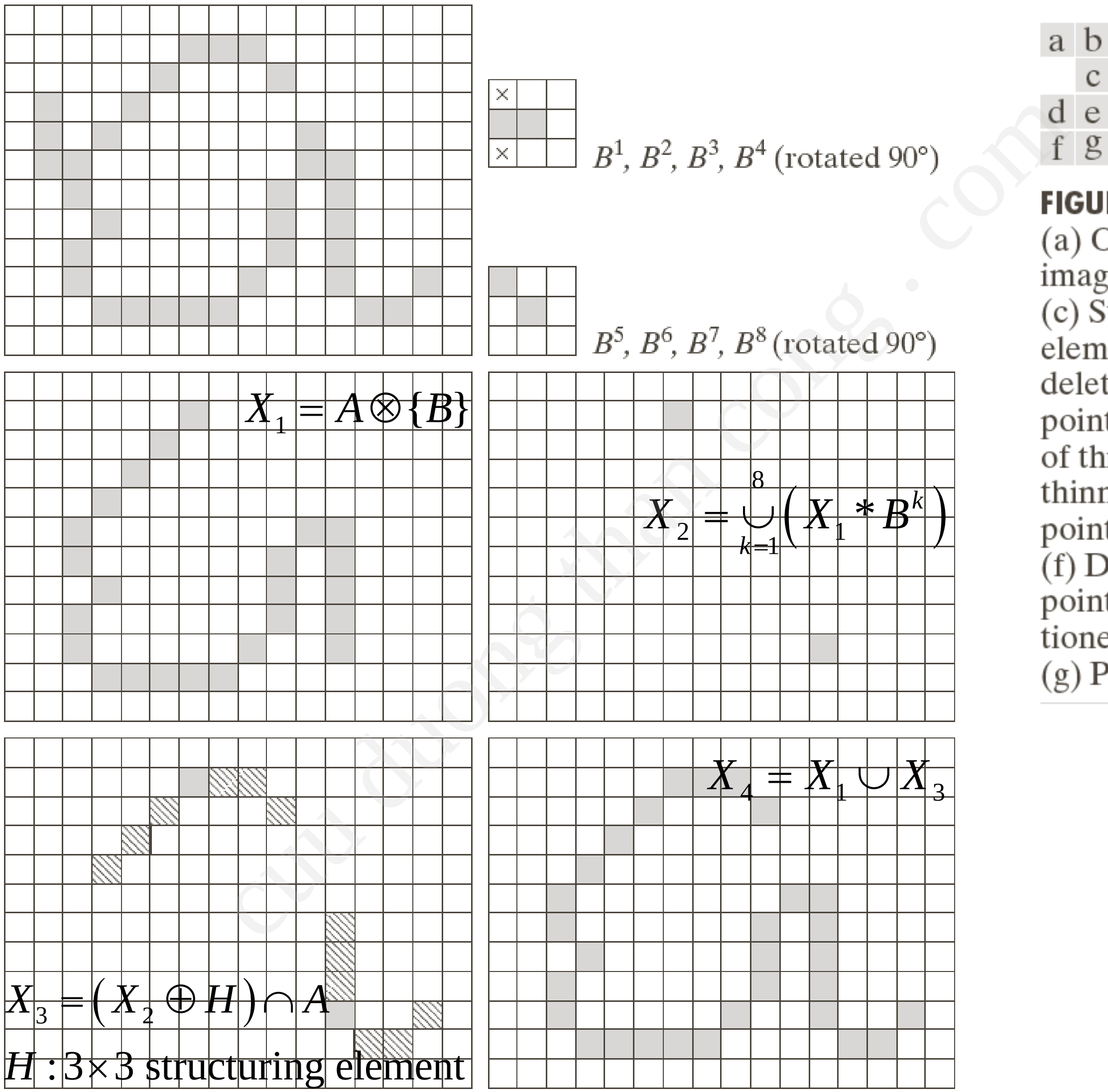


FIGURE 9.25 (a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.

4. MIP: Summary (1)

TABLE 9.2 Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \quad \text{for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \quad \text{for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\} = A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

4. MIP: Summary (2)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A; X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A; i = 1, 2, 3, 4;$ $k = 1, 2, 3, \ldots; X_0^i = A$; and $D^i = X_{\text{conv}}^i$.	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)

4. MIP: Summary (3)

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $= A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\ldots(A \odot B^1) \odot B^2 \ldots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

TABLE 9.2
Summary of morphological results and their properties. *(continued)*

4. MIP: Summary (4)

Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.